

# Computer algebra independent integration tests

3-Logarithms/3.2.1-f+g-x^m-A+B-log-e-a+b-x-over-c+d-x^n^p

Nasser M. Abbasi

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3.137	$\int \frac{(ag+bgx)^2}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx \dots\dots\dots$	630
3.138	$\int \frac{ag+bgx}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx \dots\dots\dots$	633
3.139	$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} dx \dots\dots\dots$	636
3.140	$\int \frac{1}{(ag+bgx)^2\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} dx \dots\dots\dots$	639

3.141	$\int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx \dots \dots \dots$	642
3.142	$\int \frac{(ag+bgx)^2}{\left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx \dots \dots \dots$	645
3.143	$\int \frac{ag+bgx}{\left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx \dots \dots \dots$	648
3.144	$\int \frac{1}{(ag+bgx) \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx \dots \dots \dots$	651
3.145	$\int \frac{1}{(ag+bgx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx \dots \dots \dots$	654
3.146	$\int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx \dots \dots \dots$	657
3.147	$\int (a+bx)^4 (A+B \log (e(a+bx)^n (c+dx)^{-n})) dx \dots \dots \dots$	660
3.148	$\int (a+bx)^3 (A+B \log (e(a+bx)^n (c+dx)^{-n})) dx \dots \dots \dots$	665
3.149	$\int (a+bx)^2 (A+B \log (e(a+bx)^n (c+dx)^{-n})) dx \dots \dots \dots$	669
3.150	$\int (a+bx) (A+B \log (e(a+bx)^n (c+dx)^{-n})) dx \dots \dots \dots$	673
3.151	$\int \frac{A+B \log (e(a+bx)^n (c+dx)^{-n})}{a+bx} dx \dots \dots \dots$	676
3.152	$\int \frac{A+B \log (e(a+bx)^n (c+dx)^{-n})}{(a+bx)^2} dx \dots \dots \dots$	680
3.153	$\int \frac{A+B \log (e(a+bx)^n (c+dx)^{-n})}{(a+bx)^3} dx \dots \dots \dots$	683
3.154	$\int \frac{A+B \log (e(a+bx)^n (c+dx)^{-n})}{(a+bx)^4} dx \dots \dots \dots$	687
3.155	$\int \frac{A+B \log (e(a+bx)^n (c+dx)^{-n})}{(a+bx)^5} dx \dots \dots \dots$	691
3.156	$\int (a+bx)^3 (A+B \log (e(a+bx)^n (c+dx)^{-n}))^2 dx \dots \dots \dots$	696
3.157	$\int (a+bx)^2 (A+B \log (e(a+bx)^n (c+dx)^{-n}))^2 dx \dots \dots \dots$	702
3.158	$\int (a+bx) (A+B \log (e(a+bx)^n (c+dx)^{-n}))^2 dx \dots \dots \dots$	707
3.159	$\int \frac{(A+B \log (e(a+bx)^n (c+dx)^{-n}))^2}{a+bx} dx \dots \dots \dots$	712
3.160	$\int \frac{(A+B \log (e(a+bx)^n (c+dx)^{-n}))^2}{(a+bx)^2} dx \dots \dots \dots$	716
3.161	$\int \frac{(A+B \log (e(a+bx)^n (c+dx)^{-n}))^2}{(a+bx)^3} dx \dots \dots \dots$	719
3.162	$\int \frac{(A+B \log (e(a+bx)^n (c+dx)^{-n}))^2}{(a+bx)^4} dx \dots \dots \dots$	724
3.163	$\int \frac{(A+B \log (e(a+bx)^n (c+dx)^{-n}))^2}{(a+bx)^5} dx \dots \dots \dots$	730
3.164	$\int (a+bx)^3 (A+B \log (e(a+bx)^n (c+dx)^{-n}))^3 dx \dots \dots \dots$	737
3.165	$\int (a+bx)^2 (A+B \log (e(a+bx)^n (c+dx)^{-n}))^3 dx \dots \dots \dots$	744
3.166	$\int (a+bx) (A+B \log (e(a+bx)^n (c+dx)^{-n}))^3 dx \dots \dots \dots$	750
3.167	$\int \frac{(A+B \log (e(a+bx)^n (c+dx)^{-n}))^3}{a+bx} dx \dots \dots \dots$	757
3.168	$\int \frac{(A+B \log (e(a+bx)^n (c+dx)^{-n}))^3}{(a+bx)^2} dx \dots \dots \dots$	763
3.169	$\int \frac{(A+B \log (e(a+bx)^n (c+dx)^{-n}))^3}{(a+bx)^3} dx \dots \dots \dots$	767
3.170	$\int \frac{(A+B \log (e(a+bx)^n (c+dx)^{-n}))^3}{(a+bx)^4} dx \dots \dots \dots$	773
3.171	$\int \frac{(A+B \log (e(a+bx)^n (c+dx)^{-n}))^3}{(a+bx)^5} dx \dots \dots \dots$	782
3.172	$\int \frac{1}{(ag+bgx)^2 (A+B \log (e(a+bx)^n (c+dx)^{-n}))} dx \dots \dots \dots$	793
3.173	$\int (ag+bgx)^4 \left( A+B \log \left( \frac{e(c+dx)}{a+bx} \right) \right) dx \dots \dots \dots$	795
3.174	$\int (ag+bgx)^3 \left( A+B \log \left( \frac{e(c+dx)}{a+bx} \right) \right) dx \dots \dots \dots$	800
3.175	$\int (ag+bgx)^2 \left( A+B \log \left( \frac{e(c+dx)}{a+bx} \right) \right) dx \dots \dots \dots$	804
3.176	$\int (ag+bgx) \left( A+B \log \left( \frac{e(c+dx)}{a+bx} \right) \right) dx \dots \dots \dots$	808
3.177	$\int \frac{A+B \log \left( \frac{e(c+dx)}{a+bx} \right)}{ag+bgx} dx \dots \dots \dots$	811

3.178	$\int \frac{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2} dx$	815
3.179	$\int \frac{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3} dx$	818
3.180	$\int \frac{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4} dx$	822
3.181	$\int \frac{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5} dx$	826
3.182	$\int (ag+bgx)^4 \left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$	831
3.183	$\int (ag+bgx)^3 \left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$	837
3.184	$\int (ag+bgx)^2 \left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$	842
3.185	$\int (ag+bgx) \left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 dx$	847
3.186	$\int \frac{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx} dx$	852
3.187	$\int \frac{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2} dx$	859
3.188	$\int \frac{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3} dx$	864
3.189	$\int \frac{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4} dx$	870
3.190	$\int \frac{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5} dx$	877
3.191	$\int \frac{(ag+bgx)^2}{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx$	884
3.192	$\int \frac{ag+bgx}{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx$	887
3.193	$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)} dx$	889
3.194	$\int \frac{1}{(ag+bgx)^2\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)} dx$	891
3.195	$\int \frac{1}{(ag+bgx)^3\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)} dx$	894
3.196	$\int \frac{(ag+bgx)^2}{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$	897
3.197	$\int \frac{ag+bgx}{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$	900
3.198	$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$	903
3.199	$\int \frac{1}{(ag+bgx)^2\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$	906
3.200	$\int \frac{1}{(ag+bgx)^3\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$	909
3.201	$\int (ag+bgx)^4 \left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$	912
3.202	$\int (ag+bgx)^3 \left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$	916
3.203	$\int (ag+bgx)^2 \left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$	920
3.204	$\int (ag+bgx) \left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) dx$	924
3.205	$\int \frac{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx} dx$	927
3.206	$\int \frac{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2} dx$	931



3.207	$\int \frac{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3} dx \dots\dots\dots$	934
3.208	$\int \frac{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4} dx \dots\dots\dots$	938
3.209	$\int \frac{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5} dx \dots\dots\dots$	942
3.210	$\int (ag+bgx)^4 \left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx \dots\dots\dots$	946
3.211	$\int (ag+bgx)^3 \left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx \dots\dots\dots$	952
3.212	$\int (ag+bgx)^2 \left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx \dots\dots\dots$	958
3.213	$\int (ag+bgx) \left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 dx \dots\dots\dots$	963
3.214	$\int \frac{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx} dx \dots\dots\dots$	968
3.215	$\int \frac{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^2} dx \dots\dots\dots$	975
3.216	$\int \frac{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3} dx \dots\dots\dots$	980
3.217	$\int \frac{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4} dx \dots\dots\dots$	986
3.218	$\int \frac{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5} dx \dots\dots\dots$	992
3.219	$\int \frac{(ag+bgx)^2}{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx \dots\dots\dots$	999
3.220	$\int \frac{ag+bgx}{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx \dots\dots\dots$	1002
3.221	$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} dx \dots\dots\dots$	1005
3.222	$\int \frac{1}{(ag+bgx)^2\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} dx \dots\dots\dots$	1008
3.223	$\int \frac{1}{(ag+bgx)^3\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} dx \dots\dots\dots$	1011
3.224	$\int \frac{(ag+bgx)^2}{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx \dots\dots\dots$	1014
3.225	$\int \frac{ag+bgx}{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx \dots\dots\dots$	1017
3.226	$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx \dots\dots\dots$	1020
3.227	$\int \frac{1}{(ag+bgx)^2\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx \dots\dots\dots$	1023
3.228	$\int \frac{1}{(ag+bgx)^3\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx \dots\dots\dots$	1026
3.229	$\int \frac{1}{(ag+bgx)^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))} dx \dots\dots\dots$	1029
3.230	$\int (f+gx)^4 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \dots\dots\dots$	1031
3.231	$\int (f+gx)^3 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \dots\dots\dots$	1035
3.232	$\int (f+gx)^2 \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \dots\dots\dots$	1039
3.233	$\int (f+gx) \left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \dots\dots\dots$	1044

3.234	$\int \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right) dx$	1048
3.235	$\int \frac{A+B \log \left( \frac{e(a+bx)}{c+dx} \right)}{f+gx} dx$	1051
3.236	$\int \frac{A+B \log \left( \frac{e(a+bx)}{c+dx} \right)}{(f+gx)^2} dx$	1055
3.237	$\int \frac{A+B \log \left( \frac{e(a+bx)}{c+dx} \right)}{(f+gx)^3} dx$	1059
3.238	$\int \frac{A+B \log \left( \frac{e(a+bx)}{c+dx} \right)}{(f+gx)^4} dx$	1063
3.239	$\int \frac{A+B \log \left( \frac{e(a+bx)}{c+dx} \right)}{(f+gx)^5} dx$	1067
3.240	$\int (f+gx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx$	1072
3.241	$\int (f+gx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx$	1079
3.242	$\int (f+gx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx$	1085
3.243	$\int \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx$	1090
3.244	$\int \frac{\left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{f+gx} dx$	1095
3.245	$\int \frac{\left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{(f+gx)^2} dx$	1103
3.246	$\int \frac{\left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{(f+gx)^3} dx$	1108
3.247	$\int \frac{\left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{(f+gx)^4} dx$	1113
3.248	$\int \frac{\left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{(f+gx)^5} dx$	1119
3.249	$\int \frac{\log \left( \frac{1+x}{-1+x} \right)}{x^2} dx$	1126
3.250	$\int \frac{(f+gx)^2}{A+B \log \left( \frac{e(a+bx)}{c+dx} \right)} dx$	1129
3.251	$\int \frac{f+gx}{A+B \log \left( \frac{e(a+bx)}{c+dx} \right)} dx$	1132
3.252	$\int \frac{1}{A+B \log \left( \frac{e(a+bx)}{c+dx} \right)} dx$	1134
3.253	$\int \frac{1}{(f+gx) \left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)} dx$	1136
3.254	$\int \frac{1}{(f+gx)^2 \left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)} dx$	1138
3.255	$\int \frac{1}{(f+gx)^3 \left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)} dx$	1140
3.256	$\int \frac{(f+gx)^2}{\left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2} dx$	1142
3.257	$\int \frac{f+gx}{\left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2} dx$	1145
3.258	$\int \frac{1}{\left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2} dx$	1148
3.259	$\int \frac{1}{(f+gx) \left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2} dx$	1151
3.260	$\int \frac{1}{(f+gx)^2 \left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2} dx$	1154
3.261	$\int \frac{1}{(f+gx)^3 \left( A+B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2} dx$	1157
3.262	$\int (f+gx)^4 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx$	1160

3.263	$\int (f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx$	1165
3.264	$\int (f + gx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx$	1170
3.265	$\int (f + gx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx$	1174
3.266	$\int \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx$	1178
3.267	$\int \frac{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{f+gx} dx$	1181
3.268	$\int \frac{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{(f+gx)^2} dx$	1185
3.269	$\int \frac{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{(f+gx)^3} dx$	1188
3.270	$\int \frac{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{(f+gx)^4} dx$	1192
3.271	$\int \frac{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{(f+gx)^5} dx$	1197
3.272	$\int (f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx$	1201
3.273	$\int (f + gx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx$	1208
3.274	$\int (f + gx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx$	1214
3.275	$\int \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx$	1219
3.276	$\int \frac{\left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{f+gx} dx$	1224
3.277	$\int \frac{\left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{(f+gx)^2} dx$	1231
3.278	$\int \frac{\left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{(f+gx)^3} dx$	1236
3.279	$\int \frac{\left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{(f+gx)^4} dx$	1241
3.280	$\int \frac{\left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{(f+gx)^5} dx$	1247
3.281	$\int \frac{(f+gx)^2}{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)} dx$	1254
3.282	$\int \frac{f+gx}{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)} dx$	1257
3.283	$\int \frac{1}{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)} dx$	1260
3.284	$\int \frac{1}{(f+gx) \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$	1262
3.285	$\int \frac{1}{(f+gx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$	1264
3.286	$\int \frac{1}{(f+gx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$	1267
3.287	$\int \frac{(f+gx)^2}{\left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$	1270
3.288	$\int \frac{f+gx}{\left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$	1273
3.289	$\int \frac{1}{\left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$	1276

3.290	$\int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx \dots\dots\dots$	1279
3.291	$\int \frac{1}{(f+gx)^2\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx \dots\dots\dots$	1282
3.292	$\int \frac{1}{(f+gx)^3\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx \dots\dots\dots$	1285
3.293	$\int (g+hx)^4 (A+B \log(e(a+bx)^n(c+dx)^{-n})) dx \dots\dots\dots$	1288
3.294	$\int (g+hx)^3 (A+B \log(e(a+bx)^n(c+dx)^{-n})) dx \dots\dots\dots$	1293
3.295	$\int (g+hx)^2 (A+B \log(e(a+bx)^n(c+dx)^{-n})) dx \dots\dots\dots$	1297
3.296	$\int (g+hx) (A+B \log(e(a+bx)^n(c+dx)^{-n})) dx \dots\dots\dots$	1301
3.297	$\int (A+B \log(e(a+bx)^n(c+dx)^{-n})) dx \dots\dots\dots$	1304
3.298	$\int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{g+hx} dx \dots\dots\dots$	1307
3.299	$\int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^2} dx \dots\dots\dots$	1310
3.300	$\int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^3} dx \dots\dots\dots$	1314
3.301	$\int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^4} dx \dots\dots\dots$	1319
3.302	$\int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^5} dx \dots\dots\dots$	1323
3.303	$\int (g+hx)^2 (A+B \log(e(a+bx)^n(c+dx)^{-n}))^2 dx \dots\dots\dots$	1328
3.304	$\int (g+hx) (A+B \log(e(a+bx)^n(c+dx)^{-n}))^2 dx \dots\dots\dots$	1333
3.305	$\int (A+B \log(e(a+bx)^n(c+dx)^{-n}))^2 dx \dots\dots\dots$	1338
3.306	$\int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx} dx \dots\dots\dots$	1343
3.307	$\int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2} dx \dots\dots\dots$	1348
3.308	$\int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3} dx \dots\dots\dots$	1353
3.309	$\int (g+hx)^2 (A+B \log(e(a+bx)^n(c+dx)^{-n}))^3 dx \dots\dots\dots$	1359
3.310	$\int (g+hx) (A+B \log(e(a+bx)^n(c+dx)^{-n}))^3 dx \dots\dots\dots$	1365
3.311	$\int (A+B \log(e(a+bx)^n(c+dx)^{-n}))^3 dx \dots\dots\dots$	1371
3.312	$\int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx} dx \dots\dots\dots$	1376
3.313	$\int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2} dx \dots\dots\dots$	1381
3.314	$\int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3} dx \dots\dots\dots$	1386

#### 4 Listing of Grading functions

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# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 314 ]. This is test number [ 59 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 91.72 ( 288 )	% 8.28 ( 26 )
Mathematica	% 94.9 ( 298 )	% 5.1 ( 16 )
Maple	% 59.55 ( 187 )	% 40.45 ( 127 )
Maxima	% 75.8 ( 238 )	% 24.2 ( 76 )
Fricas	% 65.61 ( 206 )	% 34.39 ( 108 )
Sympy	% 17.52 ( 55 )	% 82.48 ( 259 )
Giac	% 50. ( 157 )	% 50. ( 157 )

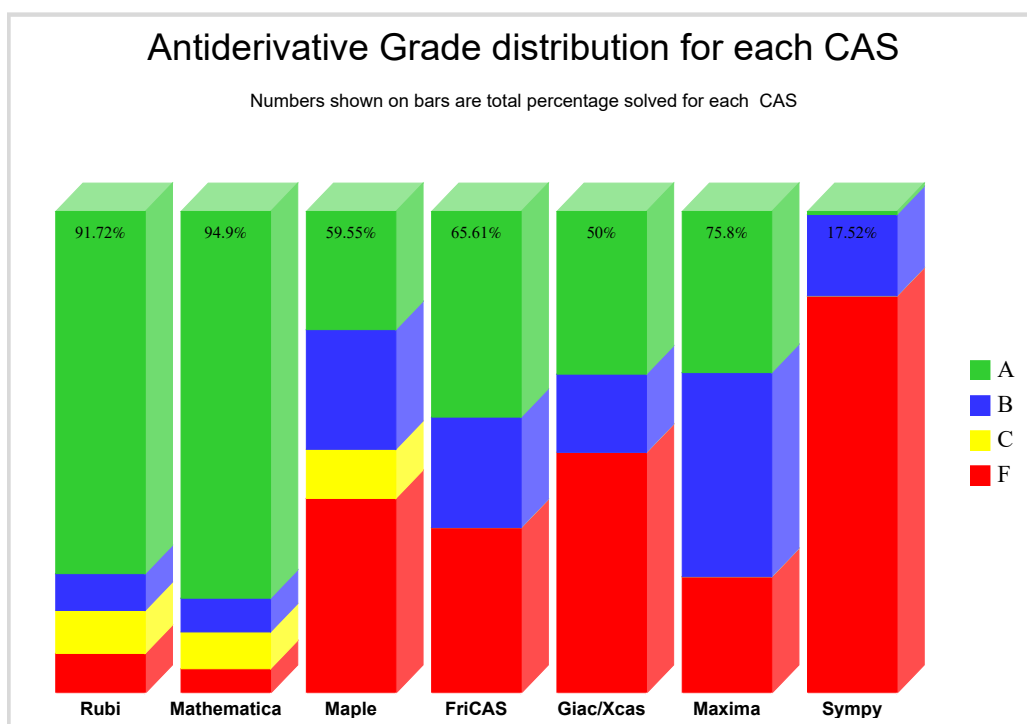
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

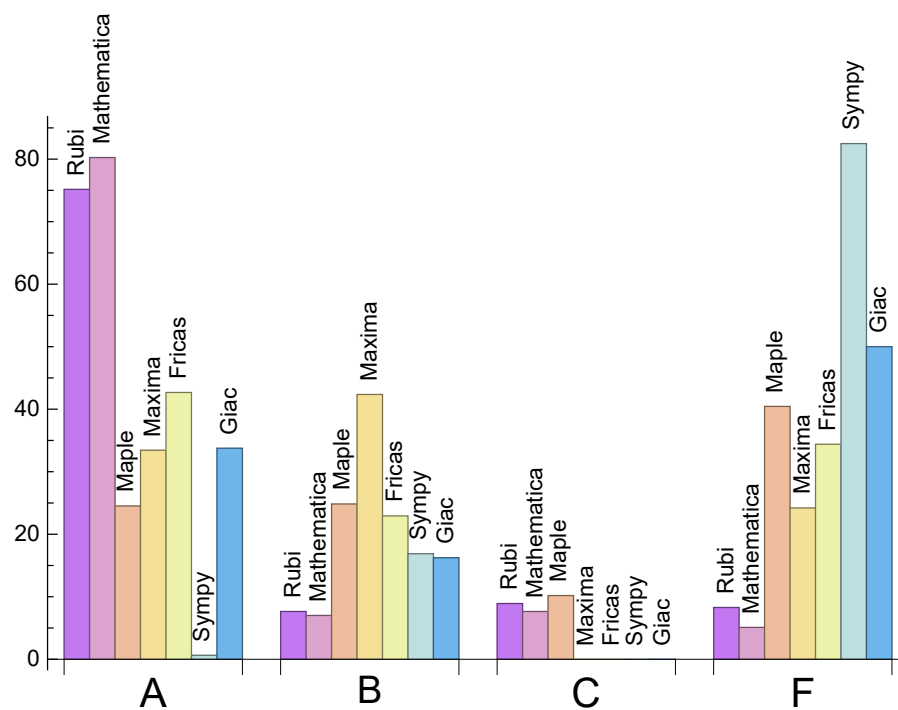
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	75.16	7.64	8.92	8.28
Mathematica	80.25	7.01	7.64	5.1
Maple	24.52	24.84	10.19	40.45
Maxima	33.44	42.36	0.	24.2
Fricas	42.68	22.93	0.	34.39
Sympy	0.64	16.88	0.	82.48
Giac	33.76	16.24	0.	50.

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



### 1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.65	340.11	1.23	179.	1.
Mathematica	0.95	374.19	1.34	142.5	0.92
Maple	1.09	5560.3	16.06	452.	3.19
Maxima	1.18	855.13	3.02	497.5	3.2
Fricas	1.12	681.37	2.96	352.5	3.25
Sympy	9.12	691.69	3.78	677.	3.75
Giac	8.42	410.36	2.02	153.	1.63

## 1.4 list of integrals that has no closed form antiderivative

{19, 20, 21, 24, 25, 26, 47, 48, 49, 52, 53, 54, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 191, 192, 193, 196, 197, 198, 219, 220, 221, 224, 225, 226, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292}

## 1.5 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {}

**Mathematica** {}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.



## 1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())
```

```
try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

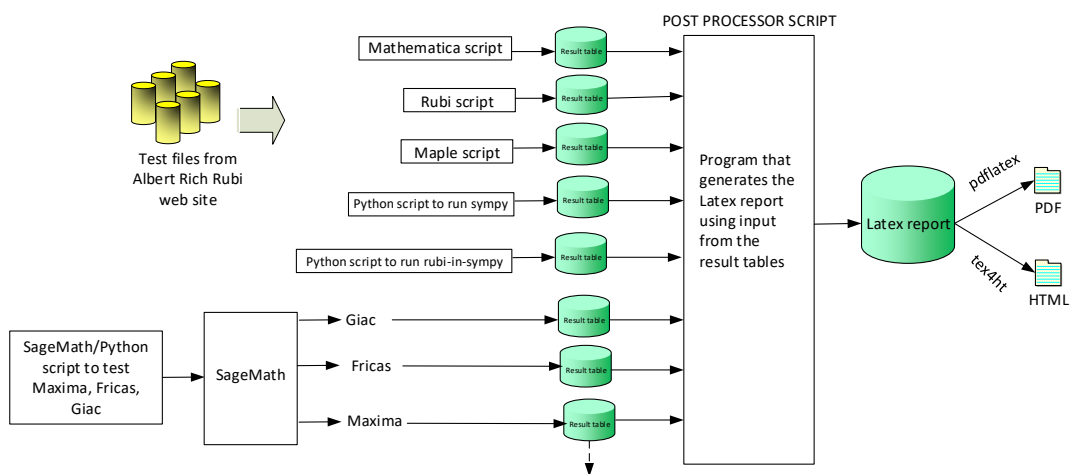
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

### High level overview of the CAS independent integration test build system

Nasser M. Abbasi  
June 22, 2018



# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 24, 25, 26, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 47, 48, 49, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 106, 107, 108, 109, 110, 111, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 137, 138, 139, 142, 143, 144, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 164, 165, 166, 168, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 191, 192, 193, 196, 197, 198, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 219, 220, 221, 224, 225, 226, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309 }

B grade: { 14, 42, 70, 71, 72, 73, 101, 132, 167, 169, 186, 214, 244, 245, 246, 276, 277, 278, 308, 310, 311, 312, 313, 314 }

C grade: { 15, 16, 17, 18, 43, 44, 45, 46, 102, 103, 104, 105, 133, 134, 135, 136, 162, 163, 170, 171, 187, 188, 189, 190, 215, 216, 217, 218 }

F grade: { 22, 23, 27, 28, 50, 51, 55, 56, 112, 113, 117, 118, 140, 141, 145, 146, 172, 194, 195, 199, 200, 222, 223, 227, 228, 229 }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 137, 138, 139, 142, 143, 144, 148, 149, 150, 151, 152, 153, 154, 155, 160, 161, 162, 163, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 219, 220, 221, 224, 225, 226, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 311 }

B grade: { 14, 42, 71, 72, 106, 107, 108, 147, 156, 157, 158, 159, 164, 165, 166, 167, 168, 245, 277, 306, 307, 308 }

C grade: { 15, 16, 17, 18, 43, 44, 45, 46, 102, 103, 104, 105, 133, 134, 135, 136, 187, 188, 189, 190, 215, 216, 217, 218 }

F grade: { 140, 141, 145, 146, 172, 222, 223, 227, 228, 229, 276, 309, 310, 312, 313, 314 }

### 2.1.3 Maple

A grade: { 19, 20, 21, 24, 25, 26, 47, 48, 49, 52, 53, 54, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 106, 107, 108, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 191, 192, 193, 194, 196, 197, 198, 219, 220, 221, 224, 225, 226, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292 }

B grade: { 61, 88, 89, 90, 91, 92, 93, 94, 95, 96, 101, 102, 103, 104, 105, 119, 120, 121, 122, 123, 124, 125, 126, 127, 133, 134, 135, 136, 173, 174, 175, 176, 177, 178, 179, 180, 181, 186, 187, 188, 189, 190, 199, 201, 202, 203, 204, 205, 206, 207, 208, 209, 215, 216, 217, 218, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 244, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 297 }

C grade: { 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 168, 169, 170, 171, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 305 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 97, 98, 99, 100, 112, 113, 117, 118, 128, 129, 130, 131, 132, 140, 141, 145, 146, 159, 164, 165, 166, 167, 172, 182, 183, 184, 185, 195, 200, 210, 211, 212, 213, 214, 222, 223, 227, 228, 229, 240, 241, 242, 243, 245, 246, 247, 248, 272, 273, 274, 275, 276, 277, 278, 279, 280, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

### 2.1.4 Maxima

A grade: { 4, 7, 19, 20, 21, 24, 25, 26, 32, 34, 35, 47, 48, 49, 52, 53, 54, 57, 58, 59, 60, 61, 63, 64, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 94, 109, 110, 111, 114, 115, 116, 137, 138, 139, 142, 143, 144, 150, 152, 153, 176, 179, 191, 192, 193, 196, 197, 198, 206, 219, 220, 221, 224, 225, 226, 230, 231, 232, 233, 234, 236, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 266, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 299 }

B grade: { 1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 15, 16, 17, 18, 29, 30, 31, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 65, 66, 67, 68, 69, 88, 89, 90, 93, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 147, 148, 149, 154, 155, 156, 157, 158, 160, 161, 162, 163, 168, 169, 170, 171, 173, 174, 175, 178, 180, 181, 182, 183, 184, 185, 187, 188, 189, 190, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 215, 216, 217, 218, 237, 238, 239, 240, 241, 242, 262, 263, 264, 265, 268, 269, 270, 271, 272, 273, 274, 294, 300, 301, 302, 303, 304 }

C grade: { }

F grade: { 5, 14, 22, 23, 27, 28, 33, 42, 50, 51, 55, 56, 62, 70, 71, 72, 73, 74, 75, 92, 101, 112, 113, 117, 118, 123, 132, 140, 141, 145, 146, 151, 159, 164, 165, 166, 167, 172, 177, 186, 194, 195, 199, 200, 205, 214, 222, 223, 227, 228, 229, 235, 243, 244, 245, 246, 247, 248, 267, 275, 276, 277, 278, 279, 280, 298, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

### 2.1.5 FriCAS

A grade: { 4, 6, 7, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 34, 35, 43, 47, 48, 49, 50, 51, 52, 53, 54, 55, 60, 61, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 93, 94, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 122, 124, 125, 133, 134, 135, 136, 137, 138, 139, 142, 143, 144, 152, 172, 176, 178, 179, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 204, 206, 207, 215, 216, 217, 219, 220, 221, 224, 225, 226, 229, 230, 232, 233, 234, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 265, 266, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 296, 297 }

B grade: { 1, 2, 3, 8, 9, 16, 17, 18, 28, 29, 30, 31, 32, 36, 37, 44, 45, 46, 56, 57, 58, 59, 63, 88, 89, 90, 95, 96, 118, 119, 120, 121, 126, 127, 147, 148, 149, 150, 153, 154, 155, 160, 161, 162, 163, 168,

169, 170, 171, 173, 174, 175, 180, 181, 190, 199, 200, 201, 202, 203, 208, 209, 218, 231, 236, 263, 264, 268, 293, 294, 295, 299 }

C grade: { }

F grade: { 5, 10, 11, 12, 13, 14, 33, 38, 39, 40, 41, 42, 62, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 123, 128, 129, 130, 131, 132, 140, 141, 145, 146, 151, 156, 157, 158, 159, 164, 165, 166, 167, 177, 182, 183, 184, 185, 186, 205, 210, 211, 212, 213, 214, 222, 223, 227, 228, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 267, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 298, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

## 2.1.6 Sympy

A grade: { 234, 249 }

B grade: { 88, 89, 90, 91, 93, 94, 95, 96, 102, 103, 104, 119, 120, 121, 122, 124, 125, 126, 127, 133, 134, 135, 173, 174, 175, 176, 178, 179, 180, 181, 187, 188, 189, 201, 202, 203, 204, 206, 207, 208, 209, 215, 216, 217, 230, 231, 232, 233, 262, 263, 264, 265, 266 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 92, 97, 98, 99, 100, 101, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 123, 128, 129, 130, 131, 132, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 182, 183, 184, 185, 186, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 205, 210, 211, 212, 213, 214, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

## 2.1.7 Giac

A grade: { 4, 6, 7, 19, 20, 21, 24, 25, 26, 32, 34, 35, 47, 48, 49, 52, 53, 54, 55, 59, 60, 61, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 91, 93, 94, 109, 110, 111, 114, 115, 116, 122, 125, 136, 137, 138, 139, 142, 143, 144, 150, 152, 153, 176, 178, 179, 191, 192, 193, 196, 197, 198, 204, 206, 207, 218, 219, 220, 221, 224, 225, 226, 232, 233, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 264, 265, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 296, 297, 299 }

B grade: { 3, 8, 9, 30, 31, 36, 37, 63, 64, 65, 66, 88, 89, 90, 95, 96, 119, 120, 121, 124, 126, 127, 133, 147, 148, 149, 154, 155, 173, 174, 175, 180, 181, 199, 201, 202, 203, 208, 209, 215, 236, 237, 238, 239, 263, 269, 270, 295, 300, 301, 302 }

C grade: { }

F grade: { 1, 2, 5, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 27, 28, 29, 33, 38, 39, 40, 41, 42, 43, 44, 45, 46, 50, 51, 56, 57, 58, 62, 67, 68, 69, 70, 71, 72, 73, 74, 75, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 112, 113, 117, 118, 123, 128, 129, 130, 131, 132, 134, 135, 140, 141, 145, 146, 151, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 177, 182, 183, 184, 185, 186, 187, 188, 189, 190, 194, 195, 200, 205, 210, 211, 212, 213, 214, 216, 217, 222, 223, 227, 228, 229, 230, 231, 234, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 262, 266, 267, 268, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 293, 294, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	146	0	913	1188	0	0
normalized size	1	1.	0.78	0.	4.86	6.32	0.	0.
time (sec)	N/A	0.143	0.129	0.546	1.28	1.183	0.	0.

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	124	0	647	883	0	0
normalized size	1	1.	0.79	0.	4.15	5.66	0.	0.
time (sec)	N/A	0.106	0.11	0.405	1.236	1.07	0.	0.

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	103	0	417	622	0	354
normalized size	1	1.	0.83	0.	3.36	5.02	0.	2.85
time (sec)	N/A	0.09	0.062	0.402	1.211	0.906	0.	26.721

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	73	0	211	360	0	167
normalized size	1	1.	0.85	0.	2.45	4.19	0.	1.94
time (sec)	N/A	0.061	0.04	0.322	1.133	0.949	0.	3.651

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	126	101	0	0	0	0	0
normalized size	1	1.5	1.2	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	0.054	0.529	0.	0.	0.	0.



Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	108	115	0	185	221	0	162
normalized size	1	1.61	1.72	0.	2.76	3.3	0.	2.42
time (sec)	N/A	0.09	0.063	0.449	1.214	0.797	0.	1.423

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	114	0	350	562	0	351
normalized size	1	1.	0.75	0.	2.32	3.72	0.	2.32
time (sec)	N/A	0.123	0.162	0.436	1.212	0.991	0.	1.571

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	145	0	583	990	0	655
normalized size	1	1.	0.79	0.	3.19	5.41	0.	3.58
time (sec)	N/A	0.152	0.185	0.437	1.263	0.879	0.	1.367

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	162	0	879	1505	0	1038
normalized size	1	1.	0.75	0.	4.09	7.	0.	4.83
time (sec)	N/A	0.189	0.26	0.437	1.348	1.066	0.	1.422

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	602	535	0	3976	0	0	0
normalized size	1	1.52	1.35	0.	10.04	0.	0.	0.
time (sec)	N/A	0.871	0.525	0.427	3.824	0.	0.	0.

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	512	411	0	2936	0	0	0
normalized size	1	1.53	1.23	0.	8.76	0.	0.	0.
time (sec)	N/A	0.707	0.36	0.433	3.801	0.	0.	0.

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	420	303	0	2026	0	0	0
normalized size	1	1.53	1.11	0.	7.39	0.	0.	0.
time (sec)	N/A	0.583	0.237	0.426	3.682	0.	0.	0.

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	309	215	0	1118	0	0	0
normalized size	1	1.58	1.1	0.	5.7	0.	0.	0.
time (sec)	N/A	0.443	0.2	0.276	3.81	0.	0.	0.

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	789	537	0	0	0	0	0
normalized size	1	5.72	3.89	0.	0.	0.	0.	0.
time (sec)	N/A	3.573	0.416	0.437	0.	0.	0.	0.

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	512	330	0	581	555	0	0
normalized size	1	3.76	2.43	0.	4.27	4.08	0.	0.
time (sec)	N/A	0.84	0.599	0.436	1.231	0.928	0.	0.

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	626	463	0	1162	1354	0	0
normalized size	1	2.17	1.61	0.	4.03	4.7	0.	0.
time (sec)	N/A	0.923	0.553	0.44	1.373	0.979	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	448	736	609	0	1933	2383	0	0
normalized size	1	1.64	1.36	0.	4.31	5.32	0.	0.
time (sec)	N/A	1.087	0.803	0.439	1.639	1.067	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	615	826	776	0	2884	3615	0	0
normalized size	1	1.34	1.26	0.	4.69	5.88	0.	0.
time (sec)	N/A	1.314	1.188	0.451	2.032	1.158	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	0.749	0.401	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.284	0.307	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.144	0.546	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	94	0	94	0	0	149	0	0
normalized size	1	0.	1.	0.	0.	1.59	0.	0.
time (sec)	N/A	0.099	0.131	0.444	0.	0.868	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	197	0	172	0	0	354	0	0
normalized size	1	0.	0.87	0.	0.	1.8	0.	0.
time (sec)	N/A	0.082	0.297	0.437	0.	0.936	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.238	0.983	0.395	0.	0.	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.934	0.254	0.	0.	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.531	0.437	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	153	0	146	0	0	598	0	0
normalized size	1	0.	0.95	0.	0.	3.91	0.	0.
time (sec)	N/A	0.105	0.183	0.441	0.	0.931	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	B	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	314	0	254	0	0	1612	0	0
normalized size	1	0.	0.81	0.	0.	5.13	0.	0.
time (sec)	N/A	0.093	0.644	0.438	0.	0.953	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	146	0	913	1188	0	0
normalized size	1	1.	0.78	0.	4.86	6.32	0.	0.
time (sec)	N/A	0.128	0.105	0.522	1.243	1.069	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	124	0	647	883	0	537
normalized size	1	1.	0.79	0.	4.15	5.66	0.	3.44
time (sec)	N/A	0.102	0.094	0.411	1.255	1.071	0.	143.004

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	101	0	417	622	0	354
normalized size	1	1.	0.81	0.	3.36	5.02	0.	2.85
time (sec)	N/A	0.081	0.06	0.404	1.195	0.975	0.	21.472

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	74	0	211	360	0	173
normalized size	1	1.	0.86	0.	2.45	4.19	0.	2.01
time (sec)	N/A	0.06	0.038	0.328	1.142	0.805	0.	3.045

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	128	101	0	0	0	0	0
normalized size	1	1.6	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	0.039	0.531	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	107	114	0	184	221	0	166
normalized size	1	1.05	1.12	0.	1.8	2.17	0.	1.63
time (sec)	N/A	0.087	0.06	0.438	1.175	0.909	0.	1.369

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	115	0	350	562	0	351
normalized size	1	1.	0.76	0.	2.32	3.72	0.	2.32
time (sec)	N/A	0.114	0.148	0.444	1.208	0.794	0.	1.4

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	146	0	585	990	0	655
normalized size	1	1.	0.8	0.	3.2	5.41	0.	3.58
time (sec)	N/A	0.143	0.177	0.449	1.259	0.934	0.	1.459

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	162	0	880	1505	0	1038
normalized size	1	1.	0.75	0.	4.09	7.	0.	4.83
time (sec)	N/A	0.176	0.237	0.455	1.255	1.	0.	1.341

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	544	634	533	0	3888	0	0	0
normalized size	1	1.17	0.98	0.	7.15	0.	0.	0.
time (sec)	N/A	0.88	0.504	0.441	3.792	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	544	409	0	2874	0	0	0
normalized size	1	1.2	0.9	0.	6.33	0.	0.	0.
time (sec)	N/A	0.684	0.339	0.429	3.632	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	361	454	303	0	1989	0	0	0
normalized size	1	1.26	0.84	0.	5.51	0.	0.	0.
time (sec)	N/A	0.567	0.24	0.43	3.604	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	307	216	0	1114	0	0	0
normalized size	1	1.4	0.98	0.	5.06	0.	0.	0.
time (sec)	N/A	0.423	0.212	0.277	3.549	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	782	537	0	0	0	0	0
normalized size	1	5.71	3.92	0.	0.	0.	0.	0.
time (sec)	N/A	3.302	0.4	0.444	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	514	331	0	578	555	0	0
normalized size	1	3.15	2.03	0.	3.55	3.4	0.	0.
time (sec)	N/A	0.774	0.446	0.441	1.26	0.88	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	626	464	0	1162	1354	0	0
normalized size	1	1.97	1.46	0.	3.67	4.27	0.	0.
time (sec)	N/A	0.915	0.492	0.441	1.383	0.97	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	736	609	0	1937	2383	0	0
normalized size	1	1.72	1.42	0.	4.52	5.55	0.	0.
time (sec)	N/A	1.098	0.775	0.441	1.62	1.062	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	F	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	536	826	776	0	2886	3615	0	0
normalized size	1	1.54	1.45	0.	5.38	6.74	0.	0.
time (sec)	N/A	1.295	1.086	0.442	2.03	1.286	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.214	0.438	0.398	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.283	0.303	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	0.174	0.54	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	96	0	96	0	0	149	0	0
normalized size	1	0.	1.	0.	0.	1.55	0.	0.
time (sec)	N/A	0.1	0.119	0.438	0.	0.904	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	199	0	174	0	0	351	0	0
normalized size	1	0.	0.87	0.	0.	1.76	0.	0.
time (sec)	N/A	0.083	0.29	0.435	0.	0.889	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	0.989	0.392	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	35	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.959	0.243	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	37	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.536	0.427	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	154	0	180	0	0	635	0	396
normalized size	1	0.	1.17	0.	0.	4.12	0.	2.57
time (sec)	N/A	0.104	0.171	0.438	0.	0.756	0.	1.459

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	B	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	256	0	288	0	0	1648	0	0
normalized size	1	0.	1.12	0.	0.	6.44	0.	0.
time (sec)	N/A	0.094	0.565	0.439	0.	0.898	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	364	348	285	0	852	1488	0	0
normalized size	1	0.96	0.78	0.	2.34	4.09	0.	0.
time (sec)	N/A	0.603	0.65	0.438	1.26	3.133	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	219	0	598	1053	0	0
normalized size	1	1.	0.93	0.	2.54	4.48	0.	0.
time (sec)	N/A	0.359	0.288	0.403	1.204	1.501	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	146	0	381	695	0	366
normalized size	1	1.	0.93	0.	2.43	4.43	0.	2.33
time (sec)	N/A	0.18	0.144	0.41	1.182	1.205	0.	81.826

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	120	0	203	390	0	182
normalized size	1	1.	1.04	0.	1.77	3.39	0.	1.58
time (sec)	N/A	0.105	0.128	0.32	1.095	0.925	0.	5.301

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	122	70	158	0	72
normalized size	1	1.	1.	2.18	1.25	2.82	0.	1.29
time (sec)	N/A	0.033	0.009	0.049	1.177	0.841	0.	1.39



Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	122	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.226	0.058	0.616	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	119	109	0	192	652	0	390
normalized size	1	1.31	1.2	0.	2.11	7.16	0.	4.29
time (sec)	N/A	0.125	0.183	0.513	1.184	17.469	0.	1.332

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	173	0	479	0	0	1165
normalized size	1	1.	0.91	0.	2.52	0.	0.	6.13
time (sec)	N/A	0.236	0.695	0.513	1.292	0.	0.	1.882

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	264	0	1150	0	0	2925
normalized size	1	1.	0.93	0.	4.06	0.	0.	10.34
time (sec)	N/A	0.458	1.021	0.507	1.549	0.	0.	3.999

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	388	388	359	0	2377	0	0	5863
normalized size	1	1.	0.93	0.	6.13	0.	0.	15.11
time (sec)	N/A	0.713	1.355	0.502	1.968	0.	0.	14.539

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	923	1060	757	0	3579	0	0	0
normalized size	1	1.15	0.82	0.	3.88	0.	0.	0.
time (sec)	N/A	1.835	1.041	0.428	3.997	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	565	699	506	0	2240	0	0	0
normalized size	1	1.24	0.9	0.	3.96	0.	0.	0.
time (sec)	N/A	1.15	0.565	0.421	3.788	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	481	362	0	1214	0	0	0
normalized size	1	1.66	1.25	0.	4.19	0.	0.	0.
time (sec)	N/A	0.831	0.328	0.274	3.5	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	275	226	0	0	0	0	0
normalized size	1	2.04	1.67	0.	0.	0.	0.	0.
time (sec)	N/A	0.617	0.17	0.309	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	2233	1441	0	0	0	0	0
normalized size	1	7.52	4.85	0.	0.	0.	0.	0.
time (sec)	N/A	5.184	0.473	0.528	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	657	418	0	0	0	0	0
normalized size	1	3.19	2.03	0.	0.	0.	0.	0.
time (sec)	N/A	1.134	0.738	0.516	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	941	615	0	0	0	0	0
normalized size	1	2.42	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	1.561	2.238	0.512	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	747	1427	918	0	0	0	0	0
normalized size	1	1.91	1.23	0.	0.	0.	0.	0.
time (sec)	N/A	2.5	4.487	0.511	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1208	1968	1476	0	0	0	0	0
normalized size	1	1.63	1.22	0.	0.	0.	0.	0.
time (sec)	N/A	3.545	7.323	0.52	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	0.414	0.396	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.279	0.305	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.015	0.282	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.933	0.625	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	1.058	0.506	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	11.991	0.53	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.207	0.935	0.418	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.696	0.253	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.015	0.593	0.321	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.078	1.609	0.549	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	4.214	0.509	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.08	36.63	0.521	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	142	8417	841	910	993	639
normalized size	1	1.	0.79	46.76	4.67	5.06	5.52	3.55
time (sec)	N/A	0.124	0.106	0.225	1.2	1.364	10.017	99.387

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	120	5556	593	664	719	458
normalized size	1	1.	0.81	37.29	3.98	4.46	4.83	3.07
time (sec)	N/A	0.096	0.098	0.18	1.222	1.137	7.131	21.887

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	99	3283	378	467	503	309
normalized size	1	1.	0.84	27.82	3.2	3.96	4.26	2.62
time (sec)	N/A	0.079	0.054	0.162	1.173	1.092	5.218	4.757

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	69	1544	194	278	257	150
normalized size	1	1.	0.85	19.06	2.4	3.43	3.17	1.85
time (sec)	N/A	0.053	0.034	0.158	1.105	1.055	4.45	1.804

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	120	95	602	0	0	0	0
normalized size	1	1.5	1.19	7.52	0.	0.	0.	0.
time (sec)	N/A	0.217	0.042	0.089	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	102	105	373	178	177	231	157
normalized size	1	1.62	1.67	5.92	2.83	2.81	3.67	2.49
time (sec)	N/A	0.079	0.059	0.047	1.159	1.074	2.56	1.36

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	110	777	344	455	422	329
normalized size	1	1.	0.76	5.4	2.39	3.16	2.93	2.28
time (sec)	N/A	0.1	0.135	0.049	1.085	1.004	4.515	1.346

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	141	1191	578	826	656	609
normalized size	1	1.	0.81	6.81	3.3	4.72	3.75	3.48
time (sec)	N/A	0.13	0.163	0.052	1.225	1.051	6.864	1.413

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	158	1607	873	1284	944	967
normalized size	1	1.	0.77	7.8	4.24	6.23	4.58	4.69
time (sec)	N/A	0.157	0.229	0.051	1.396	1.072	11.107	1.378

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	557	511	0	3225	0	0	0
normalized size	1	1.53	1.4	0.	8.84	0.	0.	0.
time (sec)	N/A	0.855	0.498	2.536	1.832	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	474	391	0	2338	0	0	0
normalized size	1	1.53	1.27	0.	7.57	0.	0.	0.
time (sec)	N/A	0.646	0.335	2.177	1.753	0.	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	253	389	287	0	1573	0	0	0
normalized size	1	1.54	1.13	0.	6.22	0.	0.	0.
time (sec)	N/A	0.553	0.221	1.991	2.033	0.	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	285	203	0	825	0	0	0
normalized size	1	1.58	1.13	0.	4.58	0.	0.	0.
time (sec)	N/A	0.456	0.177	1.642	1.545	0.	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	728	250	1186	0	0	0	0
normalized size	1	5.69	1.95	9.27	0.	0.	0.	0.
time (sec)	N/A	3.416	0.604	0.069	0.	0.	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	470	314	828	562	319	432	0
normalized size	1	3.73	2.49	6.57	4.46	2.53	3.43	0.
time (sec)	N/A	0.772	0.473	0.049	1.285	1.02	4.335	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	577	443	1715	1145	767	892	0
normalized size	1	2.15	1.65	6.4	4.27	2.86	3.33	0.
time (sec)	N/A	0.91	0.517	0.049	1.467	1.093	7.643	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	418	680	585	2624	1916	1388	1544	0
normalized size	1	1.63	1.4	6.28	4.58	3.32	3.69	0.
time (sec)	N/A	1.059	0.766	0.05	1.858	1.105	36.911	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	575	763	748	3538	2866	2152	0	0
normalized size	1	1.33	1.3	6.15	4.98	3.74	0.	0.
time (sec)	N/A	1.227	1.11	0.049	2.184	1.164	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	29	114	30	213	63	0	0
normalized size	1	1.04	4.07	1.07	7.61	2.25	0.	0.
time (sec)	N/A	0.022	0.05	0.048	1.238	0.975	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	140	15	82	53	0	0
normalized size	1	1.	9.33	1.	5.47	3.53	0.	0.
time (sec)	N/A	0.014	0.014	0.045	1.22	0.953	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	133	17	80	53	0	0
normalized size	1	1.	10.23	1.31	6.15	4.08	0.	0.
time (sec)	N/A	0.013	0.014	0.044	1.086	0.98	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.2	0.594	1.126	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.236	0.961	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.225	1.183	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	50	0	52	0	0	108	0	0
normalized size	1	0.	1.04	0.	0.	2.16	0.	0.
time (sec)	N/A	0.09	0.102	1.187	0.	1.03	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	107	0	89	0	0	288	0	0
normalized size	1	0.	0.83	0.	0.	2.69	0.	0.
time (sec)	N/A	0.073	0.165	1.283	0.	1.01	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.206	1.323	1.042	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	0.913	1.043	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.618	1.042	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	103	0	87	0	0	423	0	0
normalized size	1	0.	0.84	0.	0.	4.11	0.	0.
time (sec)	N/A	0.095	0.188	1.274	0.	1.022	0.	0.



Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	B	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	212	0	136	0	0	1187	0	0
normalized size	1	0.	0.64	0.	0.	5.6	0.	0.
time (sec)	N/A	0.084	0.757	1.43	0.	1.109	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	144	1606	1195	952	1018	670
normalized size	1	1.	0.79	8.82	6.57	5.23	5.59	3.68
time (sec)	N/A	0.11	0.096	0.423	1.381	1.303	8.432	162.17

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	122	1249	873	709	722	487
normalized size	1	1.	0.81	8.27	5.78	4.7	4.78	3.23
time (sec)	N/A	0.097	0.09	0.24	1.361	1.163	5.739	27.501

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	98	915	590	508	527	340
normalized size	1	1.	0.82	7.62	4.92	4.23	4.39	2.83
time (sec)	N/A	0.074	0.052	0.243	1.283	1.098	4.005	4.798

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	72	560	338	329	253	177
normalized size	1	1.	0.92	7.18	4.33	4.22	3.24	2.27
time (sec)	N/A	0.057	0.036	0.235	1.228	1.085	2.465	1.742

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	122	88	552	0	0	0	0
normalized size	1	1.47	1.06	6.65	0.	0.	0.	0.
time (sec)	N/A	0.292	0.037	0.375	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	105	111	157	252	228	253	254
normalized size	1	1.62	1.71	2.42	3.88	3.51	3.89	3.91
time (sec)	N/A	0.077	0.058	0.079	1.24	1.015	1.913	1.392

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	109	355	414	495	418	356
normalized size	1	1.	0.79	2.57	3.	3.59	3.03	2.58
time (sec)	N/A	0.093	0.136	0.108	1.294	1.051	3.176	1.319

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	140	579	648	869	677	639
normalized size	1	1.	0.79	3.27	3.66	4.91	3.82	3.61
time (sec)	N/A	0.115	0.135	0.148	1.238	1.072	5.492	1.333

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	162	833	944	1328	947	566
normalized size	1	1.	0.78	4.	4.54	6.38	4.55	2.72
time (sec)	N/A	0.144	0.201	0.207	1.398	1.085	8.269	1.445

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	377	569	523	0	3578	0	0	0
normalized size	1	1.51	1.39	0.	9.49	0.	0.	0.
time (sec)	N/A	0.872	0.466	1.7	2.09	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	470	402	0	2630	0	0	0
normalized size	1	1.47	1.26	0.	8.24	0.	0.	0.
time (sec)	N/A	0.76	0.328	1.577	1.99	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	397	298	0	1790	0	0	0
normalized size	1	1.56	1.17	0.	7.02	0.	0.	0.
time (sec)	N/A	0.621	0.221	1.503	1.719	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	291	207	0	981	0	0	0
normalized size	1	1.55	1.1	0.	5.22	0.	0.	0.
time (sec)	N/A	0.493	0.174	1.273	1.584	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	749	257	0	0	0	0	0
normalized size	1	5.67	1.95	0.	0.	0.	0.	0.
time (sec)	N/A	4.141	0.327	1.374	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	480	321	357	775	416	452	510
normalized size	1	3.69	2.47	2.75	5.96	3.2	3.48	3.92
time (sec)	N/A	0.888	0.451	0.09	1.482	0.998	3.797	1.476

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	579	451	815	1351	846	877	0
normalized size	1	2.13	1.66	3.	4.97	3.11	3.22	0.
time (sec)	N/A	1.046	0.49	0.139	1.857	1.046	6.77	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	429	692	598	1343	2126	1474	1561	0
normalized size	1	1.61	1.39	3.13	4.96	3.44	3.64	0.
time (sec)	N/A	1.225	0.713	0.204	2.817	1.136	35.215	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	587	757	762	1943	3077	2229	0	1180
normalized size	1	1.29	1.3	3.31	5.24	3.8	0.	2.01
time (sec)	N/A	1.394	1.029	0.297	2.26	1.208	0.	1.648

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	0.158	0.986	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.115	0.857	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.07	1.065	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.074	1.091	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	149	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.08	1.153	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.215	0.467	0.933	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.114	0.345	0.935	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.14	0.902	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	147	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.179	1.224	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	263	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.348	1.471	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	183	364	2374	906	1188	0	671
normalized size	1	1.07	2.13	13.88	5.3	6.95	0.	3.92
time (sec)	N/A	0.183	0.784	0.833	1.295	1.093	0.	13.996

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	154	273	1840	630	871	0	479
normalized size	1	1.08	1.92	12.96	4.44	6.13	0.	3.37
time (sec)	N/A	0.135	0.473	0.564	1.247	1.066	0.	5.425

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	125	194	1325	397	599	0	317
normalized size	1	1.11	1.72	11.73	3.51	5.3	0.	2.81
time (sec)	N/A	0.121	0.282	0.527	1.282	1.04	0.	2.295

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	96	126	817	208	354	0	171
normalized size	1	1.14	1.5	9.73	2.48	4.21	0.	2.04
time (sec)	N/A	0.091	0.146	0.51	1.157	1.074	0.	1.704

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	87	129	523	0	0	0	0
normalized size	1	1.1	1.63	6.62	0.	0.	0.	0.
time (sec)	N/A	0.268	0.097	1.534	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	72	89	823	157	242	0	146
normalized size	1	0.74	0.92	8.48	1.62	2.49	0.	1.51
time (sec)	N/A	0.084	0.09	0.4	1.186	1.066	0.	1.262

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	149	121	1379	311	639	0	323
normalized size	1	1.09	0.88	10.07	2.27	4.66	0.	2.36
time (sec)	N/A	0.154	0.338	0.425	1.147	1.044	0.	1.273

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	178	143	1976	540	1123	0	605
normalized size	1	1.07	0.86	11.9	3.25	6.77	0.	3.64
time (sec)	N/A	0.17	0.4	0.475	1.252	1.132	0.	1.293

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	207	165	2583	834	1697	0	959
normalized size	1	1.06	0.85	13.25	4.28	8.7	0.	4.92
time (sec)	N/A	0.193	0.396	0.509	1.78	1.18	0.	1.371

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	542	1709	26948	2526	0	0	0
normalized size	1	1.68	5.31	83.69	7.84	0.	0.	0.
time (sec)	N/A	0.772	1.551	2.524	3.86	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	263	427	1149	19969	1733	0	0	0
normalized size	1	1.62	4.37	75.93	6.59	0.	0.	0.
time (sec)	N/A	0.626	1.027	2.135	3.735	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	308	656	10210	1052	0	0	0
normalized size	1	1.58	3.36	52.36	5.39	0.	0.	0.
time (sec)	N/A	0.486	0.709	1.651	3.611	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	227	269	0	0	0	0	0
normalized size	1	1.73	2.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.504	0.18	2.042	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	189	236	10098	606	755	0	0
normalized size	1	1.47	1.83	78.28	4.7	5.85	0.	0.
time (sec)	N/A	0.182	0.371	1.365	1.255	1.115	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	411	332	17300	1214	1918	0	0
normalized size	1	1.5	1.21	63.14	4.43	7.	0.	0.
time (sec)	N/A	0.423	0.53	1.968	1.473	1.219	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	427	730	432	25057	2025	3351	0	0
normalized size	1	1.71	1.01	58.68	4.74	7.85	0.	0.
time (sec)	N/A	1.212	0.743	2.658	1.708	1.402	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	587	843	1011	33370	3021	5029	0	0
normalized size	1	1.44	1.72	56.85	5.15	8.57	0.	0.
time (sec)	N/A	1.408	1.071	3.333	2.056	1.699	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	809	1203	9054	0	0	0	0	0
normalized size	1	1.49	11.19	0.	0.	0.	0.	0.
time (sec)	N/A	2.404	9.696	5.133	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	915	5668	0	0	0	0	0
normalized size	1	1.49	9.23	0.	0.	0.	0.	0.
time (sec)	N/A	1.737	4.111	4.456	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	700	3813	0	0	0	0	0
normalized size	1	1.86	10.14	0.	0.	0.	0.	0.
time (sec)	N/A	1.183	2.971	6.112	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	424	2513	0	0	0	0	0
normalized size	1	2.28	13.51	0.	0.	0.	0.	0.
time (sec)	N/A	0.851	1.023	2.592	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	360	524	69354	1524	1808	0	0
normalized size	1	1.96	2.85	376.92	8.28	9.83	0.	0.
time (sec)	N/A	0.315	0.757	11.812	1.507	1.23	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	390	811	693	120138	3032	4618	0	0
normalized size	1	2.08	1.78	308.05	7.77	11.84	0.	0.
time (sec)	N/A	0.804	1.154	18.137	1.973	1.494	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	611	1876	1003	175812	4901	8181	0	0
normalized size	1	3.07	1.64	287.74	8.02	13.39	0.	0.
time (sec)	N/A	3.432	1.476	27.216	2.6	1.862	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	A	C	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	830	2173	1370	236754	7128	12382	0	0
normalized size	1	2.62	1.65	285.25	8.59	14.92	0.	0.
time (sec)	N/A	4.675	2.251	33.085	3.309	2.45	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	A	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	96	0	0	0	0	149	0	0
normalized size	1	0.	0.	0.	0.	1.55	0.	0.
time (sec)	N/A	0.1	0.08	3.688	0.	1.03	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	142	2930	836	910	993	635
normalized size	1	1.	0.79	16.28	4.64	5.06	5.52	3.53
time (sec)	N/A	0.124	0.101	0.201	1.298	1.352	8.532	89.529



Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	120	2191	589	664	719	460
normalized size	1	1.	0.81	14.7	3.95	4.46	4.83	3.09
time (sec)	N/A	0.101	0.08	0.158	1.168	1.157	5.867	20.928

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	99	1537	375	467	503	305
normalized size	1	1.	0.84	13.03	3.18	3.96	4.26	2.58
time (sec)	N/A	0.081	0.051	0.159	1.203	1.068	4.003	4.949

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	69	951	193	278	257	153
normalized size	1	1.	0.85	11.74	2.38	3.43	3.17	1.89
time (sec)	N/A	0.055	0.037	0.161	1.166	1.067	2.549	2.277

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	122	95	419	0	0	0	0
normalized size	1	1.51	1.17	5.17	0.	0.	0.	0.
time (sec)	N/A	0.211	0.043	0.057	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	101	86	520	181	177	231	151
normalized size	1	1.58	1.34	8.12	2.83	2.77	3.61	2.36
time (sec)	N/A	0.077	0.059	0.051	1.116	1.017	1.831	1.328

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	128	753	344	455	422	327
normalized size	1	1.	0.89	5.23	2.39	3.16	2.93	2.27
time (sec)	N/A	0.104	0.098	0.053	1.241	1.024	3.239	1.39

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	141	1012	578	826	656	609
normalized size	1	1.	0.81	5.78	3.3	4.72	3.75	3.48
time (sec)	N/A	0.131	0.158	0.056	1.267	1.02	5.23	1.398

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	166	1306	873	1284	944	967
normalized size	1	1.	0.81	6.34	4.24	6.23	4.58	4.69
time (sec)	N/A	0.149	0.202	0.053	1.169	1.108	7.936	1.357

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	503	557	512	0	3233	0	0	0
normalized size	1	1.11	1.02	0.	6.43	0.	0.	0.
time (sec)	N/A	0.818	0.524	2.313	1.847	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	420	474	392	0	2342	0	0	0
normalized size	1	1.13	0.93	0.	5.58	0.	0.	0.
time (sec)	N/A	0.648	0.355	1.958	1.69	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	335	389	290	0	1582	0	0	0
normalized size	1	1.16	0.87	0.	4.72	0.	0.	0.
time (sec)	N/A	0.556	0.239	1.943	1.761	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	284	203	0	836	0	0	0
normalized size	1	1.41	1.	0.	4.14	0.	0.	0.
time (sec)	N/A	0.418	0.186	1.7	1.69	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	719	251	906	0	0	0	0
normalized size	1	5.62	1.96	7.08	0.	0.	0.	0.
time (sec)	N/A	4.423	0.276	0.067	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	470	314	1251	562	319	430	0
normalized size	1	3.07	2.05	8.18	3.67	2.08	2.81	0.
time (sec)	N/A	0.762	0.495	0.05	1.295	1.012	3.758	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	296	578	444	1934	1143	767	892	0
normalized size	1	1.95	1.5	6.53	3.86	2.59	3.01	0.
time (sec)	N/A	0.91	0.476	0.053	1.547	1.075	6.71	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	399	680	585	2758	1917	1388	1544	0
normalized size	1	1.7	1.47	6.91	4.8	3.48	3.87	0.
time (sec)	N/A	1.074	0.79	0.055	1.781	1.302	34.789	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	498	763	748	3717	2865	2152	0	0
normalized size	1	1.53	1.5	7.46	5.75	4.32	0.	0.
time (sec)	N/A	1.264	1.061	0.054	2.29	1.387	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.196	0.587	1.224	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.237	1.047	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.235	1.347	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	A	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	53	0	50	69	0	109	0	0
normalized size	1	0.	0.94	1.3	0.	2.06	0.	0.
time (sec)	N/A	0.086	0.064	0.286	0.	0.98	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	A	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	109	0	89	0	0	286	0	0
normalized size	1	0.	0.82	0.	0.	2.62	0.	0.
time (sec)	N/A	0.072	0.158	1.425	0.	0.977	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.206	1.355	1.11	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.945	1.129	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.079	0.514	1.079	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	B	F	B	F(-1)	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	104	0	88	258	0	439	0	344
normalized size	1	0.	0.85	2.48	0.	4.22	0.	3.31
time (sec)	N/A	0.089	0.125	0.16	0.	0.97	0.	1.416

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	A	F	F	B	F(-1)	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	159	0	135	0	0	1212	0	0
normalized size	1	0.	0.85	0.	0.	7.62	0.	0.
time (sec)	N/A	0.08	0.431	1.582	0.	1.068	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	144	1030	1191	952	1018	666
normalized size	1	1.	0.79	5.66	6.54	5.23	5.59	3.66
time (sec)	N/A	0.118	0.101	0.415	1.348	1.279	8.817	162.637

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	122	788	871	709	722	491
normalized size	1	1.	0.81	5.22	5.77	4.7	4.78	3.25
time (sec)	N/A	0.098	0.074	0.243	1.373	1.181	6.026	26.115

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	98	569	589	508	527	335
normalized size	1	1.	0.82	4.74	4.91	4.23	4.39	2.79
time (sec)	N/A	0.078	0.051	0.241	1.331	1.113	4.156	4.722

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	72	340	338	329	253	173
normalized size	1	1.	0.92	4.36	4.33	4.22	3.24	2.22
time (sec)	N/A	0.052	0.037	0.242	1.249	1.105	2.545	1.744

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	121	87	265	0	0	0	0
normalized size	1	1.46	1.05	3.19	0.	0.	0.	0.
time (sec)	N/A	0.292	0.037	0.335	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	105	89	212	252	228	253	254
normalized size	1	1.03	0.87	2.08	2.47	2.24	2.48	2.49
time (sec)	N/A	0.076	0.052	0.062	1.233	1.038	1.939	1.381

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	128	300	413	495	418	350
normalized size	1	1.	0.92	2.16	2.97	3.56	3.01	2.52
time (sec)	N/A	0.101	0.096	0.06	1.22	1.005	3.269	1.342

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	140	427	648	869	677	639
normalized size	1	1.	0.79	2.41	3.66	4.91	3.82	3.61
time (sec)	N/A	0.121	0.12	0.059	1.311	1.078	5.36	1.389

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	162	587	944	1328	947	562
normalized size	1	1.	0.78	2.82	4.54	6.38	4.55	2.7
time (sec)	N/A	0.143	0.189	0.064	1.359	1.09	8.015	1.397

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	569	524	0	3591	0	0	0
normalized size	1	1.1	1.02	0.	6.97	0.	0.	0.
time (sec)	N/A	0.864	0.481	1.913	2.029	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	422	469	402	0	2633	0	0	0
normalized size	1	1.11	0.95	0.	6.24	0.	0.	0.
time (sec)	N/A	0.741	0.324	1.569	1.954	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	397	298	0	1800	0	0	0
normalized size	1	1.16	0.87	0.	5.25	0.	0.	0.
time (sec)	N/A	0.625	0.228	1.52	2.057	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	291	195	0	986	0	0	0
normalized size	1	1.38	0.92	0.	4.67	0.	0.	0.
time (sec)	N/A	0.504	0.174	1.257	1.675	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	740	257	0	0	0	0	0
normalized size	1	5.61	1.95	0.	0.	0.	0.	0.
time (sec)	N/A	4.08	0.33	1.391	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	480	322	452	774	416	450	505
normalized size	1	3.06	2.05	2.88	4.93	2.65	2.87	3.22
time (sec)	N/A	0.924	0.475	0.069	1.519	1.05	3.734	1.439

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	578	452	664	1351	846	877	0
normalized size	1	1.93	1.51	2.22	4.52	2.83	2.93	0.
time (sec)	N/A	1.081	0.484	0.07	1.557	1.09	6.775	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	407	692	598	947	2128	1474	1561	0
normalized size	1	1.7	1.47	2.33	5.23	3.62	3.84	0.
time (sec)	N/A	1.228	0.721	0.075	1.881	1.128	35.508	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	C	C	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	758	762	1285	3075	2229	0	1172
normalized size	1	1.51	1.52	2.56	6.14	4.45	0.	2.34
time (sec)	N/A	1.429	0.988	0.078	2.398	1.191	0.	1.689

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	0.157	1.127	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.115	0.962	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.07	1.298	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	91	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.087	0.077	1.228	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	151	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.074	0.081	1.371	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.46	1.	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.342	1.026	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	0.143	1.024	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	147	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.182	1.406	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	F	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	206	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.085	0.355	1.622	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	F	F	F	A	F(-1)	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	96	0	0	0	0	149	0	0
normalized size	1	0.	0.	0.	0.	1.55	0.	0.
time (sec)	N/A	0.102	0.059	0.046	0.	1.047	0.	0.



Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	355	339	279	14719	801	1283	1528	0
normalized size	1	0.95	0.79	41.46	2.26	3.61	4.3	0.
time (sec)	N/A	0.557	0.586	0.231	1.221	3.517	36.98	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	215	8605	560	895	1049	0
normalized size	1	1.	0.95	37.91	2.47	3.94	4.62	0.
time (sec)	N/A	0.341	0.262	0.2	1.202	1.837	20.558	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	142	4406	354	581	688	346
normalized size	1	1.	0.95	29.37	2.36	3.87	4.59	2.31
time (sec)	N/A	0.169	0.132	0.184	1.132	1.316	10.691	15.963

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	114	1809	189	321	325	171
normalized size	1	1.	1.05	16.6	1.73	2.94	2.98	1.57
time (sec)	N/A	0.098	0.107	0.174	1.197	1.355	4.786	2.529

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	418	73	132	83	0
normalized size	1	1.	1.	8.04	1.4	2.54	1.6	0.
time (sec)	N/A	0.027	0.008	0.149	1.168	1.304	1.166	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	115	1400	0	0	0	0
normalized size	1	1.	0.82	10.	0.	0.	0.	0.
time (sec)	N/A	0.247	0.057	0.545	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	113	105	926	186	559	0	378
normalized size	1	1.3	1.21	10.64	2.14	6.43	0.	4.34
time (sec)	N/A	0.106	0.151	0.149	1.121	21.953	0.	1.313

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	169	5274	474	0	0	1110
normalized size	1	1.	0.92	28.82	2.59	0.	0.	6.07
time (sec)	N/A	0.185	0.645	0.175	1.241	0.	0.	1.749

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	260	18285	1145	0	0	2765
normalized size	1	1.	0.95	66.49	4.16	0.	0.	10.05
time (sec)	N/A	0.396	0.927	0.203	1.538	0.	0.	4.301

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	379	379	355	44893	2372	0	0	5516
normalized size	1	1.	0.94	118.45	6.26	0.	0.	14.55
time (sec)	N/A	0.618	1.159	0.259	1.896	0.	0.	11.725

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	874	994	733	0	2889	0	0	0
normalized size	1	1.14	0.84	0.	3.31	0.	0.	0.
time (sec)	N/A	1.744	0.972	2.677	1.805	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	532	649	486	0	1755	0	0	0
normalized size	1	1.22	0.91	0.	3.3	0.	0.	0.
time (sec)	N/A	1.094	0.531	2.257	1.692	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	444	346	0	909	0	0	0
normalized size	1	1.64	1.28	0.	3.37	0.	0.	0.
time (sec)	N/A	0.82	0.314	1.851	1.585	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	246	214	0	0	0	0	0
normalized size	1	1.97	1.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.637	0.164	1.722	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	1998	431	2428	0	0	0	0
normalized size	1	7.21	1.56	8.77	0.	0.	0.	0.
time (sec)	N/A	4.9	0.857	0.336	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	612	402	0	0	0	0	0
normalized size	1	3.12	2.05	0.	0.	0.	0.	0.
time (sec)	N/A	1.119	0.959	2.067	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	883	595	0	0	0	0	0
normalized size	1	2.39	1.61	0.	0.	0.	0.	0.
time (sec)	N/A	1.481	2.067	2.909	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	714	1356	894	0	0	0	0	0
normalized size	1	1.9	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	2.381	4.224	4.183	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1159	1881	1448	0	0	0	0	0
normalized size	1	1.62	1.25	0.	0.	0.	0.	0.
time (sec)	N/A	3.403	7.311	6.561	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	34	30	46	43	77	20	39
normalized size	1	0.97	0.86	1.31	1.23	2.2	0.57	1.11
time (sec)	N/A	0.014	0.005	0.131	1.129	0.948	0.141	1.265

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	0.421	1.279	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.09	0.245	1.067	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.023	1.074	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.906	1.409	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.065	0.883	1.366	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	8.468	1.424	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.184	1.312	1.112	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	0.882	1.115	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.479	1.079	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	2.505	1.425	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	4.414	1.58	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	30.119	1.841	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	341	282	2438	1154	1328	1562	0
normalized size	1	0.96	0.79	6.83	3.23	3.72	4.38	0.
time (sec)	N/A	0.501	0.587	0.318	1.429	3.491	35.794	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	217	1783	841	938	1052	603
normalized size	1	1.	0.95	7.79	3.67	4.1	4.59	2.63
time (sec)	N/A	0.324	0.257	0.266	1.343	1.915	18.946	137.852

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	142	1188	566	621	719	377
normalized size	1	1.	0.93	7.82	3.72	4.09	4.73	2.48
time (sec)	N/A	0.161	0.131	0.237	1.322	1.333	10.373	14.604

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	118	656	332	373	321	196
normalized size	1	1.	1.13	6.31	3.19	3.59	3.09	1.88
time (sec)	N/A	0.087	0.105	0.244	1.164	1.045	4.261	2.105

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	233	77	184	104	0
normalized size	1	1.	1.	4.31	1.43	3.41	1.93	0.
time (sec)	N/A	0.027	0.023	0.224	1.23	1.019	1.145	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	119	1143	0	0	0	0
normalized size	1	1.	0.83	7.94	0.	0.	0.	0.
time (sec)	N/A	0.309	0.059	0.445	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	117	108	388	259	613	0	0
normalized size	1	1.3	1.2	4.31	2.88	6.81	0.	0.
time (sec)	N/A	0.092	0.164	0.09	1.178	22.296	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	172	1554	547	0	0	1148
normalized size	1	1.	0.98	8.88	3.13	0.	0.	6.56
time (sec)	N/A	0.17	0.656	0.174	1.315	0.	0.	1.839

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	277	277	263	4421	1215	0	0	2788
normalized size	1	1.	0.95	15.96	4.39	0.	0.	10.06
time (sec)	N/A	0.327	0.892	0.277	1.537	0.	0.	3.95

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	358	10401	2442	0	0	0
normalized size	1	1.	0.94	27.3	6.41	0.	0.	0.
time (sec)	N/A	0.553	1.135	0.438	1.927	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	869	973	746	0	3174	0	0	0
normalized size	1	1.12	0.86	0.	3.65	0.	0.	0.
time (sec)	N/A	1.808	0.967	1.77	1.95	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	659	497	0	1968	0	0	0
normalized size	1	1.22	0.92	0.	3.63	0.	0.	0.
time (sec)	N/A	1.199	0.532	1.569	1.843	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	281	450	351	0	1061	0	0	0
normalized size	1	1.6	1.25	0.	3.78	0.	0.	0.
time (sec)	N/A	0.959	0.306	1.416	1.597	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	252	220	0	0	0	0	0
normalized size	1	1.95	1.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.774	0.163	1.319	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	285	2126	0	0	0	0	0	0
normalized size	1	7.46	0.	0.	0.	0.	0.	0.
time (sec)	N/A	5.846	2.2	1.629	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	200	620	409	0	0	0	0	0
normalized size	1	3.1	2.04	0.	0.	0.	0.	0.
time (sec)	N/A	1.301	0.853	1.794	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	899	603	0	0	0	0	0
normalized size	1	2.36	1.58	0.	0.	0.	0.	0.
time (sec)	N/A	1.644	1.962	2.223	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	724	1369	909	0	0	0	0	0
normalized size	1	1.89	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	2.54	4.111	2.948	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1154	1854	1453	0	0	0	0	0
normalized size	1	1.61	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	3.543	7.253	4.122	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	0.176	1.148	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.126	0.936	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.035	0.885	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.082	1.248	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.085	1.284	0.	0.	0.	0.



Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.089	1.363	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.191	0.695	1.006	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.41	1.032	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.014	0.214	1.013	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.486	1.44	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.607	1.641	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.628	2.162	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	365	377	463	2576	906	1623	0	0
normalized size	1	1.03	1.27	7.06	2.48	4.45	0.	0.
time (sec)	N/A	0.712	0.952	0.689	1.402	1.127	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	248	314	1967	630	1149	0	0
normalized size	1	1.05	1.33	8.33	2.67	4.87	0.	0.
time (sec)	N/A	0.456	0.567	0.613	1.204	1.103	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	170	204	1389	397	753	0	402
normalized size	1	1.08	1.29	8.79	2.51	4.77	0.	2.54
time (sec)	N/A	0.24	0.337	0.576	1.192	1.09	0.	77.419

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	128	124	839	208	413	0	201
normalized size	1	1.1	1.07	7.23	1.79	3.56	0.	1.73
time (sec)	N/A	0.149	0.168	0.541	1.164	1.011	0.	5.784

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	123	80	146	0	74
normalized size	1	1.	1.	2.16	1.4	2.56	0.	1.3
time (sec)	N/A	0.03	0.011	0.058	1.127	1.031	0.	1.326

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	156	150	597	0	0	0	0
normalized size	1	1.05	1.01	4.03	0.	0.	0.	0.
time (sec)	N/A	0.189	0.095	0.783	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	132	117	1796	204	551	0	224
normalized size	1	1.1	0.98	14.97	1.7	4.59	0.	1.87
time (sec)	N/A	0.12	0.225	0.486	1.181	18.528	0.	1.471

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	203	178	4925	516	0	0	1197
normalized size	1	1.06	0.93	25.79	2.7	0.	0.	6.27
time (sec)	N/A	0.301	0.734	0.69	2.427	0.	0.	1.88

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	296	273	9645	1242	0	0	2041
normalized size	1	1.04	0.96	33.96	4.37	0.	0.	7.19
time (sec)	N/A	0.536	1.343	0.925	1.636	0.	0.	4.524

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	389	401	366	16077	2581	0	0	5925
normalized size	1	1.03	0.94	41.33	6.63	0.	0.	15.23
time (sec)	N/A	0.821	1.471	1.286	2.131	0.	0.	14.597

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	570	697	906	22955	2256	0	0	0
normalized size	1	1.22	1.59	40.27	3.96	0.	0.	0.
time (sec)	N/A	1.294	1.806	3.399	3.83	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	449	472	11007	1219	0	0	0
normalized size	1	1.53	1.61	37.44	4.15	0.	0.	0.
time (sec)	N/A	0.955	0.985	2.088	3.618	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	195	217	4749	0	0	0	0
normalized size	1	1.42	1.58	34.66	0.	0.	0.	0.
time (sec)	N/A	0.321	0.147	1.311	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	473	1082	0	0	0	0	0
normalized size	1	1.57	3.59	0.	0.	0.	0.	0.
time (sec)	N/A	0.815	0.463	2.559	0.	0.	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	343	3460	0	0	0	0	0
normalized size	1	1.65	16.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.407	2.283	2.712	0.	0.	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	B	F	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	393	968	15422	0	0	0	0	0
normalized size	1	2.46	39.24	0.	0.	0.	0.	0.
time (sec)	N/A	1.631	6.468	1.788	0.	0.	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	875	1640	0	0	0	0	0	0
normalized size	1	1.87	0.	0.	0.	0.	0.	0.
time (sec)	N/A	3.481	5.68	5.47	0.	0.	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	466	1030	0	0	0	0	0	0
normalized size	1	2.21	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.112	3.036	5.28	0.	0.	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	408	378	0	0	0	0	0
normalized size	1	2.01	1.86	0.	0.	0.	0.	0.
time (sec)	N/A	0.59	0.28	2.737	0.	0.	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	425	921	0	0	0	0	0	0
normalized size	1	2.17	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.642	1.452	3.127	0.	0.	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	302	650	0	0	0	0	0	0
normalized size	1	2.15	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.807	3.627	2.753	0.	0.	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	F	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	629	2207	0	0	0	0	0	0
normalized size	1	3.51	0.	0.	0.	0.	0.	0.
time (sec)	N/A	3.745	8.247	3.369	0.	0.	0.	0.

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [186] had the largest ratio of [ 0.75 ]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	33	0.091
2	A	4	3	1.	33	0.091
3	A	4	3	1.	33	0.091
4	A	4	3	1.	31	0.097
5	A	9	8	1.5	33	0.242
6	A	4	3	1.61	33	0.091
7	A	4	3	1.	33	0.091
8	A	4	3	1.	33	0.091
9	A	4	3	1.	33	0.091
10	A	27	13	1.52	35	0.371
11	A	23	13	1.53	35	0.371
12	A	19	13	1.53	35	0.371
13	A	15	12	1.58	33	0.364
14	B	45	23	5.72	35	0.657
15	C	24	11	3.76	35	0.314
16	C	28	11	2.17	35	0.314
17	C	32	11	1.64	35	0.314
18	C	36	11	1.34	35	0.314
19	A	0	0	0.	0	0.
20	A	0	0	0.	0	0.
21	A	0	0	0.	0	0.
22	F	0	0	N/A	0	N/A
23	F	0	0	N/A	0	N/A
24	A	0	0	0.	0	0.
25	A	0	0	0.	0	0.
26	A	0	0	0.	0	0.
27	F	0	0	N/A	0	N/A

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	F	0	0	N/A	0	N/A
29	A	4	3	1.	33	0.091
30	A	4	3	1.	33	0.091
31	A	4	3	1.	33	0.091
32	A	4	3	1.	31	0.097
33	A	9	8	1.6	33	0.242
34	A	4	3	1.05	33	0.091
35	A	4	3	1.	33	0.091
36	A	4	3	1.	33	0.091
37	A	4	3	1.	33	0.091
38	A	27	13	1.17	35	0.371
39	A	23	13	1.2	35	0.371
40	A	19	13	1.26	35	0.371
41	A	15	12	1.4	33	0.364
42	B	45	23	5.71	35	0.657
43	C	24	11	3.15	35	0.314
44	C	28	11	1.97	35	0.314
45	C	32	11	1.72	35	0.314
46	C	36	11	1.54	35	0.314
47	A	0	0	0.	0	0.
48	A	0	0	0.	0	0.
49	A	0	0	0.	0	0.
50	F	0	0	N/A	0	N/A
51	F	0	0	N/A	0	N/A
52	A	0	0	0.	0	0.
53	A	0	0	0.	0	0.
54	A	0	0	0.	0	0.
55	F	0	0	N/A	0	N/A
56	F	0	0	N/A	0	N/A
57	A	4	3	0.96	30	0.1
58	A	4	3	1.	30	0.1
59	A	4	3	1.	30	0.1
60	A	4	3	1.	28	0.107
61	A	3	2	1.	22	0.091
62	A	9	5	1.	30	0.167
63	A	4	3	1.31	30	0.1
64	A	4	3	1.	30	0.1
65	A	4	3	1.	30	0.1
66	A	4	3	1.	30	0.1
67	A	31	13	1.15	32	0.406
68	A	27	13	1.24	32	0.406
69	A	23	12	1.66	30	0.4
70	B	20	10	2.04	24	0.417
71	B	43	21	7.52	32	0.656

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
72	B	29	10	3.19	32	0.312
73	B	33	11	2.42	32	0.344
74	A	37	11	1.91	32	0.344
75	A	41	11	1.63	32	0.344
76	A	0	0	0.	0	0.
77	A	0	0	0.	0	0.
78	A	0	0	0.	0	0.
79	A	0	0	0.	0	0.
80	A	0	0	0.	0	0.
81	A	0	0	0.	0	0.
82	A	0	0	0.	0	0.
83	A	0	0	0.	0	0.
84	A	0	0	0.	0	0.
85	A	0	0	0.	0	0.
86	A	0	0	0.	0	0.
87	A	0	0	0.	0	0.
88	A	4	3	1.	30	0.1
89	A	4	3	1.	30	0.1
90	A	4	3	1.	30	0.1
91	A	4	3	1.	28	0.107
92	A	10	8	1.5	30	0.267
93	A	4	3	1.62	30	0.1
94	A	4	3	1.	30	0.1
95	A	4	3	1.	30	0.1
96	A	4	3	1.	30	0.1
97	A	28	13	1.53	32	0.406
98	A	24	13	1.53	32	0.406
99	A	20	13	1.54	32	0.406
100	A	16	12	1.58	30	0.4
101	B	46	23	5.69	32	0.719
102	C	26	11	3.73	32	0.344
103	C	30	11	2.15	32	0.344
104	C	34	11	1.63	32	0.344
105	C	38	11	1.33	32	0.344
106	A	1	1	1.04	29	0.034
107	A	1	1	1.	18	0.056
108	A	1	1	1.	20	0.05
109	A	0	0	0.	0	0.
110	A	0	0	0.	0	0.
111	A	0	0	0.	0	0.
112	F	0	0	N/A	0	N/A
113	F	0	0	N/A	0	N/A
114	A	0	0	0.	0	0.
115	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
116	A	0	0	0.	0	0.
117	F	0	0	N/A	0	N/A
118	F	0	0	N/A	0	N/A
119	A	4	3	1.	32	0.094
120	A	4	3	1.	32	0.094
121	A	4	3	1.	32	0.094
122	A	4	3	1.	30	0.1
123	A	10	8	1.47	32	0.25
124	A	4	3	1.62	32	0.094
125	A	4	3	1.	32	0.094
126	A	4	3	1.	32	0.094
127	A	4	3	1.	32	0.094
128	A	28	13	1.51	34	0.382
129	A	24	13	1.47	34	0.382
130	A	20	13	1.56	34	0.382
131	A	16	12	1.55	32	0.375
132	B	46	23	5.67	34	0.676
133	C	26	11	3.69	34	0.324
134	C	30	11	2.13	34	0.324
135	C	34	11	1.61	34	0.324
136	C	38	11	1.29	34	0.324
137	A	0	0	0.	0	0.
138	A	0	0	0.	0	0.
139	A	0	0	0.	0	0.
140	F	0	0	N/A	0	N/A
141	F	0	0	N/A	0	N/A
142	A	0	0	0.	0	0.
143	A	0	0	0.	0	0.
144	A	0	0	0.	0	0.
145	F	0	0	N/A	0	N/A
146	F	0	0	N/A	0	N/A
147	A	5	3	1.07	31	0.097
148	A	5	3	1.08	31	0.097
149	A	5	3	1.11	31	0.097
150	A	5	3	1.14	29	0.103
151	A	7	6	1.1	31	0.194
152	A	4	3	0.74	31	0.097
153	A	5	3	1.09	31	0.097
154	A	5	3	1.07	31	0.097
155	A	5	3	1.06	31	0.097
156	A	21	11	1.68	33	0.333
157	A	18	11	1.62	33	0.333
158	A	15	11	1.58	31	0.355
159	A	10	8	1.73	33	0.242

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	7	3	1.47	33	0.091
161	A	12	8	1.5	33	0.242
162	C	26	11	1.71	33	0.333
163	C	29	11	1.44	33	0.333
164	A	56	13	1.49	33	0.394
165	A	40	13	1.49	33	0.394
166	A	27	13	1.86	31	0.419
167	B	14	9	2.28	33	0.273
168	A	11	3	1.96	33	0.091
169	B	21	8	2.08	33	0.242
170	C	66	16	3.07	33	0.485
171	C	93	16	2.62	33	0.485
172	F	0	0	N/A	0	N/A
173	A	4	3	1.	30	0.1
174	A	4	3	1.	30	0.1
175	A	4	3	1.	30	0.1
176	A	4	3	1.	28	0.107
177	A	10	8	1.51	30	0.267
178	A	4	3	1.58	30	0.1
179	A	4	3	1.	30	0.1
180	A	4	3	1.	30	0.1
181	A	4	3	1.	30	0.1
182	A	28	13	1.11	32	0.406
183	A	24	13	1.13	32	0.406
184	A	20	13	1.16	32	0.406
185	A	16	12	1.41	30	0.4
186	B	47	24	5.62	32	0.75
187	C	26	11	3.07	32	0.344
188	C	30	11	1.95	32	0.344
189	C	34	11	1.7	32	0.344
190	C	38	11	1.53	32	0.344
191	A	0	0	0.	0	0.
192	A	0	0	0.	0	0.
193	A	0	0	0.	0	0.
194	F	0	0	N/A	0	N/A
195	F	0	0	N/A	0	N/A
196	A	0	0	0.	0	0.
197	A	0	0	0.	0	0.
198	A	0	0	0.	0	0.
199	F	0	0	N/A	0	N/A
200	F	0	0	N/A	0	N/A
201	A	4	3	1.	32	0.094
202	A	4	3	1.	32	0.094
203	A	4	3	1.	32	0.094

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
204	A	4	3	1.	30	0.1
205	A	10	8	1.46	32	0.25
206	A	4	3	1.03	32	0.094
207	A	4	3	1.	32	0.094
208	A	4	3	1.	32	0.094
209	A	4	3	1.	32	0.094
210	A	28	13	1.1	34	0.382
211	A	24	13	1.11	34	0.382
212	A	20	13	1.16	34	0.382
213	A	16	12	1.38	32	0.375
214	B	46	23	5.61	34	0.676
215	C	26	11	3.06	34	0.324
216	C	30	11	1.93	34	0.324
217	C	34	11	1.7	34	0.324
218	C	38	11	1.51	34	0.324
219	A	0	0	0.	0	0.
220	A	0	0	0.	0	0.
221	A	0	0	0.	0	0.
222	F	0	0	N/A	0	N/A
223	F	0	0	N/A	0	N/A
224	A	0	0	0.	0	0.
225	A	0	0	0.	0	0.
226	A	0	0	0.	0	0.
227	F	0	0	N/A	0	N/A
228	F	0	0	N/A	0	N/A
229	F	0	0	N/A	0	N/A
230	A	4	3	0.95	27	0.111
231	A	4	3	1.	27	0.111
232	A	4	3	1.	27	0.111
233	A	4	3	1.	25	0.12
234	A	3	2	1.	19	0.105
235	A	10	6	1.	27	0.222
236	A	4	3	1.3	27	0.111
237	A	4	3	1.	27	0.111
238	A	4	3	1.	27	0.111
239	A	4	3	1.	27	0.111
240	A	33	13	1.14	29	0.448
241	A	29	13	1.22	29	0.448
242	A	25	12	1.64	27	0.444
243	A	22	10	1.97	21	0.476
244	B	41	21	7.21	29	0.724
245	B	32	10	3.12	29	0.345
246	B	36	11	2.39	29	0.379
247	A	40	11	1.9	29	0.379

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
248	A	44	11	1.62	29	0.379
249	A	4	4	0.97	14	0.286
250	A	0	0	0.	0	0.
251	A	0	0	0.	0	0.
252	A	0	0	0.	0	0.
253	A	0	0	0.	0	0.
254	A	0	0	0.	0	0.
255	A	0	0	0.	0	0.
256	A	0	0	0.	0	0.
257	A	0	0	0.	0	0.
258	A	0	0	0.	0	0.
259	A	0	0	0.	0	0.
260	A	0	0	0.	0	0.
261	A	0	0	0.	0	0.
262	A	4	3	0.96	29	0.103
263	A	4	3	1.	29	0.103
264	A	4	3	1.	29	0.103
265	A	4	3	1.	27	0.111
266	A	3	2	1.	21	0.095
267	A	10	6	1.	29	0.207
268	A	4	3	1.3	29	0.103
269	A	4	3	1.	29	0.103
270	A	4	3	1.	29	0.103
271	A	4	3	1.	29	0.103
272	A	33	13	1.12	31	0.419
273	A	29	13	1.22	31	0.419
274	A	25	12	1.6	29	0.414
275	A	22	10	1.95	23	0.435
276	B	44	21	7.46	31	0.677
277	B	32	10	3.1	31	0.323
278	B	36	11	2.36	31	0.355
279	A	40	11	1.89	31	0.355
280	A	44	11	1.61	31	0.355
281	A	0	0	0.	0	0.
282	A	0	0	0.	0	0.
283	A	0	0	0.	0	0.
284	A	0	0	0.	0	0.
285	A	0	0	0.	0	0.
286	A	0	0	0.	0	0.
287	A	0	0	0.	0	0.
288	A	0	0	0.	0	0.
289	A	0	0	0.	0	0.
290	A	0	0	0.	0	0.
291	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
292	A	0	0	0.	0	0.
293	A	5	3	1.03	31	0.097
294	A	5	3	1.05	31	0.097
295	A	5	3	1.08	31	0.097
296	A	5	3	1.1	29	0.103
297	A	3	2	1.	23	0.087
298	A	9	5	1.05	31	0.161
299	A	6	4	1.1	31	0.129
300	A	5	3	1.06	31	0.097
301	A	5	3	1.04	31	0.097
302	A	5	3	1.03	31	0.097
303	A	23	11	1.22	33	0.333
304	A	20	11	1.53	31	0.355
305	A	10	8	1.42	25	0.32
306	A	16	10	1.57	33	0.303
307	A	10	7	1.65	33	0.212
308	B	29	16	2.46	33	0.485
309	A	53	13	1.87	33	0.394
310	B	35	13	2.21	31	0.419
311	B	14	10	2.01	25	0.4
312	B	25	11	2.17	33	0.333
313	B	14	9	2.15	33	0.273
314	B	49	21	3.51	33	0.636

# Chapter 3

## Listing of integrals

### 3.1

$$\int (ag + bgx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$$

**Optimal.** Leaf size=188

$$\frac{g^4(a+bx)^5 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{5b} + \frac{Bg^4nx(bc-ad)^4}{5d^4} - \frac{Bg^4n(a+bx)^2(bc-ad)^3}{10bd^3} + \frac{Bg^4n(a+bx)^3(bc-ad)^2}{15bd^2} - \frac{Bg^4n(a+bx)^4(bc-ad)}{20bd} + \frac{g^4(a+bx)^5(A+B \log[e((a+bx)/(c+dx))^n])}{5b} - \frac{(B(b*c-a*d)^4*g^4*n*x)/(5*d^4) - (B(b*c-a*d)^3*g^4*n*(a+b*x)^2)/(10*b*d^3) + (B(b*c-a*d)^2*g^4*n*(a+b*x)^3)/(15*b*d^2) - (B(b*c-a*d)*g^4*n*(a+b*x)^4)/(20*b*d) + (g^4*(a+b*x)^5*(A+B*Log[e*((a+b*x)/(c+dx))^n]))/(5*b) - (B(b*c-a*d)^5*g^4*n*Log[c+d*x])/(5*b*d^5)}{5b}$$

[Out] (B\*(b\*c - a\*d)^4\*g^4\*n\*x)/(5\*d^4) - (B\*(b\*c - a\*d)^3\*g^4\*n\*(a + b\*x)^2)/(10\*b\*d^3) + (B\*(b\*c - a\*d)^2\*g^4\*n\*(a + b\*x)^3)/(15\*b\*d^2) - (B\*(b\*c - a\*d)\*g^4\*n\*(a + b\*x)^4)/(20\*b\*d) + (g^4\*(a + b\*x)^5\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(5\*b) - (B\*(b\*c - a\*d)^5\*g^4\*n\*Log[c + d\*x])/(5\*b\*d^5)

---

**Rubi [A]** time = 0.1431, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 43}

$$\frac{g^4(a+bx)^5 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{5b} + \frac{Bg^4nx(bc-ad)^4}{5d^4} - \frac{Bg^4n(a+bx)^2(bc-ad)^3}{10bd^3} + \frac{Bg^4n(a+bx)^3(bc-ad)^2}{15bd^2} - \frac{Bg^4n(a+bx)^4(bc-ad)}{20bd} + \frac{g^4(a+bx)^5(A+B \log[e((a+bx)/(c+dx))^n])}{5b} - \frac{(B(b*c-a*d)^4*g^4*n*x)/(5*d^4) - (B(b*c-a*d)^3*g^4*n*(a+b*x)^2)/(10*b*d^3) + (B(b*c-a*d)^2*g^4*n*(a+b*x)^3)/(15*b*d^2) - (B(b*c-a*d)*g^4*n*(a+b*x)^4)/(20*b*d) + (g^4*(a+b*x)^5*(A+B*Log[e*((a+b*x)/(c+dx))^n]))/(5*b) - (B(b*c-a*d)^5*g^4*n*Log[c+d*x])/(5*b*d^5)}{5b}$$

Antiderivative was successfully verified.

[In] Int[(a\*g + b\*g\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]),x]

[Out] (B\*(b\*c - a\*d)^4\*g^4\*n\*x)/(5\*d^4) - (B\*(b\*c - a\*d)^3\*g^4\*n\*(a + b\*x)^2)/(10\*b\*d^3) + (B\*(b\*c - a\*d)^2\*g^4\*n\*(a + b\*x)^3)/(15\*b\*d^2) - (B\*(b\*c - a\*d)\*g^4\*n\*(a + b\*x)^4)/(20\*b\*d) + (g^4\*(a + b\*x)^5\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(5\*b) - (B\*(b\*c - a\*d)^5\*g^4\*n\*Log[c + d\*x])/(5\*b\*d^5)

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^n\*(d\_.) + (e\_.)\*(x\_)^(m\_.), x\_Symbol] := Simp[(((d + e\*x)^(m + 1)\*(a + b\*Log[c\*Rfx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[(((d + e\*x)^(m + 1)\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned} \int (ag + bgx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx &= \frac{g^4(a+bx)^5 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{5b} - \frac{(Bn) \int \frac{(bc-ad)g^5(a+bx)^4}{c+dx} dx}{5bg} \\ &= \frac{g^4(a+bx)^5 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{5b} - \frac{(B(bc-ad)g^4n) \int \frac{(a+bx)^4}{c+dx} dx}{5b} \\ &= \frac{g^4(a+bx)^5 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{5b} - \frac{(B(bc-ad)g^4n) \int \left( -\frac{b(bc-ad)^3}{d^4} + \frac{b(bc-ad)^2}{d^3} \right) dx}{5b} \\ &= \frac{B(bc-ad)^4 g^4 n x}{5d^4} - \frac{B(bc-ad)^3 g^4 n (a+bx)^2}{10bd^3} + \frac{B(bc-ad)^2 g^4 n (a+bx)^3}{15bd^2} \end{aligned}$$

**Mathematica [A]** time = 0.128648, size = 146, normalized size = 0.78

$$\frac{g^4 \left( (a+bx)^5 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) - \frac{Bn(bc-ad)(6d^2(a+bx)^2(bc-ad)^2 + 4d^3(a+bx)^3(ad-bc) - 12bdx(bc-ad)^3 + 12(bc-ad)^4 \log(c+dx) + 3d^4(a+bx)^4)}{12d^5}}{5b} \right)}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (B*(b*c - a*d)*n
*(-12*b*d*(b*c - a*d)^3*x + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(-(b*c)
+ a*d)*(a + b*x)^3 + 3*d^4*(a + b*x)^4 + 12*(b*c - a*d)^4*Log[c + d*x]))/(
12*d^5)))/(5*b)
```

**Maple [F]** time = 0.546, size = 0, normalized size = 0.

$$\int (bgx + ag)^4 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)), x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n)), x)
```

**Maxima [B]** time = 1.28014, size = 913, normalized size = 4.86

$$\frac{1}{5} Bb^4g^4x^5 \log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right) + \frac{1}{5} Ab^4g^4x^5 + Bab^3g^4x^4 \log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right) + Aab^3g^4x^4 + 2Ba^2b^2g^4x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out] 1/5\*B\*b^4\*g^4\*x^5\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/5\*A\*b^4\*g^4\*x^5 + B\*a\*b^3\*g^4\*x^4\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A\*a\*b^3\*g^4\*x^4 + 2\*B\*a^2\*b^2\*g^4\*x^3\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 2\*A\*a^2\*b^2\*g^4\*x^3 + 2\*B\*a^3\*b\*g^4\*x^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 2\*A\*a^3\*b\*g^4\*x^2 + 1/60\*B\*b^4\*g^4\*n\*(12\*a^5\*log(b\*x + a)/b^5 - 12\*c^5\*log(d\*x + c)/d^5 - (3\*(b^4\*c\*d^3 - a\*b^3\*d^4)\*x^4 - 4\*(b^4\*c^2\*d^2 - a^2\*b^2\*d^4)\*x^3 + 6\*(b^4\*c^3\*d - a^3\*b\*d^4)\*x^2 - 12\*(b^4\*c^4 - a^4\*d^4)\*x)/(b^4\*d^4)) - 1/6\*B\*a\*b^3\*g^4\*n\*(6\*a^4\*log(b\*x + a)/b^4 - 6\*c^4\*log(d\*x + c)/d^4 + (2\*(b^3\*c\*d^2 - a\*b^2\*d^3)\*x^3 - 3\*(b^3\*c^2\*d - a^2\*b\*d^3)\*x^2 + 6\*(b^3\*c^3 - a^3\*d^3)\*x)/(b^3\*d^3)) + B\*a^2\*b^2\*g^4\*n\*(2\*a^3\*log(b\*x + a)/b^3 - 2\*c^3\*log(d\*x + c)/d^3 - ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2)) - 2\*B\*a^3\*b\*g^4\*n\*(a^2\*log(b\*x + a)/b^2 - c^2\*log(d\*x + c)/d^2 + (b\*c - a\*d)\*x/(b\*d)) + B\*a^4\*g^4\*n\*(a\*log(b\*x + a)/b - c\*log(d\*x + c)/d) + B\*a^4\*g^4\*x\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A\*a^4\*g^4\*x

**Fricas [B]** time = 1.1829, size = 1188, normalized size = 6.32

$$12 Ab^5d^5g^4x^5 + 12 Ba^5d^5g^4n \log(bx + a) - 12 (Bb^5c^5 - 5 Bab^4c^4d + 10 Ba^2b^3c^3d^2 - 10 Ba^3b^2c^2d^3 + 5 Ba^4bcd^4)g^4n \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] 1/60\*(12\*A\*b^5\*d^5\*g^4\*x^5 + 12\*B\*a^5\*d^5\*g^4\*n\*log(b\*x + a) - 12\*(B\*b^5\*c^5 - 5\*B\*a\*b^4\*c^4\*d + 10\*B\*a^2\*b^3\*c^3\*d^2 - 10\*B\*a^3\*b^2\*c^2\*d^3 + 5\*B\*a^4\*b\*c\*d^4)\*g^4\*n\*log(d\*x + c) + 3\*(20\*A\*a\*b^4\*d^5\*g^4 - (B\*b^5\*c\*d^4 - B\*a\*b^4\*d^5)\*g^4\*n)\*x^4 + 4\*(30\*A\*a^2\*b^3\*d^5\*g^4 + (B\*b^5\*c^2\*d^3 - 5\*B\*a\*b^4\*c\*d^4 + 4\*B\*a^2\*b^3\*d^5)\*g^4\*n)\*x^3 + 6\*(20\*A\*a^3\*b^2\*d^5\*g^4 - (B\*b^5\*c^3\*d^2 - 5\*B\*a\*b^4\*c^2\*d^3 + 10\*B\*a^2\*b^3\*c\*d^4 - 6\*B\*a^3\*b^2\*d^5)\*g^4\*n)\*x^2 + 12\*(5\*A\*a^4\*b\*d^5\*g^4 + (B\*b^5\*c^4\*d - 5\*B\*a\*b^4\*c^3\*d^2 + 10\*B\*a^2\*b^3\*c^2\*d^3 - 10\*B\*a^3\*b^2\*c\*d^4 + 4\*B\*a^4\*b\*d^5)\*g^4\*n)\*x + 12\*(B\*b^5\*d^5\*g^4\*x^5 + 5\*B\*a\*b^4\*d^5\*g^4\*x^4 + 10\*B\*a^2\*b^3\*d^5\*g^4\*x^3 + 10\*B\*a^3\*b^2\*d^5\*g^4\*x^2 + 5\*B\*a^4\*b\*d^5\*g^4\*x)\*log(e) + 12\*(B\*b^5\*d^5\*g^4\*n\*x^5 + 5\*B\*a\*b^4\*d^5\*g^4\*n\*x^4 + 10\*B\*a^2\*b^3\*d^5\*g^4\*n\*x^3 + 10\*B\*a^3\*b^2\*d^5\*g^4\*n\*x^2 + 5\*B\*a^4\*b\*d^5\*g^4\*n\*x)\*log((b\*x + a)/(d\*x + c)))/(b\*d^5)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] Timed out
```



### 3.2 $\int (ag + bgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$

**Optimal.** Leaf size=156

$$\frac{g^3(a+bx)^4 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{4b} - \frac{Bg^3nx(bc-ad)^3}{4d^3} + \frac{Bg^3n(a+bx)^2(bc-ad)^2}{8bd^2} + \frac{Bg^3n(bc-ad)^4 \log(c+dx)}{4bd^4} - \frac{Bg^3n(a+bx)(bc-ad)^2}{4bd^2}$$

[Out]  $-(B*(b*c - a*d)^3*g^3*n*x)/(4*d^3) + (B*(b*c - a*d)^2*g^3*n*(a + b*x)^2)/(8*b*d^2) - (B*(b*c - a*d)*g^3*n*(a + b*x)^3)/(12*b*d) + (g^3*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b) + (B*(b*c - a*d)^4*g^3*n*Log[c + d*x])/(4*b*d^4)$

**Rubi [A]** time = 0.105831, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 43}

$$\frac{g^3(a+bx)^4 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{4b} - \frac{Bg^3nx(bc-ad)^3}{4d^3} + \frac{Bg^3n(a+bx)^2(bc-ad)^2}{8bd^2} + \frac{Bg^3n(bc-ad)^4 \log(c+dx)}{4bd^4} - \frac{Bg^3n(a+bx)(bc-ad)^2}{4bd^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]$

[Out]  $-(B*(b*c - a*d)^3*g^3*n*x)/(4*d^3) + (B*(b*c - a*d)^2*g^3*n*(a + b*x)^2)/(8*b*d^2) - (B*(b*c - a*d)*g^3*n*(a + b*x)^3)/(12*b*d) + (g^3*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b) + (B*(b*c - a*d)^4*g^3*n*Log[c + d*x])/(4*b*d^4)$

#### Rule 2525

$\text{Int}[(a + \text{Log}[(c + \text{RFX})^p])*(b + (d + e*x)^m), x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^n/(e*(m+1)), x] - \text{Dist}[(b*n*p)/(e*(m+1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^{n-1}*D[\text{RFX}, x])/RFX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[(a + (b + v)*u), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b + v)\*u] /;

#### Rule 43

$\text{Int}[(a + (b + x)^m)*(c + (d + x)^n), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int (ag + bgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx &= \frac{g^3(a+bx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{4b} - \frac{(Bn) \int \frac{(bc-ad)g^4(a+bx)^3}{c+dx} dx}{4bg} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{4b} - \frac{(B(bc-ad)g^3n) \int \frac{(a+bx)^3}{c+dx} dx}{4b} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{4b} - \frac{(B(bc-ad)g^3n) \int \left( \frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)}{d^2} \right) dx}{4b} \\
&= -\frac{B(bc-ad)^3g^3nx}{4d^3} + \frac{B(bc-ad)^2g^3n(a+bx)^2}{8bd^2} - \frac{B(bc-ad)g^3n(a+bx)^3}{12bd} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.110189, size = 124, normalized size = 0.79

$$\frac{g^3 \left( (a+bx)^4 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) - \frac{Bn(bc-ad)(3d^2(a+bx)^2(ad-bc)+6bdx(bc-ad)^2-6(bc-ad)^3 \log(c+dx)+2d^3(a+bx)^3)}{6d^4} \right)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] (g^3\*((a + b\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - (B\*(b\*c - a\*d)\*n\*(6\*b\*d\*(b\*c - a\*d)^2\*x + 3\*d^2\*(-(b\*c) + a\*d)\*(a + b\*x)^2 + 2\*d^3\*(a + b\*x)^3 - 6\*(b\*c - a\*d)^3\*Log[c + d\*x]))/(6\*d^4))/(4\*b)

**Maple [F]** time = 0.405, size = 0, normalized size = 0.

$$\int (bgx + ag)^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

[Out] int((b\*g\*x+a\*g)^3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

**Maxima [B]** time = 1.2364, size = 647, normalized size = 4.15

$$\frac{1}{4} B b^3 g^3 x^4 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + \frac{1}{4} A b^3 g^3 x^4 + B a b^2 g^3 x^3 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + A a b^2 g^3 x^3 + \frac{3}{2} B a^2 b g^3 x^2 \log \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)), x, algorithm="maxima")

[Out] 1/4\*B\*b^3\*g^3\*x^4\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/4\*A\*b^3\*g^3\*x^4 + B\*a\*b^2\*g^3\*x^3\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A\*a\*b^2\*g^3\*x^3

$$3 + 3/2*B*a^2*b*g^3*x^2*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*a^2*b*g^3*x^2 - 1/24*B*b^3*g^3*n*(6*a^4*\log(b*x + a)/b^4 - 6*c^4*\log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*a*b^2*g^3*n*(2*a^3*\log(b*x + a)/b^3 - 2*c^3*\log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3/2*B*a^2*b*g^3*n*(a^2*\log(b*x + a)/b^2 - c^2*\log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*a^3*g^3*n*(a*\log(b*x + a)/b - c*\log(d*x + c)/d) + B*a^3*g^3*x*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a^3*g^3*x$$

**Fricas [B]** time = 1.06987, size = 883, normalized size = 5.66

$$6Ab^4d^4g^3x^4 + 6Ba^4d^4g^3n \log(bx + a) + 6(Bb^4c^4 - 4Bab^3c^3d + 6Ba^2b^2c^2d^2 - 4Ba^3bcd^3)g^3n \log(dx + c) + 2(12Aa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*n*log(b*x + a) + 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*n*log(d*x + c) + 2*(12*A*a*b^3*d^4*g^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*g^3*n)*x^3 + 3*(12*A*a^2*b^2*d^4*g^3 + (B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*B*a^2*b^2*d^4)*g^3*n)*x^2 + 6*(4*A*a^3*b*d^4*g^3 - (B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*b^2*c*d^3 - 3*B*a^3*b*d^4)*g^3*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*x)*log(e) + 6*(B*b^4*d^4*g^3*n*x^4 + 4*B*a*b^3*d^4*g^3*n*x^3 + 6*B*a^2*b^2*d^4*g^3*n*x^2 + 4*B*a^3*b*d^4*g^3*n*x)*log((b*x + a)/(d*x + c)))/(b*d^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.3 $\int (ag + bgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$

**Optimal.** Leaf size=124

$$\frac{g^2(a+bx)^3 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{3b} + \frac{Bg^2nx(bc-ad)^2}{3d^2} - \frac{Bg^2n(bc-ad)^3 \log(c+dx)}{3bd^3} - \frac{Bg^2n(a+bx)^2(bc-ad)}{6bd}$$

[Out]  $(B*(b*c - a*d)^2*g^2*n*x)/(3*d^2) - (B*(b*c - a*d)*g^2*n*(a + b*x)^2)/(6*b*d) + (g^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b) - (B*(b*c - a*d)^3*g^2*n*Log[c + d*x])/(3*b*d^3)$

**Rubi [A]** time = 0.089504, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 43}

$$\frac{g^2(a+bx)^3 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{3b} + \frac{Bg^2nx(bc-ad)^2}{3d^2} - \frac{Bg^2n(bc-ad)^3 \log(c+dx)}{3bd^3} - \frac{Bg^2n(a+bx)^2(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]$

[Out]  $(B*(b*c - a*d)^2*g^2*n*x)/(3*d^2) - (B*(b*c - a*d)*g^2*n*(a + b*x)^2)/(6*b*d) + (g^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b) - (B*(b*c - a*d)^3*g^2*n*Log[c + d*x])/(3*b*d^3)$

#### Rule 2525

$\text{Int}[(a_. + \text{Log}[c_.*(\text{RFx}_.)^{(p_.)}]*(b_.))^{(n_.)}*((d_.) + (e_.)*(x_.))^{(m_.)}, x\_Symbol] :> \text{Simp}[(d + e*x)^{(m + 1)}*(a + b*\text{Log}[c*\text{RFx}^p])^n/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{(m + 1)}*(a + b*\text{Log}[c*\text{RFx}^p])^{(n - 1)}*D[\text{RFx}, x])/(\text{RFx}, x), x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] :> \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 43

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int (ag + bgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx &= \frac{g^2(a+bx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{3b} - \frac{(Bn) \int \frac{(bc-ad)g^3(a+bx)^2}{c+dx} dx}{3bg} \\
&= \frac{g^2(a+bx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{3b} - \frac{(B(bc-ad)g^2n) \int \frac{(a+bx)^2}{c+dx} dx}{3b} \\
&= \frac{g^2(a+bx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{3b} - \frac{(B(bc-ad)g^2n) \int \left( -\frac{b(bc-ad)}{d^2} + \dots \right) dx}{3b} \\
&= \frac{B(bc-ad)^2 g^2 n x}{3d^2} - \frac{B(bc-ad)g^2 n (a+bx)^2}{6bd} + \frac{g^2(a+bx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{3b}
\end{aligned}$$

**Mathematica [A]** time = 0.0621068, size = 103, normalized size = 0.83

$$\frac{g^2 \left( \frac{Bn(ad-bc)(d(a^2d+4abdx+b^2x(dx-2c))+2(bc-ad)^2 \log(c+dx))}{2d^3} + (a+bx)^3 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) \right)}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]),x]

[Out] (g^2\*((a + b\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + (B\*(-(b\*c) + a\*d)\*n\*(d\*(a^2\*d + 4\*a\*b\*d\*x + b^2\*x\*(-2\*c + d\*x)) + 2\*(b\*c - a\*d)^2\*Log[c + d\*x]))/(2\*d^3))/(3\*b)

**Maple [F]** time = 0.402, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int((b\*g\*x+a\*g)^2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

**Maxima [B]** time = 1.21147, size = 417, normalized size = 3.36

$$\frac{1}{3} B b^2 g^2 x^3 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + \frac{1}{3} A b^2 g^2 x^3 + B a b g^2 x^2 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + A a b g^2 x^2 + \frac{1}{6} B b^2 g^2 n \left( \frac{2a^2}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out] 1/3\*B\*b^2\*g^2\*x^3\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/3\*A\*b^2\*g^2\*x^3 + B\*a\*b\*g^2\*x^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A\*a\*b\*g^2\*x^2 +

$$\frac{1}{6}Bb^2g^{2n}(2a^3\log(bx+a)/b^3 - 2c^3\log(dx+c)/d^3 - ((b^2cd - ab^2d^2)x^2 - 2(b^2c^2 - a^2d^2)x)/(b^2d^2)) - B^2abg^{2n}(a^2\log(bx+a)/b^2 - c^2\log(dx+c)/d^2 + (bc - ad)x/(bd)) + B^2a^2g^{2n}(a\log(bx+a)/b - c\log(dx+c)/d) + B^2a^2g^{2n}\log(e(bx/(dx+c) + a/(dx+c)))^n + A^2a^2g^{2n}$$

**Fricas [B]** time = 0.906362, size = 622, normalized size = 5.02

$$2Ab^3d^3g^2x^3 + 2Ba^3d^3g^2n\log(bx+a) - 2(Bb^3c^3 - 3Bab^2c^2d + 3Ba^2bcd^2)g^2n\log(dx+c) + (6Aab^2d^3g^2 - Bb^3cd^2 - B^2abg^2n)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out]  $\frac{1}{6}(2A^2b^3d^3g^2x^3 + 2B^2a^3d^3g^2n\log(bx+a) - 2(B^2b^3c^3 - 3B^2a^2b^2c^2d + 3B^2a^2b^2c^2d^2)g^2n\log(dx+c) + (6A^2a^2b^2d^3g^2 - (B^2b^3c^3d^2 - B^2a^2b^2d^3)g^2n)x^2 + 2(3A^2a^2b^2d^3g^2 + (B^2b^3c^3d^2 - 3B^2a^2b^2c^2d^2 + 2B^2a^2b^2d^3)g^2n)x + 2(B^2b^3d^3g^2x^3 + 3B^2a^2b^2d^3g^2x^2 + 3B^2a^2b^2d^3g^2x)\log(e) + 2(B^2b^3d^3g^2n^2x^3 + 3B^2a^2b^2d^3g^2n^2x^2 + 3B^2a^2b^2d^3g^2n^2x)\log((bx+a)/(d*x+c)))/(b^2d^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n)),x)

[Out] Timed out

**Giac [B]** time = 26.7209, size = 354, normalized size = 2.85

$$\frac{Ba^3g^2n\log(bx+a)}{3b} + \frac{1}{3}(Ab^2g^2 + Bb^2g^2)x^3 - \frac{(Bb^2cg^2n - Babdg^2n - 6Aabdg^2 - 6Babdg^2)x^2}{6d} + \frac{1}{3}(Bb^2g^2nx^3 + 3Babg^2n)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out]  $\frac{1}{3}B^2a^3g^2n\log(bx+a)/b + \frac{1}{3}(A^2b^2g^2 + B^2b^2g^2)x^3 - \frac{1}{6}(B^2b^2c^3g^2n - B^2a^2b^2d^3g^2n - 6A^2a^2b^2d^3g^2 - 6B^2a^2b^2d^3g^2)x^2/d + \frac{1}{3}(B^2b^2g^2n^2x^3 + 3B^2a^2b^2g^2n^2x^2 + 3B^2a^2b^2g^2n^2x)\log((bx+a)/(d*x+c)) + \frac{1}{3}(B^2b^2c^2g^2n - 3B^2a^2b^2c^2d^2g^2n + 2B^2a^2d^2g^2n + 3A^2a^2d^2g^2 + 3B^2a^2d^2g^2)x/d^2 - \frac{1}{3}(B^2b^2c^3g^2n - 3B^2a^2b^2c^2d^2g^2n + 3B^2a^2c^2d^2g^2n)\log(-dx-c)/d^3$

### 3.4 $\int (ag + bgx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$

**Optimal.** Leaf size=86

$$\frac{g(a+bx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{2b} + \frac{Bgn(bc-ad)^2 \log(c+dx)}{2bd^2} - \frac{Bgnx(bc-ad)}{2d}$$

[Out]  $-(B*(b*c - a*d)*g*n*x)/(2*d) + (g*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b) + (B*(b*c - a*d)^2*g*n*Log[c + d*x])/(2*b*d^2)$

**Rubi [A]** time = 0.0614864, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {2525, 12, 43}

$$\frac{g(a+bx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{2b} + \frac{Bgn(bc-ad)^2 \log(c+dx)}{2bd^2} - \frac{Bgnx(bc-ad)}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]$

[Out]  $-(B*(b*c - a*d)*g*n*x)/(2*d) + (g*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b) + (B*(b*c - a*d)^2*g*n*Log[c + d*x])/(2*b*d^2)$

#### Rule 2525

$\text{Int}[(a_.) + \text{Log}[(c_.)*(Rfx_)^(p_.)]*(b_.)^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, m, p\}, x$  &&  $\text{RationalFunctionQ}[Rfx, x]$  &&  $\text{IGtQ}[n, 0]$  &&  $(\text{EqQ}[n, 1] \parallel \text{IntegerQ}[m])$  &&  $\text{NeQ}[m, -1]$

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$   $\text{FreeQ}[a, x]$  &&  $!\text{MatchQ}[u, (b_)*(v_)] /;$   $\text{FreeQ}[b, x]$

#### Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$   $\text{FreeQ}\{a, b, c, d, n\}, x$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{IGtQ}[m, 0]$  &&  $(!\text{IntegerQ}[n] \parallel (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) \parallel \text{LtQ}[9*m + 5*(n + 1), 0] \parallel \text{GtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned}
\int (ag + bgx) \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right) dx &= \frac{g(a + bx)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{2b} - \frac{(Bn) \int \frac{(bc - ad)g^2(a + bx)}{c + dx} dx}{2bg} \\
&= \frac{g(a + bx)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{2b} - \frac{(B(bc - ad)gn) \int \frac{a + bx}{c + dx} dx}{2b} \\
&= \frac{g(a + bx)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{2b} - \frac{(B(bc - ad)gn) \int \left( \frac{b}{d} + \frac{-bc + ad}{d(c + dx)} \right) dx}{2b} \\
&= -\frac{B(bc - ad)gnx}{2d} + \frac{g(a + bx)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{2b} + \frac{B(bc - ad)^2 gn \log}{2bd^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0395174, size = 73, normalized size = 0.85

$$\frac{g \left( (a + bx)^2 \left( B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) + A \right) + \frac{Bn(ad - bc)((ad - bc) \log(c + dx) + bdx)}{d^2} \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] (g\*((a + b\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + (B\*(-(b\*c) + a\*d)\*n\*(b\*d\*x + (-(b\*c) + a\*d)\*Log[c + d\*x]))/d^2)/(2\*b)

**Maple [F]** time = 0.322, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

[Out] int((b\*g\*x+a\*g)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

**Maxima [A]** time = 1.13311, size = 211, normalized size = 2.45

$$\frac{1}{2} Bbgx^2 \log \left( e \left( \frac{bx}{dx + c} + \frac{a}{dx + c} \right)^n \right) + \frac{1}{2} Abgx^2 - \frac{1}{2} Bbg \left( \frac{a^2 \log(bx + a)}{b^2} - \frac{c^2 \log(dx + c)}{d^2} + \frac{(bc - ad)x}{bd} \right) + Bagn \left( \frac{a \log}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)), x, algorithm="maxima")

[Out] 1/2\*B\*b\*g\*x^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/2\*A\*b\*g\*x^2 - 1/2\*B\*b\*g\*n\*(a^2\*log(b\*x + a)/b^2 - c^2\*log(d\*x + c)/d^2 + (b\*c - a\*d)\*x/(b\*d)) + B\*a\*g\*n\*(a\*log(b\*x + a)/b - c\*log(d\*x + c)/d) + B\*a\*g\*x\*log(e\*(b\*x/(d\*x



+ c) + a/(d\*x + c))^n) + A\*a\*g\*x

**Fricas [A]** time = 0.948587, size = 360, normalized size = 4.19

$$\frac{Ab^2d^2gx^2 + Ba^2d^2gn \log(bx + a) + (Bb^2c^2 - 2Babcd)gn \log(dx + c) + (2Aabd^2g - (Bb^2cd - Babd^2)gn)x + (Bb^2d^2g}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] 1/2\*(A\*b^2\*d^2\*g\*x^2 + B\*a^2\*d^2\*g\*n\*log(b\*x + a) + (B\*b^2\*c^2 - 2\*B\*a\*b\*c\*d)\*g\*n\*log(d\*x + c) + (2\*A\*a\*b\*d^2\*g - (B\*b^2\*c\*d - B\*a\*b\*d^2)\*g\*n)\*x + (B\*b^2\*d^2\*g\*x^2 + 2\*B\*a\*b\*d^2\*g\*x)\*log(e) + (B\*b^2\*d^2\*g\*n\*x^2 + 2\*B\*a\*b\*d^2\*g\*n\*x)\*log((b\*x + a)/(d\*x + c)))/(b\*d^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] Timed out

**Giac [A]** time = 3.65069, size = 167, normalized size = 1.94

$$\frac{Ba^2gn \log(bx + a)}{2b} + \frac{1}{2}(Abg + Bbg)x^2 + \frac{1}{2}(Bbgnx^2 + 2Bagnx) \log\left(\frac{bx + a}{dx + c}\right) - \frac{(Bbcgn - Badgn - 2Aadg - 2Badg)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out] 1/2\*B\*a^2\*g\*n\*log(b\*x + a)/b + 1/2\*(A\*b\*g + B\*b\*g)\*x^2 + 1/2\*(B\*b\*g\*n\*x^2 + 2\*B\*a\*g\*n\*x)\*log((b\*x + a)/(d\*x + c)) - 1/2\*(B\*b\*c\*g\*n - B\*a\*d\*g\*n - 2\*A\*a\*d\*g - 2\*B\*a\*d\*g)\*x/d + 1/2\*(B\*b\*c^2\*g\*n - 2\*B\*a\*c\*d\*g\*n)\*log(d\*x + c)/d^2

$$3.5 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag+bgx} dx$$

**Optimal.** Leaf size=84

$$\frac{Bn \operatorname{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right)}{bg} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{bg}$$

[Out]  $-\left(\operatorname{Log}\left[-\left(\frac{b*c - a*d}{d*(a + b*x)}\right)\right]*\left(A + B*\operatorname{Log}\left[e*\left(\frac{a + b*x}{c + d*x}\right)^n\right]\right)\right)/\left(b*g\right) + \left(B*n*\operatorname{PolyLog}\left[2, 1 + \left(\frac{b*c - a*d}{d*(a + b*x)}\right)\right]\right)/\left(b*g\right)$

**Rubi [A]** time = 0.209336, antiderivative size = 126, normalized size of antiderivative = 1.5, number of steps used = 9, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {2524, 2418, 2390, 12, 2301, 2394, 2393, 2391}

$$\frac{Bn \operatorname{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg} + \frac{\log(ag + bgx) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{bg} + \frac{Bn \log(ag + bgx) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{bg} - \frac{Bn \log^2(g(a + bx))}{2bg}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(A + B*\operatorname{Log}\left[e*\left(\frac{a + b*x}{c + d*x}\right)^n\right]\right)/\left(a*g + b*g*x\right), x\right]$

[Out]  $-\left(B*n*\operatorname{Log}\left[g*(a + b*x)\right]^2\right)/\left(2*b*g\right) + \left(\left(A + B*\operatorname{Log}\left[e*\left(\frac{a + b*x}{c + d*x}\right)^n\right]\right)*\operatorname{Log}\left[a*g + b*g*x\right]\right)/\left(b*g\right) + \left(B*n*\operatorname{Log}\left[\left(\frac{b*(c + d*x)}{b*c - a*d}\right)\right]*\operatorname{Log}\left[a*g + b*g*x\right]\right)/\left(b*g\right) + \left(B*n*\operatorname{PolyLog}\left[2, -\left(\frac{d*(a + b*x)}{b*c - a*d}\right)\right]\right)/\left(b*g\right)$

#### Rule 2524

$\operatorname{Int}\left[\left((a_.) + \operatorname{Log}\left[(c_.)*(RfX_)^{(p_.)}\right]*(b_.)\right)^{(n_.)} / \left((d_.) + (e_.)*(x_.)\right), x\_Symbol\right] \rightarrow \operatorname{Simp}\left[\left(\operatorname{Log}\left[d + e*x\right]*\left(a + b*\operatorname{Log}\left[c*RfX^p\right]\right)^n\right)/e, x\right] - \operatorname{Dist}\left[\left(b*n*p\right)/e, \operatorname{Int}\left[\left(\operatorname{Log}\left[d + e*x\right]*\left(a + b*\operatorname{Log}\left[c*RfX^p\right]\right)^{(n-1)}*D\left[RfX, x\right]\right)/RfX, x\right], x\right] /;$   
 $\operatorname{FreeQ}\left[\{a, b, c, d, e, p\}, x\right] \ \&\& \operatorname{RationalFunctionQ}\left[RfX, x\right] \ \&\& \operatorname{IGtQ}\left[n, 0\right]$

#### Rule 2418

$\operatorname{Int}\left[\left((a_.) + \operatorname{Log}\left[(c_.)*\left((d_.) + (e_.)*(x_.)\right)^{(n_.)}\right]*(b_.)\right)^{(p_.)}*(RfX_), x\_Symbol\right] \rightarrow \operatorname{With}\left[\{u = \operatorname{ExpandIntegrand}\left[\left(a + b*\operatorname{Log}\left[c*(d + e*x)^n\right]\right)^p, RfX, x\right]\}, \operatorname{Int}\left[u, x\right] /; \operatorname{SumQ}\left[u\right] /; \operatorname{FreeQ}\left[\{a, b, c, d, e, n\}, x\right] \ \&\& \operatorname{RationalFunctionQ}\left[RfX, x\right] \ \&\& \operatorname{IntegerQ}\left[p\right]\right]$

#### Rule 2390

$\operatorname{Int}\left[\left((a_.) + \operatorname{Log}\left[(c_.)*\left((d_.) + (e_.)*(x_.)\right)^{(n_.)}\right]*(b_.)\right)^{(p_.)}*\left((f_.) + (g_.)*(x_.)\right)^{(q_.)}, x\_Symbol\right] \rightarrow \operatorname{Dist}\left[1/e, \operatorname{Subst}\left[\operatorname{Int}\left[\left((f*x)/d\right)^q*\left(a + b*\operatorname{Log}\left[c*x^n\right]\right)^p, x\right], x, d + e*x\right], x\right] /; \operatorname{FreeQ}\left[\{a, b, c, d, e, f, g, n, p, q\}, x\right] \ \&\& \operatorname{EqQ}\left[e*f - d*g, 0\right]$

#### Rule 12

$\operatorname{Int}\left[(a_)*(u_), x\_Symbol\right] \rightarrow \operatorname{Dist}\left[a, \operatorname{Int}\left[u, x\right], x\right] /; \operatorname{FreeQ}\left[a, x\right] \ \&\& \operatorname{!MatchQ}\left[u, (b_)*(v_)\right] /; \operatorname{FreeQ}\left[b, x\right]$

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e^n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
 \int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ag + bgx} dx &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(ag + bgx)}{bg} - \frac{(Bn) \int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2} + \frac{b}{c+dx}\right) \log(ag+bgx)}{a+bx} dx}{bg} \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(ag + bgx)}{bg} - \frac{(Bn) \int \left(\frac{b \log(ag+bgx)}{a+bx} - \frac{d \log(ag+bgx)}{c+dx}\right) dx}{bg} \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(ag + bgx)}{bg} - \frac{(Bn) \int \frac{\log(ag+bgx)}{a+bx} dx}{g} + \frac{(Bdn) \int \frac{\log(ag+bgx)}{c+dx} dx}{bg} \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(ag + bgx)}{bg} + \frac{Bn \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} - (Bn) \int \frac{\log(ag+bgx)}{ag+bgx} dx \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(ag + bgx)}{bg} + \frac{Bn \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} - \frac{(Bn) \text{Subst}\left[\int \frac{\log(ag+bgx)}{ag+bgx} dx, x, a+bx\right]}{bg} \\
 &= -\frac{Bn \log^2(g(a + bx))}{2bg} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(ag + bgx)}{bg} + \frac{Bn \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag+bgx)}{bg}
 \end{aligned}$$

**Mathematica [A]** time = 0.0535529, size = 101, normalized size = 1.2

$$\frac{2Bn \text{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right) + \log(g(a + bx)) \left(2 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + Bn \log\left(\frac{b(c+dx)}{bc-ad}\right) + A\right) - Bn \log(g(a + bx))\right)}{2bg}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(a\*g + b\*g\*x), x]

[Out] (Log[g\*(a + b\*x)]\*(-(B\*n\*Log[g\*(a + b\*x)]) + 2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n] + B\*n\*Log[(b\*(c + d\*x))/(b\*c - a\*d]))) + 2\*B\*n\*PolyLog[2, (d\*(a +

$b*x))/(-(b*c) + a*d)]/(2*b*g)$

**Maple [F]** time = 0.529, size = 0, normalized size = 0.

$$\int \frac{1}{bgx + ag} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g), x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$B \left( \frac{\log(bx + a) \log((bx + a)^n) - \log(bx + a) \log((dx + c)^n)}{bg} + \int \frac{bdx \log(e) + bc \log(e) - (bcn - adn) \log(bx + a)}{b^2d gx^2 + abcg + (b^2cg + abd g)x} dx \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g), x, algorithm="maxima")

[Out] B\*((log(b\*x + a)\*log((b\*x + a)^n) - log(b\*x + a)\*log((d\*x + c)^n))/(b\*g) + integrate((b\*d\*x\*log(e) + b\*c\*log(e) - (b\*c\*n - a\*d\*n)\*log(b\*x + a))/(b^2\*d\*g\*x^2 + a\*b\*c\*g + (b^2\*c\*g + a\*b\*d\*g)\*x), x)) + A\*log(b\*g\*x + a\*g)/(b\*g)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A}{bgx + ag}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g), x, algorithm="fricas")

[Out] integral((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)/(b\*g\*x + a\*g), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c)\*\*n))/(b\*g\*x+a\*g), x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{bgx + ag} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g),x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)/(b\*g\*x + a\*g), x)

$$3.6 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2} dx$$

**Optimal.** Leaf size=67

$$-\frac{(c+dx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2(a+bx)(bc-ad)} - \frac{Bn}{bg^2(a+bx)}$$

[Out]  $-\left(\frac{Bn}{bg^2(a+bx)}\right) - \left(\frac{(c+dx)(A+B \log[e((a+bx)/(c+dx))^n])}{(bc-ad)g^2(a+bx)}\right)$

**Rubi [A]** time = 0.0904321, antiderivative size = 108, normalized size of antiderivative = 1.61, number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{bg^2(a+bx)} - \frac{Bdn \log(a+bx)}{bg^2(bc-ad)} + \frac{Bdn \log(c+dx)}{bg^2(bc-ad)} - \frac{Bn}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(a\*g + b\*g\*x)^2, x]

[Out]  $-\left(\frac{Bn}{bg^2(a+bx)}\right) - \left(\frac{Bdn \log(a+bx)}{bg^2(bc-ad)}\right) - \left(\frac{Bdn \log(c+dx)}{bg^2(bc-ad)}\right) - \left(\frac{A+B \log[e((a+bx)/(c+dx))^n]}{bg^2(a+bx)}\right)$

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^2} dx &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg^2(a + bx)} + \frac{(Bn) \int \frac{bc-ad}{g(a+bx)^2(c+dx)} dx}{bg} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg^2(a + bx)} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)^2(c+dx)} dx}{bg^2} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg^2(a + bx)} + \frac{(B(bc - ad)n) \int \left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)} + \frac{d^2}{(bc-ad)^2(c+dx)}\right) dx}{bg^2} \\
&= -\frac{Bn}{bg^2(a + bx)} - \frac{Bdn \log(a + bx)}{b(bc - ad)g^2} - \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{bg^2(a + bx)} + \frac{Bdn \log(c + dx)}{b(bc - ad)g^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0633145, size = 115, normalized size = 1.72

$$\frac{Bn(bc - ad) \left( -\frac{1}{(a+bx)(bc-ad)} - \frac{d \log(a+bx)}{(bc-ad)^2} + \frac{d \log(c+dx)}{(bc-ad)^2} \right) - \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{bg(ag + bgx)}}{bg^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(a\*g + b\*g\*x)^2,x]

[Out] -((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(b\*g\*(a\*g + b\*g\*x))) + (B\*(b\*c - a\*d)\*n\*(-(1/((b\*c - a\*d)\*(a + b\*x))) - (d\*Log[a + b\*x])/(b\*c - a\*d)^2 + (d\*Log[c + d\*x])/(b\*c - a\*d)^2))/(b\*g^2)

**Maple [F]** time = 0.449, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2} \left( A + B \ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g)^2,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g)^2,x)

**Maxima [B]** time = 1.21445, size = 185, normalized size = 2.76

$$-Bn \left( \frac{1}{b^2g^2x + abg^2} + \frac{d \log(bx + a)}{(b^2c - abd)g^2} - \frac{d \log(dx + c)}{(b^2c - abd)g^2} \right) - \frac{B \log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{b^2g^2x + abg^2} - \frac{A}{b^2g^2x + abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g)^2,x, algorithm="maxima")

[Out]  $-B*n*(1/(b^2*g^2*x + a*b*g^2) + d*\log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*\log(d*x + c)/((b^2*c - a*b*d)*g^2)) - B*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^2*g^2*x + a*b*g^2) - A/(b^2*g^2*x + a*b*g^2)$

**Fricas [A]** time = 0.796695, size = 221, normalized size = 3.3

$$\frac{Abc - Aad + (Bbc - Bad)n + (Bbc - Bad)\log(e) + (Bbdnx + Bbcn)\log\left(\frac{bx+a}{dx+c}\right)}{(b^3c - ab^2d)g^2x + (ab^2c - a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="fricas")`

[Out]  $-(A*b*c - A*a*d + (B*b*c - B*a*d)*n + (B*b*c - B*a*d)*\log(e) + (B*b*d*n*x + B*b*c*n)*\log((b*x + a)/(d*x + c)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)**2,x)`

[Out] Timed out

**Giac [A]** time = 1.42295, size = 162, normalized size = 2.42

$$\frac{Bdn \log(bx + a)}{b^2cg^2 - abdg^2} + \frac{Bdn \log(dx + c)}{b^2cg^2 - abdg^2} - \frac{Bn \log\left(\frac{bx+a}{dx+c}\right)}{b^2g^2x + abg^2} - \frac{Bn + A + B}{b^2g^2x + abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, algorithm="giac")`

[Out]  $-B*d*n*\log(b*x + a)/(b^2*c*g^2 - a*b*d*g^2) + B*d*n*\log(d*x + c)/(b^2*c*g^2 - a*b*d*g^2) - B*n*\log((b*x + a)/(d*x + c))/(b^2*g^2*x + a*b*g^2) - (B*n + A + B)/(b^2*g^2*x + a*b*g^2)$



$$3.7 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3} dx$$

**Optimal.** Leaf size=151

$$-\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{2bg^3(a+bx)^2} + \frac{Bd^2n \log(a+bx)}{2bg^3(bc-ad)^2} - \frac{Bd^2n \log(c+dx)}{2bg^3(bc-ad)^2} + \frac{Bdn}{2bg^3(a+bx)(bc-ad)} - \frac{Bn}{4bg^3(a+bx)^2}$$

[Out]  $-(B*n)/(4*b*g^3*(a+b*x)^2) + (B*d*n)/(2*b*(b*c-a*d)*g^3*(a+b*x)) + (B*d^2*n*Log[a+b*x])/(2*b*(b*c-a*d)^2*g^3) - (A+B*Log[e*((a+b*x)/(c+d*x))^n])/(2*b*g^3*(a+b*x)^2) - (B*d^2*n*Log[c+d*x])/(2*b*(b*c-a*d)^2*g^3)$

**Rubi [A]** time = 0.122887, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{2bg^3(a+bx)^2} + \frac{Bd^2n \log(a+bx)}{2bg^3(bc-ad)^2} - \frac{Bd^2n \log(c+dx)}{2bg^3(bc-ad)^2} + \frac{Bdn}{2bg^3(a+bx)(bc-ad)} - \frac{Bn}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(a\*g + b\*g\*x)^3, x]

[Out]  $-(B*n)/(4*b*g^3*(a+b*x)^2) + (B*d*n)/(2*b*(b*c-a*d)*g^3*(a+b*x)) + (B*d^2*n*Log[a+b*x])/(2*b*(b*c-a*d)^2*g^3) - (A+B*Log[e*((a+b*x)/(c+d*x))^n])/(2*b*g^3*(a+b*x)^2) - (B*d^2*n*Log[c+d*x])/(2*b*(b*c-a*d)^2*g^3)$

#### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^3} dx &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a + bx)^2} + \frac{(Bn) \int \frac{bc-ad}{g^2(a+bx)^3(c+dx)} dx}{2bg} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a + bx)^2} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)^3(c+dx)} dx}{2bg^3} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a + bx)^2} + \frac{(B(bc - ad)n) \int \left(\frac{b}{(bc-ad)(a+bx)^3} - \frac{bd}{(bc-ad)^2(a+bx)^2} + \frac{bd^2}{(bc-ad)^3(a+bx)} - \frac{d^3}{(bc-ad)^4}\right) dx}{2bg^3} \\
&= -\frac{Bn}{4bg^3(a + bx)^2} + \frac{Bdn}{2b(bc - ad)g^3(a + bx)} + \frac{Bd^2n \log(a + bx)}{2b(bc - ad)^2g^3} - \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2bg^3(a + bx)^2} - \frac{Bd^3}{2b(bc - ad)^4}
\end{aligned}$$

**Mathematica [A]** time = 0.162042, size = 114, normalized size = 0.75

$$\frac{2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right) + \frac{Bn(2d^2(a+bx)^2 \log(c+dx) + (bc-ad)(b(c-2dx) - 3ad) - 2d^2(a+bx)^2 \log(a+bx))}{(bc-ad)^2}}{4bg^3(a + bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(a\*g + b\*g\*x)^3, x]

[Out] -(2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + (B\*n\*((b\*c - a\*d)\*(-3\*a\*d + b\*(c - 2\*d\*x)) - 2\*d^2\*(a + b\*x)^2\*Log[a + b\*x] + 2\*d^2\*(a + b\*x)^2\*Log[c + d\*x]))/(b\*c - a\*d)^2/(4\*b\*g^3\*(a + b\*x)^2)

**Maple [F]** time = 0.436, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3} \left( A + B \ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g)^3, x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g)^3, x)

**Maxima [A]** time = 1.21237, size = 350, normalized size = 2.32

$$\frac{1}{4} Bn \left( \frac{2 bdx - bc + 3 ad}{(b^4c - ab^3d)g^3x^2 + 2(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{2 d^2 \log(bx + a)}{(b^3c^2 - 2 ab^2cd + a^2bd^2)g^3} - \frac{2 d^2 \log(dx + c)}{(b^3c^2 - 2 ab^2cd + a^2bd^2)g^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g)^3, x, algorithm="maxima")

```
[Out] 1/4*B*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

**Fricas [A]** time = 0.991217, size = 562, normalized size = 3.72

$$\frac{2Ab^2c^2 - 4Aabcd + 2Aa^2d^2 - 2(Bb^2cd - Babd^2)nx + (Bb^2c^2 - 4Babcd + 3Ba^2d^2)n + 2(Bb^2c^2 - 2Babcd + Ba^2d^2)}{4((b^5c^2 - 2ab^4cd + a^2b^3d^2)g^3x^2 + 2(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2)g^3x + (a^2b^4c^2 - 2a^3b^3cd + a^4b^2d^2)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*n*x + (B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2)*n + 2*(B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*log(e) - 2*(B*b^2*d^2*n*x^2 + 2*B*a*b*d^2*n*x - (B*b^2*c^2 - 2*B*a*b*c*d)*n)*log((b*x + a)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)**3,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 1.57075, size = 351, normalized size = 2.32

$$\frac{Bd^2n \log(bx + a)}{2(b^3c^2g^3 - 2ab^2cdg^3 + a^2bd^2g^3)} - \frac{Bd^2n \log(dx + c)}{2(b^3c^2g^3 - 2ab^2cdg^3 + a^2bd^2g^3)} - \frac{Bn \log\left(\frac{bx+a}{dx+c}\right)}{2(b^3g^3x^2 + 2ab^2g^3x + a^2bg^3)} + \frac{2Bbdn}{4(b^4cg^3x^2 - a^2b^3cg^3x + a^3bdg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*d^2*n*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*B*d^2*n*log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*B*n*log((b*x + a)/(d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 1/4*(2*B*b*d*n*x - B*b*c*n + 3*B*a*d*n - 2*A*b*c - 2*B*b*c + 2*A*a*d + 2*B*a*d)/(b^4*c*g^3*x^2 - a*b^3*d*g^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3*x + a^2*b^2*c*g^3 - a^3*b*d*g^3)
```

$$3.8 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4} dx$$

**Optimal.** Leaf size=183

$$\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{3bg^4(a+bx)^3} - \frac{Bd^2n}{3bg^4(a+bx)(bc-ad)^2} - \frac{Bd^3n \log(a+bx)}{3bg^4(bc-ad)^3} + \frac{Bd^3n \log(c+dx)}{3bg^4(bc-ad)^3} + \frac{Bdn}{6bg^4(a+bx)^2(bc-ad)} - \frac{9b}{9b}$$

[Out]  $-(B*n)/(9*b*g^4*(a+b*x)^3) + (B*d*n)/(6*b*(b*c-a*d)*g^4*(a+b*x)^2) - (B*d^2*n)/(3*b*(b*c-a*d)^2*g^4*(a+b*x)) - (B*d^3*n*Log[a+b*x])/(3*b*(b*c-a*d)^3*g^4) - (A+B*Log[e*((a+b*x)/(c+d*x))^n])/(3*b*g^4*(a+b*x)^3) + (B*d^3*n*Log[c+d*x])/(3*b*(b*c-a*d)^3*g^4)$

**Rubi [A]** time = 0.151906, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 44}

$$\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{3bg^4(a+bx)^3} - \frac{Bd^2n}{3bg^4(a+bx)(bc-ad)^2} - \frac{Bd^3n \log(a+bx)}{3bg^4(bc-ad)^3} + \frac{Bd^3n \log(c+dx)}{3bg^4(bc-ad)^3} + \frac{Bdn}{6bg^4(a+bx)^2(bc-ad)} - \frac{9b}{9b}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(a\*g + b\*g\*x)^4, x]

[Out]  $-(B*n)/(9*b*g^4*(a+b*x)^3) + (B*d*n)/(6*b*(b*c-a*d)*g^4*(a+b*x)^2) - (B*d^2*n)/(3*b*(b*c-a*d)^2*g^4*(a+b*x)) - (B*d^3*n*Log[a+b*x])/(3*b*(b*c-a*d)^3*g^4) - (A+B*Log[e*((a+b*x)/(c+d*x))^n])/(3*b*g^4*(a+b*x)^3) + (B*d^3*n*Log[c+d*x])/(3*b*(b*c-a*d)^3*g^4)$

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol]
:> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 44

```
Int[((a_) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^4} dx &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a + bx)^3} + \frac{(Bn) \int \frac{bc-ad}{g^3(a+bx)^4(c+dx)} dx}{3bg} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a + bx)^3} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)^4(c+dx)} dx}{3bg^4} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bg^4(a + bx)^3} + \frac{(B(bc - ad)n) \int \left(\frac{b}{(bc-ad)(a+bx)^4} - \frac{bd}{(bc-ad)^2(a+bx)^3} + \frac{bd^2}{(bc-ad)^3(a+bx)^2}\right) dx}{3bg^4} \\
&= -\frac{Bn}{9bg^4(a + bx)^3} + \frac{Bdn}{6b(bc - ad)g^4(a + bx)^2} - \frac{Bd^2n}{3b(bc - ad)^2g^4(a + bx)} - \frac{Bd^3n \log(a + bx)}{3b(bc - ad)^3g^4}
\end{aligned}$$

**Mathematica [A]** time = 0.184807, size = 145, normalized size = 0.79

$$\frac{Bn((bc-ad)(11a^2d^2+abd(15dx-7c)+b^2(2c^2-3cdx+6d^2x^2))-6d^3(a+bx)^3 \log(c+dx)+6d^3(a+bx)^3 \log(a+bx))}{(bc-ad)^3} + 6 \left( B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A \right)$$


---


$$18bg^4(a + bx)^3$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(a\*g + b\*g\*x)^4, x]

[Out] -(6\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + (B\*n\*((b\*c - a\*d)\*(11\*a^2\*d^2 + a\*b\*d\*(-7\*c + 15\*d\*x) + b^2\*(2\*c^2 - 3\*c\*d\*x + 6\*d^2\*x^2)) + 6\*d^3\*(a + b\*x)^3\*Log[a + b\*x] - 6\*d^3\*(a + b\*x)^3\*Log[c + d\*x]))/(b\*c - a\*d)^3)/(18\*b\*g^4\*(a + b\*x)^3)

**Maple [F]** time = 0.437, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^4} \left( A + B \ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g)^4, x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g)^4, x)

**Maxima [B]** time = 1.26343, size = 583, normalized size = 3.19

$$-\frac{1}{18} Bn \left( \frac{6b^2d^2x^2 + 2b^2c^2 - 7abcd + 11a^2d^2 - 3(b^2cd - 5abd^2)x}{(b^6c^2 - 2ab^5cd + a^2b^4d^2)g^4x^3 + 3(ab^5c^2 - 2a^2b^4cd + a^3b^3d^2)g^4x^2 + 3(a^2b^4c^2 - 2a^3b^3cd + a^4b^2d^2)g^4x + (a^5c^2 - 2a^4b^3cd + a^5b^2d^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g)^4, x, algorithm="maxima")

```
[Out] -1/18*B*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

**Fricas [B]** time = 0.879277, size = 990, normalized size = 5.41

$$\frac{6Ab^3c^3 - 18Aab^2c^2d + 18Aa^2bcd^2 - 6Aa^3d^3 + 6(Bb^3cd^2 - Bab^2d^3)nx^2 - 3(Bb^3c^2d - 6Bab^2cd^2 + 5Ba^2bd^3)nx + (2Bb^3c^3 - 3Aa^2b^2c^2d + 3Aa^3b^2d^2)g^4x^3 + 3(ab^6c^3 - 3Aa^2b^2c^2d + 3Aa^3b^2d^2)g^4x^2 + 3Aa^2b^2c^2d^2 - a^3b^2d^3}{18((b^7c^3 - 3ab^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3)g^4x^3 + 3(ab^6c^3 - 3Aa^2b^2c^2d + 3Aa^3b^2d^2)g^4x^2 + 3Aa^2b^2c^2d^2 - a^3b^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="fricas")
```

```
[Out] -1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*n*x + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 - 11*B*a^3*d^3)*n + 6*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 - B*a^3*d^3)*log(e) + 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log((b*x + a)/(d*x + c)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)**4,x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.36737, size = 655, normalized size = 3.58

$$\frac{Bd^3n \log(bx + a)}{3(b^4c^3g^4 - 3ab^3c^2dg^4 + 3a^2b^2cd^2g^4 - a^3bd^3g^4)} + \frac{Bd^3n \log(dx + c)}{3(b^4c^3g^4 - 3ab^3c^2dg^4 + 3a^2b^2cd^2g^4 - a^3bd^3g^4)} - \frac{B}{3(b^4g^4x^3 + 3ab^3g^4x^2 + 3a^2b^2g^4x + a^3bg^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/3*B*d^3*n*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) + 1/3*B*d^3*n*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*n*log((b*x + a)/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/18*(6*B*b^2*d^2*n*x^2 - 3*B*b^2*c*d*n*x + 15*B*a*b*d^2*n*x + 2*B*b^2*c^2*n - 7*B*a*b*c*d*n + 11*B*a^2*d^2*n + 6*A*b^2*c^2 + 6*B*b^2*c^2 - 12*A*a*b*c*d - 12*B*a*b*c*d + 6*A*a^2*d^2 + 6*B*a^2*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d^2*g^4*x^3 + 3*a*b^5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g^4*x^2 + 3*a^2*b^4*c^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x + a^3*b^3*c^2*g^4 - 2*a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

$$3.9 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^5} dx$$

**Optimal.** Leaf size=215

$$-\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{4bg^5(a+bx)^4} + \frac{Bd^3n}{4bg^5(a+bx)(bc-ad)^3} - \frac{Bd^2n}{8bg^5(a+bx)^2(bc-ad)^2} + \frac{Bd^4n \log(a+bx)}{4bg^5(bc-ad)^4} - \frac{Bd^4n \log(c+dx)}{4bg^5(bc-ad)^4} + \frac{1}{1}$$

[Out]  $-(B*n)/(16*b*g^5*(a+b*x)^4) + (B*d*n)/(12*b*(b*c-a*d)*g^5*(a+b*x)^3) - (B*d^2*n)/(8*b*(b*c-a*d)^2*g^5*(a+b*x)^2) + (B*d^3*n)/(4*b*(b*c-a*d)^3*g^5*(a+b*x)) + (B*d^4*n*Log[a+b*x])/(4*b*(b*c-a*d)^4*g^5) - (A+B*Log[e*((a+b*x)/(c+d*x))^n])/(4*b*g^5*(a+b*x)^4) - (B*d^4*n*Log[c+d*x])/(4*b*(b*c-a*d)^4*g^5)$

**Rubi [A]** time = 0.189061, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{4bg^5(a+bx)^4} + \frac{Bd^3n}{4bg^5(a+bx)(bc-ad)^3} - \frac{Bd^2n}{8bg^5(a+bx)^2(bc-ad)^2} + \frac{Bd^4n \log(a+bx)}{4bg^5(bc-ad)^4} - \frac{Bd^4n \log(c+dx)}{4bg^5(bc-ad)^4} + \frac{1}{1}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])/(a*g + b*g*x)^5, x]$

[Out]  $-(B*n)/(16*b*g^5*(a+b*x)^4) + (B*d*n)/(12*b*(b*c-a*d)*g^5*(a+b*x)^3) - (B*d^2*n)/(8*b*(b*c-a*d)^2*g^5*(a+b*x)^2) + (B*d^3*n)/(4*b*(b*c-a*d)^3*g^5*(a+b*x)) + (B*d^4*n*Log[a+b*x])/(4*b*(b*c-a*d)^4*g^5) - (A+B*Log[e*((a+b*x)/(c+d*x))^n])/(4*b*g^5*(a+b*x)^4) - (B*d^4*n*Log[c+d*x])/(4*b*(b*c-a*d)^4*g^5)$

### Rule 2525

$\text{Int}[(a + \text{Log}[c * \text{RFX}^p]) * (b + (d + e * x)^m)^n, x\_Symbol] \rightarrow \text{Simp}[(d + e * x)^{m+1} * (a + b * \text{Log}[c * \text{RFX}^p])^n / (e^{m+1}), x] - \text{Dist}[(b * n * p) / (e^{m+1}), \text{Int}[\text{SimplifyIntegrand}[(d + e * x)^{m+1} * (a + b * \text{Log}[c * \text{RFX}^p])^{n-1} * D[\text{RFX}, x]) / \text{RFX}, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

$\text{Int}[(a + (b + (c + d * x)^n)^m), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b + (c + d \* x)^n) /; FreeQ[b, x]]

### Rule 44

$\text{Int}[(a + (b + (c + d * x)^n)^m), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b * x)^m * (c + d * x)^n, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b \* c - a \* d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rubi steps



$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^5} dx &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a + bx)^4} + \frac{(Bn) \int \frac{bc-ad}{g^4(a+bx)^5(c+dx)} dx}{4bg} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a + bx)^4} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)^5(c+dx)} dx}{4bg^5} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4bg^5(a + bx)^4} + \frac{(B(bc - ad)n) \int \left(\frac{b}{(bc-ad)(a+bx)^5} - \frac{bd}{(bc-ad)^2(a+bx)^4} + \frac{bd^2}{(bc-ad)^3(a+bx)^3}\right) dx}{4bg^5} \\
&= -\frac{Bn}{16bg^5(a + bx)^4} + \frac{Bdn}{12b(bc - ad)g^5(a + bx)^3} - \frac{Bd^2n}{8b(bc - ad)^2g^5(a + bx)^2} + \frac{Bd^3n}{4b(bc - ad)^3g^5}
\end{aligned}$$

**Mathematica [A]** time = 0.259689, size = 162, normalized size = 0.75

$$\frac{Bn\left(\frac{12d^3(bc-ad)}{a+bx} - \frac{6d^2(bc-ad)^2}{(a+bx)^2} + \frac{4d(bc-ad)^3}{(a+bx)^3} - \frac{3(bc-ad)^4}{(a+bx)^4} + 12d^4 \log(a+bx) - 12d^4 \log(c+dx)\right) - \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(a+bx)^4}}{4bg^5}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(a\*g + b\*g\*x)^5,x]

[Out] (-((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(a + b\*x)^4) + (B\*n\*((-3\*(b\*c - a\*d)^4)/(a + b\*x)^4 + (4\*d\*(b\*c - a\*d)^3)/(a + b\*x)^3 - (6\*d^2\*(b\*c - a\*d)^2)/(a + b\*x)^2 + (12\*d^3\*(b\*c - a\*d))/(a + b\*x) + 12\*d^4\*Log[a + b\*x] - 12\*d^4\*Log[c + d\*x]))/(12\*(b\*c - a\*d)^4))/(4\*b\*g^5)

**Maple [F]** time = 0.437, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^5} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g)^5,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g)^5,x)

**Maxima [B]** time = 1.34799, size = 879, normalized size = 4.09

$$\frac{1}{48} Bn \left( \frac{12b^3d^3x^3 - 3b^3c^3 + 13ab^2c^2d - 23a^2bcd^2 + 2a^3c^3}{(b^8c^3 - 3ab^7c^2d + 3a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4(ab^7c^3 - 3a^2b^6c^2d + 3a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6(a^2b^6c^3 - 3a^3b^5cd^2 - a^4b^4d^3)g^5x^2 + \dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g)^5,x, algorithm="maxima")

```
[Out] 1/48*B*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 2
5*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2
+ 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^
3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3
)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^
3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d
^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g
^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4
*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*
c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*B*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b
^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^
5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

---

**Fricas [B]** time = 1.06627, size = 1505, normalized size = 7.

$$\frac{12 Ab^4c^4 - 48 Aab^3c^3d + 72 Aa^2b^2c^2d^2 - 48 Aa^3bcd^3 + 12 Aa^4d^4 - 12 (Bb^4cd^3 - Bab^3d^4)nx^3 + 6 (Bb^4c^2d^2 - 8 Bab^3cd^3 + 48 (b^9c^4 - 4ab^8c^3d + 6a^2b^7c^2d^2 - 4a^3b^6cd^3 + 4a^4b^5d^4)g^5)}{48 (b^9c^4 - 4ab^8c^3d + 6a^2b^7c^2d^2 - 4a^3b^6cd^3 + 4a^4b^5d^4)g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b*
c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 + 6*(B*b^4*c^2*
d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*n*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3
*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*n*x + (3*B*b^4*c^4 - 16*B*a
*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 48*B*a^3*b*c*d^3 + 25*B*a^4*d^4)*n + 12
*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B*a
^4*d^4)*log(e) - 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^
4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^
2*d^2 - 4*B*a^3*b*c*d^3)*n)*log((b*x + a)/(d*x + c)))/((b^9*c^4 - 4*a*b^8*c
^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^
8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4
)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^
4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b
^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^
4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)**5,x)
```

```
[Out] Timed out
```

---

**Giac [B]** time = 1.42214, size = 1038, normalized size = 4.83

$$\frac{Bd^4n \log(bx + a)}{4(b^5c^4g^5 - 4ab^4c^3dg^5 + 6a^2b^3c^2d^2g^5 - 4a^3b^2cd^3g^5 + a^4bd^4g^5)} - \frac{Bd^4n \log(dx + c)}{4(b^5c^4g^5 - 4ab^4c^3dg^5 + 6a^2b^3c^2d^2g^5 - 4a^3b^2cd^3g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(b\*g\*x+a\*g)^5,x, algorithm="giac")

[Out]  $\frac{1}{4}Bd^4n \log(bx + a) / (b^5c^4g^5 - 4a^3b^2c^3d^3g^5 + 6a^2b^3c^2d^2g^5 - 4a^4b^4c^3dg^5 + a^4bd^4g^5) - \frac{1}{4}Bd^4n \log(dx + c) / (b^5c^4g^5 - 4a^3b^2c^3d^3g^5 + 6a^2b^3c^2d^2g^5 - 4a^4b^4c^3dg^5 + a^4bd^4g^5) - \frac{1}{4}Bn \log((bx + a)/(dx + c)) / (b^5g^5x^4 + 4a^3b^2c^3d^3g^5x^3 + 6a^2b^3c^2d^2g^5x^2 + 4a^4b^4c^3dg^5x + a^4bd^4g^5) + \frac{1}{48} * (12Bb^3d^3n^3x^3 - 6Bb^3c^3d^2n^2x^2 + 42Bab^2d^3n^2x^2 + 4Bb^3c^2d^3n^2x - 20Bab^2c^2d^2n^2x + 52Ba^2b^3d^3n^2x - 3Bb^3c^3n^2 + 13Bab^2c^2d^2n - 23Ba^2b^3c^2d^2n + 25Ba^3d^3n - 12Ab^3c^3 - 12Bb^3c^3 + 36Aab^2c^2d + 36Bab^2c^2d - 36Aa^2b^3c^2d - 36Ba^2b^3c^2d^2 + 12Aa^3d^3 + 12Ba^3d^3) / (b^8c^3g^5x^4 - 3a^3b^7c^2d^3g^5x^4 + 3a^2b^6c^2d^2g^5x^4 - a^3b^5d^3g^5x^4 + 4a^4b^7c^3g^5x^3 - 12a^2b^6c^2d^2g^5x^3 + 12a^3b^5c^2d^2g^5x^3 - 4a^4b^4d^3g^5x^3 + 6a^2b^6c^3g^5x^2 - 18a^3b^5c^2d^2g^5x^2 + 18a^4b^4c^2d^2g^5x^2 - 6a^5b^3d^3g^5x^2 + 4a^3b^5c^3g^5x - 12a^4b^4c^2d^2g^5x + 12a^5b^3c^2d^2g^5x - 4a^6b^2d^3g^5x + a^4b^4c^3g^5 - 3a^5b^3c^2d^2g^5 + 3a^6b^2c^2d^2g^5 - a^7b^3d^3g^5)$

$$3.10 \quad \int (ag + bgx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx$$

**Optimal.** Leaf size=396

$$\frac{2B^2g^4n^2(bc-ad)^5 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} + \frac{Bg^4n(bc-ad)^5 \log\left(\frac{bc-ad}{b(c+dx)}\right) \left(12B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + 12A + 25Bn\right)}{30bd^5} + \frac{Bg^4n(a+bx)^2(bc-ad)^5}{5bd^5}$$

[Out]  $-(B*(b*c - a*d)*g^4*n*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(10*b*d) + (g^4*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(5*b) + (B*(b*c - a*d)^2*g^4*n*(a + b*x)^3*(4*A + B*n + 4*B*Log[e*((a + b*x)/(c + d*x))^n]))/(30*b*d^2) - (B*(b*c - a*d)^3*g^4*n*(a + b*x)^2*(12*A + 7*B*n + 12*B*Log[e*((a + b*x)/(c + d*x))^n]))/(60*b*d^3) + (B*(b*c - a*d)^4*g^4*n*(a + b*x)*(12*A + 13*B*n + 12*B*Log[e*((a + b*x)/(c + d*x))^n]))/(30*b*d^4) + (B*(b*c - a*d)^5*g^4*n*(12*A + 25*B*n + 12*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(30*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(5*b*d^5)$

**Rubi [A]** time = 0.87062, antiderivative size = 602, normalized size of antiderivative = 1.52, number of steps used = 27, number of rules used = 13, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{2B^2g^4n^2(bc-ad)^5 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{5bd^5} - \frac{2Bg^4n(bc-ad)^5 \log(c+dx) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{5bd^5} - \frac{Bg^4n(a+bx)^2(bc-ad)^5}{5bd^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]$

[Out]  $(2*A*B*(b*c - a*d)^4*g^4*n*x)/(5*d^4) + (13*B^2*(b*c - a*d)^4*g^4*n^2*x)/(30*d^4) - (7*B^2*(b*c - a*d)^3*g^4*n^2*(a + b*x)^2)/(60*b*d^3) + (B^2*(b*c - a*d)^2*g^4*n^2*(a + b*x)^3)/(30*b*d^2) + (2*B^2*(b*c - a*d)^4*g^4*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(5*b*d^4) - (B*(b*c - a*d)^3*g^4*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b*d^3) + (2*B*(b*c - a*d)^2*g^4*n*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(15*b*d^2) - (B*(b*c - a*d)*g^4*n*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(10*b*d) + (g^4*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(5*b) - (5*B^2*(b*c - a*d)^5*g^4*n^2*Log[c + d*x])/(6*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(5*b*d^5) - (2*B*(b*c - a*d)^5*g^4*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x])/(5*b*d^5) - (B^2*(b*c - a*d)^5*g^4*n^2*Log[c + d*x]^2)/(5*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(5*b*d^5)$

**Rule 2525**

$\text{Int}[(a_.) + \text{Log}[(c_.)*(RFX_)^(p_.)]*(b_.)]^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^n/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(Rfx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rubi steps

$$\begin{aligned}
 \int (ag + bgx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx &= \frac{g^4(a+bx)^5 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{5b} - \frac{(2Bn) \int \frac{(bc-ad)g^5(a+bx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{c+dx}}{5bg} \\
 &= \frac{g^4(a+bx)^5 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{5b} - \frac{(2B(bc-ad)g^4n) \int \frac{(a+bx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{c+dx}}{5b} \\
 &= \frac{g^4(a+bx)^5 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{5b} - \frac{(2B(bc-ad)g^4n) \int \left( -\frac{b(bc-ad)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{c+dx} \right)}{5b} \\
 &= \frac{g^4(a+bx)^5 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{5b} - \frac{(2B(bc-ad)g^4n) \int (a+bx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{5d} \\
 &= \frac{2AB(bc-ad)^4g^4nx}{5d^4} - \frac{B(bc-ad)^3g^4n(a+bx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{5bd^3} \\
 &= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{2B^2(bc-ad)^4g^4n(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{5bd^4} - \frac{B(bc-ad)^3g^4n^2(a+bx)}{60bd^3} \\
 &= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{2B^2(bc-ad)^4g^4n(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{5bd^4} - \frac{B(bc-ad)^3g^4n^2(a+bx)}{60bd^3} \\
 &= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30d^4} - \frac{7B^2(bc-ad)^3g^4n^2(a+bx)}{60bd^3} \\
 &= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30d^4} - \frac{7B^2(bc-ad)^3g^4n^2(a+bx)}{60bd^3} \\
 &= \frac{2AB(bc-ad)^4g^4nx}{5d^4} + \frac{13B^2(bc-ad)^4g^4n^2x}{30d^4} - \frac{7B^2(bc-ad)^3g^4n^2(a+bx)}{60bd^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.524705, size = 535, normalized size = 1.35

$$8^4 \left( \frac{Bn(bc-ad) \left( 12Bn(bc-ad)^4 \left( 2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) - 12d^2(a+bx)^2(bc-ad)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) + 8d^3(a+bx)^3(bc-ad)}{\dots} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] (g^4\*((a + b\*x)^5\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 + (B\*(b\*c - a\*d)\*n\*(24\*A\*b\*d\*(b\*c - a\*d)^3\*x + 24\*B\*d\*(b\*c - a\*d)^3\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n] - 12\*d^2\*(b\*c - a\*d)^2\*(a + b\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 8\*d^3\*(b\*c - a\*d)\*(a + b\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 6\*d^4\*(a + b\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 24\*B\*(b\*c - a\*d)^4\*n\*Log[c + d\*x] - 24\*(b\*c - a\*d)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x] + 4\*B\*(b\*c - a\*d)^2\*n\*(2\*b\*d\*(b\*c - a\*d)\*x - d^2\*(a + b\*x)^2 - 2\*(b\*c - a\*d)^2\*Log[c + d\*x]) + B\*(b\*c - a\*d)\*n\*(6\*b\*d\*(b\*c - a\*d)^2\*x + 3\*d^2\*(-(b\*c) + a\*d)\*(a + b\*x)^2 + 2\*d^3\*(a + b\*x)^3 - 6\*(b\*c - a\*d)^3\*Log[c + d\*x]) + 12\*B\*(b\*c - a\*d)^3\*n\*(b\*d\*x + (-(b\*c) + a\*d)\*Log[c + d\*x]) + 12\*B\*(b\*c - a\*d)^4\*n\*((2\*Log[(d\*(a + b\*x))/(-(b\*c) + a\*d)] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/(12\*d^5))/(5\*b)

**Maple [F]** time = 0.427, size = 0, normalized size = 0.

$$\int (bgx + ag)^4 \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^4\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((b\*g\*x+a\*g)^4\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [B]** time = 3.82424, size = 3976, normalized size = 10.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out] 2/5\*A\*B\*b^4\*g^4\*x^5\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/5\*A^2\*b^4\*g^4\*x^5 + 2\*A\*B\*a\*b^3\*g^4\*x^4\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A^2\*a\*b^3\*g^4\*x^4 + 4\*A\*B\*a^2\*b^2\*g^4\*x^3\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 2\*A^2\*a^2\*b^2\*g^4\*x^3 + 4\*A\*B\*a^3\*b\*g^4\*x^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 2\*A^2\*a^3\*b\*g^4\*x^2 + 1/30\*A\*B\*b^4\*g^4\*n\*(12\*a^5\*log(b\*x + a)/b^5 - 12\*c^5\*log(d\*x + c)/d^5 - (3\*(b^4\*c\*d^3 - a\*b^3\*d^4)\*x^4 - 4\*(b^4\*c^2\*d^2 - a^2\*b^2\*d^4)\*x^3 + 6\*(b^4\*c^3\*d - a^3\*b\*d^4)\*x^2 - 12\*(b^4\*c^4 - a^4\*d^4)\*x)/(b^4\*d^4) - 1/3\*A\*B\*a\*b^3\*g^4\*n\*(6\*a^4\*log(b\*x + a)/b^4 - 6\*c^4\*log(d\*x + c)/d^4 + (2\*(b^3\*c\*d^2 - a\*b^2\*d^3)\*x^3 - 3\*(b^3\*c^2\*d - a^2\*b\*d^3)

$$\begin{aligned}
& *x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 2*A*B*a^2*b^2*g^4*n*(2*a^3*\log \\
& (b*x + a)/b^3 - 2*c^3*\log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2* \\
& c^2 - a^2*d^2)*x)/(b^2*d^2)) - 4*A*B*a^3*b*g^4*n*(a^2*\log(b*x + a)/b^2 - c^ \\
& 2*\log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^4*g^4*n*(a*\log(b*x + a) \\
& /b - c*\log(d*x + c)/d) + 2*A*B*a^4*g^4*x*\log(e*(b*x/(d*x + c) + a/(d*x + c) \\
& )^n) + A^2*a^4*g^4*x - 1/30*((25*g^4*n^2 + 12*g^4*n*\log(e))*b^4*c^5 - (113* \\
& g^4*n^2 + 60*g^4*n*\log(e))*a*b^3*c^4*d + 4*(49*g^4*n^2 + 30*g^4*n*\log(e))*a \\
& ^2*b^2*c^3*d^2 - 12*(13*g^4*n^2 + 10*g^4*n*\log(e))*a^3*b*c^2*d^3 + 12*(4*g^ \\
& 4*n^2 + 5*g^4*n*\log(e))*a^4*c*d^4)*B^2*\log(d*x + c)/d^5 - 2/5*(b^5*c^5*g^4* \\
& n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^2 - 10*a^3*b^2*c^2*d \\
& ^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2 - a^5*d^5*g^4*n^2)*(log(b*x + a)*log((b* \\
& d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^5 \\
& ) + 1/60*(12*B^2*b^5*d^5*g^4*x^5*\log(e)^2 - 12*B^2*a^5*d^5*g^4*n^2*\log(b*x \\
& + a)^2 - 6*(b^5*c*d^4*g^4*n*\log(e) - (g^4*n*\log(e) + 10*g^4*\log(e)^2)*a*b^4 \\
& *d^5)*B^2*x^4 + 2*((g^4*n^2 + 4*g^4*n*\log(e))*b^5*c^2*d^3 - 2*(g^4*n^2 + 10 \\
& *g^4*n*\log(e))*a*b^4*c*d^4 + (g^4*n^2 + 16*g^4*n*\log(e) + 60*g^4*\log(e)^2)* \\
& a^2*b^3*d^5)*B^2*x^3 - ((7*g^4*n^2 + 12*g^4*n*\log(e))*b^5*c^3*d^2 - 3*(9*g^ \\
& 4*n^2 + 20*g^4*n*\log(e))*a*b^4*c^2*d^3 + 3*(11*g^4*n^2 + 40*g^4*n*\log(e))*a \\
& ^2*b^3*c*d^4 - (13*g^4*n^2 + 72*g^4*n*\log(e) + 120*g^4*\log(e)^2)*a^3*b^2*d^ \\
& 5)*B^2*x^2 + 24*(b^5*c^5*g^4*n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d \\
& ^2*g^4*n^2 - 10*a^3*b^2*c^2*d^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2)*B^2*\log(b* \\
& x + a)*\log(d*x + c) - 12*(b^5*c^5*g^4*n^2 - 5*a*b^4*c^4*d*g^4*n^2 + 10*a^2* \\
& b^3*c^3*d^2*g^4*n^2 - 10*a^3*b^2*c^2*d^3*g^4*n^2 + 5*a^4*b*c*d^4*g^4*n^2)*B \\
& ^2*\log(d*x + c)^2 + 2*((13*g^4*n^2 + 12*g^4*n*\log(e))*b^5*c^4*d - (59*g^4*n \\
& ^2 + 60*g^4*n*\log(e))*a*b^4*c^3*d^2 + 6*(17*g^4*n^2 + 20*g^4*n*\log(e))*a^2* \\
& b^3*c^2*d^3 - (79*g^4*n^2 + 120*g^4*n*\log(e))*a^3*b^2*c*d^4 + (23*g^4*n^2 + \\
& 48*g^4*n*\log(e) + 30*g^4*\log(e)^2)*a^4*b*d^5)*B^2*x + 2*(12*a*b^4*c^4*d*g^ \\
& 4*n^2 - 54*a^2*b^3*c^3*d^2*g^4*n^2 + 94*a^3*b^2*c^2*d^3*g^4*n^2 - 77*a^4*b* \\
& c*d^4*g^4*n^2 + (25*g^4*n^2 + 12*g^4*n*\log(e))*a^5*d^5)*B^2*\log(b*x + a) + \\
& 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4* \\
& x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x)*\log((b*x + a)^n) \\
& ^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5* \\
& g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x)*\log((d*x + c) \\
& ^n)^2 + 2*(12*B^2*b^5*d^5*g^4*x^5*\log(e) + 12*B^2*a^5*d^5*g^4*n*\log(b*x + a \\
& ) - 3*(b^5*c*d^4*g^4*n - (g^4*n + 20*g^4*\log(e))*a*b^4*d^5)*B^2*x^4 + 4*(b^ \\
& 5*c^2*d^3*g^4*n - 5*a*b^4*c*d^4*g^4*n + 2*(2*g^4*n + 15*g^4*\log(e))*a^2*b^3 \\
& *d^5)*B^2*x^3 - 6*(b^5*c^3*d^2*g^4*n - 5*a*b^4*c^2*d^3*g^4*n + 10*a^2*b^3*c \\
& *d^4*g^4*n - 2*(3*g^4*n + 10*g^4*\log(e))*a^3*b^2*d^5)*B^2*x^2 + 12*(b^5*c^4 \\
& *d*g^4*n - 5*a*b^4*c^3*d^2*g^4*n + 10*a^2*b^3*c^2*d^3*g^4*n - 10*a^3*b^2*c* \\
& d^4*g^4*n + (4*g^4*n + 5*g^4*\log(e))*a^4*b*d^5)*B^2*x - 12*(b^5*c^5*g^4*n - \\
& 5*a*b^4*c^4*d*g^4*n + 10*a^2*b^3*c^3*d^2*g^4*n - 10*a^3*b^2*c^2*d^3*g^4*n \\
& + 5*a^4*b*c*d^4*g^4*n)*B^2*\log(d*x + c))*\log((b*x + a)^n) - 2*(12*B^2*b^5*d \\
& ^5*g^4*x^5*\log(e) + 12*B^2*a^5*d^5*g^4*n*\log(b*x + a) - 3*(b^5*c*d^4*g^4*n \\
& - (g^4*n + 20*g^4*\log(e))*a*b^4*d^5)*B^2*x^4 + 4*(b^5*c^2*d^3*g^4*n - 5*a*b \\
& ^4*c*d^4*g^4*n + 2*(2*g^4*n + 15*g^4*\log(e))*a^2*b^3*d^5)*B^2*x^3 - 6*(b^5* \\
& c^3*d^2*g^4*n - 5*a*b^4*c^2*d^3*g^4*n + 10*a^2*b^3*c*d^4*g^4*n - 2*(3*g^4*n \\
& + 10*g^4*\log(e))*a^3*b^2*d^5)*B^2*x^2 + 12*(b^5*c^4*d*g^4*n - 5*a*b^4*c^3* \\
& d^2*g^4*n + 10*a^2*b^3*c^2*d^3*g^4*n - 10*a^3*b^2*c*d^4*g^4*n + (4*g^4*n + \\
& 5*g^4*\log(e))*a^4*b*d^5)*B^2*x - 12*(b^5*c^5*g^4*n - 5*a*b^4*c^4*d*g^4*n + \\
& 10*a^2*b^3*c^3*d^2*g^4*n - 10*a^3*b^2*c^2*d^3*g^4*n + 5*a^4*b*c*d^4*g^4*n)* \\
& B^2*\log(d*x + c) + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B \\
& ^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x \\
& )*\log((b*x + a)^n))*\log((d*x + c)^n))/(b*d^5)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral  $\left( A^2 b^4 g^4 x^4 + 4 A^2 a b^3 g^4 x^3 + 6 A^2 a^2 b^2 g^4 x^2 + 4 A^2 a^3 b g^4 x + A^2 a^4 g^4 + (B^2 b^4 g^4 x^4 + 4 B^2 a b^3 g^4 x^3 + 6 B^2 a^2 b^2 g^4 x^2 + \dots \right)$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log(e*((b*x + a)/(d*x + c))^n), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^4 \left( B \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^4*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

$$3.11 \quad \int (ag + bgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx$$

**Optimal.** Leaf size=335

$$\frac{B^2 g^3 n^2 (bc - ad)^4 \text{PolyLog} \left( 2, \frac{d(a+bx)}{b(c+dx)} \right)}{2bd^4} - \frac{Bg^3 n (bc - ad)^4 \log \left( \frac{bc-ad}{b(c+dx)} \right) \left( 6B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + 6A + 11Bn \right)}{12bd^4} - \frac{Bg^3 n (a + bx)}{12bd^4}$$

[Out]  $-(B*(b*c - a*d)*g^3*n*(a + b*x)^3*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]))/(6*b*d) + (g^3*(a + b*x)^4*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2)/(4*b) + (B*(b*c - a*d)^2*g^3*n*(a + b*x)^2*(3*A + B*n + 3*B*\text{Log}[e*((a + b*x)/(c + d*x))^n]))/(12*b*d^2) - (B*(b*c - a*d)^3*g^3*n*(a + b*x)*(6*A + 5*B*n + 6*B*\text{Log}[e*((a + b*x)/(c + d*x))^n]))/(12*b*d^3) - (B*(b*c - a*d)^4*g^3*n*(6*A + 11*B*n + 6*B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/(12*b*d^4) - (B^2*(b*c - a*d)^4*g^3*n^2*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b*d^4)$

**Rubi [A]** time = 0.706999, antiderivative size = 512, normalized size of antiderivative = 1.53, number of steps used = 23, number of rules used = 13, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{B^2 g^3 n^2 (bc - ad)^4 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right)}{2bd^4} + \frac{Bg^3 n (bc - ad)^4 \log(c + dx) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{2bd^4} + \frac{Bg^3 n (a + bx)^2 (bc - ad)^2}{4b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^3*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2, x]$

[Out]  $-(A*B*(b*c - a*d)^3*g^3*n*x)/(2*d^3) - (5*B^2*(b*c - a*d)^3*g^3*n^2*x)/(12*d^3) + (B^2*(b*c - a*d)^2*g^3*n^2*(a + b*x)^2)/(12*b*d^2) - (B^2*(b*c - a*d)^3*g^3*n*(a + b*x)*\text{Log}[e*((a + b*x)/(c + d*x))^n])/(2*b*d^3) + (B*(b*c - a*d)^2*g^3*n*(a + b*x)^2*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]))/(4*b*d^2) - (B*(b*c - a*d)*g^3*n*(a + b*x)^3*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]))/(6*b*d) + (g^3*(a + b*x)^4*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2)/(4*b) + (11*B^2*(b*c - a*d)^4*g^3*n^2*\text{Log}[c + d*x])/(12*b*d^4) - (B^2*(b*c - a*d)^4*g^3*n^2*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(2*b*d^4) + (B*(b*c - a*d)^4*g^3*n*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x])/(2*b*d^4) + (B^2*(b*c - a*d)^4*g^3*n^2*\text{Log}[c + d*x]^2)/(4*b*d^4) - (B^2*(b*c - a*d)^4*g^3*n^2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(2*b*d^4)$

### Rule 2525

$\text{Int}[(a_.) + \text{Log}[(c_.)*(RfX_)^(p_.)]*(b_.)]^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x\_Symbol] :> \text{Simp}[(d + e*x)^(m + 1)*(a + b*\text{Log}[c*RfX^p])^n/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^(m + 1)*(a + b*\text{Log}[c*RfX^p])^(n - 1)*D[RfX, x])/RfX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RfX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] :> \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 2528

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2524

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]

Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(Rfx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2390

```
Int[((a_.) + Log[(c_.)*(d_.) + (e_.)*(x_.)]^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.
)*(x_.))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int (ag + bgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx &= \frac{g^3(a+bx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{4b} - \frac{(Bn) \int \frac{(bc-ad)g^4(a+bx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{c+dx}}{2bg} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{4b} - \frac{(B(bc-ad)g^3n) \int \frac{(a+bx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{c+dx}}{2b} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{4b} - \frac{(B(bc-ad)g^3n) \int \left( \frac{b(bc-ad)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{d^3} \right)}{2b} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{4b} - \frac{(B(bc-ad)g^3n) \int (a+bx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2d} \\
&= -\frac{AB(bc-ad)^3 g^3 n x}{2d^3} + \frac{B(bc-ad)^2 g^3 n (a+bx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{4bd^2} \\
&= -\frac{AB(bc-ad)^3 g^3 n x}{2d^3} - \frac{B^2(bc-ad)^3 g^3 n (a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{2bd^3} + \frac{B(bc-ad)^2 g^3 n^2 (a+bx)^2}{12bd^2} \\
&= -\frac{AB(bc-ad)^3 g^3 n x}{2d^3} - \frac{B^2(bc-ad)^3 g^3 n (a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{2bd^3} + \frac{B(bc-ad)^2 g^3 n^2 (a+bx)^2}{12bd^2} \\
&= -\frac{AB(bc-ad)^3 g^3 n x}{2d^3} - \frac{5B^2(bc-ad)^3 g^3 n^2 x}{12d^3} + \frac{B^2(bc-ad)^2 g^3 n^2 (a+bx)^2}{12bd^2} \\
&= -\frac{AB(bc-ad)^3 g^3 n x}{2d^3} - \frac{5B^2(bc-ad)^3 g^3 n^2 x}{12d^3} + \frac{B^2(bc-ad)^2 g^3 n^2 (a+bx)^2}{12bd^2} \\
&= -\frac{AB(bc-ad)^3 g^3 n x}{2d^3} - \frac{5B^2(bc-ad)^3 g^3 n^2 x}{12d^3} + \frac{B^2(bc-ad)^2 g^3 n^2 (a+bx)^2}{12bd^2}
\end{aligned}$$

**Mathematica [A]** time = 0.359561, size = 411, normalized size = 1.23

$$g^3 \left( (a+bx)^4 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)^2 - \frac{Bn(bc-ad) \left( 3Bn(bc-ad)^3 \left( 2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) + 2d^3(a+bx)^3 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)^2}{12bd^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] (g^3\*((a + b\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 - (B\*(b\*c - a\*d)\*n\*(6\*A\*b\*d\*(b\*c - a\*d)^2\*x + 6\*B\*d\*(b\*c - a\*d)^2\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n] + 3\*d^2\*(-(b\*c) + a\*d)\*(a + b\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 2\*d^3\*(a + b\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 6\*B\*(b\*c - a\*d)^3\*n\*Log[c + d\*x] - 6\*(b\*c - a\*d)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x] + B\*(b\*c - a\*d)\*n\*(2\*b\*d\*(b\*c - a\*d)\*x - d^2\*(a + b\*x)^2 - 2\*(b\*c - a\*d)^2\*Log[c + d\*x]) + 3\*B\*(b\*c - a\*d)^2\*n\*(b\*d\*x + (-b\*c) + a\*d)\*Log[c + d\*x]) + 3\*B\*(b\*c - a\*d)^3\*n\*((2\*Log[(d\*(a + b\*x))/(-(b\*c) + a\*d)] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/(3\*d^4))/(4\*b)

**Maple [F]** time = 0.433, size = 0, normalized size = 0.

$$\int (bgx + ag)^3 \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((b\*g\*x+a\*g)^3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [B]** time = 3.80072, size = 2936, normalized size = 8.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out] 1/2\*A\*B\*b^3\*g^3\*x^4\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/4\*A^2\*b^3\*g^3\*x^4 + 2\*A\*B\*a\*b^2\*g^3\*x^3\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A^2\*a\*b^2\*g^3\*x^3 + 3\*A\*B\*a^2\*b\*g^3\*x^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 3/2\*A^2\*a^2\*b\*g^3\*x^2 - 1/12\*A\*B\*b^3\*g^3\*n\*(6\*a^4\*log(b\*x + a)/b^4 - 6\*c^4\*log(d\*x + c)/d^4 + (2\*(b^3\*c\*d^2 - a\*b^2\*d^3)\*x^3 - 3\*(b^3\*c^2\*d - a^2\*b\*d^3)\*x^2 + 6\*(b^3\*c^3 - a^3\*d^3)\*x)/(b^3\*d^3)) + A\*B\*a\*b^2\*g^3\*n\*(2\*a^3\*log(b\*x + a)/b^3 - 2\*c^3\*log(d\*x + c)/d^3 - ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2)) - 3\*A\*B\*a^2\*b\*g^3\*n\*(a^2\*log(b\*x + a)/b^2 - c^2\*log(d\*x + c)/d^2 + (b\*c - a\*d)\*x/(b\*d)) + 2\*A\*B\*a^3\*g^3\*n\*(a\*log(b\*x + a)/b - c\*log(d\*x + c)/d) + 2\*A\*B\*a^3\*g^3\*x\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A^2\*a^3\*g^3\*x + 1/12\*((11\*g^3\*n^2 + 6\*g^3\*n\*log(e))\*b^3\*c^4 - 2\*(19\*g^3\*n^2 + 12\*g^3\*n\*log(e))\*a\*b^2\*c^3\*d + 9\*(5\*g^3\*n^2 + 4\*g^3\*n\*log(e))\*a^2\*b\*c^2\*d^2 - 6\*(3\*g^3\*n^2 + 4\*g^3\*n\*log(e))\*a^3\*c\*d^3)\*B^2\*log(d\*x + c)/d^4 + 1/2\*(b^4\*c^4\*g^3\*n^2 - 4\*a\*b^3\*c^3\*d\*g^3\*n^2 + 6\*a^2\*b^2\*c^2\*d^2\*g^3\*n^2 - 4\*a^3\*b\*c\*d^3\*g^3\*n^2 + a^4\*d^4\*g^3\*n^2)\*(log(b\*x + a)\*log((b\*d\*x + a\*d)/(b\*c - a\*d) + 1) + dilog(-(b\*d\*x + a\*d)/(b\*c - a\*d)))\*B^2/(b\*d^4) + 1/12\*(3\*B^2\*b^4\*d^4\*g^3\*x^4\*log(e)^2 - 3\*B^2\*a^4\*d^4\*g^3\*n^2\*log(b\*x + a)^2 - 2\*(b^4\*c\*d^3\*g^3\*n\*log(e) - (g^3\*n\*log(e) + 6\*g^3\*log(e)^2)\*a\*b^3\*d^4)\*B^2\*x^3 + ((g^3\*n^2 + 3\*g^3\*n\*log(e))\*b^4\*c^2\*d^2 - 2\*(g^3\*n^2 + 6\*g^3\*n\*log(e))\*a\*b^3\*c\*d^3 + (g^3\*n^2 + 9\*g^3\*n\*log(e) + 18\*g^3\*log(e)^2)\*a^2\*b^2\*d^4)\*B^2\*x^2 - 6\*(b^4\*c^4\*g^3\*n^2 - 4\*a\*b^3\*c^3\*d\*g^3\*n^2 + 6\*a^2\*b^2\*c^2\*d^2\*g^3\*n^2

$$\begin{aligned}
& - 4a^3b^3c^3d^3g^3n^2 * B^2 * \log(bx + a) * \log(dx + c) + 3(b^4c^4g^3n^2 \\
& - 4a^3b^3c^3d^3g^3n^2 + 6a^2b^2c^2d^2g^3n^2 - 4a^3b^3c^3d^3g^3n^2) * B^2 * \log(dx + c)^2 - ((5g^3n^2 + 6g^3n * \log(e)) * b^4c^3d - (17g^3n^2 \\
& + 24g^3n * \log(e)) * a^3b^3c^2d^2 + (19g^3n^2 + 36g^3n * \log(e)) * a^2b^2c^2d^3 - (7g^3n^2 + 18g^3n * \log(e) + 12g^3 * \log(e)^2) * a^3b^3d^4) * B^2 * x \\
& - (6a^3b^3c^3d^3g^3n^2 - 21a^2b^2c^2d^2g^3n^2 + 26a^3b^3c^3d^3g^3n^2 - (11g^3n^2 + 6g^3n * \log(e)) * a^4d^4) * B^2 * \log(bx + a) + 3(B^2b^4d^4g^3x^4 + 4B^2a^2b^3d^4g^3x^3 + 6B^2a^2b^2d^4g^3x^2 + 4B^2a^3b^3d^4g^3x) * \log((bx + a)^n)^2 + 3(B^2b^4d^4g^3x^4 + 4B^2a^2b^3d^4g^3x^3 + 6B^2a^2b^2d^4g^3x^2 + 4B^2a^3b^3d^4g^3x) * \log((dx + c)^n)^2 + (6B^2b^4d^4g^3x^4 * \log(e) + 6B^2a^4d^4g^3n * \log(bx + a) - 2(b^4c^3d^3g^3n - (g^3n + 12g^3 * \log(e)) * a^3b^3d^4) * B^2 * x^3 + 3(b^4c^2d^2g^3n - 4a^3b^3c^3d^3g^3n + 3(g^3n + 4g^3 * \log(e)) * a^2b^2d^4) * B^2 * x^2 - 6(b^4c^3d^3g^3n - 4a^3b^3c^2d^2g^3n + 6a^2b^2c^3d^3g^3n - (3g^3n + 4g^3 * \log(e)) * a^3b^3d^4) * B^2 * x + 6(b^4c^4g^3n - 4a^3b^3c^3d^3g^3n + 6a^2b^2c^2d^2g^3n - 4a^3b^3c^3d^3g^3n) * B^2 * \log(dx + c)) * \log((bx + a)^n) - (6B^2b^4d^4g^3x^4 * \log(e) + 6B^2a^4d^4g^3n * \log(bx + a) - 2(b^4c^3d^3g^3n - (g^3n + 12g^3 * \log(e)) * a^3b^3d^4) * B^2 * x^3 + 3(b^4c^2d^2g^3n - 4a^3b^3c^3d^3g^3n + 3(g^3n + 4g^3 * \log(e)) * a^2b^2d^4) * B^2 * x^2 - 6(b^4c^3d^3g^3n - 4a^3b^3c^2d^2g^3n + 6a^2b^2c^3d^3g^3n - (3g^3n + 4g^3 * \log(e)) * a^3b^3d^4) * B^2 * x + 6(b^4c^4g^3n - 4a^3b^3c^3d^3g^3n + 6a^2b^2c^2d^2g^3n - 4a^3b^3c^3d^3g^3n) * B^2 * \log(dx + c) + 6(B^2b^4d^4g^3x^4 + 4B^2a^2b^3d^4g^3x^3 + 6B^2a^2b^2d^4g^3x^2 + 4B^2a^3b^3d^4g^3x) * \log((bx + a)^n)) * \log((dx + c)^n)) / (b^4d^4)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2 b^3 g^3 x^3 + 3 A^2 a b^2 g^3 x^2 + 3 A^2 a^2 b g^3 x + A^2 a^3 g^3 + (B^2 b^3 g^3 x^3 + 3 B^2 a b^2 g^3 x^2 + 3 B^2 a^2 b g^3 x + B^2 a^3 g^3) \log \left( e \left( \frac{b}{d} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out] integral(A^2\*b^3\*g^3\*x^3 + 3\*A^2\*a\*b^2\*g^3\*x^2 + 3\*A^2\*a^2\*b\*g^3\*x + A^2\*a^3\*g^3 + (B^2\*b^3\*g^3\*x^3 + 3\*B^2\*a\*b^2\*g^3\*x^2 + 3\*B^2\*a^2\*b\*g^3\*x + B^2\*a^3\*g^3)\*log(e\*((b\*x + a)/(d\*x + c))^n)^2 + 2\*(A\*B\*b^3\*g^3\*x^3 + 3\*A\*B\*a\*b^2\*g^3\*x^2 + 3\*A\*B\*a^2\*b\*g^3\*x + A\*B\*a^3\*g^3)\*log(e\*((b\*x + a)/(d\*x + c))^n), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^3 \left( B \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^3\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2, x)

$$3.12 \quad \int (ag + bgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx$$

**Optimal.** Leaf size=274

$$\frac{2B^2g^2n^2(bc-ad)^3 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} + \frac{Bg^2n(bc-ad)^3 \log\left(\frac{bc-ad}{b(c+dx)}\right) \left(2B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + 2A + 3Bn\right)}{3bd^3} + \frac{Bg^2n(a+bx)}{3bd^3}$$

[Out]  $-(B*(b*c - a*d)*g^{2*n}*(a + b*x)^{2*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])})/(3*b*d) + (g^{2*(a + b*x)^{3*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2})/(3*b) + (B*(b*c - a*d)^{2*g^{2*n}*(a + b*x)*(2*A + B*n + 2*B*\text{Log}[e*((a + b*x)/(c + d*x))^n])})/(3*b*d^2) + (B*(b*c - a*d)^{3*g^{2*n}*(2*A + 3*B*n + 2*B*\text{Log}[e*((a + b*x)/(c + d*x))^n])}*\text{Log}[(b*c - a*d)/(b*(c + d*x))])/(3*b*d^3) + (2*B^2*(b*c - a*d)^{3*g^{2*n}^2*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)$

**Rubi [A]** time = 0.582896, antiderivative size = 420, normalized size of antiderivative = 1.53, number of steps used = 19, number of rules used = 13, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{2B^2g^2n^2(bc-ad)^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3bd^3} - \frac{2Bg^2n(bc-ad)^3 \log(c+dx) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{3bd^3} + \frac{2ABg^2nx(bc-ad)^2}{3d^2} - \frac{Bg^2n(a+bx)}{3bd^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^{2*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2}, x]$

[Out]  $(2*A*B*(b*c - a*d)^{2*g^{2*n}*x})/(3*d^2) + (B^{2*(b*c - a*d)^{2*g^{2*n}^2*x})/(3*d^2) + (2*B^2*(b*c - a*d)^{2*g^{2*n}*(a + b*x)*\text{Log}[e*((a + b*x)/(c + d*x))^n])/(3*b*d^2) - (B*(b*c - a*d)*g^{2*n}*(a + b*x)^{2*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])})/(3*b*d) + (g^{2*(a + b*x)^{3*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2})/(3*b) - (B^{2*(b*c - a*d)^{3*g^{2*n}^2*\text{Log}[c + d*x]})/(b*d^3) + (2*B^2*(b*c - a*d)^{3*g^{2*n}^2*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]}*\text{Log}[c + d*x])/(3*b*d^3) - (2*B*(b*c - a*d)^{3*g^{2*n}*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]}*\text{Log}[c + d*x])/(3*b*d^3) - (B^{2*(b*c - a*d)^{3*g^{2*n}^2*\text{Log}[c + d*x]^2})/(3*b*d^3) + (2*B^2*(b*c - a*d)^{3*g^{2*n}^2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)]})/(3*b*d^3)$

### Rule 2525

$\text{Int}[(a_. + \text{Log}[(c_.)*(Rfx_)^{(p_.)}]*(b_.))^{(n_.)}*((d_.) + (e_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{(m + 1)}*(a + b*\text{Log}[c*Rfx^p])^n/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{(m + 1)}*(a + b*\text{Log}[c*Rfx^p])^{(n - 1)}*D[Rfx, x])/Rfx, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 2528

$\text{Int}[(a_. + \text{Log}[(c_.)*(Rfx_)^{(p_.)}]*(b_.))^{(n_.)}*(RGx_), x\_Symbol] \rightarrow \text{With}[\{u = \text{ExpandIntegrand}[(a + b*\text{Log}[c*Rfx^p])^n, RGx, x]\}, \text{Int}[u, x] /;$  SumQ[u



```
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_.))^(n_.), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_.)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_.)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.)
)*(x_)^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int (ag + bgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx &= \frac{g^2(a+bx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{3b} - \frac{(2Bn) \int \frac{(bc-ad)g^3(a+bx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{c+dx}}{3bg} \\ &= \frac{g^2(a+bx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{3b} - \frac{(2B(bc-ad)g^2n) \int \frac{(a+bx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{c+dx}}{3b} \\ &= \frac{g^2(a+bx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{3b} - \frac{(2B(bc-ad)g^2n) \int \left( -\frac{b(bc-ad) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{c+dx} \right)}{3b} \\ &= \frac{g^2(a+bx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{3b} - \frac{(2B(bc-ad)g^2n) \int (a+bx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{3d} \\ &= \frac{2AB(bc-ad)^2g^2nx}{3d^2} - \frac{B(bc-ad)g^2n(a+bx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{3bd} + \dots \\ &= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{2B^2(bc-ad)^2g^2n(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{3bd^2} - \frac{B(bc-ad)^2g^2n(a+bx)}{3bd^2} \\ &= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{2B^2(bc-ad)^2g^2n(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{3bd^2} - \frac{B(bc-ad)^2g^2n(a+bx)}{3bd^2} \\ &= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2n(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{3bd^2} \\ &= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2n(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{3bd^2} \\ &= \frac{2AB(bc-ad)^2g^2nx}{3d^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3d^2} + \frac{2B^2(bc-ad)^2g^2n(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{3bd^2} \end{aligned}$$

**Mathematica [A]** time = 0.237362, size = 303, normalized size = 1.11

$$g^2 \left( \frac{Bn(bc-ad) \left( Bn(bc-ad)^2 \left( 2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) - d^2(a+bx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) - 2(bc-ad)^2 \log(c+dx) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{d^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] (g^2\*((a + b\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 + (B\*(b\*c - a\*d)\*n\*(2\*A\*b\*d\*(b\*c - a\*d)\*x + 2\*B\*d\*(b\*c - a\*d)\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n] - d^2\*(a + b\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 2\*B\*(b\*c - a\*d)^2\*n\*Log[c + d\*x] - 2\*(b\*c - a\*d)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x] + B\*(b\*c - a\*d)\*n\*(b\*d\*x + (-b\*c) + a\*d)\*Log[c + d\*x]) + B\*(b\*c - a\*d)^2\*n\*((2\*Log[(d\*(a + b\*x))/(-b\*c) + a\*d] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]))/d^3)/(3\*b)

**Maple [F]** time = 0.426, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((b\*g\*x+a\*g)^2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [B]** time = 3.68197, size = 2026, normalized size = 7.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out] 2/3\*A\*B\*b^2\*g^2\*x^3\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/3\*A^2\*b^2\*g^2\*x^3 + 2\*A\*B\*a\*b\*g^2\*x^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A^2\*a\*b\*g^2\*x^2 + 1/3\*A\*B\*b^2\*g^2\*n\*(2\*a^3\*log(b\*x + a)/b^3 - 2\*c^3\*log(d\*x + c)/d^3 - ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2)) - 2\*A\*B\*a\*b\*g^2\*n\*(a^2\*log(b\*x + a)/b^2 - c^2\*log(d\*x + c)/d^2 + (b\*c - a\*d)\*x/(b\*d)) + 2\*A\*B\*a^2\*g^2\*n\*(a\*log(b\*x + a)/b - c\*log(d\*x + c)/d) + 2\*A\*B\*a^2\*g^2\*x\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A^2\*a^2\*g^2\*x - 1/3\*((3\*g^2\*n^2 + 2\*g^2\*n\*log(e))\*b^2\*c^3 - (7\*g^2\*n^2 + 6\*g^2\*n\*log(e))\*a\*b\*c^2\*d + 2\*(2\*g^2\*n^2 + 3\*g^2\*n\*log(e))\*a^2\*c\*d^2)\*B^2\*log(d\*x + c)/d^3 - 2/3\*(b^3\*c^3\*g^2\*n^2 - 3\*a\*b^2\*c^2\*d\*g^2\*n^2 + 3\*a^2\*b\*c\*d^2\*g^2\*n^2 - a^3\*d^3\*g^2\*n^2)\*(log(b\*x + a)\*log((b\*d\*x + a\*d)/(b\*c - a\*d) + 1) + dilog(-(b\*d\*x + a\*d)/(b\*c - a\*d)))\*B^2/(b\*d^3) + 1/3\*(B^2\*b^3\*d^3\*g^2\*x^3\*log(e)^2 - B^2\*a^3\*d^3\*g^2\*n^2\*log(b\*x + a)^2 - (b^3\*c\*d^2\*g^2\*n\*log(e) - (g^2\*n\*log(e) + 3\*g^2\*log(e)^2)\*a\*b^2\*d^3)\*B^2\*x^2 + 2\*(b^3\*c^3\*g^2\*n^2 - 3\*a\*b^2\*c^2\*d\*g^2\*n^2 + 3\*a^2\*b\*c\*d^2\*g^2\*n^2)\*B^2\*log(b\*x + a)\*log(d\*x + c) - (b^3\*c^3\*g^2\*n^2 - 3\*a\*b^2\*c^2\*d\*g^2\*n^2 + 3\*a^2\*b\*c\*d^2\*g^2\*n^2)\*B^2\*log(d\*x + c)^2 + ((g^2\*n^2 + 2\*g^2\*n\*log(e))\*b^3\*c^2\*d - 2\*(g^2\*n^2 + 3\*g^2\*n\*log(e))\*a\*b^2\*c\*d^2 + (g^2\*n^2 + 4\*g^2\*n\*log(e) + 3\*g^2\*log(e)^2)\*a^2\*b\*d^3)\*B^2\*x + (2\*a\*b^2\*c^2\*d\*g^2\*n^2 - 5\*a^2\*b\*c\*d^2\*g^2\*n^2 + (3\*g^2\*n^2 + 2\*g^2\*n\*log(e))\*a^3\*d^3)\*B^2\*log(b\*x + a) + (B^2\*b^3\*d^3\*g^2\*x^3 + 3\*B^2\*a\*b^2\*d^3\*g^2\*x^2 + 3\*B^2\*a^2\*b\*d^3\*g^2\*x)\*log((b\*x + a)^n)^2 + (B^2\*b^3\*d^3\*g^2\*x^3 + 3\*B^2\*a\*b^2\*d^3\*g^2\*x^2 + 3\*B^2\*a^2\*b\*d^3\*g^2\*x)\*log((d\*x + c)^n)^2 + (2\*B^2\*b^3\*d^3\*g^2\*x^3\*log(e) + 2\*B^2\*a^3\*d^3\*g^2\*n\*log(b\*x + a) - (b^3\*c\*d^2\*g^2\*n - (g^2\*n + 6\*g^2\*log(e))\*a\*b^2\*d^3)\*B^2\*x^2 + 2\*(b^3\*c^2\*d\*g^2\*n - 3\*a\*b^2\*c\*d^2\*g^2\*n + (2\*

$$g^{2n} + 3g^2 \log(e) a^2 b d^3 B^{2x} - 2(b^3 c^3 g^{2n} - 3a b^2 c^2 d g^{2n} + 3a^2 b c d^2 g^{2n}) B^2 \log(dx + c) \log((bx + a)^n) - (2B^2 b^3 d^3 g^{2x^3} \log(e) + 2B^2 a^3 d^3 g^{2n} \log(bx + a) - (b^3 c d^2 g^{2n} - (g^{2n} + 6g^2 \log(e)) a b^2 d^3) B^{2x^2} + 2(b^3 c^2 d g^{2n} - 3a b^2 c d^2 g^{2n} + (2g^{2n} + 3g^2 \log(e)) a^2 b d^3) B^{2x} - 2(b^3 c^3 g^{2n} - 3a b^2 c^2 d g^{2n} + 3a^2 b c d^2 g^{2n}) B^2 \log(dx + c) + 2(B^2 b^3 d^3 g^{2x^3} + 3B^2 a b^2 d^3 g^{2x^2} + 3B^2 a^2 b d^3 g^{2x}) \log((bx + a)^n)) \log((dx + c)^n) / (b d^3)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2 b^2 g^2 x^2 + 2 A^2 a b g^2 x + A^2 a^2 g^2 + (B^2 b^2 g^2 x^2 + 2 B^2 a b g^2 x + B^2 a^2 g^2) \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right)^2 + 2 (A B b^2 g^2 x^2 + 2 A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c)**n)))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( B \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

$$3.13 \quad \int (ag + bgx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx$$

**Optimal.** Leaf size=196

$$\frac{B^2 g n^2 (bc - ad)^2 \text{PolyLog} \left( 2, \frac{d(a+bx)}{b(c+dx)} \right)}{bd^2} - \frac{Bgn(bc - ad)^2 \log \left( \frac{bc-ad}{b(c+dx)} \right) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A + Bn \right)}{bd^2} - \frac{Bgn(a+bx)(bc - ad)}{bd^2}$$

```
[Out] -((B*(b*c - a*d)*g*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b*d
)) + (g*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b) - (B*(b
*c - a*d)^2*g*n*(A + B*n + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d
)/(b*(c + d*x))])/(b*d^2) - (B^2*(b*c - a*d)^2*g*n^2*PolyLog[2, (d*(a + b*x
))/(b*(c + d*x))])/(b*d^2)
```

**Rubi [A]** time = 0.442832, antiderivative size = 309, normalized size of antiderivative = 1.58, number of steps used = 15, number of rules used = 12, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{B^2 g n^2 (bc - ad)^2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right)}{bd^2} + \frac{Bgn(bc - ad)^2 \log(c + dx) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{bd^2} + \frac{g(a + bx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] -((A*B*(b*c - a*d)*g*n*x)/d) - (B^2*(b*c - a*d)*g*n*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n])/(b*d) + (g*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x)
])^n)^2)/(2*b) + (B^2*(b*c - a*d)^2*g*n^2*Log[c + d*x])/(b*d^2) - (B^2*(b*c
- a*d)^2*g*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d^2) + (
B*(b*c - a*d)^2*g*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x])/(b
*d^2) + (B^2*(b*c - a*d)^2*g*n^2*Log[c + d*x]^2)/(2*b*d^2) - (B^2*(b*c - a*
d)^2*g*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*d^2)
```

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
]^n))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_))^(n_.)]*(b_.)/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int (ag + bgx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx &= \frac{g(a+bx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2b} - \frac{(Bn) \int \frac{(bc-ad)g^2(a+bx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{c+dx}}{bg} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2b} - \frac{(B(bc-ad)gn) \int \frac{(a+bx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{c+dx}}{b} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2b} - \frac{(B(bc-ad)gn) \int \left( \frac{b \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{d} \right)}{b} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2b} - \frac{(B(bc-ad)gn) \int \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{d} \\
&= -\frac{AB(bc-ad)gnx}{d} + \frac{g(a+bx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2b} + \frac{B(bc-ad)}{d} \\
&= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{bd} + \frac{g(a+bx)}{d} \\
&= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{bd} + \frac{g(a+bx)}{d} \\
&= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{bd} + \frac{g(a+bx)}{d} \\
&= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{bd} + \frac{g(a+bx)}{d} \\
&= -\frac{AB(bc-ad)gnx}{d} - \frac{B^2(bc-ad)gn(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{bd} + \frac{g(a+bx)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.199556, size = 215, normalized size = 1.1

$$\frac{g \left( (a+bx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)^2 - \frac{Bn(bc-ad) \left( Bn(bc-ad) \left( 2 \operatorname{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) - 2(bc-ad) \log(c+dx)}{d^2}}{2b} \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] (g\*((a + b\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 - (B\*(b\*c - a\*d)\*n\*(2\*A\*b\*d\*x + 2\*B\*d\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n] - 2\*B\*(b\*c - a\*d)\*n\*Log[c + d\*x] - 2\*(b\*c - a\*d)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x] + B\*(b\*c - a\*d)\*n\*((2\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]))/d^2)/(2\*b)

**Maple [F]** time = 0.276, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((b\*g\*x+a\*g)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [B]** time = 3.80973, size = 1118, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out]  $A*B*b*g*x^2*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*b*g*x^2 - A*B*b*g*n*(a^2*\log(b*x + a)/b^2 - c^2*\log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a*g*n*(a*\log(b*x + a)/b - c*\log(d*x + c)/d) + 2*A*B*a*g*x*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*g*x + ((g*n^2 + g*n*\log(e))*b*c^2 - (g*n^2 + 2*g*n*\log(e))*a*c*d)*B^2*\log(d*x + c)/d^2 + (b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2 + a^2*d^2*g*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) - 1/2*(B^2*a^2*d^2*g*n^2*\log(b*x + a)^2 - B^2*b^2*d^2*g*x^2*\log(e)^2 + 2*(b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2)*B^2*\log(b*x + a)*\log(d*x + c) - (b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2)*B^2*\log(d*x + c)^2 + 2*(b^2*c*d*g*n*\log(e) - (g*n*\log(e) + g*\log(e)^2)*a*b*d^2)*B^2*x + 2*(a*b*c*d*g*n^2 - (g*n^2 + g*n*\log(e))*a^2*d^2)*B^2*\log(b*x + a) - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*\log((b*x + a)^n)^2 - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*\log((d*x + c)^n)^2 - 2*(B^2*b^2*d^2*g*x^2*\log(e) + B^2*a^2*d^2*g*n*\log(b*x + a) - (b^2*c*d*g*n - (g*n + 2*g*\log(e))*a*b*d^2)*B^2*x + (b^2*c^2*g*n - 2*a*b*c*d*g*n)*B^2*\log(d*x + c))*\log((b*x + a)^n) + 2*(B^2*b^2*d^2*g*x^2*\log(e) + B^2*a^2*d^2*g*n*\log(b*x + a) - (b^2*c*d*g*n - (g*n + 2*g*\log(e))*a*b*d^2)*B^2*x + (b^2*c^2*g*n - 2*a*b*c*d*g*n)*B^2*\log(d*x + c) + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x)*\log((b*x + a)^n))*\log((d*x + c)^n)/(b*d^2)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2bgx + A^2ag + (B^2bgx + B^2ag) \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right)^2 + 2(ABbgx + ABag) \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out] integral(A^2\*b\*g\*x + A^2\*a\*g + (B^2\*b\*g\*x + B^2\*a\*g)\*log(e\*((b\*x + a)/(d\*x + c))^n)^2 + 2\*(A\*B\*b\*g\*x + A\*B\*a\*g)\*log(e\*((b\*x + a)/(d\*x + c))^n), x)



---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag) \left( B \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2, x)

$$3.14 \quad \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx} dx$$

**Optimal.** Leaf size=138

$$\frac{2Bn \text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{bg} + \frac{2B^2n^2 \text{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right) \log\left(1 - \frac{b(c+dx)}{d(a+bx)}\right)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{bg}$$

[Out] -(((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2\*Log[1 - (b\*(c + d\*x))/(d\*(a + b\*x)]))/(b\*g)) + (2\*B\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*PolyLog[2, (b\*(c + d\*x))/(d\*(a + b\*x)]))/(b\*g) + (2\*B^2\*n^2\*PolyLog[3, (b\*(c + d\*x))/(d\*(a + b\*x)]))/(b\*g)

**Rubi [B]** time = 3.57289, antiderivative size = 789, normalized size of antiderivative = 5.72, number of steps used = 45, number of rules used = 23, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.657$ , Rules used = {2524, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 12, 6742, 2500, 2440, 2434, 2433, 2375, 2317, 2374, 6589, 2499, 2302, 30, 2396}

$$\frac{2ABn \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right) - 2B^2n \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \log((a+bx)^n) + \log((c+dx)^{-n})\right)}{bg} - \frac{2B^2n^2 \text{PolyLog}\left(3, -\frac{d(a+bx)}{bc-ad}\right)\log\left(1 - \frac{d(a+bx)}{bc-ad}\right)\left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \log((a+bx)^n) + \log((c+dx)^{-n})\right)}{bg}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(a\*g + b\*g\*x), x]

[Out] -((A\*B\*n\*Log[g\*(a + b\*x)]^2)/(b\*g)) + (B^2\*n^2\*Log[g\*(a + b\*x)]^3)/(3\*b\*g) - (B^2\*n^2\*Log[g\*(a + b\*x)]^2\*Log[-c - d\*x])/(b\*g) + (2\*B^2\*n\*Log[g\*(a + b\*x)]\*Log[(a + b\*x)^n]\*Log[-c - d\*x])/(b\*g) - (B^2\*Log[(a + b\*x)^n]^2\*Log[-c - d\*x])/(b\*g) + (B^2\*n^2\*Log[g\*(a + b\*x)]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g) + (B^2\*Log[(a + b\*x)^n]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g) + (B^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[(c + d\*x)^(-n)]^2)/(b\*g) - (B^2\*Log[g\*(a + b\*x)]\*Log[(c + d\*x)^(-n)]^2)/(b\*g) + ((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2\*Log[a\*g + b\*g\*x])/(b\*g) + (2\*A\*B\*n\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*Log[a\*g + b\*g\*x])/(b\*g) - (2\*B^2\*n\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*(Log[(a + b\*x)^n] - Log[e\*((a + b\*x)/(c + d\*x))^n] + Log[(c + d\*x)^(-n)])\*Log[a\*g + b\*g\*x])/(b\*g) - (B^2\*n\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[a\*g + b\*g\*x]^2)/(b\*g) - (B^2\*n^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*Log[a\*g + b\*g\*x]^2)/(b\*g) + (2\*A\*B\*n\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g) + (2\*B^2\*n\*Log[(a + b\*x)^n]\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g) - (2\*B^2\*n\*(Log[(a + b\*x)^n] - Log[e\*((a + b\*x)/(c + d\*x))^n] + Log[(c + d\*x)^(-n)])\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g) - (2\*B^2\*n\*Log[(c + d\*x)^(-n)]\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g) - (2\*B^2\*n^2\*PolyLog[3, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g) - (2\*B^2\*n^2\*PolyLog[3, (b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g)

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

#### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[(f*x)/d]^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e^n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 2500

```

Int[(Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))/((j_.) + (k
_.)*(x_)), x_Symbol] := Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]

```

#### Rule 2440

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :=
Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]

```

#### Rule 2434

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)))/(x_), x_Symbol] := Simp[Log[x]*(a + b
*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[e*g*m, Int[(Lo
g[x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x), x], x] - Dist[b*j*n, Int[(Log[x
]*(f + g*Log[h*(i + j*x)^m]))/(i + j*x), x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, i, j, m, n}, x] && EqQ[e*i - d*j, 0]

```

#### Rule 2433

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] := Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]

```

#### Rule 2375

```

Int[(Log[(d_.)*((e_) + (f_.)*(x_))^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_))^(n_
.))*((b_.))^(p_.))/(x_), x_Symbol] := Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]

```

#### Rule 2317

```

Int[((a_.) + Log[(c_.)*(x_))^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] := Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]

```

#### Rule 2374

```

Int[(Log[(d_.)*((e_) + (f_.)*(x_))^(m_.))]*((a_.) + Log[(c_.)*(x_))^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^
n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]

```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

#### Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]*
((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.)/((j_.) + (k_.)*(x_)),
x_Symbol]
:> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/
(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)]/
(a + b*x), x], x] - Dist[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)]/
(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r}, x]
&& NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

#### Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol]
:> Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]
```

#### Rule 30

```
Int[(x_)^(m_.), x_Symbol]
:> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)/((f_.) + (g_.)*(x_)),
x_Symbol]
:> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x]
&& NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag + bgx} dx &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2Bn) \int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2} + \frac{b}{c+dx}\right)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}}{bg} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(ag+bgx)}{(a+bx)(c+dx)} dx}{bg} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B(bc - ad)n) \int \frac{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(ag+bgx)}{(a+bx)(c+dx)}}{bg} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B(bc - ad)n) \int \left(\frac{d\left(-A-B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(ag+bgx)}{(bc-ad)(c+dx)}\right)}{bg} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2Bn) \int \frac{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(ag+bgx)}{a+bx} dx}{g} - \frac{(2B^2n) \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(ag+bgx)}{a+bx}}{g} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2Bn) \int \left(\frac{A \log(ag+bgx)}{a+bx} + \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(ag+bgx)}{a+bx}\right)}{g} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2ABn) \int \frac{\log(ag+bgx)}{a+bx} dx}{g} - \frac{(2B^2n) \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(ag+bgx)}{a+bx}}{g} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(ag + bgx)}{bg} + \frac{2ABn \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} - \frac{B^2n \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(ag + bgx)}{bg} + \frac{2ABn \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} - \frac{2B^2n \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} \\
&= -\frac{ABn \log^2(g(a + bx))}{bg} + \frac{2B^2n \log(g(a + bx)) \log((a + bx)^n) \log(-c - dx)}{bg} - \frac{B^2 \log(g(a + bx)) \log^2(-c - dx)}{bg} \\
&= -\frac{ABn \log^2(g(a + bx))}{bg} + \frac{2B^2n \log(g(a + bx)) \log((a + bx)^n) \log(-c - dx)}{bg} + \frac{B^2 \log\left(-\frac{c-dx}{a+bx}\right) \log^2(-c - dx)}{bg} \\
&= -\frac{ABn \log^2(g(a + bx))}{bg} + \frac{B^2n^2 \log^3(g(a + bx))}{3bg} - \frac{B^2n^2 \log^2(g(a + bx)) \log(-c - dx)}{bg} + \frac{B^2 \log\left(-\frac{c-dx}{a+bx}\right) \log^2(-c - dx)}{bg} \\
&= -\frac{ABn \log^2(g(a + bx))}{bg} + \frac{B^2n^2 \log^3(g(a + bx))}{3bg} - \frac{B^2n^2 \log^2(g(a + bx)) \log(-c - dx)}{bg} + \frac{B^2 \log\left(-\frac{c-dx}{a+bx}\right) \log^2(-c - dx)}{bg} \\
&= -\frac{ABn \log^2(g(a + bx))}{bg} + \frac{B^2n^2 \log^3(g(a + bx))}{3bg} - \frac{B^2n^2 \log^2(g(a + bx)) \log(-c - dx)}{bg} + \frac{B^2 \log\left(-\frac{c-dx}{a+bx}\right) \log^2(-c - dx)}{bg} \\
&= -\frac{ABn \log^2(g(a + bx))}{bg} + \frac{B^2n^2 \log^3(g(a + bx))}{3bg} - \frac{B^2n^2 \log^2(g(a + bx)) \log(-c - dx)}{bg} + \frac{B^2 \log\left(-\frac{c-dx}{a+bx}\right) \log^2(-c - dx)}{bg}
\end{aligned}$$

**Mathematica [B]** time = 0.415522, size = 537, normalized size = 3.89

$$3Bn \left( -2 \left( \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log \left( \frac{c}{d} + x \right) \log \left( \frac{d(a+bx)}{ad-bc} \right) \right) - 2 \log(a+bx) \left( -\log \left( \frac{a+bx}{c+dx} \right) + \log \left( \frac{a}{b} + x \right) - \log \left( \frac{c}{d} + x \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(a\*g + b\*g\*x), x]

[Out] (3\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n] - B\*n\*Log[(a + b\*x)/(c + d\*x)])^2 + 3\*B\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n] - B\*n\*Log[(a + b\*x)/(c + d\*x)])\*(Log[a/b + x]^2 - 2\*Log[a + b\*x]\*(Log[a/b + x] - Log[c/d + x] - Log[(a + b\*x)/(c + d\*x])) - 2\*(Log[c/d + x]\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)] + PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]) + B^2\*n^2\*(Log[a/b + x]^3 + 3\*Log[c/d + x]^2\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)] + 3\*Log[a + b\*x]\*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b\*x)/(c + d\*x)])^2 + 3\*Log[a/b + x]^2\*(-Log[c/d + x] + Log[(b\*(c + d\*x))/(b\*c - a\*d)] + 6\*Log[a/b + x]\*PolyLog[2, (d\*(a + b\*x))/(-b\*c + a\*d)] + 6\*Log[c/d + x]\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)] - 3\*(Log[a/b + x] - Log[c/d + x] - Log[(a + b\*x)/(c + d\*x)])\*(Log[a/b + x]^2 - 2\*(Log[c/d + x]\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)] + PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])) - 6\*PolyLog[3, (d\*(a + b\*x))/(-b\*c + a\*d)] - 6\*PolyLog[3, (b\*(c + d\*x))/(b\*c - a\*d)])/(3\*b\*g)

**Maple [F]** time = 0.437, size = 0, normalized size = 0.

$$\int \frac{1}{bgx + ag} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g), x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{B^2 \log(bx + a) \log((dx + c)^n)^2}{bg} + \frac{A^2 \log(bgx + ag)}{bg} - \int \frac{B^2 bc \log(e)^2 + 2 ABbc \log(e) + (B^2 bdx + B^2 bc) \log((bx + a)^n)}{bg} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g), x, algorithm="maxima")

[Out] B^2\*log(b\*x + a)\*log((d\*x + c)^n)^2/(b\*g) + A^2\*log(b\*g\*x + a\*g)/(b\*g) - integrate(-(B^2\*b\*c\*log(e)^2 + 2\*A\*B\*b\*c\*log(e) + (B^2\*b\*d\*x + B^2\*b\*c)\*log((b\*x + a)^n)^2 + (B^2\*b\*d\*log(e)^2 + 2\*A\*B\*b\*d\*log(e))\*x + 2\*(B^2\*b\*c\*log(e) + A\*B\*b\*c + (B^2\*b\*d\*log(e) + A\*B\*b\*d)\*x)\*log((b\*x + a)^n) - 2\*(B^2\*b\*c\*log(e) + A\*B\*b\*c + (B^2\*b\*d\*log(e) + A\*B\*b\*d)\*x + (B^2\*b\*d\*n\*x + B^2\*a\*d\*n)\*log(b\*x + a) + (B^2\*b\*d\*x + B^2\*b\*c)\*log((b\*x + a)^n))\*log((d\*x + c)^n)/(b^2\*d\*g\*x^2 + a\*b\*c\*g + (b^2\*c\*g + a\*b\*d\*g)\*x), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2AB \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2}{bgx + ag}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g),x, algorithm="fricas")

[Out] integral((B^2\*log(e\*((b\*x + a)/(d\*x + c))^n))^2 + 2\*A\*B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A^2)/(b\*g\*x + a\*g), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g),x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2}{bgx + ag} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g),x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2/(b\*g\*x + a\*g), x)



$$3.15 \quad \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2} dx$$

**Optimal.** Leaf size=136

$$\frac{2Bn(c+dx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{g^2(a+bx)(bc-ad)} - \frac{(c+dx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}{g^2(a+bx)(bc-ad)} - \frac{2B^2n^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

[Out]  $(-2*B^2*n^2*(c+d*x))/((b*c-a*d)*g^2*(a+b*x)) - (2*B*n*(c+d*x)*(A+B*\text{Log}[e*((a+b*x)/(c+d*x))^n]))/((b*c-a*d)*g^2*(a+b*x)) - ((c+d*x)*(A+B*\text{Log}[e*((a+b*x)/(c+d*x))^n])^2)/((b*c-a*d)*g^2*(a+b*x))$

**Rubi [C]** time = 0.840441, antiderivative size = 512, normalized size of antiderivative = 3.76, number of steps used = 24, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2B^2dn^2\text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg^2(bc-ad)} - \frac{2B^2dn^2\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bg^2(bc-ad)} - \frac{2Bdn \log(a+bx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{bg^2(bc-ad)} - \frac{2Bn\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}{g^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(a*g + b*g*x)^2, x]`

[Out]  $(-2*B^2*n^2)/(b*g^2*(a+b*x)) - (2*B^2*d*n^2*\text{Log}[a+b*x])/(b*(b*c-a*d)*g^2) + (B^2*d*n^2*\text{Log}[a+b*x]^2)/(b*(b*c-a*d)*g^2) - (2*B*n*(A+B*\text{Log}[e*((a+b*x)/(c+d*x))^n]))/(b*g^2*(a+b*x)) - (2*B*d*n*\text{Log}[a+b*x]*(A+B*\text{Log}[e*((a+b*x)/(c+d*x))^n]))/(b*(b*c-a*d)*g^2) - (A+B*\text{Log}[e*((a+b*x)/(c+d*x))^n])^2/(b*g^2*(a+b*x)) + (2*B^2*d*n^2*\text{Log}[c+d*x])/(b*(b*c-a*d)*g^2) - (2*B^2*d*n^2*\text{Log}[-((d*(a+b*x))/(b*c-a*d))]*\text{Log}[c+d*x])/(b*(b*c-a*d)*g^2) + (2*B*d*n*(A+B*\text{Log}[e*((a+b*x)/(c+d*x))^n])*\text{Log}[c+d*x])/(b*(b*c-a*d)*g^2) + (B^2*d*n^2*\text{Log}[c+d*x]^2)/(b*(b*c-a*d)*g^2) - (2*B^2*d*n^2*\text{Log}[a+b*x]*\text{Log}[(b*(c+dx))/(b*c-a*d)])/(b*(b*c-a*d)*g^2) - (2*B^2*d*n^2*\text{PolyLog}[2, -((d*(a+b*x))/(b*c-a*d))])/(b*(b*c-a*d)*g^2) - (2*B^2*d*n^2*\text{PolyLog}[2, (b*(c+dx))/(b*c-a*d)])/(b*(b*c-a*d)*g^2)$

### Rule 2525

`Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]`

### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[ {u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]
] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 44

```
Int[((a_) + (b.)*(x.))^(m.)*((c.) + (d.)*(x.))^(n.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

#### Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x.)), x_Symbol] :=
Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x.))^(n.)])*(b_.))^(p_.)*(Rfx_), x_Symbol] :=
With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x.))^(n.)])*(b_.))^(p_.)*((f_) + (g_.)*(x.))^(q.), x_Symbol] :=
Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x.)^(n.)])*(b_.))/(x.), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x.))^(n.)])*(b_.))/((f_.) + (g_.)*(x.)), x_Symbol] :=
Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x.))])*(b_.))/((f_.) + (g_.)*(x.)), x_Symbol] :=
Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x.)^(n.))]/(x.), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^2} dx &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a + bx)} + \frac{(2Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(a+bx)^2(c+dx)} dx}{bg} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a + bx)} + \frac{(2B(bc - ad)n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^2(c+dx)} dx}{bg^2} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a + bx)} + \frac{(2B(bc - ad)n) \int \left(\frac{b\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(a+bx)^2} - \frac{bd\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2(a+bx)}\right) dx}{bg^2} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a + bx)} + \frac{(2Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^2} dx}{g^2} - \frac{(2Bdn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx} dx}{(bc - ad)g^2} \\
&= -\frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a + bx)} - \frac{2Bdn \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc - ad)g^2} - \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^2} \\
&= -\frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a + bx)} - \frac{2Bdn \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc - ad)g^2} - \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^2} \\
&= -\frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a + bx)} - \frac{2Bdn \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc - ad)g^2} - \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^2} \\
&= -\frac{2B^2n^2}{bg^2(a + bx)} - \frac{2B^2dn^2 \log(a + bx)}{b(bc - ad)g^2} - \frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a + bx)} - \frac{2Bdn \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc - ad)g^2} \\
&= -\frac{2B^2n^2}{bg^2(a + bx)} - \frac{2B^2dn^2 \log(a + bx)}{b(bc - ad)g^2} + \frac{B^2dn^2 \log^2(a + bx)}{b(bc - ad)g^2} - \frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a + bx)} - \frac{2Bdn \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc - ad)g^2} \\
&= -\frac{2B^2n^2}{bg^2(a + bx)} - \frac{2B^2dn^2 \log(a + bx)}{b(bc - ad)g^2} + \frac{B^2dn^2 \log^2(a + bx)}{b(bc - ad)g^2} - \frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a + bx)} - \frac{2Bdn \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc - ad)g^2}
\end{aligned}$$

**Mathematica [C]** time = 0.598637, size = 330, normalized size = 2.43

$$\frac{Bn\left(-Bdn(a+bx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+Bdn(a+bx)\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)\right)}{b^2g^2(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(a\*g + b\*g\*x)^2,x]

[Out] -(((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 + (B\*n\*(2\*(b\*c - a\*d)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 2\*d\*(a + b\*x)\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 2\*d\*(a + b\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) \*Log[c + d\*x] + 2\*B\*n\*(b\*c - a\*d + d\*(a + b\*x)\*Log[a + b\*x] - d\*(a + b\*x)\*Log[c + d\*x]) - B\*d\*n\*(a + b\*x)\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c + a\*d)]) + B\*d\*n\*(a + b\*x)\*((2\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/(b\*c - a\*d)/(b\*g^2\*(a + b\*x))

x)))

**Maple [F]** time = 0.436, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^2,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^2,x)

**Maxima [B]** time = 1.23068, size = 581, normalized size = 4.27

$$-2 ABn \left( \frac{1}{b^2 g^2 x + a b g^2} + \frac{d \log(bx + a)}{(b^2 c - a b d) g^2} - \frac{d \log(dx + c)}{(b^2 c - a b d) g^2} \right) - \left( 2n \left( \frac{1}{b^2 g^2 x + a b g^2} + \frac{d \log(bx + a)}{(b^2 c - a b d) g^2} - \frac{d \log(dx + c)}{(b^2 c - a b d) g^2} \right) \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^2,x, algorithm="maxima")

[Out] -2\*A\*B\*n\*(1/(b^2\*g^2\*x + a\*b\*g^2) + d\*log(b\*x + a)/((b^2\*c - a\*b\*d)\*g^2) - d\*log(d\*x + c)/((b^2\*c - a\*b\*d)\*g^2)) - (2\*n\*(1/(b^2\*g^2\*x + a\*b\*g^2) + d\*log(b\*x + a)/((b^2\*c - a\*b\*d)\*g^2) - d\*log(d\*x + c)/((b^2\*c - a\*b\*d)\*g^2))\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) - ((b\*d\*x + a\*d)\*log(b\*x + a)^2 + (b\*d\*x + a\*d)\*log(d\*x + c)^2 - 2\*b\*c + 2\*a\*d - 2\*(b\*d\*x + a\*d)\*log(b\*x + a) + 2\*(b\*d\*x + a\*d - (b\*d\*x + a\*d)\*log(b\*x + a))\*log(d\*x + c))\*n^2/(a\*b^2\*c\*g^2 - a^2\*b\*d\*g^2 + (b^3\*c\*g^2 - a\*b^2\*d\*g^2)\*x)\*B^2 - B^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n)^2/(b^2\*g^2\*x + a\*b\*g^2) - 2\*A\*B\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n)/(b^2\*g^2\*x + a\*b\*g^2) - A^2/(b^2\*g^2\*x + a\*b\*g^2)

**Fricas [A]** time = 0.928436, size = 555, normalized size = 4.08

$$\frac{A^2bc - A^2ad + 2(B^2bc - B^2ad)n^2 + (B^2bc - B^2ad) \log(e)^2 + (B^2bdn^2x + B^2bcn^2) \log\left(\frac{bx+a}{dx+c}\right)^2 + 2(ABbc - ABad)n + 2(B^2b^2c - B^2a^2d) \log(e)^2 + (B^2b^2c - B^2a^2d) \log\left(\frac{bx+a}{dx+c}\right)^2 + 2(A*B*b*c - A*B*a*d)*n + 2*(A*B*b*c - A*B*a*d + (B^2*b*c - B^2*a*d)*n + (B^2*b*d*n*x + B^2*b*c*n)*\log\left(\frac{bx+a}{dx+c}\right))*\log(e) + 2*(B^2*b*c*n^2 + A*B*b*c*n + (B^2*b*d*n^2 + A*B*b*d*n)*x)*\log\left(\frac{bx+a}{dx+c}\right)}{(b^3c - ab^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^2,x, algorithm="fricas")

[Out] -(A^2\*b\*c - A^2\*a\*d + 2\*(B^2\*b\*c - B^2\*a\*d)\*n^2 + (B^2\*b\*c - B^2\*a\*d)\*log(e)^2 + (B^2\*b\*d\*n\*x + B^2\*b\*c\*n^2)\*log((b\*x + a)/(d\*x + c))^2 + 2\*(A\*B\*b\*c - A\*B\*a\*d)\*n + 2\*(A\*B\*b\*c - A\*B\*a\*d + (B^2\*b\*c - B^2\*a\*d)\*n + (B^2\*b\*d\*n\*x + B^2\*b\*c\*n)\*log((b\*x + a)/(d\*x + c)))\*log(e) + 2\*(B^2\*b\*c\*n^2 + A\*B\*b\*c\*n + (B^2\*b\*d\*n^2 + A\*B\*b\*d\*n)\*x)\*log((b\*x + a)/(d\*x + c)))/((b^3\*c - a\*b^2\*d)

) $g^2x + (ab^2c - a^2bd)g^2$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2/(b\*g\*x+a\*g)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}{(bgx + ag)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^2,x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2/(b\*g\*x + a\*g)^2, x)

$$3.16 \quad \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3} dx$$

**Optimal.** Leaf size=288

$$\frac{bBn(c+dx)^2 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{2g^3(a+bx)^2(bc-ad)^2} + \frac{2Bdn(c+dx) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{g^3(a+bx)(bc-ad)^2} - \frac{b(c+dx)^2 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}{2g^3(a+bx)^2(bc-ad)^2} +$$

[Out]  $(2*B^2*d*n^2*(c+d*x))/((b*c-a*d)^2*g^3*(a+b*x)) - (b*B^2*n^2*(c+d*x)^2)/(4*(b*c-a*d)^2*g^3*(a+b*x)^2) + (2*B*d*n*(c+d*x)*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/((b*c-a*d)^2*g^3*(a+b*x)) - (b*B*n*(c+d*x)^2*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(2*(b*c-a*d)^2*g^3*(a+b*x)^2) + (d*(c+d*x)*(A+B*Log[e*((a+b*x)/(c+d*x))^n])^2)/((b*c-a*d)^2*g^3*(a+b*x)) - (b*(c+d*x)^2*(A+B*Log[e*((a+b*x)/(c+d*x))^n])^2)/(2*(b*c-a*d)^2*g^3*(a+b*x)^2)$

**Rubi [C]** time = 0.922785, antiderivative size = 626, normalized size of antiderivative = 2.17, number of steps used = 28, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2 d^2 n^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg^3(bc-ad)^2} + \frac{B^2 d^2 n^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bg^3(bc-ad)^2} + \frac{Bd^2 n \log(a+bx) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{bg^3(bc-ad)^2} - \frac{Bd^2 n \log(c+dx)}{bg^3(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(a\*g + b\*g\*x)^3, x]

[Out]  $-(B^2*n^2)/(4*b*g^3*(a+b*x)^2) + (3*B^2*d*n^2)/(2*b*(b*c-a*d)*g^3*(a+b*x)) + (3*B^2*d^2*n^2*Log[a+b*x])/(2*b*(b*c-a*d)^2*g^3) - (B^2*d^2*n^2*Log[a+b*x]^2)/(2*b*(b*c-a*d)^2*g^3) - (B*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(2*b*g^3*(a+b*x)^2) + (B*d*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(b*(b*c-a*d)*g^3*(a+b*x)) + (B*d^2*n*Log[a+b*x]*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(b*(b*c-a*d)^2*g^3) - (A+B*Log[e*((a+b*x)/(c+d*x))^n])^2/(2*b*g^3*(a+b*x)^2) - (3*B^2*d^2*n^2*Log[c+d*x])/(2*b*(b*c-a*d)^2*g^3) + (B^2*d^2*n^2*Log[-((d*(a+b*x))/(b*c-a*d))]*Log[c+d*x])/(b*(b*c-a*d)^2*g^3) - (B*d^2*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n])*Log[c+d*x])/(b*(b*c-a*d)^2*g^3) - (B^2*d^2*n^2*Log[c+d*x]^2)/(2*b*(b*c-a*d)^2*g^3) + (B^2*d^2*n^2*Log[a+b*x]*Log[(b*(c+d*x))/(b*c-a*d)])/(b*(b*c-a*d)^2*g^3) + (B^2*d^2*n^2*PolyLog[2, -((d*(a+b*x))/(b*c-a*d))])/(b*(b*c-a*d)^2*g^3) + (B^2*d^2*n^2*PolyLog[2, (b*(c+d*x))/(b*c-a*d)])/(b*(b*c-a*d)^2*g^3)$

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^n\*(d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFX^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(RFX\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFX, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)]^(n\_.)]\*(b\_.)/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^3} dx &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a + bx)^2} + \frac{(Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^2(a+bx)^3(c+dx)} dx}{bg} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a + bx)^2} + \frac{(B(bc - ad)n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^3(c+dx)} dx}{bg^3} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a + bx)^2} + \frac{(B(bc - ad)n) \int \left(\frac{b\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(a+bx)^3} - \frac{bd\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2(a+bx)^2}\right) dx}{bg^3} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a + bx)^2} + \frac{(Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^3} dx}{g^3} + \frac{(Bd^2n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx} dx}{(bc - ad)^2g^3} \\
&= -\frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a + bx)^2} + \frac{Bdn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc - ad)g^3(a + bx)} + \frac{Bd^2n \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc - ad)g^3(a + bx)} \\
&= -\frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a + bx)^2} + \frac{Bdn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc - ad)g^3(a + bx)} + \frac{Bd^2n \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc - ad)g^3(a + bx)} \\
&= -\frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a + bx)^2} + \frac{Bdn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc - ad)g^3(a + bx)} + \frac{Bd^2n \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b(bc - ad)g^3(a + bx)} \\
&= -\frac{B^2n^2}{4bg^3(a + bx)^2} + \frac{3B^2dn^2}{2b(bc - ad)g^3(a + bx)} + \frac{3B^2d^2n^2 \log(a + bx)}{2b(bc - ad)^2g^3} - \frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a + bx)} \\
&= -\frac{B^2n^2}{4bg^3(a + bx)^2} + \frac{3B^2dn^2}{2b(bc - ad)g^3(a + bx)} + \frac{3B^2d^2n^2 \log(a + bx)}{2b(bc - ad)^2g^3} - \frac{B^2d^2n^2 \log^2(a + bx)}{2b(bc - ad)^2g^3} \\
&= -\frac{B^2n^2}{4bg^3(a + bx)^2} + \frac{3B^2dn^2}{2b(bc - ad)g^3(a + bx)} + \frac{3B^2d^2n^2 \log(a + bx)}{2b(bc - ad)^2g^3} - \frac{B^2d^2n^2 \log^2(a + bx)}{2b(bc - ad)^2g^3}
\end{aligned}$$

**Mathematica [C]** time = 0.552988, size = 463, normalized size = 1.61

$$\frac{Bn\left(2Bd^2n(a+bx)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)-2Bd^2n(a+bx)^2\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(a\*g + b\*g\*x)^3,x]

[Out] -(2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 + (B\*n\*(2\*(b\*c - a\*d)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 4\*d\*(-(b\*c) + a\*d)\*(a + b\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 4\*d^2\*(a + b\*x)^2\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(a\*g + b\*g\*x)^3



$$\begin{aligned} & (a + b*x)/(c + d*x)^n) + 4*d^2*(a + b*x)^2*(A + B*\text{Log}[e*((a + b*x)/(c + d \\ & *x))^n])*\text{Log}[c + d*x] - 4*B*d*n*(a + b*x)*(b*c - a*d + d*(a + b*x)*\text{Log}[a + \\ & b*x] - d*(a + b*x)*\text{Log}[c + d*x]) + B*n*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)* \\ & (a + b*x) - 2*d^2*(a + b*x)^2*\text{Log}[a + b*x] + 2*d^2*(a + b*x)^2*\text{Log}[c + d*x] \\ & ) + 2*B*d^2*n*(a + b*x)^2*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x)) \\ & / (b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*n*(a \\ & + b*x)^2*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \text{Log}[c + d*x])*\text{Log}[c + d*x] \\ & ] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2/(4*b*g^3*(a + \\ & b*x)^2) \end{aligned}$$

**Maple [F]** time = 0.44, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^3,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^3,x)

**Maxima [B]** time = 1.37337, size = 1162, normalized size = 4.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^3,x, algorithm="maxima")

[Out]  $\frac{1}{2}A*B*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*\log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*\log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) + 1/4*(2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*\log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*\log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3))*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - (b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(b*x + a))*\log(d*x + c))*n^2/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x)*B^2 - 1/2*B^2*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - A*B*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)$

**Fricas [B]** time = 0.979411, size = 1354, normalized size = 4.7

$$2A^2b^2c^2 - 4A^2abcd + 2A^2a^2d^2 + (B^2b^2c^2 - 8B^2abcd + 7B^2a^2d^2)n^2 + 2(B^2b^2c^2 - 2B^2abcd + B^2a^2d^2)\log(e)^2 - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (B^2*b^2*c^2 - 8*B^2*a*b*c*d + 7*B^2*a^2*d^2)*n^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d^2)*log(e)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*a*b*d^2*n^2*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A*B*b^2*c^2 - 4*A*B*a*b*c*d + 3*A*B*a^2*d^2)*n - 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2 + 2*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a^2*d^2 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x + (B^2*b^2*c^2 - 4*B^2*a*b*c*d + 3*B^2*a^2*d^2)*n - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*a*b*d^2*n*x - (B^2*b^2*c^2 - 2*B^2*a*b*c*d)*n)*log((b*x + a)/(d*x + c))*log(e) + 2*((B^2*b^2*c^2 - 4*B^2*a*b*c*d)*n^2 - (3*B^2*b^2*d^2*n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(A*B*b^2*c^2 - 2*A*B*a*b*c*d)*n - 2*(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B^2*a*b*d^2)*n^2)*x)*log((b*x + a)/(d*x + c))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2}{(bgx + ag)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x + a*g)^3, x)
```

$$3.17 \quad \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4} dx$$

**Optimal.** Leaf size=448

$$\frac{b^2(c+dx)^3 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}{3g^4(a+bx)^3(bc-ad)^3} - \frac{2b^2Bn(c+dx)^3 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{9g^4(a+bx)^3(bc-ad)^3} - \frac{d^2(c+dx) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{g^4(a+bx)(bc-ad)^3}$$

[Out]  $(-2*B^2*d^2*n^2*(c+d*x))/((b*c-a*d)^3*g^4*(a+b*x)) + (b*B^2*d*n^2*(c+d*x)^2)/(2*(b*c-a*d)^3*g^4*(a+b*x)^2) - (2*b^2*B^2*n^2*(c+d*x)^3)/(27*(b*c-a*d)^3*g^4*(a+b*x)^3) - (2*B*d^2*n*(c+d*x)*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/((b*c-a*d)^3*g^4*(a+b*x)) + (b*B*d*n*(c+d*x)^2*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/((b*c-a*d)^3*g^4*(a+b*x)^2) - (2*b^2*B*n*(c+d*x)^3*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(9*(b*c-a*d)^3*g^4*(a+b*x)^3) - (d^2*(c+d*x)*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))^2/((b*c-a*d)^3*g^4*(a+b*x)) + (b*d*(c+d*x)^2*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))^2/((b*c-a*d)^3*g^4*(a+b*x)^2) - (b^2*(c+d*x)^3*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))^2/(3*(b*c-a*d)^3*g^4*(a+b*x)^3)$

**Rubi [C]** time = 1.08656, antiderivative size = 736, normalized size of antiderivative = 1.64, number of steps used = 32, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2B^2d^3n^2\text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{3bg^4(bc-ad)^3} - \frac{2B^2d^3n^2\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3bg^4(bc-ad)^3} - \frac{2Bd^3n \log(a+bx) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{3bg^4(bc-ad)^3} + \frac{2Bd^3n \log(a+bx) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{3bg^4(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(a\*g + b\*g\*x)^4, x]

[Out]  $(-2*B^2*n^2)/(27*b*g^4*(a+b*x)^3) + (5*B^2*d*n^2)/(18*b*(b*c-a*d)*g^4*(a+b*x)^2) - (11*B^2*d^2*n^2)/(9*b*(b*c-a*d)^2*g^4*(a+b*x)) - (11*B^2*d^3*n^2*Log[a+b*x])/(9*b*(b*c-a*d)^3*g^4) + (B^2*d^3*n^2*Log[a+b*x]^2)/(3*b*(b*c-a*d)^3*g^4) - (2*B*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(9*b*g^4*(a+b*x)^3) + (B*d*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(3*b*(b*c-a*d)*g^4*(a+b*x)^2) - (2*B*d^2*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(3*b*(b*c-a*d)^2*g^4*(a+b*x)) - (2*B*d^3*n*Log[a+b*x]*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(3*b*(b*c-a*d)^3*g^4) - (A+B*Log[e*((a+b*x)/(c+d*x))^n])^2/(3*b*g^4*(a+b*x)^3) + (11*B^2*d^3*n^2*Log[c+d*x])/(9*b*(b*c-a*d)^3*g^4) - (2*B^2*d^3*n^2*Log[-((d*(a+b*x))/(b*c-a*d))]*Log[c+d*x])/(3*b*(b*c-a*d)^3*g^4) + (2*B*d^3*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n])*Log[c+d*x])/(3*b*(b*c-a*d)^3*g^4) + (B^2*d^3*n^2*Log[c+d*x]^2)/(3*b*(b*c-a*d)^3*g^4) - (2*B^2*d^3*n^2*Log[a+b*x]*Log[(b*(c+d*x))/(b*c-a*d)])/(3*b*(b*c-a*d)^3*g^4) - (2*B^2*d^3*n^2*PolyLog[2, -((d*(a+b*x))/(b*c-a*d))])/(3*b*(b*c-a*d)^3*g^4) - (2*B^2*d^3*n^2*PolyLog[2, (b*(c+d*x))/(b*c-a*d)])/(3*b*(b*c-a*d)^3*g^4)$

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^n\*(d\_.) + (e\_.)\*(x\_)]^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFX^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d

```
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x]
```

], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^4} dx &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^3(a+bx)^4(c+dx)} dx}{3bg} \\
 &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad)n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^4(c+dx)} dx}{3bg^4} \\
 &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B(bc-ad)n) \int \left(\frac{b\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2(a+bx)^4}\right) dx}{3bg^4} \\
 &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^4} dx}{3g^4} - \frac{(2Bd^3n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx} dx}{3(bc-ad)^3g^4} \\
 &= -\frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} + \frac{Bdn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b(bc-ad)^2g^4} \\
 &= -\frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} + \frac{Bdn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b(bc-ad)^2g^4} \\
 &= -\frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3} + \frac{Bdn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{2Bd^2n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b(bc-ad)^2g^4} \\
 &= -\frac{2B^2n^2}{27bg^4(a+bx)^3} + \frac{5B^2dn^2}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2n^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d^3n^2}{9b(bc-ad)^3g^4} \\
 &= -\frac{2B^2n^2}{27bg^4(a+bx)^3} + \frac{5B^2dn^2}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2n^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d^3n^2}{9b(bc-ad)^3g^4} \\
 &= -\frac{2B^2n^2}{27bg^4(a+bx)^3} + \frac{5B^2dn^2}{18b(bc-ad)g^4(a+bx)^2} - \frac{11B^2d^2n^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{11B^2d^3n^2}{9b(bc-ad)^3g^4}
 \end{aligned}$$

**Mathematica [C]** time = 0.803217, size = 609, normalized size = 1.36

$$Bn\left(-18Bd^3n(a+bx)^3\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+18Bd^3n(a+bx)^3\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)\right)-\log\left(\frac{d(a+bx)}{ad-bc}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(a\*g + b\*g\*x)^4,x]

[Out]  $-(18*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(12*(b*c - a*d)^3*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]) - 18*d*(b*c - a*d)^2*(a + b*x)*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]) + 36*d^2*(b*c - a*d)*(a + b*x)^2*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]) + 36*d^3*(a + b*x)^3*\text{Log}[a + b*x]*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]) - 36*d^3*(a + b*x)^3*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*\text{Log}[c + d*x] + 36*B*d^2*n*(a + b*x)^2*(b*c - a*d + d*(a + b*x)*\text{Log}[a + b*x] - d*(a + b*x)*\text{Log}[c + d*x]) - 9*B*d*n*(a + b*x)*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*\text{Log}[a + b*x] + 2*d^2*(a + b*x)^2*\text{Log}[c + d*x]) + 2*B*n*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*\text{Log}[a + b*x] - 6*d^3*(a + b*x)^3*\text{Log}[c + d*x]) - 18*B*d^3*n*(a + b*x)^3*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 18*B*d^3*n*(a + b*x)^3*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^3)/(54*b*g^4*(a + b*x)^3)$

**Maple [F]** time = 0.439, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^4} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^4,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^4,x)

**Maxima [B]** time = 1.63949, size = 1933, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^4,x, algorithm="maxima")

[Out]  $-1/9*A*B*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*\text{log}(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*\text{log}(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 1/54*(6*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*\text{log}(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*\text{log}(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4))*\text{log}(e*(b*x/(d*x + c) + a/(d*x + c))^n) + (4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*$

$$\log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3) * \log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3) * \log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3) * \log(b*x + a)) * \log(d*x + c) * n^2 / (a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4) * x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4) * x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*b^2*d^3*g^4) * x) * B^2 - 1/3*B^2 * \log(e*(b*x/(d*x + c) + a/(d*x + c)))^n) / (b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 2/3*A*B * \log(e*(b*x/(d*x + c) + a/(d*x + c)))^n) / (b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A^2 / (b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)$$

**Fricas [B]** time = 1.06668, size = 2383, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^4,x, algorithm="fricas")

[Out] 
$$-1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a^3*d^3 + (4*B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 108*B^2*a^2*b*c*d^2 - 85*B^2*a^3*d^3) * n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3) * n^2 + 6*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3) * n) * x^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2 - B^2*a^3*d^3) * \log(e)^2 + 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n^2*x^2 + 3*B^2*a^2*b*d^3*n^2*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2) * n^2) * \log((b*x + a)/(d*x + c))^2 + 6*(2*A*B*b^3*c^3 - 9*A*B*a*b^2*c^2*d + 18*A*B*a^2*b*c*d^2 - 11*A*B*a^3*d^3) * n - 3*((5*B^2*b^3*c^2*d - 54*B^2*a*b^2*c*d^2 + 49*B^2*a^2*b*d^3) * n^2 + 6*(A*B*b^3*c^2*d - 6*A*B*a*b^2*c*d^2 + 5*A*B*a^2*b*d^3) * n) * x + 6*(6*A*B*b^3*c^3 - 18*A*B*a*b^2*c^2*d + 18*A*B*a^2*b*c*d^2 - 6*A*B*a^3*d^3 + 6*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3) * n) * x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3) * n * x + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2 - 11*B^2*a^3*d^3) * n + 6*(B^2*b^3*d^3*n*x^3 + 3*B^2*a*b^2*d^3*n*x^2 + 3*B^2*a^2*b*d^3*n*x + (B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2) * n) * \log((b*x + a)/(d*x + c)) * \log(e) + 6*((11*B^2*b^3*d^3*n^2 + 6*A*B*b^3*d^3*n) * x^3 + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2) * n^2 + 3*(6*A*B*a*b^2*d^3*n + (2*B^2*b^3*c*d^2 + 9*B^2*a*b^2*d^3) * n^2) * x^2 + 6*(A*B*b^3*c^3 - 3*A*B*a*b^2*c^2*d + 3*A*B*a^2*b*c*d^2) * n + 3*(6*A*B*a^2*b*d^3*n - (B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 6*B^2*a^2*b*d^3) * n^2) * x) * \log((b*x + a)/(d*x + c))) / ((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3) * g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3) * g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3) * g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3) * g^4)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2/(b\*g\*x+a\*g)\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2}{(bgx + ag)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^4,x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2/(b\*g\*x + a\*g)^4, x)



$$3.18 \quad \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5} dx$$

**Optimal.** Leaf size=615

$$\frac{b^3(c+dx)^4 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}{4g^5(a+bx)^4(bc-ad)^4} - \frac{b^3 B n (c+dx)^4 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{8g^5(a+bx)^4(bc-ad)^4} + \frac{b^2 d (c+dx)^3 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{g^5(a+bx)^3(bc-ad)^4}$$

[Out]  $(2*B^2*d^3*n^2*(c+d*x))/((b*c-a*d)^4*g^5*(a+b*x)) - (3*b*B^2*d^2*n^2*(c+d*x)^2)/(4*(b*c-a*d)^4*g^5*(a+b*x)^2) + (2*b^2*B^2*d*n^2*(c+d*x)^3)/(9*(b*c-a*d)^4*g^5*(a+b*x)^3) - (b^3*B^2*n^2*(c+d*x)^4)/(32*(b*c-a*d)^4*g^5*(a+b*x)^4) + (2*B*d^3*n*(c+d*x)*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/((b*c-a*d)^4*g^5*(a+b*x)) - (3*b*B*d^2*n*(c+d*x)^2*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(2*(b*c-a*d)^4*g^5*(a+b*x)^2) + (2*b^2*B*d*n*(c+d*x)^3*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(3*(b*c-a*d)^4*g^5*(a+b*x)^3) - (b^3*B*n*(c+d*x)^4*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(8*(b*c-a*d)^4*g^5*(a+b*x)^4) + (d^3*(c+d*x)*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))^2/((b*c-a*d)^4*g^5*(a+b*x)) - (3*b*d^2*(c+d*x)^2*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))^2/(2*(b*c-a*d)^4*g^5*(a+b*x)^2) + (b^2*d*(c+d*x)^3*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))^2/((b*c-a*d)^4*g^5*(a+b*x)^3) - (b^3*(c+d*x)^4*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))^2/(4*(b*c-a*d)^4*g^5*(a+b*x)^4)$

**Rubi [C]** time = 1.31405, antiderivative size = 826, normalized size of antiderivative = 1.34, number of steps used = 36, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2 n^2 \log^2(a+bx) d^4}{4b(bc-ad)^4 g^5} - \frac{B^2 n^2 \log^2(c+dx) d^4}{4b(bc-ad)^4 g^5} + \frac{25 B^2 n^2 \log(a+bx) d^4}{24b(bc-ad)^4 g^5} + \frac{B n \log(a+bx) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) d^4}{2b(bc-ad)^4 g^5}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(a\*g + b\*g\*x)^5, x]

[Out]  $-(B^2*n^2)/(32*b*g^5*(a+b*x)^4) + (7*B^2*d*n^2)/(72*b*(b*c-a*d)*g^5*(a+b*x)^3) - (13*B^2*d^2*n^2)/(48*b*(b*c-a*d)^2*g^5*(a+b*x)^2) + (25*B^2*d^3*n^2)/(24*b*(b*c-a*d)^3*g^5*(a+b*x)) + (25*B^2*d^4*n^2*Log[a+b*x])/(24*b*(b*c-a*d)^4*g^5) - (B^2*d^4*n^2*Log[a+b*x]^2)/(4*b*(b*c-a*d)^4*g^5) - (B*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(8*b*g^5*(a+b*x)^4) + (B*d*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(6*b*(b*c-a*d)*g^5*(a+b*x)^3) - (B*d^2*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(4*b*(b*c-a*d)^2*g^5*(a+b*x)^2) + (B*d^3*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(2*b*(b*c-a*d)^3*g^5*(a+b*x)) + (B*d^4*n*Log[a+b*x]*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(2*b*(b*c-a*d)^4*g^5) - (A+B*Log[e*((a+b*x)/(c+d*x))^n])^2/(4*b*g^5*(a+b*x)^4) - (25*B^2*d^4*n^2*Log[c+d*x])/(24*b*(b*c-a*d)^4*g^5) + (B^2*d^4*n^2*Log[-((d*(a+b*x))/(b*c-a*d))]*Log[c+d*x])/(2*b*(b*c-a*d)^4*g^5) - (B*d^4*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n])*Log[c+d*x])/(2*b*(b*c-a*d)^4*g^5) - (B^2*d^4*n^2*Log[c+d*x]^2)/(4*b*(b*c-a*d)^4*g^5) + (B^2*d^4*n^2*Log[a+b*x]*Log[(b*(c+d*x))/(b*c-a*d)])/(2*b*(b*c-a*d)^4*g^5) + (B^2*d^4*n^2*PolyLog[2, -((d*(a+b*x))/(b*c-a*d))])/(2*b*(b*c-a*d)^4*g^5) + (B^2*d^4*n^2*PolyLog[2, (b*(c+d*x))/(b*c-a*d)])/(2*b*(b*c-a*d)^4*g^5)$

Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
```

, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_.)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^n)]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^5} dx &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4bg^5(a + bx)^4} + \frac{(Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^4(a+bx)^5(c+dx)} dx}{2bg} \\
 &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4bg^5(a + bx)^4} + \frac{(B(bc - ad)n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^5(c+dx)} dx}{2bg^5} \\
 &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4bg^5(a + bx)^4} + \frac{(B(bc - ad)n) \int \left(\frac{b\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(a+bx)^5} - \frac{bd\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2(a+bx)^4}\right) dx}{2bg^5} \\
 &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4bg^5(a + bx)^4} + \frac{(Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)^5} dx}{2g^5} + \frac{(Bd^4n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx} dx}{2(bc - ad)^4g^5} \\
 &= -\frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8bg^5(a + bx)^4} + \frac{Bdn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b(bc - ad)g^5(a + bx)^3} - \frac{Bd^2n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b(bc - ad)^2g^5(a + bx)^2} \\
 &= -\frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8bg^5(a + bx)^4} + \frac{Bdn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b(bc - ad)g^5(a + bx)^3} - \frac{Bd^2n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b(bc - ad)^2g^5(a + bx)^2} \\
 &= -\frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8bg^5(a + bx)^4} + \frac{Bdn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b(bc - ad)g^5(a + bx)^3} - \frac{Bd^2n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b(bc - ad)^2g^5(a + bx)^2} \\
 &= -\frac{B^2n^2}{32bg^5(a + bx)^4} + \frac{7B^2dn^2}{72b(bc - ad)g^5(a + bx)^3} - \frac{13B^2d^2n^2}{48b(bc - ad)^2g^5(a + bx)^2} + \frac{2B^2dn^2}{24b(bc - ad)g^5(a + bx)} \\
 &= -\frac{B^2n^2}{32bg^5(a + bx)^4} + \frac{7B^2dn^2}{72b(bc - ad)g^5(a + bx)^3} - \frac{13B^2d^2n^2}{48b(bc - ad)^2g^5(a + bx)^2} + \frac{2B^2dn^2}{24b(bc - ad)g^5(a + bx)} \\
 &= -\frac{B^2n^2}{32bg^5(a + bx)^4} + \frac{7B^2dn^2}{72b(bc - ad)g^5(a + bx)^3} - \frac{13B^2d^2n^2}{48b(bc - ad)^2g^5(a + bx)^2} + \frac{2B^2dn^2}{24b(bc - ad)g^5(a + bx)}
 \end{aligned}$$

**Mathematica [C]** time = 1.18765, size = 776, normalized size = 1.26

$$Bn \left( 72Bd^4n(a+bx)^4 \left( \log(a+bx) \left( \log(a+bx) - 2 \log\left(\frac{b(c+dx)}{bc-ad}\right) \right) - 2 \operatorname{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right) \right) - 72Bd^4n(a+bx)^4 \left( 2 \operatorname{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right) + \log(c+dx) \left( 2 \log\left(\frac{d(a+bx)}{ad-bc}\right) - \log(c+dx) \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(a\*g + b\*g\*x)^5,x]

[Out]  $-(72*(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(36*(b*c - a*d)^4*(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n]) + 48*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n]) + 72*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n]) + 144*d^3*(-(b*c) + a*d)*(a + b*x)^3*(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n]) - 144*d^4*(a + b*x)^4*\operatorname{Log}[a + b*x]*(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n]) + 144*d^4*(a + b*x)^4*(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n])*\operatorname{Log}[c + d*x] - 144*B*d^3*n*(a + b*x)^3*(b*c - a*d + d*(a + b*x)*\operatorname{Log}[a + b*x] - d*(a + b*x)*\operatorname{Log}[c + d*x]) + 36*B*d^2*n*(a + b*x)^2*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*\operatorname{Log}[a + b*x] + 2*d^2*(a + b*x)^2*\operatorname{Log}[c + d*x]) - 8*B*d*n*(a + b*x)*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*\operatorname{Log}[a + b*x] - 6*d^3*(a + b*x)^3*\operatorname{Log}[c + d*x]) + 3*B*n*(3*(b*c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*\operatorname{Log}[a + b*x] + 12*d^4*(a + b*x)^4*\operatorname{Log}[c + d*x]) + 72*B*d^4*n*(a + b*x)^4*(\operatorname{Log}[a + b*x]*(\operatorname{Log}[a + b*x] - 2*\operatorname{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\operatorname{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 72*B*d^4*n*(a + b*x)^4*((2*\operatorname{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \operatorname{Log}[c + d*x])*\operatorname{Log}[c + d*x] + 2*\operatorname{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^4/(288*b*g^5*(a + b*x)^4)$

**Maple [F]** time = 0.451, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^5} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^5,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^5,x)

**Maxima [B]** time = 2.03168, size = 2884, normalized size = 4.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^5,x, algorithm="maxima")

[Out]  $\frac{1}{24}A*B*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*$

$$\begin{aligned}
& d^3)g^5x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5 \\
& + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) + 1/288*(12*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a))*log(d*x + c))*n^2/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 - 4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 - 1/4*B^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/2*A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
\end{aligned}$$

**Fricas [B]** time = 1.15835, size = 3615, normalized size = 5.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^5,x, algorithm="fricas")

[Out]  $-1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 288*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 - 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*n^2 + 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (9*B^2*b^4*c^4 - 64*B^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 576*B^2*a^3*b*c*d^3 + 415*B^2*a^4*d^4)*n^2 + 6*((13*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 163*B^2*a^2*b^2*d^4)*n^2 + 12*(A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)*n)*x^2 + 72*(B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3$

```

*b*c*d^3 + B^2*a^4*d^4)*log(e)^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^
4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n^2*x^2 + 4*B^2*a^3*b*d^4*n^2*x - (B^2*b^4*c^
4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n^2)*log
((b*x + a)/(d*x + c))^2 + 12*(3*A*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d + 36*A*B*a
^2*b^2*c^2*d^2 - 48*A*B*a^3*b*c*d^3 + 25*A*B*a^4*d^4)*n - 4*((7*B^2*b^4*c^3
*d - 60*B^2*a*b^3*c^2*d^2 + 324*B^2*a^2*b^2*c*d^3 - 271*B^2*a^3*b*d^4)*n^2
+ 12*(A*B*b^4*c^3*d - 6*A*B*a*b^3*c^2*d^2 + 18*A*B*a^2*b^2*c*d^3 - 13*A*B*a
^3*b*d^4)*n)*x + 12*(12*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 72*A*B*a^2*b^2*c
^2*d^2 - 48*A*B*a^3*b*c*d^3 + 12*A*B*a^4*d^4 - 12*(B^2*b^4*c*d^3 - B^2*a*b^
3*d^4)*n*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 7*B^2*a^2*b^2*d^4)*
n*x^2 - 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 - 13*
B^2*a^3*b*d^4)*n*x + (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c
^2*d^2 - 48*B^2*a^3*b*c*d^3 + 25*B^2*a^4*d^4)*n - 12*(B^2*b^4*d^4*n*x^4 + 4
*B^2*a*b^3*d^4*n*x^3 + 6*B^2*a^2*b^2*d^4*n*x^2 + 4*B^2*a^3*b*d^4*n*x - (B^2
*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3)*n
)*log((b*x + a)/(d*x + c))*log(e) - 12*((25*B^2*b^4*d^4*n^2 + 12*A*B*b^4*d
^4*n)*x^4 + 4*(12*A*B*a*b^3*d^4*n + (3*B^2*b^4*c*d^3 + 22*B^2*a*b^3*d^4)*n^
2)*x^3 - (3*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c^2*d^2 - 48*
B^2*a^3*b*c*d^3)*n^2 + 6*(12*A*B*a^2*b^2*d^4*n - (B^2*b^4*c^2*d^2 - 8*B^2*a
*b^3*c*d^3 - 18*B^2*a^2*b^2*d^4)*n^2)*x^2 - 12*(A*B*b^4*c^4 - 4*A*B*a*b^3*c
^3*d + 6*A*B*a^2*b^2*c^2*d^2 - 4*A*B*a^3*b*c*d^3)*n + 4*(12*A*B*a^3*b*d^4*n
+ (B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 12*B^2*a^3
*b*d^4)*n^2)*x)*log((b*x + a)/(d*x + c))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2
*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^
2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 +
6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^
6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 -
4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*
a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)

```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*n))\*\*2/(b\*g\*x+a\*g)\*\*5,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}{(bgx + ag)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(b\*g\*x+a\*g)^5,x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2/(b\*g\*x + a\*g)^5, x)

$$3.19 \quad \int \frac{(ag+bgx)^2}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

**Optimal.** Leaf size=37

$$\text{Unintegrable}\left[\frac{(ag + bgx)^2}{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}, x\right]$$

[Out] Unintegrable[(a\*g + b\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

**Rubi [A]** time = 0.232895, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(ag + bgx)^2}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]),x]

[Out] a^2\*g^2\*Defer[Int][(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^(-1), x] + 2\*a\*b\*g^2\*Defer[Int][x/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x] + b^2\*g^2\*Defer[Int][x^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

Rubi steps

$$\begin{aligned} \int \frac{(ag + bgx)^2}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx &= \int \left( \frac{a^2g^2}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{2abg^2x}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{b^2g^2x^2}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} \right) dx \\ &= (a^2g^2) \int \frac{1}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx + (2abg^2) \int \frac{x}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx + (b^2g^2) \int \frac{x^2}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.748981, size = 0, normalized size = 0.

$$\int \frac{(ag + bgx)^2}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]),x]

[Out] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

**Maple [A]** time = 0.401, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out] integrate((b\*g\*x + a\*g)^2/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b^2 g^2 x^2 + 2 a b g^2 x + a^2 g^2}{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] integral((b^2\*g^2\*x^2 + 2\*a\*b\*g^2\*x + a^2\*g^2)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n)),x)

[Out] Timed out



**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^2/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

$$3.20 \quad \int \frac{ag+bgx}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

**Optimal.** Leaf size=35

$$\text{Unintegrable}\left(\frac{ag + bgx}{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

**Rubi [A]** time = 0.114719, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{ag + bgx}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] a\*g\*Defer[Int][(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]^(-1), x] + b\*g\*Defer[Int][x/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

Rubi steps

$$\begin{aligned} \int \frac{ag + bgx}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx &= \int \left( \frac{ag}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{bgx}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} \right) dx \\ &= (ag) \int \frac{1}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx + (bg) \int \frac{x}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.284075, size = 0, normalized size = 0.

$$\int \frac{ag + bgx}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

**Maple [A]** time = 0.307, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int((b\*g\*x+a\*g)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bgx + ag}{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out] integrate((b\*g\*x + a\*g)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{bgx + ag}{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] integral((b\*g\*x + a\*g)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c)\*\*n))),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bgx + ag}{B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

$$3.21 \quad \int \frac{1}{(ag+bgx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx$$

**Optimal.** Leaf size=37

$$\text{Unintegrable}\left(\frac{1}{(ag + bgx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}, x\right)$$

[Out] Unintegrable[1/((a\*g + b\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

**Rubi [A]** time = 0.081441, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag + bgx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

Rubi steps

$$\int \frac{1}{(ag + bgx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx = \int \frac{1}{(ag + bgx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx$$

**Mathematica [A]** time = 0.144034, size = 0, normalized size = 0.

$$\int \frac{1}{(ag + bgx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

[Out] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

**Maple [A]** time = 0.546, size = 0, normalized size = 0.

$$\int \frac{1}{bgx + ag}\left(A + B \ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

[Out] `int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)\left(B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")`

[Out] `integrate(1/((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{Abgx + Aag + (Bbgx + Bag)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")`

[Out] `integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log(e*((b*x + a)/(d*x + c))^n)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)\left(B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

$$3.22 \quad \int \frac{1}{(ag+bgx)^2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

**Optimal.** Leaf size=94

$$\frac{e^{\frac{A}{Bn}}(c+dx) \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{\frac{1}{n}} \operatorname{Ei} \left( -\frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right)}{Bg^2n(a+bx)(bc-ad)}$$

[Out] (E^(A/(B\*n)))\*(e\*((a + b\*x)/(c + d\*x))^n)^n^(-1)\*(c + d\*x)\*ExpIntegralEi[-((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(B\*n))]/(B\*(b\*c - a\*d)\*g^2\*n\*(a + b\*x))

**Rubi [F]** time = 0.0990798, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag + bgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])),x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

Rubi steps

$$\int \frac{1}{(ag + bgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx = \int \frac{1}{(ag + bgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

**Mathematica [A]** time = 0.131033, size = 94, normalized size = 1.

$$\frac{e^{\frac{A}{Bn}}(c+dx) \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{\frac{1}{n}} \operatorname{Ei} \left( -\frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right)}{Bg^2n(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])),x]

[Out] (E^(A/(B\*n)))\*(e\*((a + b\*x)/(c + d\*x))^n)^n^(-1)\*(c + d\*x)\*ExpIntegralEi[-((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(B\*n))]/(B\*(b\*c - a\*d)\*g^2\*n\*(a + b\*x))



**Maple [F]** time = 0.444, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out] integrate(1/((b\*g\*x + a\*g)^2\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)), x)

**Fricas [A]** time = 0.867845, size = 149, normalized size = 1.59

$$\frac{e^{\left(\frac{B \log(e)+A}{Bn}\right)} \log\_integral \left( \frac{(dx+c)e^{\left(-\frac{B \log(e)+A}{Bn}\right)}}{bx+a} \right)}{(Bbc - Bad)g^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] e^((B\*log(e) + A)/(B\*n))\*log\_integral((d\*x + c)\*e^(-(B\*log(e) + A)/(B\*n))/(b\*x + a))/((B\*b\*c - B\*a\*d)\*g^2\*n)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)\*\*2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out] integrate(1/((b\*g\*x + a\*g)^2\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)), x)

$$3.23 \quad \int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

**Optimal.** Leaf size=197

$$\frac{be^{\frac{2A}{Bn}}(c+dx)^2 \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{2/n} \operatorname{Ei} \left( -\frac{2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{Bn} \right)}{Bg^3 n(a+bx)^2 (bc-ad)^2} - \frac{de^{\frac{A}{Bn}}(c+dx) \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{\frac{1}{n}} \operatorname{Ei} \left( -\frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right)}{Bg^3 n(a+bx)(bc-ad)^2}$$

[Out] (b\*E^((2\*A)/(B\*n))\*(e\*((a+b\*x)/(c+d\*x))^n)^(2/n)\*(c+d\*x)^2\*ExpIntegralEi[(-2\*(A+B\*Log[e\*((a+b\*x)/(c+d\*x))^n])/(B\*n))]/(B\*(b\*c-a\*d)^2\*g^3\*n\*(a+b\*x)^2) - (d\*E^(A/(B\*n))\*(e\*((a+b\*x)/(c+d\*x))^n)^(1/n)\*(c+d\*x)\*ExpIntegralEi[-((A+B\*Log[e\*((a+b\*x)/(c+d\*x))^n])/(B\*n))])/(B\*(b\*c-a\*d)^2\*g^3\*n\*(a+b\*x))

**Rubi [F]** time = 0.0821707, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx = \int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

**Mathematica [A]** time = 0.2969, size = 172, normalized size = 0.87

$$\frac{e^{\frac{A}{Bn}}(c+dx) \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{\frac{1}{n}} \left( be^{\frac{A}{Bn}}(c+dx) \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{\frac{1}{n}} \operatorname{Ei} \left( -\frac{2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{Bn} \right) - d(a+bx) \operatorname{Ei} \left( -\frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right) \right)}{Bg^3 n(a+bx)^2 (bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

[Out] (E^(A/(B\*n))\*(e\*((a+b\*x)/(c+d\*x))^n)^(1/n)\*(c+d\*x)\*(b\*E^(A/(B\*n))\*(e\*((a+b\*x)/(c+d\*x))^n)^(1/n)\*(c+d\*x)\*ExpIntegralEi[(-2\*(A+B\*Log[e\*((a+b\*x)/(c+d\*x))^n])/(B\*n)] - d\*(a+b\*x)\*ExpIntegralEi[-((A+B\*Log[e\*((a+b\*x)/(c+d\*x))^n])/(B\*n))])/(B\*(b\*c-a\*d)^2\*g^3\*n\*(a+b\*x)^2)

---

**Maple [F]** time = 0.437, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^3/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int(1/(b\*g\*x+a\*g)^3/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out] integrate(1/((b\*g\*x + a\*g)^3\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)), x)

---

**Fricas [A]** time = 0.936258, size = 354, normalized size = 1.8

$$\frac{de^{\left(\frac{B \log(e)+A}{Bn}\right)} \log\_integral\left(\frac{(dx+c)e^{\left(-\frac{B \log(e)+A}{Bn}\right)}}{bx+a}\right) - be^{\left(\frac{2(B \log(e)+A)}{Bn}\right)} \log\_integral\left(\frac{(d^2x^2+2cdx+c^2)e^{\left(-\frac{2(B \log(e)+A)}{Bn}\right)}}{b^2x^2+2abx+a^2}\right)}{(Bb^2c^2 - 2Babcd + Ba^2d^2)g^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] -(d\*e^((B\*log(e) + A)/(B\*n))\*log\_integral((d\*x + c)\*e^(-(B\*log(e) + A)/(B\*n)))/(b\*x + a)) - b\*e^(2\*(B\*log(e) + A)/(B\*n))\*log\_integral((d^2\*x^2 + 2\*c\*d\*x + c^2)\*e^(-2\*(B\*log(e) + A)/(B\*n))/(b^2\*x^2 + 2\*a\*b\*x + a^2)))/((B\*b^2\*c^2 - 2\*B\*a\*b\*c\*d + B\*a^2\*d^2)\*g^3\*n)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)\*\*3/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out] integrate(1/((b\*g\*x + a\*g)^3\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)), x)

$$3.24 \quad \int \frac{(ag+bgx)^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

**Optimal.** Leaf size=37

$$\text{Unintegrable}\left[\frac{(ag + bgx)^2}{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}, x\right]$$

[Out] Unintegrable[(a\*g + b\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

**Rubi [A]** time = 0.237531, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(ag + bgx)^2}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] a^2\*g^2\*Defer[Int][(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^(-2), x] + 2\*a\*b\*g^2\*Defer[Int][x/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x] + b^2\*g^2\*Defer[Int][x^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{(ag + bgx)^2}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx &= \int \left( \frac{a^2g^2}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{2abg^2x}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{b^2g^2x^2}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} \right) dx \\ &= (a^2g^2) \int \frac{1}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx + (2abg^2) \int \frac{x}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx + (b^2g^2) \int \frac{x^2}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.983394, size = 0, normalized size = 0.

$$\int \frac{(ag + bgx)^2}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

---

**Maple [A]** time = 0.395, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{b^3 dg^2 x^4 + a^3 cg^2 + (b^3 cg^2 + 3 ab^2 dg^2) x^3 + 3 (ab^2 cg^2 + a^2 bdg^2) x^2 + (3 a^2 bcg^2 + a^3 dg^2) x}{(bcn - adn) B^2 \log((bx + a)^n) - (bcn - adn) B^2 \log((dx + c)^n) + (bcn - adn) AB + (bcn \log(e) - adn \log(e)) B^2} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out]  $-(b^3 d g^2 x^4 + a^3 c g^2 + (b^3 c g^2 + 3 a b^2 d g^2) x^3 + 3 (a b^2 c g^2 + a^2 b d g^2) x^2 + (3 a^2 b c g^2 + a^3 d g^2) x) / ((b c n - a d n) B^2 \log((b x + a)^n) - (b c n - a d n) B^2 \log((d x + c)^n) + (b c n - a d n) A B + (b c n \log(e) - a d n \log(e)) B^2) + \text{integrate}((4 b^3 d g^2 x^3 + 3 a^2 b c g^2 + a^3 d g^2 + 3 (b^3 c g^2 + 3 a b^2 d g^2) x^2 + 6 (a b^2 c g^2 + a^2 b d g^2) x) / ((b c n - a d n) B^2 \log((b x + a)^n) - (b c n - a d n) B^2 \log((d x + c)^n) + (b c n - a d n) A B + (b c n \log(e) - a d n \log(e)) B^2), x)$

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b^2 g^2 x^2 + 2 a b g^2 x + a^2 g^2}{B^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2 A B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out] integral((b^2\*g^2\*x^2 + 2\*a\*b\*g^2\*x + a^2\*g^2)/(B^2\*log(e\*((b\*x + a)/(d\*x + c))^n)^2 + 2\*A\*B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{\left(B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^2/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2, x)



$$3.25 \quad \int \frac{ag+bgx}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

**Optimal.** Leaf size=35

$$\text{Unintegrable} \left( \frac{ag + bgx}{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}, x \right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

**Rubi [A]** time = 0.125693, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{ag + bgx}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

[Out] a\*g\*Defer[Int][(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^(-2), x] + b\*g\*Defer[Int][x/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{ag + bgx}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx &= \int \left( \frac{ag}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{bgx}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} \right) dx \\ &= (ag) \int \frac{1}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx + (bg) \int \frac{x}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.93374, size = 0, normalized size = 0.

$$\int \frac{ag + bgx}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

[Out] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

**Maple [A]** time = 0.254, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((b\*g\*x+a\*g)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{b^2 dx^3 + a^2 cg + (b^2 cg + 2 abdg)x^2 + (2 abcg + a^2 dg)x}{(bcn - adn)B^2 \log((bx + a)^n) - (bcn - adn)B^2 \log((dx + c)^n) + (bcn - adn)AB + (bcn \log(e) - adn \log(e))B^2} + \int \frac{bcn - adn}{(bcn - adn)B^2 \log((bx + a)^n) - (bcn - adn)B^2 \log((dx + c)^n) + (bcn - adn)AB + (bcn \log(e) - adn \log(e))B^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out] -(b^2\*d\*g\*x^3 + a^2\*c\*g + (b^2\*c\*g + 2\*a\*b\*d\*g)\*x^2 + (2\*a\*b\*c\*g + a^2\*d\*g)\*x)/((b\*c\*n - a\*d\*n)\*B^2\*log((b\*x + a)^n) - (b\*c\*n - a\*d\*n)\*B^2\*log((d\*x + c)^n) + (b\*c\*n - a\*d\*n)\*A\*B + (b\*c\*n\*log(e) - a\*d\*n\*log(e))\*B^2) + integrate((3\*b^2\*d\*g\*x^2 + 2\*a\*b\*c\*g + a^2\*d\*g + 2\*(b^2\*c\*g + 2\*a\*b\*d\*g)\*x)/((b\*c\*n - a\*d\*n)\*B^2\*log((b\*x + a)^n) - (b\*c\*n - a\*d\*n)\*B^2\*log((d\*x + c)^n) + (b\*c\*n - a\*d\*n)\*A\*B + (b\*c\*n\*log(e) - a\*d\*n\*log(e))\*B^2), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{bgx + ag}{B^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2AB \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out] integral((b\*g\*x + a\*g)/(B^2\*log(e\*((b\*x + a)/(d\*x + c))^n))^2 + 2\*A\*B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c)\*\*n))\*\*2,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bgx + ag}{\left(B \log\left(e \left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2, x)

$$3.26 \quad \int \frac{1}{(ag+bgx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

**Optimal.** Leaf size=37

$$\text{Unintegrable}\left(\frac{1}{(ag+bgx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[1/((a\*g + b\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

**Rubi [A]** time = 0.0908582, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx = \int \frac{1}{(ag+bgx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

**Mathematica [A]** time = 0.530636, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

[Out] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

**Maple [A]** time = 0.437, size = 0, normalized size = 0.

$$\int \frac{1}{bgx+ag}\left(A+B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$d \int \frac{1}{(bcgn - adgn)B^2 \log((bx + a)^n) - (bcgn - adgn)B^2 \log((dx + c)^n) + (bcgn - adgn)AB + (bcgn \log(e) - adgn \log(e))B^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
```

```
[Out] d*integrate(1/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d*g*n*log(e))*B^2), x) - (d*x + c)/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d*g*n*log(e))*B^2)
```

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2bgx + A^2ag + (B^2bgx + B^2ag) \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2(ABbgx + ABag) \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*ln(e*((b*x+a)/(d*x+c)**n))**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag) \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate(1/((b\*g\*x + a\*g)\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2), x)

$$3.27 \quad \int \frac{1}{(ag+bgx)^2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

**Optimal.** Leaf size=153

$$\frac{e^{\frac{A}{Bn}}(c+dx) \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{\frac{1}{n}} \operatorname{Ei} \left( -\frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right)}{B^2 g^2 n^2 (a+bx)(bc-ad)} - \frac{c+dx}{Bg^2 n(a+bx)(bc-ad) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}$$

[Out]  $-\left(\left(E^{\left(\frac{A}{Bn}\right)}\right)\left(e^{\left(\frac{a+bx}{c+dx}\right)^n}\right)^{\frac{1}{n}}\right)\left(-\frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn}\right) \operatorname{ExpIntegralEi}\left[-\frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn}\right]\right) / \left(B^2 (bc-ad) g^2 n^2 (a+bx)\right) - \frac{(c+dx)}{Bg^2 n(a+bx)(bc-ad) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}$

**Rubi [F]** time = 0.104589, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)^2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2),x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)^2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx = \int \frac{1}{(ag+bgx)^2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

**Mathematica [A]** time = 0.182995, size = 146, normalized size = 0.95

$$\frac{(c+dx) \left( e^{\frac{A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{\frac{1}{n}} \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) \operatorname{Ei} \left( -\frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right) + Bn \right)}{B^2 g^2 n^2 (a+bx)(bc-ad) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2),x]

[Out]  $-\left(\left((c+dx)\left(Bn+E^{\left(\frac{A}{Bn}\right)}\right)\left(e^{\left(\frac{a+bx}{c+dx}\right)^n}\right)^{\frac{1}{n}}\right)\left(-\frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn}\right) \operatorname{ExpIntegralEi}\left[-\frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn}\right]\right)\left(A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A\right) / \left(B^2 (bc-ad) g^2 n^2 (a+bx)\right) - \frac{(c+dx)}{Bg^2 n(a+bx)(bc-ad) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}$

$b*x)/(c + d*x))^n]))))$

**Maple [F]** time = 0.441, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$(abcg^2n - a^2dg^2n)AB + (abcg^2n \log(e) - a^2dg^2n \log(e))B^2 + ((b^2cg^2n - abdg^2n)AB + (b^2cg^2n \log(e) - abdg^2n \log(e))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out]  $-(d*x + c)/((a*b*c*g^2*n - a^2*d*g^2*n)*A*B + (a*b*c*g^2*n*\log(e) - a^2*d*g^2*n*\log(e))*B^2 + ((b^2*c*g^2*n - a*b*d*g^2*n)*A*B + (b^2*c*g^2*n*\log(e) - a*b*d*g^2*n*\log(e))*B^2)*x + ((b^2*c*g^2*n - a*b*d*g^2*n)*B^2*x + (a*b*c*g^2*n - a^2*d*g^2*n)*B^2)*\log((b*x + a)^n) - ((b^2*c*g^2*n - a*b*d*g^2*n)*B^2*x + (a*b*c*g^2*n - a^2*d*g^2*n)*B^2)*\log((d*x + c)^n) + \text{integrate}(-1/(B^2*a^2*g^2*n*\log(e) + A*B*a^2*g^2*n + (B^2*b^2*g^2*n*\log(e) + A*B*b^2*g^2*n)*x^2 + 2*(B^2*a*b*g^2*n*\log(e) + A*B*a*b*g^2*n)*x + (B^2*b^2*g^2*n*x^2 + 2*B^2*a*b*g^2*n*x + B^2*a^2*g^2*n)*\log((b*x + a)^n) - (B^2*b^2*g^2*n*x^2 + 2*B^2*a*b*g^2*n*x + B^2*a^2*g^2*n)*\log((d*x + c)^n)), x)$

**Fricas [A]** time = 0.930972, size = 598, normalized size = 3.91

$$Bdnx + Bcn + \left( Abx + Aa + (Bbx + Ba) \log(e) + (Bbnx + Ban) \log\left(\frac{bx+a}{dx+c}\right) \right) e^{\left(\frac{B \log(e)+A}{Bn}\right)} \log_i$$

$$(AB^2b^2c - AB^2abd)g^2n^2x + (AB^2abc - AB^2a^2d)g^2n^2 + ((B^3b^2c - B^3abd)g^2n^2x + (B^3abc - B^3a^2d)g^2n^2) \log(e) + ((B^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out]  $-(B*d*n*x + B*c*n + (A*b*x + A*a + (B*b*x + B*a)*\log(e) + (B*b*n*x + B*a*n)*\log((b*x + a)/(d*x + c)))*e^{((B*\log(e) + A)/(B*n))*\log\_integral((d*x + c)*e^{-(B*\log(e) + A)/(B*n)})/(b*x + a))}/((A*B^2*b^2*c - A*B^2*a*b*d)*g^2*n^2*x + (A*B^2*a*b*c - A*B^2*a^2*d)*g^2*n^2 + ((B^3*b^2*c - B^3*a*b*d)*g^2*n^2*x + (B^3*a*b*c - B^3*a^2*d)*g^2*n^2)*\log(e) + ((B^3*b^2*c - B^3*a*b*d)*g^2*$



$$n^3*x + (B^3*a*b*c - B^3*a^2*d)*g^2*n^3)*\log((b*x + a)/(d*x + c)))$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)\*\*2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate(1/((b\*g\*x + a\*g)^2\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2), x)

$$3.28 \quad \int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

**Optimal.** Leaf size=314

$$\frac{2be^{\frac{2A}{Bn}}(c+dx)^2 \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{2/n} \operatorname{Ei} \left( -\frac{2(A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right))}{Bn} \right)}{B^2 g^3 n^2 (a+bx)^2 (bc-ad)^2} + \frac{de^{\frac{A}{Bn}}(c+dx) \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{\frac{1}{n}} \operatorname{Ei} \left( -\frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right)}{B^2 g^3 n^2 (a+bx)(bc-ad)^2} - \frac{1}{Bg^3 n(a+bx)}$$

[Out]  $(-2*b*E^{\frac{2A}{Bn}}*(e^{\frac{a+bx}{c+dx}})^{2/n}*(c+dx)^2*\operatorname{ExpIntegralEi}[-\frac{2*(A+B*\log(e^{\frac{a+bx}{c+dx}})^n)}{Bn}])/(B^2*(b*c-a*d)^2*g^3*n^2*(a+bx)^2) + (d*E^{\frac{A}{Bn}}*(e^{\frac{a+bx}{c+dx}})^{1/n}*(c+dx)*\operatorname{ExpIntegralEi}[-\frac{A+B*\log(e^{\frac{a+bx}{c+dx}})^n}{Bn}])/(B^2*(b*c-a*d)^2*g^3*n^2*(a+bx)) + (d*(c+dx))/(B*(b*c-a*d)^2*g^3*n*(a+bx)*(A+B*\log(e^{\frac{a+bx}{c+dx}})^n)) - (b*(c+dx)^2)/(B*(b*c-a*d)^2*g^3*n*(a+bx)^2*(A+B*\log(e^{\frac{a+bx}{c+dx}})^n))$

**Rubi [F]** time = 0.0928482, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[1/((a*g + b*g*x)^3*(A + B*\log[e^{\frac{a+bx}{c+dx}}]^n))^2, x]$

[Out]  $\operatorname{Defer}[\operatorname{Int}[1/((a*g + b*g*x)^3*(A + B*\log[e^{\frac{a+bx}{c+dx}}]^n))^2, x]]$

Rubi steps

$$\int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx = \int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

**Mathematica [A]** time = 0.643753, size = 254, normalized size = 0.81

$$\frac{(c+dx) \left( -2be^{\frac{2A}{Bn}}(c+dx) \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{2/n} \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) \operatorname{Ei} \left( -\frac{2(A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right))}{Bn} \right) + d(a+bx)e^{\frac{A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{\frac{1}{n}} \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) \right)}{B^2 g^3 n^2 (a+bx)^2 (bc-ad)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[1/((a*g + b*g*x)^3*(A + B*\log[e^{\frac{a+bx}{c+dx}}]^n))^2, x]$

```
[Out] ((c + d*x)*(B*(-(b*c) + a*d)*n - 2*b*E^((2*A)/(B*n))*(e*((a + b*x)/(c + d*x))
)^n)^(2/n)*(c + d*x)*ExpIntegralEi[(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + d*E^(A/(B*n))*(a + b*x)
)*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*ExpIntegralEi[-((A + B*Log[e*((a + b*x)
)/(c + d*x))^n)/(B*n))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])]/(B^2*(b*c
- a*d)^2*g^3*n^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
```

**Maple [F]** time = 0.438, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(b*g*x+a*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="
maxima")
```

```
[Out] -(d*x + c)/((a^2*b*c*g^3*n - a^3*d*g^3*n)*A*B + (a^2*b*c*g^3*n*log(e) - a^3
*d*g^3*n*log(e))*B^2 + ((b^3*c*g^3*n - a*b^2*d*g^3*n)*A*B + (b^3*c*g^3*n*log
(e) - a*b^2*d*g^3*n*log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3*n - a^2*b*d*g^3*n)*
A*B + (a*b^2*c*g^3*n*log(e) - a^2*b*d*g^3*n*log(e))*B^2)*x + ((b^3*c*g^3*n
- a*b^2*d*g^3*n)*B^2*x^2 + 2*(a*b^2*c*g^3*n - a^2*b*d*g^3*n)*B^2*x + (a^2*b
*c*g^3*n - a^3*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b^3*c*g^3*n - a*b^2*d*g^3
*n)*B^2*x^2 + 2*(a*b^2*c*g^3*n - a^2*b*d*g^3*n)*B^2*x + (a^2*b*c*g^3*n - a
^3*d*g^3*n)*B^2)*log((d*x + c)^n) - integrate((b*d*x + 2*b*c - a*d)/((b^4*
c*g^3*n - a*b^3*d*g^3*n)*A*B + (b^4*c*g^3*n*log(e) - a*b^3*d*g^3*n*log(e))*
B^2)*x^3 + (a^3*b*c*g^3*n - a^4*d*g^3*n)*A*B + (a^3*b*c*g^3*n*log(e) - a^4*
d*g^3*n*log(e))*B^2 + 3*((a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)*A*B + (a*b^3*c*g
^3*n*log(e) - a^2*b^2*d*g^3*n*log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3*n - a^3*
b*d*g^3*n)*A*B + (a^2*b^2*c*g^3*n*log(e) - a^3*b*d*g^3*n*log(e))*B^2)*x + (
(b^4*c*g^3*n - a*b^3*d*g^3*n)*B^2*x^3 + 3*(a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)
*B^2*x^2 + 3*(a^2*b^2*c*g^3*n - a^3*b*d*g^3*n)*B^2*x + (a^3*b*c*g^3*n - a^4
*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b^4*c*g^3*n - a*b^3*d*g^3*n)*B^2*x^3 +
3*(a*b^3*c*g^3*n - a^2*b^2*d*g^3*n)*B^2*x^2 + 3*(a^2*b^2*c*g^3*n - a^3*b*d*
g^3*n)*B^2*x + (a^3*b*c*g^3*n - a^4*d*g^3*n)*B^2)*log((d*x + c)^n)), x)
```

**Fricas [B]** time = 0.953429, size = 1612, normalized size = 5.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] -((B*b*c*d - B*a*d^2)*n*x - (A*b^2*d*x^2 + 2*A*a*b*d*x + A*a^2*d + (B*b^2*d*x^2 + 2*B*a*b*d*x + B*a^2*d)*log(e) + (B*b^2*d*n*x^2 + 2*B*a*b*d*n*x + B*a^2*d*n)*log((b*x + a)/(d*x + c)))*e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(-(B*log(e) + A)/(B*n)))/(b*x + a)) + 2*(A*b^3*x^2 + 2*A*a*b^2*x + A*a^2*b + (B*b^3*x^2 + 2*B*a*b^2*x + B*a^2*b)*log(e) + (B*b^3*n*x^2 + 2*B*a*b^2*n*x + B*a^2*b*n)*log((b*x + a)/(d*x + c)))*e^(2*(B*log(e) + A)/(B*n))*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-(B*log(e) + A)/(B*n)))/(b^2*x^2 + 2*a*b*x + a^2)) + (B*b*c^2 - B*a*c*d)*n)/((A*B^2*b^4*c^2 - 2*A*B^2*a*b^3*c*d + A*B^2*a^2*b^2*d^2)*g^3*n^2*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^2*c*d + A*B^2*a^3*b*d^2)*g^3*n^2*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d + A*B^2*a^4*d^2)*g^3*n^2 + ((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*g^3*n^2*x^2 + 2*(B^3*a*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*g^3*n^2*x + (B^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*g^3*n^2)*log(e) + ((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*g^3*n^3*x^2 + 2*(B^3*a*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*g^3*n^3*x + (B^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*g^3*n^3)*log((b*x + a)/(d*x + c)))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

$$3.29 \quad \int (cg + dgx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$$

**Optimal.** Leaf size=188

$$\frac{g^4(c+dx)^5 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{5d} - \frac{Bg^4nx(bc-ad)^4}{5b^4} - \frac{Bg^4n(c+dx)^2(bc-ad)^3}{10b^3d} - \frac{Bg^4n(c+dx)^3(bc-ad)^2}{15b^2d} - \frac{Bg^4n(c+dx)^4(bc-ad)}{20b^2d} - \frac{Bg^4n(c+dx)^5(bc-ad)}{25b^2d}$$

[Out]  $-(B*(b*c - a*d)^4*g^4*n*x)/(5*b^4) - (B*(b*c - a*d)^3*g^4*n*(c + d*x)^2)/(10*b^3*d) - (B*(b*c - a*d)^2*g^4*n*(c + d*x)^3)/(15*b^2*d) - (B*(b*c - a*d)*g^4*n*(c + d*x)^4)/(20*b*d) - (B*(b*c - a*d)^5*g^4*n*Log[a + b*x])/(5*b^5*d) + (g^4*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d)$

**Rubi [A]** time = 0.127825, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 43}

$$\frac{g^4(c+dx)^5 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{5d} - \frac{Bg^4nx(bc-ad)^4}{5b^4} - \frac{Bg^4n(c+dx)^2(bc-ad)^3}{10b^3d} - \frac{Bg^4n(c+dx)^3(bc-ad)^2}{15b^2d} - \frac{Bg^4n(c+dx)^4(bc-ad)}{20b^2d} - \frac{Bg^4n(c+dx)^5(bc-ad)}{25b^2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]$

[Out]  $-(B*(b*c - a*d)^4*g^4*n*x)/(5*b^4) - (B*(b*c - a*d)^3*g^4*n*(c + d*x)^2)/(10*b^3*d) - (B*(b*c - a*d)^2*g^4*n*(c + d*x)^3)/(15*b^2*d) - (B*(b*c - a*d)*g^4*n*(c + d*x)^4)/(20*b*d) - (B*(b*c - a*d)^5*g^4*n*Log[a + b*x])/(5*b^5*d) + (g^4*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d)$

#### Rule 2525

$\text{Int}[(a + \text{Log}[(c + \text{RFX})^p])*(b + (d + e*x)^m)^n, x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^n/(e^{m+1}), x] - \text{Dist}[(b*n*p)/(e^{m+1}), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^{n-1}*D[\text{RFX}, x])/RFX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[(a + (b + (c + d*x)^n)^m), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b + (c + d\*x)^n)^m] /;

#### Rule 43

$\text{Int}[(a + (b + (c + d*x)^n)^m), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int (cg + dgx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx &= \frac{g^4(c+dx)^5 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{5d} - \frac{(Bn) \int \frac{(bc-ad)g^5(c+dx)^4}{a+bx} dx}{5dg} \\
&= \frac{g^4(c+dx)^5 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{5d} - \frac{(B(bc-ad)g^4n) \int \frac{(c+dx)^4}{a+bx} dx}{5d} \\
&= \frac{g^4(c+dx)^5 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{5d} - \frac{(B(bc-ad)g^4n) \int \left( \frac{d(bc-ad)^3}{b^4} + \frac{(bc-a)}{b^4(a+bx)} \right) dx}{5d} \\
&= -\frac{B(bc-ad)^4g^4nx}{5b^4} - \frac{B(bc-ad)^3g^4n(c+dx)^2}{10b^3d} - \frac{B(bc-ad)^2g^4n(c+dx)^3}{15b^2d}
\end{aligned}$$

**Mathematica [A]** time = 0.105292, size = 146, normalized size = 0.78

$$\frac{g^4 \left( (c+dx)^5 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) - \frac{Bn(bc-ad)(6b^2(c+dx)^2(bc-ad)^2+4b^3(c+dx)^3(bc-ad)+12bdx(bc-ad)^3+12(bc-ad)^4 \log(a+bx)+3b^4(c+dx)^4)}{12b^5} \right)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*g + d\*g\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] (g^4\*(-(B\*(b\*c - a\*d)\*n\*(12\*b\*d\*(b\*c - a\*d)^3\*x + 6\*b^2\*(b\*c - a\*d)^2\*(c + d\*x)^2 + 4\*b^3\*(b\*c - a\*d)\*(c + d\*x)^3 + 3\*b^4\*(c + d\*x)^4 + 12\*(b\*c - a\*d)^4\*Log[a + b\*x]))/(12\*b^5) + (c + d\*x)^5\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(5\*d)

**Maple [F]** time = 0.522, size = 0, normalized size = 0.

$$\int (d gx + cg)^4 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*g\*x+c\*g)^4\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

[Out] int((d\*g\*x+c\*g)^4\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

**Maxima [B]** time = 1.24272, size = 913, normalized size = 4.86

$$\frac{1}{5} B d^4 g^4 x^5 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + \frac{1}{5} A d^4 g^4 x^5 + B c d^3 g^4 x^4 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + A c d^3 g^4 x^4 + 2 B c^2 d^2 g^4 x^3 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^4\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)), x, algorithm="maxima")

[Out] 1/5\*B\*d^4\*g^4\*x^5\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/5\*A\*d^4\*g^4\*x^5 + B\*c\*d^3\*g^4\*x^4\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A\*c\*d^3\*g^4\*x^4

$$4 + 2Bc^2d^2g^4x^3 \log(e*(bx/(dx+c) + a/(dx+c))^n) + 2Ac^2d^2g^4x^3 + 2Bc^3d^2g^4x^2 \log(e*(bx/(dx+c) + a/(dx+c))^n) + 2Ac^3d^2g^4x^2 + 1/60Bd^4g^4n*(12a^5 \log(bx+a)/b^5 - 12c^5 \log(dx+c)/d^5 - (3*(b^4c^3d^3 - a*b^3d^4)*x^4 - 4*(b^4c^2d^2 - a^2b^2d^4)*x^3 + 6*(b^4c^3d - a^3b^3d^4)*x^2 - 12*(b^4c^4 - a^4d^4)*x)/(b^4d^4)) - 1/6Bc^3d^3g^4n*(6a^4 \log(bx+a)/b^4 - 6c^4 \log(dx+c)/d^4 + (2*(b^3c^2d^2 - a*b^2d^3)*x^3 - 3*(b^3c^2d - a^2b^2d^3)*x^2 + 6*(b^3c^3 - a^3d^3)*x)/(b^3d^3)) + Bc^2d^2g^4n*(2a^3 \log(bx+a)/b^3 - 2c^3 \log(dx+c)/d^3 - ((b^2c^2d - a*b^2d^2)*x^2 - 2*(b^2c^2 - a^2d^2)*x)/(b^2d^2)) - 2Bc^3d^2g^4n*(a^2 \log(bx+a)/b^2 - c^2 \log(dx+c)/d^2 + (bc - a*d)*x/(b*d)) + Bc^4g^4n*(a \log(bx+a)/b - c \log(dx+c)/d) + Bc^4g^4x \log(e*(bx/(dx+c) + a/(dx+c))^n) + Ac^4g^4x$$

**Fricas [B]** time = 1.06928, size = 1188, normalized size = 6.32

$$12 Ab^5 d^5 g^4 x^5 - 12 B b^5 c^5 g^4 n \log(dx+c) + 12 (5 Bab^4 c^4 d - 10 Ba^2 b^3 c^3 d^2 + 10 Ba^3 b^2 c^2 d^3 - 5 Ba^4 b c d^4 + Ba^5 d^5) g^4 n \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^4\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out]  $1/60*(12A*b^5*d^5*g^4*x^5 - 12B*b^5*c^5*g^4*n*\log(dx+c) + 12*(5B*a*b^4*c^4*d - 10B*a^2*b^3*c^3*d^2 + 10B*a^3*b^2*c^2*d^3 - 5B*a^4*b*c*d^4 + B*a^5*d^5)*g^4*n*\log(bx+a) + 3*(20A*b^5*c*d^4*g^4 - (B*b^5*c*d^4 - B*a*b^4*d^5)*g^4*n)*x^4 + 4*(30A*b^5*c^2*d^3*g^4 - (4B*b^5*c^2*d^3 - 5B*a*b^4*c*d^4 + B*a^2*b^3*d^5)*g^4*n)*x^3 + 6*(20A*b^5*c^3*d^2*g^4 - (6B*b^5*c^3*d^2 - 10B*a*b^4*c^2*d^3 + 5B*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*g^4*n)*x^2 + 12*(5A*b^5*c^4*d*g^4 - (4B*b^5*c^4*d - 10B*a*b^4*c^3*d^2 + 10B*a^2*b^3*c^2*d^3 - 5B*a^3*b^2*c*d^4 + B*a^4*b*d^5)*g^4*n)*x + 12*(B*b^5*d^5*g^4*x^5 + 5B*b^5*c*d^4*g^4*x^4 + 10B*b^5*c^2*d^3*g^4*x^3 + 10B*b^5*c^3*d^2*g^4*x^2 + 5B*b^5*c^4*d*g^4*x)*\log(e) + 12*(B*b^5*d^5*g^4*n*x^5 + 5B*b^5*c*d^4*g^4*n*x^4 + 10B*b^5*c^2*d^3*g^4*n*x^3 + 10B*b^5*c^3*d^2*g^4*n*x^2 + 5B*b^5*c^4*d*g^4*n*x)*\log((bx+a)/(dx+c)))/(b^5*d)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)\*\*4\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n)),x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] Timed out
```



$$3.30 \quad \int (cg + dgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$$

**Optimal.** Leaf size=156

$$\frac{g^3(c+dx)^4 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{4d} - \frac{Bg^3nx(bc-ad)^3}{4b^3} - \frac{Bg^3n(c+dx)^2(bc-ad)^2}{8b^2d} - \frac{Bg^3n(bc-ad)^4 \log(a+bx)}{4b^4d} - \frac{Bg^3n(bc-ad)^4 \log(a+bx)}{4b^4d}$$

```
[Out] -(B*(b*c - a*d)^3*g^3*n*x)/(4*b^3) - (B*(b*c - a*d)^2*g^3*n*(c + d*x)^2)/(8*b^2*d) - (B*(b*c - a*d)*g^3*n*(c + d*x)^3)/(12*b*d) - (B*(b*c - a*d)^4*g^3*n*Log[a + b*x])/(4*b^4*d) + (g^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d)
```

**Rubi [A]** time = 0.1021, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 43}

$$\frac{g^3(c+dx)^4 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{4d} - \frac{Bg^3nx(bc-ad)^3}{4b^3} - \frac{Bg^3n(c+dx)^2(bc-ad)^2}{8b^2d} - \frac{Bg^3n(bc-ad)^4 \log(a+bx)}{4b^4d} - \frac{Bg^3n(bc-ad)^4 \log(a+bx)}{4b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

```
[Out] -(B*(b*c - a*d)^3*g^3*n*x)/(4*b^3) - (B*(b*c - a*d)^2*g^3*n*(c + d*x)^2)/(8*b^2*d) - (B*(b*c - a*d)*g^3*n*(c + d*x)^3)/(12*b*d) - (B*(b*c - a*d)^4*g^3*n*Log[a + b*x])/(4*b^4*d) + (g^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d)
```

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^n_.*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
\int (cg + dgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx &= \frac{g^3(c+dx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{4d} - \frac{(Bn) \int \frac{(bc-ad)g^4(c+dx)^3}{a+bx} dx}{4dg} \\
&= \frac{g^3(c+dx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{4d} - \frac{(B(bc-ad)g^3n) \int \frac{(c+dx)^3}{a+bx} dx}{4d} \\
&= \frac{g^3(c+dx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{4d} - \frac{(B(bc-ad)g^3n) \int \left( \frac{d(bc-ad)^2}{b^3} + \frac{(bc-a)}{b^3(a+bx)} \right) dx}{4d} \\
&= -\frac{B(bc-ad)^3g^3nx}{4b^3} - \frac{B(bc-ad)^2g^3n(c+dx)^2}{8b^2d} - \frac{B(bc-ad)g^3n(c+dx)^3}{12bd}
\end{aligned}$$

**Mathematica [A]** time = 0.0937343, size = 124, normalized size = 0.79

$$\frac{g^3 \left( (c+dx)^4 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) - \frac{Bn(bc-ad)(3b^2(c+dx)^2(bc-ad)+6bdx(bc-ad)^2+6(bc-ad)^3 \log(a+bx)+2b^3(c+dx)^3)}{6b^4} \right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*g + d\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] (g^3\*(-(B\*(b\*c - a\*d)\*n\*(6\*b\*d\*(b\*c - a\*d)^2\*x + 3\*b^2\*(b\*c - a\*d)\*(c + d\*x)^2 + 2\*b^3\*(c + d\*x)^3 + 6\*(b\*c - a\*d)^3\*Log[a + b\*x]))/(6\*b^4) + (c + d\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(4\*d)

**Maple [F]** time = 0.411, size = 0, normalized size = 0.

$$\int (d gx + c g)^3 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*g\*x+c\*g)^3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

[Out] int((d\*g\*x+c\*g)^3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

**Maxima [B]** time = 1.25501, size = 647, normalized size = 4.15

$$\frac{1}{4} B d^3 g^3 x^4 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + \frac{1}{4} A d^3 g^3 x^4 + B c d^2 g^3 x^3 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + A c d^2 g^3 x^3 + \frac{3}{2} B c^2 d g^3 x^2 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^3\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)), x, algorithm="maxima")

[Out] 1/4\*B\*d^3\*g^3\*x^4\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/4\*A\*d^3\*g^3\*x^4 + B\*c\*d^2\*g^3\*x^3\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A\*c\*d^2\*g^3\*x^3

$$3 + 3/2*B*c^2*d*g^3*x^2*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*c^2*d*g^3*x^2 - 1/24*B*d^3*g^3*n*(6*a^4*\log(b*x + a)/b^4 - 6*c^4*\log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*c*d^2*g^3*n*(2*a^3*\log(b*x + a)/b^3 - 2*c^3*\log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3/2*B*c^2*d*g^3*n*(a^2*\log(b*x + a)/b^2 - c^2*\log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^3*g^3*n*(a*\log(b*x + a)/b - c*\log(d*x + c)/d) + B*c^3*g^3*x*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c^3*g^3*x$$

**Fricas [B]** time = 1.07087, size = 883, normalized size = 5.66

$$6Ab^4d^4g^3x^4 - 6Bb^4c^4g^3n \log(dx + c) + 6(4Bab^3c^3d - 6Ba^2b^2c^2d^2 + 4Ba^3bcd^3 - Ba^4d^4)g^3n \log(bx + a) + 2(12Ab$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^3\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] 1/24\*(6\*A\*b^4\*d^4\*g^3\*x^4 - 6\*B\*b^4\*c^4\*g^3\*n\*log(d\*x + c) + 6\*(4\*B\*a\*b^3\*c^3\*d - 6\*B\*a^2\*b^2\*c^2\*d^2 + 4\*B\*a^3\*b\*c\*d^3 - B\*a^4\*d^4)\*g^3\*n\*log(b\*x + a) + 2\*(12\*A\*b^4\*c\*d^3\*g^3 - (B\*b^4\*c\*d^3 - B\*a\*b^3\*d^4)\*g^3\*n)\*x^3 + 3\*(12\*A\*b^4\*c^2\*d^2\*g^3 - (3\*B\*b^4\*c^2\*d^2 - 4\*B\*a\*b^3\*c\*d^3 + B\*a^2\*b^2\*d^4)\*g^3\*n)\*x^2 + 6\*(4\*A\*b^4\*c^3\*d\*g^3 - (3\*B\*b^4\*c^3\*d - 6\*B\*a\*b^3\*c^2\*d^2 + 4\*B\*a^2\*b^2\*c\*d^3 - B\*a^3\*b\*d^4)\*g^3\*n)\*x + 6\*(B\*b^4\*d^4\*g^3\*x^4 + 4\*B\*b^4\*c\*d^3\*g^3\*x^3 + 6\*B\*b^4\*c^2\*d^2\*g^3\*x^2 + 4\*B\*b^4\*c^3\*d\*g^3\*x)\*log(e) + 6\*(B\*b^4\*d^4\*g^3\*n\*x^4 + 4\*B\*b^4\*c\*d^3\*g^3\*n\*x^3 + 6\*B\*b^4\*c^2\*d^2\*g^3\*n\*x^2 + 4\*B\*b^4\*c^3\*d\*g^3\*n\*x)\*log((b\*x + a)/(d\*x + c)))/(b^4\*d)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)\*\*3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] Timed out

**Giac [B]** time = 143.004, size = 537, normalized size = 3.44

$$-\frac{Bc^4g^3n \log(-dx - c)}{4d} + \frac{1}{4}(Ad^3g^3 + Bd^3g^3)x^4 - \frac{(Bbcd^2g^3n - Bad^3g^3n - 12Abcd^2g^3 - 12Bbcd^2g^3)x^3}{12b} + \frac{1}{4}(Bd^3g^3nx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^3\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out] -1/4\*B\*c^4\*g^3\*n\*log(-d\*x - c)/d + 1/4\*(A\*d^3\*g^3 + B\*d^3\*g^3)\*x^4 - 1/12\*(B\*b\*c\*d^2\*g^3\*n - B\*a\*d^3\*g^3\*n - 12\*A\*b\*c\*d^2\*g^3 - 12\*B\*b\*c\*d^2\*g^3)\*x^3/

$$\begin{aligned}
& b + \frac{1}{4} * (B * d^3 * g^3 * n * x^4 + 4 * B * c * d^2 * g^3 * n * x^3 + 6 * B * c^2 * d * g^3 * n * x^2 + 4 * B * \\
& c^3 * g^3 * n * x) * \log((b * x + a) / (d * x + c)) - \frac{1}{8} * (3 * B * b^2 * c^2 * d * g^3 * n - 4 * B * a * b * \\
& c * d^2 * g^3 * n + B * a^2 * d^3 * g^3 * n - 12 * A * b^2 * c^2 * d * g^3 - 12 * B * b^2 * c^2 * d * g^3) * x^2 / b^2 \\
& - \frac{1}{4} * (3 * B * b^3 * c^3 * g^3 * n - 6 * B * a * b^2 * c^2 * d * g^3 * n + 4 * B * a^2 * b * c * d^2 * g^3 * n \\
& - B * a^3 * d^3 * g^3 * n - 4 * A * b^3 * c^3 * g^3 - 4 * B * b^3 * c^3 * g^3) * x / b^3 + \frac{1}{4} * (4 * B * \\
& a * b^3 * c^3 * g^3 * n - 6 * B * a^2 * b^2 * c^2 * d * g^3 * n + 4 * B * a^3 * b * c * d^2 * g^3 * n - B * a^4 * \\
& d^3 * g^3 * n) * \log(b * x + a) / b^4
\end{aligned}$$

$$3.31 \quad \int (cg + dgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$$

**Optimal.** Leaf size=124

$$\frac{g^2(c+dx)^3 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{3d} - \frac{Bg^2nx(bc-ad)^2}{3b^2} - \frac{Bg^2n(bc-ad)^3 \log(a+bx)}{3b^3d} - \frac{Bg^2n(c+dx)^2(bc-ad)}{6bd}$$

[Out]  $-(B*(b*c - a*d)^2*g^2*n*x)/(3*b^2) - (B*(b*c - a*d)*g^2*n*(c + d*x)^2)/(6*b*d) - (B*(b*c - a*d)^3*g^2*n*Log[a + b*x])/(3*b^3*d) + (g^2*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d)$

**Rubi [A]** time = 0.0813244, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 43}

$$\frac{g^2(c+dx)^3 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{3d} - \frac{Bg^2nx(bc-ad)^2}{3b^2} - \frac{Bg^2n(bc-ad)^3 \log(a+bx)}{3b^3d} - \frac{Bg^2n(c+dx)^2(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]$

[Out]  $-(B*(b*c - a*d)^2*g^2*n*x)/(3*b^2) - (B*(b*c - a*d)*g^2*n*(c + d*x)^2)/(6*b*d) - (B*(b*c - a*d)^3*g^2*n*Log[a + b*x])/(3*b^3*d) + (g^2*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d)$

#### Rule 2525

$\text{Int}[(a_. + \text{Log}[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int (cg + dgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx &= \frac{g^2(c+dx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{3d} - \frac{(Bn) \int \frac{(bc-ad)g^3(c+dx)^2}{a+bx} dx}{3dg} \\
&= \frac{g^2(c+dx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{3d} - \frac{(B(bc-ad)g^2n) \int \frac{(c+dx)^2}{a+bx} dx}{3d} \\
&= \frac{g^2(c+dx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{3d} - \frac{(B(bc-ad)g^2n) \int \left( \frac{d(bc-ad)}{b^2} + \frac{(bc-ad)}{b^2(a+bx)} \right) dx}{3d} \\
&= -\frac{B(bc-ad)^2g^2nx}{3b^2} - \frac{B(bc-ad)g^2n(c+dx)^2}{6bd} - \frac{B(bc-ad)^3g^2n \log(a+bx)}{3b^3d}
\end{aligned}$$

**Mathematica [A]** time = 0.0597527, size = 101, normalized size = 0.81

$$\frac{g^2 \left( (c+dx)^3 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) - \frac{Bn(bc-ad)(2bdx(bc-ad)+2(bc-ad)^2 \log(a+bx)+b^2(c+dx)^2)}{2b^3} \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*g + d\*g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] (g^2\*(-(B\*(b\*c - a\*d)\*n\*(2\*b\*d\*(b\*c - a\*d)\*x + b^2\*(c + d\*x)^2 + 2\*(b\*c - a\*d)^2\*Log[a + b\*x]))/(2\*b^3) + (c + d\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(3\*d)

**Maple [F]** time = 0.404, size = 0, normalized size = 0.

$$\int (d gx + c g)^2 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*g\*x+c\*g)^2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

[Out] int((d\*g\*x+c\*g)^2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

**Maxima [B]** time = 1.19467, size = 417, normalized size = 3.36

$$\frac{1}{3} B d^2 g^2 x^3 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + \frac{1}{3} A d^2 g^2 x^3 + B c d g^2 x^2 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + A c d g^2 x^2 + \frac{1}{6} B d^2 g^2 n \left( \frac{2 a^3 \log(a+bx)}{b^2(a+bx)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)), x, algorithm="maxima")

[Out] 1/3\*B\*d^2\*g^2\*x^3\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/3\*A\*d^2\*g^2\*x^3 + B\*c\*d\*g^2\*x^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A\*c\*d\*g^2\*x^2 +

$$\frac{1}{6}B*d^2*g^2*n*(2*a^3*\log(b*x + a)/b^3 - 2*c^3*\log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*c*d*g^2*n*(a^2*\log(b*x + a)/b^2 - c^2*\log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^2*g^2*n*(a*\log(b*x + a)/b - c*\log(d*x + c)/d) + B*c^2*g^2*x*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c^2*g^2*x$$

**Fricas [B]** time = 0.974927, size = 622, normalized size = 5.02

$$2Ab^3d^3g^2x^3 - 2Bb^3c^3g^2n\log(dx + c) + 2(3Bab^2c^2d - 3Ba^2bcd^2 + Ba^3d^3)g^2n\log(bx + a) + (6Ab^3cd^2g^2 - (Bb^3cd^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out]  $\frac{1}{6}(2A*b^3*d^3*g^2*x^3 - 2B*b^3*c^3*g^2*n*\log(d*x + c) + 2*(3B*a*b^2*c^2*d - 3B*a^2*b*c*d^2 + B*a^3*d^3)*g^2*n*\log(b*x + a) + (6A*b^3*c*d^2*g^2 - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2*n)*x^2 + 2*(3A*b^3*c^2*d*g^2 - (2B*b^3*c^2*d - 3B*a*b^2*c*d^2 + B*a^2*b*d^3)*g^2*n)*x + 2*(B*b^3*d^3*g^2*x^3 + 3B*b^3*c*d^2*g^2*x^2 + 3B*b^3*c^2*d*g^2*x)*\log(e) + 2*(B*b^3*d^3*g^2*n*x^3 + 3B*b^3*c*d^2*g^2*n*x^2 + 3B*b^3*c^2*d*g^2*n*x)*\log((b*x + a)/(d*x + c)))/(b^3*d)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)\*\*2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] Timed out

**Giac [B]** time = 21.4717, size = 354, normalized size = 2.85

$$-\frac{Bc^3g^2n\log(-dx - c)}{3d} + \frac{1}{3}(Ad^2g^2 + Bd^2g^2)x^3 - \frac{(Bbcdg^2n - Bad^2g^2n - 6Abcdg^2 - 6Bbcdg^2)x^2}{6b} + \frac{1}{3}(Bd^2g^2nx^3 + 3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out]  $-\frac{1}{3}B*c^3*g^2*n*\log(-d*x - c)/d + \frac{1}{3}(A*d^2*g^2 + B*d^2*g^2)*x^3 - \frac{1}{6}(B*b*c*d*g^2*n - B*a*d^2*g^2*n - 6*A*b*c*d*g^2 - 6*B*b*c*d*g^2)*x^2/b + \frac{1}{3}(B*d^2*g^2*n*x^3 + 3*B*c*d*g^2*n*x^2 + 3*B*c^2*g^2*n*x)*\log((b*x + a)/(d*x + c)) - \frac{1}{3}(2*B*b^2*c^2*g^2*n - 3*B*a*b*c*d*g^2*n + B*a^2*d^2*g^2*n - 3*A*b^2*c^2*g^2 - 3*B*b^2*c^2*g^2)*x/b^2 + \frac{1}{3}(3*B*a*b^2*c^2*g^2*n - 3*B*a^2*b*c*d*g^2*n + B*a^3*d^2*g^2*n)*\log(b*x + a)/b^3$

### 3.32 $\int (cg + dgx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$

**Optimal.** Leaf size=86

$$\frac{g(c+dx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{2d} - \frac{Bgn(bc-ad)^2 \log(a+bx)}{2b^2d} - \frac{Bgnx(bc-ad)}{2b}$$

[Out]  $-(B*(b*c - a*d)*g*n*x)/(2*b) - (B*(b*c - a*d)^2*g*n*Log[a + b*x])/(2*b^2*d) + (g*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d)$

**Rubi [A]** time = 0.0599644, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {2525, 12, 43}

$$\frac{g(c+dx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{2d} - \frac{Bgn(bc-ad)^2 \log(a+bx)}{2b^2d} - \frac{Bgnx(bc-ad)}{2b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*g + d*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]$

[Out]  $-(B*(b*c - a*d)*g*n*x)/(2*b) - (B*(b*c - a*d)^2*g*n*Log[a + b*x])/(2*b^2*d) + (g*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d)$

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps



$$\begin{aligned}
\int (cg + dgx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx &= \frac{g(c+dx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{2d} - \frac{(Bn) \int \frac{(bc-ad)g^2(c+dx)}{a+bx} dx}{2dg} \\
&= \frac{g(c+dx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{2d} - \frac{(B(bc-ad)gn) \int \frac{c+dx}{a+bx} dx}{2d} \\
&= \frac{g(c+dx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{2d} - \frac{(B(bc-ad)gn) \int \left( \frac{d}{b} + \frac{bc-ad}{b(a+bx)} \right) dx}{2d} \\
&= -\frac{B(bc-ad)gnx}{2b} - \frac{B(bc-ad)^2 gn \log(a+bx)}{2b^2 d} + \frac{g(c+dx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{2d}
\end{aligned}$$

**Mathematica [A]** time = 0.0380534, size = 74, normalized size = 0.86

$$\frac{g \left( (c+dx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) - \frac{Bn(bc-ad)((bc-ad) \log(a+bx) + bdx)}{b^2} \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(c\*g + d\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]),x]

[Out] (g\*(-((B\*(b\*c - a\*d)\*n\*(b\*d\*x + (b\*c - a\*d)\*Log[a + b\*x]))/b^2) + (c + d\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(2\*d)

**Maple [F]** time = 0.328, size = 0, normalized size = 0.

$$\int (dgx + cg) \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*g\*x+c\*g)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int((d\*g\*x+c\*g)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

**Maxima [A]** time = 1.14195, size = 211, normalized size = 2.45

$$\frac{1}{2} Bdgx^2 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + \frac{1}{2} Adgx^2 - \frac{1}{2} Bdgn \left( \frac{a^2 \log(bx+a)}{b^2} - \frac{c^2 \log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd} \right) + Bcgn \left( \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out] 1/2\*B\*d\*g\*x^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/2\*A\*d\*g\*x^2 - 1/2\*B\*d\*g\*n\*(a^2\*log(b\*x + a)/b^2 - c^2\*log(d\*x + c)/d^2 + (b\*c - a\*d)\*x/(b\*d)) + B\*c\*g\*n\*(a\*log(b\*x + a)/b - c\*log(d\*x + c)/d) + B\*c\*g\*x\*log(e\*(b\*x/(d\*x



$$3.33 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg+dgx} dx$$

**Optimal.** Leaf size=80

$$\frac{Bn \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{dg} - \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{dg}$$

[Out] -(((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[(b\*c - a\*d)/(b\*(c + d\*x))])/(d\*g)) - (B\*n\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(d\*g)

**Rubi [A]** time = 0.20138, antiderivative size = 128, normalized size of antiderivative = 1.6, number of steps used = 9, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {2524, 2418, 2394, 2393, 2391, 2390, 12, 2301}

$$\frac{Bn \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{dg} + \frac{\log(cg + dgx) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{dg} - \frac{Bn \log(cg + dgx) \log\left(-\frac{d(a+bx)}{bc-ad}\right)}{dg} + \frac{Bn \log^2(cg + dgx)}{2dg}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(c\*g + d\*g\*x), x]

[Out] (B\*n\*Log[g\*(c + d\*x)]^2)/(2\*d\*g) - (B\*n\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c\*g + d\*g\*x])/(d\*g) + ((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c\*g + d\*g\*x])/(d\*g) - (B\*n\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/(d\*g)

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*(Rfx\_), x\_Symbol] :> With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_.)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_.)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{cg + dgx} dx &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(cg + dgx)}{dg} - \frac{(Bn) \int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2} + \frac{b}{c+dx}\right) \log(cg+dgx)}{a+bx} dx}{dg} \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(cg + dgx)}{dg} - \frac{(Bn) \int \left(\frac{b \log(cg+dgx)}{a+bx} - \frac{d \log(cg+dgx)}{c+dx}\right) dx}{dg} \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(cg + dgx)}{dg} + \frac{(Bn) \int \frac{\log(cg+dgx)}{c+dx} dx}{g} - \frac{(bBn) \int \frac{\log(cg+dgx)}{a+bx} dx}{dg} \\
 &= -\frac{Bn \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(cg + dgx)}{dg} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(cg + dgx)}{dg} + (Bn) \int \frac{\log(cg + dgx)}{cg + dgx} dx \\
 &= -\frac{Bn \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(cg + dgx)}{dg} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(cg + dgx)}{dg} + \frac{(Bn) \text{Subst}\left[\int \frac{\log(cg + dgx)}{cg + dgx} dx, x, \frac{c+dx}{g}\right]}{g} \\
 &= \frac{Bn \log^2(g(c + dx))}{2dg} - \frac{Bn \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(cg + dgx)}{dg} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(cg + dgx)}{dg}
 \end{aligned}$$

**Mathematica [A]** time = 0.039353, size = 101, normalized size = 1.26

$$\frac{\log(g(c + dx)) \left( 2B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - 2Bn \log\left(\frac{d(a+bx)}{ad-bc}\right) + 2A + Bn \log(g(c + dx)) \right) - 2Bn \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{2dg}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(c\*g + d\*g\*x), x]

[Out] (Log[g\*(c + d\*x)]\*(2\*A - 2\*B\*n\*Log[(d\*(a + b\*x))/(-(b\*c) + a\*d)] + 2\*B\*Log[e\*((a + b\*x)/(c + d\*x))^n] + B\*n\*Log[g\*(c + d\*x)]) - 2\*B\*n\*PolyLog[2, (b\*(c

+ d\*x))/(b\*c - a\*d)]/(2\*d\*g)

**Maple [F]** time = 0.531, size = 0, normalized size = 0.

$$\int \frac{1}{d g x + c g} \left( A + B \ln \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g), x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2} B \left( \frac{2 n \log (b x + a) \log (d x + c) - n \log (d x + c)^2 - 2 \log (d x + c) \log \left( (b x + a)^n \right) + 2 \log (d x + c) \log \left( (d x + c)^n \right)}{d g} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g), x, algorithm="maxima")

[Out] -1/2\*B\*((2\*n\*log(b\*x + a)\*log(d\*x + c) - n\*log(d\*x + c)^2 - 2\*log(d\*x + c)\*log((b\*x + a)^n) + 2\*log(d\*x + c)\*log((d\*x + c)^n))/(d\*g) - 2\*integrate((n\*log(b\*x + a) + log(e))/(d\*g\*x + c\*g), x)) + A\*log(d\*g\*x + c\*g)/(d\*g)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A}{d g x + c g}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g), x, algorithm="fricas")

[Out] integral((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)/(d\*g\*x + c\*g), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c)\*\*n))/(d\*g\*x+c\*g), x)

[Out] Timed out

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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{d gx + c g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g),x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)/(d\*g\*x + c\*g), x)

$$3.34 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^2} dx$$

**Optimal.** Leaf size=102

$$\frac{A(a+bx)}{g^2(c+dx)(bc-ad)} + \frac{B(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2(c+dx)(bc-ad)} - \frac{Bn(a+bx)}{g^2(c+dx)(bc-ad)}$$

[Out] (A\*(a + b\*x))/((b\*c - a\*d)\*g^2\*(c + d\*x)) - (B\*n\*(a + b\*x))/((b\*c - a\*d)\*g^2\*(c + d\*x)) + (B\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/((b\*c - a\*d)\*g^2\*(c + d\*x))

**Rubi [A]** time = 0.0873029, antiderivative size = 107, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{dg^2(c+dx)} + \frac{bBn \log(a+bx)}{dg^2(bc-ad)} - \frac{bBn \log(c+dx)}{dg^2(bc-ad)} + \frac{Bn}{dg^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(c\*g + d\*g\*x)^2,x]

[Out] (B\*n)/(d\*g^2\*(c + d\*x)) + (b\*B\*n\*Log[a + b\*x])/(d\*(b\*c - a\*d)\*g^2) - (A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(d\*g^2\*(c + d\*x)) - (b\*B\*n\*Log[c + d\*x])/(d\*(b\*c - a\*d)\*g^2)

#### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg + dgx)^2} dx &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg^2(c + dx)} + \frac{(Bn) \int \frac{bc-ad}{g(a+bx)(c+dx)^2} dx}{dg} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg^2(c + dx)} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)(c+dx)^2} dx}{dg^2} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg^2(c + dx)} + \frac{(B(bc - ad)n) \int \left(\frac{b^2}{(bc-ad)^2(a+bx)} - \frac{d}{(bc-ad)(c+dx)^2} - \frac{bd}{(bc-ad)^2(c+dx)}\right) dx}{dg^2} \\
&= \frac{Bn}{dg^2(c + dx)} + \frac{bBn \log(a + bx)}{d(bc - ad)g^2} - \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{dg^2(c + dx)} - \frac{bBn \log(c + dx)}{d(bc - ad)g^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0602251, size = 114, normalized size = 1.12

$$\frac{Bn(bc - ad) \left( \frac{1}{(c+dx)(bc-ad)} + \frac{b \log(a+bx)}{(bc-ad)^2} - \frac{b \log(c+dx)}{(bc-ad)^2} \right) - \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{dg(cg + dgx)}}{dg^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(c\*g + d\*g\*x)^2, x]

[Out] -((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(d\*g\*(c\*g + d\*g\*x))) + (B\*(b\*c - a\*d)\*n\*(1/((b\*c - a\*d)\*(c + d\*x)) + (b\*Log[a + b\*x])/(b\*c - a\*d)^2 - (b\*Log[c + d\*x])/(b\*c - a\*d)^2))/(d\*g^2)

**Maple [F]** time = 0.438, size = 0, normalized size = 0.

$$\int \frac{1}{(d gx + c g)^2} \left( A + B \ln \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^2, x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^2, x)

**Maxima [A]** time = 1.17496, size = 184, normalized size = 1.8

$$Bn \left( \frac{1}{d^2 g^2 x + c d g^2} + \frac{b \log(bx + a)}{(bcd - ad^2) g^2} - \frac{b \log(dx + c)}{(bcd - ad^2) g^2} \right) - \frac{B \log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{d^2 g^2 x + c d g^2} - \frac{A}{d^2 g^2 x + c d g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^2, x, algorithm="maxima")



[Out]  $B*n*(1/(d^2*g^2*x + c*d*g^2) + b*\log(b*x + a)/((b*c*d - a*d^2)*g^2) - b*\log(d*x + c)/((b*c*d - a*d^2)*g^2)) - B*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^2*g^2*x + c*d*g^2) - A/(d^2*g^2*x + c*d*g^2)$

**Fricas [A]** time = 0.908957, size = 221, normalized size = 2.17

$$\frac{A b c - A a d - (B b c - B a d) n + (B b c - B a d) \log(e) - (B b d n x + B a d n) \log\left(\frac{b x + a}{d x + c}\right)}{(b c d^2 - a d^3) g^2 x + (b c^2 d - a c d^2) g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^2,x, algorithm="fricas")

[Out]  $-(A*b*c - A*a*d - (B*b*c - B*a*d)*n + (B*b*c - B*a*d)*\log(e) - (B*b*d*n*x + B*a*d*n)*\log((b*x + a)/(d*x + c)))/((b*c*d^2 - a*d^3)*g^2*x + (b*c^2*d - a*c*d^2)*g^2)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)\*\*2,x)

[Out] Timed out

**Giac [A]** time = 1.36895, size = 166, normalized size = 1.63

$$\frac{B b n \log(b x + a)}{b c d g^2 - a d^2 g^2} - \frac{B b n \log(d x + c)}{b c d g^2 - a d^2 g^2} - \frac{B n \log\left(\frac{b x + a}{d x + c}\right)}{d^2 g^2 x + c d g^2} + \frac{B n - A - B}{d^2 g^2 x + c d g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^2,x, algorithm="giac")

[Out]  $B*b*n*\log(b*x + a)/(b*c*d*g^2 - a*d^2*g^2) - B*b*n*\log(d*x + c)/(b*c*d*g^2 - a*d^2*g^2) - B*n*\log((b*x + a)/(d*x + c))/(d^2*g^2*x + c*d*g^2) + (B*n - A - B)/(d^2*g^2*x + c*d*g^2)$

$$3.35 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^3} dx$$

**Optimal.** Leaf size=151

$$-\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{2dg^3(c+dx)^2} + \frac{b^2Bn \log(a+bx)}{2dg^3(bc-ad)^2} - \frac{b^2Bn \log(c+dx)}{2dg^3(bc-ad)^2} + \frac{bBn}{2dg^3(c+dx)(bc-ad)} + \frac{Bn}{4dg^3(c+dx)^2}$$

[Out] (B\*n)/(4\*d\*g^3\*(c + d\*x)^2) + (b\*B\*n)/(2\*d\*(b\*c - a\*d)\*g^3\*(c + d\*x)) + (b^2\*B\*n\*Log[a + b\*x])/(2\*d\*(b\*c - a\*d)^2\*g^3) - (A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(2\*d\*g^3\*(c + d\*x)^2) - (b^2\*B\*n\*Log[c + d\*x])/(2\*d\*(b\*c - a\*d)^2\*g^3)

**Rubi [A]** time = 0.114301, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{2dg^3(c+dx)^2} + \frac{b^2Bn \log(a+bx)}{2dg^3(bc-ad)^2} - \frac{b^2Bn \log(c+dx)}{2dg^3(bc-ad)^2} + \frac{bBn}{2dg^3(c+dx)(bc-ad)} + \frac{Bn}{4dg^3(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(c\*g + d\*g\*x)^3,x]

[Out] (B\*n)/(4\*d\*g^3\*(c + d\*x)^2) + (b\*B\*n)/(2\*d\*(b\*c - a\*d)\*g^3\*(c + d\*x)) + (b^2\*B\*n\*Log[a + b\*x])/(2\*d\*(b\*c - a\*d)^2\*g^3) - (A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(2\*d\*g^3\*(c + d\*x)^2) - (b^2\*B\*n\*Log[c + d\*x])/(2\*d\*(b\*c - a\*d)^2\*g^3)

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg + dgx)^3} dx &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c + dx)^2} + \frac{(Bn) \int \frac{bc-ad}{g^2(a+bx)(c+dx)^3} dx}{2dg} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c + dx)^2} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)(c+dx)^3} dx}{2dg^3} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c + dx)^2} + \frac{(B(bc - ad)n) \int \left(\frac{b^3}{(bc-ad)^3(a+bx)} - \frac{d}{(bc-ad)(c+dx)^3} - \frac{bd}{(bc-ad)^2(c+dx)^2}\right) dx}{2dg^3} \\
&= \frac{Bn}{4dg^3(c + dx)^2} + \frac{bBn}{2d(bc - ad)g^3(c + dx)} + \frac{b^2Bn \log(a + bx)}{2d(bc - ad)^2g^3} - \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2dg^3(c + dx)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.147845, size = 115, normalized size = 0.76

$$\frac{Bn(2b^2(c+dx)^2 \log(a+bx) + (bc-ad)(-ad+3bc+2bdx) - 2b^2(c+dx)^2 \log(c+dx))}{(bc-ad)^2} - 2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)$$


---


$$4dg^3(c + dx)^2$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(c\*g + d\*g\*x)^3, x]

[Out] (-2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + (B\*n\*((b\*c - a\*d)\*(3\*b\*c - a\*d + 2\*b\*d\*x) + 2\*b^2\*(c + d\*x)^2\*Log[a + b\*x] - 2\*b^2\*(c + d\*x)^2\*Log[c + d\*x]))/(b\*c - a\*d)^2/(4\*d\*g^3\*(c + d\*x)^2)

**Maple [F]** time = 0.444, size = 0, normalized size = 0.

$$\int \frac{1}{(d gx + cg)^3} \left( A + B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^3, x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^3, x)

**Maxima [A]** time = 1.20754, size = 350, normalized size = 2.32

$$\frac{1}{4} Bn \left( \frac{2 b dx + 3 bc - ad}{(bcd^3 - ad^4)g^3x^2 + 2(bc^2d^2 - acd^3)g^3x + (bc^3d - ac^2d^2)g^3} + \frac{2 b^2 \log(bx + a)}{(b^2c^2d - 2 abcd^2 + a^2d^3)g^3} - \frac{2 b^2 \log(dx + c)}{(b^2c^2d - 2 abcd^2 + a^2d^3)g^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^3, x, algorithm="maxima")

```
[Out] 1/4*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*g^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*g^3*x + (b*c^3*d - a*c^2*d^2)*g^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g^3) - 1/2*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^3*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) - 1/2*A/(d^3*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3)
```

**Fricas [A]** time = 0.794291, size = 562, normalized size = 3.72

$$\frac{2Ab^2c^2 - 4Aabcd + 2Aa^2d^2 - 2(Bb^2cd - Babd^2)nx - (3Bb^2c^2 - 4Babcd + Ba^2d^2)n + 2(Bb^2c^2 - 2Babcd + Ba^2d^2)\log(e)}{4((b^2c^2d^3 - 2abcd^4 + a^2d^5)g^3x^2 + 2(b^2c^3d^2 - 2abc^2d^3 + a^2cd^4)g^3x + (b^2c^4d - 2a^2b^2c^2d^2 + a^2c^2d^3)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*n*x - (3*B*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2)*n + 2*(B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*log(e) - 2*(B*b^2*d^2*n*x^2 + 2*B*b^2*c*d*n*x + (2*B*a*b*c*d - B*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*g^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*g^3*x + (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*g^3)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)**3,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 1.40003, size = 351, normalized size = 2.32

$$\frac{Bb^2n \log(bx + a)}{2(b^2c^2dg^3 - 2abcd^2g^3 + a^2d^3g^3)} - \frac{Bb^2n \log(dx + c)}{2(b^2c^2dg^3 - 2abcd^2g^3 + a^2d^3g^3)} - \frac{Bn \log\left(\frac{bx+a}{dx+c}\right)}{2(d^3g^3x^2 + 2cd^2g^3x + c^2dg^3)} + \frac{2Bbdnx + a^2d^3g^3}{4(bcd^3g^3x^2 - a^2d^3g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*b^2*n*log(b*x + a)/(b^2*c^2*d*g^3 - 2*a*b*c*d^2*g^3 + a^2*d^3*g^3) - 1/2*B*b^2*n*log(d*x + c)/(b^2*c^2*d*g^3 - 2*a*b*c*d^2*g^3 + a^2*d^3*g^3) - 1/2*B*n*log((b*x + a)/(d*x + c))/(d^3*g^3*x^2 + 2*c*d^2*g^3*x + c^2*d*g^3) + 1/4*(2*B*b*d*n*x + 3*B*b*c*n - B*a*d*n - 2*A*b*c - 2*B*b*c + 2*A*a*d + 2*B*a*d)/(b*c*d^3*g^3*x^2 - a*d^4*g^3*x^2 + 2*b*c^2*d^2*g^3*x - 2*a*c*d^3*g^3*x + b*c^3*d*g^3 - a*c^2*d^2*g^3)
```

$$3.36 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^4} dx$$

**Optimal.** Leaf size=183

$$-\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{3dg^4(c+dx)^3} + \frac{b^2Bn}{3dg^4(c+dx)(bc-ad)^2} + \frac{b^3Bn \log(a+bx)}{3dg^4(bc-ad)^3} - \frac{b^3Bn \log(c+dx)}{3dg^4(bc-ad)^3} + \frac{bBn}{6dg^4(c+dx)^2(bc-ad)} +$$

[Out] (B\*n)/(9\*d\*g^4\*(c + d\*x)^3) + (b\*B\*n)/(6\*d\*(b\*c - a\*d)\*g^4\*(c + d\*x)^2) + (b^2\*B\*n)/(3\*d\*(b\*c - a\*d)^2\*g^4\*(c + d\*x)) + (b^3\*B\*n\*Log[a + b\*x])/(3\*d\*(b\*c - a\*d)^3\*g^4) - (A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(3\*d\*g^4\*(c + d\*x)^3) - (b^3\*B\*n\*Log[c + d\*x])/(3\*d\*(b\*c - a\*d)^3\*g^4)

**Rubi [A]** time = 0.143203, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{3dg^4(c+dx)^3} + \frac{b^2Bn}{3dg^4(c+dx)(bc-ad)^2} + \frac{b^3Bn \log(a+bx)}{3dg^4(bc-ad)^3} - \frac{b^3Bn \log(c+dx)}{3dg^4(bc-ad)^3} + \frac{bBn}{6dg^4(c+dx)^2(bc-ad)} +$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(c\*g + d\*g\*x)^4, x]

[Out] (B\*n)/(9\*d\*g^4\*(c + d\*x)^3) + (b\*B\*n)/(6\*d\*(b\*c - a\*d)\*g^4\*(c + d\*x)^2) + (b^2\*B\*n)/(3\*d\*(b\*c - a\*d)^2\*g^4\*(c + d\*x)) + (b^3\*B\*n\*Log[a + b\*x])/(3\*d\*(b\*c - a\*d)^3\*g^4) - (A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(3\*d\*g^4\*(c + d\*x)^3) - (b^3\*B\*n\*Log[c + d\*x])/(3\*d\*(b\*c - a\*d)^3\*g^4)

#### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg + dgx)^4} dx &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c + dx)^3} + \frac{(Bn) \int \frac{bc-ad}{g^3(a+bx)(c+dx)^4} dx}{3dg} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c + dx)^3} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)(c+dx)^4} dx}{3dg^4} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3dg^4(c + dx)^3} + \frac{(B(bc - ad)n) \int \left(\frac{b^4}{(bc-ad)^4(a+bx)} - \frac{d}{(bc-ad)(c+dx)^4} - \frac{bd}{(bc-ad)^2(c+dx)^3} - \frac{d^2}{(bc-ad)^3(c+dx)^2}\right) dx}{3dg^4} \\
&= \frac{Bn}{9dg^4(c + dx)^3} + \frac{bBn}{6d(bc - ad)g^4(c + dx)^2} + \frac{b^2Bn}{3d(bc - ad)^2g^4(c + dx)} + \frac{b^3Bn \log(a + bx)}{3d(bc - ad)^3g^4} - \frac{d^2Bn}{3d(bc - ad)^3g^4}
\end{aligned}$$

**Mathematica [A]** time = 0.176667, size = 146, normalized size = 0.8

$$\frac{Bn((bc-ad)(2a^2d^2-abd(7c+3dx)+b^2(11c^2+15cdx+6d^2x^2))+6b^3(c+dx)^3 \log(a+bx)-6b^3(c+dx)^3 \log(c+dx))}{(bc-ad)^3} - 6 \left( B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A \right)$$


---


$$18dg^4(c + dx)^3$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(c\*g + d\*g\*x)^4, x]

[Out] (-6\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + (B\*n\*((b\*c - a\*d)\*(2\*a^2\*d^2 - a\*b\*d\*(7\*c + 3\*d\*x) + b^2\*(11\*c^2 + 15\*c\*d\*x + 6\*d^2\*x^2)) + 6\*b^3\*(c + d\*x)^3\*Log[a + b\*x] - 6\*b^3\*(c + d\*x)^3\*Log[c + d\*x]))/(b\*c - a\*d)^3)/(18\*d\*g^4\*(c + d\*x)^3)

**Maple [F]** time = 0.449, size = 0, normalized size = 0.

$$\int \frac{1}{(dgx + cg)^4} \left( A + B \ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^4, x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^4, x)

**Maxima [B]** time = 1.25888, size = 585, normalized size = 3.2

$$\frac{1}{18} Bn \left( \frac{6b^2d^2x^2 + 11b^2c^2 - 7abcd + 2a^2d^2 + 3(5b^2cd - abd^2)x}{(b^2c^2d^4 - 2abcd^5 + a^2d^6)g^4x^3 + 3(b^2c^3d^3 - 2abc^2d^4 + a^2cd^5)g^4x^2 + 3(b^2c^4d^2 - 2abc^3d^3 + a^2c^2d^4)g^4x + (b^2c^5d - abc^4d^2 + a^2cd^5)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^4, x, algorithm="maxima")

```
[Out] 1/18*B*n*((6*b^2*d^2*x^2 + 11*b^2*c^2 - 7*a*b*c*d + 2*a^2*d^2 + 3*(5*b^2*c*d - a*b*d^2)*x)/((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*g^4*x^3 + 3*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*g^4*x^2 + 3*(b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*g^4*x + (b^2*c^5*d - 2*a*b*c^4*d^2 + a^2*c^3*d^3)*g^4) + 6*b^3*log(b*x + a)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*g^4) - 6*b^3*log(d*x + c)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*g^4)) - 1/3*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*g^4*x^3 + 3*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4) - 1/3*A/(d^4*g^4*x^3 + 3*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4)
```

---

**Fricas [B]** time = 0.9341, size = 990, normalized size = 5.41

$$\frac{6Ab^3c^3 - 18Aab^2c^2d + 18Aa^2bcd^2 - 6Aa^3d^3 - 6(Bb^3cd^2 - Bab^2d^3)nx^2 - 3(5Bb^3c^2d - 6Bab^2cd^2 + Ba^2bd^3)nx - 18((b^3c^3d^4 - 3ab^2c^2d^5 + 3a^2bcd^6 - a^3d^7)g^4x^3 + 3(b^3c^4d^3 - 3a^2b^2c^3d^4 + 3a^2b^2c^2d^5 - a^3c^2d^6)g^4x^2 + 3(b^3c^5d^2 - 3a^2b^2c^4d^3 + 3a^2b^2c^3d^4 - a^3c^2d^5)g^4x + (b^3c^6d - 3a^2b^2c^5d^2 + 3a^2b^2c^4d^3 - a^3c^3d^4)g^4)}{18((b^3c^3d^4 - 3a^2b^2c^2d^5 + 3a^2b^2c^2d^6 - a^3d^7)g^4x^3 + 3(b^3c^4d^3 - 3a^2b^2c^3d^4 + 3a^2b^2c^2d^5 - a^3c^2d^6)g^4x^2 + 3(b^3c^5d^2 - 3a^2b^2c^4d^3 + 3a^2b^2c^3d^4 - a^3c^2d^5)g^4x + (b^3c^6d - 3a^2b^2c^5d^2 + 3a^2b^2c^4d^3 - a^3c^3d^4)g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="fricas")
```

```
[Out] -1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(5*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + B*a^2*b*d^3)*n*x - (11*B*b^3*c^3 - 18*B*a*b^2*c^2*d + 9*B*a^2*b*c*d^2 - 2*B*a^3*d^3)*n + 6*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 - B*a^3*d^3)*log(e) - 6*(B*b^3*d^3*n*x^3 + 3*B*b^3*c*d^2*n*x^2 + 3*B*b^3*c^2*d*n*x + (3*B*a*b^2*c^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*n)*log((b*x + a)/(d*x + c)))/((b^3*c^3*d^4 - 3*a*b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*g^4*x^3 + 3*(b^3*c^4*d^3 - 3*a*b^2*c^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*g^4*x^2 + 3*(b^3*c^5*d^2 - 3*a*b^2*c^4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*g^4*x + (b^3*c^6*d - 3*a*b^2*c^5*d^2 + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*g^4)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)**4,x)
```

```
[Out] Timed out
```

---

**Giac [B]** time = 1.45879, size = 655, normalized size = 3.58

$$\frac{Bb^3n \log(bx + a)}{3(b^3c^3dg^4 - 3ab^2c^2d^2g^4 + 3a^2bcd^3g^4 - a^3d^4g^4)} - \frac{Bb^3n \log(dx + c)}{3(b^3c^3dg^4 - 3ab^2c^2d^2g^4 + 3a^2bcd^3g^4 - a^3d^4g^4)} - \frac{1}{3(d^4g^4x^3 + 3c^2d^2g^4x + c^3dg^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^4,x, algorithm="giac")
```

```
[Out] 1/3*B*b^3*n*log(b*x + a)/(b^3*c^3*d*g^4 - 3*a*b^2*c^2*d^2*g^4 + 3*a^2*b*c*d^3*g^4 - a^3*d^4*g^4) - 1/3*B*b^3*n*log(d*x + c)/(b^3*c^3*d*g^4 - 3*a*b^2*c^2*d^2*g^4 + 3*a^2*b*c*d^3*g^4 - a^3*d^4*g^4) - 1/3*B*n*log((b*x + a)/(d*x + c))/(d^4*g^4*x^3 + 3*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4) + 1/18*(6*B*b^2*d^2*n*x^2 + 15*B*b^2*c*d*n*x - 3*B*a*b*d^2*n*x + 11*B*b^2*c^2*n - 7*B*a*b*c*d*n + 2*B*a^2*d^2*n - 6*A*b^2*c^2 - 6*B*b^2*c^2 + 12*A*a*b*c*d + 12*B*a*b*c*d - 6*A*a^2*d^2 - 6*B*a^2*d^2)/(b^2*c^2*d^4*g^4*x^3 - 2*a*b*c*d^5*g^4*x^3 + a^2*d^6*g^4*x^3 + 3*b^2*c^3*d^3*g^4*x^2 - 6*a*b*c^2*d^4*g^4*x^2 + 3*a^2*c*d^5*g^4*x^2 + 3*b^2*c^4*d^2*g^4*x - 6*a*b*c^3*d^3*g^4*x + 3*a^2*c^2*d^4*g^4*x + b^2*c^5*d*g^4 - 2*a*b*c^4*d^2*g^4 + a^2*c^3*d^3*g^4)
```



$$3.37 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg+dgx)^5} dx$$

**Optimal.** Leaf size=215

$$\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{4dg^5(c+dx)^4} + \frac{b^3Bn}{4dg^5(c+dx)(bc-ad)^3} + \frac{b^2Bn}{8dg^5(c+dx)^2(bc-ad)^2} + \frac{b^4Bn \log(a+bx)}{4dg^5(bc-ad)^4} - \frac{b^4Bn \log(c+dx)}{4dg^5(bc-ad)^4}$$

[Out] (B\*n)/(16\*d\*g^5\*(c + d\*x)^4) + (b\*B\*n)/(12\*d\*(b\*c - a\*d)\*g^5\*(c + d\*x)^3) + (b^2\*B\*n)/(8\*d\*(b\*c - a\*d)^2\*g^5\*(c + d\*x)^2) + (b^3\*B\*n)/(4\*d\*(b\*c - a\*d)^3\*g^5\*(c + d\*x)) + (b^4\*B\*n\*Log[a + b\*x])/(4\*d\*(b\*c - a\*d)^4\*g^5) - (A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(4\*d\*g^5\*(c + d\*x)^4) - (b^4\*B\*n\*Log[c + d\*x])/(4\*d\*(b\*c - a\*d)^4\*g^5)

**Rubi [A]** time = 0.175639, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2525, 12, 44}

$$\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{4dg^5(c+dx)^4} + \frac{b^3Bn}{4dg^5(c+dx)(bc-ad)^3} + \frac{b^2Bn}{8dg^5(c+dx)^2(bc-ad)^2} + \frac{b^4Bn \log(a+bx)}{4dg^5(bc-ad)^4} - \frac{b^4Bn \log(c+dx)}{4dg^5(bc-ad)^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(c\*g + d\*g\*x)^5, x]

[Out] (B\*n)/(16\*d\*g^5\*(c + d\*x)^4) + (b\*B\*n)/(12\*d\*(b\*c - a\*d)\*g^5\*(c + d\*x)^3) + (b^2\*B\*n)/(8\*d\*(b\*c - a\*d)^2\*g^5\*(c + d\*x)^2) + (b^3\*B\*n)/(4\*d\*(b\*c - a\*d)^3\*g^5\*(c + d\*x)) + (b^4\*B\*n\*Log[a + b\*x])/(4\*d\*(b\*c - a\*d)^4\*g^5) - (A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(4\*d\*g^5\*(c + d\*x)^4) - (b^4\*B\*n\*Log[c + d\*x])/(4\*d\*(b\*c - a\*d)^4\*g^5)

#### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^n\_.\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*Rfx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(cg + dgx)^5} dx &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c + dx)^4} + \frac{(Bn) \int \frac{bc-ad}{g^4(a+bx)(c+dx)^5} dx}{4dg} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c + dx)^4} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)(c+dx)^5} dx}{4dg^5} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4dg^5(c + dx)^4} + \frac{(B(bc - ad)n) \int \left(\frac{b^5}{(bc-ad)^5(a+bx)} - \frac{d}{(bc-ad)(c+dx)^5} - \frac{bd}{(bc-ad)^2(c+dx)^4} - \frac{1}{(bc-ad)^3(c+dx)^3}\right) dx}{4dg^5} \\
&= \frac{Bn}{16dg^5(c + dx)^4} + \frac{bBn}{12d(bc - ad)g^5(c + dx)^3} + \frac{b^2Bn}{8d(bc - ad)^2g^5(c + dx)^2} + \frac{b^3Bn}{4d(bc - ad)^3g^5(c + dx)}
\end{aligned}$$

**Mathematica [A]** time = 0.237246, size = 162, normalized size = 0.75

$$\frac{Bn\left(\frac{12b^3(bc-ad)}{c+dx} + \frac{6b^2(bc-ad)^2}{(c+dx)^2} + 12b^4 \log(a+bx) + \frac{4b(bc-ad)^3}{(c+dx)^3} + \frac{3(bc-ad)^4}{(c+dx)^4} - 12b^4 \log(c+dx)\right) - \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(c+dx)^4}}{4dg^5}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(c\*g + d\*g\*x)^5, x]

[Out] (-((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(c + d\*x)^4) + (B\*n\*((3\*(b\*c - a\*d)^4)/(c + d\*x)^4 + (4\*b\*(b\*c - a\*d)^3)/(c + d\*x)^3 + (6\*b^2\*(b\*c - a\*d)^2)/(c + d\*x)^2 + (12\*b^3\*(b\*c - a\*d))/(c + d\*x) + 12\*b^4\*Log[a + b\*x] - 12\*b^4\*Log[c + d\*x]))/(12\*(b\*c - a\*d)^4)/(4\*d\*g^5)

**Maple [F]** time = 0.455, size = 0, normalized size = 0.

$$\int \frac{1}{(dgx + cg)^5} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^5, x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^5, x)

**Maxima [B]** time = 1.25488, size = 880, normalized size = 4.09

$$\frac{1}{48} Bn \left( \frac{12b^3d^3x^3 + 25b^3c^3 - 23ab^2c^2d + 13a^2bcd^2 - 3a^3}{(b^3c^3d^5 - 3ab^2c^2d^6 + 3a^2bcd^7 - a^3d^8)g^5x^4 + 4(b^3c^4d^4 - 3ab^2c^3d^5 + 3a^2bc^2d^6 - a^3cd^7)g^5x^3 + 6(b^3c^5d^3 - 3ab^2c^4d^4 - 3a^2bcd^5 + a^3d^6)g^5x^2 + 6(b^3c^6d^2 - 3ab^2c^5d^3 - 3a^2bcd^6 + a^3d^7)g^5x + 6(b^3c^7d - 3ab^2c^6d^2 - 3a^2bcd^7 + a^3d^8)g^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^5, x, algorithm="maxima")

```
[Out] 1/48*B*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2 -
3*a^3*d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2*c*d
^2 + a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^
8)*g^5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7
)*g^5*x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^
6)*g^5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d
^5)*g^5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*g
^5) + 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3
- 4*a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4*d - 4*a*b^
3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5)) - 1/4*B*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d
^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c^4*d*g^5) - 1/4*A/(d^5*g^5*x^4 + 4*c*d^4*g^
5*x^3 + 6*c^2*d^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c^4*d*g^5)
```

---

**Fricas [B]** time = 0.999619, size = 1505, normalized size = 7.

$$\frac{12 Ab^4c^4 - 48 Aab^3c^3d + 72 Aa^2b^2c^2d^2 - 48 Aa^3bcd^3 + 12 Aa^4d^4 - 12 (Bb^4cd^3 - Bab^3d^4)nx^3 - 6 (7Bb^4c^2d^2 - 8Bab^3c^2d^2 + 6Aa^2b^2c^2d^2 - 4Aa^3b^2c^2d^2 + 6Aa^4d^4)nx^2 - 4 (13Bb^4c^3d - 18Bab^3c^3d + 6Aa^2b^2c^3d - 6Aa^3b^2c^3d + 3Aa^4d^4)nx + 12 (Bb^4c^4 - 4Bab^3c^4 + 6Aa^2b^2c^4 - 4Aa^3b^2c^4 + Bb^4d^4)log(e) - 12 (Bb^4d^4n^4 + 4Bb^4c^3d^3n^3 + 6Bb^4c^2d^2n^2 + 4Bb^4c^3d^3n^2 + 4Bb^4c^3d^3n^2 + (4Bb^4c^3d^3n^2 + 4Bb^4c^3d^3n^2 + 4Bb^4c^3d^3n^2 + 4Bb^4c^3d^3n^2)log((b*x + a)/(d*x + c)))/((b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b^2*c^2*d^7 + a^4*d^9)*g^5*x^4 + 4*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b^2*c^2*d^7 + a^4*c*d^8)*g^5*x^3 + 6*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^4*d^5 - 4*a^3*b^2*c^3*d^6 + a^4*c^2*d^7)*g^5*x^2 + 4*(b^4*c^7*d^2 - 4*a*b^3*c^6*d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b^2*c^4*d^5 + a^4*c^3*d^6)*g^5*x + (b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b^2*c^5*d^4 + a^4*c^4*d^5)*g^5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)^5,x, algorithm="fric
as")
```

```
[Out] -1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b*
c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 - 6*(7*B*b^4*c^
2*d^2 - 8*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n*x^2 - 4*(13*B*b^4*c^3*d - 18*B*a
*b^3*c^2*d^2 + 6*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*n*x - (25*B*b^4*c^4 - 48*B*
a*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 16*B*a^3*b*c*d^3 + 3*B*a^4*d^4)*n + 12
*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B*a
^4*d^4)*log(e) - 12*(B*b^4*d^4*n^4 + 4*B*b^4*c^3*d^3*n^3 + 6*B*b^4*c^2*d^
2*n^2 + 4*B*b^4*c^3*d^3*n^2 + 4*B*b^4*c^3*d^3*n^2 + 4*B*b^4*c^3*d^3*n^2 + 4*B
a^3*b*c*d^3 - B*a^4*d^4)*n*log((b*x + a)/(d*x + c)))/((b^4*c^4*d^5 - 4*a*b
^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b^2*c^2*d^7 + a^4*d^9)*g^5*x^4 + 4*(b^4*
c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b^2*c^2*d^7 + a^4*c*d^8
)*g^5*x^3 + 6*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^4*d^5 - 4*a^3*b*
c^3*d^6 + a^4*c^2*d^7)*g^5*x^2 + 4*(b^4*c^7*d^2 - 4*a*b^3*c^6*d^3 + 6*a^2*b
^2*c^5*d^4 - 4*a^3*b^2*c^4*d^5 + a^4*c^3*d^6)*g^5*x + (b^4*c^8*d - 4*a*b^3*c^
7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b^2*c^5*d^4 + a^4*c^4*d^5)*g^5)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*g*x+c*g)**5,x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.34134, size = 1038, normalized size = 4.83

$$\frac{Bb^4n \log(bx + a)}{4(b^4c^4dg^5 - 4ab^3c^3d^2g^5 + 6a^2b^2c^2d^3g^5 - 4a^3bcd^4g^5 + a^4d^5g^5)} - \frac{Bb^4n \log(dx + c)}{4(b^4c^4dg^5 - 4ab^3c^3d^2g^5 + 6a^2b^2c^2d^3g^5 - 4a^3bcd^4g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(d\*g\*x+c\*g)^5,x, algorithm="giac")

[Out]  $\frac{1}{4}Bb^4n \log(bx + a) / (b^4c^4dg^5 - 4a^3b^3c^3d^2g^5 + 6a^2b^2c^2d^3g^5 - 4a^3bcd^4g^5 + a^4d^5g^5) - \frac{1}{4}Bb^4n \log(dx + c) / (b^4c^4dg^5 - 4a^3b^3c^3d^2g^5 + 6a^2b^2c^2d^3g^5 - 4a^3bcd^4g^5 + a^4d^5g^5) - \frac{1}{4}Bn \log((bx + a)/(dx + c)) / (d^5g^5x^4 + 4cd^4g^5x^3 + 6c^2d^3g^5x^2 + 4c^3d^2g^5x + c^4dg^5) + \frac{1}{48} (12Bb^3d^3nx^3 + 42Bb^3cd^2nx^2 - 6Bab^2d^3nx^2 + 52Bb^3c^2d^2nx - 20Bab^2cd^2nx + 4Ba^2bd^3nx + 25Bb^3c^3n - 23Bab^2c^2d^2n + 13Ba^2b^2cd^2n - 3Ba^3d^3n - 12Aab^3c^3 - 12Bb^3c^3 + 36Aab^2c^2d + 36Bab^2c^2d - 36Aa^2b^2cd^2 - 36Ba^2b^2cd^2 + 12Aa^3d^3 + 12Ba^3d^3) / (b^3c^3d^5g^5x^4 - 3ab^2c^2d^6g^5x^4 + 3a^2b^2cd^7g^5x^4 - a^3d^8g^5x^4 + 4b^3c^4d^4g^5x^3 - 12ab^2c^3d^5g^5x^3 + 12a^2b^2cd^6g^5x^3 - 4a^3cd^7g^5x^3 + 6b^3c^5d^3g^5x^2 - 18ab^2c^4d^4g^5x^2 + 18a^2b^2cd^5g^5x^2 - 6a^3c^2d^6g^5x^2 + 4b^3c^6d^2g^5x - 12ab^2c^5d^3g^5x + 12a^2b^2cd^4d^4g^5x - 4a^3c^3d^5g^5x + b^3c^7dg^5 - 3ab^2c^6d^2g^5 + 3a^2b^2cd^5d^3g^5 - a^3c^4d^4g^5)$

$$3.38 \quad \int (cg + dgx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx$$

**Optimal.** Leaf size=544

$$\frac{2B^2g^4n^2(bc-ad)^5 \text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right)}{5b^5d} + \frac{2Bg^4n(bc-ad)^5 \log\left(1 - \frac{b(c+dx)}{d(a+bx)}\right) \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{5b^5d} - \frac{2Bg^4n(a+bx)^2(bc-ad)^5 \text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right)}{5b^5d}$$

```
[Out] (13*B^2*(b*c - a*d)^4*g^4*n^2*x)/(30*b^4) + (7*B^2*(b*c - a*d)^3*g^4*n^2*(c + d*x)^2)/(60*b^3*d) + (B^2*(b*c - a*d)^2*g^4*n^2*(c + d*x)^3)/(30*b^2*d) - (2*B*(b*c - a*d)^4*g^4*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(5*b^5) - (B*(b*c - a*d)^3*g^4*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b^3*d) - (2*B*(b*c - a*d)^2*g^4*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(15*b^2*d) - (B*(b*c - a*d)*g^4*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(10*b*d) + (g^4*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(5*d) + (13*B^2*(b*c - a*d)^5*g^4*n^2*Log[(a + b*x)/(c + d*x)]/(30*b^5*d) + (5*B^2*(b*c - a*d)^5*g^4*n^2*Log[c + d*x])/(6*b^5*d) + (2*B*(b*c - a*d)^5*g^4*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(5*b^5*d) - (2*B^2*(b*c - a*d)^5*g^4*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(5*b^5*d)
```

**Rubi [A]** time = 0.880468, antiderivative size = 634, normalized size of antiderivative = 1.17, number of steps used = 27, number of rules used = 13, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$ , Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 43}

$$\frac{2B^2g^4n^2(bc-ad)^5 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{5b^5d} - \frac{2Bg^4n(bc-ad)^5 \log(a+bx) \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{5b^5d} - \frac{Bg^4n(c+dx)^2(bc-ad)^5 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{5b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2, x]
```

```
[Out] (-2*A*B*(b*c - a*d)^4*g^4*n*x)/(5*b^4) + (13*B^2*(b*c - a*d)^4*g^4*n^2*x)/(30*b^4) + (7*B^2*(b*c - a*d)^3*g^4*n^2*(c + d*x)^2)/(60*b^3*d) + (B^2*(b*c - a*d)^2*g^4*n^2*(c + d*x)^3)/(30*b^2*d) + (13*B^2*(b*c - a*d)^5*g^4*n^2*Log[a + b*x])/(30*b^5*d) + (B^2*(b*c - a*d)^5*g^4*n^2*Log[a + b*x]^2)/(5*b^5*d) - (2*B^2*(b*c - a*d)^4*g^4*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(5*b^5) - (B*(b*c - a*d)^3*g^4*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b^3*d) - (2*B*(b*c - a*d)^2*g^4*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(15*b^2*d) - (B*(b*c - a*d)*g^4*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(10*b*d) - (2*B*(b*c - a*d)^5*g^4*n*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b^5*d) + (g^4*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(5*d) + (2*B^2*(b*c - a*d)^5*g^4*n^2*Log[c + d*x])/(5*b^5*d) - (2*B^2*(b*c - a*d)^5*g^4*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(5*b^5*d) - (2*B^2*(b*c - a*d)^5*g^4*n^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(5*b^5*d)
```

**Rule 2525**

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d
```

, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x)

, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))])\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
 \int (cg + dgx)^4 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2 dx &= \frac{g^4(c + dx)^5 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{5d} - \frac{(2Bn) \int \frac{(bc - ad)g^5(c + dx)^4 (A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right))^2}{a + bx} dx}{5dg} \\
 &= \frac{g^4(c + dx)^5 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{5d} - \frac{(2B(bc - ad)g^4n) \int \frac{(c + dx)^4 (A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right))^2}{a + bx} dx}{5d} \\
 &= \frac{g^4(c + dx)^5 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{5d} - \frac{(2B(bc - ad)g^4n) \int \frac{d(bc - ad)^3 (c + dx)^3 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{a + bx} dx}{5d} \\
 &= \frac{g^4(c + dx)^5 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{5d} - \frac{(2B(bc - ad)g^4n) \int (c + dx)^3 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2 dx}{5b} \\
 &= -\frac{2AB(bc - ad)^4 g^4 n x}{5b^4} - \frac{B(bc - ad)^3 g^4 n (c + dx)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{5b^3 d} \\
 &= -\frac{2AB(bc - ad)^4 g^4 n x}{5b^4} - \frac{2B^2(bc - ad)^4 g^4 n (a + bx) \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right)}{5b^5} + \frac{B^2(bc - ad)^4 g^4 n^2 (a + bx)^2}{5b^5} \\
 &= -\frac{2AB(bc - ad)^4 g^4 n x}{5b^4} - \frac{2B^2(bc - ad)^4 g^4 n (a + bx) \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right)}{5b^5} + \frac{B^2(bc - ad)^4 g^4 n^2 (a + bx)^2}{5b^5} \\
 &= -\frac{2AB(bc - ad)^4 g^4 n x}{5b^4} + \frac{13B^2(bc - ad)^4 g^4 n^2 x}{30b^4} + \frac{7B^2(bc - ad)^3 g^4 n^2 (c + dx)^2}{60b^3 d} \\
 &= -\frac{2AB(bc - ad)^4 g^4 n x}{5b^4} + \frac{13B^2(bc - ad)^4 g^4 n^2 x}{30b^4} + \frac{7B^2(bc - ad)^3 g^4 n^2 (c + dx)^2}{60b^3 d} \\
 &= -\frac{2AB(bc - ad)^4 g^4 n x}{5b^4} + \frac{13B^2(bc - ad)^4 g^4 n^2 x}{30b^4} + \frac{7B^2(bc - ad)^3 g^4 n^2 (c + dx)^2}{60b^3 d}
 \end{aligned}$$

**Mathematica [A]** time = 0.504332, size = 533, normalized size = 0.98

$$g^4 \left( (c + dx)^5 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)^2 - \frac{Bn(bc-ad) \left( -12Bn(bc-ad)^4 \left( \log(a+bx) \left( \log(a+bx) - 2 \log \left( \frac{b(c+dx)}{bc-ad} \right) \right) - 2 \text{PolyLog} \left( 2, \frac{d(a+bx)}{ad-bc} \right) \right) + 12b^2(c+dx)^2 (bc-ad)}{5d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c\*g + d\*g\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] (g^4\*((c + d\*x)^5\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 - (B\*(b\*c - a\*d)\*n\*(24\*A\*b\*d\*(b\*c - a\*d)^3\*x - 12\*B\*(b\*c - a\*d)^3\*n\*(b\*d\*x + (b\*c - a\*d)\*Log[a + b\*x]) - 4\*B\*(b\*c - a\*d)^2\*n\*(2\*b\*d\*(b\*c - a\*d)\*x + b^2\*(c + d\*x)^2 + 2\*(b\*c - a\*d)^2\*Log[a + b\*x]) - B\*(b\*c - a\*d)\*n\*(6\*b\*d\*(b\*c - a\*d)^2\*x + 3\*b^2\*(b\*c - a\*d)\*(c + d\*x)^2 + 2\*b^3\*(c + d\*x)^3 + 6\*(b\*c - a\*d)^3\*Log[a + b\*x]) + 24\*B\*d\*(b\*c - a\*d)^3\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n] + 12\*b^2\*(b\*c - a\*d)^2\*(c + d\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 8\*b^3\*(b\*c - a\*d)\*(c + d\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 6\*b^4\*(c + d\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 24\*(b\*c - a\*d)^4\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 24\*B\*(b\*c - a\*d)^4\*n\*Log[(c + d\*x)/(b\*c - a\*d)] - 12\*B\*(b\*c - a\*d)^4\*n\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-(b\*c) + a\*d)])))/(12\*b^5))/(5\*d)

**Maple [F]** time = 0.441, size = 0, normalized size = 0.

$$\int (d g x + c g)^4 \left( A + B \ln \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*g\*x+c\*g)^4\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((d\*g\*x+c\*g)^4\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [B]** time = 3.79233, size = 3888, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^4\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out] 2/5\*A\*B\*d^4\*g^4\*x^5\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/5\*A^2\*d^4\*g^4\*x^5 + 2\*A\*B\*c\*d^3\*g^4\*x^4\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A^2\*c\*d^3\*g^4\*x^4 + 4\*A\*B\*c^2\*d^2\*g^4\*x^3\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 2\*A^2\*c^2\*d^2\*g^4\*x^3 + 4\*A\*B\*c^3\*d\*g^4\*x^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 2\*A^2\*c^3\*d\*g^4\*x^2 + 1/30\*A\*B\*d^4\*g^4\*n\*(12\*a^5\*log(b\*x + a)/b^5 - 12\*c^5\*log(d\*x + c)/d^5 - (3\*(b^4\*c\*d^3 - a\*b^3\*d^4)\*x^4 - 4\*(b^4\*c^2\*d^2 - a^2\*b^2\*d^4)\*x^3 + 6\*(b^4\*c^3\*d - a^3\*b\*d^4)\*x^2 - 12\*(b^4\*c^4 - a^4\*d^4)\*x)/(b^4\*d^4) - 1/3\*A\*B\*c\*d^3\*g^4\*n\*(6\*a^4\*log(b\*x + a)/b^4 - 6\*c^4\*log(d\*x + c)/d^4 + (2\*(b^3\*c\*d^2 - a\*b^2\*d^3)\*x^3 - 3\*(b^3\*c^2\*d - a^2\*b\*d^3)



$$\begin{aligned}
& *x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 2*A*B*c^2*d^2*g^4*n*(2*a^3*\log \\
& (b*x + a)/b^3 - 2*c^3*\log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2* \\
& c^2 - a^2*d^2)*x)/(b^2*d^2)) - 4*A*B*c^3*d*g^4*n*(a^2*\log(b*x + a)/b^2 - c^ \\
& 2*\log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^4*g^4*n*(a*\log(b*x + a) \\
& /b - c*\log(d*x + c)/d) + 2*A*B*c^4*g^4*x*\log(e*(b*x/(d*x + c) + a/(d*x + c) \\
& )^n) + A^2*c^4*g^4*x - 1/30*(77*a*b^3*c^4*d*g^4*n^2 - 94*a^2*b^2*c^3*d^2*g^ \\
& 4*n^2 + 54*a^3*b*c^2*d^3*g^4*n^2 - 12*a^4*c*d^4*g^4*n^2 - (25*g^4*n^2 - 12* \\
& g^4*n*\log(e))*b^4*c^5)*B^2*\log(d*x + c)/(b^4*d) - 2/5*(b^5*c^5*g^4*n^2 - 5* \\
& a*b^4*c^4*d*g^4*n^2 + 10*a^2*b^3*c^3*d^2*g^4*n^2 - 10*a^3*b^2*c^2*d^3*g^4*n \\
& ^2 + 5*a^4*b*c*d^4*g^4*n^2 - a^5*d^5*g^4*n^2)*(log(b*x + a)*log((b*d*x + a* \\
& d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))*B^2/(b^5*d) + 1/60 \\
& *(12*B^2*b^5*d^5*g^4*x^5*\log(e)^2 + 24*B^2*b^5*c^5*g^4*n^2*\log(b*x + a)*log \\
& (d*x + c) - 12*B^2*b^5*c^5*g^4*n^2*\log(d*x + c)^2 + 6*(a*b^4*d^5*g^4*n*\log(e) \\
& - (g^4*n*\log(e) - 10*g^4*\log(e)^2)*b^5*c*d^4)*B^2*x^4 + 2*((g^4*n^2 - 16 \\
& *g^4*n*\log(e) + 60*g^4*\log(e)^2)*b^5*c^2*d^3 - 2*(g^4*n^2 - 10*g^4*n*\log(e) \\
& )*a*b^4*c*d^4 + (g^4*n^2 - 4*g^4*n*\log(e))*a^2*b^3*d^5)*B^2*x^3 + ((13*g^4* \\
& n^2 - 72*g^4*n*\log(e) + 120*g^4*\log(e)^2)*b^5*c^3*d^2 - 3*(11*g^4*n^2 - 40* \\
& g^4*n*\log(e))*a*b^4*c^2*d^3 + 3*(9*g^4*n^2 - 20*g^4*n*\log(e))*a^2*b^3*c*d^4 \\
& - (7*g^4*n^2 - 12*g^4*n*\log(e))*a^3*b^2*d^5)*B^2*x^2 - 12*(5*a*b^4*c^4*d*g \\
& ^4*n^2 - 10*a^2*b^3*c^3*d^2*g^4*n^2 + 10*a^3*b^2*c^2*d^3*g^4*n^2 - 5*a^4*b* \\
& c*d^4*g^4*n^2 + a^5*d^5*g^4*n^2)*B^2*\log(b*x + a)^2 + 2*((23*g^4*n^2 - 48*g \\
& ^4*n*\log(e) + 30*g^4*\log(e)^2)*b^5*c^4*d - (79*g^4*n^2 - 120*g^4*n*\log(e))* \\
& a*b^4*c^3*d^2 + 6*(17*g^4*n^2 - 20*g^4*n*\log(e))*a^2*b^3*c^2*d^3 - (59*g^4* \\
& n^2 - 60*g^4*n*\log(e))*a^3*b^2*c*d^4 + (13*g^4*n^2 - 12*g^4*n*\log(e))*a^4*b \\
& *d^5)*B^2*x - 2*(12*(4*g^4*n^2 - 5*g^4*n*\log(e))*a*b^4*c^4*d - 12*(13*g^4*n \\
& ^2 - 10*g^4*n*\log(e))*a^2*b^3*c^3*d^2 + 4*(49*g^4*n^2 - 30*g^4*n*\log(e))*a^ \\
& 3*b^2*c^2*d^3 - (113*g^4*n^2 - 60*g^4*n*\log(e))*a^4*b*c*d^4 + (25*g^4*n^2 - \\
& 12*g^4*n*\log(e))*a^5*d^5)*B^2*\log(b*x + a) + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B \\
& ^2*b^5*c*d^4*g^4*x^4 + 10*B^2*b^5*c^2*d^3*g^4*x^3 + 10*B^2*b^5*c^3*d^2*g^4* \\
& x^2 + 5*B^2*b^5*c^4*d*g^4*x)*log((b*x + a)^n)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + \\
& 5*B^2*b^5*c*d^4*g^4*x^4 + 10*B^2*b^5*c^2*d^3*g^4*x^3 + 10*B^2*b^5*c^3*d^2* \\
& g^4*x^2 + 5*B^2*b^5*c^4*d*g^4*x)*log((d*x + c)^n)^2 + 2*(12*B^2*b^5*d^5*g^4 \\
& *x^5*\log(e) - 12*B^2*b^5*c^5*g^4*n*\log(d*x + c) + 3*(a*b^4*d^5*g^4*n - (g^4 \\
& *n - 20*g^4*\log(e))*b^5*c*d^4)*B^2*x^4 + 4*(5*a*b^4*c*d^4*g^4*n - a^2*b^3*d \\
& ^5*g^4*n - 2*(2*g^4*n - 15*g^4*\log(e))*b^5*c^2*d^3)*B^2*x^3 + 6*(10*a*b^4*c \\
& ^2*d^3*g^4*n - 5*a^2*b^3*c*d^4*g^4*n + a^3*b^2*d^5*g^4*n - 2*(3*g^4*n - 10* \\
& g^4*\log(e))*b^5*c^3*d^2)*B^2*x^2 + 12*(10*a*b^4*c^3*d^2*g^4*n - 10*a^2*b^3* \\
& c^2*d^3*g^4*n + 5*a^3*b^2*c*d^4*g^4*n - a^4*b*d^5*g^4*n - (4*g^4*n - 5*g^4* \\
& \log(e))*b^5*c^4*d)*B^2*x + 12*(5*a*b^4*c^4*d*g^4*n - 10*a^2*b^3*c^3*d^2*g^4 \\
& *n + 10*a^3*b^2*c^2*d^3*g^4*n - 5*a^4*b*c*d^4*g^4*n + a^5*d^5*g^4*n)*B^2*lo \\
& g(b*x + a)*log((b*x + a)^n) - 2*(12*B^2*b^5*d^5*g^4*x^5*\log(e) - 12*B^2*b^ \\
& 5*c^5*g^4*n*\log(d*x + c) + 3*(a*b^4*d^5*g^4*n - (g^4*n - 20*g^4*\log(e))*b^5 \\
& *c*d^4)*B^2*x^4 + 4*(5*a*b^4*c*d^4*g^4*n - a^2*b^3*d^5*g^4*n - 2*(2*g^4*n - \\
& 15*g^4*\log(e))*b^5*c^2*d^3)*B^2*x^3 + 6*(10*a*b^4*c^2*d^3*g^4*n - 5*a^2*b^ \\
& 3*c*d^4*g^4*n + a^3*b^2*d^5*g^4*n - 2*(3*g^4*n - 10*g^4*\log(e))*b^5*c^3*d^2 \\
& )*B^2*x^2 + 12*(10*a*b^4*c^3*d^2*g^4*n - 10*a^2*b^3*c^2*d^3*g^4*n + 5*a^3*b \\
& ^2*c*d^4*g^4*n - a^4*b*d^5*g^4*n - (4*g^4*n - 5*g^4*\log(e))*b^5*c^4*d)*B^2* \\
& x + 12*(5*a*b^4*c^4*d*g^4*n - 10*a^2*b^3*c^3*d^2*g^4*n + 10*a^3*b^2*c^2*d^3 \\
& *g^4*n - 5*a^4*b*c*d^4*g^4*n + a^5*d^5*g^4*n)*B^2*\log(b*x + a) + 12*(B^2*b^ \\
& 5*d^5*g^4*x^5 + 5*B^2*b^5*c*d^4*g^4*x^4 + 10*B^2*b^5*c^2*d^3*g^4*x^3 + 10*B \\
& ^2*b^5*c^3*d^2*g^4*x^2 + 5*B^2*b^5*c^4*d*g^4*x)*log((b*x + a)^n)*log((d*x \\
& + c)^n))/(b^5*d)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral  $\left( A^2 d^4 g^4 x^4 + 4 A^2 c d^3 g^4 x^3 + 6 A^2 c^2 d^2 g^4 x^2 + 4 A^2 c^3 d g^4 x + A^2 c^4 g^4 + (B^2 d^4 g^4 x^4 + 4 B^2 c d^3 g^4 x^3 + 6 B^2 c^2 d^2 g^4 x^2 + 4 B^2 c^3 d g^4 x + B^2 c^4 g^4) \log((b*x + a)^n) \right) / (b^5*d)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*d^4*g^4*x^4 + 4*A^2*c*d^3*g^4*x^3 + 6*A^2*c^2*d^2*g^4*x^2 + 4*A^2*c^3*d*g^4*x + A^2*c^4*g^4 + (B^2*d^4*g^4*x^4 + 4*B^2*c*d^3*g^4*x^3 + 6*B^2*c^2*d^2*g^4*x^2 + 4*B^2*c^3*d*g^4*x + B^2*c^4*g^4)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^4*g^4*x^4 + 4*A*B*c*d^3*g^4*x^3 + 6*A*B*c^2*d^2*g^4*x^2 + 4*A*B*c^3*d*g^4*x + A*B*c^4*g^4)*log(e*((b*x + a)/(d*x + c))^n), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**4*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d g x + c g)^4 \left( B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((d*g*x + c*g)^4*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

$$3.39 \quad \int (cg + dgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx$$

**Optimal.** Leaf size=454

$$\frac{B^2 g^3 n^2 (bc - ad)^4 \text{PolyLog} \left( 2, \frac{b(c+dx)}{d(a+bx)} \right)}{2b^4 d} + \frac{B g^3 n (bc - ad)^4 \log \left( 1 - \frac{b(c+dx)}{d(a+bx)} \right) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{2b^4 d} - \frac{B g^3 n (a + bx)(bc - ad)^4}{2b^4 d}$$

[Out]  $(5*B^2*(b*c - a*d)^3*g^3*n^2*x)/(12*b^3) + (B^2*(b*c - a*d)^2*g^3*n^2*(c + d*x)^2)/(12*b^2*d) - (B*(b*c - a*d)^3*g^3*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b^4) - (B*(b*c - a*d)^2*g^3*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b^2*d) - (B*(b*c - a*d)*g^3*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*b*d) + (g^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*d) + (5*B^2*(b*c - a*d)^4*g^3*n^2*Log[(a + b*x)/(c + d*x)])/(12*b^4*d) + (11*B^2*(b*c - a*d)^4*g^3*n^2*Log[c + d*x])/(12*b^4*d) + (B*(b*c - a*d)^4*g^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(2*b^4*d) - (B^2*(b*c - a*d)^4*g^3*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(2*b^4*d)$

**Rubi [A]** time = 0.684356, antiderivative size = 544, normalized size of antiderivative = 1.2, number of steps used = 23, number of rules used = 13, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$ , Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 43}

$$\frac{B^2 g^3 n^2 (bc - ad)^4 \text{PolyLog} \left( 2, -\frac{d(a+bx)}{bc-ad} \right)}{2b^4 d} - \frac{B g^3 n (bc - ad)^4 \log(a + bx) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{2b^4 d} - \frac{B g^3 n (c + dx)^2 (bc - ad)^4}{2b^4 d}$$

Antiderivative was successfully verified.

[In] Int[(c\*g + d\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out]  $-(A*B*(b*c - a*d)^3*g^3*n*x)/(2*b^3) + (5*B^2*(b*c - a*d)^3*g^3*n^2*x)/(12*b^3) + (B^2*(b*c - a*d)^2*g^3*n^2*(c + d*x)^2)/(12*b^2*d) + (5*B^2*(b*c - a*d)^4*g^3*n^2*Log[a + b*x])/(12*b^4*d) + (B^2*(b*c - a*d)^4*g^3*n^2*Log[a + b*x]^2)/(4*b^4*d) - (B^2*(b*c - a*d)^3*g^3*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(2*b^4) - (B*(b*c - a*d)^2*g^3*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b^2*d) - (B*(b*c - a*d)*g^3*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*b*d) - (B*(b*c - a*d)^4*g^3*n*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b^4*d) + (g^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*d) + (B^2*(b*c - a*d)^4*g^3*n^2*Log[c + d*x])/(2*b^4*d) - (B^2*(b*c - a*d)^4*g^3*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(2*b^4*d) - (B^2*(b*c - a*d)^4*g^3*n^2*PolyLog[2, -(d*(a + b*x))/(b*c - a*d)])/(2*b^4*d)$

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x]]/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(Rfx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[(f\*x)/d]^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
 \int (cg + dgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx &= \frac{g^3(c+dx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{4d} - \frac{(Bn) \int \frac{(bc-ad)g^4(c+dx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{a+bx} dx}{2dg} \\
 &= \frac{g^3(c+dx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{4d} - \frac{(B(bc-ad)g^3n) \int \frac{(c+dx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{a+bx} dx}{2d} \\
 &= \frac{g^3(c+dx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{4d} - \frac{(B(bc-ad)g^3n) \int \frac{d(bc-ad)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{a+bx} dx}{2d} \\
 &= \frac{g^3(c+dx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{4d} - \frac{(B(bc-ad)g^3n) \int (c+dx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx}{2b} \\
 &= -\frac{AB(bc-ad)^3 g^3 nx}{2b^3} - \frac{B(bc-ad)^2 g^3 n (c+dx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{4b^2 d} \\
 &= -\frac{AB(bc-ad)^3 g^3 nx}{2b^3} - \frac{B^2(bc-ad)^3 g^3 n (a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{2b^4} - \frac{B(bc-ad)^2 g^3 n^2 (c+dx)^2}{12b^2 d} \\
 &= -\frac{AB(bc-ad)^3 g^3 nx}{2b^3} - \frac{B^2(bc-ad)^3 g^3 n (a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{2b^4} - \frac{B(bc-ad)^2 g^3 n^2 (c+dx)^2}{12b^2 d} \\
 &= -\frac{AB(bc-ad)^3 g^3 nx}{2b^3} + \frac{5B^2(bc-ad)^3 g^3 n^2 x}{12b^3} + \frac{B^2(bc-ad)^2 g^3 n^2 (c+dx)^2}{12b^2 d} \\
 &= -\frac{AB(bc-ad)^3 g^3 nx}{2b^3} + \frac{5B^2(bc-ad)^3 g^3 n^2 x}{12b^3} + \frac{B^2(bc-ad)^2 g^3 n^2 (c+dx)^2}{12b^2 d} \\
 &= -\frac{AB(bc-ad)^3 g^3 nx}{2b^3} + \frac{5B^2(bc-ad)^3 g^3 n^2 x}{12b^3} + \frac{B^2(bc-ad)^2 g^3 n^2 (c+dx)^2}{12b^2 d}
 \end{aligned}$$

**Mathematica [A]** time = 0.339475, size = 409, normalized size = 0.9

$$g^3 \left( (c + dx)^4 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)^2 - \frac{Bn(bc-ad) \left( -3Bn(bc-ad)^3 \left( \log(a+bx) \left( \log(a+bx) - 2 \log \left( \frac{b(c+dx)}{bc-ad} \right) \right) - 2 \text{PolyLog} \left( 2, \frac{d(a+bx)}{ad-bc} \right) \right) + 3b^2(c+dx)^2(bc-d)}{4d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c\*g + d\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] (g^3\*((c + d\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 - (B\*(b\*c - a\*d)\*n\*(6\*A\*b\*d\*(b\*c - a\*d)^2\*x - 3\*B\*(b\*c - a\*d)^2\*n\*(b\*d\*x + (b\*c - a\*d)\*Log[a + b\*x]) - B\*(b\*c - a\*d)\*n\*(2\*b\*d\*(b\*c - a\*d)\*x + b^2\*(c + d\*x)^2 + 2\*(b\*c - a\*d)^2\*Log[a + b\*x]) + 6\*B\*d\*(b\*c - a\*d)^2\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n] + 3\*b^2\*(b\*c - a\*d)\*(c + d\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 2\*b^3\*(c + d\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 6\*(b\*c - a\*d)^3\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 6\*B\*(b\*c - a\*d)^3\*n\*Log[c + d\*x] - 3\*B\*(b\*c - a\*d)^3\*n\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-(b\*c) + a\*d)])))/(3\*b^4))/(4\*d)

**Maple [F]** time = 0.429, size = 0, normalized size = 0.

$$\int (d g x + c g)^3 \left( A + B \ln \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*g\*x+c\*g)^3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((d\*g\*x+c\*g)^3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [B]** time = 3.63244, size = 2874, normalized size = 6.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^3\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out] 1/2\*A\*B\*d^3\*g^3\*x^4\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/4\*A^2\*d^3\*g^3\*x^4 + 2\*A\*B\*c\*d^2\*g^3\*x^3\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A^2\*c\*d^2\*g^3\*x^3 + 3\*A\*B\*c^2\*d\*g^3\*x^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 3/2\*A^2\*c^2\*d\*g^3\*x^2 - 1/12\*A\*B\*d^3\*g^3\*n\*(6\*a^4\*log(b\*x + a)/b^4 - 6\*c^4\*log(d\*x + c)/d^4 + (2\*(b^3\*c\*d^2 - a\*b^2\*d^3)\*x^3 - 3\*(b^3\*c^2\*d - a^2\*b\*d^3)\*x^2 + 6\*(b^3\*c^3 - a^3\*d^3)\*x)/(b^3\*d^3)) + A\*B\*c\*d^2\*g^3\*n\*(2\*a^3\*log(b\*x + a)/b^3 - 2\*c^3\*log(d\*x + c)/d^3 - ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2)) - 3\*A\*B\*c^2\*d\*g^3\*n\*(a^2\*log(b\*x + a)/b^2 - c^2\*log(d\*x + c)/d^2 + (b\*c - a\*d)\*x/(b\*d)) + 2\*A\*B\*c^3\*g^3\*n\*(a\*log(b\*x + a)/b - c\*log(d\*x + c)/d) + 2\*A\*B\*c^3\*g^3\*x\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A^2\*c^3\*g^3\*x - 1/12\*(26\*a\*b^2\*c^3\*d\*g^3\*n^2 - 21\*a^2\*b\*c^2\*d^2\*g^3\*n^2 + 6\*a^3\*c\*d^3\*g^3\*n^2 - (11\*g^3\*n^2 - 6\*g^3\*n\*log(e))\*b^3\*c^4)\*B^2\*log(d

$$\begin{aligned}
& x + c)/(b^3*d) - 1/2*(b^4*c^4*g^3*n^2 - 4*a*b^3*c^3*d*g^3*n^2 + 6*a^2*b^2*c^2*d^2*g^3*n^2 - 4*a^3*b*c*d^3*g^3*n^2 + a^4*d^4*g^3*n^2)*(\log(b*x + a)*\log((b*d*x + a*d)/(b*c - a*d) + 1) + \text{dilog}(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*\log(e)^2 + 6*B^2*b^4*c^4*g^3*n^2*\log(b*x + a)*\log(d*x + c) - 3*B^2*b^4*c^4*g^3*n^2*\log(d*x + c)^2 + 2*(a*b^3*d^4*g^3*n*\log(e) - (g^3*n*\log(e) - 6*g^3*\log(e)^2)*b^4*c*d^3)*B^2*x^3 + ((g^3*n^2 - 9*g^3*n*\log(e) + 18*g^3*\log(e)^2)*b^4*c^2*d^2 - 2*(g^3*n^2 - 6*g^3*n*\log(e))*a*b^3*c*d^3 + (g^3*n^2 - 3*g^3*n*\log(e))*a^2*b^2*d^4)*B^2*x^2 - 3*(4*a*b^3*c^3*d*g^3*n^2 - 6*a^2*b^2*c^2*d^2*g^3*n^2 + 4*a^3*b*c*d^3*g^3*n^2 - a^4*d^4*g^3*n^2)*B^2*\log(b*x + a)^2 + ((7*g^3*n^2 - 18*g^3*n*\log(e) + 12*g^3*\log(e)^2)*b^4*c^3*d - (19*g^3*n^2 - 36*g^3*n*\log(e))*a*b^3*c^2*d^2 + (17*g^3*n^2 - 24*g^3*n*\log(e))*a^2*b^2*c*d^3 - (5*g^3*n^2 - 6*g^3*n*\log(e))*a^3*b*d^4)*B^2*x - (6*(3*g^3*n^2 - 4*g^3*n*\log(e))*a*b^3*c^3*d - 9*(5*g^3*n^2 - 4*g^3*n*\log(e))*a^2*b^2*c^2*d^2 + 2*(19*g^3*n^2 - 12*g^3*n*\log(e))*a^3*b*c*d^3 - (11*g^3*n^2 - 6*g^3*n*\log(e))*a^4*d^4)*B^2*\log(b*x + a) + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*c*d^3*g^3*x^3 + 6*B^2*b^4*c^2*d^2*g^3*x^2 + 4*B^2*b^4*c^3*d*g^3*x)*\log((b*x + a)^n)^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*c*d^3*g^3*x^3 + 6*B^2*b^4*c^2*d^2*g^3*x^2 + 4*B^2*b^4*c^3*d*g^3*x)*\log((d*x + c)^n)^2 + (6*B^2*b^4*d^4*g^3*x^4*\log(e) - 6*B^2*b^4*c^4*g^3*n*\log(d*x + c) + 2*(a*b^3*d^4*g^3*n - (g^3*n - 12*g^3*\log(e))*b^4*c*d^3)*B^2*x^3 + 3*(4*a*b^3*c*d^3*g^3*n - a^2*b^2*d^4*g^3*n - 3*(g^3*n - 4*g^3*\log(e))*b^4*c^2*d^2)*B^2*x^2 + 6*(6*a*b^3*c^2*d^2*g^3*n - 4*a^2*b^2*c*d^3*g^3*n + a^3*b*d^4*g^3*n - (3*g^3*n - 4*g^3*\log(e))*b^4*c^3*d)*B^2*x + 6*(4*a*b^3*c^3*d*g^3*n - 6*a^2*b^2*c^2*d^2*g^3*n + 4*a^3*b*c*d^3*g^3*n - a^4*d^4*g^3*n)*B^2*\log(b*x + a))*\log((b*x + a)^n) - (6*B^2*b^4*d^4*g^3*x^4*\log(e) - 6*B^2*b^4*c^4*g^3*n*\log(d*x + c) + 2*(a*b^3*d^4*g^3*n - (g^3*n - 12*g^3*\log(e))*b^4*c*d^3)*B^2*x^3 + 3*(4*a*b^3*c*d^3*g^3*n - a^2*b^2*d^4*g^3*n - 3*(g^3*n - 4*g^3*\log(e))*b^4*c^2*d^2)*B^2*x^2 + 6*(6*a*b^3*c^2*d^2*g^3*n - 4*a^2*b^2*c*d^3*g^3*n + a^3*b*d^4*g^3*n - (3*g^3*n - 4*g^3*\log(e))*b^4*c^3*d)*B^2*x + 6*(4*a*b^3*c^3*d*g^3*n - 6*a^2*b^2*c^2*d^2*g^3*n + 4*a^3*b*c*d^3*g^3*n - a^4*d^4*g^3*n)*B^2*\log(b*x + a) + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*c*d^3*g^3*x^3 + 6*B^2*b^4*c^2*d^2*g^3*x^2 + 4*B^2*b^4*c^3*d*g^3*x)*\log((b*x + a)^n))*\log((d*x + c)^n))/(b^4*d)
\end{aligned}$$


---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(A^2d^3g^3x^3 + 3A^2cd^2g^3x^2 + 3A^2c^2dg^3x + A^2c^3g^3 + (B^2d^3g^3x^3 + 3B^2cd^2g^3x^2 + 3B^2c^2dg^3x + B^2c^3g^3)\log(e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^3\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out] integral(A^2\*d^3\*g^3\*x^3 + 3\*A^2\*c\*d^2\*g^3\*x^2 + 3\*A^2\*c^2\*d\*g^3\*x + A^2\*c^3\*g^3 + (B^2\*d^3\*g^3\*x^3 + 3\*B^2\*c\*d^2\*g^3\*x^2 + 3\*B^2\*c^2\*d\*g^3\*x + B^2\*c^3\*g^3)\*log(e\*((b\*x + a)/(d\*x + c))^n)^2 + 2\*(A\*B\*d^3\*g^3\*x^3 + 3\*A\*B\*c\*d^2\*g^3\*x^2 + 3\*A\*B\*c^2\*d\*g^3\*x + A\*B\*c^3\*g^3)\*log(e\*((b\*x + a)/(d\*x + c))^n), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (dgx + cg)^3 \left( B \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((d*g*x + c*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```



$$3.40 \quad \int (cg + dgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx$$

**Optimal.** Leaf size=361

$$\frac{2B^2g^2n^2(bc-ad)^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d} + \frac{2Bg^2n(bc-ad)^3 \log\left(1 - \frac{b(c+dx)}{d(a+bx)}\right) \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{3b^3d} - \frac{2Bg^2n(a+bx)}{3b^2d}$$

```
[Out] (B^2*(b*c - a*d)^2*g^2*n^2*x)/(3*b^2) - (2*B*(b*c - a*d)^2*g^2*n*(a + b*x)*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*b^3) - (B*(b*c - a*d)*g^2*n*(c +
d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*b*d) + (g^2*(c + d*x)^3*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*d) + (B^2*(b*c - a*d)^3*g^2*n^
2*Log[(a + b*x)/(c + d*x)]/(3*b^3*d) + (B^2*(b*c - a*d)^3*g^2*n^2*Log[c +
d*x])/(b^3*d) + (2*B*(b*c - a*d)^3*g^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))]/(3*b^3*d) - (2*B^2*(b*c - a*d)^3
*g^2*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))]/(3*b^3*d)
```

**Rubi [A]** time = 0.566623, antiderivative size = 454, normalized size of antiderivative = 1.26, number of steps used = 19, number of rules used = 13, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$ , Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 43}

$$\frac{2B^2g^2n^2(bc-ad)^3 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{3b^3d} - \frac{2Bg^2n(bc-ad)^3 \log(a+bx) \left(B \log\left(e \left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{3b^3d} - \frac{2ABg^2nx(bc-ad)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c*g + d*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] (-2*A*B*(b*c - a*d)^2*g^2*n*x)/(3*b^2) + (B^2*(b*c - a*d)^2*g^2*n^2*x)/(3*b
^2) + (B^2*(b*c - a*d)^3*g^2*n^2*Log[a + b*x])/(3*b^3*d) + (B^2*(b*c - a*d)
^3*g^2*n^2*Log[a + b*x]^2)/(3*b^3*d) - (2*B^2*(b*c - a*d)^2*g^2*n*(a + b*x)
*Log[e*((a + b*x)/(c + d*x))^n])/(3*b^3) - (B*(b*c - a*d)*g^2*n*(c + d*x)^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*b*d) - (2*B*(b*c - a*d)^3*g^2*n
*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*b^3*d) + (g^2*(c +
d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*d) + (2*B^2*(b*c - a*d)
^3*g^2*n^2*Log[c + d*x])/(3*b^3*d) - (2*B^2*(b*c - a*d)^3*g^2*n^2*Log[a +
b*x]*Log[(b*(c + d*x))/(b*c - a*d)]/(3*b^3*d) - (2*B^2*(b*c - a*d)^3*g^2*n
^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))]/(3*b^3*d)
```

**Rule 2525**

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^n_.)*((d_.) + (e_.)*(x_.))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

**Rule 12**

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x]]/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 43

Int[((a\_.) + (b\_.)\*(x\_)^(m\_.))\*((c\_.) + (d\_.)\*(x\_)^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rubi steps

$$\begin{aligned}
 \int (cg + dgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx &= \frac{g^2(c+dx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{3d} - \frac{(2Bn) \int \frac{(bc-ad)g^3(c+dx)^2 (A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right))^2}{a+bx}}{3dg} \\
 &= \frac{g^2(c+dx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{3d} - \frac{(2B(bc-ad)g^2n) \int \frac{(c+dx)^2 (A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right))^2}{a+bx}}{3d} \\
 &= \frac{g^2(c+dx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{3d} - \frac{(2B(bc-ad)g^2n) \int \left( \frac{d(bc-ad)(A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right))^2}{a+bx} \right)}{3d} \\
 &= \frac{g^2(c+dx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{3d} - \frac{(2B(bc-ad)g^2n) \int (c+dx) \left( \frac{d(bc-ad)(A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right))^2}{a+bx} \right)}{3b} \\
 &= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} - \frac{B(bc-ad)g^2n(c+dx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{3bd} \\
 &= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} - \frac{2B^2(bc-ad)^2g^2n(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{3b^3} + \frac{B^2(bc-ad)^2g^2n^2 \log(a+bx)}{3b^3} \\
 &= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} - \frac{2B^2(bc-ad)^2g^2n(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{3b^3} + \frac{B^2(bc-ad)^2g^2n^2 \log(a+bx)}{3b^3} \\
 &= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3b^2} + \frac{B^2(bc-ad)^3g^2n^2 \log(a+bx)}{3b^3d} \\
 &= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3b^2} + \frac{B^2(bc-ad)^3g^2n^2 \log(a+bx)}{3b^3d} \\
 &= -\frac{2AB(bc-ad)^2g^2nx}{3b^2} + \frac{B^2(bc-ad)^2g^2n^2x}{3b^2} + \frac{B^2(bc-ad)^3g^2n^2 \log(a+bx)}{3b^3d}
 \end{aligned}$$

**Mathematica [A]** time = 0.239755, size = 303, normalized size = 0.84

$$g^2 \left( (c+dx)^3 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) - \frac{Bn(bc-ad) \left( -Bn(bc-ad)^2 \left( \log(a+bx) \left( \log(a+bx) - 2 \log \left( \frac{b(c+dx)}{bc-ad} \right) \right) - 2 \text{PolyLog} \left( 2, \frac{d(a+bx)}{ad-bc} \right) \right) + b^2(c+dx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)^2 \right)}{3b^3d} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(c\*g + d\*g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] (g^2\*((c + d\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 - (B\*(b\*c - a\*d)\*n\*(2\*A\*b\*d\*(b\*c - a\*d)\*x - B\*(b\*c - a\*d)\*n\*(b\*d\*x + (b\*c - a\*d)\*Log[a + b\*x]) + 2\*B\*d\*(b\*c - a\*d)\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n] + b^2\*(c + d\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 2\*(b\*c - a\*d)^2\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 2\*B\*(b\*c - a\*d)^2\*n\*Log[c + d\*x] - B\*(b\*c - a\*d)^2\*n\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-(b\*c) + a\*d)])))/b^3)/(3\*d)

**Maple [F]** time = 0.43, size = 0, normalized size = 0.

$$\int (d g x + c g)^2 \left( A + B \ln \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*g\*x+c\*g)^2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((d\*g\*x+c\*g)^2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [B]** time = 3.60364, size = 1989, normalized size = 5.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out] 2/3\*A\*B\*d^2\*g^2\*x^3\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/3\*A^2\*d^2\*g^2\*x^3 + 2\*A\*B\*c\*d\*g^2\*x^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A^2\*c\*d\*g^2\*x^2 + 1/3\*A\*B\*d^2\*g^2\*n\*(2\*a^3\*log(b\*x + a)/b^3 - 2\*c^3\*log(d\*x + c)/d^3 - ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2)) - 2\*A\*B\*c\*d\*g^2\*n\*(a^2\*log(b\*x + a)/b^2 - c^2\*log(d\*x + c)/d^2 + (b\*c - a\*d)\*x/(b\*d)) + 2\*A\*B\*c^2\*g^2\*n\*(a\*log(b\*x + a)/b - c\*log(d\*x + c)/d) + 2\*A\*B\*c^2\*g^2\*x\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A^2\*c^2\*g^2\*x - 1/3\*(5\*a\*b\*c^2\*d\*g^2\*n^2 - 2\*a^2\*c\*d^2\*g^2\*n^2 - (3\*g^2\*n^2 - 2\*g^2\*n\*log(e))\*b^2\*c^3)\*B^2\*log(d\*x + c)/(b^2\*d) - 2/3\*(b^3\*c^3\*g^2\*n^2 - 3\*a\*b^2\*c^2\*d\*g^2\*n^2 + 3\*a^2\*b\*c\*d^2\*g^2\*n^2 - a^3\*d^3\*g^2\*n^2)\*(log(b\*x + a)\*log((b\*d\*x + a\*d)/(b\*c - a\*d) + 1) + dilog(-(b\*d\*x + a\*d)/(b\*c - a\*d)))\*B^2/(b^3\*d) + 1/3\*(B^2\*b^3\*d^3\*g^2\*x^3\*log(e)^2 + 2\*B^2\*b^3\*c^3\*g^2\*n^2\*log(b\*x + a)\*log(d\*x + c) - B^2\*b^3\*c^3\*g^2\*n^2\*log(d\*x + c)^2 + (a\*b^2\*d^3\*g^2\*n\*log(e) - (g^2\*n\*log(e) - 3\*g^2\*log(e)^2)\*b^3\*c\*d^2)\*B^2\*x^2 - (3\*a\*b^2\*c^2\*d\*g^2\*n^2 - 3\*a^2\*b\*c\*d^2\*g^2\*n^2 + a^3\*d^3\*g^2\*n^2)\*B^2\*log(b\*x + a)^2 + ((g^2\*n^2 - 4\*g^2\*n\*log(e) + 3\*g^2\*log(e)^2)\*b^3\*c^2\*d - 2\*(g^2\*n^2 - 3\*g^2\*n\*log(e))\*a\*b^2\*c\*d^2 + (g^2\*n^2 - 2\*g^2\*n\*log(e))\*a^2\*b\*d^3)\*B^2\*x - (2\*(2\*g^2\*n^2 - 3\*g^2\*n\*log(e))\*a\*b^2\*c^2\*d - (7\*g^2\*n^2 - 6\*g^2\*n\*log(e))\*a^2\*b\*c\*d^2 + (3\*g^2\*n^2 - 2\*g^2\*n\*log(e))\*a^3\*d^3)\*B^2\*log(b\*x + a) + (B^2\*b^3\*d^3\*g^2\*x^3 + 3\*B^2\*b^3\*c\*d^2\*g^2\*x^2 + 3\*B^2\*b^3\*c^2\*d\*g^2\*x)\*log((b\*x + a)^n)^2 + (B^2\*b^3\*d^3\*g^2\*x^3 + 3\*B^2\*b^3\*c\*d^2\*g^2\*x^2 + 3\*B^2\*b^3\*c^2\*d\*g^2\*x)\*log((d\*x + c)^n)^2 + (2\*B^2\*b^3\*d^3\*g^2\*x^3\*log(e) - 2\*B^2\*b^3\*c^3\*g^2\*n\*log(d\*x + c) + (a\*b^2\*d^3\*g^2\*n - (g^2\*n - 6\*g^2\*log(e))\*b^3\*c\*d^2)\*B^2\*x^2 + 2\*(3\*a\*b^2\*c\*d^2\*g^2\*n - a^2\*b\*d^3\*g^2\*n - (2\*g^2\*n - 3\*g^2\*log(e))\*b^3\*c^2\*d)\*B^2\*x + 2\*(3\*

$$a*b^2*c^2*d*g^2*n - 3*a^2*b*c*d^2*g^2*n + a^3*d^3*g^2*n)*B^2*\log(b*x + a))*\log((b*x + a)^n) - (2*B^2*b^3*d^3*g^2*x^3*\log(e) - 2*B^2*b^3*c^3*g^2*n*\log(d*x + c) + (a*b^2*d^3*g^2*n - (g^2*n - 6*g^2*\log(e))*b^3*c*d^2)*B^2*x^2 + 2*(3*a*b^2*c*d^2*g^2*n - a^2*b*d^3*g^2*n - (2*g^2*n - 3*g^2*\log(e))*b^3*c^2*d)*B^2*x + 2*(3*a*b^2*c^2*d*g^2*n - 3*a^2*b*c*d^2*g^2*n + a^3*d^3*g^2*n)*B^2*\log(b*x + a) + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*c*d^2*g^2*x^2 + 3*B^2*b^3*c^2*d*g^2*x)*\log((b*x + a)^n))*\log((d*x + c)^n))/(b^3*d)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(A^2d^2g^2x^2 + 2A^2cdg^2x + A^2c^2g^2 + \left(B^2d^2g^2x^2 + 2B^2cdg^2x + B^2c^2g^2\right)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2\left(ABd^2g^2x^2 + 2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out] integral(A^2\*d^2\*g^2\*x^2 + 2\*A^2\*c\*d\*g^2\*x + A^2\*c^2\*g^2 + (B^2\*d^2\*g^2\*x^2 + 2\*B^2\*c\*d\*g^2\*x + B^2\*c^2\*g^2)\*log(e\*((b\*x + a)/(d\*x + c))^n)^2 + 2\*(A\*B\*d^2\*g^2\*x^2 + 2\*A\*B\*c\*d\*g^2\*x + A\*B\*c^2\*g^2)\*log(e\*((b\*x + a)/(d\*x + c))^n), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)\*\*2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))\*\*2,x)

[Out] Timed out

**Giacc [F]** time = 0., size = 0, normalized size = 0.

$$\int (d gx + c g)^2 \left( B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate((d\*g\*x + c\*g)^2\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2, x)

$$3.41 \quad \int (cg + dgx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx$$

**Optimal.** Leaf size=220

$$\frac{B^2 g n^2 (bc - ad)^2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{d(a+bx)} \right)}{b^2 d} + \frac{B g n (bc - ad)^2 \log \left( 1 - \frac{b(c+dx)}{d(a+bx)} \right) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{b^2 d} - \frac{B g n (a + bx)(bc - ad)}{b^2 d}$$

[Out]  $-\left(\left(B*(b*c - a*d)*g*n*(a + b*x)*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])\right)/b^2\right)$   
 $+ (g*(c + d*x)^2*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2)/(2*d) + (B^2*(b*c - a*d)^2*g*n^2*\text{Log}[c + d*x])/(b^2*d) + (B*(b*c - a*d)^2*g*n*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*\text{Log}[1 - (b*(c + d*x))/(d*(a + b*x))])/(b^2*d) - (B^2*(b*c - a*d)^2*g*n^2*\text{PolyLog}[2, (b*(c + d*x))/(d*(a + b*x))])/(b^2*d)$

**Rubi [A]** time = 0.422598, antiderivative size = 307, normalized size of antiderivative = 1.4, number of steps used = 15, number of rules used = 12, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2 g n^2 (bc - ad)^2 \text{PolyLog} \left( 2, -\frac{d(a+bx)}{bc-ad} \right)}{b^2 d} - \frac{B g n (bc - ad)^2 \log(a + bx) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{b^2 d} + \frac{g(c + dx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c*g + d*g*x)*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2, x]$

[Out]  $-\left(\left(A*B*(b*c - a*d)*g*n*x\right)/b\right) + (B^2*(b*c - a*d)^2*g*n^2*\text{Log}[a + b*x]^2)/(2*b^2*d) - (B^2*(b*c - a*d)*g*n*(a + b*x)*\text{Log}[e*((a + b*x)/(c + d*x))^n])/b^2 - (B*(b*c - a*d)^2*g*n*\text{Log}[a + b*x]*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])/(b^2*d) + (g*(c + d*x)^2*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2)/(2*d) + (B^2*(b*c - a*d)^2*g*n^2*\text{Log}[c + d*x])/(b^2*d) - (B^2*(b*c - a*d)^2*g*n^2*\text{Log}[a + b*x]*\text{Log}[(b*(c + d*x))/(b*c - a*d)])/(b^2*d) - (B^2*(b*c - a*d)^2*g*n^2*\text{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))])/(b^2*d)$

#### Rule 2525

$\text{Int}[(a_. + \text{Log}[c_.*(\text{RFx}_.)^{(p_.)}]*(b_.))^{(n_.)}*((d_.) + (e_.)*(x_.))^{(m_.)}, x\_Symbol] :> \text{Simp}[(d + e*x)^{(m + 1)}*(a + b*\text{Log}[c*\text{RFx}^p])^n/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{(m + 1)}*(a + b*\text{Log}[c*\text{RFx}^p])^{(n - 1)}*D[\text{RFx}, x])/(\text{RFx}, x)], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] :> \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2528

$\text{Int}[(a_. + \text{Log}[c_.*(\text{RFx}_.)^{(p_.)}]*(b_.))^{(n_.)}*(\text{RGx}_.), x\_Symbol] :> \text{With}[\{u = \text{ExpandIntegrand}[(a + b*\text{Log}[c*\text{RFx}^p])^n, \text{RGx}, x]\}, \text{Int}[u, x] /;$  SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

Rule 2486

Int[Log[(e\_.)\*((f\_.)\*(a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))  
^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)  
^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c +  
d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s  
}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x,  
x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_S  
ymbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e  
, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /;  
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(RFx\_), x\_Sy  
mbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]},  
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[  
RFx, x] && IntegerQ[p]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.  
)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^  
n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E  
qQ[e\*f - d\*g, 0]

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Lo  
g[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_  
)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)  
^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x)  
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_  
Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x  
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*  
(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2  
, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int (cg + dgx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx &= \frac{g(c+dx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2d} - \frac{(Bn) \int \frac{(bc-ad)g^2(c+dx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{a+bx} dg}{d} \\
&= \frac{g(c+dx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2d} - \frac{(B(bc-ad)gn) \int \frac{(c+dx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{a+bx} d}{d} \\
&= \frac{g(c+dx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2d} - \frac{(B(bc-ad)gn) \int \left( \frac{d \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{b} \right) d}{d} \\
&= \frac{g(c+dx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2d} - \frac{(B(bc-ad)gn) \int \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) d}{b} \\
&= -\frac{AB(bc-ad)gnx}{b} - \frac{B(bc-ad)^2gn \log(a+bx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{b^2d} + \dots \\
&= -\frac{AB(bc-ad)gnx}{b} - \frac{B^2(bc-ad)gn(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b^2} - \frac{B(bc-ad)^2g}{b^2} \\
&= -\frac{AB(bc-ad)gnx}{b} - \frac{B^2(bc-ad)gn(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b^2} - \frac{B(bc-ad)^2g}{b^2} \\
&= -\frac{AB(bc-ad)gnx}{b} - \frac{B^2(bc-ad)gn(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b^2} - \frac{B(bc-ad)^2g}{b^2} \\
&= -\frac{AB(bc-ad)gnx}{b} + \frac{B^2(bc-ad)^2gn^2 \log^2(a+bx)}{2b^2d} - \frac{B^2(bc-ad)gn(a+bx)}{b^2} \\
&= -\frac{AB(bc-ad)gnx}{b} + \frac{B^2(bc-ad)^2gn^2 \log^2(a+bx)}{2b^2d} - \frac{B^2(bc-ad)gn(a+bx)}{b^2}
\end{aligned}$$

**Mathematica [A]** time = 0.21219, size = 216, normalized size = 0.98

$$g \left( \frac{Bn(bc-ad) \left( 2Bn(ad-bc) \text{PolyLog} \left( 2, \frac{d(a+bx)}{ad-bc} \right) - 2(bc-ad) \log(a+bx) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + Bn \log \left( \frac{b(c+dx)}{bc-ad} \right) + A \right) - 2 \left( Bd(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + Bn(ad-bc) \log(c+dx) + \dots \right)}{b^2} \right)$$


---


$$2d$$

Antiderivative was successfully verified.

[In] Integrate[(c\*g + d\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] (g\*((c + d\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 + (B\*(b\*c - a\*d)\*n\*(B\*(b\*c - a\*d)\*n\*Log[a + b\*x]^2 - 2\*(A\*b\*d\*x + B\*d\*(a + b\*x))\*Log[e\*((a + b\*x)/(c + d\*x))^n] + B\*(-(b\*c) + a\*d)\*n\*Log[c + d\*x]) - 2\*(b\*c - a\*d)\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n] + B\*n\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) + 2\*B\*(-(b\*c) + a\*d)\*n\*PolyLog[2, (d\*(a + b\*x))/(-(b\*c) + a\*d)])/b^2)



**Maple [F]** time = 0.277, size = 0, normalized size = 0.

$$\int (d gx + c g) \left( A + B \ln \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*g\*x+c\*g)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((d\*g\*x+c\*g)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [B]** time = 3.54851, size = 1114, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out]  $A*B*d*g*x^2*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*d*g*x^2 - A*B*d*g*n*(a^2*\log(b*x + a)/b^2 - c^2*\log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c*g*n*(a*\log(b*x + a)/b - c*\log(d*x + c)/d) + 2*A*B*c*g*x*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*g*x - (a*c*d*g*n^2 - (g*n^2 - g*n*\log(e))*b*c^2)*B^2*\log(d*x + c)/(b*d) - (b^2*c^2*g*n^2 - 2*a*b*c*d*g*n^2 + a^2*d^2*g*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d) + 1/2*(2*B^2*b^2*c^2*g*n^2*\log(b*x + a)*log(d*x + c) - B^2*b^2*c^2*g*n^2*\log(d*x + c)^2 + B^2*b^2*d^2*g*x^2*\log(e)^2 - (2*a*b*c*d*g*n^2 - a^2*d^2*g*n^2)*B^2*\log(b*x + a)^2 + 2*(a*b*d^2*g*n*log(e) - (g*n*log(e) - g*log(e)^2)*b^2*c*d)*B^2*x - 2*((g*n^2 - 2*g*n*log(e))*a*b*c*d - (g*n^2 - g*n*log(e))*a^2*d^2)*B^2*\log(b*x + a) + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*c*d*g*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*c*d*g*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*g*x^2*\log(e) - B^2*b^2*c^2*g*n*log(d*x + c) + (a*b*d^2*g*n - (g*n - 2*g*log(e))*b^2*c*d)*B^2*x + (2*a*b*c*d*g*n - a^2*d^2*g*n)*B^2*\log(b*x + a))*log((b*x + a)^n) - 2*(B^2*b^2*d^2*g*x^2*\log(e) - B^2*b^2*c^2*g*n*log(d*x + c) + (a*b*d^2*g*n - (g*n - 2*g*log(e))*b^2*c*d)*B^2*x + (2*a*b*c*d*g*n - a^2*d^2*g*n)*B^2*\log(b*x + a) + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*c*d*g*x)*log((b*x + a)^n))*log((d*x + c)^n)/(b^2*d)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2 d g x + A^2 c g + (B^2 d g x + B^2 c g) \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right)^2 + 2 (A B d g x + A B c g) \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out] integral(A^2\*d\*g\*x + A^2\*c\*g + (B^2\*d\*g\*x + B^2\*c\*g)\*log(e\*((b\*x + a)/(d\*x + c))^n)^2 + 2\*(A\*B\*d\*g\*x + A\*B\*c\*g)\*log(e\*((b\*x + a)/(d\*x + c))^n), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (d gx + c g) \left( B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate((d\*g\*x + c\*g)\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2, x)

$$3.42 \quad \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg+dx} dx$$

**Optimal.** Leaf size=137

$$\frac{2Bn \operatorname{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{dg} + \frac{2B^2 n^2 \operatorname{PolyLog}\left(3, \frac{d(a+bx)}{b(c+dx)}\right) \log\left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{dg}$$

[Out] -(((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2\*Log[(b\*c - a\*d)/(b\*(c + d\*x))])/(d\*g)) - (2\*B\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(d\*g) + (2\*B^2\*n^2\*PolyLog[3, (d\*(a + b\*x))/(b\*(c + d\*x))])/(d\*g)

**Rubi [B]** time = 3.30232, antiderivative size = 782, normalized size of antiderivative = 5.71, number of steps used = 45, number of rules used = 23, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.657$ , Rules used = {2524, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 12, 6742, 2499, 2396, 2433, 2374, 6589, 2302, 30, 2500, 2375, 2317, 2440, 2434}

$$\frac{2ABn \operatorname{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{dg} + \frac{2B^2 n \operatorname{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right) \left(-\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + \log((a + bx)^n) + \log((c + dx)^{-n})\right)}{dg}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(c\*g + d\*g\*x), x]

[Out] (B^2\*Log[(a + b\*x)^n]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]/(d\*g) - (B^2\*Log[(a + b\*x)^n]^2\*Log[g\*(c + d\*x)]/(d\*g) + (A\*B\*n\*Log[g\*(c + d\*x)]^2)/(d\*g) - (B^2\*n^2\*Log[a + b\*x]\*Log[g\*(c + d\*x)]^2)/(d\*g) + (B^2\*n^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[g\*(c + d\*x)]^2)/(d\*g) + (B^2\*n^2\*Log[g\*(c + d\*x)]^3)/(3\*d\*g) - (2\*B^2\*n\*Log[a + b\*x]\*Log[g\*(c + d\*x)]\*Log[(c + d\*x)^(-n)]/(d\*g) - (B^2\*Log[a + b\*x]\*Log[(c + d\*x)^(-n)]^2)/(d\*g) + (B^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[(c + d\*x)^(-n)]^2)/(d\*g) - (2\*A\*B\*n\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c\*g + d\*g\*x])/(d\*g) + ((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2\*Log[c\*g + d\*g\*x])/(d\*g) + (2\*B^2\*n\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[(a + b\*x)^n] - Log[e\*((a + b\*x)/(c + d\*x))^n] + Log[(c + d\*x)^(-n)])\*Log[c\*g + d\*g\*x])/(d\*g) - (B^2\*n^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c\*g + d\*g\*x]^2)/(d\*g) + (B^2\*n\*Log[e\*((a + b\*x)/(c + d\*x))^n]\*Log[c\*g + d\*g\*x]^2)/(d\*g) + (2\*B^2\*n\*Log[(a + b\*x)^n]\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(d\*g) - (2\*A\*B\*n\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]/(d\*g) - (2\*B^2\*n\*Log[(c + d\*x)^(-n)]\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]/(d\*g) + (2\*B^2\*n\*(Log[(a + b\*x)^n] - Log[e\*((a + b\*x)/(c + d\*x))^n] + Log[(c + d\*x)^(-n)])\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]/(d\*g) - (2\*B^2\*n^2\*PolyLog[3, -((d\*(a + b\*x))/(b\*c - a\*d))])/(d\*g) - (2\*B^2\*n^2\*PolyLog[3, (b\*(c + d\*x))/(b\*c - a\*d)]/(d\*g))

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^n\_]/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]

#### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[ {u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]
] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Symbol] := With[ {u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u] ] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 2499

Int[(Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.))\*((c\_.) + (d\_.)\*(x\_.))^(q\_.)]^(r\_.)]\*((s\_.) + Log[(i\_.)\*((g\_.) + (h\_.)\*(x\_.))^(n\_.)]\*(t\_.))^(m\_.))/((j\_.) + (k\_.)\*(x\_.)), x\_Symbol] := Simp[((s + t\*Log[i\*(g + h\*x)^n])^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)]/(k\*n\*t\*(m + 1)), x] + (-Dist[(b\*p\*r)/(k\*n\*t\*(m + 1)), Int[(s + t\*Log[i\*(g + h\*x)^n])^(m + 1)/(a + b\*x), x], x] - Dist[(d\*q\*r)/(k\*n\*t\*(m + 1)), Int[(s + t\*Log[i\*(g + h\*x)^n])^(m + 1)/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r}, x] && NeQ[b\*c - a\*d, 0] && EqQ[h\*j - g\*k, 0] && IGtQ[m, 0]

#### Rule 2396

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.))/((f\_.) + (g\_.)\*(x\_.)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^p)/g, x] - Dist[(b\*e\*n\*p)/g, Int[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])^(p - 1))/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e\*f - d\*g, 0] && IGtQ[p, 1]

#### Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.))\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_.))^(m\_.)]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_.))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[(k\*x)/d]^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

#### Rule 2374

Int[(Log[(d\_.)\*((e\_.) + (f\_.)\*(x\_.))^(m\_.))]\*((a\_.) + Log[(c\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.))/(x\_), x\_Symbol] := -Simp[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^p)/m, x] + Dist[(b\*n\*p)/m, Int[(PolyLog[2, -(d\*f\*x^m)]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d\*e, 1]

#### Rule 6589

Int[PolyLog[n\_, (c\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := Simp[PolyLog[n + 1, c\*(a + b\*x)^p]/(e\*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b\*d, a\*e]

#### Rule 2302

Int[((a\_.) + Log[(c\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Dist[1/(b\*n), Subst[Int[x^p, x], x, a + b\*Log[c\*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]

#### Rule 30

Int[(x\_)^(m\_.), x\_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

#### Rule 2500

Int[(Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.))\*((c\_.) + (d\_.)\*(x\_.))^(q\_.)]^(r\_.)]\*((s\_.) + Log[(i\_.)\*((g\_.) + (h\_.)\*(x\_.))^(n\_.)]\*(t\_.)))/((j\_.) + (k\_.)\*(x\_.)), x\_Symbol] := Dist[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r] - Log[(a + b\*x)^(p\*r)] - Log[(c + d\*x)^(q\*r)], Int[(s + t\*Log[i\*(g + h\*x)^n])/(j + k\*x), x], x] + (Int[(Log[(a + b\*x)^(p\*r)]\*(s + t\*Log[i\*(g + h\*x)^n]))/(j + k\*x), x] + Int[(Log[(c + d\*x)^(q\*r)]\*(s + t\*Log[i\*(g + h\*x)^n]))/(j + k\*x), x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ

[b\*c - a\*d, 0]

### Rule 2375

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.))/(x_), x_Symbol] := Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m - 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]
```

### Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol] := Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e, Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]
```

### Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(m_.)]*(b_.))*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_)^(m_.)]*(g_.))*((k_) + (l_.)*(x_)^(r_.)), x_Symbol] := Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f + g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

### Rule 2434

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))^(m_.)]*(b_.))*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_)^(m_.)]*(g_.)))/(x_), x_Symbol] := Simp[Log[x]*(a + b*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[e*g*m, Int[(Log[x]*(a + b*Log[c*(d + e*x)^n])]/(d + e*x), x], x] - Dist[b*j*n, Int[(Log[x]*(f + g*Log[h*(i + j*x)^m])]/(i + j*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && EqQ[e*i - d*j, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{cg + dgx} dx &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(cg + dgx)}{dg} - \frac{(2Bn) \int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2} + \frac{b}{c+dx}\right)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx} dx}{dg} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(cg + dgx)}{dg} - \frac{(2Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(cg+dgx)}{(a+bx)(c+dx)} dx}{dg} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(cg + dgx)}{dg} - \frac{(2B(bc-ad)n) \int \frac{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(cg+dgx)}{(a+bx)(c+dx)} dx}{dg} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(cg + dgx)}{dg} - \frac{(2B(bc-ad)n) \int \frac{d\left(-A-B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(cg+dgx)}{(bc-ad)(c+dx)} dx}{dg} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(cg + dgx)}{dg} - \frac{(2Bn) \int \frac{\left(-A-B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(cg+dgx)}{c+dx} dx}{g} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(cg + dgx)}{dg} - \frac{(2Bn) \int \left(\frac{A \log(cg+dgx)}{-c-dx} + \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(cg+dgx)}{-c-dx}\right) dx}{g} \\
&= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(cg + dgx)}{dg} - \frac{(2ABn) \int \frac{\log(cg+dgx)}{-c-dx} dx}{g} - \frac{(2B^2n) \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(cg+dgx)}{-c-dx} dx}{g} \\
&= -\frac{2ABn \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(cg + dgx)}{dg} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(cg + dgx)}{dg} + \frac{2B^2n \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(cg + dgx)}{dg} \\
&= -\frac{2ABn \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(cg + dgx)}{dg} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(cg + dgx)}{dg} + \frac{2B^2n \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(cg + dgx)}{dg} \\
&= -\frac{B^2 \log^2\left((a+bx)^n\right) \log(g(c+dx))}{dg} + \frac{ABn \log^2(g(c+dx))}{dg} - \frac{2B^2n \log(a+bx) \log(g(c+dx))}{dg} \\
&= \frac{B^2 \log^2\left((a+bx)^n\right) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{dg} - \frac{B^2 \log^2\left((a+bx)^n\right) \log(g(c+dx))}{dg} + \frac{ABn \log^2(g(c+dx))}{dg} \\
&= \frac{B^2 \log^2\left((a+bx)^n\right) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{dg} - \frac{B^2 \log^2\left((a+bx)^n\right) \log(g(c+dx))}{dg} + \frac{ABn \log^2(g(c+dx))}{dg} \\
&= \frac{B^2 \log^2\left((a+bx)^n\right) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{dg} - \frac{B^2 \log^2\left((a+bx)^n\right) \log(g(c+dx))}{dg} + \frac{ABn \log^2(g(c+dx))}{dg} \\
&= \frac{B^2 \log^2\left((a+bx)^n\right) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{dg} - \frac{B^2 \log^2\left((a+bx)^n\right) \log(g(c+dx))}{dg} + \frac{ABn \log^2(g(c+dx))}{dg} \\
&= \frac{B^2 \log^2\left((a+bx)^n\right) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{dg} - \frac{B^2 \log^2\left((a+bx)^n\right) \log(g(c+dx))}{dg} + \frac{ABn \log^2(g(c+dx))}{dg}
\end{aligned}$$

**Mathematica [B]** time = 0.399526, size = 537, normalized size = 3.92

$$-3Bn \left( -2 \left( \text{PolyLog} \left( 2, \frac{d(a+bx)}{ad-bc} \right) + \log \left( \frac{a}{b} + x \right) \log \left( \frac{b(c+dx)}{bc-ad} \right) \right) + 2 \log(c+dx) \left( -\log \left( \frac{a+bx}{c+dx} \right) + \log \left( \frac{a}{b} + x \right) - \log \left( \frac{c}{d} + x \right) \right) \right) +$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(c\*g + d\*g\*x), x]

[Out] (3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n] - B\*n\*Log[(a + b\*x)/(c + d\*x)])^2\*Log[c + d\*x] - 3\*B\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n] - B\*n\*Log[(a + b\*x)/(c + d\*x)])\*(Log[c/d + x]^2 + 2\*(Log[a/b + x] - Log[c/d + x] - Log[(a + b\*x)/(c + d\*x)])\*Log[c + d\*x] - 2\*(Log[a/b + x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] + PolyLog[2, (d\*(a + b\*x))/(-(b\*c) + a\*d)])) + B^2\*n^2\*(Log[c/d + x]^3 + 3\*Log[c/d + x]^2\*(-Log[a/b + x] + Log[(d\*(a + b\*x))/(-(b\*c) + a\*d)]) + 3\*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b\*x)/(c + d\*x)])^2\*Log[c + d\*x] + 3\*Log[a/b + x]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] + 6\*Log[a/b + x]\*PolyLog[2, (d\*(a + b\*x))/(-(b\*c) + a\*d)] + 3\*(Log[a/b + x] - Log[c/d + x] - Log[(a + b\*x)/(c + d\*x)])\*(Log[c/d + x]^2 - 2\*(Log[a/b + x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] + PolyLog[2, (d\*(a + b\*x))/(-(b\*c) + a\*d)])) + 6\*Log[c/d + x]\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)] - 6\*PolyLog[3, (d\*(a + b\*x))/(-(b\*c) + a\*d)] - 6\*PolyLog[3, (b\*(c + d\*x))/(b\*c - a\*d)]))/(3\*d\*g)

**Maple [F]** time = 0.444, size = 0, normalized size = 0.

$$\int \frac{1}{dgx + cg} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g), x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{B^2 \log(dx + c) \log((dx + c)^n)^2}{dg} + \frac{A^2 \log(dgx + cg)}{dg} - \int -\frac{B^2 \log((bx + a)^n)^2 + B^2 \log(e)^2 + 2AB \log(e) + 2(B^2 \log(e) \log(dx + c) \log((dx + c)^n)^2 + 2A^2 \log(dgx + cg) \log((dx + c)^n)^2 + 2AB \log(e) \log((dx + c)^n)^2 + 2A^2 \log(dgx + cg) \log((dx + c)^n)^2 + 2AB \log(e) \log((dx + c)^n)^2 + 2A^2 \log(dgx + cg) \log((dx + c)^n)^2)}{dg} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g), x, algorithm="maxima")

[Out] B^2\*log(d\*x + c)\*log((d\*x + c)^n)^2/(d\*g) + A^2\*log(d\*g\*x + c\*g)/(d\*g) - integrate(-(B^2\*log((b\*x + a)^n)^2 + B^2\*log(e)^2 + 2\*A\*B\*log(e) + 2\*(B^2\*log(e) + A\*B)\*log((b\*x + a)^n) - 2\*(B^2\*n\*log(d\*x + c) + B^2\*log((b\*x + a)^n) + B^2\*log(e) + A\*B)\*log((d\*x + c)^n))/(d\*g\*x + c\*g), x)



**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2AB \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2}{d gx + c g}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g),x, algorithm="fricas")

[Out] integral((B^2\*log(e\*((b\*x + a)/(d\*x + c))^n)^2 + 2\*A\*B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A^2)/(d\*g\*x + c\*g), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2}{d gx + c g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g),x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2/(d\*g\*x + c\*g), x)

$$3.43 \quad \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^2} dx$$

**Optimal.** Leaf size=163

$$\frac{(a+bx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}{g^2(c+dx)(bc-ad)} - \frac{2ABn(a+bx)}{g^2(c+dx)(bc-ad)} - \frac{2B^2n(a+bx) \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g^2(c+dx)(bc-ad)} + \frac{2B^2n^2(a+bx)}{g^2(c+dx)(bc-ad)}$$

[Out]  $(-2*A*B*n*(a+b*x))/((b*c-a*d)*g^2*(c+d*x)) + (2*B^2*n^2*(a+b*x))/((b*c-a*d)*g^2*(c+d*x)) - (2*B^2*n*(a+b*x)*\text{Log}[e*((a+b*x)/(c+d*x))^n])/((b*c-a*d)*g^2*(c+d*x)) + ((a+b*x)*(A+B*\text{Log}[e*((a+b*x)/(c+d*x))^n])^2)/((b*c-a*d)*g^2*(c+d*x))$

**Rubi [C]** time = 0.773967, antiderivative size = 514, normalized size of antiderivative = 3.15, number of steps used = 24, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 44}

$$\frac{2bB^2n^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{dg^2(bc-ad)} + \frac{2bB^2n^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{dg^2(bc-ad)} + \frac{2bBn \log(a+bx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{dg^2(bc-ad)} + \frac{2Bn\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}{dg^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Log}[e*((a+b*x)/(c+d*x))^n])^2/(c*g + d*g*x)^2, x]$

[Out]  $(-2*B^2*n^2)/(d*g^2*(c+d*x)) - (2*b*B^2*n^2*\text{Log}[a+b*x])/(d*(b*c-a*d)*g^2) - (b*B^2*n^2*\text{Log}[a+b*x]^2)/(d*(b*c-a*d)*g^2) + (2*B*n*(A+B*\text{Log}[e*((a+b*x)/(c+d*x))^n]))/(d*g^2*(c+d*x)) + (2*b*B*n*\text{Log}[a+b*x]*(A+B*\text{Log}[e*((a+b*x)/(c+d*x))^n]))/(d*(b*c-a*d)*g^2) - (A+B*\text{Log}[e*((a+b*x)/(c+d*x))^n])^2/(d*g^2*(c+d*x)) + (2*b*B^2*n^2*\text{Log}[c+d*x])/(d*(b*c-a*d)*g^2) + (2*b*B^2*n^2*\text{Log}[-((d*(a+b*x))/(b*c-a*d))]*\text{Log}[c+d*x])/(d*(b*c-a*d)*g^2) - (2*b*B*n*(A+B*\text{Log}[e*((a+b*x)/(c+d*x))^n])* \text{Log}[c+d*x])/(d*(b*c-a*d)*g^2) - (b*B^2*n^2*\text{Log}[c+d*x]^2)/(d*(b*c-a*d)*g^2) + (2*b*B^2*n^2*\text{Log}[a+b*x]*\text{Log}[(b*(c+d*x))/(b*c-a*d)])/(d*(b*c-a*d)*g^2) + (2*b*B^2*n^2*\text{PolyLog}[2, -((d*(a+b*x))/(b*c-a*d))])/(d*(b*c-a*d)*g^2) + (2*b*B^2*n^2*\text{PolyLog}[2, (b*(c+d*x))/(b*c-a*d)])/(d*(b*c-a*d)*g^2)$

**Rule 2525**

$\text{Int}[(a_. + \text{Log}[c_.*(RfX_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^(m+1)*(a + b*\text{Log}[c*RfX^p])^n/(e*(m+1)), x] - \text{Dist}[(b*n*p)/(e*(m+1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^(m+1)*(a + b*\text{Log}[c*RfX^p])^(n-1)*D[RfX, x])/RfX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RfX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

**Rule 12**

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 2528**

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[(f*x)/d]^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg + dgx)^2} dx &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{dg^2(c + dx)} + \frac{(2Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(a+bx)(c+dx)^2} dx}{dg} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{dg^2(c + dx)} + \frac{(2B(bc - ad)n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)^2} dx}{dg^2} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{dg^2(c + dx)} + \frac{(2B(bc - ad)n) \int \left(\frac{b^2\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2(a+bx)} - \frac{d\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(c+dx)^2}\right) dx}{dg^2} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{dg^2(c + dx)} - \frac{(2Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^2} dx}{g^2} - \frac{(2bBn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} dx}{(bc - ad)g^2} \\
&= \frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c + dx)} + \frac{2bBn \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc - ad)g^2} - \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2} \\
&= \frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c + dx)} + \frac{2bBn \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc - ad)g^2} - \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2} \\
&= \frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c + dx)} + \frac{2bBn \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc - ad)g^2} - \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2} \\
&= -\frac{2B^2n^2}{dg^2(c + dx)} - \frac{2bB^2n^2 \log(a + bx)}{d(bc - ad)g^2} + \frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c + dx)} + \frac{2bBn \log(a + bx)}{d(bc - ad)g^2} \\
&= -\frac{2B^2n^2}{dg^2(c + dx)} - \frac{2bB^2n^2 \log(a + bx)}{d(bc - ad)g^2} - \frac{bB^2n^2 \log^2(a + bx)}{d(bc - ad)g^2} + \frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c + dx)} \\
&= -\frac{2B^2n^2}{dg^2(c + dx)} - \frac{2bB^2n^2 \log(a + bx)}{d(bc - ad)g^2} - \frac{bB^2n^2 \log^2(a + bx)}{d(bc - ad)g^2} + \frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{dg^2(c + dx)}
\end{aligned}$$

**Mathematica [C]** time = 0.446407, size = 331, normalized size = 2.03

$$\frac{Bn\left(-bBn(c+dx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+bBn(c+dx)\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)+2(bc-d)g^2\right)}{d^2g^2(c+dx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(c\*g + d\*g\*x)^2,x]

[Out]  $(-(A + B \text{Log}[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B \text{Log}[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*\text{Log}[a + b*x]*(A + B \text{Log}[e*((a + b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B \text{Log}[e*((a + b*x)/(c + d*x))^n]) * \text{Log}[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*x)*\text{Log}[a + b*x] - b*(c + d*x)*\text{Log}[c + d*x]) - b*B*n*(c + d*x)*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-b*c + a*d)]) + b*B*n*(c + d*x)*((2*\text{Log}[(d*(a + b*x))/(-b*c + a*d)] - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(d*g^2*(c + d*x)^2)$

))

---

**Maple [F]** time = 0.441, size = 0, normalized size = 0.

$$\int \frac{1}{(d gx + c g)^2} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^2,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^2,x)

---

**Maxima [B]** time = 1.26026, size = 578, normalized size = 3.55

$$2 ABn \left( \frac{1}{d^2 g^2 x + c d g^2} + \frac{b \log(bx + a)}{(bcd - ad^2)g^2} - \frac{b \log(dx + c)}{(bcd - ad^2)g^2} \right) + \left( 2n \left( \frac{1}{d^2 g^2 x + c d g^2} + \frac{b \log(bx + a)}{(bcd - ad^2)g^2} - \frac{b \log(dx + c)}{(bcd - ad^2)g^2} \right) \log \left( \frac{bx + a}{dx + c} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^2,x, algorithm="maxima")

[Out] 2\*A\*B\*n\*(1/(d^2\*g^2\*x + c\*d\*g^2) + b\*log(b\*x + a)/((b\*c\*d - a\*d^2)\*g^2) - b\*log(d\*x + c)/((b\*c\*d - a\*d^2)\*g^2)) + (2\*n\*(1/(d^2\*g^2\*x + c\*d\*g^2) + b\*log(b\*x + a)/((b\*c\*d - a\*d^2)\*g^2) - b\*log(d\*x + c)/((b\*c\*d - a\*d^2)\*g^2))\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) - ((b\*d\*x + b\*c)\*log(b\*x + a)^2 + (b\*d\*x + b\*c)\*log(d\*x + c)^2 + 2\*b\*c - 2\*a\*d + 2\*(b\*d\*x + b\*c)\*log(b\*x + a) - 2\*(b\*d\*x + b\*c + (b\*d\*x + b\*c)\*log(b\*x + a))\*log(d\*x + c))\*n^2/(b\*c^2\*d\*g^2 - a\*c\*d^2\*g^2 + (b\*c\*d^2\*g^2 - a\*d^3\*g^2)\*x)\*B^2 - B^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n)^2/(d^2\*g^2\*x + c\*d\*g^2) - 2\*A\*B\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n)/(d^2\*g^2\*x + c\*d\*g^2) - A^2/(d^2\*g^2\*x + c\*d\*g^2)

---

**Fricas [A]** time = 0.879876, size = 555, normalized size = 3.4

$$\frac{A^2bc - A^2ad + 2(B^2bc - B^2ad)n^2 + (B^2bc - B^2ad)\log(e)^2 - (B^2bdn^2x + B^2adn^2)\log\left(\frac{bx+a}{dx+c}\right)^2 - 2(ABbc - ABad)n}{(bcd^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^2,x, algorithm="fricas")

[Out] -(A^2\*b\*c - A^2\*a\*d + 2\*(B^2\*b\*c - B^2\*a\*d)\*n^2 + (B^2\*b\*c - B^2\*a\*d)\*log(e)^2 - (B^2\*b\*d\*n^2\*x + B^2\*a\*d\*n^2)\*log((b\*x + a)/(d\*x + c))^2 - 2\*(A\*B\*b\*c - A\*B\*a\*d)\*n + 2\*(A\*B\*b\*c - A\*B\*a\*d - (B^2\*b\*c - B^2\*a\*d)\*n - (B^2\*b\*d\*n\*x + B^2\*a\*d\*n)\*log((b\*x + a)/(d\*x + c)))\*log(e) + 2\*(B^2\*a\*d\*n^2 - A\*B\*a\*d\*n + (B^2\*b\*d\*n^2 - A\*B\*b\*d\*n)\*x)\*log((b\*x + a)/(d\*x + c)))/((b\*c\*d^2 - a\*d^3

) $g^2x + (b^2c^2d - a^2cd^2)g^2$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2/(d\*g\*x+c\*g)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2}{(d gx + c g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^2,x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2/(d\*g\*x + c\*g)^2, x)

$$3.44 \quad \int \frac{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^3} dx$$

**Optimal.** Leaf size=317

$$\frac{Bdn(a+bx)^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2g^3(c+dx)^2(bc-ad)^2} + \frac{b(a+bx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{g^3(c+dx)(bc-ad)^2} - \frac{d(a+bx)^2\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2g^3(c+dx)^2(bc-ad)^2} - \dots$$

[Out]  $-(B^2*d*n^2*(a+b*x)^2)/(4*(b*c-a*d)^2*g^3*(c+d*x)^2) - (2*A*b*B*n*(a+b*x))/((b*c-a*d)^2*g^3*(c+d*x)) + (2*b*B^2*n^2*(a+b*x))/((b*c-a*d)^2*g^3*(c+d*x)) - (2*b*B^2*n*(a+b*x)*Log[e*((a+b*x)/(c+d*x))^n])/((b*c-a*d)^2*g^3*(c+d*x)) + (B*d*n*(a+b*x)^2*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(2*(b*c-a*d)^2*g^3*(c+d*x)^2) - (d*(a+b*x)^2*(A+B*Log[e*((a+b*x)/(c+d*x))^n])^2)/(2*(b*c-a*d)^2*g^3*(c+d*x)^2) + (b*(a+b*x)*(A+B*Log[e*((a+b*x)/(c+d*x))^n])^2)/((b*c-a*d)^2*g^3*(c+d*x))$

**Rubi [C]** time = 0.915045, antiderivative size = 626, normalized size of antiderivative = 1.97, number of steps used = 28, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 44}

$$\frac{b^2 B^2 n^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{dg^3(bc-ad)^2} + \frac{b^2 B^2 n^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{dg^3(bc-ad)^2} + \frac{b^2 B n \log(a+bx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{dg^3(bc-ad)^2} - \dots$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(c\*g + d\*g\*x)^3, x]

[Out]  $-(B^2*n^2)/(4*d*g^3*(c+d*x)^2) - (3*b*B^2*n^2)/(2*d*(b*c-a*d)*g^3*(c+d*x)) - (3*b^2*B^2*n^2*Log[a+b*x])/(2*d*(b*c-a*d)^2*g^3) - (b^2*B^2*n^2*Log[a+b*x]^2)/(2*d*(b*c-a*d)^2*g^3) + (B*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(2*d*g^3*(c+d*x)^2) + (b*B*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(d*(b*c-a*d)*g^3*(c+d*x)) + (b^2*B*n*Log[a+b*x]*(A+B*Log[e*((a+b*x)/(c+d*x))^n]))/(d*(b*c-a*d)^2*g^3) - (A+B*Log[e*((a+b*x)/(c+d*x))^n])^2/(2*d*g^3*(c+d*x)^2) + (3*b^2*B^2*n^2*Log[c+d*x])/(2*d*(b*c-a*d)^2*g^3) + (b^2*B^2*n^2*Log[-((d*(a+b*x))/(b*c-a*d))]*Log[c+d*x])/(d*(b*c-a*d)^2*g^3) - (b^2*B*n*(A+B*Log[e*((a+b*x)/(c+d*x))^n])*Log[c+d*x])/(d*(b*c-a*d)^2*g^3) - (b^2*B^2*n^2*Log[c+d*x]^2)/(2*d*(b*c-a*d)^2*g^3) + (b^2*B^2*n^2*Log[a+b*x]*Log[(b*(c+d*x))/(b*c-a*d)])/(d*(b*c-a*d)^2*g^3) + (b^2*B^2*n^2*PolyLog[2, -((d*(a+b*x))/(b*c-a*d))])/(d*(b*c-a*d)^2*g^3) + (b^2*B^2*n^2*PolyLog[2, (b*(c+d*x))/(b*c-a*d)])/(d*(b*c-a*d)^2*g^3)$

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

**Rule 12**

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
```



& NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg + dgx)^3} dx &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2dg^3(c + dx)^2} + \frac{(Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^2(a+bx)(c+dx)^3} dx}{dg} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2dg^3(c + dx)^2} + \frac{(B(bc - ad)n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)^3} dx}{dg^3} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2dg^3(c + dx)^2} + \frac{(B(bc - ad)n) \int \left(\frac{b^3\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3(a+bx)} - \frac{d\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(c+dx)^3}\right) dx}{dg^3} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2dg^3(c + dx)^2} - \frac{(Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^3} dx}{g^3} - \frac{(b^2Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} dx}{(bc - ad)^2g^3} \\
&= \frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2dg^3(c + dx)^2} + \frac{bBn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc - ad)g^3(c + dx)} + \frac{b^2Bn \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc - ad)^2g^3} \\
&= \frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2dg^3(c + dx)^2} + \frac{bBn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc - ad)g^3(c + dx)} + \frac{b^2Bn \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc - ad)^2g^3} \\
&= \frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2dg^3(c + dx)^2} + \frac{bBn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc - ad)g^3(c + dx)} + \frac{b^2Bn \log(a + bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(bc - ad)^2g^3} \\
&= -\frac{B^2n^2}{4dg^3(c + dx)^2} - \frac{3bB^2n^2}{2d(bc - ad)g^3(c + dx)} - \frac{3b^2B^2n^2 \log(a + bx)}{2d(bc - ad)^2g^3} + \frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2dg^3(c + dx)} \\
&= -\frac{B^2n^2}{4dg^3(c + dx)^2} - \frac{3bB^2n^2}{2d(bc - ad)g^3(c + dx)} - \frac{3b^2B^2n^2 \log(a + bx)}{2d(bc - ad)^2g^3} - \frac{b^2B^2n^2 \log^2(a + bx)}{2d(bc - ad)^2g^3} \\
&= -\frac{B^2n^2}{4dg^3(c + dx)^2} - \frac{3bB^2n^2}{2d(bc - ad)g^3(c + dx)} - \frac{3b^2B^2n^2 \log(a + bx)}{2d(bc - ad)^2g^3} - \frac{b^2B^2n^2 \log^2(a + bx)}{2d(bc - ad)^2g^3}
\end{aligned}$$

**Mathematica [C]** time = 0.49214, size = 464, normalized size = 1.46

$$Bn\left(-2b^2Bn(c+dx)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+2b^2Bn(c+dx)^2\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(c\*g + d\*g\*x)^3,x]

[Out] (-2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 + (B\*n\*(2\*(b\*c - a\*d)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 4\*b\*(b\*c - a\*d)\*(c + d\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 4\*b^2\*(c + d\*x)^2\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])

+ b\*x)/(c + d\*x))^n]) - 4\*b^2\*(c + d\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x] - 4\*b\*B\*n\*(c + d\*x)\*(b\*c - a\*d + b\*(c + d\*x)\*Log[a + b\*x] - b\*(c + d\*x)\*Log[c + d\*x]) - B\*n\*((b\*c - a\*d)^2 + 2\*b\*(b\*c - a\*d)\*(c + d\*x) + 2\*b^2\*(c + d\*x)^2\*Log[a + b\*x] - 2\*b^2\*(c + d\*x)^2\*Log[c + d\*x]) - 2\*b^2\*B\*n\*(c + d\*x)^2\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c + a\*d)]) + 2\*b^2\*B\*n\*(c + d\*x)^2\*((2\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/(b\*c - a\*d)^2/(4\*d\*g^3\*(c + d\*x)^2)

**Maple [F]** time = 0.441, size = 0, normalized size = 0.

$$\int \frac{1}{(dgx + cg)^3} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^3,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^3,x)

**Maxima [B]** time = 1.38282, size = 1162, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^3,x, algorithm="maxima")

[Out] 1/2\*A\*B\*n\*((2\*b\*d\*x + 3\*b\*c - a\*d)/((b\*c\*d^3 - a\*d^4)\*g^3\*x^2 + 2\*(b\*c^2\*d^2 - a\*c\*d^3)\*g^3\*x + (b\*c^3\*d - a\*c^2\*d^2)\*g^3) + 2\*b^2\*log(b\*x + a)/((b^2\*c^2\*d - 2\*a\*b\*c\*d^2 + a^2\*d^3)\*g^3) - 2\*b^2\*log(d\*x + c)/((b^2\*c^2\*d - 2\*a\*b\*c\*d^2 + a^2\*d^3)\*g^3) + 1/4\*(2\*n\*((2\*b\*d\*x + 3\*b\*c - a\*d)/((b\*c\*d^3 - a\*d^4)\*g^3\*x^2 + 2\*(b\*c^2\*d^2 - a\*c\*d^3)\*g^3\*x + (b\*c^3\*d - a\*c^2\*d^2)\*g^3) + 2\*b^2\*log(b\*x + a)/((b^2\*c^2\*d - 2\*a\*b\*c\*d^2 + a^2\*d^3)\*g^3) - 2\*b^2\*log(d\*x + c)/((b^2\*c^2\*d - 2\*a\*b\*c\*d^2 + a^2\*d^3)\*g^3))\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) - (7\*b^2\*c^2 - 8\*a\*b\*c\*d + a^2\*d^2 + 2\*(b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2)\*log(b\*x + a)^2 + 2\*(b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2)\*log(d\*x + c)^2 + 6\*(b^2\*c\*d - a\*b\*d^2)\*x + 6\*(b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2)\*log(b\*x + a) - 2\*(3\*b^2\*d^2\*x^2 + 6\*b^2\*c\*d\*x + 3\*b^2\*c^2 + 2\*(b^2\*d^2\*x^2 + 2\*b^2\*c\*d\*x + b^2\*c^2)\*log(b\*x + a))\*log(d\*x + c))\*n^2/(b^2\*c^4\*d\*g^3 - 2\*a\*b\*c^3\*d^2\*g^3 + a^2\*c^2\*d^3\*g^3 + (b^2\*c^2\*d^3\*g^3 - 2\*a\*b\*c\*d^4\*g^3 + a^2\*d^5\*g^3)\*x^2 + 2\*(b^2\*c^3\*d^2\*g^3 - 2\*a\*b\*c^2\*d^3\*g^3 + a^2\*c\*d^4\*g^3)\*x)\*B^2 - 1/2\*B^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n)^2/(d^3\*g^3\*x^2 + 2\*c\*d^2\*g^3\*x + c^2\*d\*g^3) - A\*B\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n)/(d^3\*g^3\*x^2 + 2\*c\*d^2\*g^3\*x + c^2\*d\*g^3) - 1/2\*A^2/(d^3\*g^3\*x^2 + 2\*c\*d^2\*g^3\*x + c^2\*d\*g^3)

**Fricas [B]** time = 0.969725, size = 1354, normalized size = 4.27

$$2 A^2 b^2 c^2 - 4 A^2 a b c d + 2 A^2 a^2 d^2 + (7 B^2 b^2 c^2 - 8 B^2 a b c d + B^2 a^2 d^2) n^2 + 2 (B^2 b^2 c^2 - 2 B^2 a b c d + B^2 a^2 d^2) \log(e)^2 - 2 (B^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (7*B^2*b^2*c^2 - 8*B^2*a*b*c*d + B^2*a^2*d^2)*n^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d^2)*log(e)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*b^2*c*d*n^2*x + (2*B^2*a*b*c*d - B^2*a^2*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(3*A*B*b^2*c^2 - 4*A*B*a*b*c*d + A*B*a^2*d^2)*n + 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2 - 2*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a^2*d^2 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x - (3*B^2*b^2*c^2 - 4*B^2*a*b*c*d + B^2*a^2*d^2)*n - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*b^2*c*d*n*x + (2*B^2*a*b*c*d - B^2*a^2*d^2)*n)*log((b*x + a)/(d*x + c))*log(e) + 2*((4*B^2*a*b*c*d - B^2*a^2*d^2)*n^2 + (3*B^2*b^2*d^2*n^2 - 2*A*B*b^2*d^2*n)*x^2 - 2*(2*A*B*a*b*c*d - A*B*a^2*d^2)*n - 2*(2*A*B*b^2*c*d*n - (2*B^2*b^2*c*d + B^2*a*b*d^2)*n^2)*x)*log((b*x + a)/(d*x + c))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*d^5)*g^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*g^3*x + (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*g^3)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2}{(dgx + cg)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*g*x+c*g)^3,x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*g*x + c*g)^3, x)
```

$$3.45 \quad \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^4} dx$$

**Optimal.** Leaf size=429

$$\frac{2b^3 B n \log\left(\frac{a+bx}{c+dx}\right) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{3dg^4(bc-ad)^3} - \frac{2b^2 B n(a+bx) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{g^4(c+dx)(bc-ad)^3} - \frac{2Bd^2 n(a+bx)^3 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9g^4(c+dx)^3(bc-ad)^3}$$

[Out]  $(2*B^2*d^2*n^2*(a + b*x)^3)/(27*(b*c - a*d)^3*g^4*(c + d*x)^3) - (b*B^2*d*n^2*(a + b*x)^2)/(2*(b*c - a*d)^3*g^4*(c + d*x)^2) + (2*b^2*B^2*n^2*(a + b*x))/((b*c - a*d)^3*g^4*(c + d*x)) - (2*B*d^2*n*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(b*c - a*d)^3*g^4*(c + d*x)^3) + (b*B*d*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^3*g^4*(c + d*x)^2) - (2*b^2*B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)^3*g^4*(c + d*x)) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(3*d*g^4*(c + d*x)^3) + (2*b^3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)])/(3*d*(b*c - a*d)^3*g^4) - (b^3*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2)/(3*d*(b*c - a*d)^3*g^4)$

**Rubi [C]** time = 1.098, antiderivative size = 736, normalized size of antiderivative = 1.72, number of steps used = 32, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 44}

$$\frac{2b^3 B^2 n^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{3dg^4(bc-ad)^3} + \frac{2b^3 B^2 n^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3dg^4(bc-ad)^3} + \frac{2b^3 B n \log(a+bx) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{3dg^4(bc-ad)^3} - \frac{2b^3 B n \log\left(\frac{a+bx}{c+dx}\right) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{3dg^4(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(c\*g + d\*g\*x)^4, x]

[Out]  $(-2*B^2*n^2)/(27*d*g^4*(c + d*x)^3) - (5*b*B^2*n^2)/(18*d*(b*c - a*d)*g^4*(c + d*x)^2) - (11*b^2*B^2*n^2)/(9*d*(b*c - a*d)^2*g^4*(c + d*x)) - (11*b^3*B^2*n^2*Log[a + b*x])/(9*d*(b*c - a*d)^3*g^4) - (b^3*B^2*n^2*Log[a + b*x]^2)/(3*d*(b*c - a*d)^3*g^4) + (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*d*g^4*(c + d*x)^3) + (b*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d*(b*c - a*d)*g^4*(c + d*x)^2) + (2*b^2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d*(b*c - a*d)^2*g^4*(c + d*x)) + (2*b^3*B*n*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d*(b*c - a*d)^3*g^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(3*d*g^4*(c + d*x)^3) + (11*b^3*B^2*n^2*Log[c + d*x])/(9*d*(b*c - a*d)^3*g^4) + (2*b^3*B^2*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*d*(b*c - a*d)^3*g^4) - (2*b^3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x])/(3*d*(b*c - a*d)^3*g^4) - (b^3*B^2*n^2*Log[c + d*x]^2)/(3*d*(b*c - a*d)^3*g^4) + (2*b^3*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*d*(b*c - a*d)^3*g^4) + (2*b^3*B^2*n^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*d*(b*c - a*d)^3*g^4) + (2*b^3*B^2*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*d*(b*c - a*d)^3*g^4)$

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d

, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

## Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] & NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

## Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg + dgx)^4} dx &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c + dx)^3} + \frac{(2Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^3(a+bx)(c+dx)^4} dx}{3dg} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c + dx)^3} + \frac{(2B(bc - ad)n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)^4} dx}{3dg^4} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c + dx)^3} + \frac{(2B(bc - ad)n) \int \left(\frac{b^4\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4(a+bx)} - \frac{d\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(c+dx)^4}\right) dx}{3dg^4} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3dg^4(c + dx)^3} - \frac{(2Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^4} dx}{3g^4} - \frac{(2b^3Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} dx}{3(bc - ad)^3g^4} \\
&= \frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9dg^4(c + dx)^3} + \frac{bBn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d(bc - ad)g^4(c + dx)^2} + \frac{2b^2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d(bc - ad)^2g^4(c + dx)} \\
&= \frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9dg^4(c + dx)^3} + \frac{bBn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d(bc - ad)g^4(c + dx)^2} + \frac{2b^2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d(bc - ad)^2g^4(c + dx)} \\
&= \frac{2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9dg^4(c + dx)^3} + \frac{bBn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d(bc - ad)g^4(c + dx)^2} + \frac{2b^2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d(bc - ad)^2g^4(c + dx)} \\
&= -\frac{2B^2n^2}{27dg^4(c + dx)^3} - \frac{5bB^2n^2}{18d(bc - ad)g^4(c + dx)^2} - \frac{11b^2B^2n^2}{9d(bc - ad)^2g^4(c + dx)} - \frac{11b^3B^2n^2 \log\left(\frac{a+bx}{c+dx}\right)}{9d(bc - ad)^3g^4} \\
&= -\frac{2B^2n^2}{27dg^4(c + dx)^3} - \frac{5bB^2n^2}{18d(bc - ad)g^4(c + dx)^2} - \frac{11b^2B^2n^2}{9d(bc - ad)^2g^4(c + dx)} - \frac{11b^3B^2n^2 \log\left(\frac{a+bx}{c+dx}\right)}{9d(bc - ad)^3g^4} \\
&= -\frac{2B^2n^2}{27dg^4(c + dx)^3} - \frac{5bB^2n^2}{18d(bc - ad)g^4(c + dx)^2} - \frac{11b^2B^2n^2}{9d(bc - ad)^2g^4(c + dx)} - \frac{11b^3B^2n^2 \log\left(\frac{a+bx}{c+dx}\right)}{9d(bc - ad)^3g^4}
\end{aligned}$$

**Mathematica [C]** time = 0.774851, size = 609, normalized size = 1.42

$$Bn\left(-18b^3Bn(c+dx)^3\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+18b^3Bn(c+dx)^3\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)\right)-\log(c+dx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(c\*g + d\*g\*x)^4,x]

[Out] (-18\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 + (B\*n\*(12\*(b\*c - a\*d)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 18\*b\*(b\*c - a\*d)^2\*(c + d\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 36\*b^2\*(b\*c - a\*d)\*(c + d\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 36\*b^3\*(c + d\*x)^3\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 36\*b^3\*(c + d\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x] - 36\*b^2\*B\*n\*(c + d\*x)^2\*(b\*c - a\*d + b\*(c + d\*x)\*Log[a + b\*x] - b\*(c + d\*x)\*Log[c + d\*x]) - 9\*b\*B\*n\*(c + d\*x)\*((b\*c - a\*d)^2 + 2\*b\*(b\*c - a\*d)\*(c + d\*x) + 2\*b^2\*(c + d\*x)^2\*Log[a + b\*x] - 2\*b^2\*(c + d\*x)^2\*Log[c + d\*x]) - 2\*B\*n\*(2\*(b\*c - a\*d)^3 + 3\*b\*(b\*c - a\*d)^2\*(c + d\*x) + 6\*b^2\*(b\*c - a\*d)\*(c + d\*x)^2 + 6\*b^3\*(c + d\*x)^3\*Log[a + b\*x] - 6\*b^3\*(c + d\*x)^3\*Log[c + d\*x]) - 18\*b^3\*B\*n\*(c + d\*x)^3\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-(b\*c) + a\*d)]) + 18\*b^3\*B\*n\*(c + d\*x)^3\*((2\*Log[(d\*(a + b\*x))/(-(b\*c) + a\*d)] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/(b\*c - a\*d)^3)/(54\*d\*g^4\*(c + d\*x)^3)

**Maple [F]** time = 0.441, size = 0, normalized size = 0.

$$\int \frac{1}{(d g x + c g)^4} \left( A + B \ln \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^4,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^4,x)

**Maxima [B]** time = 1.61998, size = 1937, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^4,x, algorithm="maxima")

[Out] 1/9\*A\*B\*n\*((6\*b^2\*d^2\*x^2 + 11\*b^2\*c^2 - 7\*a\*b\*c\*d + 2\*a^2\*d^2 + 3\*(5\*b^2\*c\*d - a\*b\*d^2)\*x)/((b^2\*c^2\*d^4 - 2\*a\*b\*c\*d^5 + a^2\*d^6)\*g^4\*x^3 + 3\*(b^2\*c^3\*d^3 - 2\*a\*b\*c^2\*d^4 + a^2\*c\*d^5)\*g^4\*x^2 + 3\*(b^2\*c^4\*d^2 - 2\*a\*b\*c^3\*d^3 + a^2\*c^2\*d^4)\*g^4\*x + (b^2\*c^5\*d - 2\*a\*b\*c^4\*d^2 + a^2\*c^3\*d^3)\*g^4) + 6\*b^3\*log(b\*x + a)/((b^3\*c^3\*d - 3\*a\*b^2\*c^2\*d^2 + 3\*a^2\*b\*c\*d^3 - a^3\*d^4)\*g^4) - 6\*b^3\*log(d\*x + c)/((b^3\*c^3\*d - 3\*a\*b^2\*c^2\*d^2 + 3\*a^2\*b\*c\*d^3 - a^3\*d^4)\*g^4) + 1/54\*(6\*n\*((6\*b^2\*d^2\*x^2 + 11\*b^2\*c^2 - 7\*a\*b\*c\*d + 2\*a^2\*d^2 + 3\*(5\*b^2\*c\*d - a\*b\*d^2)\*x)/((b^2\*c^2\*d^4 - 2\*a\*b\*c\*d^5 + a^2\*d^6)\*g^4\*x^3 + 3\*(b^2\*c^3\*d^3 - 2\*a\*b\*c^2\*d^4 + a^2\*c\*d^5)\*g^4\*x^2 + 3\*(b^2\*c^4\*d^2 - 2\*a\*b\*c^3\*d^3 + a^2\*c^2\*d^4)\*g^4\*x + (b^2\*c^5\*d - 2\*a\*b\*c^4\*d^2 + a^2\*c^3\*d^3)\*g^4) + 6\*b^3\*log(b\*x + a)/((b^3\*c^3\*d - 3\*a\*b^2\*c^2\*d^2 + 3\*a^2\*b\*c\*d^3 - a^3\*d^4)\*g^4) - 6\*b^3\*log(d\*x + c)/((b^3\*c^3\*d - 3\*a\*b^2\*c^2\*d^2 + 3\*a^2\*b\*c\*d^3 - a^3\*d^4)\*g^4))\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) - (85\*b^3\*c^3 - 108\*a\*b^2\*c^2\*d + 27\*a^2\*b\*c\*d^2 - 4\*a^3\*d^3 + 66\*(b^3\*c\*d^2 - a\*b^2\*d^3)\*x^2 + 18\*(b^3\*d^3\*x^3 + 3\*b^3\*c\*d^2\*x^2 + 3\*b^3\*c^2\*d\*x + b^3\*c^3)\*x

$$\begin{aligned} & \log(b*x + a)^2 + 18*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3) \\ & )*\log(d*x + c)^2 + 3*(49*b^3*c^2*d - 54*a*b^2*c*d^2 + 5*a^2*b*d^3)*x + 66*( \\ & b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\log(b*x + a) - 6*( \\ & 11*b^3*d^3*x^3 + 33*b^3*c*d^2*x^2 + 33*b^3*c^2*d*x + 11*b^3*c^3 + 6*(b^3*d^ \\ & 3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*\log(b*x + a))*\log(d*x + \\ & c))*n^2/(b^3*c^6*d*g^4 - 3*a*b^2*c^5*d^2*g^4 + 3*a^2*b*c^4*d^3*g^4 - a^3*c^ \\ & 3*d^4*g^4 + (b^3*c^3*d^4*g^4 - 3*a*b^2*c^2*d^5*g^4 + 3*a^2*b*c*d^6*g^4 - a^ \\ & 3*d^7*g^4)*x^3 + 3*(b^3*c^4*d^3*g^4 - 3*a*b^2*c^3*d^4*g^4 + 3*a^2*b*c^2*d^5 \\ & *g^4 - a^3*c*d^6*g^4)*x^2 + 3*(b^3*c^5*d^2*g^4 - 3*a*b^2*c^4*d^3*g^4 + 3*a^ \\ & 2*b*c^3*d^4*g^4 - a^3*c^2*d^5*g^4)*x)*B^2 - 1/3*B^2*\log(e*(b*x/(d*x + c) + \\ & a/(d*x + c)))^n)^2/(d^4*g^4*x^3 + 3*c*d^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d \\ & *g^4) - 2/3*A*B*\log(e*(b*x/(d*x + c) + a/(d*x + c)))^n)/(d^4*g^4*x^3 + 3*c*d \\ & ^3*g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4) - 1/3*A^2/(d^4*g^4*x^3 + 3*c*d^3* \\ & g^4*x^2 + 3*c^2*d^2*g^4*x + c^3*d*g^4) \end{aligned}$$

**Fricas [B]** time = 1.06222, size = 2383, normalized size = 5.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^4,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/54*(18*A^2*b^3*c^3 - 54*A^2*a*b^2*c^2*d + 54*A^2*a^2*b*c*d^2 - 18*A^2*a^ \\ & 3*d^3 + (85*B^2*b^3*c^3 - 108*B^2*a*b^2*c^2*d + 27*B^2*a^2*b*c*d^2 - 4*B^2* \\ & a^3*d^3)*n^2 + 6*(11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^2 - 6*(A*B*b^3*c*d^2 \\ & - A*B*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2* \\ & b*c*d^2 - B^2*a^3*d^3)*\log(e)^2 - 18*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*b^3*c*d^2 \\ & *n^2*x^2 + 3*B^2*b^3*c^2*d*n^2*x + (3*B^2*a*b^2*c^2*d - 3*B^2*a^2*b*c*d^2 + \\ & B^2*a^3*d^3)*n^2)*\log((b*x + a)/(d*x + c))^2 - 6*(11*A*B*b^3*c^3 - 18*A*B* \\ & a*b^2*c^2*d + 9*A*B*a^2*b*c*d^2 - 2*A*B*a^3*d^3)*n + 3*((49*B^2*b^3*c^2*d - \\ & 54*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3)*n^2 - 6*(5*A*B*b^3*c^2*d - 6*A*B*a*b \\ & ^2*c*d^2 + A*B*a^2*b*d^3)*n)*x + 6*(6*A*B*b^3*c^3 - 18*A*B*a*b^2*c^2*d + 18 \\ & *A*B*a^2*b*c*d^2 - 6*A*B*a^3*d^3 - 6*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n*x^2 \\ & - 3*(5*B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*n*x - (11*B^2*b^3 \\ & *c^3 - 18*B^2*a*b^2*c^2*d + 9*B^2*a^2*b*c*d^2 - 2*B^2*a^3*d^3)*n - 6*(B^2*b \\ & ^3*d^3*n*x^3 + 3*B^2*b^3*c*d^2*n*x^2 + 3*B^2*b^3*c^2*d*n*x + (3*B^2*a*b^2*c \\ & ^2*d - 3*B^2*a^2*b*c*d^2 + B^2*a^3*d^3)*n)*\log((b*x + a)/(d*x + c))*\log(e) \\ & + 6*((11*B^2*b^3*d^3*n^2 - 6*A*B*b^3*d^3*n)*x^3 + (18*B^2*a*b^2*c^2*d - 9* \\ & B^2*a^2*b*c*d^2 + 2*B^2*a^3*d^3)*n^2 - 3*(6*A*B*b^3*c*d^2*n - (9*B^2*b^3*c* \\ & d^2 + 2*B^2*a*b^2*d^3)*n^2)*x^2 - 6*(3*A*B*a*b^2*c^2*d - 3*A*B*a^2*b*c*d^2 \\ & + A*B*a^3*d^3)*n - 3*(6*A*B*b^3*c^2*d*n - (6*B^2*b^3*c^2*d + 6*B^2*a*b^2*c* \\ & d^2 - B^2*a^2*b*d^3)*n^2)*x)*\log((b*x + a)/(d*x + c)))/((b^3*c^3*d^4 - 3*a* \\ & b^2*c^2*d^5 + 3*a^2*b*c*d^6 - a^3*d^7)*g^4*x^3 + 3*(b^3*c^4*d^3 - 3*a*b^2*c^ \\ & ^3*d^4 + 3*a^2*b*c^2*d^5 - a^3*c*d^6)*g^4*x^2 + 3*(b^3*c^5*d^2 - 3*a*b^2*c^ \\ & 4*d^3 + 3*a^2*b*c^3*d^4 - a^3*c^2*d^5)*g^4*x + (b^3*c^6*d - 3*a*b^2*c^5*d^2 \\ & + 3*a^2*b*c^4*d^3 - a^3*c^3*d^4)*g^4) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2/(d\*g\*x+c\*g)\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2}{(d gx + cg)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^4,x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2/(d\*g\*x + c\*g)^4, x)

$$3.46 \quad \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg+dgx)^5} dx$$

**Optimal.** Leaf size=536

$$\frac{b^4 B n \log\left(\frac{a+bx}{c+dx}\right) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{2d g^5 (bc - ad)^4} - \frac{2b^3 B n (a + bx) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{g^5 (c + dx) (bc - ad)^4} + \frac{3b^2 B d n (a + bx)^2 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2g^5 (c + dx)^2 (bc - ad)^4}$$

[Out]  $-(B^2 d^3 n^2 (a + b x)^4) / (32 (b c - a d)^4 g^5 (c + d x)^4) + (2 b B^2 d^2 n^2 (a + b x)^3) / (9 (b c - a d)^4 g^5 (c + d x)^3) - (3 b^2 B^2 d n^2 (a + b x)^2) / (4 (b c - a d)^4 g^5 (c + d x)^2) + (2 b^3 B^2 n^2 (a + b x)) / ((b c - a d)^4 g^5 (c + d x)) + (B d^3 n (a + b x)^4 (A + B \text{Log}[e((a + b x) / (c + d x))^n])) / (8 (b c - a d)^4 g^5 (c + d x)^4) - (2 b B d^2 n (a + b x)^3 (A + B \text{Log}[e((a + b x) / (c + d x))^n])) / (3 (b c - a d)^4 g^5 (c + d x)^3) + (3 b^2 B d n (a + b x)^2 (A + B \text{Log}[e((a + b x) / (c + d x))^n])) / (2 (b c - a d)^4 g^5 (c + d x)^2) - (2 b^3 B n (a + b x) (A + B \text{Log}[e((a + b x) / (c + d x))^n])) / ((b c - a d)^4 g^5 (c + d x)) - (A + B \text{Log}[e((a + b x) / (c + d x))^n])^2 / (4 d g^5 (c + d x)^4) + (b^4 B n (A + B \text{Log}[e((a + b x) / (c + d x))^n]) \text{Log}[(a + b x) / (c + d x)]) / (2 d (b c - a d)^4 g^5) - (b^4 B^2 n^2 \text{Log}[(a + b x) / (c + d x)]^2) / (4 d (b c - a d)^4 g^5)$

**Rubi [C]** time = 1.29528, antiderivative size = 826, normalized size of antiderivative = 1.54, number of steps used = 36, number of rules used = 11, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 44}

$$\frac{B^2 n^2 \log^2(a + bx) b^4}{4d(bc - ad)^4 g^5} - \frac{B^2 n^2 \log^2(c + dx) b^4}{4d(bc - ad)^4 g^5} - \frac{25 B^2 n^2 \log(a + bx) b^4}{24d(bc - ad)^4 g^5} + \frac{B n \log(a + bx) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) b^4}{2d(bc - ad)^4 g^5} + \frac{25 B^2 n^2 \log^2(a + bx) b^4}{24d(bc - ad)^4 g^5}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(c\*g + d\*g\*x)^5, x]

[Out]  $-(B^2 n^2) / (32 d g^5 (c + d x)^4) - (7 b B^2 n^2) / (72 d (b c - a d) g^5 (c + d x)^3) - (13 b^2 B^2 n^2) / (48 d (b c - a d)^2 g^5 (c + d x)^2) - (25 b^3 B^2 n^2) / (24 d (b c - a d)^3 g^5 (c + d x)) - (25 b^4 B^2 n^2 \text{Log}[a + b x]) / (24 d (b c - a d)^4 g^5) - (b^4 B^2 n^2 \text{Log}[a + b x]^2) / (4 d (b c - a d)^4 g^5) + (B n (A + B \text{Log}[e((a + b x) / (c + d x))^n])) / (8 d g^5 (c + d x)^4) + (b B n (A + B \text{Log}[e((a + b x) / (c + d x))^n])) / (6 d (b c - a d) g^5 (c + d x)^3) + (b^2 B n (A + B \text{Log}[e((a + b x) / (c + d x))^n])) / (4 d (b c - a d)^2 g^5 (c + d x)^2) + (b^3 B n (A + B \text{Log}[e((a + b x) / (c + d x))^n])) / (2 d (b c - a d)^3 g^5 (c + d x)) + (b^4 B n \text{Log}[a + b x] (A + B \text{Log}[e((a + b x) / (c + d x))^n])) / (2 d (b c - a d)^4 g^5) - (A + B \text{Log}[e((a + b x) / (c + d x))^n])^2 / (4 d g^5 (c + d x)^4) + (25 b^4 B^2 n^2 \text{Log}[c + d x]) / (24 d (b c - a d)^4 g^5) + (b^4 B^2 n^2 \text{Log}[-((d(a + b x)) / (b c - a d))] \text{Log}[c + d x]) / (2 d (b c - a d)^4 g^5) - (b^4 B n (A + B \text{Log}[e((a + b x) / (c + d x))^n]) \text{Log}[c + d x]) / (2 d (b c - a d)^4 g^5) - (b^4 B^2 n^2 \text{Log}[c + d x]^2) / (4 d (b c - a d)^4 g^5) + (b^4 B^2 n^2 \text{Log}[a + b x] \text{Log}[(b(c + d x)) / (b c - a d)]) / (2 d (b c - a d)^4 g^5) + (b^4 B^2 n^2 \text{PolyLog}[2, -((d(a + b x)) / (b c - a d))]) / (2 d (b c - a d)^4 g^5) + (b^4 B^2 n^2 \text{PolyLog}[2, (b(c + d x)) / (b c - a d)]) / (2 d (b c - a d)^4 g^5)$

Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_.))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_.)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))]*(b_.))/((f_.) + (g_.)*(x_.)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*(d_) + (e_.)*(x_)^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(cg + dgx)^5} dx &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c + dx)^4} + \frac{(Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g^4(a+bx)(c+dx)^5} dx}{2dg} \\ &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c + dx)^4} + \frac{(B(bc - ad)n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)^5} dx}{2dg^5} \\ &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c + dx)^4} + \frac{(B(bc - ad)n) \int \left(\frac{b^5\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5(a+bx)} - \frac{d\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(c+dx)^5}\right) dx}{2dg^5} \\ &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4dg^5(c + dx)^4} - \frac{(Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(c+dx)^5} dx}{2g^5} - \frac{(b^4Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} dx}{2(bc - ad)^4g^5} \\ &= \frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8dg^5(c + dx)^4} + \frac{bBn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d(bc - ad)g^5(c + dx)^3} + \frac{b^2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d(bc - ad)^2g^5(c + dx)^2} \\ &= \frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8dg^5(c + dx)^4} + \frac{bBn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d(bc - ad)g^5(c + dx)^3} + \frac{b^2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d(bc - ad)^2g^5(c + dx)^2} \\ &= \frac{Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8dg^5(c + dx)^4} + \frac{bBn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d(bc - ad)g^5(c + dx)^3} + \frac{b^2Bn\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d(bc - ad)^2g^5(c + dx)^2} \\ &= \frac{B^2n^2}{32dg^5(c + dx)^4} - \frac{7bB^2n^2}{72d(bc - ad)g^5(c + dx)^3} - \frac{13b^2B^2n^2}{48d(bc - ad)^2g^5(c + dx)^2} - \frac{25b^3}{24d(bc - ad)g^5(c + dx)} \\ &= \frac{B^2n^2}{32dg^5(c + dx)^4} - \frac{7bB^2n^2}{72d(bc - ad)g^5(c + dx)^3} - \frac{13b^2B^2n^2}{48d(bc - ad)^2g^5(c + dx)^2} - \frac{25b^3}{24d(bc - ad)g^5(c + dx)} \\ &= \frac{B^2n^2}{32dg^5(c + dx)^4} - \frac{7bB^2n^2}{72d(bc - ad)g^5(c + dx)^3} - \frac{13b^2B^2n^2}{48d(bc - ad)^2g^5(c + dx)^2} - \frac{25b^3}{24d(bc - ad)g^5(c + dx)} \end{aligned}$$

**Mathematica [C]** time = 1.08577, size = 776, normalized size = 1.45

---


$$Bn\left(-72b^4Bn(c+dx)^4\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+72b^4Bn(c+dx)^4\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)\right)-\log(c+dx)\right)$$


---

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(c\*g + d\*g\*x)^5,x]

[Out] (-72\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 + (B\*n\*(36\*(b\*c - a\*d)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 48\*b\*(b\*c - a\*d)^3\*(c + d\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 72\*b^2\*(b\*c - a\*d)^2\*(c + d\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 144\*b^3\*(b\*c - a\*d)\*(c + d\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 144\*b^4\*(c + d\*x)^4\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 144\*b^4\*(c + d\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x] - 144\*b^3\*B\*n\*(c + d\*x)^3\*(b\*c - a\*d + b\*(c + d\*x))\*Log[a + b\*x] - b\*(c + d\*x)\*Log[c + d\*x]) - 36\*b^2\*B\*n\*(c + d\*x)^2\*((b\*c - a\*d)^2 + 2\*b\*(b\*c - a\*d)\*(c + d\*x) + 2\*b^2\*(c + d\*x)^2\*Log[a + b\*x] - 2\*b^2\*(c + d\*x)^2\*Log[c + d\*x]) - 8\*b\*B\*n\*(c + d\*x)\*(2\*(b\*c - a\*d)^3 + 3\*b\*(b\*c - a\*d)^2\*(c + d\*x) + 6\*b^2\*(b\*c - a\*d)\*(c + d\*x)^2 + 6\*b^3\*(c + d\*x)^3\*Log[a + b\*x] - 6\*b^3\*(c + d\*x)^3\*Log[c + d\*x]) - 3\*B\*n\*(3\*(b\*c - a\*d)^4 + 4\*b\*(b\*c - a\*d)^3\*(c + d\*x) + 6\*b^2\*(b\*c - a\*d)^2\*(c + d\*x)^2 + 12\*b^3\*(b\*c - a\*d)\*(c + d\*x)^3 + 12\*b^4\*(c + d\*x)^4\*Log[a + b\*x] - 12\*b^4\*(c + d\*x)^4\*Log[c + d\*x]) - 72\*b^4\*B\*n\*(c + d\*x)^4\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-(b\*c) + a\*d)]) + 72\*b^4\*B\*n\*(c + d\*x)^4\*((2\*Log[(d\*(a + b\*x))/(-(b\*c) + a\*d)] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/(b\*c - a\*d)^4/(288\*d\*g^5\*(c + d\*x)^4)

**Maple [F]** time = 0.442, size = 0, normalized size = 0.

$$\int \frac{1}{(dgx + cg)^5} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^5,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^5,x)

**Maxima [B]** time = 2.03036, size = 2886, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^5,x, algorithm="maxima")

[Out] 1/24\*A\*B\*n\*((12\*b^3\*d^3\*x^3 + 25\*b^3\*c^3 - 23\*a\*b^2\*c^2\*d + 13\*a^2\*b\*c\*d^2 - 3\*a^3\*d^3 + 6\*(7\*b^3\*c\*d^2 - a\*b^2\*d^3)\*x^2 + 4\*(13\*b^3\*c^2\*d - 5\*a\*b^2\*c\*d^2 + a^2\*b\*d^3)\*x)/((b^3\*c^3\*d^5 - 3\*a\*b^2\*c^2\*d^6 + 3\*a^2\*b\*c\*d^7 - a^3\*d^8)\*g^5\*x^4 + 4\*(b^3\*c^4\*d^4 - 3\*a\*b^2\*c^3\*d^5 + 3\*a^2\*b\*c^2\*d^6 - a^3\*c\*d^7)\*g^5\*x^3 + 6\*(b^3\*c^5\*d^3 - 3\*a\*b^2\*c^4\*d^4 + 3\*a^2\*b\*c^3\*d^5 - a^3\*c^2\*d^6)\*g^5\*x^2 + 4\*(b^3\*c^6\*d^2 - 3\*a\*b^2\*c^5\*d^3 + 3\*a^2\*b\*c^4\*d^4 - a^3\*c^3\*d^5)\*g^5\*x + (b^3\*c^7\*d - 3\*a\*b^2\*c^6\*d^2 + 3\*a^2\*b\*c^5\*d^3 - a^3\*c^4\*d^4)\*g^5) + 12\*b^4\*log(b\*x + a)/((b^4\*c^4\*d - 4\*a\*b^3\*c^3\*d^2 + 6\*a^2\*b^2\*c^2\*d^3 - 4\*a^3\*b\*c\*d^4 + a^4\*d^5)\*g^5) - 12\*b^4\*log(d\*x + c)/((b^4\*c^4\*d - 4\*a\*

$$\begin{aligned}
& b^3c^3d^2 + 6a^2b^2c^2d^3 - 4a^3b^2c^2d^4 + a^4d^5)g^5)) + 1/288*(1 \\
& 2*n*((12*b^3*d^3*x^3 + 25*b^3*c^3 - 23*a*b^2*c^2*d + 13*a^2*b*c*d^2 - 3*a^3 \\
& *d^3 + 6*(7*b^3*c*d^2 - a*b^2*d^3)*x^2 + 4*(13*b^3*c^2*d - 5*a*b^2*c*d^2 + \\
& a^2*b*d^3)*x)/((b^3*c^3*d^5 - 3*a*b^2*c^2*d^6 + 3*a^2*b*c*d^7 - a^3*d^8)*g^ \\
& 5*x^4 + 4*(b^3*c^4*d^4 - 3*a*b^2*c^3*d^5 + 3*a^2*b*c^2*d^6 - a^3*c*d^7)*g^5 \\
& *x^3 + 6*(b^3*c^5*d^3 - 3*a*b^2*c^4*d^4 + 3*a^2*b*c^3*d^5 - a^3*c^2*d^6)*g^ \\
& 5*x^2 + 4*(b^3*c^6*d^2 - 3*a*b^2*c^5*d^3 + 3*a^2*b*c^4*d^4 - a^3*c^3*d^5)*g^ \\
& 5*x + (b^3*c^7*d - 3*a*b^2*c^6*d^2 + 3*a^2*b*c^5*d^3 - a^3*c^4*d^4)*g^5) + \\
& 12*b^4*log(b*x + a)/((b^4*c^4*d - 4*a*b^3*c^3*d^2 + 6*a^2*b^2*c^2*d^3 - 4* \\
& a^3*b*c*d^4 + a^4*d^5)*g^5) - 12*b^4*log(d*x + c)/((b^4*c^4*d - 4*a*b^3*c^3 \\
& *d^2 + 6*a^2*b^2*c^2*d^3 - 4*a^3*b*c*d^4 + a^4*d^5)*g^5))*log(e*(b*x/(d*x + \\
& c) + a/(d*x + c))^n) - (415*b^4*c^4 - 576*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^ \\
& 2 - 64*a^3*b*c*d^3 + 9*a^4*d^4 + 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(163*b \\
& ^4*c^2*d^2 - 176*a*b^3*c*d^3 + 13*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4*b^ \\
& 4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(b*x + a)^2 + \\
& 72*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^ \\
& 4*c^4)*log(d*x + c)^2 + 4*(271*b^4*c^3*d - 324*a*b^3*c^2*d^2 + 60*a^2*b^2*c \\
& *d^3 - 7*a^3*b*d^4)*x + 300*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2* \\
& x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(b*x + a) - 12*(25*b^4*d^4*x^4 + 100*b^4* \\
& c*d^3*x^3 + 150*b^4*c^2*d^2*x^2 + 100*b^4*c^3*d*x + 25*b^4*c^4 + 12*(b^4*d^ \\
& 4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log( \\
& b*x + a))*log(d*x + c))^n^2/(b^4*c^8*d*g^5 - 4*a*b^3*c^7*d^2*g^5 + 6*a^2*b^ \\
& 2*c^6*d^3*g^5 - 4*a^3*b*c^5*d^4*g^5 + a^4*c^4*d^5*g^5 + (b^4*c^4*d^5*g^5 - \\
& 4*a*b^3*c^3*d^6*g^5 + 6*a^2*b^2*c^2*d^7*g^5 - 4*a^3*b*c*d^8*g^5 + a^4*d^9*g^ \\
& 5)*x^4 + 4*(b^4*c^5*d^4*g^5 - 4*a*b^3*c^4*d^5*g^5 + 6*a^2*b^2*c^3*d^6*g^5 \\
& - 4*a^3*b*c^2*d^7*g^5 + a^4*c*d^8*g^5)*x^3 + 6*(b^4*c^6*d^3*g^5 - 4*a*b^3*c \\
& ^5*d^4*g^5 + 6*a^2*b^2*c^4*d^5*g^5 - 4*a^3*b*c^3*d^6*g^5 + a^4*c^2*d^7*g^5) \\
& *x^2 + 4*(b^4*c^7*d^2*g^5 - 4*a*b^3*c^6*d^3*g^5 + 6*a^2*b^2*c^5*d^4*g^5 - 4 \\
& *a^3*b*c^4*d^5*g^5 + a^4*c^3*d^6*g^5)*x))B^2 - 1/4*B^2*log(e*(b*x/(d*x + c \\
& ) + a/(d*x + c))^n)^2/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 + \\
& 4*c^3*d^2*g^5*x + c^4*d*g^5) - 1/2*A*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^ \\
& n)/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 + 4*c^3*d^2*g^5*x + c \\
& ^4*d*g^5) - 1/4*A^2/(d^5*g^5*x^4 + 4*c*d^4*g^5*x^3 + 6*c^2*d^3*g^5*x^2 + 4* \\
& c^3*d^2*g^5*x + c^4*d*g^5)
\end{aligned}$$

**Fricas [B]** time = 1.28624, size = 3615, normalized size = 6.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^5,x, algorithm="fricas")

[Out]  $-1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 288*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 + 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*n^2 - 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (415*B^2*b^4*c^4 - 576*B^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 64*B^2*a^3*b*c*d^3 + 9*B^2*a^4*d^4)*n^2 + 6*((163*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 13*B^2*a^2*b^2*d^4)*n^2 - 12*(7*A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + A*B*a^2*b^2*d^4)*n)*x^2 + 72*(B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - 4*B^2*a^3*b*c*d^3 + B^2*a^4*d^4)*log(e)^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*b^4*c*d^3*n^2*x^3 + 6*B^2*b^4*c^2*d^2*n^2*x^2 + 4*B^2*b^4*c^3*d*n^2*x + (4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*B^2*a^3*b*c*d^3 - B^2*a^4*d^4)*n^2)*log((b*x + a)/(d*x + c))^2 - 12*(25*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 36*A*B*a^2*b^2*c^2*d^2 - 16*A*B*a^3*b*c*d^3 + 3*A*B*a^4*d^4)*n + 4*((271*B^2*b^4*c^3*d - 324*B^2*a*b^3*c^2*d^2 + 60*B^2*a^2*b^2*c*d^3 - 7*B^2*a^3*b*d^4)*n^2$

$$\begin{aligned}
& - 12*(13*A*B*b^4*c^3*d - 18*A*B*a*b^3*c^2*d^2 + 6*A*B*a^2*b^2*c*d^3 - A*B*a^3*b*d^4)*n)*x + 12*(12*A*B*b^4*c^4 - 48*A*B*a*b^3*c^3*d + 72*A*B*a^2*b^2*c^2*d^2 - 48*A*B*a^3*b*c*d^3 + 12*A*B*a^4*d^4 - 12*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*n*x^3 - 6*(7*B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*n*x^2 - 4*(13*B^2*b^4*c^3*d - 18*B^2*a*b^3*c^2*d^2 + 6*B^2*a^2*b^2*c*d^3 - B^2*a^3*b*d^4)*n*x - (25*B^2*b^4*c^4 - 48*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c^2*d^2 - 16*B^2*a^3*b*c*d^3 + 3*B^2*a^4*d^4)*n - 12*(B^2*b^4*d^4*n*x^4 + 4*B^2*b^4*c*d^3*n*x^3 + 6*B^2*b^4*c^2*d^2*n*x^2 + 4*B^2*b^4*c^3*d*n*x + (4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*B^2*a^3*b*c*d^3 - B^2*a^4*d^4)*n)*log((b*x + a)/(d*x + c))*log(e) + 12*((25*B^2*b^4*d^4*n^2 - 12*A*B*b^4*d^4*n)*x^4 - 4*(12*A*B*b^4*c*d^3*n - (22*B^2*b^4*c*d^3 + 3*B^2*a*b^3*d^4)*n^2)*x^3 + (48*B^2*a*b^3*c^3*d - 36*B^2*a^2*b^2*c^2*d^2 + 16*B^2*a^3*b*c*d^3 - 3*B^2*a^4*d^4)*n^2 - 6*(12*A*B*b^4*c^2*d^2*n - (18*B^2*b^4*c^2*d^2 + 8*B^2*a*b^3*c*d^3 - B^2*a^2*b^2*d^4)*n^2)*x^2 - 12*(4*A*B*a*b^3*c^3*d - 6*A*B*a^2*b^2*c^2*d^2 + 4*A*B*a^3*b*c*d^3 - A*B*a^4*d^4)*n - 4*(12*A*B*b^4*c^3*d*n - (12*B^2*b^4*c^3*d + 18*B^2*a*b^3*c^2*d^2 - 6*B^2*a^2*b^2*c*d^3 + B^2*a^3*b*d^4)*n^2)*x)*log((b*x + a)/(d*x + c))/((b^4*c^4*d^5 - 4*a*b^3*c^3*d^6 + 6*a^2*b^2*c^2*d^7 - 4*a^3*b*c*d^8 + a^4*d^9)*g^5*x^4 + 4*(b^4*c^5*d^4 - 4*a*b^3*c^4*d^5 + 6*a^2*b^2*c^3*d^6 - 4*a^3*b*c^2*d^7 + a^4*c*d^8)*g^5*x^3 + 6*(b^4*c^6*d^3 - 4*a*b^3*c^5*d^4 + 6*a^2*b^2*c^4*d^5 - 4*a^3*b*c^3*d^6 + a^4*c^2*d^7)*g^5*x^2 + 4*(b^4*c^7*d^2 - 4*a*b^3*c^6*d^3 + 6*a^2*b^2*c^5*d^4 - 4*a^3*b*c^4*d^5 + a^4*c^3*d^6)*g^5*x + (b^4*c^8*d - 4*a*b^3*c^7*d^2 + 6*a^2*b^2*c^6*d^3 - 4*a^3*b*c^5*d^4 + a^4*c^4*d^5)*g^5)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2/(d\*g\*x+c\*g)\*\*5,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2}{(d gx + c g)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(d\*g\*x+c\*g)^5,x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2/(d\*g\*x + c\*g)^5, x)

$$3.47 \quad \int \frac{(cg+dgx)^2}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

**Optimal.** Leaf size=37

$$\text{Unintegrable}\left(\frac{(cg + dgx)^2}{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}, x\right)$$

[Out] Unintegrable[(c\*g + d\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

**Rubi [A]** time = 0.214294, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(cg + dgx)^2}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(c\*g + d\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] c^2\*g^2\*Defer[Int][(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^(-1), x] + 2\*c\*d\*g^2\*Defer[Int][x/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x] + d^2\*g^2\*Defer[Int][x^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

Rubi steps

$$\begin{aligned} \int \frac{(cg + dgx)^2}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx &= \int \left( \frac{c^2g^2}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{2cdg^2x}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{d^2g^2x^2}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} \right) dx \\ &= (c^2g^2) \int \frac{1}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx + (2cdg^2) \int \frac{x}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx + (d^2g^2) \int \frac{x^2}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.437876, size = 0, normalized size = 0.

$$\int \frac{(cg + dgx)^2}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c\*g + d\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] Integrate[(c\*g + d\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]



**Maple [A]** time = 0.398, size = 0, normalized size = 0.

$$\int (d gx + c g)^2 \left( A + B \ln \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*g\*x+c\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int((d\*g\*x+c\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d gx + c g)^2}{B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out] integrate((d\*g\*x + c\*g)^2/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{d^2 g^2 x^2 + 2 c d g^2 x + c^2 g^2}{B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] integral((d^2\*g^2\*x^2 + 2\*c\*d\*g^2\*x + c^2\*g^2)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)\*\*2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c)\*\*n)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d gx + c g)^2}{B \log\left(e\left(\frac{b x + a}{d x + c}\right)^n\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate((d*g*x + c*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

$$3.48 \quad \int \frac{cg+dgx}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

**Optimal.** Leaf size=35

$$\text{Unintegrable}\left(\frac{cg + dgx}{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}, x\right)$$

[Out] Unintegrable[(c\*g + d\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

**Rubi [A]** time = 0.114506, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{cg + dgx}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(c\*g + d\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] c\*g\*Defer[Int][(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^(-1), x] + d\*g\*Defer[Int][x/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

Rubi steps

$$\begin{aligned} \int \frac{cg + dgx}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx &= \int \left( \frac{cg}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{dgx}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} \right) dx \\ &= (cg) \int \frac{1}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx + (dg) \int \frac{x}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.2834, size = 0, normalized size = 0.

$$\int \frac{cg + dgx}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c\*g + d\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] Integrate[(c\*g + d\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

**Maple [A]** time = 0.303, size = 0, normalized size = 0.

$$\int (d gx + c g) \left( A + B \ln \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*g\*x+c\*g)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int((d\*g\*x+c\*g)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{d gx + c g}{B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out] integrate((d\*g\*x + c\*g)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{d gx + c g}{B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] integral((d\*g\*x + c\*g)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{d gx + c g}{B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out] integrate((d\*g\*x + c\*g)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

$$3.49 \quad \int \frac{1}{(cg+dgx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx$$

**Optimal.** Leaf size=37

$$\text{Unintegrable}\left(\frac{1}{(cg + dgx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}, x\right)$$

[Out] Unintegrable[1/((c\*g + d\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

**Rubi [A]** time = 0.0831093, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(cg + dgx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c\*g + d\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

[Out] Defer[Int][1/((c\*g + d\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

Rubi steps

$$\int \frac{1}{(cg + dgx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx = \int \frac{1}{(cg + dgx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx$$

**Mathematica [A]** time = 0.17414, size = 0, normalized size = 0.

$$\int \frac{1}{(cg + dgx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c\*g + d\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

[Out] Integrate[1/((c\*g + d\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

**Maple [A]** time = 0.54, size = 0, normalized size = 0.

$$\int \frac{1}{dgx + cg}\left(A + B \ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

[Out] `int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d gx + c g) \left( B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")`

[Out] `integrate(1/((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A d g x + A c g + (B d g x + B c g) \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")`

[Out] `integral(1/(A*d*g*x + A*c*g + (B*d*g*x + B*c*g)*log(e*((b*x + a)/(d*x + c))^n)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d gx + c g) \left( B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*g*x + c*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```



$$3.50 \quad \int \frac{1}{(cg+dgx)^2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

**Optimal.** Leaf size=96

$$\frac{(a+bx)e^{-\frac{A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{-1/n} \operatorname{Ei} \left( \frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right)}{Bg^2n(c+dx)(bc-ad)}$$

[Out] ((a + b\*x)\*ExpIntegralEi[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]/(B\*n)))/(B\*(b\*c - a\*d)\*E^(A/(B\*n))\*g^2\*n\*(e\*((a + b\*x)/(c + d\*x))^n)^n^(-1)\*(c + d\*x))

**Rubi [F]** time = 0.100225, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(cg + dgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c\*g + d\*g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])),x]

[Out] Defer[Int][1/((c\*g + d\*g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

Rubi steps

$$\int \frac{1}{(cg + dgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx = \int \frac{1}{(cg + dgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

**Mathematica [A]** time = 0.119399, size = 96, normalized size = 1.

$$\frac{(a+bx)e^{-\frac{A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{-1/n} \operatorname{Ei} \left( \frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right)}{Bg^2n(c+dx)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*g + d\*g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])),x]

[Out] ((a + b\*x)\*ExpIntegralEi[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]/(B\*n)))/(B\*(b\*c - a\*d)\*E^(A/(B\*n))\*g^2\*n\*(e\*((a + b\*x)/(c + d\*x))^n)^n^(-1)\*(c + d\*x))

**Maple [F]** time = 0.438, size = 0, normalized size = 0.

$$\int \frac{1}{(d gx + c g)^2 \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

[Out] `int(1/(d*g*x+c*g)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d gx + c g)^2 \left( B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")`

[Out] `integrate(1/((d*g*x + c*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)`

**Fricas [A]** time = 0.904063, size = 149, normalized size = 1.55

$$\frac{e^{\left(-\frac{B \log(e)+A}{B n}\right)} \log\_integral \left( \frac{(b x + a) e^{\left(\frac{B \log(e)+A}{B n}\right)}}{d x + c} \right)}{(B b c - B a d) g^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")`

[Out] `e^(-(B*log(e) + A)/(B*n))*log_integral((b*x + a)*e^((B*log(e) + A)/(B*n))/(d*x + c))/((B*b*c - B*a*d)*g^2*n)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d gx + c g)^2 \left( B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*g*x + c*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

$$3.51 \quad \int \frac{1}{(cg+dgx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

**Optimal.** Leaf size=199

$$\frac{b(a+bx)e^{-\frac{A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{-1/n} \operatorname{Ei} \left( \frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right)}{Bg^3n(c+dx)(bc-ad)^2} - \frac{d(a+bx)^2 e^{-\frac{2A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{-2/n} \operatorname{Ei} \left( \frac{2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{Bn} \right)}{Bg^3n(c+dx)^2(bc-ad)^2}$$

[Out] (b\*(a + b\*x)\*ExpIntegralEi[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])]/(B\*n))/ (B\*(b\*c - a\*d)^2\*E^(A/(B\*n))\*g^3\*n\*(e\*((a + b\*x)/(c + d\*x))^n)^n^(-1)\*(c + d\*x)) - (d\*(a + b\*x)^2\*ExpIntegralEi[(2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])]/(B\*n))]/(B\*(b\*c - a\*d)^2\*E^((2\*A)/(B\*n))\*g^3\*n\*(e\*((a + b\*x)/(c + d\*x))^n)^n^(2/n)\*(c + d\*x)^2)

**Rubi [F]** time = 0.0828902, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(cg + dgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c\*g + d\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])),x]

[Out] Defer[Int][1/((c\*g + d\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

Rubi steps

$$\int \frac{1}{(cg + dgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx = \int \frac{1}{(cg + dgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

**Mathematica [A]** time = 0.290127, size = 174, normalized size = 0.87

$$\frac{(a+bx)e^{-\frac{2A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{-2/n} \left( be^{\frac{A}{Bn}}(c+dx) \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{\frac{1}{n}} \operatorname{Ei} \left( \frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right) - d(a+bx) \operatorname{Ei} \left( \frac{2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{Bn} \right) \right)}{Bg^3n(c+dx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*g + d\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])),x]

[Out] ((a + b\*x)\*(b\*E^(A/(B\*n))\*(e\*((a + b\*x)/(c + d\*x))^n)^n^(-1)\*(c + d\*x)\*ExpIntegralEi[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])]/(B\*n)] - d\*(a + b\*x)\*ExpIntegralEi[(2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])]/(B\*n))]/(B\*(b\*c - a\*d)^2\*E^((2\*A)/(B\*n))\*g^3\*n\*(e\*((a + b\*x)/(c + d\*x))^n)^n^(2/n)\*(c + d\*x)^2)

---

**Maple [F]** time = 0.435, size = 0, normalized size = 0.

$$\int \frac{1}{(dgx + cg)^3} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*g\*x+c\*g)^3/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int(1/(d\*g\*x+c\*g)^3/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dgx + cg)^3 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*g\*x+c\*g)^3/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out] integrate(1/((d\*g\*x + c\*g)^3\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)), x)

---

**Fricas [A]** time = 0.88901, size = 351, normalized size = 1.76

$$\frac{\left( be^{\frac{B \log(e)+A}{Bn}} \log\_integral \left( \frac{(bx+a)e^{\frac{B \log(e)+A}{Bn}}}{dx+c} \right) - d \log\_integral \left( \frac{(b^2x^2+2abx+a^2)e^{\frac{2(B \log(e)+A)}{Bn}}}{d^2x^2+2cdx+c^2} \right) \right) e^{\frac{-2(B \log(e)+A)}{Bn}}}{(Bb^2c^2 - 2Babcd + Ba^2d^2)g^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*g\*x+c\*g)^3/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] (b\*e^((B\*log(e) + A)/(B\*n))\*log\_integral((b\*x + a)\*e^((B\*log(e) + A)/(B\*n))/(d\*x + c)) - d\*log\_integral((b^2\*x^2 + 2\*a\*b\*x + a^2)\*e^(2\*(B\*log(e) + A)/(B\*n))/(d^2\*x^2 + 2\*c\*d\*x + c^2)))\*e^(-2\*(B\*log(e) + A)/(B\*n))/((B\*b^2\*c^2 - 2\*B\*a\*b\*c\*d + B\*a^2\*d^2)\*g^3\*n)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*g\*x+c\*g)\*\*3/(A+B\*ln(e\*((b\*x+a)/(d\*x+c)\*\*n)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d^2x + c^2)^3 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*g\*x+c\*g)^3/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out] integrate(1/((d\*g\*x + c\*g)^3\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)), x)

$$3.52 \quad \int \frac{(cg+dgx)^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Optimal. Leaf size=37

$$\text{Unintegrable}\left[\frac{(cg+dgx)^2}{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}, x\right]$$

[Out] Unintegrable[(c\*g + d\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

**Rubi [A]** time = 0.232878, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(cg+dgx)^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(c\*g + d\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

[Out] c^2\*g^2\*Defer[Int][(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^(-2), x] + 2\*c\*d\*g^2\*Defer[Int][x/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x] + d^2\*g^2\*Defer[Int][x^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{(cg+dgx)^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx &= \int \left[ \frac{c^2g^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{2cdg^2x}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{d^2g^2x^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} \right] dx \\ &= (c^2g^2) \int \frac{1}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx + (2cdg^2) \int \frac{x}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx + \left( \int \frac{d^2g^2x^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx \right) \end{aligned}$$

**Mathematica [A]** time = 0.989418, size = 0, normalized size = 0.

$$\int \frac{(cg+dgx)^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c\*g + d\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

[Out] Integrate[(c\*g + d\*g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

---

**Maple [A]** time = 0.392, size = 0, normalized size = 0.

$$\int (d g x + c g)^2 \left( A + B \ln \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*g\*x+c\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((d\*g\*x+c\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^3g^2x^4 + ac^3g^2 + (3bcd^2g^2 + ad^3g^2)x^3 + 3(bc^2dg^2 + acd^2g^2)x^2 + (bc^3g^2 + 3ac^2dg^2)x}{(bcn - adn)B^2 \log((bx + a)^n) - (bcn - adn)B^2 \log((dx + c)^n) + (bcn - adn)AB + (bcn \log(e) - adn \log(e))B^2} + \int \frac{bc}{(bcn - adn)B^2 \log((bx + a)^n) - (bcn - adn)B^2 \log((dx + c)^n) + (bcn - adn)AB + (bcn \log(e) - adn \log(e))B^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out] -(b\*d^3\*g^2\*x^4 + a\*c^3\*g^2 + (3\*b\*c\*d^2\*g^2 + a\*d^3\*g^2)\*x^3 + 3\*(b\*c^2\*d\*g^2 + a\*c\*d^2\*g^2)\*x^2 + (b\*c^3\*g^2 + 3\*a\*c^2\*d\*g^2)\*x)/((b\*c\*n - a\*d\*n)\*B^2\*log((b\*x + a)^n) - (b\*c\*n - a\*d\*n)\*B^2\*log((d\*x + c)^n) + (b\*c\*n - a\*d\*n)\*A\*B + (b\*c\*n\*log(e) - a\*d\*n\*log(e))\*B^2) + integrate((4\*b\*d^3\*g^2\*x^3 + b\*c^3\*g^2 + 3\*a\*c^2\*d\*g^2 + 3\*(3\*b\*c\*d^2\*g^2 + a\*d^3\*g^2)\*x^2 + 6\*(b\*c^2\*d\*g^2 + a\*c\*d^2\*g^2)\*x)/((b\*c\*n - a\*d\*n)\*B^2\*log((b\*x + a)^n) - (b\*c\*n - a\*d\*n)\*B^2\*log((d\*x + c)^n) + (b\*c\*n - a\*d\*n)\*A\*B + (b\*c\*n\*log(e) - a\*d\*n\*log(e))\*B^2), x)

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{d^2 g^2 x^2 + 2 c d g^2 x + c^2 g^2}{B^2 \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right)^2 + 2 A B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out] integral((d^2\*g^2\*x^2 + 2\*c\*d\*g^2\*x + c^2\*g^2)/(B^2\*log(e\*((b\*x + a)/(d\*x + c))^n))^2 + 2\*A\*B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dgx + cg)^2}{\left(B \log\left(e \left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*g*x+c*g)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((d*g*x + c*g)^2/(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

$$3.53 \quad \int \frac{cg+dgx}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

**Optimal.** Leaf size=35

$$\text{Unintegrable}\left[\frac{cg + dgx}{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}, x\right]$$

[Out] Unintegrable[(c\*g + d\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

**Rubi [A]** time = 0.123848, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{cg + dgx}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(c\*g + d\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

[Out] c\*g\*Defer[Int][(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^(-2), x] + d\*g\*Defer[Int][x/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{cg + dgx}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx &= \int \left[ \frac{cg}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{dgx}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} \right] dx \\ &= (cg) \int \frac{1}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx + (dg) \int \frac{x}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.959053, size = 0, normalized size = 0.

$$\int \frac{cg + dgx}{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(c\*g + d\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

[Out] Integrate[(c\*g + d\*g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

**Maple [A]** time = 0.243, size = 0, normalized size = 0.

$$\int (d gx + c g) \left( A + B \ln \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*g\*x+c\*g)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((d\*g\*x+c\*g)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bd^2gx^3 + ac^2g + (2bcdg + ad^2g)x^2 + (bc^2g + 2acdg)x}{(bcn - adn)B^2 \log((bx + a)^n) - (bcn - adn)B^2 \log((dx + c)^n) + (bcn - adn)AB + (bcn \log(e) - adn \log(e))B^2} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out] -(b\*d^2\*g\*x^3 + a\*c^2\*g + (2\*b\*c\*d\*g + a\*d^2\*g)\*x^2 + (b\*c^2\*g + 2\*a\*c\*d\*g)\*x)/((b\*c\*n - a\*d\*n)\*B^2\*log((b\*x + a)^n) - (b\*c\*n - a\*d\*n)\*B^2\*log((d\*x + c)^n) + (b\*c\*n - a\*d\*n)\*A\*B + (b\*c\*n\*log(e) - a\*d\*n\*log(e))\*B^2) + integrate((3\*b\*d^2\*g\*x^2 + b\*c^2\*g + 2\*a\*c\*d\*g + 2\*(2\*b\*c\*d\*g + a\*d^2\*g)\*x)/((b\*c\*n - a\*d\*n)\*B^2\*log((b\*x + a)^n) - (b\*c\*n - a\*d\*n)\*B^2\*log((d\*x + c)^n) + (b\*c\*n - a\*d\*n)\*A\*B + (b\*c\*n\*log(e) - a\*d\*n\*log(e))\*B^2), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{d gx + c g}{B^2 \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right)^2 + 2 A B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out] integral((d\*g\*x + c\*g)/(B^2\*log(e\*((b\*x + a)/(d\*x + c))^n))^2 + 2\*A\*B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A^2), x)

**Sympy [F-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c)\*\*n))\*\*2,x)

[Out] Timed out

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**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{d gx + c g}{\left( B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*g\*x+c\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate((d\*g\*x + c\*g)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2, x)

$$3.54 \quad \int \frac{1}{(cg+dgx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

**Optimal.** Leaf size=37

$$\text{Unintegrable} \left( \frac{1}{(cg + dgx) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)^2}, x \right)$$

[Out] Unintegrable[1/((c\*g + d\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

**Rubi [A]** time = 0.0903306, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(cg + dgx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c\*g + d\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2),x]

[Out] Defer[Int][1/((c\*g + d\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

Rubi steps

$$\int \frac{1}{(cg + dgx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx = \int \frac{1}{(cg + dgx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

**Mathematica [A]** time = 0.535532, size = 0, normalized size = 0.

$$\int \frac{1}{(cg + dgx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((c\*g + d\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2),x]

[Out] Integrate[1/((c\*g + d\*g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

**Maple [A]** time = 0.427, size = 0, normalized size = 0.

$$\int \frac{1}{dgx + cg} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)`

[Out] `int(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$b \int \frac{1}{(bcgn - adgn)B^2 \log((bx + a)^n) - (bcgn - adgn)B^2 \log((dx + c)^n) + (bcgn - adgn)AB + (bcgn \log(e) - adgn \log(e))B^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")`

[Out] `b*integrate(1/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d*g*n*log(e))*B^2), x) - (b*x + a)/((b*c*g*n - a*d*g*n)*B^2*log((b*x + a)^n) - (b*c*g*n - a*d*g*n)*B^2*log((d*x + c)^n) + (b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d*g*n*log(e))*B^2)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2 d g x + A^2 c g + (B^2 d g x + B^2 c g) \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right)^2 + 2 (A B d g x + A B c g) \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*g*x+c*g)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")`

[Out] `integral(1/(A^2*d*g*x + A^2*c*g + (B^2*d*g*x + B^2*c*g)*log(e*((b*x + a)/(d*x + c))^n))^2 + 2*(A*B*d*g*x + A*B*c*g)*log(e*((b*x + a)/(d*x + c))^n)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*g*x+c*g)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(dgx + cg) \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*g\*x+c\*g)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate(1/((d\*g\*x + c\*g)\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2), x)

$$3.55 \quad \int \frac{1}{(cg+dgx)^2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

**Optimal.** Leaf size=154

$$\frac{(a+bx)e^{-\frac{A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{-1/n} \operatorname{Ei} \left( \frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right)}{B^2 g^2 n^2 (c+dx)(bc-ad)} - \frac{a+bx}{Bg^2 n (c+dx)(bc-ad) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}$$

[Out]  $((a + b*x)*\operatorname{ExpIntegralEi}[(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n]]/(B*n)))/(B^2*(b*c - a*d)*E^{(A/(B*n))*g^2*n^2*(e*((a + b*x)/(c + d*x))^n)^{-1}*(c + d*x)} - (a + b*x)/(B*(b*c - a*d)*g^2*n*(c + d*x)*(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n]))$

**Rubi [F]** time = 0.103582, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(cg + dgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[1/((c*g + d*g*x)^2*(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n])^2), x]$

[Out]  $\operatorname{Defer}[\operatorname{Int}[1/((c*g + d*g*x)^2*(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n])^2), x]$

Rubi steps

$$\int \frac{1}{(cg + dgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx = \int \frac{1}{(cg + dgx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

**Mathematica [A]** time = 0.171179, size = 180, normalized size = 1.17

$$\frac{(a+bx)e^{-\frac{A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{-1/n} \left( Bne^{\frac{A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{\frac{1}{n}} - \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) \operatorname{Ei} \left( \frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right) \right)}{B^2 g^2 n^2 (c+dx)(bc-ad) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[1/((c*g + d*g*x)^2*(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n])^2), x]$

[Out]  $-(((a + b*x)*(B*E^{(A/(B*n))}*n*(e*((a + b*x)/(c + d*x))^n)^{-1} - \operatorname{ExpIntegralEi}[(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n]]/(B*n)]*(A + B*\operatorname{Log}[e*((a + b*x)/(c + d*x))^n])))/(B^2*(b*c - a*d)*E^{(A/(B*n))*g^2*n^2*(e*((a + b*x)/(c + d*x))^n)^{-1}*(c + d*x)}$



$x))^n)^{-1} * (c + dx) * (A + B * \text{Log}[e * ((a + bx) / (c + dx))^n]))$

**Maple [F]** time = 0.438, size = 0, normalized size = 0.

$$\int \frac{1}{(d^2gx + c^2g)^2} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d\*g\*x+c\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int(1/(d\*g\*x+c\*g)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(bc^2g^2n - acdg^2n)AB + (bc^2g^2n \log(e) - acdg^2n \log(e))B^2 + ((bcdg^2n - ad^2g^2n)AB + (bcdg^2n \log(e) - ad^2g^2n \log(e))B^2)}{(bc^2g^2n - acdg^2n)AB + (bc^2g^2n \log(e) - acdg^2n \log(e))B^2 + ((bcdg^2n - ad^2g^2n)AB + (bcdg^2n \log(e) - ad^2g^2n \log(e))B^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*g\*x+c\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out]  $-(b*x + a) / ((b*c^2*g^2*n - a*c*d*g^2*n)*A*B + (b*c^2*g^2*n*\log(e) - a*c*d*g^2*n*\log(e))*B^2 + ((b*c*d*g^2*n - a*d^2*g^2*n)*A*B + (b*c*d*g^2*n*\log(e) - a*d^2*g^2*n*\log(e))*B^2)*x + ((b*c*d*g^2*n - a*d^2*g^2*n)*B^2*x + (b*c^2*g^2*n - a*c*d*g^2*n)*B^2)*\log((b*x + a)^n) - ((b*c*d*g^2*n - a*d^2*g^2*n)*B^2*x + (b*c^2*g^2*n - a*c*d*g^2*n)*B^2)*\log((d*x + c)^n) - \text{integrate}(-1/(B^2*c^2*g^2*n*\log(e) + A*B*c^2*g^2*n + (B^2*d^2*g^2*n*\log(e) + A*B*d^2*g^2*n)*x^2 + 2*(B^2*c*d*g^2*n*\log(e) + A*B*c*d*g^2*n)*x + (B^2*d^2*g^2*n*x^2 + 2*B^2*c*d*g^2*n*x + B^2*c^2*g^2*n)*\log((b*x + a)^n) - (B^2*d^2*g^2*n*x^2 + 2*B^2*c*d*g^2*n*x + B^2*c^2*g^2*n)*\log((d*x + c)^n)), x)$

**Fricas [A]** time = 0.756198, size = 635, normalized size = 4.12

$$\left( (Bbnx + Ban)e^{\frac{B \log(e) + A}{Bn}} \right) - \left( Adx + Ac + (Bdx + Bc) \log(e) + (Bdnx + Bcn) \log\left(\frac{bx+a}{dx+c}\right) \right) \log_{\text{int}}$$

$$\frac{(AB^2bcd - AB^2ad^2)g^2n^2x + (AB^2bc^2 - AB^2acd)g^2n^2 + ((B^3bcd - B^3ad^2)g^2n^2x + (B^3bc^2 - B^3acd)g^2n^2) \log(e) + ((B^3bcd - B^3ad^2)g^2n^2x + (B^3bc^2 - B^3acd)g^2n^2) \log\left(\frac{bx+a}{dx+c}\right)}{(AB^2bcd - AB^2ad^2)g^2n^2x + (AB^2bc^2 - AB^2acd)g^2n^2 + ((B^3bcd - B^3ad^2)g^2n^2x + (B^3bc^2 - B^3acd)g^2n^2) \log(e) + ((B^3bcd - B^3ad^2)g^2n^2x + (B^3bc^2 - B^3acd)g^2n^2) \log\left(\frac{bx+a}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*g\*x+c\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out]  $-\left( (B*b*n*x + B*a*n) * e^{\frac{B*\log(e) + A}{B*n}} - (A*d*x + A*c + (B*d*x + B*c) * \log(e) + (B*d*n*x + B*c*n) * \log\left(\frac{b*x + a}{d*x + c}\right)) * \log_{\text{integral}}\left(\frac{b*x + a}{d*x + c}\right) * e^{\frac{B*\log(e) + A}{B*n}} \right) / \left( (A*B^2*b*c*d - A*B^2*a*d^2) * g^2*n^2*x + (A*B^2*b*c^2 - A*B^2*a*c*d) * g^2*n^2 + ((B^3*b*c*d - B^3*a*d^2) * g^2*n^2*x + (B^3*b*c^2 - B^3*a*c*d) * g^2*n^2) * \log(e) + ((B^3*b*c*d - B^3*a*d^2) * g^2*n^3*x + (B^3*b*c^2 - B^3*a*c*d) * g^2*n^3) * \log\left(\frac{b*x + a}{d*x + c}\right) \right)$

$x + a)/(d*x + c))$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*g\*x+c\*g)\*\*2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 1.45868, size = 396, normalized size = 2.57

$$\frac{\operatorname{Ei}\left(\frac{A}{Bn} + \frac{1}{n} + \log\left(\frac{bx+a}{dx+c}\right)\right) e^{\left(-\frac{A}{Bn} - \frac{1}{n}\right)}}{B^2bcg^2n^2 - B^2adg^2n^2} - \frac{B^2bcdg^2n^2x \log\left(\frac{bx+a}{dx+c}\right) - B^2ad^2g^2n^2x \log\left(\frac{bx+a}{dx+c}\right) + B^2bc^2g^2n^2 \log\left(\frac{bx+a}{dx+c}\right) - B^2acd^2g^2n^2}{B^2bcg^2n^2 - B^2adg^2n^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*g\*x+c\*g)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] Ei(A/(B\*n) + 1/n + log((b\*x + a)/(d\*x + c)))\*e^(-A/(B\*n) - 1/n)/(B^2\*b\*c\*g^2\*n^2 - B^2\*a\*d\*g^2\*n^2) - (b\*x + a)/(B^2\*b\*c\*d\*g^2\*n^2\*x\*log((b\*x + a)/(d\*x + c)) - B^2\*a\*d^2\*g^2\*n^2\*x\*log((b\*x + a)/(d\*x + c)) + B^2\*b\*c^2\*g^2\*n^2\*log((b\*x + a)/(d\*x + c)) - B^2\*a\*c\*d\*g^2\*n^2\*log((b\*x + a)/(d\*x + c)) + A\*B\*b\*c\*d\*g^2\*n\*x + B^2\*b\*c\*d\*g^2\*n\*x - A\*B\*a\*d^2\*g^2\*n\*x - B^2\*a\*d^2\*g^2\*n\*x + A\*B\*b\*c^2\*g^2\*n + B^2\*b\*c^2\*g^2\*n - A\*B\*a\*c\*d\*g^2\*n - B^2\*a\*c\*d\*g^2\*n)

$$3.56 \quad \int \frac{1}{(cg+dgx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

**Optimal.** Leaf size=256

$$\frac{2d(a+bx)^2 e^{-\frac{2A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{-2/n} \operatorname{Ei} \left( \frac{2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{Bn} \right)}{B^2 g^3 n^2 (c+dx)^2 (bc-ad)^2} + \frac{b(a+bx) e^{-\frac{A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{-1/n} \operatorname{Ei} \left( \frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right)}{B^2 g^3 n^2 (c+dx)(bc-ad)^2} - \frac{1}{Bg^3 n^2}$$

[Out] (b\*(a + b\*x)\*ExpIntegralEi[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(B\*n)])/(B^2\*(b\*c - a\*d)^2\*E^(A/(B\*n))\*g^3\*n^2\*(e\*((a + b\*x)/(c + d\*x))^n)^(-1)\*(c + d\*x)) - (2\*d\*(a + b\*x)^2\*ExpIntegralEi[(2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(B\*n)])/(B^2\*(b\*c - a\*d)^2\*E^((2\*A)/(B\*n))\*g^3\*n^2\*(e\*((a + b\*x)/(c + d\*x))^n)^(2/n)\*(c + d\*x)^2) - (a + b\*x)/(B\*(b\*c - a\*d)\*g^3\*n\*(c + d\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))

**Rubi [F]** time = 0.0943265, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(cg + dgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((c\*g + d\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

[Out] Defer[Int][1/((c\*g + d\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

Rubi steps

$$\int \frac{1}{(cg + dgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx = \int \frac{1}{(cg + dgx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

**Mathematica [A]** time = 0.565389, size = 288, normalized size = 1.12

$$\frac{(a+bx) e^{-\frac{2A}{Bn}} \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{-2/n} \left( b e^{\frac{A}{Bn}} (c+dx) \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)^{\frac{1}{n}} \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) \operatorname{Ei} \left( \frac{A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{Bn} \right) - 2d(a+bx) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{B^2 g^3 n^2 (c+dx)^2 (bc-ad)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((c\*g + d\*g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

[Out] ((a + b\*x)\*(-(B\*(b\*c - a\*d)\*E^((2\*A)/(B\*n))\*n\*(e\*((a + b\*x)/(c + d\*x))^n)^(2/n)) + b\*E^(A/(B\*n))\*(e\*((a + b\*x)/(c + d\*x))^n)^(-1)\*(c + d\*x)\*ExpInteg

```

ralEi[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*ExpIntegralEi[(2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B^2*(b*c - a*d)^2*E^((2*A)/(B*n))*g^3*n^2*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))

```

**Maple [F]** time = 0.439, size = 0, normalized size = 0.

$$\int \frac{1}{(d*gx + c*g)^3} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(d*g*x+c*g)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*g*x+c*g)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")
```

```
[Out] -(b*x + a)/((b*c^3*g^3*n - a*c^2*d*g^3*n)*A*B + (b*c^3*g^3*n*log(e) - a*c^2*d*g^3*n*log(e))*B^2 + ((b*c*d^2*g^3*n - a*d^3*g^3*n)*A*B + (b*c*d^2*g^3*n*log(e) - a*d^3*g^3*n*log(e))*B^2)*x^2 + 2*((b*c^2*d*g^3*n - a*c*d^2*g^3*n)*A*B + (b*c^2*d*g^3*n*log(e) - a*c*d^2*g^3*n*log(e))*B^2)*x + ((b*c*d^2*g^3*n - a*d^3*g^3*n)*B^2*x^2 + 2*(b*c^2*d*g^3*n - a*c*d^2*g^3*n)*B^2*x + (b*c^3*g^3*n - a*c^2*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b*c*d^2*g^3*n - a*d^3*g^3*n)*B^2*x^2 + 2*(b*c^2*d*g^3*n - a*c*d^2*g^3*n)*B^2*x + (b*c^3*g^3*n - a*c^2*d*g^3*n)*B^2)*log((d*x + c)^n) - integrate((b*d*x - b*c + 2*a*d)/((b*c*d^3*g^3*n - a*d^4*g^3*n)*A*B + (b*c*d^3*g^3*n*log(e) - a*d^4*g^3*n*log(e))*B^2)*x^3 + (b*c^4*g^3*n - a*c^3*d*g^3*n)*A*B + (b*c^4*g^3*n*log(e) - a*c^3*d*g^3*n*log(e))*B^2 + 3*((b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)*A*B + (b*c^2*d^2*g^3*n*log(e) - a*c*d^3*g^3*n*log(e))*B^2)*x^2 + 3*((b*c^3*d*g^3*n - a*c^2*d^2*g^3*n)*A*B + (b*c^3*d*g^3*n*log(e) - a*c^2*d^2*g^3*n*log(e))*B^2)*x + ((b*c*d^3*g^3*n - a*d^4*g^3*n)*B^2*x^3 + 3*(b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)*B^2*x^2 + 3*(b*c^3*d*g^3*n - a*c^2*d^2*g^3*n)*B^2*x + (b*c^4*g^3*n - a*c^3*d*g^3*n)*B^2)*log((b*x + a)^n) - ((b*c*d^3*g^3*n - a*d^4*g^3*n)*B^2*x^3 + 3*(b*c^2*d^2*g^3*n - a*c*d^3*g^3*n)*B^2*x^2 + 3*(b*c^3*d*g^3*n - a*c^2*d^2*g^3*n)*B^2*x + (b*c^4*g^3*n - a*c^3*d*g^3*n)*B^2)*log((d*x + c)^n)), x)

```

**Fricas [B]** time = 0.897675, size = 1648, normalized size = 6.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*g\*x+c\*g)^3/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out] ((A\*b\*d^2\*x^2 + 2\*A\*b\*c\*d\*x + A\*b\*c^2 + (B\*b\*d^2\*x^2 + 2\*B\*b\*c\*d\*x + B\*b\*c^2)\*log(e) + (B\*b\*d^2\*n\*x^2 + 2\*B\*b\*c\*d\*n\*x + B\*b\*c^2\*n)\*log((b\*x + a)/(d\*x + c)))\*e^((B\*log(e) + A)/(B\*n))\*log\_integral((b\*x + a)\*e^((B\*log(e) + A)/(B\*n))/(d\*x + c)) - ((B\*b^2\*c - B\*a\*b\*d)\*n\*x + (B\*a\*b\*c - B\*a^2\*d)\*n)\*e^(2\*(B\*log(e) + A)/(B\*n)) - 2\*(A\*d^3\*x^2 + 2\*A\*c\*d^2\*x + A\*c^2\*d + (B\*d^3\*x^2 + 2\*B\*c\*d^2\*x + B\*c^2\*d)\*log(e) + (B\*d^3\*n\*x^2 + 2\*B\*c\*d^2\*n\*x + B\*c^2\*d\*n)\*log((b\*x + a)/(d\*x + c)))\*log\_integral((b^2\*x^2 + 2\*a\*b\*x + a^2)\*e^(2\*(B\*log(e) + A)/(B\*n))/(d^2\*x^2 + 2\*c\*d\*x + c^2))\*e^(-2\*(B\*log(e) + A)/(B\*n))/((A\*B^2\*b^2\*c^2\*d^2 - 2\*A\*B^2\*a\*b\*c\*d^3 + A\*B^2\*a^2\*d^4)\*g^3\*n^2\*x^2 + 2\*(A\*B^2\*b^2\*c^3\*d - 2\*A\*B^2\*a\*b\*c^2\*d^2 + A\*B^2\*a^2\*c\*d^3)\*g^3\*n^2\*x + (A\*B^2\*b^2\*c^4 - 2\*A\*B^2\*a\*b\*c^3\*d + A\*B^2\*a^2\*c^2\*d^2)\*g^3\*n^2 + ((B^3\*b^2\*c^2\*d^2 - 2\*B^3\*a\*b\*c\*d^3 + B^3\*a^2\*d^4)\*g^3\*n^2\*x^2 + 2\*(B^3\*b^2\*c^3\*d - 2\*B^3\*a\*b\*c^2\*d^2 + B^3\*a^2\*c\*d^3)\*g^3\*n^2\*x + (B^3\*b^2\*c^4 - 2\*B^3\*a\*b\*c^3\*d + B^3\*a^2\*c^2\*d^2)\*g^3\*n^2)\*log(e) + ((B^3\*b^2\*c^2\*d^2 - 2\*B^3\*a\*b\*c\*d^3 + B^3\*a^2\*d^4)\*g^3\*n^3\*x^2 + 2\*(B^3\*b^2\*c^3\*d - 2\*B^3\*a\*b\*c^2\*d^2 + B^3\*a^2\*c\*d^3)\*g^3\*n^3\*x + (B^3\*b^2\*c^4 - 2\*B^3\*a\*b\*c^3\*d + B^3\*a^2\*c^2\*d^2)\*g^3\*n^3)\*log((b\*x + a)/(d\*x + c)))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*g\*x+c\*g)\*\*3/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d g x + c g)^3 \left( B \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d\*g\*x+c\*g)^3/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate(1/((d\*g\*x + c\*g)^3\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2), x)

### 3.57 $\int (f + gx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$

**Optimal.** Leaf size=364

$$\frac{Bg^2nx^2(bc - ad)(a^2d^2g^2 - abdg(5df - cg) + b^2(c^2g^2 - 5cdfg + 10d^2f^2))}{10b^3d^3} + \frac{Bgnx(bc - ad)(-a^2bd^2g^2(5df - cg) + a^3d^4g^2)}{10b^3d^3}$$

[Out] (B\*(b\*c - a\*d)\*g\*(a^3\*d^3\*g^3 - a^2\*b\*d^2\*g^2\*(5\*d\*f - c\*g) + a\*b^2\*d\*g\*(10\*d^2\*f^2 - 5\*c\*d\*f\*g + c^2\*g^2) - b^3\*(10\*d^3\*f^3 - 10\*c\*d^2\*f^2\*g + 5\*c^2\*d\*f\*g^2 - c^3\*g^3))\*n\*x)/(5\*b^4\*d^4) - (B\*(b\*c - a\*d)\*g^2\*(a^2\*d^2\*g^2 - a\*b\*d\*g\*(5\*d\*f - c\*g) + b^2\*(10\*d^2\*f^2 - 5\*c\*d\*f\*g + c^2\*g^2))\*n\*x^2)/(10\*b^3\*d^3) - (B\*(b\*c - a\*d)\*g^3\*(5\*b\*d\*f - b\*c\*g - a\*d\*g)\*n\*x^3)/(15\*b^2\*d^2) - (B\*(b\*c - a\*d)\*g^4\*n\*x^4)/(20\*b\*d) - (B\*(b\*f - a\*g)^5\*n\*Log[a + b\*x])/(5\*b^5\*g) + ((f + g\*x)^5\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(5\*g) + (B\*(d\*f - c\*g)^5\*n\*Log[c + d\*x])/(5\*d^5\*g)

**Rubi [A]** time = 0.602933, antiderivative size = 348, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 72}

$$\frac{Bg^2nx^2(bc - ad)(a^2d^2g^2 - abdg(5df - cg) + b^2(c^2g^2 - 5cdfg + 10d^2f^2))}{10b^3d^3} + \frac{Bgnx(-10a^2b^2d^4f^2g + 5a^3bd^4fg^2 - a^4d^4g^2)}{10b^3d^3}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] (B\*g\*(10\*a\*b^3\*d^4\*f^3 - 10\*a^2\*b^2\*d^4\*f^2\*g + 5\*a^3\*b\*d^4\*f\*g^2 - a^4\*d^4\*g^3 - b^4\*c\*(10\*d^3\*f^3 - 10\*c\*d^2\*f^2\*g + 5\*c^2\*d\*f\*g^2 - c^3\*g^3))\*n\*x)/(5\*b^4\*d^4) - (B\*(b\*c - a\*d)\*g^2\*(a^2\*d^2\*g^2 - a\*b\*d\*g\*(5\*d\*f - c\*g) + b^2\*(10\*d^2\*f^2 - 5\*c\*d\*f\*g + c^2\*g^2))\*n\*x^2)/(10\*b^3\*d^3) - (B\*(b\*c - a\*d)\*g^3\*(5\*b\*d\*f - b\*c\*g - a\*d\*g)\*n\*x^3)/(15\*b^2\*d^2) - (B\*(b\*c - a\*d)\*g^4\*n\*x^4)/(20\*b\*d) - (B\*(b\*f - a\*g)^5\*n\*Log[a + b\*x])/(5\*b^5\*g) + ((f + g\*x)^5\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(5\*g) + (B\*(d\*f - c\*g)^5\*n\*Log[c + d\*x])/(5\*d^5\*g)

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

;/ FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

### Rubi steps

$$\begin{aligned} \int (f + gx)^4 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx &= \frac{(f + gx)^5 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{5g} - \frac{(Bn) \int \frac{(bc-ad)(f+gx)^5}{(a+bx)(c+dx)} dx}{5g} \\ &= \frac{(f + gx)^5 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{5g} - \frac{(B(bc - ad)n) \int \frac{(f+gx)^5}{(a+bx)(c+dx)} dx}{5g} \\ &= \frac{(f + gx)^5 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{5g} - \frac{(B(bc - ad)n) \int \left( \frac{g^2(-a^3d^3g^3 + a^2bd^2g^2(5d^2f^2 - 5cdfg + c^2g^2))}{(a+bx)(c+dx)} \right) dx}{5g} \\ &= \frac{B(bc - ad)g \left( a^3d^3g^3 - a^2bd^2g^2(5df - cg) + ab^2dg(10d^2f^2 - 5cdfg + c^2g^2) \right)}{5b^4d^4} \end{aligned}$$

**Mathematica [A]** time = 0.649985, size = 285, normalized size = 0.78

$$\frac{Bg^2nx(ad-bc)(6a^2bd^2g^2(-2cg+10df+dgx)-12a^3d^3g^3-2ab^2dg(6c^2g^2-3cdg(10f+gx)+d^2(60f^2+15fgx+2g^2x^2)))+b^3(6c^2dg^2(10f+gx)-12c^3g^3-2cd^2g(60f^2+15fgx+2g^2x^2))}{12b^4d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]),x]

[Out] ((B\*(-(b\*c) + a\*d)\*g^2\*n\*x\*(-12\*a^3\*d^3\*g^3 + 6\*a^2\*b\*d^2\*g^2\*(10\*d\*f - 2\*c\*g + d\*g\*x) - 2\*a\*b^2\*d\*g\*(6\*c^2\*g^2 - 3\*c\*d\*g\*(10\*f + g\*x) + d^2\*(60\*f^2 + 15\*f\*g\*x + 2\*g^2\*x^2)) + b^3\*(-12\*c^3\*g^3 + 6\*c^2\*d\*g^2\*(10\*f + g\*x) - 2\*c\*d^2\*g\*(60\*f^2 + 15\*f\*g\*x + 2\*g^2\*x^2) + d^3\*(120\*f^3 + 60\*f^2\*g\*x + 20\*f\*g^2\*x^2 + 3\*g^3\*x^3)))/(12\*b^4\*d^4) - (B\*(b\*f - a\*g)^5\*n\*Log[a + b\*x])/b^5 + (f + g\*x)^5\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + (B\*(d\*f - c\*g)^5\*n\*Log[c + d\*x])/d^5)/(5\*g)

**Maple [F]** time = 0.438, size = 0, normalized size = 0.

$$\int (gx + f)^4 \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^4\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int((g\*x+f)^4\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

**Maxima [A]** time = 1.26038, size = 852, normalized size = 2.34

$$\frac{1}{5} Bg^4x^5 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + \frac{1}{5} Ag^4x^5 + Bfg^3x^4 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + Afg^3x^4 + 2Bf^2g^2x^3 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^4\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out]  $\frac{1}{5}B*g^4*x^5*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + \frac{1}{5}A*g^4*x^5 + B*f*g^3*x^4*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g^3*x^4 + 2*B*f^2*g^2*x^3*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*f^2*g^2*x^3 + 2*B*f^3*g*x^2*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*f^3*g*x^2 + \frac{1}{60}B*g^4*n*(12*a^5*\log(b*x + a)/b^5 - 12*c^5*\log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4) - \frac{1}{6}B*f*g^3*n*(6*a^4*\log(b*x + a)/b^4 - 6*c^4*\log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3) + B*f^2*g^2*n*(2*a^3*\log(b*x + a)/b^3 - 2*c^3*\log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2) - 2*B*f^3*g*n*(a^2*\log(b*x + a)/b^2 - c^2*\log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^4*n*(a*\log(b*x + a)/b - c*\log(d*x + c)/d) + B*f^4*x*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f^4*x$

---

**Fricas [B]** time = 3.13323, size = 1488, normalized size = 4.09

$12 Ab^5 d^5 g^4 x^5 + 3 (20 Ab^5 d^5 f g^3 - (Bb^5 cd^4 - Bab^4 d^5) g^4 n) x^4 + 4 (30 Ab^5 d^5 f^2 g^2 - (5 (Bb^5 cd^4 - Bab^4 d^5) f g^3 - (Bb^5 c^2 d^3 -$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^4\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out]  $\frac{1}{60}*(12*A*b^5*d^5*g^4*x^5 + 3*(20*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^4*d^5)*d^5)*g^4*n*x^4 + 4*(30*A*b^5*d^5*f^2*g^2 - (5*(B*b^5*c*d^4 - B*a*b^4*d^5)*f*g^3 - (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*n)*x^3 + 6*(20*A*b^5*d^5*f^3*g - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 - 5*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f*g^3 + (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*n)*x^2 + 12*(5*B*a*b^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*b^2*d^5*f^2*g^2 - 5*B*a^4*b*d^5*f*g^3 + B*a^5*d^5*g^4)*n*\log(b*x + a) - 12*(5*B*b^5*c*d^4*f^4 - 10*B*b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*B*b^5*c^4*d*f*g^3 + B*b^5*c^5*g^4)*n*\log(d*x + c) + 12*(5*A*b^5*d^5*f^4 - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^3*g - 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f^2*g^2 + 5*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*f*g^3 - (B*b^5*c^4*d - B*a^4*b*d^5)*g^4)*n)*x + 12*(B*b^5*d^5*g^4*x^5 + 5*B*b^5*d^5*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*g*x^2 + 5*B*b^5*d^5*f^4*x)*\log(e) + 12*(B*b^5*d^5*g^4*n*x^5 + 5*B*b^5*d^5*f*g^3*n*x^4 + 10*B*b^5*d^5*f^2*g^2*n*x^3 + 10*B*b^5*d^5*f^3*g*n*x^2 + 5*B*b^5*d^5*f^4*n*x)*\log((b*x + a)/(d*x + c))/(b^5*d^5)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*4\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n)),x)

[Out] Timed out



---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.58 $\int (f + gx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$

**Optimal.** Leaf size=235

$$\frac{Bgnx(bc - ad)(a^2d^2g^2 - abdg(4df - cg) + b^2(c^2g^2 - 4cdfg + 6d^2f^2))}{4b^3d^3} + \frac{(f + gx)^4 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{4g} - \frac{Bg^2nx^2}{4g}$$

[Out]  $-(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n*x)/(4*b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*n*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*g^3*n*x^3)/(12*b*d) - (B*(b*f - a*g)^4*n*Log[a + b*x])/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*g) + (B*(d*f - c*g)^4*n*Log[c + d*x])/(4*d^4*g)$

**Rubi [A]** time = 0.359478, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 72}

$$\frac{Bgnx(bc - ad)(a^2d^2g^2 - abdg(4df - cg) + b^2(c^2g^2 - 4cdfg + 6d^2f^2))}{4b^3d^3} + \frac{(f + gx)^4 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{4g} - \frac{Bg^2nx^2}{4g}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]$

[Out]  $-(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n*x)/(4*b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*n*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*g^3*n*x^3)/(12*b*d) - (B*(b*f - a*g)^4*n*Log[a + b*x])/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*g) + (B*(d*f - c*g)^4*n*Log[c + d*x])/(4*d^4*g)$

#### Rule 2525

$\text{Int}[(a + \text{Log}[c \cdot \text{RFX}]^p) \cdot (b + (d + e \cdot x)^m)^n], x\_Symbol] \rightarrow \text{Simp}[(d + e \cdot x)^{m+1} \cdot (a + b \cdot \text{Log}[c \cdot \text{RFX}]^p)^n / (e^{m+1}), x] - \text{Dist}[(b \cdot n \cdot p) / (e^{m+1}), \text{Int}[\text{SimplifyIntegrand}[(d + e \cdot x)^{m+1} \cdot (a + b \cdot \text{Log}[c \cdot \text{RFX}]^p)^{n-1} \cdot D[\text{RFX}, x]) / \text{RFX}, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[a \cdot (u), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b) \cdot (v) /; FreeQ[b, x]]

#### Rule 72

$\text{Int}[(e + f \cdot x)^p / ((a + b \cdot x) \cdot (c + d \cdot x))], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(e + f \cdot x)^p / ((a + b \cdot x) \cdot (c + d \cdot x)), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

#### Rubi steps

$$\begin{aligned}
\int (f + gx)^3 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right) dx &= \frac{(f + gx)^4 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{4g} - \frac{(Bn) \int \frac{(bc - ad)(f + gx)^4}{(a + bx)(c + dx)} dx}{4g} \\
&= \frac{(f + gx)^4 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{4g} - \frac{(B(bc - ad)n) \int \frac{(f + gx)^4}{(a + bx)(c + dx)} dx}{4g} \\
&= \frac{(f + gx)^4 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{4g} - \frac{(B(bc - ad)n) \int \left( \frac{g^2(a^2 d^2 g^2 - abdg(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))}{(a + bx)(c + dx)} \right) dx}{4g} \\
&= - \frac{B(bc - ad)g(a^2 d^2 g^2 - abdg(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))}{4b^3 d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.28813, size = 219, normalized size = 0.93

$$\frac{(f + gx)^4 \left( B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) + A \right) - \frac{Bn(6bdg^2x(bc - ad)(a^2 d^2 g^2 + abdg(cg - 4df) + b^2(c^2 g^2 - 4cdfg + 6d^2 f^2)) + 3b^2 d^2 g^3 x^2 (bc - ad)(-adg - bcg + 4bdf) + 2b^3 d^3 (b^2 c - a^2 d)g^4 x^3 + 6d^4 (b^2 f - a^2 g)^4 \log[a + bx] - 6b^4 (d^2 f - c^2 g)^4 \log[c + dx])}{6b^4 d^4}}{4g}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] ((f + g\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - (B\*n\*(6\*b\*d\*(b\*c - a\*d)\*g^2\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(-4\*d\*f + c\*g) + b^2\*(6\*d^2\*f^2 - 4\*c\*d\*f\*g + c^2\*g^2))\*x + 3\*b^2\*d^2\*(b\*c - a\*d)\*g^3\*(4\*b\*d\*f - b\*c\*g - a\*d\*g)\*x^2 + 2\*b^3\*d^3\*(b\*c - a\*d)\*g^4\*x^3 + 6\*d^4\*(b\*f - a\*g)^4\*Log[a + b\*x] - 6\*b^4\*(d\*f - c\*g)^4\*Log[c + d\*x]))/(6\*b^4\*d^4)/(4\*g)

**Maple [F]** time = 0.403, size = 0, normalized size = 0.

$$\int (gx + f)^3 \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

[Out] int((g\*x+f)^3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

**Maxima [A]** time = 1.20435, size = 598, normalized size = 2.54

$$\frac{1}{4} B g^3 x^4 \log \left( e \left( \frac{bx}{dx + c} + \frac{a}{dx + c} \right)^n \right) + \frac{1}{4} A g^3 x^4 + B f g^2 x^3 \log \left( e \left( \frac{bx}{dx + c} + \frac{a}{dx + c} \right)^n \right) + A f g^2 x^3 + \frac{3}{2} B f^2 g x^2 \log \left( e \left( \frac{bx}{dx + c} + \frac{a}{dx + c} \right)^n \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)), x, algorithm="maxima")

```
[Out] 1/4*B*g^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*g^3*x^4 + B*f*
g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*f*g^2*x^3 + 3/2*B*f^2*g*
x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*f^2*g*x^2 - 1/24*B*g^3*n
*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d
^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3
)) + 1/2*B*f*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3/2*B*f^2*g*n*(
a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f^3*
n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*f^3*x*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + A*f^3*x
```

**Fricas [B]** time = 1.50081, size = 1053, normalized size = 4.48

$$6Ab^4d^4g^3x^4 + 2(12Ab^4d^4fg^2 - (Bb^4cd^3 - Bab^3d^4)g^3n)x^3 + 3(12Ab^4d^4f^2g - (4(Bb^4cd^3 - Bab^3d^4)fg^2 - (Bb^4c^2d^2 - B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 2*(12*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3*
d^4)*g^3*n)*x^3 + 3*(12*A*b^4*d^4*f^2*g - (4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*
g^2 - (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*n)*x^2 + 6*(4*B*a*b^3*d^4*f^3 -
6*B*a^2*b^2*d^4*f^2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*n*log(b*x + a)
- 6*(4*B*b^4*c*d^3*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b
^4*c^4*g^3)*n*log(d*x + c) + 6*(4*A*b^4*d^4*f^3 - (6*(B*b^4*c*d^3 - B*a*b^3
*d^4)*f^2*g - 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 + (B*b^4*c^3*d - B*a^
3*b*d^4)*g^3)*n)*x + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4
*d^4*f^2*g*x^2 + 4*B*b^4*d^4*f^3*x)*log(e) + 6*(B*b^4*d^4*g^3*n*x^4 + 4*B*b
^4*d^4*f*g^2*n*x^3 + 6*B*b^4*d^4*f^2*g*n*x^2 + 4*B*b^4*d^4*f^3*n*x)*log((b*
x + a)/(d*x + c)))/(b^4*d^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.59 \quad \int (f + gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$$

**Optimal.** Leaf size=157

$$\frac{(f + gx)^3 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{3g} - \frac{Bgnx(bc - ad)(-adg - bcg + 3bdf)}{3b^2d^2} - \frac{Bn(bf - ag)^3 \log(a + bx)}{3b^3g} - \frac{Bg^2nx^2(bc - a)}{6bd}$$

[Out]  $-(B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*n*x)/(3*b^2*d^2) - (B*(b*c - a*d)*g^2*n*x^2)/(6*b*d) - (B*(b*f - a*g)^3*n*Log[a + b*x])/(3*b^3*g) + ((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*g) + (B*(d*f - c*g)^3*n*Log[c + d*x])/(3*d^3*g)$

**Rubi [A]** time = 0.180485, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 72}

$$\frac{(f + gx)^3 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{3g} - \frac{Bgnx(bc - ad)(-adg - bcg + 3bdf)}{3b^2d^2} - \frac{Bn(bf - ag)^3 \log(a + bx)}{3b^3g} - \frac{Bg^2nx^2(bc - a)}{6bd}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]),x]

[Out]  $-(B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*n*x)/(3*b^2*d^2) - (B*(b*c - a*d)*g^2*n*x^2)/(6*b*d) - (B*(b*f - a*g)^3*n*Log[a + b*x])/(3*b^3*g) + ((f + g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*g) + (B*(d*f - c*g)^3*n*Log[c + d*x])/(3*d^3*g)$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*Rfx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 72

Int[((e\_.) + (f\_.)\*(x\_))^(p\_.)/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

### Rubi steps

$$\begin{aligned}
\int (f + gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx &= \frac{(f + gx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{3g} - \frac{(Bn) \int \frac{(bc-ad)(f+gx)^3}{(a+bx)(c+dx)} dx}{3g} \\
&= \frac{(f + gx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{3g} - \frac{(B(bc - ad)n) \int \frac{(f+gx)^3}{(a+bx)(c+dx)} dx}{3g} \\
&= \frac{(f + gx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{3g} - \frac{(B(bc - ad)n) \int \left( \frac{g^2(3bdf - bcg - adg)}{b^2d^2} + \frac{g^3x}{bd} \right) dx}{3g} \\
&= -\frac{B(bc - ad)g(3bdf - bcg - adg)nx}{3b^2d^2} - \frac{B(bc - ad)g^2nx^2}{6bd} - \frac{B(bf - ag)^3n \log(c+dx)}{3b^3g}
\end{aligned}$$

**Mathematica [A]** time = 0.143738, size = 146, normalized size = 0.93

$$\frac{(f + gx)^3 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right) - \frac{Bn(b^2d^2g^3x^2(bc-ad) + 2bdg^2x(bc-ad)(-adg - bcg + 3bdf) + 2d^3(bf-ag)^3 \log(a+bx) - 2b^3(df-cg)^3 \log(c+dx))}{2b^3d^3}}{3g}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] ((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - (B\*n\*(2\*b\*d\*(b\*c - a\*d)\*g^2\*(3\*b\*d\*f - b\*c\*g - a\*d\*g)\*x + b^2\*d^2\*(b\*c - a\*d)\*g^3\*x^2 + 2\*d^3\*(b\*f - a\*g)^3\*Log[a + b\*x] - 2\*b^3\*(d\*f - c\*g)^3\*Log[c + d\*x]))/(2\*b^3\*d^3)/(3\*g)

**Maple [F]** time = 0.41, size = 0, normalized size = 0.

$$\int (gx + f)^2 \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

[Out] int((g\*x+f)^2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)), x)

**Maxima [A]** time = 1.18207, size = 381, normalized size = 2.43

$$\frac{1}{3} Bg^2x^3 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + \frac{1}{3} Ag^2x^3 + Bfgx^2 \log \left( e \left( \frac{bx}{dx+c} + \frac{a}{dx+c} \right)^n \right) + Afgx^2 + \frac{1}{6} Bg^2n \left( \frac{2a^3 \log(bx+a)}{b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)), x, algorithm="maxima")

[Out] 1/3\*B\*g^2\*x^3\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + 1/3\*A\*g^2\*x^3 + B\*f\*g\*x^2\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n) + A\*f\*g\*x^2 + 1/6\*B\*g^2\*n\*(2\*a

$$\begin{aligned} &^3 \log(bx + a)/b^3 - 2c^3 \log(dx + c)/d^3 - ((b^2cd - a^2d^2)x^2 - 2 \\ &*(b^2c^2 - a^2d^2)x)/(b^2d^2) - Bf^2g^2n(a^2 \log(bx + a)/b^2 - c^2 \log \\ &g(dx + c)/d^2 + (bc - ad)x/(bd)) + Bf^2n(a \log(bx + a)/b - c \log(d \\ & * x + c)/d) + Bf^2x \log(e*(bx/(dx + c) + a/(dx + c))^n) + Af^2x \end{aligned}$$

**Fricas [B]** time = 1.20537, size = 695, normalized size = 4.43

$$2Ab^3d^3g^2x^3 + (6Ab^3d^3fg - (Bb^3cd^2 - Bab^2d^3)g^2n)x^2 + 2(3Bab^2d^3f^2 - 3Ba^2bd^3fg + Ba^3d^3g^2)n \log(bx + a) - 2(3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out]  $\frac{1}{6}(2Ab^3d^3g^2x^3 + (6Ab^3d^3fg - (Bb^3cd^2 - Bab^2d^3)g^2n)x^2 + 2(3Bab^2d^3f^2 - 3Ba^2bd^3fg + Ba^3d^3g^2)n \log(bx + a) - 2(3Bb^3cd^2 - 3Bab^2d^3)g^2n \log(dx + c) + 2(3Aab^3d^3f^2 - (3(Bb^3cd^2 - Ba^2bd^3)fg - (Bb^3c^2d - Ba^2bd^3)g^2)n)x + 2(Bb^3d^3g^2x^3 + 3Bb^3d^3fg^2x^2 + 3Bb^3d^3f^2x) \log(e) + 2(Bb^3d^3g^2nx^3 + 3Bb^3d^3fg^2nx^2 + 3Bb^3d^3f^2nx) \log((bx + a)/(dx + c)))/b^3d^3)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n)),x)

[Out] Timed out

**Giac [A]** time = 81.8263, size = 366, normalized size = 2.33

$$\frac{1}{3}(Ag^2 + Bg^2)x^3 + \frac{1}{3}(Bg^2nx^3 + 3Bfgnx^2 + 3Bf^2nx) \log\left(\frac{bx + a}{dx + c}\right) - \frac{(Bbcg^2n - Badg^2n - 6Abdfg - 6Bbdfg)x^2}{6bd} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out]  $\frac{1}{3}(Ag^2 + Bg^2)x^3 + \frac{1}{3}(Bg^2nx^3 + 3Bfgnx^2 + 3Bf^2nx) \log\left(\frac{bx + a}{dx + c}\right) - \frac{1}{6}(Bb^3cd^2g^2n - Ba^2d^2g^2n - 6Aab^2d^2fg - 6Bb^2d^2fg)x^2/(bd) + \frac{1}{3}(3Baa^2b^2f^2n - 3Baa^2b^2fg^2n + Ba^3g^2n) \log(bx + a)/b^3 - \frac{1}{3}(3Bc^2d^2f^2n - 3Bc^2d^2fg^2n + Bc^3g^2n) \log(-dx - c)/d^3 - \frac{1}{3}(3Bb^2cd^2fg^2n - 3Baa^2bd^2fg^2n - Bb^2c^2g^2n + Ba^2d^2g^2n - 3Aab^2d^2f^2 - 3Bb^2d^2f^2)x/(b^2d^2)$

### 3.60 $\int (f + gx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$

**Optimal.** Leaf size=115

$$\frac{(f + gx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{2g} - \frac{Bn(bf - ag)^2 \log(a + bx)}{2b^2g} - \frac{Bgnx(bc - ad)}{2bd} + \frac{Bn(df - cg)^2 \log(c + dx)}{2d^2g}$$

[Out]  $-(B*(b*c - a*d)*g*n*x)/(2*b*d) - (B*(b*f - a*g)^2*n*Log[a + b*x])/(2*b^2*g) + ((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*g) + (B*(d*f - c*g)^2*n*Log[c + d*x])/(2*d^2*g)$

**Rubi [A]** time = 0.104979, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$ , Rules used = {2525, 12, 72}

$$\frac{(f + gx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}{2g} - \frac{Bn(bf - ag)^2 \log(a + bx)}{2b^2g} - \frac{Bgnx(bc - ad)}{2bd} + \frac{Bn(df - cg)^2 \log(c + dx)}{2d^2g}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]), x]$

[Out]  $-(B*(b*c - a*d)*g*n*x)/(2*b*d) - (B*(b*f - a*g)^2*n*Log[a + b*x])/(2*b^2*g) + ((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*g) + (B*(d*f - c*g)^2*n*Log[c + d*x])/(2*d^2*g)$

#### Rule 2525

$\text{Int}[(a + \text{Log}[c * (\text{RFx})^p]) * (b + (d + (e * x))^m)], x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1} * (a + b * \text{Log}[c * \text{RFx}^p])^n / (e^{m+1}), x] - \text{Dist}[(b * n * p) / (e^{m+1}), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1} * (a + b * \text{Log}[c * \text{RFx}^p])^{n-1} * D[\text{RFx}, x]) / \text{RFx}, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[(a * u), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b \* v) /; FreeQ[b, x]]

#### Rule 72

$\text{Int}[(e + (f * x))^p / ((a + (b * x)) * (c + (d * x)))], x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(e + f*x)^p / ((a + b*x) * (c + d*x)), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

#### Rubi steps



$$\begin{aligned}
\int (f + gx) \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right) dx &= \frac{(f + gx)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{2g} - \frac{(Bn) \int \frac{(bc - ad)(f + gx)^2}{(a + bx)(c + dx)} dx}{2g} \\
&= \frac{(f + gx)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{2g} - \frac{(B(bc - ad)n) \int \frac{(f + gx)^2}{(a + bx)(c + dx)} dx}{2g} \\
&= \frac{(f + gx)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{2g} - \frac{(B(bc - ad)n) \int \left( \frac{g^2}{bd} + \frac{(bf - ag)^2}{b(bc - ad)(a + bx)} \right) dx}{2g} \\
&= -\frac{B(bc - ad)gnx}{2bd} - \frac{B(bf - ag)^2 n \log(a + bx)}{2b^2g} + \frac{(f + gx)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{2g}
\end{aligned}$$

**Mathematica [A]** time = 0.128372, size = 120, normalized size = 1.04

$$\frac{b \left( d \left( Bg^2nx(ad - bc) + Abd(f + gx)^2 \right) + bBd^2(f + gx)^2 \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) + bBn(df - cg)^2 \log(c + dx) \right) - Bd^2n(bf - ag)}{2b^2d^2g}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]),x]

[Out]  $(-(B*d^2*(b*f - a*g)^{2*n}*Log[a + b*x]) + b*(d*(B*(-(b*c) + a*d)*g^{2*n}*x + A*b*d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + b*B*(d*f - c*g)^{2*n}*Log[c + d*x]))/(2*b^2*d^2*g)$

**Maple [F]** time = 0.32, size = 0, normalized size = 0.

$$\int (gx + f) \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int((g\*x+f)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

**Maxima [A]** time = 1.09531, size = 203, normalized size = 1.77

$$\frac{1}{2} Bgx^2 \log \left( e \left( \frac{bx}{dx + c} + \frac{a}{dx + c} \right)^n \right) + \frac{1}{2} Agx^2 - \frac{1}{2} Bgn \left( \frac{a^2 \log(bx + a)}{b^2} - \frac{c^2 \log(dx + c)}{d^2} + \frac{(bc - ad)x}{bd} \right) + Bfn \left( \frac{a \log(bx + a)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out]  $1/2*B*g*x^2*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*g*x^2 - 1/2*B*g*n*(a^2*\log(b*x + a)/b^2 - c^2*\log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*f*n*(a*\log(b*x + a)/b - c*\log(d*x + c)/d) + B*f*x*\log(e*(b*x/(d*x + c) + a/$

$d*x + c))^n) + A*f*x$

**Fricas [A]** time = 0.925426, size = 390, normalized size = 3.39

$$\frac{Ab^2d^2gx^2 + (2Babd^2f - Ba^2d^2g)n \log(bx + a) - (2Bb^2cdf - Bb^2c^2g)n \log(dx + c) + (2Ab^2d^2f - (Bb^2cd - Babd^2)gn)}{2b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] 1/2\*(A\*b^2\*d^2\*g\*x^2 + (2\*B\*a\*b\*d^2\*f - B\*a^2\*d^2\*g)\*n\*log(b\*x + a) - (2\*B\*b^2\*c\*d\*f - B\*b^2\*c^2\*g)\*n\*log(d\*x + c) + (2\*A\*b^2\*d^2\*f - (B\*b^2\*c\*d - B\*a\*b\*d^2)\*g\*n)\*x + (B\*b^2\*d^2\*g\*x^2 + 2\*B\*b^2\*d^2\*f\*x)\*log(e) + (B\*b^2\*d^2\*g\*n\*x^2 + 2\*B\*b^2\*d^2\*f\*n\*x)\*log((b\*x + a)/(d\*x + c)))/(b^2\*d^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] Timed out

**Giac [A]** time = 5.30086, size = 182, normalized size = 1.58

$$\frac{1}{2}(Ag + Bg)x^2 + \frac{1}{2}(Bgnx^2 + 2Bfnx) \log\left(\frac{bx + a}{dx + c}\right) - \frac{(Bbcgn - Badgn - 2Abdf - 2Bddf)x}{2bd} + \frac{(2Babfn - Ba^2gn) \log\left(\frac{bx + a}{dx + c}\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out] 1/2\*(A\*g + B\*g)\*x^2 + 1/2\*(B\*g\*n\*x^2 + 2\*B\*f\*n\*x)\*log((b\*x + a)/(d\*x + c)) - 1/2\*(B\*b\*c\*g\*n - B\*a\*d\*g\*n - 2\*A\*b\*d\*f - 2\*B\*b\*d\*f)\*x/(b\*d) + 1/2\*(2\*B\*a\*b\*f\*n - B\*a^2\*g\*n)\*log(b\*x + a)/b^2 - 1/2\*(2\*B\*c\*d\*f\*n - B\*c^2\*g\*n)\*log(-d\*x - c)/d^2

$$3.61 \quad \int \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx$$

**Optimal.** Leaf size=56

$$\frac{B(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b} - \frac{Bn(bc-ad) \log(c+dx)}{bd} + Ax$$

[Out] A\*x + (B\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/b - (B\*(b\*c - a\*d)\*n\*Log[c + d\*x])/(b\*d)

**Rubi [A]** time = 0.0327508, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {2486, 31}

$$\frac{B(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b} - \frac{Bn(bc-ad) \log(c+dx)}{bd} + Ax$$

Antiderivative was successfully verified.

[In] Int[A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n], x]

[Out] A\*x + (B\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/b - (B\*(b\*c - a\*d)\*n\*Log[c + d\*x])/(b\*d)

#### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_ - 1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned} \int \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) dx &= Ax + B \int \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) dx \\ &= Ax + \frac{B(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b} - \frac{(B(bc-ad)n) \int \frac{1}{c+dx} dx}{b} \\ &= Ax + \frac{B(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b} - \frac{B(bc-ad)n \log(c+dx)}{bd} \end{aligned}$$

**Mathematica [A]** time = 0.0090986, size = 56, normalized size = 1.

$$\frac{B(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b} - \frac{Bn(bc-ad) \log(c+dx)}{bd} + Ax$$

Antiderivative was successfully verified.

[In] Integrate[A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n], x]

[Out] A\*x + (B\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/b - (B\*(b\*c - a\*d)\*n\*Log[c + d\*x])/(b\*d)

**Maple [B]** time = 0.049, size = 122, normalized size = 2.2

$$Ax + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) x + \frac{Bna^2 \ln(bx + a) d}{b(ad - bc)} - \frac{Bna \ln(bx + a) c}{ad - bc} - \frac{Bnc \ln(dx + c) a}{ad - bc} + \frac{Bnc^2 \ln(dx + c) b}{d(ad - bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n), x)

[Out] A\*x+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)\*x+B\*n\*a^2/(a\*d-b\*c)/b\*ln(b\*x+a)\*d-B\*n\*a/(a\*d-b\*c)\*ln(b\*x+a)\*c-B\*n\*c/(a\*d-b\*c)\*ln(d\*x+c)\*a+B\*n\*c^2/(a\*d-b\*c)/d\*ln(d\*x+c)\*b

**Maxima [A]** time = 1.17749, size = 70, normalized size = 1.25

$$Bn \left( \frac{a \log(bx + a)}{b} - \frac{c \log(dx + c)}{d} \right) + Bx \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) + Ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n), x, algorithm="maxima")

[Out] B\*n\*(a\*log(b\*x + a)/b - c\*log(d\*x + c)/d) + B\*x\*log(e\*((b\*x + a)/(d\*x + c))^n) + A\*x

**Fricas [A]** time = 0.841066, size = 158, normalized size = 2.82

$$\frac{Bbdnx \log \left( \frac{bx+a}{dx+c} \right) + Badn \log(bx + a) - Bbcn \log(dx + c) + Bbdx \log(e) + Abdx}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n), x, algorithm="fricas")

[Out] (B\*b\*d\*n\*x\*log((b\*x + a)/(d\*x + c)) + B\*a\*d\*n\*log(b\*x + a) - B\*b\*c\*n\*log(d\*x + c) + B\*b\*d\*x\*log(e) + A\*b\*d\*x)/(b\*d)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*ln(e*((b*x+a)/(d*x+c))**n),x)
```

```
[Out] Timed out
```

**Giac [A]** time = 1.39047, size = 72, normalized size = 1.29

$$\left( nx \log\left(\frac{bx+a}{dx+c}\right) + \frac{an \log(bx+a)}{b} - \frac{cn \log(-dx-c)}{d} + x \right) B + Ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(A+B*log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
```

```
[Out] (n*x*log((b*x + a)/(d*x + c)) + a*n*log(b*x + a)/b - c*n*log(-d*x - c)/d + x)*B + A*x
```

$$3.62 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx} dx$$

**Optimal.** Leaf size=147

$$\frac{Bn \operatorname{PolyLog}\left(2, \frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{Bn \operatorname{PolyLog}\left(2, \frac{d(f+gx)}{df-cg}\right)}{g} + \frac{\log(f+gx) \left( B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A \right)}{g} - \frac{Bn \log(f+gx) \log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g}$$

[Out]  $-\left(\frac{Bn \operatorname{Log}\left[-\left(\frac{g(a+bx)}{bf-ag}\right)\right] \operatorname{Log}[f+gx]}{g}\right) + \left(\frac{(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] \operatorname{Log}[f+gx]}{g} + \frac{Bn \operatorname{Log}\left[-\left(\frac{g(c+dx)}{df-cg}\right)\right] \operatorname{Log}[f+gx]}{g} - \frac{Bn \operatorname{PolyLog}\left[2, \frac{b(f+gx)}{bf-ag}\right]}{g} + \frac{Bn \operatorname{PolyLog}\left[2, \frac{d(f+gx)}{df-cg}\right]}{g}\right)}{g}$

**Rubi [A]** time = 0.225662, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$ , Rules used = {2524, 2418, 2394, 2393, 2391}

$$\frac{Bn \operatorname{PolyLog}\left(2, \frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{Bn \operatorname{PolyLog}\left(2, \frac{d(f+gx)}{df-cg}\right)}{g} + \frac{\log(f+gx) \left( B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A \right)}{g} - \frac{Bn \log(f+gx) \log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right]}{f+gx}, x\right]$

[Out]  $-\left(\frac{Bn \operatorname{Log}\left[-\left(\frac{g(a+bx)}{bf-ag}\right)\right] \operatorname{Log}[f+gx]}{g}\right) + \left(\frac{(A+B \operatorname{Log}\left[e\left(\frac{a+bx}{c+dx}\right)^n\right] \operatorname{Log}[f+gx]}{g} + \frac{Bn \operatorname{Log}\left[-\left(\frac{g(c+dx)}{df-cg}\right)\right] \operatorname{Log}[f+gx]}{g} - \frac{Bn \operatorname{PolyLog}\left[2, \frac{b(f+gx)}{bf-ag}\right]}{g} + \frac{Bn \operatorname{PolyLog}\left[2, \frac{d(f+gx)}{df-cg}\right]}{g}\right)}{g}$

#### Rule 2524

$\operatorname{Int}\left[\frac{(a_.) + \operatorname{Log}\left[(c_.) \operatorname{RFX}_\right]^{\left(p_.\right)} \left(b_.\right)^{\left(n_.\right)}}{\left(d_.\right) + \left(e_.\right) \left(x_.\right)}, x\_Symbol\right] \rightarrow \operatorname{Simp}\left[\frac{\operatorname{Log}\left[d + e x\right] \left(a + b \operatorname{Log}\left[c \operatorname{RFX}^p\right]\right)^n}{e}, x\right] - \operatorname{Dist}\left[\frac{b^n p}{e}, \operatorname{Int}\left[\frac{\operatorname{Log}\left[d + e x\right] \left(a + b \operatorname{Log}\left[c \operatorname{RFX}^p\right]\right)^{n-1} D\left[\operatorname{RFX}, x\right]}{\operatorname{RFX}}, x\right], x\right] /;$   
 $\operatorname{FreeQ}\left[\{a, b, c, d, e, p\}, x\right] \&\& \operatorname{RationalFunctionQ}\left[\operatorname{RFX}, x\right] \&\& \operatorname{IGtQ}\left[n, 0\right]$

#### Rule 2418

$\operatorname{Int}\left[\frac{(a_.) + \operatorname{Log}\left[(c_.) \left((d_.) + (e_.) \left(x_.\right)^{\left(n_.\right)}\right) \left(b_.\right)^{\left(p_.\right)} \operatorname{RFX}_\right]}{\left(d_.\right) + \left(e_.\right) \left(x_.\right)}, x\_Symbol\right] \rightarrow \operatorname{With}\left[\{u = \operatorname{ExpandIntegrand}\left[\left(a + b \operatorname{Log}\left[c \left(d + e x\right)^n\right]\right)^p, \operatorname{RFX}, x\right]\}, \operatorname{Int}\left[u, x\right] /; \operatorname{SumQ}\left[u\right] /; \operatorname{FreeQ}\left[\{a, b, c, d, e, n\}, x\right] \&\& \operatorname{RationalFunctionQ}\left[\operatorname{RFX}, x\right] \&\& \operatorname{IntegerQ}\left[p\right]\right]$

#### Rule 2394

$\operatorname{Int}\left[\frac{(a_.) + \operatorname{Log}\left[(c_.) \left((d_.) + (e_.) \left(x_.\right)^{\left(n_.\right)}\right) \left(b_.\right)\right]}{\left(f_.\right) + \left(g_.\right) \left(x_.\right)}, x\_Symbol\right] \rightarrow \operatorname{Simp}\left[\frac{\operatorname{Log}\left[\frac{e(f+gx)}{e f-d g}\right] \left(a + b \operatorname{Log}\left[c \left(d + e x\right)^n\right]\right)}{g}, x\right] - \operatorname{Dist}\left[\frac{b e^n}{g}, \operatorname{Int}\left[\frac{\operatorname{Log}\left[\frac{e(f+gx)}{e f-d g}\right]}{d + e x}, x\right], x\right] /; \operatorname{FreeQ}\left[\{a, b, c, d, e, f, g, n\}, x\right] \&\& \operatorname{NeQ}\left[e f-d g, 0\right]$

#### Rule 2393

$\operatorname{Int}\left[\frac{(a_.) + \operatorname{Log}\left[(c_.) \left((d_.) + (e_.) \left(x_.\right)\right) \left(b_.\right)\right]}{\left(f_.\right) + \left(g_.\right) \left(x_.\right)}, x\_Symbol\right] \rightarrow \operatorname{Dist}\left[\frac{1}{g}, \operatorname{Subst}\left[\operatorname{Int}\left[\frac{a + b \operatorname{Log}\left[1 + (c e x)\right]}{x}, x\right], x, f + g x\right]\right]$

], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{f+gx} dx &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(f+gx)}{g} - \frac{(Bn) \int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2} + \frac{b}{c+dx}\right) \log(f+gx)}{a+bx} dx}{g} \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(f+gx)}{g} - \frac{(Bn) \int \left(\frac{b \log(f+gx)}{a+bx} - \frac{d \log(f+gx)}{c+dx}\right) dx}{g} \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(f+gx)}{g} - \frac{(bBn) \int \frac{\log(f+gx)}{a+bx} dx}{g} + \frac{(Bdn) \int \frac{\log(f+gx)}{c+dx} dx}{g} \\
 &= -\frac{Bn \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(f+gx)}{g} + \frac{Bn \log\left(-\frac{g(c+dx)}{df-cg}\right) \log(f+gx)}{g} \\
 &= -\frac{Bn \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(f+gx)}{g} + \frac{Bn \log\left(-\frac{g(c+dx)}{df-cg}\right) \log(f+gx)}{g} \\
 &= -\frac{Bn \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(f+gx)}{g} + \frac{Bn \log\left(-\frac{g(c+dx)}{df-cg}\right) \log(f+gx)}{g}
 \end{aligned}$$

**Mathematica [A]** time = 0.0576367, size = 122, normalized size = 0.83

$$\frac{-Bn \text{PolyLog}\left(2, \frac{b(f+gx)}{bf-ag}\right) + Bn \text{PolyLog}\left(2, \frac{d(f+gx)}{df-cg}\right) + \log(f+gx) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) - Bn \log\left(\frac{g(a+bx)}{ag-bf}\right) + A + Bn \log\left(-\frac{g(c+dx)}{df-cg}\right)\right)}{g}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x), x]

[Out] ((A - B\*n\*Log[(g\*(a + b\*x))/(-(b\*f) + a\*g)] + B\*Log[e\*((a + b\*x)/(c + d\*x))^n] + B\*n\*Log[(g\*(c + d\*x))/(-(d\*f) + c\*g)])\*Log[f + g\*x] - B\*n\*PolyLog[2, (b\*(f + g\*x))/(b\*f - a\*g)] + B\*n\*PolyLog[2, (d\*(f + g\*x))/(d\*f - c\*g)]/g

**Maple [F]** time = 0.616, size = 0, normalized size = 0.

$$\int \frac{1}{gx+f} \left( A + B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f), x)

[Out] `int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-B \int -\frac{\log((bx+a)^n) - \log((dx+c)^n) + \log(e)}{gx+f} dx + \frac{A \log(gx+f)}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="maxima")`

[Out] `-B*integrate(-log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(g*x + f), x) + A*log(g*x + f)/g`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A}{gx+f}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="fricas")`

[Out] `integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(g*x + f), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A}{gx+f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f),x, algorithm="giac")`

[Out] `integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(g*x + f), x)`



$$3.63 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2} dx$$

**Optimal.** Leaf size=91

$$\frac{(a+bx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(f+gx)(bf-ag)} + \frac{Bn(bc-ad) \log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

[Out] ((a + b\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/((b\*f - a\*g)\*(f + g\*x)) + (B\*(b\*c - a\*d)\*n\*Log[(f + g\*x)/(c + d\*x]])/((b\*f - a\*g)\*(d\*f - c\*g))

**Rubi [A]** time = 0.124769, antiderivative size = 119, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 72}

$$-\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{g(f+gx)} + \frac{Bn(bc-ad) \log(f+gx)}{(bf-ag)(df-cg)} + \frac{bBn \log(a+bx)}{g(bf-ag)} - \frac{Bdn \log(c+dx)}{g(df-cg)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x)^2, x]

[Out] (b\*B\*n\*Log[a + b\*x])/(g\*(b\*f - a\*g)) - (A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/((g\*(f + g\*x)) - (B\*d\*n\*Log[c + d\*x])/(g\*(d\*f - c\*g)) + (B\*(b\*c - a\*d)\*n\*Log[f + g\*x])/(b\*f - a\*g)\*(d\*f - c\*g))

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^2} dx &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(f+gx)} + \frac{(Bn) \int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)} dx}{g} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(f+gx)} + \frac{(B(bc-ad)n) \int \frac{1}{(a+bx)(c+dx)(f+gx)} dx}{g} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(f+gx)} + \frac{(B(bc-ad)n) \int \left(\frac{b^2}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2}{(bc-ad)(-df+cg)(c+dx)} + \frac{1}{(bf-ag)(f+gx)}\right) dx}{g} \\
&= \frac{bBn \log(a+bx)}{g(bf-ag)} - \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(f+gx)} - \frac{Bdn \log(c+dx)}{g(df-cg)} + \frac{B(bc-ad)n \log(f+gx)}{(bf-ag)(df-cg)}
\end{aligned}$$

**Mathematica [A]** time = 0.182617, size = 109, normalized size = 1.2

$$\frac{\frac{Bn(b \log(a+bx)(df-cg) + \log(c+dx)(adg-bdf) + g(bc-ad) \log(f+gx))}{(bf-ag)(df-cg)} - \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{f+gx}}{g}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x)^2, x]

[Out] (-((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x)) + (B\*n\*(b\*(d\*f - c\*g)\*Log[a + b\*x] + (- (b\*d\*f) + a\*d\*g)\*Log[c + d\*x] + (b\*c - a\*d)\*g\*Log[f + g\*x]))/(b\*f - a\*g)\*(d\*f - c\*g))/g

**Maple [F]** time = 0.513, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^2} \left( A + B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^2, x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^2, x)

**Maxima [A]** time = 1.184, size = 192, normalized size = 2.11

$$Bn \left( \frac{b \log(bx+a)}{bfg-ag^2} - \frac{d \log(dx+c)}{dfg-cg^2} + \frac{(bc-ad) \log(gx+f)}{bdf^2+acg^2-(bc+ad)fg} \right) - \frac{B \log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right)}{g^2x+fg} - \frac{A}{g^2x+fg}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^2, x, algorithm="maxima")

[Out]  $B*n*(b*\log(b*x + a)/(b*f*g - a*g^2) - d*\log(d*x + c)/(d*f*g - c*g^2) + (b*c - a*d)*\log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g)) - B*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(g^2*x + f*g) - A/(g^2*x + f*g)$

**Fricas [B]** time = 17.4685, size = 652, normalized size = 7.16

$$\frac{Abdf^2 + Aacg^2 - (Abc + Aad)fg + (Bbdf^2 + Bacg^2 - (Bbc + Bad)fg)n \log\left(\frac{bx+a}{dx+c}\right) - ((Bbdfg - Bbcg^2)nx + (Bbdfg - Bbcg^2))}{(g^2x + fg)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^2,x, algorithm="fricas")

[Out]  $-(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*n*\log((b*x + a)/(d*x + c)) - ((B*b*d*f*g - B*b*c*g^2)*n*x + (B*b*d*f^2 - B*b*c*f*g)*n)*\log(b*x + a) + ((B*b*d*f*g - B*a*d*g^2)*n*x + (B*b*d*f^2 - B*a*d*f*g)*n)*\log(d*x + c) - ((B*b*c - B*a*d)*g^2*n*x + (B*b*c - B*a*d)*f*g*n)*\log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*\log(e))/(b*d*f^3*g + a*c*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*g^2 + a*c*g^4 - (b*c + a*d)*f*g^3)*x$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)\*\*2,x)

[Out] Timed out

**Giac [B]** time = 1.33159, size = 390, normalized size = 4.29

$$\frac{Bn \log\left(\frac{bx+a}{dx+c}\right)}{g^2x + fg} + \frac{(Bbcn - Badn) \log(gx + f)}{bdf^2 - bcfg - adfg + acg^2} - \frac{(Bbcn - Badn) \log(|bdx^2 + bcx + adx + ac|)}{2(bdf^2 - bcfg - adfg + acg^2)} - \frac{A + B}{g^2x + fg} + \frac{(2Bb^2cd - Bbcn - Badn)}{(bdf^2 - bcfg - adfg + acg^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^2,x, algorithm="giac")

[Out]  $-B*n*\log((b*x + a)/(d*x + c))/(g^2*x + f*g) + (B*b*c*n - B*a*d*n)*\log(g*x + f)/(b*d*f^2 - b*c*f*g - a*d*f*g + a*c*g^2) - 1/2*(B*b*c*n - B*a*d*n)*\log(\text{abs}(b*d*x^2 + b*c*x + a*d*x + a*c))/(b*d*f^2 - b*c*f*g - a*d*f*g + a*c*g^2) - (A + B)/(g^2*x + f*g) + 1/2*(2*B*b^2*c*d*f*n - 2*B*a*b*d^2*f*n - B*b^2*c^2*g*n + B*a^2*d^2*g*n)*\log(\text{abs}((2*b*d*x + b*c + a*d - \text{abs}(-b*c + a*d))/(2*b*d*x + b*c + a*d + \text{abs}(-b*c + a*d))))/((b*d*f^2*g - b*c*f*g^2 - a*d*f*g^2 + a*c*g^3)*\text{abs}(-b*c + a*d))$

$$3.64 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3} dx$$

**Optimal.** Leaf size=190

$$-\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{2g(f+gx)^2} + \frac{b^2 B n \log(a+bx)}{2g(bf-ag)^2} - \frac{B n(bc-ad)}{2(f+gx)(bf-ag)(df-cg)} + \frac{B n(bc-ad) \log(f+gx)(-adg-bcg+2bd)}{2(bf-ag)^2(df-cg)^2}$$

[Out]  $-(B*(b*c - a*d)*n)/(2*(b*f - a*g)*(d*f - c*g)*(f + g*x)) + (b^2*B*n*Log[a + b*x])/(2*g*(b*f - a*g)^2) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(2*g*(f + g*x)^2) - (B*d^2*n*Log[c + d*x])/(2*g*(d*f - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n*Log[f + g*x])/(2*(b*f - a*g)^2*(d*f - c*g)^2)$

**Rubi [A]** time = 0.235799, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 72}

$$-\frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{2g(f+gx)^2} + \frac{b^2 B n \log(a+bx)}{2g(bf-ag)^2} - \frac{B n(bc-ad)}{2(f+gx)(bf-ag)(df-cg)} + \frac{B n(bc-ad) \log(f+gx)(-adg-bcg+2bd)}{2(bf-ag)^2(df-cg)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x)^3, x]

[Out]  $-(B*(b*c - a*d)*n)/(2*(b*f - a*g)*(d*f - c*g)*(f + g*x)) + (b^2*B*n*Log[a + b*x])/(2*g*(b*f - a*g)^2) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(2*g*(f + g*x)^2) - (B*d^2*n*Log[c + d*x])/(2*g*(d*f - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n*Log[f + g*x])/(2*(b*f - a*g)^2*(d*f - c*g)^2)$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 72

Int[((e\_.) + (f\_.)\*(x\_.))^(p\_.)/(((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))), x\_Symbol] :> Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^3} dx &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2} + \frac{(Bn) \int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)^2} dx}{2g} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2} + \frac{(B(bc-ad)n) \int \frac{1}{(a+bx)(c+dx)(f+gx)^2} dx}{2g} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2} + \frac{(B(bc-ad)n) \int \left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d^3}{(bc-ad)(-df+cg)^2(c+dx)}\right) dx}{2g} \\
&= -\frac{B(bc-ad)n}{2(bf-ag)(df-cg)(f+gx)} + \frac{b^2 Bn \log(a+bx)}{2g(bf-ag)^2} - \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2g(f+gx)^2} - \frac{Bd^2 n \log(c+dx)}{2g(df-cg)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.69519, size = 173, normalized size = 0.91

$$\frac{Bn(bc-ad) \left( \frac{b^2 \log(a+bx)}{(bc-ad)(bf-ag)^2} + \frac{\frac{d^2 \log(c+dx)}{ad-bc} + \frac{g(cg-df)}{(f+gx)(bf-ag)} - \frac{g \log(f+gx)(adg+bcg-2bdf)}{(bf-ag)^2}}{(df-cg)^2} \right) - \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{(f+gx)^2}}{2g}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x)^3,x]

[Out] (-((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x)^2) + B\*(b\*c - a\*d)\*n\*((b^2\*Log[a + b\*x])/((b\*c - a\*d)\*(b\*f - a\*g)^2) + ((g\*(-d\*f) + c\*g))/((b\*f - a\*g)\*(f + g\*x)) + (d^2\*Log[c + d\*x])/(-b\*c + a\*d) - (g\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g)\*Log[f + g\*x])/(b\*f - a\*g)^2)/(d\*f - c\*g)^2)/(2\*g)

**Maple [F]** time = 0.513, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^3} \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^3,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^3,x)

**Maxima [A]** time = 1.29185, size = 479, normalized size = 2.52

$$\frac{1}{2} \left( \frac{b^2 \log(bx+a)}{b^2 f^2 g - 2 abf g^2 + a^2 g^3} - \frac{d^2 \log(dx+c)}{d^2 f^2 g - 2 cdf g^2 + c^2 g^3} + \frac{(2(b^2 cd - abd^2)f - (b^2 c^2 - a^2 d^2)g) \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}{b^2 d^2 f^4 + a^2 c^2 g^4 - 2(b^2 cd + abd^2)f^3 g + (b^2 c^2 + 4 abcd + a^2 d^2)g^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^3,x, algorithm="maxima")

```
[Out] 1/2*(b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - d^2*log(d*x + c)
)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + (2*(b^2*c*d - a*b*d^2)*f - (b^2*c^2
- a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b*d
^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*d)
*f*g^3) - (b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f^2*g
+ a*c*g^3 - (b*c + a*d)*f*g^2)*x))*B^n - 1/2*B*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n)/(g^3*x^2 + 2*f*g^2*x + f^2*g) - 1/2*A/(g^3*x^2 + 2*f*g^2*x + f^
2*g)
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)**3,x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.88225, size = 1165, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^3,x, algorithm="giac")
```

```
[Out] -1/2*B^n*log((b*x + a)/(d*x + c))/(g^3*x^2 + 2*f*g^2*x + f^2*g) + 1/2*(2*B*
b^2*c*d*f^n - 2*B*a*b*d^2*f^n - B*b^2*c^2*g^n + B*a^2*d^2*g^n)*log(g*x + f)
/(b^2*d^2*f^4 - 2*b^2*c*d*f^3*g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b
*c*d*f^2*g^2 + a^2*d^2*f^2*g^2 - 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^
2*g^4) - 1/4*(2*B*b^2*c*d*f^n - 2*B*a*b*d^2*f^n - B*b^2*c^2*g^n + B*a^2*d^2
*g^n)*log(abs(b*d*x^2 + b*c*x + a*d*x + a*c))/(b^2*d^2*f^4 - 2*b^2*c*d*f^3*
g - 2*a*b*d^2*f^3*g + b^2*c^2*f^2*g^2 + 4*a*b*c*d*f^2*g^2 + a^2*d^2*f^2*g^2
- 2*a*b*c^2*f*g^3 - 2*a^2*c*d*f*g^3 + a^2*c^2*g^4) - 1/2*(B*b*c*g^2*n*x -
B*a*d*g^2*n*x + B*b*c*f*g^n - B*a*d*f*g^n + A*b*d*f^2 + B*b*d*f^2 - A*b*c*f
*g - B*b*c*f*g - A*a*d*f*g - B*a*d*f*g + A*a*c*g^2 + B*a*c*g^2)/(b*d*f^2*g^
3*x^2 - b*c*f*g^4*x^2 - a*d*f*g^4*x^2 + a*c*g^5*x^2 + 2*b*d*f^3*g^2*x - 2*b
*c*f^2*g^3*x - 2*a*d*f^2*g^3*x + 2*a*c*f*g^4*x + b*d*f^4*g - b*c*f^3*g^2 -
a*d*f^3*g^2 + a*c*f^2*g^3) + 1/4*(2*B*b^3*c*d^2*f^2*n - 2*B*a*b^2*d^3*f^2*n
- 2*B*b^3*c^2*d*f*g^n + 2*B*a^2*b*d^3*f*g^n + B*b^3*c^3*g^2*n - B*a*b^2*c^
2*d*g^2*n + B*a^2*b*c*d^2*g^2*n - B*a^3*d^3*g^2*n)*log(abs((2*b*d*x + b*c +
```

$$\frac{a*d - \text{abs}(-b*c + a*d)}{(2*b*d*x + b*c + a*d + \text{abs}(-b*c + a*d))} / ((b^2*d^2 * f^4*g - 2*b^2*c*d*f^3*g^2 - 2*a*b*d^2*f^3*g^2 + b^2*c^2*f^2*g^3 + 4*a*b*c*d*f^2*g^3 + a^2*d^2*f^2*g^3 - 2*a*b*c^2*f*g^4 - 2*a^2*c*d*f*g^4 + a^2*c^2*g^5) * \text{abs}(-b*c + a*d))$$

$$3.65 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4} dx$$

**Optimal.** Leaf size=283

$$\frac{Bn(bc - ad) \log(f + gx) (a^2 d^2 g^2 - abdg(3df - cg) + b^2 (c^2 g^2 - 3cdfg + 3d^2 f^2))}{3(bf - ag)^3 (df - cg)^3} - \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{3g(f + gx)^3} + \frac{b^3 Bn \log(a + bx)}{3g(bf - ag)}$$

[Out]  $-(B*(b*c - a*d)*n)/(6*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*B*n*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*g*(f + g*x)^3) - (B*d^3*n*Log[c + d*x])/(3*g*(d*f - c*g)^3) + (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)$

**Rubi [A]** time = 0.457508, antiderivative size = 283, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 72}

$$\frac{Bn(bc - ad) \log(f + gx) (a^2 d^2 g^2 - abdg(3df - cg) + b^2 (c^2 g^2 - 3cdfg + 3d^2 f^2))}{3(bf - ag)^3 (df - cg)^3} - \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}{3g(f + gx)^3} + \frac{b^3 Bn \log(a + bx)}{3g(bf - ag)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x)^4, x]

[Out]  $-(B*(b*c - a*d)*n)/(6*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*B*n*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*g*(f + g*x)^3) - (B*d^3*n*Log[c + d*x])/(3*g*(d*f - c*g)^3) + (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^n]\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 72

Int[((e\_.) + (f\_.)\*(x\_))^(p\_.)/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]



Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^4} dx &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3} + \frac{(Bn) \int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)^3} dx}{3g} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3} + \frac{(B(bc-ad)n) \int \frac{1}{(a+bx)(c+dx)(f+gx)^3} dx}{3g} \\
&= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3g(f+gx)^3} + \frac{(B(bc-ad)n) \int \left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)} + \frac{d^4}{(bc-ad)(-df+cg)^3(c+dx)} + \dots\right) dx}{3g} \\
&= -\frac{B(bc-ad)n}{6(bf-ag)(df-cg)(f+gx)^2} - \frac{B(bc-ad)(2bdf-bcg-adg)n}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^3 Bn \log(a+bx)}{3g(bf-ag)^3} + \dots
\end{aligned}$$

**Mathematica [A]** time = 1.02113, size = 264, normalized size = 0.93

$$\frac{Bn(bc-ad) \left( \frac{g \log(f+gx)(a^2 d^2 g^2 + abdg(cg-3df) + b^2(c^2 g^2 - 3cdfg + 3d^2 f^2))}{(bf-ag)^3(df-cg)^3} + \frac{b^3 \log(a+bx)}{(bc-ad)(bf-ag)^3} + \frac{d^3 \log(c+dx)}{(bc-ad)(cg-df)^3} + \frac{g(adg+bcg-2bdf)}{(f+gx)(bf-ag)^2(df-cg)^2} - \dots \right)}{3g}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x)^4, x]

[Out] (-((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(f + g\*x)^3) + B\*(b\*c - a\*d)\*n\*(-g/(2\*(b\*f - a\*g)\*(d\*f - c\*g)\*(f + g\*x)^2) + (g\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g))/( (b\*f - a\*g)^2\*(d\*f - c\*g)^2\*(f + g\*x)) + (b^3\*Log[a + b\*x])/((b\*c - a\*d)\*(b\*f - a\*g)^3) + (d^3\*Log[c + d\*x])/((b\*c - a\*d)\*(-d\*f) + c\*g)^3) + (g\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(-3\*d\*f + c\*g) + b^2\*(3\*d^2\*f^2 - 3\*c\*d\*f\*g + c^2\*g^2))\*Log[f + g\*x])/((b\*f - a\*g)^3\*(d\*f - c\*g)^3))/(3\*g)

**Maple [F]** time = 0.507, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^4} \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^4, x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^4, x)

**Maxima [B]** time = 1.54857, size = 1150, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^4, x, algorithm="maxima")

```
[Out] 1/6*(2*b^3*log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*g^4) - 2*d^3*log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g + (b^3*c^3 - a^3*d^3)*g^2)*log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x)*B*n - 1/3*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) - 1/3*A/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)**4,x)
```

```
[Out] Timed out
```

**Giac [B]** time = 3.99896, size = 2925, normalized size = 10.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^4,x, algorithm="giac")
```

```
[Out] -1/3*B*n*log((b*x + a)/(d*x + c))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 1/3*(3*B*b^3*c*d^2*f^2*n - 3*B*a*b^2*d^3*f^2*n - 3*B*b^3*c^2*d*f*g*n + 3*B*a^2*b*d^3*f*g*n + B*b^3*c^3*g^2*n - B*a^3*d^3*g^2*n)*log(g*x + f)/(b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g + 3*b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2 - b^3*c^3*f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^3*f^3*g^3 + 3*a*b^2*c^3*f^2*g^4
```

$$\begin{aligned}
& + 9a^2b^2c^2d^2f^2g^4 + 3a^3c^2d^2f^2g^4 - 3a^2b^2c^3f^2g^5 - 3a^3c^2d^2f^2g^5 + a^3c^3g^6) - 1/6(3B^2b^3c^2d^2f^2n - 3B^2a^2b^2d^3f^2n \\
& - 3B^2b^3c^2d^2f^2g^n + 3B^2a^2b^2d^3f^2g^n + B^2b^3c^3g^2n - B^2a^3d^3g^2n) * \log(\text{abs}(b^2d^2x^2 + b^2c^2x + a^2d^2x + a^2c^2)) / (b^3d^3f^6 - 3b^3c^2d^2f^5g \\
& - 3a^2b^2d^3f^5g + 3b^3c^2d^2f^4g^2 + 9a^2b^2c^2d^2f^4g^2 + 3a^2b^2d^3f^4g^2 - b^3c^3f^3g^3 - 9a^2b^2c^2d^2f^3g^3 - 9a^2b^2c^2d^2f^3g^3 \\
& - a^3d^3f^3g^3 + 3a^2b^2c^3f^2g^4 + 9a^2b^2c^2d^2f^2g^4 + 3a^3c^2d^2f^2g^4 - 3a^2b^2c^3f^2g^5 - 3a^3c^2d^2f^2g^5 + a^3c^3g^6) - \\
& 1/6(4B^2b^2c^2d^2f^2g^3n^2x^2 - 4B^2a^2b^2d^2f^2g^3n^2x^2 - 2B^2b^2c^2g^4n^2x^2 + 2B^2a^2d^2g^4n^2x^2 + 9B^2b^2c^2d^2f^2g^2n^2x - 9B^2a^2b^2d^2f^2g^2n^2x \\
& - 5B^2b^2c^2f^2g^3n^2x + 5B^2a^2d^2f^2g^3n^2x + B^2a^2b^2c^2g^4n^2x - B^2a^2c^2d^2g^4n^2x + 5B^2b^2c^2d^2f^3g^n - 5B^2a^2b^2d^2f^3g^n - 3B^2b^2c^2 \\
& 2f^2g^2n + 3B^2a^2d^2f^2g^2n + B^2a^2b^2c^2f^2g^3n - B^2a^2c^2d^2f^2g^3n + 2A^2b^2d^2f^4 + 2B^2b^2d^2f^4 - 4A^2b^2c^2d^2f^3g - 4B^2b^2c^2d^2f^3g \\
& - 4A^2a^2b^2d^2f^3g - 4B^2a^2b^2d^2f^3g + 2A^2b^2c^2f^2g^2 + 2B^2b^2c^2f^2g^2 + 8A^2a^2b^2c^2d^2f^2g^2 + 8B^2a^2b^2c^2d^2f^2g^2 + 2A^2a^2d^2f^2g^2 \\
& 2 + 2B^2a^2d^2f^2g^2 - 4A^2a^2b^2c^2f^2g^3 - 4B^2a^2b^2c^2f^2g^3 - 4A^2a^2c^2d^2f^2g^3 - 4B^2a^2c^2d^2f^2g^3 + 2A^2a^2c^2g^4 + 2B^2a^2c^2g^4) / (b^2d^2f^4g^4x^3 \\
& - 2b^2c^2d^2f^3g^5x^3 - 2a^2b^2d^2f^3g^5x^3 + b^2c^2f^2g^6x^3 + 4a^2b^2c^2d^2f^2g^6x^3 + a^2d^2f^2g^6x^3 - 2a^2b^2c^2f^2g^7x^3 \\
& - 2a^2c^2d^2f^2g^7x^3 + a^2c^2g^8x^3 + 3b^2d^2f^5g^3x^2 - 6b^2c^2d^2f^4g^4x^2 - 6a^2b^2d^2f^4g^4x^2 + 3b^2c^2f^3g^5x^2 + 12a^2b^2c^2d^2f^3g^5x^2 \\
& + 3a^2d^2f^3g^5x^2 - 6a^2b^2c^2f^2g^6x^2 - 6a^2c^2d^2f^2g^6x^2 + 3a^2c^2f^2g^7x^2 + 3b^2d^2f^6g^2x - 6b^2c^2d^2f^5g^3x - 6a^2b^2d^2f^5g^3x \\
& + 3b^2c^2f^4g^4x + 12a^2b^2c^2d^2f^4g^4x + 3a^2d^2f^4g^4x - 6a^2b^2c^2f^3g^5x - 6a^2c^2d^2f^3g^5x + 3a^2c^2f^2g^6x + b^2d^2f^7g - 2b^2c^2d^2f^6g^2 \\
& - 2a^2b^2d^2f^6g^2 + b^2c^2f^5g^3 + 4a^2b^2c^2d^2f^5g^3 + a^2d^2f^5g^3 - 2a^2b^2c^2f^4g^4 - 2a^2c^2d^2f^4g^4 + a^2c^2f^3g^5) + 1/6(2B^2b^4c^2d^3f^3n - 2B^2a^2b^3d^4f^3n - \\
& 3B^2b^4c^2d^2f^2g^n + 3B^2a^2b^2d^4f^2g^n + 3B^2b^4c^3d^2f^2g^2n - 3B^2a^2b^3c^2d^2f^2g^2n + 3B^2a^2b^2c^3d^3f^2g^2n - 3B^2a^3b^2d^4f^2g^2n \\
& - B^2b^4c^4g^3n + B^2a^2b^3c^3d^3g^3n - B^2a^3b^2c^3d^3g^3n + B^2a^4d^4g^3n) * \log(\text{abs}((2b^2d^2x + b^2c + a^2d - \text{abs}(-b^2c + a^2d)) / (2b^2d^2x + b^2c + \\
& a^2d + \text{abs}(-b^2c + a^2d)))) / ((b^3d^3f^6g - 3b^3c^2d^2f^5g^2 - 3a^2b^2d^3f^5g^2 + 3b^3c^2d^2f^4g^3 + 9a^2b^2c^2d^2f^4g^3 + 3a^2b^2d^3f^4g^3 \\
& - b^3c^3f^3g^4 - 9a^2b^2c^2d^2f^3g^4 - 9a^2b^2c^2d^2f^3g^4 - a^3d^3f^3g^4 + 3a^2b^2c^3f^2g^5 + 9a^2b^2c^2d^2f^2g^5 + 3a^3c^2d^2f^2g^5 - 3a^2b^2c^3f^2g^6 \\
& - 3a^3c^2d^2f^2g^6 + a^3c^3g^7) * \text{abs}(-b^2c + a^2d))
\end{aligned}$$

$$3.66 \quad \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5} dx$$

**Optimal.** Leaf size=388

$$\frac{Bn(bc-ad)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2))}{4(f+gx)(bf-ag)^3(df-cg)^3} - \frac{Bn(bc-ad)\log(f+gx)(-adg-bcg+2bdf)(-a)}{4(bf-ag)^3}$$

```
[Out] -(B*(b*c - a*d)*n)/(12*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)
)*(2*b*d*f - b*c*g - a*d*g)*n)/(8*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2)
- (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*
c*d*f*g + c^2*g^2))*n)/(4*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*n
*Log[a + b*x])/(4*g*(b*f - a*g)^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])
/(4*g*(f + g*x)^4) - (B*d^4*n*Log[c + d*x])/(4*g*(d*f - c*g)^4) - (B*(b*c -
a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f
^2 - 2*c*d*f*g + c^2*g^2))*n*Log[f + g*x])/(4*(b*f - a*g)^4*(d*f - c*g)^4)
```

**Rubi [A]** time = 0.713149, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 72}

$$\frac{Bn(bc-ad)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2))}{4(f+gx)(bf-ag)^3(df-cg)^3} - \frac{Bn(bc-ad)\log(f+gx)(-adg-bcg+2bdf)(-a)}{4(bf-ag)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^5, x]
```

```
[Out] -(B*(b*c - a*d)*n)/(12*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)
)*(2*b*d*f - b*c*g - a*d*g)*n)/(8*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2)
- (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*
c*d*f*g + c^2*g^2))*n)/(4*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*n
*Log[a + b*x])/(4*g*(b*f - a*g)^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])
/(4*g*(f + g*x)^4) - (B*d^4*n*Log[c + d*x])/(4*g*(d*f - c*g)^4) - (B*(b*c -
a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f
^2 - 2*c*d*f*g + c^2*g^2))*n*Log[f + g*x])/(4*(b*f - a*g)^4*(d*f - c*g)^4)
```

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^n]*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 72

```
Int[((e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))),
  x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
  /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

### Rubi steps

$$\begin{aligned} \int \frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(f+gx)^5} dx &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4} + \frac{(Bn) \int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)^4} dx}{4g} \\ &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4} + \frac{(B(bc-ad)n) \int \frac{1}{(a+bx)(c+dx)(f+gx)^4} dx}{4g} \\ &= -\frac{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{4g(f+gx)^4} + \frac{(B(bc-ad)n) \int \left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5}{(bc-ad)(-df+cg)^4(c+dx)}\right) dx}{4g} \\ &= -\frac{B(bc-ad)n}{12(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)n}{8(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{B(bc-ad)(a^2d^2g)}{4g} \end{aligned}$$

**Mathematica [A]** time = 1.35501, size = 359, normalized size = 0.93

$$\frac{Bn(bc-ad) \left( -\frac{g(a^2d^2g^2+abdg(cg-3df)+b^2(c^2g^2-3cdfg+3d^2f^2))}{(f+gx)(bf-ag)^3(df-cg)^3} - \frac{g \log(f+gx)(adg+bcg-2bdf)(a^2d^2g^2-2abd^2fg+b^2(c^2g^2-2cdfg+2d^2f^2))}{(bf-ag)^4(df-cg)^4} \right) + \frac{b^4}{(bc-4g}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^5, x]
```

```
[Out] (-(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(f + g*x)^4) + B*(b*c - a*d)*n*(-g/(3*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) + (g*(-2*b*d*f + b*c*g + a*d*g))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/((b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^4) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) - (g*(-2*b*d*f + b*c*g + a*d*g)*(-2*a*b*d^2*f*g + a^2*d^2*g^2 + b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4))/(4*g)
```

**Maple [F]** time = 0.502, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^5} \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5, x)
```

```
[Out] int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(g*x+f)^5, x)
```

**Maxima [B]** time = 1.96833, size = 2377, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^5,x, algorithm="maxima")

[Out]  $\frac{1}{24} \cdot (6b^4 \log(bx+a) / (b^4 f^4 g - 4ab^3 f^3 g^2 + 6a^2 b^2 f^2 g^3 - 4a^3 b f g^4 + a^4 g^5) - 6d^4 \log(dx+c) / (d^4 f^4 g - 4cd^3 f^3 g^2 + 6c^2 d^2 f^2 g^3 - 4c^3 d f g^4 + c^4 g^5) + 6(4(b^4 c d^3 - ab^3 d^4) f^3 - 6(b^4 c^2 d^2 - a^2 b^2 d^4) f^2 g + 4(b^4 c^3 d - a^3 b d^4) f g^2 - (b^4 c^4 - a^4 d^4) g^3) \log(gx+f) / (b^4 d^4 f^8 + a^4 c^4 g^8 - 4(b^4 c d^3 + ab^3 d^4) f^7 g + 2(3b^4 c^2 d^2 + 8ab^3 c d^3 + 3a^2 b^2 d^4) f^6 g^2 - 4(b^4 c^3 d + 6ab^3 c^2 d^2 + 6a^2 b^2 c d^3 + a^3 b d^4) f^5 g^3 + (b^4 c^4 + 16ab^3 c^3 d + 36a^2 b^2 c^2 d^2 + 16a^3 b c d^3 + a^4 d^4) f^4 g^4 - 4(ab^3 c^4 + 6a^2 b^2 c^3 d + 6a^3 b c^2 d^2 + a^4 c d^3) f^3 g^5 + 2(3a^2 b^2 c^4 + 8a^3 b c^3 d + 3a^4 c^2 d^2) f^2 g^6 - 4(a^3 b c^4 + a^4 c^3 d) f g^7) - (26(b^3 c d^2 - ab^2 d^3) f^4 - 31(b^3 c^2 d - a^2 b d^3) f^3 g + (11b^3 c^3 + 15ab^2 c^2 d - 15a^2 b c d^2 - 11a^3 d^3) f^2 g^2 - 7(ab^2 c^3 - a^3 c d^2) f g^3 + 2(a^2 b c^3 - a^3 c^2 d) g^4 + 6(3(b^3 c d^2 - ab^2 d^3) f^2 g^2 - 3(b^3 c^2 d - a^2 b d^3) f g^3 + (b^3 c^3 - a^3 d^3) g^4) x^2 + 3(14(b^3 c d^2 - ab^2 d^3) f^3 g - 15(b^3 c^2 d - a^2 b d^3) f^2 g^2 + (5b^3 c^3 + 3ab^2 c^2 d - 3a^2 b c d^2 - 5a^3 d^3) f g^3 - (ab^2 c^3 - a^3 c d^2) g^4) x) / (b^3 d^3 f^9 + a^3 c^3 f^3 g^6 - 3(b^3 c d^2 + ab^2 d^3) f^8 g + 3(b^3 c^2 d + 3ab^2 c d^2 + a^2 b d^3) f^7 g^2 - (b^3 c^3 + 9ab^2 c^2 d + 9a^2 b c d^2 + a^3 d^3) f^6 g^3 + 3(ab^2 c^3 + 3a^2 b c^2 d + a^3 c d^2) f^5 g^4 - 3(a^2 b c^3 + a^3 c^2 d) f^4 g^5 + (b^3 d^3 f^6 g^3 + a^3 c^3 g^9 - 3(b^3 c d^2 + ab^2 d^3) f^5 g^4 + 3(b^3 c^2 d + 3ab^2 c d^2 + a^2 b d^3) f^4 g^5 - (b^3 c^3 + 9ab^2 c^2 d + 9a^2 b c d^2 + a^3 d^3) f^3 g^6 + 3(ab^2 c^3 + 3a^2 b c^2 d + a^3 c d^2) f^2 g^7 - 3(a^2 b c^3 + a^3 c^2 d) f g^8) x^3 + 3(b^3 d^3 f^7 g^2 + a^3 c^3 f g^8 - 3(b^3 c d^2 + ab^2 d^3) f^6 g^3 + 3(b^3 c^2 d + 3ab^2 c d^2 + a^2 b d^3) f^5 g^4 - (b^3 c^3 + 9ab^2 c^2 d + 9a^2 b c d^2 + a^3 d^3) f^4 g^5 + 3(ab^2 c^3 + 3a^2 b c^2 d + a^3 c d^2) f^3 g^6 - 3(a^2 b c^3 + a^3 c^2 d) f^2 g^7) x^2 + 3(b^3 d^3 f^8 g + a^3 c^3 f^2 g^7 - 3(b^3 c d^2 + ab^2 d^3) f^7 g^2 + 3(b^3 c^2 d + 3ab^2 c d^2 + a^2 b d^3) f^6 g^3 - (b^3 c^3 + 9ab^2 c^2 d + 9a^2 b c d^2 + a^3 d^3) f^5 g^4 + 3(ab^2 c^3 + 3a^2 b c^2 d + a^3 c d^2) f^4 g^5 - 3(a^2 b c^3 + a^3 c^2 d) f^3 g^6) x) * B^n - 1/4 * B * log(e*(bx/(dx+c) + a/(dx+c))^n) / (g^5 x^4 + 4f g^4 x^3 + 6f^2 g^3 x^2 + 4f^3 g^2 x + f^4 g) - 1/4 * A / (g^5 x^4 + 4f g^4 x^3 + 6f^2 g^3 x^2 + 4f^3 g^2 x + f^4 g)$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^5,x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))/(g\*x+f)\*\*5,x)

[Out] Timed out

**Giac [B]** time = 14.5389, size = 5863, normalized size = 15.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))/(g\*x+f)^5,x, algorithm="giac")

[Out] 
$$-1/4*B*n*\log((b*x + a)/(d*x + c))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g) + 1/4*(4*B*b^4*c*d^3*f^3*n - 4*B*a*b^3*d^4*f^3*n - 6*B*b^4*c^2*d^2*f^2*g*n + 6*B*a^2*b^2*d^4*f^2*g*n + 4*B*b^4*c^3*d*f*g^2*n - 4*B*a^3*b*d^4*f*g^2*n - B*b^4*c^4*g^3*n + B*a^4*d^4*g^3*n)*\log(g*x + f)/(b^4*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 6*b^4*c^2*d^2*f^6*g^2 + 16*a*b^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*g^2 - 4*b^4*c^3*d*f^5*g^3 - 24*a*b^3*c^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^5*g^3 - 4*a^3*b*d^4*f^5*g^3 + b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 36*a^2*b^2*c^2*d^2*f^4*g^4 + 16*a^3*b*c*d^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a*b^3*c^4*f^3*g^5 - 24*a^2*b^2*c^3*d*f^3*g^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4*a^4*c*d^3*f^3*g^5 + 6*a^2*b^2*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^4*c^2*d^2*f^2*g^6 - 4*a^3*b*c^4*f*g^7 - 4*a^4*c^3*d*f*g^7 + a^4*c^4*g^8) - 1/8*(4*B*b^4*c*d^3*f^3*n - 4*B*a*b^3*d^4*f^3*n - 6*B*b^4*c^2*d^2*f^2*g*n + 6*B*a^2*b^2*d^4*f^2*g*n + 4*B*b^4*c^3*d*f*g^2*n - 4*B*a^3*b*d^4*f*g^2*n - B*b^4*c^4*g^3*n + B*a^4*d^4*g^3*n)*\log(\text{abs}(b*d*x^2 + b*c*x + a*d*x + a*c))/(b^4*d^4*f^8 - 4*b^4*c*d^3*f^7*g - 4*a*b^3*d^4*f^7*g + 6*b^4*c^2*d^2*f^6*g^2 + 16*a*b^3*c*d^3*f^6*g^2 + 6*a^2*b^2*d^4*f^6*g^2 - 4*b^4*c^3*d*f^5*g^3 - 24*a*b^3*c^2*d^2*f^5*g^3 - 24*a^2*b^2*c*d^3*f^5*g^3 - 4*a^3*b*d^4*f^5*g^3 + b^4*c^4*f^4*g^4 + 16*a*b^3*c^3*d*f^4*g^4 + 36*a^2*b^2*c^2*d^2*f^4*g^4 + 16*a^3*b*c*d^3*f^4*g^4 + a^4*d^4*f^4*g^4 - 4*a*b^3*c^4*f^3*g^5 - 24*a^2*b^2*c^3*d*f^3*g^5 - 24*a^3*b*c^2*d^2*f^3*g^5 - 4*a^4*c*d^3*f^3*g^5 + 6*a^2*b^2*c^4*f^2*g^6 + 16*a^3*b*c^3*d*f^2*g^6 + 6*a^4*c^2*d^2*f^2*g^6 - 4*a^3*b*c^4*f*g^7 - 4*a^4*c^3*d*f*g^7 + a^4*c^4*g^8) - 1/24*(18*B*b^3*c*d^2*f^2*g^4*n*x^3 - 18*B*a*b^2*d^3*f^2*g^4*n*x^3 - 18*B*b^3*c^2*d*f*g^5*n*x^3 + 18*B*a^2*b*d^3*f*g^5*n*x^3 + 6*B*b^3*c^3*g^6*n*x^3 - 6*B*a^3*d^3*g^6*n*x^3 + 60*B*b^3*c*d^2*f^3*g^3*n*x^2 - 60*B*a*b^2*d^3*f^3*g^3*n*x^2 - 63*B*b^3*c^2*d*f^2*g^4*n*x^2 + 63*B*a^2*b*d^3*f^2*g^4*n*x^2 + 21*B*b^3*c^3*f*g^5*n*x^2 + 9*B*a*b^2*c^2*d*f*g^5*n*x^2 - 9*B*a^2*b*c*d^2*f*g^5*n*x^2 - 21*B*a^3*d^3*f*g^5*n*x^2 - 3*B*a*b^2*c^3*g^6*n*x^2 + 3*B*a^3*c*d^2*g^6*n*x^2 + 68*B*b^3*c*d^2*f^4*g^2*n*x - 68*B*a*b^2*d^3*f^4*g^2*n*x - 76*B*b^3*c^2*d*f^3*g^3*n*x + 76*B*a^2*b*d^3*f^3*g^3*n*x + 26*B*b^3*c^3*f^2*g^4*n*x + 24*B*a*b^2*c^2*d*f^2*g^4*n*x - 24*B*a^2*b*c*d^2*f^2*g^4*n*x - 26*B*a^3*d^3*f^2*g^4*n*x - 10*B*a*b^2*c^3*f*g^5*n*x + 10*B*a^3*c*d^2*f*g^5*n*x + 2*B*a^2*b*c^3*g^6*n*x - 2*B*a^3*c^2*d*g^6*n*x + 26*B*b^3*c*d^2*f^5*g*n - 26*B*a*b^2*d^3*f^5*g*n - 31*B*b^3*c^2*d*f^4*g^2*n + 31*B*a^2*b*d^3*f^4*g^2*n + 11*B*b^3*c^3*f^3*g^3*n + 15*B*a*b^2*c^2*d*f^3*g^3*n - 15*B*a^2*b*c*d^2*f^3*g^3*n - 11*B*a^3*d^3*f^3*g^3*n - 7*B*a*b^2*c^3*f^2*g^4*n + 7*B*a^3*c*d^2*f^2*g^4*n + 2*B*a^2*b*c^3*f*g^5*n - 2*B*a^3*c^2*d*f*g^5*n + 6*A*b^3*d^3*f^6 + 6*B*b^3*d^3*f^6 - 18*A*b^3*c*d^2*f^5*g - 18*B*b^3*c*d^2*f^5*g - 18*A$$

$$\begin{aligned}
& a^2 b^2 d^3 f^5 g - 18 B a^2 b^2 d^3 f^5 g + 18 A^2 b^3 c^2 d f^4 g^2 + 18 B^2 b^3 c^2 d f^4 g^2 + 54 A^2 a^2 b^2 c^2 d^2 f^4 g^2 + 54 B^2 a^2 b^2 c^2 d^2 f^4 g^2 + 18 A^2 a^2 b^2 d^3 f^4 g^2 + 18 B^2 a^2 b^2 d^3 f^4 g^2 - 6 A^2 b^3 c^3 f^3 g^3 - 6 B^2 b^3 c^3 f^3 g^3 - 54 A^2 a^2 b^2 c^2 d^2 f^3 g^3 - 54 B^2 a^2 b^2 c^2 d^2 f^3 g^3 - 54 A^2 a^2 b^2 c^2 d^2 f^3 g^3 - 54 B^2 a^2 b^2 c^2 d^2 f^3 g^3 - 6 A^2 a^3 d^3 f^3 g^3 - 6 B^2 a^3 d^3 f^3 g^3 + 18 A^2 a^2 b^2 c^3 f^2 g^4 + 18 B^2 a^2 b^2 c^3 f^2 g^4 + 54 A^2 a^2 b^2 c^2 d^2 f^2 g^4 + 54 B^2 a^2 b^2 c^2 d^2 f^2 g^4 + 18 A^2 a^3 c^2 d^2 f^2 g^4 + 18 B^2 a^3 c^2 d^2 f^2 g^4 - 18 A^2 a^2 b^2 c^3 f^2 g^5 - 18 B^2 a^2 b^2 c^3 f^2 g^5 - 18 A^2 a^3 c^2 d^2 f^2 g^5 - 18 B^2 a^3 c^2 d^2 f^2 g^5 + 6 A^2 a^3 c^3 g^6 + 6 B^2 a^3 c^3 g^6) / (b^3 d^3 f^6 g^5 x^4 - 3 b^3 c^2 d^2 f^5 g^6 x^4 - 3 a^2 b^2 d^3 f^5 g^6 x^4 + 3 b^3 c^2 d^2 f^4 g^7 x^4 + 9 a^2 b^2 c^2 d^2 f^4 g^7 x^4 + 3 a^2 b^2 d^3 f^4 g^7 x^4 - b^3 c^3 f^3 g^8 x^4 - 9 a^2 b^2 c^2 d^2 f^3 g^8 x^4 - 9 a^2 b^2 c^2 d^2 f^3 g^8 x^4 - a^3 d^3 f^3 g^8 x^4 + 3 a^2 b^2 c^3 f^2 g^9 x^4 + 9 a^2 b^2 c^2 d^2 f^2 g^9 x^4 + 3 a^3 c^2 d^2 f^2 g^9 x^4 - 3 a^2 b^2 c^3 f^2 g^10 x^4 - 3 a^3 c^2 d^2 f^2 g^10 x^4 + a^3 c^3 g^11 x^4 + 4 b^3 d^3 f^7 g^4 x^3 - 12 b^3 c^2 d^2 f^6 g^5 x^3 - 12 a^2 b^2 d^3 f^6 g^5 x^3 + 12 b^3 c^2 d^2 f^5 g^6 x^3 + 36 a^2 b^2 c^2 d^2 f^5 g^6 x^3 + 12 a^2 b^2 d^3 f^5 g^6 x^3 - 4 b^3 c^3 f^4 g^7 x^3 - 36 a^2 b^2 c^2 d^2 f^4 g^7 x^3 - 36 a^2 b^2 c^2 d^2 f^4 g^7 x^3 - 4 a^3 d^3 f^4 g^7 x^3 + 12 a^2 b^2 c^3 f^3 g^8 x^3 + 36 a^2 b^2 c^2 d^2 f^3 g^8 x^3 + 12 a^3 c^2 d^2 f^3 g^8 x^3 - 12 a^2 b^2 c^3 f^2 g^9 x^3 - 12 a^3 c^2 d^2 f^2 g^9 x^3 + 4 a^3 c^3 f^2 g^10 x^3 + 6 b^3 d^3 f^8 g^3 x^2 - 18 b^3 c^2 d^2 f^7 g^4 x^2 - 18 a^2 b^2 d^3 f^7 g^4 x^2 + 18 b^3 c^2 d^2 f^6 g^5 x^2 + 54 a^2 b^2 c^2 d^2 f^6 g^5 x^2 + 18 a^2 b^2 d^3 f^6 g^5 x^2 - 6 b^3 c^3 f^5 g^6 x^2 - 54 a^2 b^2 c^2 d^2 f^5 g^6 x^2 - 54 a^2 b^2 c^2 d^2 f^5 g^6 x^2 - 6 a^3 d^3 f^5 g^6 x^2 + 18 a^2 b^2 c^3 f^4 g^7 x^2 + 54 a^2 b^2 c^2 d^2 f^4 g^7 x^2 + 18 a^3 c^2 d^2 f^4 g^7 x^2 - 18 a^2 b^2 c^3 f^3 g^8 x^2 - 18 a^3 c^2 d^2 f^3 g^8 x^2 + 6 a^3 c^3 f^2 g^9 x^2 + 4 b^3 d^3 f^9 g^2 x - 12 b^3 c^2 d^2 f^8 g^3 x - 12 a^2 b^2 d^3 f^8 g^3 x + 12 b^3 c^2 d^2 f^7 g^4 x + 36 a^2 b^2 c^2 d^2 f^7 g^4 x + 12 a^2 b^2 d^3 f^7 g^4 x - 4 b^3 c^3 f^6 g^5 x - 36 a^2 b^2 c^2 d^2 f^6 g^5 x - 36 a^2 b^2 c^2 d^2 f^6 g^5 x - 4 a^3 d^3 f^6 g^5 x + 12 a^2 b^2 c^3 f^5 g^6 x + 36 a^2 b^2 c^2 d^2 f^5 g^6 x + 12 a^3 c^2 d^2 f^5 g^6 x x - 12 a^2 b^2 c^3 f^4 g^7 x - 12 a^3 c^2 d^2 f^4 g^7 x + 4 a^3 c^3 f^3 g^8 x + b^3 d^3 f^10 g - 3 b^3 c^2 d^2 f^9 g^2 - 3 a^2 b^2 d^3 f^9 g^2 + 3 b^3 c^2 d^2 f^8 g^3 + 9 a^2 b^2 c^2 d^2 f^8 g^3 + 3 a^2 b^2 d^3 f^8 g^3 - b^3 c^3 f^7 g^4 - 9 a^2 b^2 c^2 d^2 f^7 g^4 - 9 a^2 b^2 c^2 d^2 f^7 g^4 - a^3 d^3 f^7 g^4 + 3 a^2 b^2 c^3 f^6 g^5 + 9 a^2 b^2 c^2 d^2 f^6 g^5 + 3 a^3 c^2 d^2 f^6 g^5 - 3 a^2 b^2 c^3 f^5 g^6 - 3 a^3 c^2 d^2 f^5 g^6 + a^3 c^3 f^4 g^7) + 1/8 * (2 B^2 b^5 c^2 d^4 f^4 g^n - 2 B^2 a^2 b^4 d^5 f^4 g^n - 4 B^2 b^5 c^2 d^3 f^3 g^n + 4 B^2 a^2 b^3 d^5 f^3 g^n + 6 B^2 b^5 c^3 d^2 f^2 g^2 n - 6 B^2 a^2 b^4 c^2 d^3 f^2 g^2 n + 6 B^2 a^2 b^3 c^2 d^4 f^2 g^2 n - 6 B^2 a^3 b^2 d^5 f^2 g^2 n - 4 B^2 b^5 c^4 d^2 f g^3 n + 4 B^2 a^2 b^4 c^3 d^2 f g^3 n - 4 B^2 a^3 b^2 c^4 d^2 f g^3 n + 4 B^2 a^4 b^2 d^5 f g^3 n + B^2 b^5 c^5 g^4 n - B^2 a^2 b^4 c^4 d^2 g^4 n + B^2 a^4 b^2 c^4 d^2 g^4 n - B^2 a^5 d^5 g^4 n) * log(abs((2 b^2 d^2 x + b^2 c + a^2 d - abs(-b^2 c + a^2 d)) / (2 b^2 d^2 x + b^2 c + a^2 d + abs(-b^2 c + a^2 d)))) / ((b^4 d^4 f^8 g - 4 b^4 c^2 d^3 f^7 g^2 - 4 a^2 b^3 d^4 f^7 g^2 + 6 b^4 c^3 d^2 f^5 g^4 - 24 a^2 b^3 c^2 d^2 f^5 g^4 - 24 a^2 b^2 c^2 d^3 f^5 g^4 - 4 a^3 b^2 d^4 f^5 g^4 + b^4 c^4 f^4 g^5 + 16 a^2 b^3 c^3 d^2 f^4 g^5 + 36 a^2 b^2 c^2 d^2 f^4 g^5 + 16 a^3 b^2 c^2 d^3 f^4 g^5 + a^4 d^4 f^4 g^5 - 4 a^2 b^3 c^4 f^3 g^6 - 24 a^2 b^2 c^3 d^2 f^3 g^6 - 24 a^3 b^2 c^2 d^2 f^3 g^6 - 4 a^4 c^2 d^3 f^3 g^6 + 6 a^2 b^2 c^4 f^2 g^7 + 16 a^3 b^2 c^3 d^2 f^2 g^7 + 6 a^4 c^2 d^2 f^2 g^7 - 4 a^3 b^2 c^4 f g^8 - 4 a^4 c^3 d^2 f g^8 + a^4 c^4 g^9) * abs(-b^2 c + a^2 d))
\end{aligned}$$



$$3.67 \quad \int (f + gx)^3 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx$$

**Optimal.** Leaf size=923

$$\frac{B^2 g^3 n^2 \log \left( \frac{a+bx}{c+dx} \right) (bc - ad)^4}{6b^4 d^4} + \frac{B^2 g^3 n^2 \log(c + dx) (bc - ad)^4}{6b^4 d^4} + \frac{B^2 g^3 n^2 x (bc - ad)^3}{6b^3 d^3} + \frac{B^2 g^2 (4bdf - 3bcg - adg) n^2 \log \left( \frac{b(c+dx)}{bc-ad} \right)}{4b^4 d^4}$$

[Out] (B^2\*(b\*c - a\*d)^3\*g^3\*n^2\*x)/(6\*b^3\*d^3) + (B^2\*(b\*c - a\*d)^2\*g^2\*(4\*b\*d\*f - 3\*b\*c\*g - a\*d\*g)\*n^2\*x)/(4\*b^3\*d^3) + (B^2\*(b\*c - a\*d)^2\*g^3\*n^2\*(c + d\*x)^2)/(12\*b^2\*d^4) - (B\*(b\*c - a\*d)\*g\*(a^2\*d^2\*g^2 - 2\*a\*b\*d\*g\*(2\*d\*f - c\*g) + b^2\*(6\*d^2\*f^2 - 8\*c\*d\*f\*g + 3\*c^2\*g^2))\*n\*(a + b\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(2\*b^4\*d^3) - (B\*(b\*c - a\*d)\*g^2\*(4\*b\*d\*f - 3\*b\*c\*g - a\*d\*g)\*n\*(c + d\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(4\*b^2\*d^4) - (B\*(b\*c - a\*d)\*g^3\*n\*(c + d\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(6\*b\*d^4) - ((b\*f - a\*g)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2)/(4\*b^4\*g) + ((f + g\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2)/(4\*g) - (B\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*(2\*a\*b\*d^2\*f\*g - a^2\*d^2\*g^2 - b^2\*(2\*d^2\*f^2 - 2\*c\*d\*f\*g + c^2\*g^2))\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[(b\*c - a\*d)/(b\*(c + d\*x))])/(2\*b^4\*d^4) + (B^2\*(b\*c - a\*d)^4\*g^3\*n^2\*Log[(a + b\*x)/(c + d\*x)])/(6\*b^4\*d^4) + (B^2\*(b\*c - a\*d)^3\*g^2\*(4\*b\*d\*f - 3\*b\*c\*g - a\*d\*g)\*n^2\*Log[(a + b\*x)/(c + d\*x)])/(4\*b^4\*d^4) + (B^2\*(b\*c - a\*d)^4\*g^3\*n^2\*Log[c + d\*x])/(6\*b^4\*d^4) + (B^2\*(b\*c - a\*d)^3\*g^2\*(4\*b\*d\*f - 3\*b\*c\*g - a\*d\*g)\*n^2\*Log[c + d\*x])/(4\*b^4\*d^4) + (B^2\*(b\*c - a\*d)^2\*g\*(a^2\*d^2\*g^2 - 2\*a\*b\*d\*g\*(2\*d\*f - c\*g) + b^2\*(6\*d^2\*f^2 - 8\*c\*d\*f\*g + 3\*c^2\*g^2))\*n^2\*Log[c + d\*x])/(2\*b^4\*d^4) - (B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*(2\*a\*b\*d^2\*f\*g - a^2\*d^2\*g^2 - b^2\*(2\*d^2\*f^2 - 2\*c\*d\*f\*g + c^2\*g^2))\*n^2\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(2\*b^4\*d^4)

**Rubi [A]** time = 1.83533, antiderivative size = 1060, normalized size of antiderivative = 1.15, number of steps used = 31, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {2525, 12, 2528, 2486, 31, 72, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2 n^2 \log^2(a + bx) (bf - ag)^4}{4b^4 g} - \frac{Bn \log(a + bx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right) (bf - ag)^4}{2b^4 g} - \frac{B^2 n^2 \log(a + bx) \log \left( \frac{b(c+dx)}{bc-ad} \right) (bf - ag)^4}{2b^4 g}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] -(A\*B\*(b\*c - a\*d)\*g\*(a^2\*d^2\*g^2 - a\*b\*d\*g\*(4\*d\*f - c\*g) + b^2\*(6\*d^2\*f^2 - 4\*c\*d\*f\*g + c^2\*g^2))\*n\*x)/(2\*b^3\*d^3) - (B^2\*(b\*c - a\*d)^2\*(b\*c + a\*d)\*g^3\*n^2\*x)/(6\*b^3\*d^3) + (B^2\*(b\*c - a\*d)^2\*g^2\*(4\*b\*d\*f - b\*c\*g - a\*d\*g)\*n^2\*x)/(4\*b^3\*d^3) + (B^2\*(b\*c - a\*d)^2\*g^3\*n^2\*x^2)/(12\*b^2\*d^4) - (a^3\*B^2\*(b\*c - a\*d)\*g^3\*n^2\*Log[a + b\*x])/(6\*b^4\*d) + (a^2\*B^2\*(b\*c - a\*d)\*g^2\*(4\*b\*d\*f - b\*c\*g - a\*d\*g)\*n^2\*Log[a + b\*x])/(4\*b^4\*d^2) + (B^2\*(b\*f - a\*g)^4\*n^2\*Log[a + b\*x]^2)/(4\*b^4\*g) - (B^2\*(b\*c - a\*d)\*g\*(a^2\*d^2\*g^2 - a\*b\*d\*g\*(4\*d\*f - c\*g) + b^2\*(6\*d^2\*f^2 - 4\*c\*d\*f\*g + c^2\*g^2))\*n\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(2\*b^4\*d^3) - (B\*(b\*c - a\*d)\*g^2\*(4\*b\*d\*f - b\*c\*g - a\*d\*g)\*n\*x^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(4\*b^2\*d^2) - (B\*(b\*c - a\*d)\*g^3\*n\*x^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(6\*b\*d) - (B\*(b\*f - a\*g)^4\*n\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(2\*b^4\*g) + ((f + g\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2)/(4\*g) + (B^2\*c^3\*(b\*c

$$\begin{aligned}
& - a*d)*g^{3*n^2*\text{Log}[c + d*x]}/(6*b*d^4) - (B^2*c^2*(b*c - a*d)*g^{2*(4*b*d*f \\
& - b*c*g - a*d*g)*n^2*\text{Log}[c + d*x]}/(4*b^2*d^4) + (B^2*(b*c - a*d)^2*g*(a^2* \\
& d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n^ \\
& 2*\text{Log}[c + d*x]}/(2*b^4*d^4) - (B^2*(d*f - c*g)^4*n^2*\text{Log}[-((d*(a + b*x))/(b \\
& *c - a*d))]*\text{Log}[c + d*x]}/(2*d^4*g) + (B*(d*f - c*g)^4*n*(A + B*\text{Log}[e*((a + \\
& b*x)/(c + d*x))^n]*\text{Log}[c + d*x]}/(2*d^4*g) + (B^2*(d*f - c*g)^4*n^2*\text{Log}[c \\
& + d*x]^2)/(4*d^4*g) - (B^2*(b*f - a*g)^4*n^2*\text{Log}[a + b*x]*\text{Log}[(b*(c + d*x) \\
& )/(b*c - a*d)])/(2*b^4*g) - (B^2*(b*f - a*g)^4*n^2*\text{PolyLog}[2, -((d*(a + b*x) \\
& )/(b*c - a*d))])/(2*b^4*g) - (B^2*(d*f - c*g)^4*n^2*\text{PolyLog}[2, (b*(c + d*x) \\
& )/(b*c - a*d)])/(2*d^4*g)
\end{aligned}$$

### Rule 2525

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]

```

### Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

### Rule 2528

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]

```

### Rule 2486

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)
^q)^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

```

### Rule 31

```

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]

```

### Rule 72

```

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

```

### Rule 2524

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^p_.)*(RFx_), x_Sy
mbol] :=> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n]^p, RFx, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :=> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n]^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :=> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :=> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e^n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :=> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :=> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps

$$\begin{aligned}
\int (f + gx)^3 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2 dx &= \frac{(f + gx)^4 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{4g} - \frac{(Bn) \int \frac{(bc - ad)(f + gx)^4 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{(a + bx)(c + dx)} dx}{2g} \\
&= \frac{(f + gx)^4 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{4g} - \frac{(B(bc - ad)n) \int \frac{(f + gx)^4 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{(a + bx)(c + dx)} dx}{2g} \\
&= \frac{(f + gx)^4 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{4g} - \frac{(B(bc - ad)n) \int \frac{g^2(a^2d^2g^2 - abdg(4df - cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2))}{(a + bx)(c + dx)} dx}{2g} \\
&= \frac{(f + gx)^4 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{4g} - \frac{(B(bc - ad)g^3n) \int x^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2 dx}{2bd} \\
&= -\frac{AB(bc - ad)g(a^2d^2g^2 - abdg(4df - cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2))nx}{2b^3d^3} \\
&= -\frac{AB(bc - ad)g(a^2d^2g^2 - abdg(4df - cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2))nx}{2b^3d^3} \\
&= -\frac{AB(bc - ad)g(a^2d^2g^2 - abdg(4df - cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2))nx}{2b^3d^3} \\
&= -\frac{AB(bc - ad)g(a^2d^2g^2 - abdg(4df - cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2))nx}{2b^3d^3} \\
&= -\frac{AB(bc - ad)g(a^2d^2g^2 - abdg(4df - cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2))nx}{2b^3d^3} \\
&= -\frac{AB(bc - ad)g(a^2d^2g^2 - abdg(4df - cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2))nx}{2b^3d^3} \\
&= -\frac{AB(bc - ad)g(a^2d^2g^2 - abdg(4df - cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2))nx}{2b^3d^3} \\
&= -\frac{AB(bc - ad)g(a^2d^2g^2 - abdg(4df - cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2))nx}{2b^3d^3}
\end{aligned}$$

**Mathematica [A]** time = 1.04064, size = 757, normalized size = 0.82

$$(f + gx)^4 \left( B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) + A \right)^2 - \frac{Bn \left( 3b^4 Bn(df - cg)^4 \left( 2 \text{PolyLog} \left( 2, \frac{b(c + dx)}{bc - ad} \right) + \log(c + dx) \left( 2 \log \left( \frac{d(a + bx)}{ad - bc} \right) - \log(c + dx) \right) \right) - 3Bd^4 n(bf - ag)^4 \left( \log(a + bx) \right)}{2b^3d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] ((f + g\*x)^4\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 - (B\*n\*(6\*A\*b\*d\*(b\*c - a\*d)\*g^2\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(-4\*d\*f + c\*g) + b^2\*(6\*d^2\*f^2 - 4\*c\*d\*f\*g + c^2\*g^2))\*x + 6\*B\*d\*(b\*c - a\*d)\*g^2\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(-4\*d\*f + c\*g) + b^2\*(6\*d^2\*f^2 - 4\*c\*d\*f\*g + c^2\*g^2))\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n] + 3\*b^2\*d^2\*(b\*c - a\*d)\*g^3\*(4\*b\*d\*f - b\*c\*g - a\*d\*g)\*x^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 2\*b^3\*d^3\*(b\*c - a\*d)\*g^4\*x^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 6\*d^4\*(b\*f - a\*g)^4\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 6\*B\*(b\*c - a\*d)^2\*g^2\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*

$$(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n*\text{Log}[c + d*x] - 6*b^4*(d*f - c*g)^4*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*\text{Log}[c + d*x] + B*(b*c - a*d)*g^4*n*(b*d*(b*c - a*d)*x*(2*b*c + 2*a*d - b*d*x) + 2*a^3*d^3*\text{Log}[a + b*x] - 2*b^3*c^3*\text{Log}[c + d*x]) - 3*B*(b*c - a*d)*g^3*(-4*b*d*f + b*c*g + a*d*g)*n*(-(a^2*d^2*\text{Log}[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*\text{Log}[c + d*x])) - 3*B*d^4*(b*f - a*g)^4*n*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 3*b^4*B*(d*f - c*g)^4*n*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \text{Log}[c + d*x])* \text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(3*b^4*d^4)/(4*g)$$

**Maple [F]** time = 0.428, size = 0, normalized size = 0.

$$\int (gx + f)^3 \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((g\*x+f)^3\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [B]** time = 3.99702, size = 3579, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out]  $\frac{1}{2}A*B*g^3*x^4*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + \frac{1}{4}A^2*g^3*x^4 + 2*A*B*f*g^2*x^3*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f*g^2*x^3 + 3*A*B*f^2*g*x^2*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + \frac{3}{2}A^2*f^2*g*x^2 - \frac{1}{12}A*B*g^3*n*(6*a^4*\log(b*x + a)/b^4 - 6*c^4*\log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*f*g^2*n*(2*a^3*\log(b*x + a)/b^3 - 2*c^3*\log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*f^2*g*n*(a^2*\log(b*x + a)/b^2 - c^2*\log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*f^3*n*(a*\log(b*x + a)/b - c*\log(d*x + c)/d) + 2*A*B*f^3*x*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f^3*x - \frac{1}{12}(6*a^3*c*d^3*g^3*n^2 - 3*(8*c*d^3*f*g^2*n^2 - c^2*d^2*g^3*n^2)*a^2*b + 2*(18*c*d^3*f^2*g*n^2 - 6*c^2*d^2*f*g^2*n^2 + c^3*d*g^3*n^2)*a*b^2 + (24*c*d^3*f^3*n*\log(e) - (11*g^3*n^2 + 6*g^3*n*\log(e))*c^4 + 12*(3*f*g^2*n^2 + 2*f*g^2*n*\log(e))*c^3*d - 36*(f^2*g*n^2 + f^2*g*n*\log(e))*c^2*d^2)*b^3)*B^2*\log(d*x + c)/(b^3*d^4) + \frac{1}{2}(4*a*b^3*d^4*f^3*n^2 - 6*a^2*b^2*d^4*f^2*g*n^2 + 4*a^3*b*d^4*f*g^2*n^2 - a^4*d^4*g^3*n^2 - (4*c*d^3*f^3*n^2 - 6*c^2*d^2*f^2*g*n^2 + 4*c^3*d*f*g^2*n^2 - c^4*g^3*n^2)*b^4)*(\log(b*x + a)*\log((b*d*x + a*d)/(b*c - a*d) + 1) + \text{dilog}(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d^4) + \frac{1}{12}(3*B^2*b^4*d^4*g^3*x^4*\log(e)^2 + 6*(4*c*d^3*f^3*n^2 - 6*c^2*d^2*f^2*g*n^2 + 4*c^3*d*f*g^2*n^2 - c^4*g^3*n^2)*B^2*b^4*\log(b*x + a)*\log(d*x + c) - 3*(4*c*d^3*f^3*n^2 - 6*c^2*d^2*f^2*g*n^2 + 4*c^3*d*f*g^2*n^2 - c^4*g^3*n^2)*B^2*b^4*\log(d*x + c)^2 + 2*(a*b^3*d^4*g^3*n*\log(e) - (c*d^3*g^3*n*\log(e) - 6*d^4*f*g^2*\log(e)^2)*b^4)*B^2*x^3 + ((g^3*n^2 - 3*g^3*n*\log(e))*a^2*b^2*d^4 - 2*(c*d^3*g^3*n^2 - 6$

```

*d^4*f*g^2*n*log(e))*a*b^3 - (12*c*d^3*f*g^2*n*log(e) - 18*d^4*f^2*g*log(e)
^2 - (g^3*n^2 + 3*g^3*n*log(e))*c^2*d^2)*b^4)*B^2*x^2 - 3*(4*a*b^3*d^4*f^3*
n^2 - 6*a^2*b^2*d^4*f^2*g*n^2 + 4*a^3*b*d^4*f*g^2*n^2 - a^4*d^4*g^3*n^2)*B^
2*log(b*x + a)^2 - ((5*g^3*n^2 - 6*g^3*n*log(e))*a^3*b*d^4 - (5*c*d^3*g^3*n
^2 + 12*(f*g^2*n^2 - 2*f*g^2*n*log(e))*d^4)*a^2*b^2 + (24*c*d^3*f*g^2*n^2 -
5*c^2*d^2*g^3*n^2 - 36*d^4*f^2*g*n*log(e))*a*b^3 + (36*c*d^3*f^2*g*n*log(e)
) - 12*d^4*f^3*log(e)^2 + (5*g^3*n^2 + 6*g^3*n*log(e))*c^3*d - 12*(f*g^2*n^
2 + 2*f*g^2*n*log(e))*c^2*d^2)*b^4)*B^2*x + ((11*g^3*n^2 - 6*g^3*n*log(e))*
a^4*d^4 - 2*(c*d^3*g^3*n^2 + 6*(3*f*g^2*n^2 - 2*f*g^2*n*log(e))*d^4)*a^3*b
+ 3*(4*c*d^3*f*g^2*n^2 - c^2*d^2*g^3*n^2 + 12*(f^2*g*n^2 - f^2*g*n*log(e))*
d^4)*a^2*b^2 - 6*(6*c*d^3*f^2*g*n^2 - 4*c^2*d^2*f*g^2*n^2 + c^3*d*g^3*n^2 -
4*d^4*f^3*n*log(e))*a*b^3)*B^2*log(b*x + a) + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B
^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x)*log((
b*x + a)^n)^2 + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^
4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x)*log((d*x + c)^n)^2 + (6*B^2*b^4*d^4*
g^3*x^4*log(e) - 6*(4*c*d^3*f^3*n - 6*c^2*d^2*f^2*g*n + 4*c^3*d*f*g^2*n - c
^4*g^3*n)*B^2*b^4*log(d*x + c) + 2*(a*b^3*d^4*g^3*n - (c*d^3*g^3*n - 12*d^4
*f*g^2*log(e))*b^4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2*n - a^2*b^2*d^4*g^3*n -
(4*c*d^3*f*g^2*n - c^2*d^2*g^3*n - 12*d^4*f^2*g*log(e))*b^4)*B^2*x^2 + 6*(6
*a*b^3*d^4*f^2*g*n - 4*a^2*b^2*d^4*f*g^2*n + a^3*b*d^4*g^3*n - (6*c*d^3*f^2
*g*n - 4*c^2*d^2*f*g^2*n + c^3*d*g^3*n - 4*d^4*f^3*log(e))*b^4)*B^2*x + 6*(
4*a*b^3*d^4*f^3*n - 6*a^2*b^2*d^4*f^2*g*n + 4*a^3*b*d^4*f*g^2*n - a^4*d^4*g
^3*n)*B^2*log(b*x + a))*log((b*x + a)^n) - (6*B^2*b^4*d^4*g^3*x^4*log(e) -
6*(4*c*d^3*f^3*n - 6*c^2*d^2*f^2*g*n + 4*c^3*d*f*g^2*n - c^4*g^3*n)*B^2*b^4
*log(d*x + c) + 2*(a*b^3*d^4*g^3*n - (c*d^3*g^3*n - 12*d^4*f*g^2*log(e))*b^
4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2*n - a^2*b^2*d^4*g^3*n - (4*c*d^3*f*g^2*n
- c^2*d^2*g^3*n - 12*d^4*f^2*g*log(e))*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g*
n - 4*a^2*b^2*d^4*f*g^2*n + a^3*b*d^4*g^3*n - (6*c*d^3*f^2*g*n - 4*c^2*d^2*
f*g^2*n + c^3*d*g^3*n - 4*d^4*f^3*log(e))*b^4)*B^2*x + 6*(4*a*b^3*d^4*f^3*
n - 6*a^2*b^2*d^4*f^2*g*n + 4*a^3*b*d^4*f*g^2*n - a^4*d^4*g^3*n)*B^2*log(b*x
+ a) + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^
2*g*x^2 + 4*B^2*b^4*d^4*f^3*x)*log((b*x + a)^n))*log((d*x + c)^n))/(b^4*d^4
)

```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2 g^3 x^3 + 3 A^2 f g^2 x^2 + 3 A^2 f^2 g x + A^2 f^3 + (B^2 g^3 x^3 + 3 B^2 f g^2 x^2 + 3 B^2 f^2 g x + B^2 f^3) \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right) \right)^2 + 2 \left( \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^3
*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log(e*((b*x + a)/(d*x + c)
))^n)^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*log(e
*((b*x + a)/(d*x + c))^n), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (gx + f)^3 \left( B \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

$$3.68 \quad \int (f + gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx$$

**Optimal.** Leaf size=565

$$\frac{2B^2n^2(bc - ad)(a^2d^2g^2 - abdg(3df - cg) + b^2(c^2g^2 - 3cdfg + 3d^2f^2)) \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right) + 2Bn(bc - ad)(a^2d^2g^2 - a}{3b^3d^3}$$

[Out] (B^2\*(b\*c - a\*d)^2\*g^2\*n^2\*x)/(3\*b^2\*d^2) - (2\*B\*(b\*c - a\*d)\*g\*(3\*b\*d\*f - 2\*b\*c\*g - a\*d\*g)\*n\*(a + b\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(3\*b^3\*d^2) - (B\*(b\*c - a\*d)\*g^2\*n\*(c + d\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(3\*b\*d^3) - ((b\*f - a\*g)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2)/(3\*b^3\*g) + ((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2)/(3\*g) + (2\*B\*(b\*c - a\*d)\*(a^2\*d^2\*g^2 - a\*b\*d\*g\*(3\*d\*f - c\*g) + b^2\*(3\*d^2\*f^2 - 3\*c\*d\*f\*g + c^2\*g^2))\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[(b\*c - a\*d)/(b\*(c + d\*x)))]/(3\*b^3\*d^3) + (B^2\*(b\*c - a\*d)^3\*g^2\*n^2\*Log[(a + b\*x)/(c + d\*x)])/(3\*b^3\*d^3) + (B^2\*(b\*c - a\*d)^3\*g^2\*n^2\*Log[c + d\*x])/(3\*b^3\*d^3) + (2\*B^2\*(b\*c - a\*d)^2\*g\*(3\*b\*d\*f - 2\*b\*c\*g - a\*d\*g)\*n^2\*Log[c + d\*x])/(3\*b^3\*d^3) + (2\*B^2\*(b\*c - a\*d)\*(a^2\*d^2\*g^2 - a\*b\*d\*g\*(3\*d\*f - c\*g) + b^2\*(3\*d^2\*f^2 - 3\*c\*d\*f\*g + c^2\*g^2))\*n^2\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(3\*b^3\*d^3)

**Rubi [A]** time = 1.15004, antiderivative size = 699, normalized size of antiderivative = 1.24, number of steps used = 27, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {2525, 12, 2528, 2486, 31, 72, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2B^2n^2(bf - ag)^3 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right) - 2B^2n^2(df - cg)^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right) + a^2B^2g^2n^2(bc - ad) \log(a + bx) - 2AB}{3b^3g} - \frac{2B^2n^2(df - cg)^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right) + a^2B^2g^2n^2(bc - ad) \log(a + bx) - 2AB}{3d^3g} + \frac{a^2B^2g^2n^2(bc - ad) \log(a + bx) - 2AB}{3b^3d} - \frac{2AB}{3b^3d}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] (-2\*A\*B\*(b\*c - a\*d)\*g\*(3\*b\*d\*f - b\*c\*g - a\*d\*g)\*n\*x)/(3\*b^2\*d^2) + (B^2\*(b\*c - a\*d)^2\*g^2\*n^2\*x)/(3\*b^2\*d^2) + (a^2\*B^2\*(b\*c - a\*d)\*g^2\*n^2\*Log[a + b\*x])/(3\*b^3\*d) + (B^2\*(b\*f - a\*g)^3\*n^2\*Log[a + b\*x]^2)/(3\*b^3\*g) - (2\*B^2\*(b\*c - a\*d)\*g\*(3\*b\*d\*f - b\*c\*g - a\*d\*g)\*n\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n])/(3\*b^3\*d^2) - (B\*(b\*c - a\*d)\*g^2\*n\*x^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(3\*b\*d) - (2\*B\*(b\*f - a\*g)^3\*n\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(3\*b^3\*g) + ((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2)/(3\*g) - (B^2\*c^2\*(b\*c - a\*d)\*g^2\*n^2\*Log[c + d\*x])/(3\*b\*d^3) + (2\*B^2\*(b\*c - a\*d)^2\*g\*(3\*b\*d\*f - b\*c\*g - a\*d\*g)\*n^2\*Log[c + d\*x])/(3\*b^3\*d^3) - (2\*B^2\*(d\*f - c\*g)^3\*n^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x])/(3\*d^3\*g) + (2\*B\*(d\*f - c\*g)^3\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x])/(3\*d^3\*g) + (B^2\*(d\*f - c\*g)^3\*n^2\*Log[c + d\*x]^2)/(3\*d^3\*g) - (2\*B^2\*(b\*f - a\*g)^3\*n^2\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(3\*b^3\*g) - (2\*B^2\*(b\*f - a\*g)^3\*n^2\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(3\*b^3\*g) - (2\*B^2\*(d\*f - c\*g)^3\*n^2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/(3\*d^3\*g)

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1))



, x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 2528

Int[((a\_) + Log[(c\_)\*(RFx\_)^(p\_)]\*(b\_))^(n\_)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

### Rule 2486

Int[Log[(e\_)\*((f\_)\*(a\_) + (b\_)\*(x\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(q\_)]^(r\_)]^(s\_), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 72

Int[((e\_) + (f\_)\*(x\_))^(p\_)/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

### Rule 2524

Int[((a\_) + Log[(c\_)\*(RFx\_)^(p\_)]\*(b\_))^(n\_)/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

### Rule 2418

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)]\*(b\_))^(p\_)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

### Rule 2390

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)]\*(b\_))^(p\_)\*((f\_) + (g\_)\*(x\_))^(q\_), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps

$$\begin{aligned}
\int (f + gx)^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2 dx &= \frac{(f + gx)^3 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{3g} - \frac{(2Bn) \int \frac{(bc - ad)(f + gx)^3 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{(a + bx)(c + dx)}}{3g} \\
&= \frac{(f + gx)^3 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{3g} - \frac{(2B(bc - ad)n) \int \frac{(f + gx)^3 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{(a + bx)(c + dx)}}{3g} \\
&= \frac{(f + gx)^3 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{3g} - \frac{(2B(bc - ad)n) \int \left( \frac{g^2(3bdf - bcg - adg)}{b^2} \right)}{3g} \\
&= \frac{(f + gx)^3 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)^2}{3g} - \frac{(2B(bc - ad)g^2n) \int x \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{3bd} \\
&= -\frac{2AB(bc - ad)g(3bdf - bcg - adg)nx}{3b^2d^2} - \frac{B(bc - ad)g^2nx^2 \left( A + B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) \right)}{3bd} \\
&= -\frac{2AB(bc - ad)g(3bdf - bcg - adg)nx}{3b^2d^2} - \frac{2B^2(bc - ad)g(3bdf - bcg - adg)nx^2}{3b^3d} \\
&= -\frac{2AB(bc - ad)g(3bdf - bcg - adg)nx}{3b^2d^2} - \frac{2B^2(bc - ad)g(3bdf - bcg - adg)nx^2}{3b^3d} \\
&= -\frac{2AB(bc - ad)g(3bdf - bcg - adg)nx}{3b^2d^2} + \frac{B^2(bc - ad)^2g^2n^2x}{3b^2d^2} + \frac{a^2B^2(bc - ad)}{3b^3d} \\
&= -\frac{2AB(bc - ad)g(3bdf - bcg - adg)nx}{3b^2d^2} + \frac{B^2(bc - ad)^2g^2n^2x}{3b^2d^2} + \frac{a^2B^2(bc - ad)}{3b^3d} \\
&= -\frac{2AB(bc - ad)g(3bdf - bcg - adg)nx}{3b^2d^2} + \frac{B^2(bc - ad)^2g^2n^2x}{3b^2d^2} + \frac{a^2B^2(bc - ad)}{3b^3d}
\end{aligned}$$

**Mathematica [A]** time = 0.565092, size = 506, normalized size = 0.9

$$\frac{(f + gx)^3 \left( B \log \left( e \left( \frac{a + bx}{c + dx} \right)^n \right) + A \right)^2 - \frac{Bn \left( b^3 B n (df - cg)^3 \left( 2 \text{PolyLog} \left( 2, \frac{b(c + dx)}{bc - ad} \right) + \log(c + dx) \left( 2 \log \left( \frac{d(a + bx)}{ad - bc} \right) - \log(c + dx) \right) \right) - B d^3 n (bf - ag)^3 \left( \log(a + bx) \right)}{3b^2d^2}}{3b^2d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2,x]

[Out] ((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 - (B\*n\*(2\*A\*b\*d\*(b\*c - a\*d)\*g^2\*(3\*b\*d\*f - b\*c\*g - a\*d\*g)\*x + 2\*B\*d\*(b\*c - a\*d)\*g^2\*(3\*b\*d\*f - b\*c\*g - a\*d\*g)\*(a + b\*x)\*Log[e\*((a + b\*x)/(c + d\*x))^n] + b^2\*d^2\*(b\*c - a\*d)\*g^3\*x^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 2\*d^3\*(b\*f - a\*g)^3\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 2\*B\*(b\*c - a\*d)^2\*g^2\*(-3\*b\*d\*f + b\*c\*g + a\*d\*g)\*n\*Log[c + d\*x] - 2\*b^3\*(d\*f - c\*g)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x] - B\*(b\*c - a\*d)\*g^3\*n\*(a^2\*d^2\*Log[a + b\*x] - b\*(d\*(-(b\*c) + a\*d)\*x + b\*c^2\*Log[c + d\*x])) - B\*d^3\*(b\*f - a\*g)^3

```
*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Poly
Log[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b^3*B*(d*f - c*g)^3*n*((2*Log[(d*(a
+ b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)])))/(b^3*d^3))/(3*g)
```

**Maple [F]** time = 0.421, size = 0, normalized size = 0.

$$\int (gx + f)^2 \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int((g*x+f)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Maxima [B]** time = 3.78847, size = 2240, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima
")
```

```
[Out] 2/3*A*B*g^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*g^2*x^3 +
2*A*B*f*g*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f*g*x^2 + 1/3*A*
B*g^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*
d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*A*B*f*g*n*(a^2*log(b*x +
a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*f^2*n*(a*log(
b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*f^2*x*log(e*(b*x/(d*x + c) + a/(d*x
+ c))^n) + A^2*f^2*x + 1/3*(2*a^2*c*d^2*g^2*n^2 - (6*c*d^2*f*g*n^2 - c^2*d*
g^2*n^2)*a*b - (6*c*d^2*f^2*n*log(e) + (3*g^2*n^2 + 2*g^2*n*log(e))*c^3 - 6
*(f*g*n^2 + f*g*n*log(e))*c^2*d)*b^2)*B^2*log(d*x + c)/(b^2*d^3) + 2/3*(3*a
*b^2*d^3*f^2*n^2 - 3*a^2*b*d^3*f*g*n^2 + a^3*d^3*g^2*n^2 - (3*c*d^2*f^2*n^2
- 3*c^2*d*f*g*n^2 + c^3*g^2*n^2)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
- a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1/3*(B^2*
b^3*d^3*g^2*x^3*log(e)^2 + 2*(3*c*d^2*f^2*n^2 - 3*c^2*d*f*g*n^2 + c^3*g^2*n
^2)*B^2*b^3*log(b*x + a)*log(d*x + c) - (3*c*d^2*f^2*n^2 - 3*c^2*d*f*g*n^2
+ c^3*g^2*n^2)*B^2*b^3*log(d*x + c)^2 + (a*b^2*d^3*g^2*n*log(e) - (c*d^2*g^
2*n*log(e) - 3*d^3*f*g*log(e)^2)*b^3)*B^2*x^2 - (3*a*b^2*d^3*f^2*n^2 - 3*a^
2*b*d^3*f*g*n^2 + a^3*d^3*g^2*n^2)*B^2*log(b*x + a)^2 + ((g^2*n^2 - 2*g^2*n
*log(e))*a^2*b*d^3 - 2*(c*d^2*g^2*n^2 - 3*d^3*f*g*n*log(e))*a*b^2 - (6*c*d^
2*f*g*n*log(e) - 3*d^3*f^2*log(e)^2 - (g^2*n^2 + 2*g^2*n*log(e))*c^2*d)*b^3
)*B^2*x - ((3*g^2*n^2 - 2*g^2*n*log(e))*a^3*d^3 - (c*d^2*g^2*n^2 + 6*(f*g*n
^2 - f*g*n*log(e))*d^3)*a^2*b + 2*(3*c*d^2*f*g*n^2 - c^2*d*g^2*n^2 - 3*d^3*
f^2*n*log(e))*a*b^2)*B^2*log(b*x + a) + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^
3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*g^2*x^3
+ 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x)*log((d*x + c)^n)^2 + (2*B^2*
b^3*d^3*g^2*x^3*log(e) - 2*(3*c*d^2*f^2*n - 3*c^2*d*f*g*n + c^3*g^2*n)*B^2*
b^3*log(d*x + c) + (a*b^2*d^3*g^2*n - (c*d^2*g^2*n - 6*d^3*f*g*log(e))*b^3)
*B^2*x^2 + 2*(3*a*b^2*d^3*f*g*n - a^2*b*d^3*g^2*n - (3*c*d^2*f*g*n - c^2*d*
g^2*n - 3*d^3*f^2*log(e))*b^3)*B^2*x + 2*(3*a*b^2*d^3*f^2*n - 3*a^2*b*d^3*f
*g*n + a^3*d^3*g^2*n)*B^2*log(b*x + a))*log((b*x + a)^n) - (2*B^2*b^3*d^3*g
```

$$\begin{aligned} & ^2*x^3*\log(e) - 2*(3*c*d^2*f^2*n - 3*c^2*d*f*g*n + c^3*g^2*n)*B^2*b^3*\log(d \\ & *x + c) + (a*b^2*d^3*g^2*n - (c*d^2*g^2*n - 6*d^3*f*g*\log(e))*b^3)*B^2*x^2 \\ & + 2*(3*a*b^2*d^3*f*g*n - a^2*b*d^3*g^2*n - (3*c*d^2*f*g*n - c^2*d*g^2*n - 3 \\ & *d^3*f^2*\log(e))*b^3)*B^2*x + 2*(3*a*b^2*d^3*f^2*n - 3*a^2*b*d^3*f*g*n + a^ \\ & 3*d^3*g^2*n)*B^2*\log(b*x + a) + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g* \\ & x^2 + 3*B^2*b^3*d^3*f^2*x)*\log((b*x + a)^n)*\log((d*x + c)^n)/(b^3*d^3) \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(A^2g^2x^2 + 2A^2fgx + A^2f^2 + (B^2g^2x^2 + 2B^2fgx + B^2f^2)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2(ABg^2x^2 + 2ABfgx + ABf^2)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*x + B^2*f^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*x + A*B*f^2)*log(e*((b*x + a)/(d*x + c))^n), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (gx + f)^2 \left( B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

$$3.69 \quad \int (f + gx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx$$

**Optimal.** Leaf size=290

$$\frac{B^2 n^2 (bc - ad)(-adg - bcg + 2bdf) \text{PolyLog} \left( 2, \frac{d(a+bx)}{b(c+dx)} \right)}{b^2 d^2} + \frac{Bn(bc - ad)(-adg - bcg + 2bdf) \log \left( \frac{bc-ad}{b(c+dx)} \right) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{b^2 d^2}$$

[Out]  $-\left(\left(B*(b*c - a*d)*g*n*(a + b*x)*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])\right)\right)/(b^2*d) - \left(\left(b*f - a*g\right)^2*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2\right)/(2*b^2*g) + \left(\left(f + g*x\right)^2*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2\right)/(2*g) + (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*\text{Log}[(b*c - a*d)/(b*(c + d*x))])/(b^2*d^2) + (B^2*(b*c - a*d)^2*g*n^2*\text{Log}[c + d*x])/(b^2*d^2) + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n^2*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)$

**Rubi [A]** time = 0.831266, antiderivative size = 481, normalized size of antiderivative = 1.66, number of steps used = 23, number of rules used = 12, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2 n^2 (bf - ag)^2 \text{PolyLog} \left( 2, -\frac{d(a+bx)}{bc-ad} \right)}{b^2 g} - \frac{B^2 n^2 (df - cg)^2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right)}{d^2 g} - \frac{Bn(bf - ag)^2 \log(a + bx) \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{b^2 g}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2, x]$

[Out]  $-\left(\left(A*B*(b*c - a*d)*g*n*x\right)\right)/(b*d) + (B^2*(b*f - a*g)^2*n^2*\text{Log}[a + b*x]^2)/(2*b^2*g) - (B^2*(b*c - a*d)*g*n*(a + b*x)*\text{Log}[e*((a + b*x)/(c + d*x))^n])/(b^2*d) - (B*(b*f - a*g)^2*n*\text{Log}[a + b*x]*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]))/(b^2*g) + ((f + g*x)^2*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2)/(2*g) + (B^2*(b*c - a*d)^2*g*n^2*\text{Log}[c + d*x])/(b^2*d^2) - (B^2*(d*f - c*g)^2*n^2*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(d^2*g) + (B*(d*f - c*g)^2*n*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*\text{Log}[c + d*x])/(d^2*g) + (B^2*(d*f - c*g)^2*n^2*\text{Log}[c + d*x]^2)/(2*d^2*g) - (B^2*(b*f - a*g)^2*n^2*\text{Log}[a + b*x]*\text{Log}[(b*(c + d*x))/(b*c - a*d)])/(b^2*g) - (B^2*(b*f - a*g)^2*n^2*\text{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))])/(b^2*g) - (B^2*(d*f - c*g)^2*n^2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(d^2*g)$

### Rule 2525

$\text{Int}[(a_.) + \text{Log}[(c_.)*(RfX_)^(p_.)]*(b_.)]^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x\_Symbol] :> \text{Simp}[(d + e*x)^(m + 1)*(a + b*\text{Log}[c*RfX^p])^n/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^(m + 1)*(a + b*\text{Log}[c*RfX^p])^(n - 1)*D[RfX, x])/RfX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RfX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] :> \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 2528

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2524

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]

Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(Rfx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

**Rule 2391**

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned} \int (f + gx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx &= \frac{(f + gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2g} - \frac{(Bn) \int \frac{(bc-ad)(f+gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{(a+bx)(c+dx)} dx}{g} \\ &= \frac{(f + gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2g} - \frac{(B(bc - ad)n) \int \frac{(f+gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{(a+bx)(c+dx)} dx}{g} \\ &= \frac{(f + gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2g} - \frac{(B(bc - ad)n) \int \left( \frac{g^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{bd} + \dots \right)}{g} \\ &= \frac{(f + gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2}{2g} - \frac{(B(bc - ad)gn) \int \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{bd} \\ &= -\frac{AB(bc - ad)gnx}{bd} - \frac{B(bf - ag)^2 n \log(a + bx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{b^2 g} + \dots \\ &= -\frac{AB(bc - ad)gnx}{bd} - \frac{B^2(bc - ad)gn(a + bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b^2 d} - \frac{B(bf - ag)^2 n \log(a + bx)}{b^2 g} \\ &= -\frac{AB(bc - ad)gnx}{bd} - \frac{B^2(bc - ad)gn(a + bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b^2 d} - \frac{B(bf - ag)^2 n \log(a + bx)}{b^2 g} \\ &= -\frac{AB(bc - ad)gnx}{bd} - \frac{B^2(bc - ad)gn(a + bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b^2 d} - \frac{B(bf - ag)^2 n \log(a + bx)}{b^2 g} \\ &= -\frac{AB(bc - ad)gnx}{bd} + \frac{B^2(bf - ag)^2 n^2 \log^2(a + bx)}{2b^2 g} - \frac{B^2(bc - ad)gn(a + bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b^2 d} \\ &= -\frac{AB(bc - ad)gnx}{bd} + \frac{B^2(bf - ag)^2 n^2 \log^2(a + bx)}{2b^2 g} - \frac{B^2(bc - ad)gn(a + bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{b^2 d} \end{aligned}$$

**Mathematica [A]** time = 0.32811, size = 362, normalized size = 1.25

$$(f + gx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)^2 - \frac{Bn \left( b^2 Bn(df - cg)^2 \left( 2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) - Bd^2 n(bf - ag)^2 \left( \log(a+bx) \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + \dots \right)}{b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

```
[Out] ((f + g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*n*(2*A*b*d*(b*c - a*d)*g^2*x + 2*B*d*(b*c - a*d)*g^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])
```



$$n] + 2*d^2*(b*f - a*g)^2*\text{Log}[a + b*x]*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*g^2*n*\text{Log}[c + d*x] - 2*b^2*(d*f - c*g)^2*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*\text{Log}[c + d*x] - B*d^2*(b*f - a*g)^2*n*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*n*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b^2*d^2)/(2*g)$$

**Maple [F]** time = 0.274, size = 0, normalized size = 0.

$$\int (gx + f) \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((g\*x+f)\*(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [B]** time = 3.50015, size = 1214, normalized size = 4.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out]  $A*B*g*x^2*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*g*x^2 - A*B*g*n*(a^2*\log(b*x + a)/b^2 - c^2*\log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*f*n*(a*\log(b*x + a)/b - c*\log(d*x + c)/d) + 2*A*B*f*x*\log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*f*x - (a*c*d*g*n^2 + (2*c*d*f*n*\log(e) - (g*n^2 + g*n*\log(e))*c^2)*b)*B^2*\log(d*x + c)/(b*d^2) + (2*a*b*d^2*f*n^2 - a^2*d^2*g*n^2 - (2*c*d*f*n^2 - c^2*g*n^2)*b^2)*(\log(b*x + a)*\log((b*d*x + a*d)/(b*c - a*d) + 1) + \text{dilog}(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*\log(e)^2 + 2*(2*c*d*f*n^2 - c^2*g*n^2)*B^2*b^2*\log(b*x + a)*\log(d*x + c) - (2*c*d*f*n^2 - c^2*g*n^2)*B^2*b^2*\log(d*x + c)^2 - (2*a*b*d^2*f*n^2 - a^2*d^2*g*n^2)*B^2*\log(b*x + a)^2 + 2*(a*b*d^2*g*n*\log(e) - (c*d*g*n*\log(e) - d^2*f*\log(e)^2)*b^2)*B^2*x + 2*((g*n^2 - g*n*\log(e))*a^2*d^2 - (c*d*g*n^2 - 2*d^2*f*n*\log(e))*a*b)*B^2*\log(b*x + a) + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x)*\log((b*x + a)^n)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x)*\log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*g*x^2*\log(e) - (2*c*d*f*n - c^2*g*n)*B^2*b^2*\log(d*x + c) + (a*b*d^2*g*n - (c*d*g*n - 2*d^2*f*\log(e))*b^2)*B^2*x + (2*a*b*d^2*f*n - a^2*d^2*g*n)*B^2*\log(b*x + a))*\log((b*x + a)^n) - 2*(B^2*b^2*d^2*g*x^2*\log(e) - (2*c*d*f*n - c^2*g*n)*B^2*b^2*\log(d*x + c) + (a*b*d^2*g*n - (c*d*g*n - 2*d^2*f*\log(e))*b^2)*B^2*x + (2*a*b*d^2*f*n - a^2*d^2*g*n)*B^2*\log(b*x + a) + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x)*\log((b*x + a)^n))*\log((d*x + c)^n))/(b^2*d^2)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2gx + A^2f + (B^2gx + B^2f) \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right)^2 + 2(ABgx + ABf) \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log(e*((b*x + a)/(d*x + c))^n)
^2 + 2*(A*B*g*x + A*B*f)*log(e*((b*x + a)/(d*x + c))^n), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (gx + f) \left( B \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

$$3.70 \quad \int \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx$$

**Optimal.** Leaf size=135

$$\frac{2B^2n^2(bc-ad)\text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd} + \frac{2Bn(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{bd} + \frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{b}$$

[Out] ((a + b\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2)/b + (2\*B\*(b\*c - a\*d)\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[(b\*c - a\*d)/(b\*(c + d\*x))])/(b\*d) + (2\*B^2\*(b\*c - a\*d)\*n^2\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*d)

**Rubi [B]** time = 0.617333, antiderivative size = 275, normalized size of antiderivative = 2.04, number of steps used = 20, number of rules used = 10, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$ , Rules used = {2523, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2aB^2n^2\text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{b} + \frac{2B^2cn^2\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{d} + \frac{2aBn\log(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{b} - \frac{2Bcn\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

[Out] -((a\*B^2\*n^2\*Log[a + b\*x]^2)/b) + (2\*a\*B\*n\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/b + x\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 + (2\*B^2\*c\*n^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x])/d - (2\*B\*c\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x])/d - (B^2\*c\*n^2\*Log[c + d\*x]^2)/d + (2\*a\*B^2\*n^2\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]/b) + (2\*a\*B^2\*n^2\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))]/b) + (2\*B^2\*c\*n^2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]/d)

#### Rule 2523

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^n, x\_Symbol] := Simp[x\*(a + b\*Log[c\*RFx^p])^n, x] - Dist[b\*n\*p, Int[SimplifyIntegrand[(x\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^n\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^n/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e

```
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :=> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(q_.), x_Symbol] :=> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :=> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :=> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :=> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :=> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps

$$\begin{aligned}
\int \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 dx &= x \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 - (2Bn) \int \frac{(bc-ad)x \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{(a+bx)(c+dx)} dx \\
&= x \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 - (2B(bc-ad)n) \int \frac{x \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{(a+bx)(c+dx)} dx \\
&= x \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 - (2B(bc-ad)n) \int \left[ -\frac{a \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{(bc-ad)(a+bx)} \right] + \\
&= x \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 + (2aBn) \int \frac{A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right)}{a+bx} dx - (2Bcn) \int \frac{A}{a+bx} dx \\
&= \frac{2aBn \log(a+bx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{b} + x \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 - \frac{2Bcn}{b} \int \frac{A}{a+bx} dx \\
&= \frac{2aBn \log(a+bx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{b} + x \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 - \frac{2Bcn}{b} \int \frac{A}{a+bx} dx \\
&= \frac{2aBn \log(a+bx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{b} + x \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 - \frac{2Bcn}{b} \int \frac{A}{a+bx} dx \\
&= \frac{2aBn \log(a+bx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{b} + x \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 - \frac{2B^2cn}{b} \int \frac{A}{a+bx} dx \\
&= -\frac{aB^2n^2 \log^2(a+bx)}{b} + \frac{2aBn \log(a+bx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{b} + x \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2 \\
&= -\frac{aB^2n^2 \log^2(a+bx)}{b} + \frac{2aBn \log(a+bx) \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)}{b} + x \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2
\end{aligned}$$

**Mathematica [A]** time = 0.170024, size = 226, normalized size = 1.67

$$\frac{Bn \left( -aBdn \left( \log(a+bx) \left( \log(a+bx) - 2 \log \left( \frac{b(c+dx)}{bc-ad} \right) \right) - 2 \text{PolyLog} \left( 2, \frac{d(a+bx)}{ad-bc} \right) \right) + bBcn \left( 2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log \right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

[Out] x\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 + (B\*n\*(2\*a\*d\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 2\*b\*c\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x] - a\*B\*d\*n\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c + a\*d)]) + b\*B\*c\*n\*(2\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]))/(b\*d)

**Maple [F]** time = 0.309, size = 0, normalized size = 0.

$$\int \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2ABn \left( \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) + 2ABx \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2x + B^2 \left( \frac{2bcn^2 \log(bx+a) \log(dx+c) - bcn^2 \log^2(dx+c)}{b^2d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out] 2\*A\*B\*n\*(a\*log(b\*x + a)/b - c\*log(d\*x + c)/d) + 2\*A\*B\*x\*log(e\*((b\*x + a)/(d\*x + c))^n) + A^2\*x + B^2\*((2\*b\*c\*n^2\*log(b\*x + a)\*log(d\*x + c) - b\*c\*n^2\*log(d\*x + c)^2 + b\*d\*x\*log((b\*x + a)^n)^2 + b\*d\*x\*log((d\*x + c)^n)^2 + 2\*(a\*d\*n\*log(b\*x + a) - b\*c\*n\*log(d\*x + c) + b\*d\*x\*log(e))\*log((b\*x + a)^n) - 2\*(a\*d\*n\*log(b\*x + a) - b\*c\*n\*log(d\*x + c) + b\*d\*x\*log((b\*x + a)^n) + b\*d\*x\*log(e))\*log((d\*x + c)^n))/(b\*d) - integrate(-(b^2\*d\*x^2\*log(e)^2 + a\*b\*c\*log(e)^2 - ((2\*n\*log(e) - log(e)^2)\*b^2\*c - (2\*n\*log(e) + log(e)^2)\*a\*b\*d)\*x - 2\*(b^2\*c\*n^2\*x + 2\*a\*b\*c\*n^2 - a^2\*d\*n^2)\*log(b\*x + a))/(b^2\*d\*x^2 + a\*b\*c + (b^2\*c + a\*b\*d)\*x), x))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( B^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2AB \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out] integral(B^2\*log(e\*((b\*x + a)/(d\*x + c))^n)^2 + 2\*A\*B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c)\*\*n))\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2, x)

$$3.71 \quad \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx} dx$$

**Optimal.** Leaf size=297

$$\frac{2Bn \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{g} - \frac{2Bn \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{g} - 2B^2 n^2 \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)$$

[Out] -(((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2\*Log[(b\*c - a\*d)/(b\*(c + d\*x))])/g) + ((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2\*Log[1 - ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/g - (2\*B\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/g + (2\*B\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*PolyLog[2, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/g + (2\*B^2\*n^2\*PolyLog[3, (d\*(a + b\*x))/(b\*(c + d\*x))])/g - (2\*B^2\*n^2\*PolyLog[3, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/g

**Rubi [B]** time = 5.18411, antiderivative size = 2233, normalized size of antiderivative = 7.52, number of steps used = 43, number of rules used = 21, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.656$ , Rules used = {2524, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 12, 6742, 2500, 2433, 2375, 2317, 2374, 6589, 2440, 2437, 2435, 2315}

result too large to display

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(f + g\*x), x]

[Out] (-2\*A\*B\*n\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*Log[f + g\*x])/g - (B^2\*Log[(a + b\*x)^n]^2\*Log[f + g\*x])/g + ((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2\*Log[f + g\*x])/g + (2\*B^2\*n^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x]\*Log[f + g\*x])/g + (2\*B^2\*n^2\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*Log[f + g\*x])/g + (2\*A\*B\*n\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*Log[f + g\*x])/g - (2\*B^2\*n\*(n\*Log[a + b\*x] - Log[(a + b\*x)^n])\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*Log[f + g\*x])/g - (B^2\*Log[(c + d\*x)^(-n)]^2\*Log[f + g\*x])/g + (2\*B^2\*n\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*(Log[(a + b\*x)^n] - Log[e\*((a + b\*x)/(c + d\*x))^n] + Log[(c + d\*x)^(-n)]\*Log[f + g\*x])/g - (2\*B^2\*n\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*(Log[(a + b\*x)^n] - Log[e\*((a + b\*x)/(c + d\*x))^n] + Log[(c + d\*x)^(-n)]\*Log[f + g\*x])/g - (2\*B^2\*n\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*(n\*Log[c + d\*x] + Log[(c + d\*x)^(-n)]\*Log[f + g\*x])/g + (B^2\*Log[(a + b\*x)^n]^2\*Log[(b\*(f + g\*x))/(b\*f - a\*g))]/g + (B^2\*Log[(c + d\*x)^(-n)]^2\*Log[(d\*(f + g\*x))/(d\*f - c\*g)]/g + (B^2\*n^2\*(Log[(b\*(c + d\*x))/(b\*c - a\*d)] + Log[(b\*f - a\*g)/(b\*(f + g\*x))] - Log[((b\*f - a\*g)\*(c + d\*x))/((b\*c - a\*d)\*(f + g\*x))])\*Log[-(((b\*c - a\*d)\*(f + g\*x))/((d\*f - c\*g)\*(a + b\*x)))]^2)/g - (B^2\*n^2\*(Log[(b\*(c + d\*x))/(b\*c - a\*d)] - Log[-((g\*(c + d\*x))/(d\*f - c\*g))])\*(Log[a + b\*x] + Log[-(((b\*c - a\*d)\*(f + g\*x))/((d\*f - c\*g)\*(a + b\*x)))]^2)/g + (B^2\*n^2\*(Log[-((d\*(a + b\*x))/(b\*c - a\*d))] + Log[(d\*f - c\*g)/(d\*(f + g\*x))] - Log[-(((d\*f - c\*g)\*(a + b\*x))/((b\*c - a\*d)\*(f + g\*x)))]\*Log[(b\*c - a\*d)\*(f + g\*x))/((b\*f - a\*g)\*(c + d\*x))]^2)/g - (B^2\*n^2\*(Log[-((d\*(a + b\*x))/(b\*c - a\*d))] - Log[-((g\*(a + b\*x))/(b\*f - a\*g))])\*Log[c + d\*x] + Log[(b\*c - a\*d)\*(f + g\*x))/((b\*f - a\*g)\*(c + d\*x))]^2)/g + (2\*B^2\*n^2\*(Log[f + g\*x] - Log[-(((b\*c - a\*d)\*(f + g\*x))/((d\*f - c\*g)\*(a + b\*x)))]\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/g + (2\*B^2\*n\*Log[(a + b\*x)^n]\*PolyLog[2, -((g\*(a + b\*x))/(b\*f - a\*g))])/g + (2\*B^2\*n^2\*(Log[f + g\*x] - Log[(b\*c - a\*d)\*(f + g\*x))/((b\*f - a\*g)\*(c + d\*x))])\*PolyLog[2, (b\*(c + d\*x))/(b\*



$$\begin{aligned}
& c - a*d))/g - (2*B^2*n*Log[(c + d*x)^{-n}]*PolyLog[2, -((g*(c + d*x))/(d*f \\
& - c*g))])/g - (2*B^2*n^2*Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b \\
& *x)))]*PolyLog[2, (g*(a + b*x))/(b*(f + g*x))])/g + (2*B^2*n^2*Log[-(((b*c \\
& - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))]*PolyLog[2, -(((d*f - c*g)*(a + \\
& b*x))/((b*c - a*d)*(f + g*x)))])/g - (2*B^2*n^2*Log[((b*c - a*d)*(f + g*x)) \\
& /((b*f - a*g)*(c + d*x))]*PolyLog[2, (g*(c + d*x))/(d*(f + g*x))])/g + (2*B \\
& ^2*n^2*Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))]*PolyLog[2, ((b* \\
& f - a*g)*(c + d*x))/((b*c - a*d)*(f + g*x))])/g - (2*A*B*n*PolyLog[2, (b*(f \\
& + g*x))/(b*f - a*g)])/g + (2*B^2*n*(Log[(a + b*x)^n] - Log[e*((a + b*x)/(c \\
& + d*x))^n] + Log[(c + d*x)^{-n}])*PolyLog[2, (b*(f + g*x))/(b*f - a*g)]/g \\
& - (2*B^2*n*(n*Log[c + d*x] + Log[(c + d*x)^{-n}])*PolyLog[2, (b*(f + g*x)) \\
& /((b*f - a*g))])/g + (2*B^2*n^2*(Log[c + d*x] + Log[((b*c - a*d)*(f + g*x))/ \\
& (b*f - a*g)*(c + d*x)])*PolyLog[2, (b*(f + g*x))/(b*f - a*g)]/g + (2*A*B* \\
& n*PolyLog[2, (d*(f + g*x))/(d*f - c*g)]/g - (2*B^2*n*(n*Log[a + b*x] - Log \\
& [(a + b*x)^n] - Log[e*((a + b*x)/(c + d*x))^n] + Log[(c + d*x)^{-n}])*PolyLog[2 \\
& , (d*(f + g*x))/(d*f - c*g)]/g + (2*B^2*n^2*(Log[a + b*x] + Log[-(((b*c - \\
& a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))])*PolyLog[2, (d*(f + g*x))/(d*f - \\
& c*g)]/g - (2*B^2*n^2*PolyLog[3, -(((d*(a + b*x))/(b*c - a*d))])/g - (2*B^2*n \\
& ^2*PolyLog[3, -((g*(a + b*x))/(b*f - a*g))])/g - (2*B^2*n^2*PolyLog[3, (b* \\
& (c + d*x))/(b*c - a*d)]/g - (2*B^2*n^2*PolyLog[3, -((g*(c + d*x))/(d*f - c \\
& *g))])/g - (2*B^2*n^2*PolyLog[3, (g*(a + b*x))/(b*(f + g*x))])/g + (2*B^2*n \\
& ^2*PolyLog[3, -(((d*f - c*g)*(a + b*x))/((b*c - a*d)*(f + g*x)))]/g - (2*B \\
& ^2*n^2*PolyLog[3, (g*(c + d*x))/(d*(f + g*x))])/g + (2*B^2*n^2*PolyLog[3, ( \\
& (b*f - a*g)*(c + d*x))/((b*c - a*d)*(f + g*x))])/g - (2*B^2*n^2*PolyLog[3, \\
& (b*(f + g*x))/(b*f - a*g)]/g - (2*B^2*n^2*PolyLog[3, (d*(f + g*x))/(d*f - \\
& c*g)]/g
\end{aligned}$$

#### Rule 2524

```

Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Simp[(Log[d + e*x]*(a + b*Log[c*RFX^p])^n)/e, x] - Dist[(b*n*p)/e,
Int[(Log[d + e*x]*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]

```

#### Rule 2528

```

Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol]
:> With[{u = ExpandIntegrand[(a + b*Log[c*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /;
FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

```

#### Rule 2418

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*(RFX_), x_Symbol]
:> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFX, x]}, Int[u, x] /; SumQ[u]] /;
FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFX, x] && IntegerQ[p]

```

#### Rule 2390

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.)
*(x_.))^(q_.), x_Symbol]
:> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /;
FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]

```

#### Rule 2301

```

Int[((a_.) + Log[(c_.)*(x_.)^(n_.)]*(b_.))/(x_), x_Symbol]
:> Simp[(a + b*Lo

```

$g[c*x^n]^2/(2*b*n), x] /; \text{FreeQ}\{a, b, c, n\}, x]$

#### Rule 2394

$\text{Int}[(a_.) + \text{Log}[c_.]*((d_.) + (e_.)*(x_.))^{(n_.)}*(b_.)]/((f_.) + (g_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n]))/g, x] - \text{Dist}[(b*e^n)/g, \text{Int}[\text{Log}[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n\}, x] \&\& \text{NeQ}[e*f - d*g, 0]$

#### Rule 2393

$\text{Int}[(a_.) + \text{Log}[c_.]*((d_.) + (e_.)*(x_.))]*(b_.)]/((f_.) + (g_.)*(x_.)), x\_Symbol] \rightarrow \text{Dist}[1/g, \text{Subst}[\text{Int}[(a + b*\text{Log}[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{EqQ}[g + c*(e*f - d*g), 0]$

#### Rule 2391

$\text{Int}[\text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^{(n_.)}]]/(x_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

#### Rule 6688

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}\{v = \text{SimplifyIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SimplerIntegrandQ}[v, u, x]$

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$

#### Rule 2500

$\text{Int}[(\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^{(p_.)}*((c_.) + (d_.)*(x_.))^{(q_.)})^{(r_.)}*((s_.) + \text{Log}[(i_.)*((g_.) + (h_.)*(x_.))^{(n_.)}*(t_.))]/((j_.) + (k_.)*(x_.))), x\_Symbol] \rightarrow \text{Dist}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - \text{Log}[(a + b*x)^{p*r}] - \text{Log}[(c + d*x)^{q*r}], \text{Int}[(s + t*\text{Log}[i*(g + h*x)^n])/(j + k*x), x], x] + (\text{Int}[(\text{Log}[(a + b*x)^{p*r}])*(s + t*\text{Log}[i*(g + h*x)^n])/(j + k*x), x] + \text{Int}[(\text{Log}[(c + d*x)^{q*r}])*(s + t*\text{Log}[i*(g + h*x)^n])/(j + k*x), x]) /; \text{FreeQ}\{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

#### Rule 2433

$\text{Int}[(a_.) + \text{Log}[c_.]*((d_.) + (e_.)*(x_.))^{(n_.)}*(b_.)]^{(p_.)}*((f_.) + \text{Log}[(h_.)*((i_.) + (j_.)*(x_.))^{(m_.)}*(g_.)]*(k_.) + (l_.)*(x_.))^{(r_.)}, x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(k*x)/d]^r*(a + b*\text{Log}[c*x^n])^p*(f + g*\text{Log}[h*(e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r\}, x] \&\& \text{EqQ}[e*k - d*l, 0]$

#### Rule 2375

$\text{Int}[(\text{Log}[(d_.)*((e_.) + (f_.)*(x_.))^{(m_.)})^{(r_.)}*((a_.) + \text{Log}[c_.*(x_.))^{(n_.)}*(b_.)]^{(p_.)}]/(x_), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[d*(e + f*x^m)^r]*(a + b*\text{Log}[\text{Log}[c*x^n])^p)]/x, x]$

$c*x^n)^{(p+1)}/(b*n*(p+1)), x] - \text{Dist}[(f*m*r)/(b*n*(p+1)), \text{Int}[(x^{(m-1)}*(a+b*\text{Log}[c*x^n])^{(p+1)})/(e+f*x^m), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, r, m, n\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{NeQ}[d*e, 1]$

#### Rule 2317

$\text{Int}[(a + \text{Log}[c*x^n])*(b*x^p)/(d + e*x), x\_Symbol] :> \text{Simp}[(\text{Log}[1 + (e*x)/d]*(a + b*\text{Log}[c*x^n])^p)/e, x] - \text{Dist}[(b*n*p)/e, \text{Int}[(\text{Log}[1 + (e*x)/d]*(a + b*\text{Log}[c*x^n])^{(p-1)})/x, x], x] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \&\& \text{IGtQ}[p, 0]$

#### Rule 2374

$\text{Int}[(\text{Log}[d*(e + f*x^m)]*(a + \text{Log}[c*x^n])*(b*x^p))/x, x\_Symbol] :> -\text{Simp}[(\text{PolyLog}[2, -(d*f*x^m)]*(a + b*\text{Log}[c*x^n])^p)/m, x] + \text{Dist}[(b*n*p)/m, \text{Int}[(\text{PolyLog}[2, -(d*f*x^m)]*(a + b*\text{Log}[c*x^n])^{(p-1)})/x, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[d*e, 1]$

#### Rule 6589

$\text{Int}[\text{PolyLog}[n, c*(a + b*x)^p]/(d + e*x), x\_Symbol] :> \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}[\{a, b, c, d, e, n, p\}, x] \&\& \text{EqQ}[b*d, a*e]$

#### Rule 2440

$\text{Int}[(a + \text{Log}[c*(d + e*x)^n])*(b*x^p)*((f + \text{Log}[h*(i + j*x)^m])*(g*(k + l*x)^r)), x\_Symbol] :> \text{Dist}[1/l, \text{Subst}[\text{Int}[x^r*(a + b*\text{Log}[c*(-((e*k - d*1)/1) + (e*x)/1)^n])*(f + g*\text{Log}[h*(-((j*k - i*1)/1) + (j*x)/1)^m]), x], x, k + l*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, i, j, k, l, m, n\}, x] \&\& \text{IntegerQ}[r]$

#### Rule 2437

$\text{Int}[(\text{Log}[c*(d + e*x)^n]*\text{Log}[h*(i + j*x)^m])/x, x\_Symbol] :> \text{Dist}[m, \text{Int}[(\text{Log}[i + j*x]*\text{Log}[c*(d + e*x)^n])/x, x], x] - \text{Dist}[m*\text{Log}[i + j*x] - \text{Log}[h*(i + j*x)^m], \text{Int}[(\text{Log}[c*(d + e*x)^n])/x, x], x] /; \text{FreeQ}[\{c, d, e, h, i, j, m, n\}, x] \&\& \text{NeQ}[e*i - d*j, 0] \&\& \text{NeQ}[i + j*x, h*(i + j*x)^m]$

#### Rule 2435

$\text{Int}[(\text{Log}[a + b*x]*\text{Log}[c + d*x])/x, x\_Symbol] :> \text{Simp}[\text{Log}[-((b*x)/a)]*\text{Log}[a + b*x]*\text{Log}[c + d*x], x] + (\text{Simp}[(1*(\text{Log}[-((b*x)/a)] - \text{Log}[-((b*c - a*d)*x)/(a*(c + d*x))]) + \text{Log}[(b*c - a*d)/(b*(c + d*x))])*\text{Log}[(a*(c + d*x))/(c*(a + b*x))]^2/2, x] - \text{Simp}[(1*(\text{Log}[-((b*x)/a)] - \text{Log}[-((d*x)/c)])*(\text{Log}[a + b*x] + \text{Log}[(a*(c + d*x))/(c*(a + b*x))])^2/2, x] + \text{Simp}[(\text{Log}[c + d*x] - \text{Log}[(a*(c + d*x))/(c*(a + b*x))])*\text{PolyLog}[2, 1 + (b*x)/a], x] + \text{Simp}[(\text{Log}[a + b*x] + \text{Log}[(a*(c + d*x))/(c*(a + b*x))])*\text{PolyLog}[2, 1 + (d*x)/c], x] + \text{Simp}[\text{Log}[(a*(c + d*x))/(c*(a + b*x))]*\text{PolyLog}[2, (c*(a + b*x))/(a*(c + d*x))], x] - \text{Simp}[\text{Log}[(a*(c + d*x))/(c*(a + b*x))]*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))], x] - \text{Simp}[\text{PolyLog}[3, 1 + (b*x)/a], x] - \text{Simp}[\text{PolyLog}[3, 1 + (d*x)/c], x] + \text{Simp}[\text{PolyLog}[3, (c*(a + b*x))/(a*(c + d*x))], x] - \text{Simp}[\text{PolyLog}[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{f+gx} dx &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(f+gx)}{g} - \frac{(2Bn) \int \frac{(c+dx)\left(-\frac{d(a+bx)}{(c+dx)^2} + \frac{b}{c+dx}\right)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(f+gx)}{a+bx} dx}{g} \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(f+gx)}{g} - \frac{(2Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(f+gx)}{(a+bx)(c+dx)} dx}{g} \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(f+gx)}{g} - \frac{(2B(bc-ad)n) \int \frac{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(f+gx)}{(a+bx)(c+dx)} dx}{g} \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(f+gx)}{g} - \frac{(2B(bc-ad)n) \int \frac{b\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(f+gx)}{(bc-ad)(a+bx)} dx}{g} \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(f+gx)}{g} - \frac{(2bBn) \int \frac{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(f+gx)}{a+bx} dx}{g} + \frac{(2Bd) \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(f+gx)}{a+bx} dx}{g} \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(f+gx)}{g} - \frac{(2bBn) \int \left(\frac{A \log(f+gx)}{a+bx} + \frac{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(f+gx)}{a+bx}\right) dx}{g} \\
 &= \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(f+gx)}{g} - \frac{(2AbBn) \int \frac{\log(f+gx)}{a+bx} dx}{g} - \frac{(2bB^2n) \int \frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) \log(f+gx)}{a+bx} dx}{g} \\
 &= -\frac{2ABn \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(f+gx)}{g} + \frac{2ABn \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} \\
 &= -\frac{2ABn \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(f+gx)}{g} + \frac{2ABn \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} \\
 &= -\frac{2ABn \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} - \frac{B^2 \log^2\left((a+bx)^n\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(f+gx)}{g} \\
 &= -\frac{2ABn \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} - \frac{B^2 \log^2\left((a+bx)^n\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(f+gx)}{g} \\
 &= -\frac{2ABn \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} - \frac{B^2 \log^2\left((a+bx)^n\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(f+gx)}{g} \\
 &= -\frac{2ABn \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} - \frac{B^2 \log^2\left((a+bx)^n\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \log(f+gx)}{g}
 \end{aligned}$$

**Mathematica [B]** time = 0.473154, size = 1441, normalized size = 4.85

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(f + g\*x), x]

[Out] 
$$\begin{aligned} & \left( -\left( B^2 n^2 \operatorname{Log}\left[ \frac{-(b*c) + a*d}{d*(a + b*x)} \right] \right) * \operatorname{Log}\left[ \frac{(b*f - a*g)*(c + d*x)}{(d*f - c*g)*(a + b*x)} \right] \right)^2 + A^2 * \operatorname{Log}[f + g*x] - 2*A*B*n * \operatorname{Log}[a/b + x] * \operatorname{Log}[f + g*x] \\ & + B^2 n^2 * \operatorname{Log}[a/b + x]^2 * \operatorname{Log}[f + g*x] + 2*A*B*n * \operatorname{Log}[c/d + x] * \operatorname{Log}[f + g*x] - 2*B^2 n^2 * \operatorname{Log}[a/b + x] * \operatorname{Log}[c/d + x] * \operatorname{Log}[f + g*x] \\ & + B^2 n^2 * \operatorname{Log}[c/d + x]^2 * \operatorname{Log}[f + g*x] + 2*A*B * \operatorname{Log}\left[ e^{\left( \frac{a + b*x}{c + d*x} \right)^n} \right] * \operatorname{Log}[f + g*x] - 2*B^2 n * \operatorname{Log}[a/b + x] * \operatorname{Log}\left[ e^{\left( \frac{a + b*x}{c + d*x} \right)^n} \right] * \operatorname{Log}[f + g*x] \\ & + 2*B^2 n * \operatorname{Log}[c/d + x] * \operatorname{Log}\left[ e^{\left( \frac{a + b*x}{c + d*x} \right)^n} \right] * \operatorname{Log}[f + g*x] + B^2 * \operatorname{Log}\left[ e^{\left( \frac{a + b*x}{c + d*x} \right)^n} \right]^2 * \operatorname{Log}[f + g*x] \\ & + 2*A*B*n * \operatorname{Log}[a/b + x] * \operatorname{Log}\left[ \frac{b*(f + g*x)}{b*f - a*g} \right] - B^2 n^2 * \operatorname{Log}[a/b + x]^2 * \operatorname{Log}\left[ \frac{b*(f + g*x)}{b*f - a*g} \right] + 2*B^2 n * \operatorname{Log}[a/b + x] * \operatorname{Log}\left[ \frac{b*(f + g*x)}{b*f - a*g} \right] \\ & + 2*B^2 n^2 * \operatorname{Log}[a/b + x] * \operatorname{Log}\left[ \frac{g*(c + d*x)}{-(d*f) + c*g} \right] * \operatorname{Log}\left[ \frac{b*(f + g*x)}{b*f - a*g} \right] - B^2 n^2 * \operatorname{Log}\left[ \frac{g*(c + d*x)}{-(d*f) + c*g} \right]^2 * \operatorname{Log}\left[ \frac{b*(f + g*x)}{b*f - a*g} \right] \\ & + 2*B^2 n^2 * \operatorname{Log}\left[ \frac{g*(c + d*x)}{-(d*f) + c*g} \right] * \operatorname{Log}\left[ \frac{(b*f - a*g)*(c + d*x)}{(d*f - c*g)*(a + b*x)} \right] * \operatorname{Log}\left[ \frac{b*(f + g*x)}{b*f - a*g} \right] - B^2 n^2 * \operatorname{Log}\left[ \frac{(b*f - a*g)*(c + d*x)}{(d*f - c*g)*(a + b*x)} \right]^2 * \operatorname{Log}\left[ \frac{b*(f + g*x)}{b*f - a*g} \right] \\ & - 2*A*B*n * \operatorname{Log}[c/d + x] * \operatorname{Log}\left[ \frac{d*(f + g*x)}{d*f - c*g} \right] + 2*B^2 n^2 * \operatorname{Log}[a/b + x] * \operatorname{Log}[c/d + x] * \operatorname{Log}\left[ \frac{d*(f + g*x)}{d*f - c*g} \right] - B^2 n^2 * \operatorname{Log}[c/d + x]^2 * \operatorname{Log}\left[ \frac{d*(f + g*x)}{d*f - c*g} \right] \\ & - 2*B^2 n * \operatorname{Log}[c/d + x] * \operatorname{Log}\left[ e^{\left( \frac{a + b*x}{c + d*x} \right)^n} \right] * \operatorname{Log}\left[ \frac{d*(f + g*x)}{d*f - c*g} \right] - 2*B^2 n^2 * \operatorname{Log}[a/b + x] * \operatorname{Log}\left[ \frac{g*(c + d*x)}{-(d*f) + c*g} \right] * \operatorname{Log}\left[ \frac{d*(f + g*x)}{d*f - c*g} \right] \\ & + B^2 n^2 * \operatorname{Log}\left[ \frac{g*(c + d*x)}{-(d*f) + c*g} \right]^2 * \operatorname{Log}\left[ \frac{d*(f + g*x)}{d*f - c*g} \right] - 2*B^2 n^2 * \operatorname{Log}\left[ \frac{g*(c + d*x)}{-(d*f) + c*g} \right] * \operatorname{Log}\left[ \frac{(b*f - a*g)*(c + d*x)}{(d*f - c*g)*(a + b*x)} \right] * \operatorname{Log}\left[ \frac{d*(f + g*x)}{d*f - c*g} \right] \\ & + B^2 n^2 * \operatorname{Log}\left[ \frac{(b*f - a*g)*(c + d*x)}{(d*f - c*g)*(a + b*x)} \right]^2 * \operatorname{Log}\left[ \frac{d*(f + g*x)}{d*f - c*g} \right] + B^2 n^2 * \operatorname{Log}\left[ \frac{(b*f - a*g)*(c + d*x)}{(d*f - c*g)*(a + b*x)} \right] * \operatorname{PolyLog}\left[ 2, \frac{g*(a + b*x)}{-(b*f) + a*g} \right] \\ & - 2*B*n*(A + B * \operatorname{Log}\left[ e^{\left( \frac{a + b*x}{c + d*x} \right)^n} \right] + B*n * \operatorname{Log}\left[ \frac{(b*f - a*g)*(c + d*x)}{(d*f - c*g)*(a + b*x)} \right]) * \operatorname{PolyLog}\left[ 2, \frac{g*(c + d*x)}{-(d*f) + c*g} \right] \\ & - 2*B^2 n^2 * \operatorname{Log}\left[ \frac{(b*f - a*g)*(c + d*x)}{(d*f - c*g)*(a + b*x)} \right] * \operatorname{PolyLog}\left[ 2, \frac{b*(c + d*x)}{d*(a + b*x)} \right] + 2*B^2 n^2 * \operatorname{Log}\left[ \frac{(b*f - a*g)*(c + d*x)}{(d*f - c*g)*(a + b*x)} \right] * \operatorname{PolyLog}\left[ 2, \frac{(b*f - a*g)*(c + d*x)}{(d*f - c*g)*(a + b*x)} \right] \\ & + 2*B^2 n^2 * \operatorname{PolyLog}\left[ 3, \frac{b*(c + d*x)}{d*(a + b*x)} \right] - 2*B^2 n^2 * \operatorname{PolyLog}\left[ 3, \frac{(b*f - a*g)*(c + d*x)}{(d*f - c*g)*(a + b*x)} \right] \end{aligned}$$

**Maple [F]** time = 0.528, size = 0, normalized size = 0.

$$\int \frac{1}{gx + f} \left( A + B \ln \left( e^{\left( \frac{bx + a}{dx + c} \right)^n} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f), x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{A^2 \log(gx + f)}{g} + \int \frac{B^2 \log((bx + a)^n)^2 + B^2 \log((dx + c)^n)^2 + B^2 \log(e)^2 + 2AB \log(e) + 2(B^2 \log(e) + AB) \log((bx + a)^n)}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f),x, algorithm="maxima")

[Out] A^2\*log(g\*x + f)/g + integrate((B^2\*log((b\*x + a)^n)^2 + B^2\*log((d\*x + c)^n)^2 + B^2\*log(e)^2 + 2\*A\*B\*log(e) + 2\*(B^2\*log(e) + A\*B)\*log((b\*x + a)^n) - 2\*(B^2\*log((b\*x + a)^n) + B^2\*log(e) + A\*B)\*log((d\*x + c)^n))/(g\*x + f), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2AB \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2}{gx + f}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f),x, algorithm="fricas")

[Out] integral((B^2\*log(e\*((b\*x + a)/(d\*x + c))^n)^2 + 2\*A\*B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A^2)/(g\*x + f), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f),x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2/(g\*x + f), x)

$$3.72 \quad \int \frac{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2} dx$$

**Optimal.** Leaf size=206

$$\frac{2B^2n^2(bc-ad)\text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{(bf-ag)(df-cg)} + \frac{2Bn(bc-ad) \log\left(1 - \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{(bf-ag)(df-cg)} + \frac{(a+bx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{(f+gx)^2}$$

```
[Out] ((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/((b*f - a*g)*(f + g*x)
) + (2*B*(b*c - a*d)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((d*f
- c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)*(d*f - c*g)) + (2
*B^2*(b*c - a*d)*n^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d
*x))])/((b*f - a*g)*(d*f - c*g))
```

**Rubi [B]** time = 1.13374, antiderivative size = 657, normalized size of antiderivative = 3.19, number of steps used = 29, number of rules used = 10, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2bB^2n^2\text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{g(bf-ag)} + \frac{2B^2dn^2\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{g(df-cg)} - \frac{2B^2n^2(bc-ad)\text{PolyLog}\left(2, \frac{b(f+gx)}{bf-ag}\right)}{(bf-ag)(df-cg)} + \frac{2B^2n^2(bc-ad)}{(bf-ag)(df-cg)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^2, x]
```

```
[Out] -((b*B^2*n^2*Log[a + b*x]^2)/(g*(b*f - a*g))) + (2*b*B*n*Log[a + b*x]*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(b*f - a*g)) - (A + B*Log[e*((a + b*x)
)/(c + d*x))^n]^2/(g*(f + g*x)) + (2*B^2*d*n^2*Log[-((d*(a + b*x))/(b*c -
a*d))]*Log[c + d*x])/(g*(d*f - c*g)) - (2*B*d*n*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n])*Log[c + d*x])/(g*(d*f - c*g)) - (B^2*d*n^2*Log[c + d*x]^2)/(g*(
d*f - c*g)) + (2*b*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]/(g*
(b*f - a*g)) - (2*B^2*(b*c - a*d)*n^2*Log[-((g*(a + b*x))/(b*f - a*g))]*Log
[f + g*x])/((b*f - a*g)*(d*f - c*g)) + (2*B*(b*c - a*d)*n*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n])*Log[f + g*x])/((b*f - a*g)*(d*f - c*g)) + (2*B^2*(b*c
- a*d)*n^2*Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/((b*f - a*g)*(d
*f - c*g)) + (2*b*B^2*n^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))]/(g*(b*f
- a*g)) + (2*B^2*d*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]/(g*(d*f - c*
g)) - (2*B^2*(b*c - a*d)*n^2*PolyLog[2, (b*(f + g*x))/(b*f - a*g)]/((b*f -
a*g)*(d*f - c*g)) + (2*B^2*(b*c - a*d)*n^2*PolyLog[2, (d*(f + g*x))/(d*f -
c*g)]/((b*f - a*g)*(d*f - c*g)))
```

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^n_)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps



$$\begin{aligned}
\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^2} dx &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} + \frac{(2Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)(c+dx)(f+gx)} dx}{g} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} + \frac{(2B(bc-ad)n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)(f+gx)} dx}{g} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} + \frac{(2B(bc-ad)n) \int \left(\frac{b^2\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(-df+cg)}\right) dx}{g} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} + \frac{(2b^2Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx} dx}{g(bf-ag)} - \frac{(2Bd^2n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} dx}{g(df-cg)} \\
&= \frac{2bBn \log(a+bx) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} - \frac{2Bdn \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(df-cg)} \\
&= \frac{2bBn \log(a+bx) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} - \frac{2Bdn \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(df-cg)} \\
&= \frac{2bBn \log(a+bx) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} - \frac{2Bdn \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(df-cg)} \\
&= \frac{2bBn \log(a+bx) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} + \frac{2B^2dn^2 \log^2(a+bx)}{g(bf-ag)} \\
&= -\frac{bB^2n^2 \log^2(a+bx)}{g(bf-ag)} + \frac{2bBn \log(a+bx) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)} \\
&= -\frac{bB^2n^2 \log^2(a+bx)}{g(bf-ag)} + \frac{2bBn \log(a+bx) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)}
\end{aligned}$$

**Mathematica [B]** time = 0.7378, size = 418, normalized size = 2.03

$$\frac{Bn\left(-bBn(df-cg)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+Bdn(bf-ag)\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{g(f+gx)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(f + g\*x)^2,x]

[Out] (-((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(f + g\*x)) + (B\*n\*(2\*b\*(d\*f - c\*g)\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 2\*d\*(b\*f - a\*g)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x] + 2\*(b\*c - a\*d)\*g\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[f + g\*x] - b\*B\*(d\*f - c\*g)\*n\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c) + a\*d]) + B\*d\*(b\*f - a\*g)\*n\*((2\*Log[(d\*(a + b\*x))/(-b\*c) + a\*d]) - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d]) - 2\*B\*(b\*c - a\*d)\*g\*n\*((Log[(g\*(a + b\*x))/(-b\*f) + a\*g]) - Log[(g\*(c

$+ d*x))/(-d*f + c*g)]*Log[f + g*x] + PolyLog[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*g))/g$

**Maple [F]** time = 0.516, size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^2} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^2,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2ABn \left( \frac{b \log(bx + a)}{bfg - ag^2} - \frac{d \log(dx + c)}{dfg - cg^2} + \frac{(bc - ad) \log(gx + f)}{bdf^2 + acg^2 - (bc + ad)fg} \right) - B^2 \left( \frac{\log((dx + c)^n)^2}{g^2x + fg} + \int -\frac{d g x \log(e)^2 + c g \log(e)}{g^2x + fg} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^2,x, algorithm="maxima")

[Out] 2\*A\*B\*n\*(b\*log(b\*x + a)/(b\*f\*g - a\*g^2) - d\*log(d\*x + c)/(d\*f\*g - c\*g^2) + (b\*c - a\*d)\*log(g\*x + f)/(b\*d\*f^2 + a\*c\*g^2 - (b\*c + a\*d)\*f\*g) - B^2\*(log((d\*x + c)^n)^2/(g^2\*x + f\*g) + integrate(-(d\*g\*x\*log(e)^2 + c\*g\*log(e)^2 + (d\*g\*x + c\*g)\*log((b\*x + a)^n)^2 + 2\*(d\*g\*x\*log(e) + c\*g\*log(e))\*log((b\*x + a)^n) + 2\*(d\*f\*n + (g\*n - g\*log(e))\*d\*x - c\*g\*log(e) - (d\*g\*x + c\*g)\*log((b\*x + a)^n))\*log((d\*x + c)^n)/(d\*g^3\*x^3 + c\*f^2\*g + (2\*d\*f\*g^2 + c\*g^3)\*x^2 + (d\*f^2\*g + 2\*c\*f\*g^2)\*x), x)) - 2\*A\*B\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n)/(g^2\*x + f\*g) - A^2/(g^2\*x + f\*g)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2AB \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2}{g^2x^2 + 2fgx + f^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^2,x, algorithm="fricas")

[Out] integral((B^2\*log(e\*((b\*x + a)/(d\*x + c))^n)^2 + 2\*A\*B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A^2)/(g^2\*x^2 + 2\*f\*g\*x + f^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2/(g\*x+f)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2}{(gx+f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^2,x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2/(g\*x + f)^2, x)

$$3.73 \quad \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3} dx$$

**Optimal.** Leaf size=389

$$\frac{B^2 n^2 (bc - ad)(-adg - bcg + 2bdf) \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{(bf-ag)^2(df-cg)^2} + \frac{b^2 \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}{2g(bf-ag)^2} + \frac{Bgn(a+bx)(bc-ad) \left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{(f+gx)(bf-ag)}$$

[Out] (B\*(b\*c - a\*d)\*g\*n\*(a + b\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/((b\*f - a\*g)^2\*(d\*f - c\*g)\*(f + g\*x)) + (b^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2)/(2\*g\*(b\*f - a\*g)^2) - (A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(2\*g\*(f + g\*x)^2) + (B^2\*(b\*c - a\*d)^2\*g\*n^2\*Log[(f + g\*x)/(c + d\*x)])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (B\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[1 - ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*n^2\*PolyLog[2, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2)

**Rubi [B]** time = 1.56109, antiderivative size = 941, normalized size of antiderivative = 2.42, number of steps used = 33, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

$$\frac{B^2 n^2 \log^2(a+bx)b^2}{2g(bf-ag)^2} + \frac{Bn \log(a+bx) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) b^2}{g(bf-ag)^2} + \frac{B^2 n^2 \log(a+bx) \log\left(\frac{b(c+dx)}{bc-ad}\right) b^2}{g(bf-ag)^2} + \frac{B^2 n^2 \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right) b^2}{g(bf-ag)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(f + g\*x)^3, x]

[Out] (b\*B^2\*(b\*c - a\*d)\*n^2\*Log[a + b\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)) - (b^2\*B^2\*n^2\*Log[a + b\*x]^2)/(2\*g\*(b\*f - a\*g)^2) - (B\*(b\*c - a\*d)\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/((b\*f - a\*g)\*(d\*f - c\*g)\*(f + g\*x)) + (b^2\*B\*n\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(g\*(b\*f - a\*g)^2) - (A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(2\*g\*(f + g\*x)^2) - (B^2\*d\*(b\*c - a\*d)\*n^2\*Log[c + d\*x])/((b\*f - a\*g)\*(d\*f - c\*g)^2) + (B^2\*d^2\*n^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x])/((b\*f - a\*g)\*(d\*f - c\*g)^2) - (B\*d^2\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x])/((b\*f - a\*g)\*(d\*f - c\*g)^2) - (B^2\*d^2\*n^2\*Log[c + d\*x]^2)/(2\*g\*(d\*f - c\*g)^2) + (b^2\*B^2\*n^2\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(g\*(b\*f - a\*g)^2) + (B^2\*(b\*c - a\*d)^2\*g\*n^2\*Log[f + g\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) - (B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*n^2\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*Log[f + g\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (B\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*n\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[f + g\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*n^2\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*Log[f + g\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (b^2\*B^2\*n^2\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/((b\*f - a\*g)^2) + (B^2\*d^2\*n^2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/(g\*(d\*f - c\*g)^2) - (B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*n^2\*PolyLog[2, (b\*(f + g\*x))/(b\*f - a\*g)])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*n^2\*PolyLog[2, (d\*(f + g\*x))/(d\*f - c\*g)])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2)

Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.)/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2391

Int[Log[(c\_.)\*(d\_) + (e\_.)\*(x\_)^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 72

Int[((e\_.) + (f\_.)\*(x\_)^(p\_.))/((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^3} dx &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)(c+dx)(f+gx)^2} dx}{g} \\
 &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(B(bc-ad)n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)(f+gx)^2} dx}{g} \\
 &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(B(bc-ad)n) \int \left(\frac{b^3\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d^3\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(-df+cg)^2(c+dx)}\right) dx}{g} \\
 &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2g(f+gx)^2} + \frac{(b^3Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx} dx}{g(bf-ag)^2} - \frac{(Bd^3n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} dx}{g(df-cg)^2} \\
 &= -\frac{B(bc-ad)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2Bn \log(a+bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)^2} - \frac{Bd^3n \log(a+bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(df-cg)^2} \\
 &= -\frac{B(bc-ad)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2Bn \log(a+bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)^2} - \frac{Bd^3n \log(a+bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(df-cg)^2} \\
 &= -\frac{B(bc-ad)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2Bn \log(a+bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(bf-ag)^2} - \frac{Bd^3n \log(a+bx)\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{g(df-cg)^2} \\
 &= \frac{bB^2(bc-ad)n^2 \log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{B(bc-ad)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2Bn \log(a+bx)}{g(bf-ag)^2} \\
 &= \frac{bB^2(bc-ad)n^2 \log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{b^2B^2n^2 \log^2(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)} \\
 &= \frac{bB^2(bc-ad)n^2 \log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{b^2B^2n^2 \log^2(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bf-ag)(df-cg)(f+gx)}
 \end{aligned}$$

**Mathematica [A]** time = 2.23813, size = 615, normalized size = 1.58

$$\frac{Bn(f+gx)\left(b^2Bn(f+gx)(df-cg)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)-Bd^2n(f+gx)(bf-ag)^2\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)\right)+\log(c+dx)\left(2\log\left(\frac{a+bx}{c+dx}\right)\right)}{g^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(f + g\*x)^3,x]

[Out] -((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 + (B\*n\*(f + g\*x)\*(2\*(b\*c - a\*d)\*g\*(b\*f - a\*g)\*(d\*f - c\*g)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 2\*b^2\*(d\*f - c\*g)^2\*(f + g\*x)\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 2\*d^2\*(b\*f - a\*g)^2\*(f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) \* Log[c + d\*x] + 2\*(b\*c - a\*d)\*g\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g)\*(f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) \* Log[f + g\*x] - 2\*B\*(b\*c - a\*d)\*g\*n\*(f + g\*x)\*(b\*(d\*f - c\*g)\*Log[a + b\*x] + (-b\*d\*f) + a\*d\*g)\*Log[c + d\*x] + (b\*c - a\*d)\*g\*Log[f + g\*x]) + b^2\*B\*(d\*f - c\*g)^2\*n\*(f + g\*x)\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c) + a\*d]) - B\*d^2\*(b\*f - a\*g)^2\*n\*(f + g\*x)\*((2\*Log[(d\*(a + b\*x))/(-b\*c) + a\*d]) - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*B\*(b\*c - a\*d)\*g\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g)\*n\*(f + g\*x)\*((Log[(g\*(a + b\*x))/(-b\*f) + a\*g]) - Log[(g\*(c + d\*x))/(-d\*f) + c\*g])\*Log[f + g\*x] + PolyLog[2, (b\*(f + g\*x))/(b\*f - a\*g]) - PolyLog[2, (d\*(f + g\*x))/(d\*f - c\*g)])/(b\*f - a\*g)^2\*(d\*f - c\*g)^2)/(2\*g\*(f + g\*x)^2)

**Maple [F]** time = 0.512, size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^3} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^3,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left( \frac{b^2 \log(bx + a)}{b^2 f^2 g - 2 abf g^2 + a^2 g^3} - \frac{d^2 \log(dx + c)}{d^2 f^2 g - 2 cdf g^2 + c^2 g^3} + \frac{(2(b^2 cd - abd^2)f - (b^2 c^2 - a^2 d^2)g) \log(gx + f)}{b^2 d^2 f^4 + a^2 c^2 g^4 - 2(b^2 cd + abd^2)f^3 g + (b^2 c^2 + 4 abcd + a^2 d^2)g^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^3,x, algorithm="maxima")

[Out] (b^2\*log(b\*x + a)/(b^2\*f^2\*g - 2\*a\*b\*f\*g^2 + a^2\*g^3) - d^2\*log(d\*x + c)/(d^2\*f^2\*g - 2\*c\*d\*f\*g^2 + c^2\*g^3) + (2\*(b^2\*c\*d - a\*b\*d^2)\*f - (b^2\*c^2 - a^2\*d^2)\*g)\*log(g\*x + f)/(b^2\*d^2\*f^4 + a^2\*c^2\*g^4 - 2\*(b^2\*c\*d + a\*b\*d^2)\*f^3\*g + (b^2\*c^2 + 4\*a\*b\*c\*d + a^2\*d^2)\*f^2\*g^2 - 2\*(a\*b\*c^2 + a^2\*c\*d)\*f\*g^3 - (b\*c - a\*d)/(b\*d\*f^3 + a\*c\*f\*g^2 - (b\*c + a\*d)\*f^2\*g + (b\*d\*f^2\*g + a\*c\*g^3 - (b\*c + a\*d)\*f\*g^2)\*x))\*A\*B\*n - 1/2\*B^2\*(log((d\*x + c)^n))^2/(g^3\*x^2 + 2\*f\*g^2\*x + f^2\*g) + 2\*integrate(-(d\*g\*x\*log(e)^2 + c\*g\*log(e)^2 + (d\*g\*x + c\*g)\*log((b\*x + a)^n))^2 + 2\*(d\*g\*x\*log(e) + c\*g\*log(e))\*log((b\*x + a)^n) + (d\*f\*n + (g\*n - 2\*g\*log(e))\*d\*x - 2\*c\*g\*log(e) - 2\*(d\*g\*x + c\*g)\*log((b\*x + a)^n))\*log((d\*x + c)^n)/(d\*g^4\*x^4 + c\*f^3\*g + (3\*d\*f\*g^3 + c\*g^4)\*x^3 + 3\*(d\*f^2\*g^2 + c\*f\*g^3)\*x^2 + (d\*f^3\*g + 3\*c\*f^2\*g^2)\*x), x) - A\*B\*log(e\*(b\*x/(d\*x + c) + a/(d\*x + c))^n)/(g^3\*x^2 + 2\*f\*g^2\*x + f^2\*g) - 1/2\*A^2

$$2/(g^3x^2 + 2fg^2x + f^2g)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2AB \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2}{g^3x^3 + 3fg^2x^2 + 3f^2gx + f^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c)))^n))^2/(g\*x+f)^3,x, algorithm="fricas")

[Out] integral((B^2\*log(e\*((b\*x + a)/(d\*x + c)))^n)^2 + 2\*A\*B\*log(e\*((b\*x + a)/(d\*x + c)))^n + A^2)/(g^3\*x^3 + 3\*f\*g^2\*x^2 + 3\*f^2\*g\*x + f^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c)))^n))^2/(g\*x+f)^3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2}{(gx + f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c)))^n))^2/(g\*x+f)^3,x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c)))^n + A)^2/(g\*x + f)^3, x)



$$3.74 \quad \int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4} dx$$

**Optimal.** Leaf size=747

$$\frac{2B^2n^2(bc-ad)\left(a^2d^2g^2 - abdg(3df - cg) + b^2(c^2g^2 - 3cdfg + 3d^2f^2)\right) \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right) + 2Bn(bc-ad)\left(a^2d^2g^2 - abdg(3df - cg) + b^2(c^2g^2 - 3cdfg + 3d^2f^2)\right)}{3(bf-ag)^3(df-cg)^3}$$

```
[Out] (B^2*(b*c - a*d)^2*g^2*n^2*(c + d*x))/(3*(b*f - a*g)^2*(d*f - c*g)^3*(f + g*x)) - (B*(b*c - a*d)*g^2*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*f - a*g)*(d*f - c*g)^3*(f + g*x)^2) + (2*B*(b*c - a*d)*g*(3*b*d*f - b*c*g - 2*a*d*g)*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*f - a*g)^3*(d*f - c*g)^2*(f + g*x)) + (b^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*g*(b*f - a*g)^3) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(3*g*(f + g*x)^3) + (B^2*(b*c - a*d)^3*g^2*n^2*Log[(a + b*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g)^3) - (B^2*(b*c - a*d)^3*g^2*n^2*Log[(f + g*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)^2*g*(3*b*d*f - b*c*g - 2*a*d*g)*n^2*Log[(f + g*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x)))]/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x)))]/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

**Rubi [A]** time = 2.5001, antiderivative size = 1427, normalized size of antiderivative = 1.91, number of steps used = 37, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

$$\frac{B^2n^2 \log^2(a+bx)b^3}{3g(bf-ag)^3} + \frac{2Bn \log(a+bx) \left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) b^3}{3g(bf-ag)^3} + \frac{2B^2n^2 \log(a+bx) \log\left(\frac{b(c+dx)}{bc-ad}\right) b^3}{3g(bf-ag)^3} + \frac{2B^2n^2 \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right) b^3}{3g(bf-ag)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^4, x]
```

```
[Out] -(B^2*(b*c - a*d)^2*g*n^2)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^2*B^2*(b*c - a*d)*n^2*Log[a + b*x])/(3*(b*f - a*g)^3*(d*f - c*g)) + (2*b*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n^2*Log[a + b*x])/(3*(b*f - a*g)^3*(d*f - c*g)^2) - (b^3*B^2*n^2*Log[a + b*x]^2)/(3*g*(b*f - a*g)^3) - (B*(b*c - a*d)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (2*B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (2*b^3*B*n*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*g*(b*f - a*g)^3) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(3*g*(f + g*x)^3) - (B^2*d^2*(b*c - a*d)*n^2*Log[c + d*x])/(3*(b*f - a*g)*(d*f - c*g)^3) - (2*B^2*d*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n^2*Log[c + d*x])/(3*(b*f - a*g)^2*(d*f - c*g)^3) + (2*B^2*d^3*n^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*g*(d*f - c*g)^3) - (2*B*d^3*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x])/(3*g*(d*f - c*g)^3) - (B^2*d^3*n^2*Log[c + d*x]^2)/(3*g*(d*f - c*g)^3) + (2*b^3*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(3*g*(b*f - a*g)^3) + (B^2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g)*n^2*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3) - (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x)))]/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

```

*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*Log[-((g*(a
+ b*x))/(b*f - a*g))]*Log[f + g*x]]/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B
*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*
f*g + c^2*g^2))*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[f + g*x]]/(3*(
b*f - a*g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*
d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*Log[-((g*(c + d*x))
/(d*f - c*g))]*Log[f + g*x]]/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*b^3*B^2*n
^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))]/(3*g*(b*f - a*g)^3) + (2*B^2*d
^3*n^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]/(3*g*(d*f - c*g)^3) - (2*B^2*
(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f
*g + c^2*g^2))*n^2*PolyLog[2, (b*(f + g*x))/(b*f - a*g)]/(3*(b*f - a*g)^3*
(d*f - c*g)^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) +
b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*PolyLog[2, (d*(f + g*x))/(d*f -
c*g)]/(3*(b*f - a*g)^3*(d*f - c*g)^3)

```

### Rule 2525

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]

```

### Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

### Rule 2528

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]

```

### Rule 2524

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

```

### Rule 2418

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^p_.)*(RFx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]

```

### Rule 2390

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]

```

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 72

Int[((e\_.) + (f\_.)\*(x\_)^(p\_.))/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4} dx &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)(c+dx)(f+gx)^3} dx}{3g} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2B(bc-ad)n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)(f+gx)^3} dx}{3g} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2B(bc-ad)n) \int \left(\frac{b^4\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(bf-ag)^3(a+bx)} + \frac{d^4\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(-df+cg)^3(c+dx)}\right) dx}{3g} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3g(f+gx)^3} + \frac{(2b^4Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx} dx}{3g(bf-ag)^3} - \frac{(2Bd^4n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} dx}{3g(df-cg)^3} \\
&= -\frac{B(bc-ad)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)^2(df-cg)^2(f+gx)} \\
&= -\frac{B(bc-ad)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)^2(df-cg)^2(f+gx)} \\
&= -\frac{B(bc-ad)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bf-ag)^2(df-cg)^2(f+gx)} \\
&= -\frac{B^2(bc-ad)^2gn^2}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)n^2 \log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-ad)(2bdf-bcg-adg)n^2 \log(a+bx)}{3(bf-ag)^3(df-cg)} \\
&= -\frac{B^2(bc-ad)^2gn^2}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)n^2 \log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-ad)(2bdf-bcg-adg)n^2 \log(a+bx)}{3(bf-ag)^3(df-cg)} \\
&= -\frac{B^2(bc-ad)^2gn^2}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)n^2 \log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-ad)(2bdf-bcg-adg)n^2 \log(a+bx)}{3(bf-ag)^3(df-cg)}
\end{aligned}$$

**Mathematica [A]** time = 4.48669, size = 918, normalized size = 1.23

$$\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^4} + \frac{Bn(f+gx)\left(2d^3(f+gx)^2\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log(c+dx)(bf-ag)^3 - Bd^3n(f+gx)^2\left(\left(2 \log\left(\frac{d(a+bx)}{ad-bc}\right) - \log(c+dx)\right) \log(c+dx) + 2\right)\right)}{3g(bf-ag)^2(df-cg)^2(f+gx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(f + g\*x)^4, x]

[Out] -((A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2 + (B\*n\*(f + g\*x)\*((b\*c - a\*d)\*g\*(b\*f - a\*g)^2\*(d\*f - c\*g)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 2\*(b\*c - a\*d)\*g\*(b\*f - a\*g)\*(-(d\*f) + c\*g)\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g)\*(f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) - 2\*b^3\*(d\*f - c\*g)^3\*(f + g\*x)^2\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]) + 2\*d^3\*(b\*f - a\*g)^3\*(f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[c + d\*x] - 2\*(b\*c - a\*d)\*g\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(-3\*d\*f + c\*g) + b^2\*(3\*d^2\*f^2 - 3\*c\*d\*f\*g + c^2\*g^2))\*(f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])\*Log[f + g\*x] - 2\*

$$\begin{aligned}
& B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*n*(f + g*x)^2*(b*(d*f - c*g)*\text{Log}[ \\
& a + b*x] + (-b*d*f) + a*d*g)*\text{Log}[c + d*x] + (b*c - a*d)*g*\text{Log}[f + g*x]) + \\
& B*(b*c - a*d)*g*n*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d \\
& *f - c*g)^2*(f + g*x)*\text{Log}[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*\text{Log}[c + d* \\
& x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*\text{Log}[f + g*x]) + b^3 \\
& *B*(d*f - c*g)^3*n*(f + g*x)^2*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + \\
& d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-b*c) + a*d]) - B*d^3*( \\
& b*f - a*g)^3*n*(f + g*x)^2*((2*\text{Log}[(d*(a + b*x))/(-b*c) + a*d]) - \text{Log}[c + \\
& d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)]) + 2*B*(b*c - \\
& a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + \\
& c^2*g^2))*n*(f + g*x)^2*((\text{Log}[(g*(a + b*x))/(-b*f) + a*g]) - \text{Log}[(g*(c + \\
& d*x))/(-d*f) + c*g]))*\text{Log}[f + g*x] + \text{PolyLog}[2, (b*(f + g*x))/(b*f - a*g)] \\
& - \text{PolyLog}[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)^3*(d*f - c*g)^3))/ \\
& (3*g*(f + g*x)^3)
\end{aligned}$$

**Maple [F]** time = 0.511, size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^4} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^4,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^4,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^4,x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& 1/3*(2*b^3*\text{log}(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3* \\
& g^4) - 2*d^3*\text{log}(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^3* \\
& 3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g + \\
& (b^3*c^3 - a^3*d^3)*g^2)*\text{log}(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3* \\
& c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^2 \\
& - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2* \\
& c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5) \\
& - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2* \\
& c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2* \\
& d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b* \\
& c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 + \\
& a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2) \\
& ^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2 \\
& *f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^3 \\
& *g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x)*A*B*n - 1/3*B^2*(\text{log}((d*x + c)^n) \\
& )^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*\text{integrate}(-1/3*(3*d*g \\
& *x*\text{log}(e)^2 + 3*c*g*\text{log}(e)^2 + 3*(d*g*x + c*g)*\text{log}((b*x + a)^n)^2 + 6*(d*g* \\
& x*\text{log}(e) + c*g*\text{log}(e))*\text{log}((b*x + a)^n) + 2*(d*f*n + (g*n - 3*g*\text{log}(e))*d*x \\
& - 3*c*g*\text{log}(e) - 3*(d*g*x + c*g)*\text{log}((b*x + a)^n))*\text{log}((d*x + c)^n))/(d*g^
\end{aligned}$$

$5x^5 + cf^4g + (4df^4g^4 + cg^5)x^4 + 2(3df^2g^3 + 2cf^4g^4)x^3 + 2(2df^3g^2 + 3cf^2g^3)x^2 + (df^4g + 4cf^3g^2)x$ ,  $x$ ) -  $2/3AB \log(e^{(bx/(dx+c) + a/(dx+c))^n}) / (g^4x^3 + 3f^3g^3x^2 + 3f^2g^2x + f^3g) - 1/3A^2 / (g^4x^3 + 3f^3g^3x^2 + 3f^2g^2x + f^3g)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2AB \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2}{g^4x^4 + 4fg^3x^3 + 6f^2g^2x^2 + 4f^3gx + f^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^4,x, algorithm="fricas")

[Out] integral((B^2\*log(e\*((b\*x + a)/(d\*x + c))^n))^2 + 2\*A\*B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A^2)/(g^4\*x^4 + 4\*f\*g^3\*x^3 + 6\*f^2\*g^2\*x^2 + 4\*f^3\*g\*x + f^4), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2}{(gx + f)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^4,x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2/(g\*x + f)^4, x)

$$3.75 \quad \int \frac{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5} dx$$

**Optimal.** Leaf size=1208

result too large to display

```
[Out] -(B^2*(b*c - a*d)^2*g^3*n^2*(c + d*x)^2)/(12*(b*f - a*g)^2*(d*f - c*g)^4*(f
+ g*x)^2) - (B^2*(b*c - a*d)^3*g^3*n^2*(c + d*x))/(6*(b*f - a*g)^3*(d*f -
c*g)^4*(f + g*x)) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*n^2*
(c + d*x))/(4*(b*f - a*g)^3*(d*f - c*g)^4*(f + g*x)) + (B*(b*c - a*d)*g^3*n
*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*(b*f - a*g)*(d*f -
c*g)^4*(f + g*x)^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*n*(c +
d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*(b*f - a*g)^2*(d*f - c*g
)^4*(f + g*x)^2) + (B*(b*c - a*d)*g*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g
) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*n*(a + b*x)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(2*(b*f - a*g)^4*(d*f - c*g)^3*(f + g*x)) + (b^4*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*g*(b*f - a*g)^4) - (A + B*Log[e*((a
+ b*x)/(c + d*x))^n])^2/(4*g*(f + g*x)^4) - (B^2*(b*c - a*d)^4*g^3*n^2*Log[
(a + b*x)/(c + d*x)])/(6*(b*f - a*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^3*
g^2*(4*b*d*f - b*c*g - 3*a*d*g)*n^2*Log[(a + b*x)/(c + d*x)])/(4*(b*f - a*g
)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^4*g^3*n^2*Log[(f + g*x)/(c + d*x)])/(
6*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - b*c*g -
3*a*d*g)*n^2*Log[(f + g*x)/(c + d*x)])/(4*(b*f - a*g)^4*(d*f - c*g)^4) + (B
^2*(b*c - a*d)^2*g*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^
2 - 4*c*d*f*g + c^2*g^2))*n^2*Log[(f + g*x)/(c + d*x)])/(2*(b*f - a*g)^4*(d
*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^
2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n*(A + B*Log[e*((a + b*x
)/(c + d*x))^n])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/
(2*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g
)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n^2
*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(2*(b*f - a*g
)^4*(d*f - c*g)^4)
```

---

**Rubi [A]** time = 3.54545, antiderivative size = 1968, normalized size of antiderivative = 1.63, number of steps used = 41, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(f + g*x)^5, x]
```

```
[Out] -(B^2*(b*c - a*d)^2*g*n^2)/(12*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (
5*B^2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g)*n^2)/(12*(b*f - a*g)^3*(d*f
- c*g)^3*(f + g*x)) + (b^3*B^2*(b*c - a*d)*n^2*Log[a + b*x])/(6*(b*f - a*g
)^4*(d*f - c*g)) + (b^2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n^2*Log[a
+ b*x])/(4*(b*f - a*g)^4*(d*f - c*g)^2) + (b*B^2*(b*c - a*d)*(a^2*d^2*g^2
- a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*Log[a
+ b*x])/(2*(b*f - a*g)^4*(d*f - c*g)^3) - (b^4*B^2*n^2*Log[a + b*x]^2)/(4*g
*(b*f - a*g)^4) - (B*(b*c - a*d)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(6*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)*(2*b*d*f - b*c*g -
a*d*g)*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*(b*f - a*g)^2*(d*f - c
*g)^2*(f + g*x)^2) - (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) +
```

$$\begin{aligned}
& b^2(3d^2f^2 - 3cd*fg + c^2g^2)*n*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]) / (2*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*n*\text{Log}[a + b*x]*(A \\
& + B*\text{Log}[e*((a + b*x)/(c + d*x))^n]) / (2*g*(b*f - a*g)^4) - (A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])^2 / (4*g*(f + g*x)^4) - (B^2*d^3*(b*c - a*d)*n^2*\text{Log}[c \\
& + d*x]) / (6*(b*f - a*g)*(d*f - c*g)^4) - (B^2*d^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*n^2*\text{Log}[c + d*x]) / (4*(b*f - a*g)^2*(d*f - c*g)^4) - (B^2*d*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*\text{Log}[c + d*x]) / (2*(b*f - a*g)^3*(d*f - c*g)^4) + (B^2*d^4*n^2*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x]) / (2*g*(d*f - c*g)^4) - (B*d^4*n*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*\text{Log}[c + d*x]) / (2*g*(d*f - c*g)^4) - (B^2*d^4*n^2*\text{Log}[c + d*x]^2) / (4*g*(d*f - c*g)^4) + (b^4*B^2*n^2*\text{Log}[a + b*x]*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) / (2*g*(b*f - a*g)^4) + (B^2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g)^2*n^2*\text{Log}[f + g*x]) / (4*(b*f - a*g)^4*(d*f - c*g)^4) + (2*B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n^2*\text{Log}[f + g*x]) / (3*(b*f - a*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n^2*\text{Log}[-((g*(a + b*x))/(b*f - a*g))]*\text{Log}[f + g*x]) / (2*(b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*\text{Log}[f + g*x]) / (2*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n^2*\text{Log}[-((g*(c + d*x))/(d*f - c*g))]*\text{Log}[f + g*x]) / (2*(b*f - a*g)^4*(d*f - c*g)^4) + (b^4*B^2*n^2*\text{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))]) / (2*g*(b*f - a*g)^4) + (B^2*d^4*n^2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)]) / (2*g*(d*f - c*g)^4) + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n^2*\text{PolyLog}[2, (b*(f + g*x))/(b*f - a*g)]) / (2*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*n^2*\text{PolyLog}[2, (d*(f + g*x))/(d*f - c*g)]) / (2*(b*f - a*g)^4*(d*f - c*g)^4)
\end{aligned}$$

### Rule 2525

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

```

### Rule 12

```

Int[(a_)*(u_), x_Symbol]
:> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

```

### Rule 2528

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol]
:> With[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

```

### Rule 2524

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

```



Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*(RFx_), x_Sy
mbo1] :=> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_) + (g_.
)*(x_)^(q_.), x_Symbol] :=> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :=> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :=> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n])/g, x] - Dist[(b*e^n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :=> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :=> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_)^(p_.))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :=> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(f+gx)^5} dx &= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4} + \frac{(Bn) \int \frac{(bc-ad)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)(c+dx)(f+gx)^4} dx}{2g} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad)n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)(f+gx)^4} dx}{2g} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad)n) \int \left(\frac{b^5\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)(-df+cg)^4(c+dx)}\right) dx}{2g} \\
&= -\frac{\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4g(f+gx)^4} + \frac{(b^5Bn) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx} dx}{2g(bf-ag)^4} - \frac{(Bd^5n) \int \frac{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} dx}{2g(df-cg)^4} \\
&= -\frac{B(bc-ad)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&= -\frac{B(bc-ad)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&= -\frac{B(bc-ad)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)n\left(A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&= -\frac{B^2(bc-ad)^2gn^2}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)n^2}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3B^2(bc-ad)^2gn^2}{6(bf-ag)^3(df-cg)^3(f+gx)} \\
&= -\frac{B^2(bc-ad)^2gn^2}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)n^2}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3B^2(bc-ad)^2gn^2}{6(bf-ag)^3(df-cg)^3(f+gx)} \\
&= -\frac{B^2(bc-ad)^2gn^2}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)n^2}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3B^2(bc-ad)^2gn^2}{6(bf-ag)^3(df-cg)^3(f+gx)}
\end{aligned}$$

**Mathematica [A]** time = 7.32275, size = 1476, normalized size = 1.22

$$B(bc-ad)n \left( \frac{\log(a+bx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^4}{(bc-ad)(bf-ag)^4} - \frac{Bn\left(\log^2(a+bx)-2 \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(a+bx)-2 \text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)\right)b^4}{2(bc-ad)(bf-ag)^4} - \frac{g\left((3d^2f^2-3cdgf+c^2g^2)b^2-adg\right)}{(bf-ag)^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(f + g\*x)^5,x]

[Out] -(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2/(4\*g\*(f + g\*x)^4) + (B\*(b\*c - a\*d)\*n\*(-(g\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(3\*(b\*f - a\*g)\*(d\*f - c\*g)\*(f + g\*x)^3) - (g\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/(2\*(b\*f - a\*g)^2\*(d\*f - c\*g)^2\*(f + g\*x)^2) - (g\*(a^2\*d^2\*g^2 - a\*b\*d\*g\*(3\*d\*f - c\*g) + b^2\*(3\*d^2\*f^2 - 3\*c\*d\*f\*g + c^2\*g^2))\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/((b\*f - a\*g)^3\*(d\*f - c\*g)^3\*(f + g\*x)) + (b^4\*Log[a + b\*x]\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]))/((b\*c - a\*d)\*(b\*f - a\*g

$$\begin{aligned} &)^4) - (d^4*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*\text{Log}[c + d*x])/((b*c - a*d)*(d*f - c*g)^4) + (g*(2*b*d*f - b*c*g - a*d*g)*(2*b^2*d^2*f^2 - 2*b^2*c*d*f*g - 2*a*b*d^2*f*g + b^2*c^2*g^2 + a^2*d^2*g^2)*(A + B*\text{Log}[e*((a + b*x)/(c + d*x))^n])*\text{Log}[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4) + (B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*n*((b*\text{Log}[a + b*x])/((b*c - a*d)*(b*f - a*g)) - (d*\text{Log}[c + d*x])/((b*c - a*d)*(d*f - c*g)) + (g*\text{Log}[f + g*x])/((b*f - a*g)*(d*f - c*g))))/((b*f - a*g)^3*(d*f - c*g)^3) - (B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*n*(g/((b*f - a*g)*(d*f - c*g)*(f + g*x)) - (b^2*\text{Log}[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + (d^2*\text{Log}[c + d*x])/((b*c - a*d)*(d*f - c*g)^2) - (g*(2*b*d*f - b*c*g - a*d*g)*\text{Log}[f + g*x])/((b*f - a*g)^2*(d*f - c*g)^2)))/(2*(b*f - a*g)^2*(d*f - c*g)^2) - (B*(b*c - a*d)*g*n*(g/((b*f - a*g)*(d*f - c*g)*(f + g*x))^2) + (2*g*(2*b*d*f - b*c*g - a*d*g))/((b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) - (2*b^3*\text{Log}[a + b*x])/((b*c - a*d)*(b*f - a*g)^3) + (2*d^3*\text{Log}[c + d*x])/((b*c - a*d)*(d*f - c*g)^3) - (2*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*\text{Log}[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3))/((6*(b*f - a*g)*(d*f - c*g)) - (b^4*B*n*(\text{Log}[a + b*x]^2 - 2*\text{Log}[a + b*x]*\text{Log}[(b*(c + d*x))/(b*c - a*d]) - 2*\text{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))]))/(2*(b*c - a*d)*(b*f - a*g)^4) + (B*d^4*n*(2*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x] - \text{Log}[c + d*x]^2 + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)]))/(2*(b*c - a*d)*(d*f - c*g)^4) - (B*g*(2*b*d*f - b*c*g - a*d*g)*(2*b^2*d^2*f^2 - 2*b^2*c*d*f*g - 2*a*b*d^2*f*g + b^2*c^2*g^2 + a^2*d^2*g^2)*n*(\text{Log}[-((g*(a + b*x))/(b*f - a*g))]*\text{Log}[f + g*x] - \text{Log}[-((g*(c + d*x))/(d*f - c*g))]*\text{Log}[f + g*x] + \text{PolyLog}[2, (b*(f + g*x))/(b*f - a*g)] - \text{PolyLog}[2, (d*(f + g*x))/(d*f - c*g)]))/((b*f - a*g)^4*(d*f - c*g)^4))/(2*g) \end{aligned}$$

**Maple [F]** time = 0.52, size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^5} \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^5,x)

[Out] int((A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^5,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^5,x, algorithm="maxima")

[Out]  $\frac{1}{12}*(6*b^4*\text{log}(b*x + a)/(b^4*f^4*g - 4*a*b^3*f^3*g^2 + 6*a^2*b^2*f^2*g^3 - 4*a^3*b*f*g^4 + a^4*g^5) - 6*d^4*\text{log}(d*x + c)/(d^4*f^4*g - 4*c*d^3*f^3*g^2 + 6*c^2*d^2*f^2*g^3 - 4*c^3*d*f*g^4 + c^4*g^5) + 6*(4*(b^4*c*d^3 - a*b^3*d^4)*f^3 - 6*(b^4*c^2*d^2 - a^2*b^2*d^4)*f^2*g + 4*(b^4*c^3*d - a^3*b*d^4)*f*g^2 - (b^4*c^4 - a^4*d^4)*g^3)*\text{log}(g*x + f)/(b^4*d^4*f^8 + a^4*c^4*g^8 - 4*(b^4*c*d^3 + a*b^3*d^4)*f^7*g + 2*(3*b^4*c^2*d^2 + 8*a*b^3*c*d^3 + 3*a^2*b^2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*d^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c$

```

d^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2
*g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b
*c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c
^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2
*d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^
3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*
d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b
*c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g
^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3
*(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3
)*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3
*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d
)*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^
3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^
3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*
c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a
^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f
^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x)))*A*B*n - 1/4*B^2*(log((d*x +
c)^n)^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g) + 4*
integrate(-1/2*(2*d*g*x*log(e)^2 + 2*c*g*log(e)^2 + 2*(d*g*x + c*g)*log((b*
x + a)^n)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log((b*x + a)^n) + (d*f*n + (g*
n - 4*g*log(e))*d*x - 4*c*g*log(e) - 4*(d*g*x + c*g)*log((b*x + a)^n))*log(
(d*x + c)^n))/(d*g^6*x^6 + c*f^5*g + (5*d*f*g^5 + c*g^6)*x^5 + 5*(2*d*f^2*g
^4 + c*f*g^5)*x^4 + 10*(d*f^3*g^3 + c*f^2*g^4)*x^3 + 5*(d*f^4*g^2 + 2*c*f^3
*g^3)*x^2 + (d*f^5*g + 5*c*f^4*g^2)*x), x) - 1/2*A*B*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^
4*g) - 1/4*A^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g
)

```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2AB \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2}{g^5 x^5 + 5fg^4 x^4 + 10f^2 g^3 x^3 + 10f^3 g^2 x^2 + 5f^4 gx + f^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(g*x+f)^5,x, algorithm="fricas
")

```

```

[Out] integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*
x + c))^n) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2
+ 5*f^4*g*x + f^5), x)

```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2/(g\*x+f)\*\*5,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2}{(gx+f)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2/(g\*x+f)^5,x, algorithm="giac")

[Out] integrate((B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2/(g\*x + f)^5, x)

$$3.76 \quad \int \frac{(f+gx)^2}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{(f+gx)^2}{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}, x\right)$$

[Out] Unintegrable[(f + g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

**Rubi [A]** time = 0.188514, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(f+gx)^2}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] f^2\*Defer[Int][(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]^(-1), x] + 2\*f\*g\*Defer[Int][x/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x] + g^2\*Defer[Int][x^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

Rubi steps

$$\begin{aligned} \int \frac{(f+gx)^2}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx &= \int \left( \frac{f^2}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{2fgx}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{g^2x^2}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} \right) dx \\ &= f^2 \int \frac{1}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx + (2fg) \int \frac{x}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx + g^2 \int \frac{x^2}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.414442, size = 0, normalized size = 0.

$$\int \frac{(f+gx)^2}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] Integrate[(f + g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

**Maple [A]** time = 0.396, size = 0, normalized size = 0.

$$\int (gx + f)^2 \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int((g\*x+f)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^2}{B \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out] integrate((g\*x + f)^2/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{g^2 x^2 + 2 f g x + f^2}{B \log \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) + A}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] integral((g^2\*x^2 + 2\*f\*g\*x + f^2)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^2}{B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="giac")

[Out] integrate((g\*x + f)^2/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)



$$3.77 \quad \int \frac{f+gx}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

**Optimal.** Leaf size=32

$$\text{Unintegrable}\left(\frac{f+gx}{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}, x\right)$$

[Out] Unintegrable[(f + g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

**Rubi [A]** time = 0.101737, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f+gx}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] f\*Defer[Int][(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^(-1), x] + g\*Defer[Int][x/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

Rubi steps

$$\begin{aligned} \int \frac{f+gx}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx &= \int \left( \frac{f}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} + \frac{gx}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} \right) dx \\ &= f \int \frac{1}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx + g \int \frac{x}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.279493, size = 0, normalized size = 0.

$$\int \frac{f+gx}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

[Out] Integrate[(f + g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]), x]

**Maple [A]** time = 0.305, size = 0, normalized size = 0.

$$\int (gx + f) \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

[Out] int((g\*x+f)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="maxima")

[Out] integrate((g\*x + f)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{gx + f}{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n)),x, algorithm="fricas")

[Out] integral((g\*x + f)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)
```

$$3.78 \quad \int \frac{1}{A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

**Optimal.** Leaf size=26

$$\text{Unintegrable}\left(\frac{1}{B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A}, x\right)$$

[Out] Unintegrable[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]^(-1), x]

**Rubi [A]** time = 0.0150083, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]^(-1), x]

[Out] Defer[Int] [(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]^(-1), x]

Rubi steps

$$\int \frac{1}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \int \frac{1}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

**Mathematica [A]** time = 0.015118, size = 0, normalized size = 0.

$$\int \frac{1}{A + B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]^(-1), x]

[Out] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]^(-1), x]

**Maple [A]** time = 0.282, size = 0, normalized size = 0.

$$\int \left(A + B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

[Out] `int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")`

[Out] `integrate(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")`

[Out] `integral(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")`

[Out] `integrate(1/(B*log(e*((b*x + a)/(d*x + c))^n) + A), x)`

$$3.79 \quad \int \frac{1}{(f+gx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left[\frac{1}{(f+gx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}, x\right]$$

[Out] Unintegrable[1/((f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

**Rubi [A]** time = 0.0708478, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

[Out] Defer[Int][1/((f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx = \int \frac{1}{(f+gx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx$$

**Mathematica [A]** time = 0.933164, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

[Out] Integrate[1/((f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

**Maple [A]** time = 0.625, size = 0, normalized size = 0.

$$\int \frac{1}{gx+f}\left(A+B \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

[Out] `int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f) \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")`

[Out] `integrate(1/((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{Agx + Af + (Bgx + Bf) \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")`

[Out] `integral(1/(A*g*x + A*f + (B*g*x + B*f)*log(e*((b*x + a)/(d*x + c))^n)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f) \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")`

[Out] `integrate(1/((g*x + f)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)`

$$3.80 \quad \int \frac{1}{(f+gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable} \left( \frac{1}{(f+gx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}, x \right)$$

[Out] Unintegrable[1/((f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

**Rubi [A]** time = 0.0748452, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

[Out] Defer[Int][1/((f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

Rubi steps

$$\int \frac{1}{(f+gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx = \int \frac{1}{(f+gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

**Mathematica [A]** time = 1.05775, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)^2 \left( A + B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

[Out] Integrate[1/((f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

**Maple [A]** time = 0.506, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^2 \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

[Out] `int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^2 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")`

[Out] `integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{Ag^2x^2 + 2Afgx + Af^2 + (Bg^2x^2 + 2Bfgx + Bf^2) \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")`

[Out] `integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2)*log(e*((b*x + a)/(d*x + c))^n)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^2 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

$$3.81 \quad \int \frac{1}{(f+gx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable} \left( \frac{1}{(f+gx)^3 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)}, x \right)$$

[Out] Unintegrable[1/((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

**Rubi [A]** time = 0.0724352, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])),x]

[Out] Defer[Int][1/((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

Rubi steps

$$\int \frac{1}{(f+gx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx = \int \frac{1}{(f+gx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

**Mathematica [A]** time = 11.9913, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])),x]

[Out] Integrate[1/((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])), x]

**Maple [A]** time = 0.53, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^3 \left( A+B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

[Out] `int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^3 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxima")`

[Out] `integrate(1/((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{Ag^3x^3 + 3Afg^2x^2 + 3Af^2gx + Af^3 + (Bg^3x^3 + 3Bfg^2x^2 + 3Bf^2gx + Bf^3) \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fricas")`

[Out] `integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 + 3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log(e*((b*x + a)/(d*x + c))^n)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)**3/(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^3 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)), x)
```

$$3.82 \quad \int \frac{(f+gx)^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left[\frac{(f+gx)^2}{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}, x\right]$$

[Out] Unintegrable[(f + g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

**Rubi [A]** time = 0.207001, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(f+gx)^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

[Out] f^2\*Defer[Int][(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^(-2), x] + 2\*f\*g\*Defer[Int][x/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x] + g^2\*Defer[Int][x^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{(f+gx)^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx &= \int \left[ \frac{f^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{2fgx}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{g^2x^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} \right] dx \\ &= f^2 \int \frac{1}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx + (2fg) \int \frac{x}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx + g^2 \int \frac{x^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.935178, size = 0, normalized size = 0.

$$\int \frac{(f+gx)^2}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

[Out] Integrate[(f + g\*x)^2/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

---

**Maple [A]** time = 0.418, size = 0, normalized size = 0.

$$\int (gx + f)^2 \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((g\*x+f)^2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

---

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bdg^2x^4 + acf^2 + (adg^2 + (2dfg + cg^2)b)x^3 + ((2dfg + cg^2)a + (df^2 + 2cfcg)b)x^2 + (bcf^2 + (df^2 + 2cfcg)a)x}{(bcn - adn)B^2 \log((bx + a)^n) - (bcn - adn)B^2 \log((dx + c)^n) + (bcn - adn)AB + (bcn \log(e) - adn \log(e))B^2} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out]  $-(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*x) / ((b*c*n - a*d*n)*B^2*\log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*\log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*\log(e) - a*d*n*\log(e))*B^2) + \text{integrate}((4*b*d*g^2*x^3 + b*c*f^2 + 3*(a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^2 + (d*f^2 + 2*c*f*g)*a + 2*((2*d*f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x) / ((b*c*n - a*d*n)*B^2*\log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*\log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*\log(e) - a*d*n*\log(e))*B^2), x)$

---

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{g^2x^2 + 2fgx + f^2}{B^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2AB \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out] integral((g^2\*x^2 + 2\*f\*g\*x + f^2)/(B^2\*log(e\*((b\*x + a)/(d\*x + c))^n))^2 + 2\*A\*B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))\*\*n))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^2}{\left(B \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate((g\*x + f)^2/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2, x)



$$3.83 \quad \int \frac{f+gx}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

**Optimal.** Leaf size=32

$$\text{Unintegrable}\left(\frac{f+gx}{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[(f + g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

**Rubi [A]** time = 0.113645, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f+gx}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

[Out] f\*Defer[Int][(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^(-2), x] + g\*Defer[Int][x/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{f+gx}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx &= \int \left( \frac{f}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{gx}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} \right) dx \\ &= f \int \frac{1}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx + g \int \frac{x}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.695821, size = 0, normalized size = 0.

$$\int \frac{f+gx}{\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

[Out] Integrate[(f + g\*x)/(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2, x]

**Maple [A]** time = 0.253, size = 0, normalized size = 0.

$$\int (gx + f) \left( A + B \ln \left( e \left( \frac{bx + a}{dx + c} \right)^n \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] int((g\*x+f)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bdgx^3 + acf + (adg + (df + cg)b)x^2 + (bcf + (df + cg)a)x}{(bcn - adn)B^2 \log((bx + a)^n) - (bcn - adn)B^2 \log((dx + c)^n) + (bcn - adn)AB + (bcn \log(e) - adn \log(e))B^2} + \int \frac{bc}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="maxima")

[Out]  $-(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g)*a)*x) / ((b*c*n - a*d*n)*B^2*\log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*\log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*\log(e) - a*d*n*\log(e))*B^2) + \text{integrate}((3*b*d*g*x^2 + b*c*f + (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x) / ((b*c*n - a*d*n)*B^2*\log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*\log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*\log(e) - a*d*n*\log(e))*B^2), x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{gx + f}{B^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2AB \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="fricas")

[Out] integral((g\*x + f)/(B^2\*log(e\*((b\*x + a)/(d\*x + c))^n))^2 + 2\*A\*B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))^2,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{\left(B \log\left(e \left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate((g\*x + f)/(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2, x)

$$3.84 \quad \int \frac{1}{\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

**Optimal.** Leaf size=26

$$\text{Unintegrable} \left( \frac{1}{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}, x \right)$$

[Out] Unintegrable[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]^(-2), x]

**Rubi [A]** time = 0.0147984, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]^(-2), x]

[Out] Defer[Int][(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]^(-2), x]

Rubi steps

$$\int \frac{1}{\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx = \int \frac{1}{\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

**Mathematica [A]** time = 0.59263, size = 0, normalized size = 0.

$$\int \frac{1}{\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]^(-2), x]

[Out] Integrate[(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n]]^(-2), x]

**Maple [A]** time = 0.321, size = 0, normalized size = 0.

$$\int \left(A+B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)\right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)`

[Out] `int(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx^2 + ac + (bc + ad)x}{(bcn - adn)B^2 \log((bx + a)^n) - (bcn - adn)B^2 \log((dx + c)^n) + (bcn - adn)AB + (bcn \log(e) - adn \log(e))B^2} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")`

[Out] `-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2) + integrate((2*b*d*x + b*c + a*d)/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x + c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{B^2 \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2AB \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")`

[Out] `integral(1/(B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d*x + c))^n) + A^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^(-2), x)
```

$$3.85 \quad \int \frac{1}{(f+gx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable} \left( \frac{1}{(f+gx)\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}, x \right)$$

[Out] Unintegrable[1/((f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

**Rubi [A]** time = 0.0782827, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

[Out] Defer[Int][1/((f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx = \int \frac{1}{(f+gx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

**Mathematica [A]** time = 1.60863, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)\left(A+B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

[Out] Integrate[1/((f + g\*x)\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

**Maple [A]** time = 0.549, size = 0, normalized size = 0.

$$\int \frac{1}{gx+f} \left( A + B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)`

[Out] `int(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx^2 + ac + (bc + bcfn - adfn)AB + (bcfn \log(e) - adfn \log(e))B^2 + ((bcgn - adgn)AB + (bcgn \log(e) - adgn \log(e))B^2)x + ((bcgn - adgn)AB + (bcgn \log(e) - adgn \log(e))B^2)}{(bcfn - adfn)AB + (bcfn \log(e) - adfn \log(e))B^2 + ((bcgn - adgn)AB + (bcgn \log(e) - adgn \log(e))B^2)x + ((bcgn - adgn)AB + (bcgn \log(e) - adgn \log(e))B^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")`

[Out] `-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f*n - a*d*f*n)*A*B + (b*c*f*n*log(e) - a*d*f*n*log(e))*B^2 + ((b*c*g*n - a*d*g*n)*A*B + (b*c*g*n*log(e) - a*d*g*n*log(e))*B^2)*x + ((b*c*g*n - a*d*g*n)*B^2*x + (b*c*f*n - a*d*f*n)*B^2)*log((b*x + a)^n) - ((b*c*g*n - a*d*g*n)*B^2*x + (b*c*f*n - a*d*f*n)*B^2)*log((d*x + c)^n) + integrate((b*d*g*x^2 + 2*b*d*f*x + b*c*f + (d*f - c*g)*a)/((b*c*f^2*n - a*d*f^2*n)*A*B + (b*c*f^2*n*log(e) - a*d*f^2*n*log(e))*B^2 + ((b*c*g^2*n - a*d*g^2*n)*A*B + (b*c*g^2*n*log(e) - a*d*g^2*n*log(e))*B^2)*x^2 + 2*((b*c*f*g*n - a*d*f*g*n)*A*B + (b*c*f*g*n*log(e) - a*d*f*g*n*log(e))*B^2)*x + ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((b*x + a)^n) - ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((d*x + c)^n), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2gx + A^2f + (B^2gx + B^2f) \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)^2 + 2(ABgx + ABf) \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")`

[Out] `integral(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*log(e*((b*x + a)/(d*x + c))^n))^2 + 2*(A*B*g*x + A*B*f)*log(e*((b*x + a)/(d*x + c))^n), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)`



[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f) \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate(1/((g\*x + f)\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2), x)

$$3.86 \quad \int \frac{1}{(f+gx)^2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable} \left( \frac{1}{(f+gx)^2 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)^2}, x \right)$$

[Out] Unintegrable[1/((f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

**Rubi [A]** time = 0.0813881, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)^2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

[Out] Defer[Int][1/((f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

Rubi steps

$$\int \frac{1}{(f+gx)^2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx = \int \frac{1}{(f+gx)^2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

**Mathematica [A]** time = 4.21421, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)^2 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

[Out] Integrate[1/((f + g\*x)^2\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

**Maple [A]** time = 0.509, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^2 \left( A+B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)^{-2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

```
[Out] int(1/(g*x+f)^2/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxi
ma")
```

```
[Out] -(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2*n - a*d*f^2*n)*A*B + (b*c*f^2*n*
log(e) - a*d*f^2*n*log(e))*B^2 + ((b*c*g^2*n - a*d*g^2*n)*A*B + (b*c*g^2*n*
log(e) - a*d*g^2*n*log(e))*B^2)*x^2 + 2*((b*c*f*g*n - a*d*f*g*n)*A*B + (b*c
*f*g*n*log(e) - a*d*f*g*n*log(e))*B^2)*x + ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2
+ 2*(b*c*f*g*n - a*d*f*g*n)*B^2*x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((b*x
+ a)^n) - ((b*c*g^2*n - a*d*g^2*n)*B^2*x^2 + 2*(b*c*f*g*n - a*d*f*g*n)*B^2*
x + (b*c*f^2*n - a*d*f^2*n)*B^2)*log((d*x + c)^n) - integrate(-(b*c*f + (d
*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*x)/(((b*c*g^3*n - a*d*g^3*n)*A*B
+ (b*c*g^3*n*log(e) - a*d*g^3*n*log(e))*B^2)*x^3 + (b*c*f^3*n - a*d*f^3*n)*
A*B + (b*c*f^3*n*log(e) - a*d*f^3*n*log(e))*B^2 + 3*((b*c*f*g^2*n - a*d*f*g
^2*n)*A*B + (b*c*f*g^2*n*log(e) - a*d*f*g^2*n*log(e))*B^2)*x^2 + 3*((b*c*f^
2*g*n - a*d*f^2*g*n)*A*B + (b*c*f^2*g*n*log(e) - a*d*f^2*g*n*log(e))*B^2)*x
+ ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*B^2*x^2
+ 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2)*log((
b*x + a)^n) - ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2
*n)*B^2*x^2 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)
*B^2)*log((d*x + c)^n)), x)
```

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2 g^2 x^2 + 2 A^2 f g x + A^2 f^2 + (B^2 g^2 x^2 + 2 B^2 f g x + B^2 f^2) \log \left( e \left( \frac{b x + a}{d x + c} \right)^n \right)^2 + 2 (A B g^2 x^2 + 2 A B f g x + A B f^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fric
as")
```

```
[Out] integral(1/(A^2*g^2*x^2 + 2*A^2*f*g*x + A^2*f^2 + (B^2*g^2*x^2 + 2*B^2*f*g*
x + B^2*f^2)*log(e*((b*x + a)/(d*x + c))^n))^2 + 2*(A*B*g^2*x^2 + 2*A*B*f*g*
x + A*B*f^2)*log(e*((b*x + a)/(d*x + c))^n)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^2 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2), x)
```

$$3.87 \quad \int \frac{1}{(f+gx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable} \left( \frac{1}{(f+gx)^3 \left( B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) + A \right)^2}, x \right)$$

[Out] Unintegrable[1/((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

**Rubi [A]** time = 0.0798446, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

[Out] Defer[Int][1/((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

Rubi steps

$$\int \frac{1}{(f+gx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx = \int \frac{1}{(f+gx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

**Mathematica [A]** time = 36.6298, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)^3 \left( A+B \log \left( e \left( \frac{a+bx}{c+dx} \right)^n \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

[Out] Integrate[1/((f + g\*x)^3\*(A + B\*Log[e\*((a + b\*x)/(c + d\*x))^n])^2), x]

**Maple [A]** time = 0.521, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^3 \left( A+B \ln \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) \right)^{-2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)`

[Out] `int(1/(g*x+f)^3/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="maxima")`

[Out] 
$$-(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3*n - a*d*g^3*n)*A*B + (b*c*g^3*n * \log(e) - a*d*g^3*n*\log(e))*B^2)*x^3 + (b*c*f^3*n - a*d*f^3*n)*A*B + (b*c*f^3*n*\log(e) - a*d*f^3*n*\log(e))*B^2 + 3*((b*c*f*g^2*n - a*d*f*g^2*n)*A*B + (b*c*f*g^2*n*\log(e) - a*d*f*g^2*n*\log(e))*B^2)*x^2 + 3*((b*c*f^2*g*n - a*d*f^2*g*n)*A*B + (b*c*f^2*g*n*\log(e) - a*d*f^2*g*n*\log(e))*B^2)*x + ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*B^2*x^2 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2)*\log((b*x + a)^n) - ((b*c*g^3*n - a*d*g^3*n)*B^2*x^3 + 3*(b*c*f*g^2*n - a*d*f*g^2*n)*B^2*x^2 + 3*(b*c*f^2*g*n - a*d*f^2*g*n)*B^2*x + (b*c*f^3*n - a*d*f^3*n)*B^2)*\log((d*x + c)^n) - \int (b*d*g*x^2 - b*c*f - (d*f - 3*c*g)*a + 2*(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4*n - a*d*g^4*n)*A*B + (b*c*g^4*n*\log(e) - a*d*g^4*n*\log(e))*B^2)*x^4 + 4*((b*c*f*g^3*n - a*d*f*g^3*n)*A*B + (b*c*f*g^3*n*\log(e) - a*d*f*g^3*n*\log(e))*B^2)*x^3 + (b*c*f^4*n - a*d*f^4*n)*A*B + (b*c*f^4*n*\log(e) - a*d*f^4*n*\log(e))*B^2 + 6*((b*c*f^2*g^2*n - a*d*f^2*g^2*n)*A*B + (b*c*f^2*g^2*n*\log(e) - a*d*f^2*g^2*n*\log(e))*B^2)*x^2 + 4*((b*c*f^3*g*n - a*d*f^3*g*n)*A*B + (b*c*f^3*g*n*\log(e) - a*d*f^3*g*n*\log(e))*B^2)*x + ((b*c*g^4*n - a*d*g^4*n)*B^2*x^4 + 4*(b*c*f*g^3*n - a*d*f*g^3*n)*B^2*x^3 + 6*(b*c*f^2*g^2*n - a*d*f^2*g^2*n)*B^2*x^2 + 4*(b*c*f^3*g*n - a*d*f^3*g*n)*B^2*x + (b*c*f^4*n - a*d*f^4*n)*B^2)*\log((b*x + a)^n) - ((b*c*g^4*n - a*d*g^4*n)*B^2*x^4 + 4*(b*c*f*g^3*n - a*d*f*g^3*n)*B^2*x^3 + 6*(b*c*f^2*g^2*n - a*d*f^2*g^2*n)*B^2*x^2 + 4*(b*c*f^3*g*n - a*d*f^3*g*n)*B^2*x + (b*c*f^4*n - a*d*f^4*n)*B^2)*\log((d*x + c)^n)), x)$$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

integral 
$$\frac{1}{A^2g^3x^3 + 3A^2fg^2x^2 + 3A^2f^2gx + A^2f^3 + (B^2g^3x^3 + 3B^2fg^2x^2 + 3B^2f^2gx + B^2f^3)\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2 + 2(AB$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^3/(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fricas")`

[Out] `integral(1/(A^2*g^3*x^3 + 3*A^2*f*g^2*x^2 + 3*A^2*f^2*g*x + A^2*f^3 + (B^2*g^3*x^3 + 3*B^2*f*g^2*x^2 + 3*B^2*f^2*g*x + B^2*f^3)*log(e*((b*x + a)/(d*x + c))^n))^2 + 2*(A*B*g^3*x^3 + 3*A*B*f*g^2*x^2 + 3*A*B*f^2*g*x + A*B*f^3)*lo`

$g(e*((b*x + a)/(d*x + c))^n), x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)\*\*3/(A+B\*ln(e\*((b\*x+a)/(d\*x+c))^n))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^3 \left( B \log \left( e \left( \frac{bx+a}{dx+c} \right)^n \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)^3/(A+B\*log(e\*((b\*x+a)/(d\*x+c))^n))^2,x, algorithm="giac")

[Out] integrate(1/((g\*x + f)^3\*(B\*log(e\*((b\*x + a)/(d\*x + c))^n) + A)^2), x)

### 3.88 $\int (ag + bgx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right) dx$

**Optimal.** Leaf size=180

$$\frac{g^4(a+bx)^5 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right)}{5b} + \frac{Bg^4x(bc-ad)^4}{5d^4} - \frac{Bg^4(a+bx)^2(bc-ad)^3}{10bd^3} + \frac{Bg^4(a+bx)^3(bc-ad)^2}{15bd^2} - \frac{Bg^4(bc-ad)^5}{5bd^5}$$

[Out] (B\*(b\*c - a\*d)^4\*g^4\*x)/(5\*d^4) - (B\*(b\*c - a\*d)^3\*g^4\*(a + b\*x)^2)/(10\*b\*d^3) + (B\*(b\*c - a\*d)^2\*g^4\*(a + b\*x)^3)/(15\*b\*d^2) - (B\*(b\*c - a\*d)\*g^4\*(a + b\*x)^4)/(20\*b\*d) + (g^4\*(a + b\*x)^5\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]))/(5\*b) - (B\*(b\*c - a\*d)^5\*g^4\*Log[c + d\*x])/(5\*b\*d^5)

**Rubi [A]** time = 0.123745, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 43}

$$\frac{g^4(a+bx)^5 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right)}{5b} + \frac{Bg^4x(bc-ad)^4}{5d^4} - \frac{Bg^4(a+bx)^2(bc-ad)^3}{10bd^3} + \frac{Bg^4(a+bx)^3(bc-ad)^2}{15bd^2} - \frac{Bg^4(bc-ad)^5}{5bd^5}$$

Antiderivative was successfully verified.

[In] Int[(a\*g + b\*g\*x)^4\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]),x]

[Out] (B\*(b\*c - a\*d)^4\*g^4\*x)/(5\*d^4) - (B\*(b\*c - a\*d)^3\*g^4\*(a + b\*x)^2)/(10\*b\*d^3) + (B\*(b\*c - a\*d)^2\*g^4\*(a + b\*x)^3)/(15\*b\*d^2) - (B\*(b\*c - a\*d)\*g^4\*(a + b\*x)^4)/(20\*b\*d) + (g^4\*(a + b\*x)^5\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]))/(5\*b) - (B\*(b\*c - a\*d)^5\*g^4\*Log[c + d\*x])/(5\*b\*d^5)

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol]
:> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps



$$\begin{aligned}
\int (ag + bgx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right) dx &= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{5b} - \frac{B \int \frac{(bc-ad)g^5(a+bx)^4}{c+dx} dx}{5bg} \\
&= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{5b} - \frac{(B(bc-ad)g^4) \int \frac{(a+bx)^4}{c+dx} dx}{5b} \\
&= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{5b} - \frac{(B(bc-ad)g^4) \int \left( -\frac{b(bc-ad)^3}{d^4} + \frac{b(bc-ad)}{c} \right) dx}{5b} \\
&= \frac{B(bc-ad)^4 g^4 x}{5d^4} - \frac{B(bc-ad)^3 g^4 (a+bx)^2}{10bd^3} + \frac{B(bc-ad)^2 g^4 (a+bx)^3}{15bd^2} - \frac{B(bc-ad) g^4 (a+bx)^4}{20bd} + \frac{B(bc-ad) g^4 (a+bx)^5}{25b}
\end{aligned}$$

**Mathematica [A]** time = 0.1062, size = 142, normalized size = 0.79

$$\frac{g^4 \left( (a+bx)^5 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right) - \frac{B(bc-ad)(6d^2(a+bx)^2(bc-ad)^2 + 4d^3(a+bx)^3(ad-bc) - 12bdx(bc-ad)^3 + 12(bc-ad)^4 \log(c+dx) + 3d^4(a+bx)^4)}{12d^5} \right)}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^4\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]),x]

[Out] (g^4\*((a + b\*x)^5\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]) - (B\*(b\*c - a\*d)\*(-12\*b\*d\*(b\*c - a\*d)^3\*x + 6\*d^2\*(b\*c - a\*d)^2\*(a + b\*x)^2 + 4\*d^3\*(-(b\*c) + a\*d)\*(a + b\*x)^3 + 3\*d^4\*(a + b\*x)^4 + 12\*(b\*c - a\*d)^4\*Log[c + d\*x]))/(12\*d^5))/(5\*b)

**Maple [B]** time = 0.225, size = 8417, normalized size = 46.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^4\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c))),x)

[Out] result too large to display

**Maxima [B]** time = 1.2001, size = 841, normalized size = 4.67

$$\frac{1}{5} Ab^4 g^4 x^5 + Aab^3 g^4 x^4 + 2Aa^2 b^2 g^4 x^3 + 2Aa^3 b g^4 x^2 + \left( x \log \left( \frac{bex}{dx+c} + \frac{ae}{dx+c} \right) + \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) Ba$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="maxima")

[Out] 1/5\*A\*b^4\*g^4\*x^5 + A\*a\*b^3\*g^4\*x^4 + 2\*A\*a^2\*b^2\*g^4\*x^3 + 2\*A\*a^3\*b\*g^4\*x^2 + (x\*log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c)) + a\*log(b\*x + a)/b - c\*log(d\*x + c)/d)\*B\*a^4\*g^4 + 2\*(x^2\*log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c)) - a^2\*log(b\*x + a)/b^2 + c^2\*log(d\*x + c)/d^2 - (b\*c - a\*d)\*x/(b\*d))\*B\*a^3\*b\*g^4 + (2\*x^3\*log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c)) + 2\*a^3\*log(b\*x + a)/b^3 - 2\*c^3\*

$$\log(dx + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) * B*a^2*b^2*g^4 + 1/6*(6*x^4*\log(b*e*x/(dx + c)) + a*e/(dx + c)) - 6*a^4*\log(b*x + a)/b^4 + 6*c^4*\log(dx + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) * B*a*b^3*g^4 + 1/60*(12*x^5*\log(b*e*x/(dx + c)) + a*e/(dx + c)) + 12*a^5*\log(b*x + a)/b^5 - 12*c^5*\log(dx + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) * B*b^4*g^4 + A*a^4*g^4*x$$

**Fricas [B]** time = 1.36423, size = 910, normalized size = 5.06

$$12 Ab^5 d^5 g^4 x^5 + 12 Ba^5 d^5 g^4 \log(bx + a) - 3 (Bb^5 cd^4 - (20A + B)ab^4 d^5) g^4 x^4 + 4 (Bb^5 c^2 d^3 - 5 Bab^4 cd^4 + 2(15A + 2B)a^2 d^5) g^4 x^3 - 6 (Bb^5 c^3 d^2 - 5Bab^4 c^2 d^3 + 10B^2 a^2 b^3 c d^4 - 2(10A + 3B)a^3 b^2 d^5) g^4 x^2 + 12 (Bb^5 c^4 d - 5B^2 a^2 b^4 c^3 d^2 + 10B^2 a^2 b^3 c^2 d^3 - 10B^2 a^3 b^2 c d^4 + (5A + 4B)a^4 b d^5) g^4 x - 12 (Bb^5 c^5 - 5B^2 a^4 b^4 c^4 d + 10B^2 a^2 b^3 c^3 d^2 - 10B^2 a^3 b^2 c^2 d^3 + 5B^2 a^4 b^2 c d^4) g^4 \log(dx + c) + 12 (Bb^5 d^5 g^4 x^5 + 5B^2 a^4 b^4 d^5 g^4 x^4 + 10B^2 a^2 b^3 d^5 g^4 x^3 + 10B^2 a^3 b^2 d^5 g^4 x^2 + 5B^2 a^4 b^2 d^5 g^4 x) \log((b*e*x + a*e)/(dx + c)) / (b*d^5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 + 12*B*a^5*d^5*g^4*log(b*x + a) - 3*(B*b^5*c*d^4 - (20*A + B)*a*b^4*d^5)*g^4*x^4 + 4*(B*b^5*c^2*d^3 - 5*B*a*b^4*c*d^4 + 2*(15*A + 2*B)*a^2*b^3*d^5)*g^4*x^3 - 6*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 10*B*a^2*b^3*c*d^4 - 2*(10*A + 3*B)*a^3*b^2*d^5)*g^4*x^2 + 12*(B*b^5*c^4*d - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 10*B*a^3*b^2*c*d^4 + (5*A + 4*B)*a^4*b*d^5)*g^4*x - 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*log(dx + c) + 12*(B*b^5*d^5*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*log((b*e*x + a*e)/(d*x + c))/(b*d^5)
```

**Sympy [B]** time = 10.0173, size = 993, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**4*g**4*x**5/5 + B*a**5*g**4*log(x + (B*a**6*d**5*g**4/b + 5*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b**2*c**3*d**2*g**4 - 5*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4)/(B*a**5*d**5*g**4 + 5*B*a**4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d**2*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*b) - B*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)*log(x + (6*B*a**5*c*d**4*g**4 - 10*B*a**4*b*c**2*d**3*g**4 + 10*B*a**3*b**2*c**3*d**2*g**4 - 5*B*a**2*b**3*c**4*d*g**4 + B*a*b**4*c**5*g**4 - B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4) + B*b*c**2*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/(B*a**5*d**5*g**4 + 5*B*a**4*b*c*d**4*g**4 - 10*B*a**3*b**2*c**2*d**3*g**4 + 10*B*a**2*b**3*c**3*d**2*g**4 - 5*B*a*b**4*c**4*d*g**4 + B*b**5*c**5*g**4))/(5*d**5) + (B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x**3 + B*a*b**3*g**4*x**4 + B*b**4*g**4*x**5/5)*log(e*(a + b*x)/(c + d*x)) + x**4*(20*A*a*b**3*d*g**4 + B*a*b**3*d*g**4 - B*b**4*c*g**4)/(20*d) + x**3*(30*A*a**2*b**2*d**2*g**4 + 4*B*a**2*b**2*d**2*g**4 - 5*B*a*b**3*c*d*g**4 + B*b**4*c**2*g**4)/(15*d**2) + x**2*(20*A*a**3*b*d**3*g**4 + 6*B*a**3*b*d**3*g**4 - 10*B*a**2*b**2*c
```

$(d^2g^4 + 5Bab^3c^2dg^4 - Bb^4c^3g^4)/(10d^3) + x(5Aa^4d^4g^4 + 4Baa^4d^4g^4 - 10Baa^3b^2cd^3g^4 + 10Baa^2b^2c^2d^2g^4 - 5Bab^3c^2dg^4 + Bb^4c^3g^4)/(5d^4)$

**Giac [B]** time = 99.387, size = 639, normalized size = 3.55

$$\frac{Ba^5g^4 \log(bx + a)}{5b} + \frac{1}{5}(Ab^4g^4 + Bb^4g^4)x^5 - \frac{(Bb^4cg^4 - 20Aab^3dg^4 - 21Bab^3dg^4)x^4}{20d} + \frac{(Bb^4c^2g^4 - 5Bab^3cdg^4 + 30Aa^4d^4g^4)}{15d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="giac")

[Out]  $\frac{1}{5}B^5a^5g^4 \log(bx + a)/b + \frac{1}{5}(A^5b^4g^4 + B^5b^4g^4)x^5 - \frac{1}{20}(B^5b^4c^2g^4 - 5B^5ab^3c^2dg^4 + 30A^5a^2b^2d^2g^4 + 34B^5a^2b^2d^2g^4)x^3/d^2 + \frac{1}{5}(B^5b^4g^4x^5 + 5B^5ab^3g^4x^4 + 10B^5a^2b^2g^4x^3 + 10B^5a^3b^2g^4x^2 + 5B^5a^4g^4x) \log((bx + a)/(d*x + c)) - \frac{1}{10}(B^5b^4c^3g^4 - 5B^5ab^3c^2dg^4 + 10B^5a^2b^2c^2d^2g^4 - 20A^5a^3b^2d^3g^4 - 26B^5a^3b^2d^3g^4)x^2/d^3 + \frac{1}{5}(B^5b^4c^4g^4 - 5B^5ab^3c^3dg^4 + 10B^5a^2b^2c^2d^2g^4 - 10B^5a^3b^2c^2d^2g^4 - 10B^5a^3b^2c^2d^2g^4 + 5A^5a^4d^4g^4 + 9B^5a^4d^4g^4)x/d^4 - \frac{1}{5}(B^5b^4c^5g^4 - 5B^5ab^3c^4dg^4 + 10B^5a^2b^2c^3d^2g^4 - 10B^5a^3b^2c^3d^2g^4 + 5B^5a^4c^3d^2g^4) \log(-d*x - c)/d^5$

$$3.89 \quad \int (ag + bgx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right) dx$$

**Optimal.** Leaf size=149

$$\frac{g^3(a+bx)^4 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right)}{4b} - \frac{Bg^3x(bc-ad)^3}{4d^3} + \frac{Bg^3(a+bx)^2(bc-ad)^2}{8bd^2} + \frac{Bg^3(bc-ad)^4 \log(c+dx)}{4bd^4} - \frac{Bg^3(a+bx)}{12b}$$

[Out]  $-(B*(b*c - a*d)^3*g^3*x)/(4*d^3) + (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(8*b*d^2) - (B*(b*c - a*d)*g^3*(a + b*x)^3)/(12*b*d) + (g^3*(a + b*x)^4*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(4*b) + (B*(b*c - a*d)^4*g^3*\text{Log}[c + d*x])/(4*b*d^4)$

**Rubi [A]** time = 0.0962254, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 43}

$$\frac{g^3(a+bx)^4 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right)}{4b} - \frac{Bg^3x(bc-ad)^3}{4d^3} + \frac{Bg^3(a+bx)^2(bc-ad)^2}{8bd^2} + \frac{Bg^3(bc-ad)^4 \log(c+dx)}{4bd^4} - \frac{Bg^3(a+bx)}{12b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^3*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]), x]$

[Out]  $-(B*(b*c - a*d)^3*g^3*x)/(4*d^3) + (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(8*b*d^2) - (B*(b*c - a*d)*g^3*(a + b*x)^3)/(12*b*d) + (g^3*(a + b*x)^4*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(4*b) + (B*(b*c - a*d)^4*g^3*\text{Log}[c + d*x])/(4*b*d^4)$

### Rule 2525

$\text{Int}[(a + \text{Log}[(c + \text{RFX})^p])*(b*x)^n*((d + e*x)^m), x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^n/(e^{m+1}), x] - \text{Dist}[(b*n*p)/(e^{m+1}), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^{n-1}*D[\text{RFX}, x])/RFX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

$\text{Int}[(a)*(u), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b)\*(v)] /; FreeQ[b, x]

### Rule 43

$\text{Int}[(a + (b*x)^m)*((c + d*x)^n), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
\int (ag + bgx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right) dx &= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{4b} - \frac{B \int \frac{(bc-ad)g^4(a+bx)^3}{c+dx} dx}{4bg} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{4b} - \frac{(B(bc-ad)g^3) \int \frac{(a+bx)^3}{c+dx} dx}{4b} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{4b} - \frac{(B(bc-ad)g^3) \int \left( \frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} \right) dx}{4b} \\
&= -\frac{B(bc-ad)^3 g^3 x}{4d^3} + \frac{B(bc-ad)^2 g^3 (a+bx)^2}{8bd^2} - \frac{B(bc-ad)g^3 (a+bx)^3}{12bd} + \frac{g^3}{4b}
\end{aligned}$$

**Mathematica [A]** time = 0.0977591, size = 120, normalized size = 0.81

$$\frac{g^3 \left( (a+bx)^4 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right) - \frac{B(bc-ad)(3d^2(a+bx)^2(ad-bc) + 6bdx(bc-ad)^2 - 6(bc-ad)^3 \log(c+dx) + 2d^3(a+bx)^3)}{6d^4} \right)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]),x]

[Out] (g^3\*((a + b\*x)^4\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]) - (B\*(b\*c - a\*d)\*(6\*b\*d\*(b\*c - a\*d)^2\*x + 3\*d^2\*(-(b\*c) + a\*d)\*(a + b\*x)^2 + 2\*d^3\*(a + b\*x)^3 - 6\*(b\*c - a\*d)^3\*Log[c + d\*x]))/(6\*d^4))/(4\*b)

**Maple [B]** time = 0.18, size = 5556, normalized size = 37.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^3\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c))),x)

[Out] result too large to display

**Maxima [B]** time = 1.22249, size = 593, normalized size = 3.98

$$\frac{1}{4} Ab^3 g^3 x^4 + Aab^2 g^3 x^3 + \frac{3}{2} Aa^2 b g^3 x^2 + \left( x \log \left( \frac{bex}{dx+c} + \frac{ae}{dx+c} \right) + \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) Ba^3 g^3 + \frac{3}{2} \left( x^2 \log \left( \frac{bex}{dx+c} + \frac{ae}{dx+c} \right) + \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="maxima")

[Out] 1/4\*A\*b^3\*g^3\*x^4 + A\*a\*b^2\*g^3\*x^3 + 3/2\*A\*a^2\*b\*g^3\*x^2 + (x\*log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c)) + a\*log(b\*x + a)/b - c\*log(d\*x + c)/d)\*B\*a^3\*g^3 + 3/2\*(x^2\*log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c)) - a^2\*log(b\*x + a)/b^2 + c^2\*log(d\*x + c)/d^2 - (b\*c - a\*d)\*x/(b\*d))\*B\*a^2\*b\*g^3 + 1/2\*(2\*x^3\*log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c)) + 2\*a^3\*log(b\*x + a)/b^3 - 2\*c^3\*log(d\*x + c)/d^3 - ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2))\*B\*a\*b^2

$$*g^3 + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^3 + A*a^3*g^3*x$$

**Fricas [B]** time = 1.13677, size = 664, normalized size = 4.46

$$6 Ab^4 d^4 g^3 x^4 + 6 Ba^4 d^4 g^3 \log(bx + a) - 2 (Bb^4 cd^3 - (12 A + B)ab^3 d^4)g^3 x^3 + 3 (Bb^4 c^2 d^2 - 4 Bab^3 cd^3 + 3 (4 A + B)a^2 b^2 d^4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 6*B*a^4*d^4*g^3*log(b*x + a) - 2*(B*b^4*c*d^3 - (12*A + B)*a*b^3*d^4)*g^3*x^3 + 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*(4*A + B)*a^2*b^2*d^4)*g^3*x^2 - 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*b^2*c*d^3 - (4*A + 3*B)*a^3*b*d^4)*g^3*x + 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 6*(B*b^4*d^4*g^3*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*x)*log((b*e*x + a*e)/(d*x + c))/(b*d^4)
```

**Sympy [B]** time = 7.13052, size = 719, normalized size = 4.83

$$\frac{Ab^3 g^3 x^4}{4} + \frac{Ba^4 g^3 \log\left(x + \frac{Ba^5 d^4 g^3 + 4Ba^4 cd^3 g^3 - 6Ba^3 bc^2 d^2 g^3 + 4Ba^2 b^2 c^3 dg^3 - Bab^3 c^4 g^3}{Ba^4 d^4 g^3 + 4Ba^3 bcd^3 g^3 - 6Ba^2 b^2 c^2 d^2 g^3 + 4Bab^3 c^3 dg^3 - Bb^4 c^4 g^3}\right)}{4b} - \frac{Bcg^3 (2ad - bc) (2a^2 d^2 - 2abcd + b^2 c^2) \log\left(\dots\right)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b**3*g**3*x**4/4 + B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*b) - B*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c**4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) + B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(4*d**4) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(a + b*x)/(c + d*x)) + x**3*(12*A*a*b**2*d*g**3 + B*a*b**2*d*g**3 - B*b**3*c*g**3)/(12*d) + x**2*(12*A*a**2*b*d**2*g**3 + 3*B*a**2*b*d**2*g**3 - 4*B*a*b**2*c*d*g**3 + B*b**3*c**2*g**3)/(8*d**2) + x*(4*A*a**3*d**3*g**3 + 3*B*a**3*d**3*g**3 - 6*B*a**2*b*c*d**2*g**3 + 4*B*a*b**2*c**2*d*g**3 - B*b**3*c**3*g**3)/(4*d**3)
```

**Giac [B]** time = 21.8869, size = 458, normalized size = 3.07

$$\frac{Ba^4 g^3 \log(bx + a)}{4b} + \frac{1}{4} (Ab^3 g^3 + Bb^3 g^3) x^4 - \frac{(Bb^3 cg^3 - 12 Aab^2 dg^3 - 13 Bab^2 dg^3) x^3}{12d} + \frac{1}{4} (Bb^3 g^3 x^4 + 4 Bab^2 g^3 x^3 + 6 Ba^2 b g^3 x^2 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="giac")

[Out]  $\frac{1}{4}B*a^4*g^3*\log(b*x + a)/b + \frac{1}{4}(A*b^3*g^3 + B*b^3*g^3)*x^4 - \frac{1}{12}(B*b^3*c*g^3 - 12*A*a*b^2*d*g^3 - 13*B*a*b^2*d*g^3)*x^3/d + \frac{1}{4}(B*b^3*g^3*x^4 + 4*B*a*b^2*g^3*x^3 + 6*B*a^2*b*g^3*x^2 + 4*B*a^3*g^3*x)*\log((b*x + a)/(d*x + c)) + \frac{1}{8}(B*b^3*c^2*g^3 - 4*B*a*b^2*c*d*g^3 + 12*A*a^2*b*d^2*g^3 + 15*B*a^2*b*d^2*g^3)*x^2/d^2 - \frac{1}{4}(B*b^3*c^3*g^3 - 4*B*a*b^2*c^2*d*g^3 + 6*B*a^2*b*c*d^2*g^3 - 4*A*a^3*d^3*g^3 - 7*B*a^3*d^3*g^3)*x/d^3 + \frac{1}{4}(B*b^3*c^4*g^3 - 4*B*a*b^2*c^3*d*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a^3*c*d^3*g^3)*\log(d*x + c)/d^4$

### 3.90 $\int (ag + bgx)^2 \left( A + B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) \right) dx$

**Optimal.** Leaf size=118

$$\frac{g^2(a+bx)^3 \left( B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) + A \right)}{3b} + \frac{Bg^2x(bc-ad)^2}{3d^2} - \frac{Bg^2(bc-ad)^3 \log(c+dx)}{3bd^3} - \frac{Bg^2(a+bx)^2(bc-ad)}{6bd}$$

[Out]  $(B*(b*c - a*d)^2*g^2*x)/(3*d^2) - (B*(b*c - a*d)*g^2*(a + b*x)^2)/(6*b*d) + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b) - (B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3)$

**Rubi [A]** time = 0.0792376, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 43}

$$\frac{g^2(a+bx)^3 \left( B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) + A \right)}{3b} + \frac{Bg^2x(bc-ad)^2}{3d^2} - \frac{Bg^2(bc-ad)^3 \log(c+dx)}{3bd^3} - \frac{Bg^2(a+bx)^2(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]$

[Out]  $(B*(b*c - a*d)^2*g^2*x)/(3*d^2) - (B*(b*c - a*d)*g^2*(a + b*x)^2)/(6*b*d) + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b) - (B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3)$

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol]
:> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps



$$\begin{aligned}
\int (ag + bgx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right) dx &= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{3b} - \frac{B \int \frac{(bc-ad)g^3(a+bx)^2}{c+dx} dx}{3bg} \\
&= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{3b} - \frac{(B(bc-ad)g^2) \int \frac{(a+bx)^2}{c+dx} dx}{3b} \\
&= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{3b} - \frac{(B(bc-ad)g^2) \int \left( -\frac{b(bc-ad)}{d^2} + \frac{b(a+bx)}{d} \right) dx}{3b} \\
&= \frac{B(bc-ad)^2 g^2 x}{3d^2} - \frac{B(bc-ad)g^2(a+bx)^2}{6bd} + \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{3b}
\end{aligned}$$

**Mathematica [A]** time = 0.0537513, size = 99, normalized size = 0.84

$$\frac{g^2 \left( \frac{B(ad-bc)(d(a^2d+4abdxb^2x(dx-2c))+2(bc-ad)^2 \log(c+dx))}{2d^3} + (a+bx)^3 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right) \right)}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]),x]

[Out] (g^2\*((a + b\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]) + (B\*(-(b\*c) + a\*d)\*(d\*(a^2\*d + 4\*a\*b\*d\*x + b^2\*x\*(-2\*c + d\*x)) + 2\*(b\*c - a\*d)^2\*Log[c + d\*x]))/(2\*d^3)))/(3\*b)

**Maple [B]** time = 0.162, size = 3283, normalized size = 27.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c))),x)

[Out] e\*B\*g^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)\*a^3+1/d\*B\*g^2\*ln(d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))-b\*e)\*a^2\*c+1/3/d^3\*B\*g^2\*b^2\*ln(d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))-b\*e)\*c^3+1/3\*e^3\*A\*g^2\*b^2/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)^3\*a^3+e^2\*A\*g^2\*b/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)^2\*a^3+1/6\*e^2\*B\*g^2\*b/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)^2\*a^3+5\*e^3\*d\*B\*g^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)^3\*a^4/(d\*x+c)^3\*c^2\*b+5\*e^3/d\*B\*g^2\*b^3\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)^3\*a^2/(d\*x+c)^3\*c^4-5\*e^2/d^2\*B\*g^2\*b^3\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)^2\*c^4/(d\*x+c)^2\*a+10\*e^2/d\*B\*g^2\*b^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)^2\*c^3/(d\*x+c)^2\*a^2+6\*e/d\*B\*g^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)\*a^2/(d\*x+c)\*c^2\*b-2\*e^3/d^2\*B\*g^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)^3\*a\*c^5/(d\*x+c)^3\*b^4-4\*e/d^2\*B\*g^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*b^2/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)\*c^3/(d\*x+c)\*a-3\*e/d\*B\*g^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)\*a^2\*c\*b+e/d^3\*B\*g^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*b^3/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)\*c^4/(d\*x+c)+e\*d\*B\*g^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/b/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)\*a^4/(d\*x+c)+5\*e^2\*d\*B\*g^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)^2\*a^4/(d\*x+c)^2\*c+e^3/d^2\*B\*g^2\*b^4\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)^3\*c^2\*a-2\*e^3\*d^2\*B\*g^2\*ln(b\*e/d+(a\*d

$$-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^5/(d*x+c)^3*c+1/3*e^3/d^3*B*g^2*b^5*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*c^6/(d*x+c)^3-e^2*d^2*B*g^2/b*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^5/(d*x+c)^2+e^2/d^3*B*g^2*b^4*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^5/(d*x+c)^2+1/3*e^3*d^3*B*g^2/b*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^6/(d*x+c)^3-3*e^2/d*B*g^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2*b^2*c-e^3/d*B*g^2*b^3*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^2*c+3*e^2/d^2*B*g^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a*b^3*c^2+3*e/d^2*B*g^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^2*a-20/3*e^3*B*g^2*b^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^3/(d*x+c)^3*c^3-10*e^2*B*g^2*b*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^2/(d*x+c)^2*a^3+2/3*e*B*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^3+e*A*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^3-1/3*B*g^2/b*\ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*a^3-3*e^2/d*A*g^2*b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2*c+2*e/d^2*B*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a*b^2*c^2+e^3/d^2*A*g^2*b^4/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a*c^2+3*e^2/d^2*A*g^2*b^3/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a*c^2-3*e/d*A*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^2*b*c+3*e/d^2*A*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*b^2*c^2*a-2*e/d*B*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^2*b*c+1/2*e^2/d^2*B*g^2*b^3/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a*c^2-e^3/d*A*g^2*b^3/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^2*c-4*e*B*g^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^3/(d*x+c)*c-1/3*e^3/d^3*B*g^2*b^5*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*c^3-e^2/d^3*B*g^2*b^4*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^3-e/d^3*B*g^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b^3/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^3-1/2*e^2/d*B*g^2*b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2*c-e/d^3*A*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*b^3*c^3-2/3*e/d^3*B*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*b^3*c^3-1/3*e^3/d^3*A*g^2*b^5/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*c^3-e^2/d^3*A*g^2*b^4/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^3-1/6*e^2/d^3*B*g^2*b^4/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^3+e^2*B*g^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^3*b-1/d^2*B*g^2*\ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*a*c^2*b+1/3*e^3*B*g^2*b^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^3$$

**Maxima [B]** time = 1.17322, size = 378, normalized size = 3.2

$$\frac{1}{3} Ab^2g^2x^3 + Aabg^2x^2 + \left( x \log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) Ba^2g^2 + \left( x^2 \log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="maxima")

[Out] 
$$\frac{1}{3}A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*\log(b*e*x/(d*x+c) + a*e/(d*x+c)) + a*\log(b*x+a)/b - c*\log(d*x+c)/d)*B*a^2*g^2 + (x^2*\log(b*e*x/(d*x+c) + a*e/(d*x+c)) - a^2*\log(b*x+a)/b^2 + c^2*\log(d*x+c)/d^2 - (b*c-a*d)*x/(b*d))*B*a*b*g^2 + 1/6*(2*x^3*\log(b*e*x/(d*x+c) + a*e/(d*x+c)) + 2*a^3*\log(b*x+a)/b^3 - 2*c^3*\log(d*x+c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x$$

**Fricas [B]** time = 1.09247, size = 467, normalized size = 3.96

$$2 Ab^3d^3g^2x^3 + 2 Ba^3d^3g^2 \log(bx+a) - (Bb^3cd^2 - (6A+B)ab^2d^3)g^2x^2 + 2 (Bb^3c^2d - 3 Bab^2cd^2 + (3A+2B)a^2bd^3)g^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="fricas")

[Out]  $\frac{1}{6}*(2*A*b^3*d^3*g^2*x^3 + 2*B*a^3*d^3*g^2*\log(b*x + a) - (B*b^3*c*d^2 - (6*A + B)*a*b^2*d^3)*g^2*x^2 + 2*(B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + (3*A + 2*B)*a^2*b*d^3)*g^2*x - 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*\log(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*g^2*x)*\log((b*e*x + a*e)/(d*x + c)))/(b*d^3)$

**Sympy [B]** time = 5.21757, size = 503, normalized size = 4.26

$$\frac{Ab^2g^2x^3}{3} + \frac{Ba^3g^2 \log\left(x + \frac{Ba^4d^3g^2 + 3Ba^3cd^2g^2 - 3Ba^2bc^2dg^2 + Bab^2c^3g^2}{Ba^3d^3g^2 + 3Ba^2bcd^2g^2 - 3Bab^2c^2dg^2 + Bb^3c^3g^2}\right)}{3b} - \frac{Bcg^2(3a^2d^2 - 3abcd + b^2c^2) \log\left(x + \frac{4Ba^3cd^2g^2 - 3Ba^2bc^2dg^2 + Bb^3c^3g^2}{Ba^3d^3g^2 + 3Ba^2bcd^2g^2 - 3Bab^2c^2dg^2 + Bb^3c^3g^2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*2\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c))),x)

[Out]  $A*b**2*g**2*x**3/3 + B*a**3*g**2*\log(x + (B*a**4*d**3*g**2/b + 3*B*a**3*c*d**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2)/(B*a**3*d**3*g**2 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**2 + B*b**3*c**3*g**2))/(3*b) - B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*\log(x + (4*B*a**3*c*d**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2 - B*a*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2) + B*b*c**2*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)/d)/(B*a**3*d**3*g**2 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**2 + B*b**3*c**3*g**2))/(3*d**3) + (B*a**2*g**2*x + B*a*b*g**2*x**2 + B*b**2*g**2*x**3/3)*\log(e*(a + b*x)/(c + d*x)) + x**2*(6*A*a*b*d*g**2 + B*a*b*d*g**2 - B*b**2*c*g**2)/(6*d) + x*(3*A*a**2*d**2*g**2 + 2*B*a**2*d**2*g**2 - 3*B*a*b*c*d*g**2 + B*b**2*c**2*g**2)/(3*d**2)$

**Giac [B]** time = 4.75747, size = 309, normalized size = 2.62

$$\frac{Ba^3g^2 \log(bx + a)}{3b} + \frac{1}{3}(Ab^2g^2 + Bb^2g^2)x^3 - \frac{(Bb^2cg^2 - 6Aabdg^2 - 7Babdg^2)x^2}{6d} + \frac{1}{3}(Bb^2g^2x^3 + 3Babg^2x^2 + 3Ba^2g^2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="giac")

[Out]  $\frac{1}{3}B*a^3*g^2*\log(b*x + a)/b + \frac{1}{3}*(A*b^2*g^2 + B*b^2*g^2)*x^3 - \frac{1}{6}*(B*b^2*c*g^2 - 6*A*a*b*d*g^2 - 7*B*a*b*d*g^2)*x^2/d + \frac{1}{3}*(B*b^2*g^2*x^3 + 3*B*a*b*g^2*x^2 + 3*B*a^2*g^2*x)*\log((b*x + a)/(d*x + c)) + \frac{1}{3}*(B*b^2*c^2*g^2 - 3*B*a*b*c*d*g^2 + 3*A*a^2*d^2*g^2 + 5*B*a^2*d^2*g^2)*x/d^2 - \frac{1}{3}*(B*b^2*c^3*g^2 - 3*B*a*b*c^2*d*g^2 + 3*B*a^2*c*d^2*g^2)*\log(-d*x - c)/d^3$

### 3.91 $\int (ag + bgx) \left( A + B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) \right) dx$

**Optimal.** Leaf size=81

$$\frac{g(a+bx)^2 \left( B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) + A \right)}{2b} + \frac{Bg(bc-ad)^2 \log(c+dx)}{2bd^2} - \frac{Bgx(bc-ad)}{2d}$$

[Out]  $-(B*(b*c - a*d)*g*x)/(2*d) + (g*(a + b*x)^2*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(2*b) + (B*(b*c - a*d)^2*g*\text{Log}[c + d*x])/(2*b*d^2)$

**Rubi [A]** time = 0.0528924, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$ , Rules used = {2525, 12, 43}

$$\frac{g(a+bx)^2 \left( B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) + A \right)}{2b} + \frac{Bg(bc-ad)^2 \log(c+dx)}{2bd^2} - \frac{Bgx(bc-ad)}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]), x]$

[Out]  $-(B*(b*c - a*d)*g*x)/(2*d) + (g*(a + b*x)^2*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(2*b) + (B*(b*c - a*d)^2*g*\text{Log}[c + d*x])/(2*b*d^2)$

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol]
:> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
\int (ag + bgx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right) dx &= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{2b} - \frac{B \int \frac{(bc-ad)g^2(a+bx)}{c+dx} dx}{2bg} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{2b} - \frac{(B(bc-ad)g) \int \frac{a+bx}{c+dx} dx}{2b} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{2b} - \frac{(B(bc-ad)g) \int \left( \frac{b}{d} + \frac{-bc+ad}{d(c+dx)} \right) dx}{2b} \\
&= -\frac{B(bc-ad)gx}{2d} + \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{2b} + \frac{B(bc-ad)^2 g \log(c+dx)}{2bd^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0342394, size = 69, normalized size = 0.85

$$\frac{g \left( (a+bx)^2 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right) + \frac{B(ad-bc)((ad-bc) \log(c+dx)+bdx)}{d^2} \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]),x]

[Out] (g\*((a + b\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]) + (B\*(-(b\*c) + a\*d)\*(b\*d\*x + (-b\*c) + a\*d)\*Log[c + d\*x]))/d^2)/(2\*b)

**Maple [B]** time = 0.158, size = 1544, normalized size = 19.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c))),x)

[Out] 
$$\begin{aligned}
& -1/2*e^2/d^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^4/(d*x+c)^2*b^3-1/2*e^2*d^2*B*g/b*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^4/(d*x+c)^2-e^2/d*B*g*b^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a*c+2*e^2*d*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^3/(d*x+c)^2*c-3*e^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2/(d*x+c)^2*c^2*b-e/d^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^3/(d*x+c)*b^2+e*d*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^3/(d*x+c)-2*e/d*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a*b*c+1/d*B*g*ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*a*c+e*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^2-1/2/d^2*B*g*ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*c^2*b+1/2*e^2*A*g*b/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2+3*e/d*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^2/(d*x+c)*b*a-e/d*B*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a*b*c-2*e/d*A*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*b*c*a+e/d^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^2*b^2+1/2*e^2/d^2*A*g*b^3/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^2+e/d^2*A*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*b^2*c^2+1/2*e^2*B*g*b*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2+e*A*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^2+2*e^2/d*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^3/(d*x+c)^2*b^2*
\end{aligned}$$

$$a+1/2*e*B*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^2-1/2*B*g/b*\ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*a^2+1/2*e^2/d^2*B*g*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^2*b^3-3*e*B*g*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^2/(d*x+c)*c-e^2/d*A*g*b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c*a$$

**Maxima [A]** time = 1.10547, size = 194, normalized size = 2.4

$$\frac{1}{2} Abgx^2 + \left( x \log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) Bag + \frac{1}{2} \left( x^2 \log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2 \log(bx+a)}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/2*A*b*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a*g + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*b*g + A*a*g*x
```

**Fricas [A]** time = 1.05546, size = 278, normalized size = 3.43

$$\frac{Ab^2d^2gx^2 + Ba^2d^2g \log(bx+a) - (Bb^2cd - (2A+B)abd^2)gx + (Bb^2c^2 - 2Babcd)g \log(dx+c) + (Bb^2d^2gx^2 + 2Babd^2g)}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + B*a^2*d^2*g*log(b*x + a) - (B*b^2*c*d - (2*A + B)*a*b*d^2)*g*x + (B*b^2*c^2 - 2*B*a*b*c*d)*g*log(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*a*b*d^2*g*x)*log((b*e*x + a*e)/(d*x + c)))/(b*d^2)
```

**Sympy [B]** time = 4.44988, size = 257, normalized size = 3.17

$$\frac{Abgx^2}{2} + \frac{Ba^2g \log\left(x + \frac{Ba^3d^2g + 2Ba^2cdg - Babc^2g}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{2b} - \frac{Bcg(2ad - bc) \log\left(x + \frac{3Ba^2cdg - Babc^2g - Bacg(2ad - bc) + \frac{Bbc^2g(2ad - bc)}{d}}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{2d^2} + (Bagx +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*b*g*x**2/2 + B*a**2*g*log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*b) - B*c*g*(2*a*d - b*c)*log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*d**2) + (B*a*g*x + B*b*g*x**2/2)*log(e*(a + b*x)/(c + d*x)) + x*(2*A*a*d*g + B*a*d*g - B*b*c*g)/(2*d)
```

**Giac [A]** time = 1.80428, size = 150, normalized size = 1.85

$$\frac{Ba^2g \log(bx + a)}{2b} + \frac{1}{2}(Abg + Bbg)x^2 + \frac{1}{2}(Bbgx^2 + 2Bagx) \log\left(\frac{bx + a}{dx + c}\right) - \frac{(Bbcg - 2Aadg - 3Badg)x}{2d} + \frac{(Bbc^2g - 2Aad^2g - 3Bad^2g)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="giac")

[Out] 1/2\*B\*a^2\*g\*log(b\*x + a)/b + 1/2\*(A\*b\*g + B\*b\*g)\*x^2 + 1/2\*(B\*b\*g\*x^2 + 2\*B\*a\*g\*x)\*log((b\*x + a)/(d\*x + c)) - 1/2\*(B\*b\*c\*g - 2\*A\*a\*d\*g - 3\*B\*a\*d\*g)\*x/d + 1/2\*(B\*b\*c^2\*g - 2\*B\*a\*c\*d\*g)\*log(d\*x + c)/d^2

$$3.92 \quad \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{ag+bgx} dx$$

**Optimal.** Leaf size=80

$$\frac{BPolyLog\left(2, \frac{bc-ad}{d(a+bx)} + 1\right)}{bg} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{bg}$$

[Out]  $-(\text{Log}[-((b*c - a*d)/(d*(a + b*x)))]*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/$   
 $(b*g) + (B*\text{PolyLog}[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b*g)$

**Rubi [A]** time = 0.217414, antiderivative size = 120, normalized size of antiderivative = 1.5, number of steps used = 10, number of rules used = 8, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2524, 12, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{BPolyLog\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg} + \frac{\log(ag + bgx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{bg} + \frac{B \log(ag + bgx)\log\left(\frac{b(c+dx)}{bc-ad}\right)}{bg} - \frac{B \log^2(g(a + bx))}{2bg}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Log}[(e*(a + b*x))/(c + d*x]])/(a*g + b*g*x), x]$

[Out]  $-(B*\text{Log}[g*(a + b*x)]^2)/(2*b*g) + ((A + B*\text{Log}[(e*(a + b*x))/(c + d*x]])*\text{Log}$   
 $[a*g + b*g*x])/(b*g) + (B*\text{Log}[(b*(c + d*x))/(b*c - a*d)]*\text{Log}[a*g + b*g*x])/$   
 $(b*g) + (B*\text{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g)$

### Rule 2524

$\text{Int}[(a_. + \text{Log}[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[d + e*x]*(a + b*\text{Log}[c*Rfx^p])^n)/e, x] - \text{Dist}[(b*n*p)/e,$   
 $\text{Int}[(\text{Log}[d + e*x]*(a + b*\text{Log}[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, e, p\}, x\} \&\& \text{RationalFunctionQ}[Rfx, x] \&\& \text{IGtQ}[n, 0]$

### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$   $\text{FreeQ}[a, x] \&\& !\text{Match}$   
 $Q[u, (b_)*(v_)] /;$   $\text{FreeQ}[b, x]$

### Rule 2418

$\text{Int}[(a_. + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*(Rfx_), x\_Symbol] \rightarrow \text{With}\{u = \text{ExpandIntegrand}[(a + b*\text{Log}[c*(d + e*x)^n])^p, Rfx, x]\},$   
 $\text{Int}[u, x] /;$   $\text{SumQ}[u] /;$   $\text{FreeQ}\{a, b, c, d, e, n\}, x\} \&\& \text{RationalFunctionQ}[$   
 $Rfx, x] \&\& \text{IntegerQ}[p]$

### Rule 2390

$\text{Int}[(a_. + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.$   
 $)*(x_.))^(q_.), x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(f*x)/d]^q*(a + b*\text{Log}[c*x^n])^p, x], x,$   
 $d + e*x], x] /;$   $\text{FreeQ}\{a, b, c, d, e, f, g, n, p, q\}, x\} \&\& \text{EQ}[e*f - d*g, 0]$

### Rule 2301



Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e^n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
 \int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{ag + bgx} dx &= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(ag + bgx)}{bg} - \frac{B \int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2} + \frac{be}{c+dx}\right) \log(ag+bgx)}{e(a+bx)} dx}{bg} \\
 &= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(ag + bgx)}{bg} - \frac{B \int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2} + \frac{be}{c+dx}\right) \log(ag+bgx)}{a+bx} dx}{beg} \\
 &= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(ag + bgx)}{bg} - \frac{B \int \left(\frac{be \log(ag+bgx)}{a+bx} - \frac{de \log(ag+bgx)}{c+dx}\right) dx}{beg} \\
 &= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(ag + bgx)}{bg} - \frac{B \int \frac{\log(ag+bgx)}{a+bx} dx}{g} + \frac{(Bd) \int \frac{\log(ag+bgx)}{c+dx} dx}{bg} \\
 &= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(ag + bgx)}{bg} + \frac{B \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} - B \int \frac{\log\left(\frac{bg(c+dx)}{bcg-adg}\right)}{ag + bgx} dx \\
 &= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(ag + bgx)}{bg} + \frac{B \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} - \frac{B \text{Subst}\left(\int \frac{\log(x)}{x} dx\right)}{bg} \\
 &= -\frac{B \log^2(g(a + bx))}{2bg} + \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(ag + bgx)}{bg} + \frac{B \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg}
 \end{aligned}$$

**Mathematica [A]** time = 0.0420439, size = 95, normalized size = 1.19

$$\frac{2B \text{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right) + \log(g(a + bx)) \left(2 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + B \log\left(\frac{b(c+dx)}{bc-ad}\right) + A\right) - B \log(g(a + bx))\right)}{2bg}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])/(a\*g + b\*g\*x), x]

[Out]  $(\text{Log}[g*(a + b*x)]*(-(B*\text{Log}[g*(a + b*x)])) + 2*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)] + B*\text{Log}[(b*(c + d*x))/(b*c - a*d)])) + 2*B*\text{PolyLog}[2, (d*(a + b*x))/(b*c + a*d)]/(2*b*g)$

**Maple [B]** time = 0.089, size = 602, normalized size = 7.5

$$\frac{dAa}{g(ad-bc)b} \ln\left(\frac{be}{d} + \frac{e(ad-bc)}{(dx+c)d}\right) - \frac{Ac}{g(ad-bc)} \ln\left(\frac{be}{d} + \frac{e(ad-bc)}{(dx+c)d}\right) - \frac{dAa}{g(ad-bc)b} \ln\left(d\left(\frac{be}{d} + \frac{e(ad-bc)}{(dx+c)d}\right) - be\right) + \frac{A}{g(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g), x)`

[Out]  $d/g/(a*d-b*c)*A/b*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^a-1/g/(a*d-b*c)*A*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*c-d/g/(a*d-b*c)*A/b*\ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*a+1/g/(a*d-b*c)*A*\ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*c+1/2*d/g/(a*d-b*c)*B/b*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^{2*a-1/2}/g/(a*d-b*c)*B*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^{2*c-d}/g/(a*d-b*c)*B/b*\text{dilog}(-(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)/b/e)*a+1/g/(a*d-b*c)*B*\text{dilog}(-(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)/b/e)*c-d/g/(a*d-b*c)*B/b*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*\ln(-(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)/b/e)*a+1/g/(a*d-b*c)*B*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*\ln(-(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)/b/e)*c$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-B\left(\frac{\log(bx+a)\log(dx+c)}{bg} - \int \frac{bdx \log(e) + bc \log(e) + (2bdx + bc + ad) \log(bx+a)}{b^2d gx^2 + abcg + (b^2cg + abdg)x} dx\right) + \frac{A \log(bgx + ag)}{bg}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g), x, algorithm="maxima")`

[Out]  $-B*(\log(b*x + a)*\log(d*x + c))/(b*g) - \text{integrate}((b*d*x*\log(e) + b*c*\log(e) + (2*b*d*x + b*c + a*d)*\log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x) + A*\log(b*g*x + a*g)/(b*g)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B \log\left(\frac{bex+ae}{dx+c}\right) + A}{bgx + ag}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g), x, algorithm="fricas")`

[Out] `integral((B*log((b*e*x + a*e)/(d*x + c)) + A)/(b*g*x + a*g), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A}{bgx + ag} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g),x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)\*e/(d\*x + c)) + A)/(b\*g\*x + a\*g), x)

$$3.93 \quad \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2} dx$$

**Optimal.** Leaf size=63

$$-\frac{(c+dx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2(a+bx)(bc-ad)} - \frac{B}{bg^2(a+bx)}$$

[Out]  $-(B/(b*g^2*(a + b*x))) - ((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*c - a*d)*g^2*(a + b*x))$

**Rubi [A]** time = 0.078977, antiderivative size = 102, normalized size of antiderivative = 1.62, number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(a+bx)}{c+dx}\right)+A}{bg^2(a+bx)} - \frac{Bd \log(a+bx)}{bg^2(bc-ad)} + \frac{Bd \log(c+dx)}{bg^2(bc-ad)} - \frac{B}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Log}[(e*(a + b*x))/(c + d*x]])/(a*g + b*g*x)^2, x]$

[Out]  $-(B/(b*g^2*(a + b*x))) - (B*d*\text{Log}[a + b*x])/(b*(b*c - a*d)*g^2) - (A + B*\text{Log}[(e*(a + b*x))/(c + d*x]])/(b*g^2*(a + b*x)) + (B*d*\text{Log}[c + d*x])/(b*(b*c - a*d)*g^2)$

#### Rule 2525

$\text{Int}[(a_.) + \text{Log}[(c_.)*(\text{RFX}_.)^{(p_.)}*(b_.)^{(n_.)}*((d_.) + (e_.)*(x_))^{(m_.)}], x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{(m+1)}*(a + b*\text{Log}[c*\text{RFX}^p])^n/(e*(m+1)), x] - \text{Dist}[(b*n*p)/(e*(m+1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{(m+1)}*(a + b*\text{Log}[c*\text{RFX}^p])^{(n-1)}*D[\text{RFX}, x])/RFX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 44

$\text{Int}[(a_.) + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e^{(a+bx)}}{c+dx}\right)}{(ag + bgx)^2} dx &= -\frac{A + B \log\left(\frac{e^{(a+bx)}}{c+dx}\right)}{bg^2(a + bx)} + \frac{B \int \frac{bc-ad}{g(a+bx)^2(c+dx)} dx}{bg} \\
&= -\frac{A + B \log\left(\frac{e^{(a+bx)}}{c+dx}\right)}{bg^2(a + bx)} + \frac{(B(bc - ad)) \int \frac{1}{(a+bx)^2(c+dx)} dx}{bg^2} \\
&= -\frac{A + B \log\left(\frac{e^{(a+bx)}}{c+dx}\right)}{bg^2(a + bx)} + \frac{(B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)} + \frac{d^2}{(bc-ad)^2(c+dx)}\right) dx}{bg^2} \\
&= -\frac{B}{bg^2(a + bx)} - \frac{Bd \log(a + bx)}{b(bc - ad)g^2} - \frac{A + B \log\left(\frac{e^{(a+bx)}}{c+dx}\right)}{bg^2(a + bx)} + \frac{Bd \log(c + dx)}{b(bc - ad)g^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0591164, size = 105, normalized size = 1.67

$$\frac{aAd + (aBd - bBc) \log\left(\frac{e^{(a+bx)}}{c+dx}\right) - Bd(a + bx) \log(a + bx) + aBd \log(c + dx) + aBd - Abc + bBdx \log(c + dx) - bBc}{bg^2(a + bx)(bc - ad)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))/(a\*g + b\*g\*x)^2,x]

[Out]  $(-(A*b*c) - b*B*c + a*A*d + a*B*d - B*d*(a + b*x)*\text{Log}[a + b*x] + (-(b*B*c) + a*B*d)*\text{Log}[(e*(a + b*x))/(c + d*x)] + a*B*d*\text{Log}[c + d*x] + b*B*d*x*\text{Log}[c + d*x])/(b*(b*c - a*d)*g^2*(a + b*x))$

**Maple [B]** time = 0.047, size = 373, normalized size = 5.9

$$\frac{deAa}{(ad - bc)^2 g^2} \left(\frac{be}{d} + \frac{ae}{dx + c} - \frac{bec}{(dx + c)d}\right)^{-1} - \frac{eAbc}{(ad - bc)^2 g^2} \left(\frac{be}{d} + \frac{ae}{dx + c} - \frac{bec}{(dx + c)d}\right)^{-1} + \frac{deBa}{(ad - bc)^2 g^2} \ln\left(\frac{be}{d} + \frac{e(a+bx)}{c+dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)^2,x)

[Out]  $e*d/(a*d-b*c)^2/g^2*A/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*a-e/(a*d-b*c)^2/g^2*A/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*b*c+e*d/(a*d-b*c)^2/g^2*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a-e/(a*d-b*c)^2/g^2*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b*c+e*d/(a*d-b*c)^2/g^2*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*a-e/(a*d-b*c)^2/g^2*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*b*c$

**Maxima [B]** time = 1.15851, size = 178, normalized size = 2.83

$$-B \left( \frac{\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)}{b^2g^2x + abg^2} + \frac{1}{b^2g^2x + abg^2} + \frac{d \log(bx + a)}{(b^2c - abd)g^2} - \frac{d \log(dx + c)}{(b^2c - abd)g^2} \right) - \frac{A}{b^2g^2x + abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)^2,x, algorithm="maxima")

[Out]  $-B \cdot (\log(b \cdot e^x / (d \cdot x + c)) + a \cdot e / (d \cdot x + c)) / (b^2 \cdot g^2 \cdot x + a \cdot b \cdot g^2) + 1 / (b^2 \cdot g^2 \cdot x + a \cdot b \cdot g^2) + d \cdot \log(b \cdot x + a) / ((b^2 \cdot c - a \cdot b \cdot d) \cdot g^2) - d \cdot \log(d \cdot x + c) / ((b^2 \cdot c - a \cdot b \cdot d) \cdot g^2) - A / (b^2 \cdot g^2 \cdot x + a \cdot b \cdot g^2)$

**Fricas [A]** time = 1.07407, size = 177, normalized size = 2.81

$$\frac{(A + B)bc - (A + B)ad + (Bbdx + Bbc) \log\left(\frac{bex+ae}{dx+c}\right)}{(b^3c - ab^2d)g^2x + (ab^2c - a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="fricas")`

[Out]  $-((A + B) \cdot b \cdot c - (A + B) \cdot a \cdot d + (B \cdot b \cdot d \cdot x + B \cdot b \cdot c) \cdot \log((b \cdot e^x + a \cdot e) / (d \cdot x + c))) / ((b^3 \cdot c - a \cdot b^2 \cdot d) \cdot g^2 \cdot x + (a \cdot b^2 \cdot c - a^2 \cdot b \cdot d) \cdot g^2)$

**Sympy [B]** time = 2.56011, size = 231, normalized size = 3.67

$$\frac{B \log\left(\frac{e(a+bx)}{c+dx}\right)}{abg^2 + b^2g^2x} - \frac{Bd \log\left(x + \frac{\frac{Ba^2d^3}{ad-bc} + \frac{2Babcd^2}{ad-bc} + Bad^2 - \frac{Bb^2c^2d}{ad-bc} + Bbcd}{2Bbd^2}\right)}{bg^2(ad-bc)} + \frac{Bd \log\left(x + \frac{\frac{Ba^2d^3}{ad-bc} - \frac{2Babcd^2}{ad-bc} + Bad^2 + \frac{Bb^2c^2d}{ad-bc} + Bbcd}{2Bbd^2}\right)}{bg^2(ad-bc)} - \frac{A+B}{abg^2 + b^2g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)`

[Out]  $-B \cdot \log(e \cdot (a + b \cdot x) / (c + d \cdot x)) / (a \cdot b \cdot g^2 + b^2 \cdot g^2 \cdot x) - B \cdot d \cdot \log(x + (-B \cdot a \cdot d^3 / (a \cdot d - b \cdot c) + 2 \cdot B \cdot a \cdot b \cdot c \cdot d^2 / (a \cdot d - b \cdot c) + B \cdot a \cdot d^2 - B \cdot b^2 \cdot c^2 \cdot d / (a \cdot d - b \cdot c) + B \cdot b \cdot c \cdot d) / (2 \cdot B \cdot b \cdot d^2)) / (b \cdot g^2 \cdot (a \cdot d - b \cdot c)) + B \cdot d \cdot \log(x + (B \cdot a \cdot d^3 / (a \cdot d - b \cdot c) - 2 \cdot B \cdot a \cdot b \cdot c \cdot d^2 / (a \cdot d - b \cdot c) + B \cdot a \cdot d^2 + B \cdot b^2 \cdot c^2 \cdot d / (a \cdot d - b \cdot c) + B \cdot b \cdot c \cdot d) / (2 \cdot B \cdot b \cdot d^2)) / (b \cdot g^2 \cdot (a \cdot d - b \cdot c)) - (A + B) / (a \cdot b \cdot g^2 + b^2 \cdot g^2 \cdot x)$

**Giac [A]** time = 1.35968, size = 157, normalized size = 2.49

$$\frac{Bd \log(bx + a)}{b^2cg^2 - abdg^2} + \frac{Bd \log(dx + c)}{b^2cg^2 - abdg^2} - \frac{B \log\left(\frac{bx+a}{dx+c}\right)}{b^2g^2x + abg^2} - \frac{A + 2B}{b^2g^2x + abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algorithm="giac")`

[Out]  $-B \cdot d \cdot \log(b \cdot x + a) / (b^2 \cdot c \cdot g^2 - a \cdot b \cdot d \cdot g^2) + B \cdot d \cdot \log(d \cdot x + c) / (b^2 \cdot c \cdot g^2 - a \cdot b \cdot d \cdot g^2) - B \cdot \log((b \cdot x + a) / (d \cdot x + c)) / (b^2 \cdot g^2 \cdot x + a \cdot b \cdot g^2) - (A + 2 \cdot B) / (b^2 \cdot g^2 \cdot x + a \cdot b \cdot g^2)$

$$3.94 \quad \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3} dx$$

**Optimal.** Leaf size=144

$$-\frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{2bg^3(a+bx)^2} + \frac{Bd^2 \log(a+bx)}{2bg^3(bc-ad)^2} - \frac{Bd^2 \log(c+dx)}{2bg^3(bc-ad)^2} + \frac{Bd}{2bg^3(a+bx)(bc-ad)} - \frac{B}{4bg^3(a+bx)^2}$$

[Out]  $-B/(4*b*g^3*(a + b*x)^2) + (B*d)/(2*b*(b*c - a*d)*g^3*(a + b*x)) + (B*d^2*\text{Log}[a + b*x])/(2*b*(b*c - a*d)^2*g^3) - (A + B*\text{Log}[(e*(a + b*x))/(c + d*x]))/(2*b*g^3*(a + b*x)^2) - (B*d^2*\text{Log}[c + d*x])/(2*b*(b*c - a*d)^2*g^3)$

**Rubi [A]** time = 0.0999325, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{2bg^3(a+bx)^2} + \frac{Bd^2 \log(a+bx)}{2bg^3(bc-ad)^2} - \frac{Bd^2 \log(c+dx)}{2bg^3(bc-ad)^2} + \frac{Bd}{2bg^3(a+bx)(bc-ad)} - \frac{B}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Log}[(e*(a + b*x))/(c + d*x]))/(a*g + b*g*x)^3, x]$

[Out]  $-B/(4*b*g^3*(a + b*x)^2) + (B*d)/(2*b*(b*c - a*d)*g^3*(a + b*x)) + (B*d^2*\text{Log}[a + b*x])/(2*b*(b*c - a*d)^2*g^3) - (A + B*\text{Log}[(e*(a + b*x))/(c + d*x]))/(2*b*g^3*(a + b*x)^2) - (B*d^2*\text{Log}[c + d*x])/(2*b*(b*c - a*d)^2*g^3)$

#### Rule 2525

$\text{Int}[(a + \text{Log}[(c + \text{RFX})^p])*(b + (d + e*x)^m)^n, x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^n/(e^{m+1}), x] - \text{Dist}[(b*n*p)/(e^{m+1}), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^{n-1}*D[\text{RFX}, x])/RFX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[(a)*(u), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b)\*(v) /; FreeQ[b, x]]

#### Rule 44

$\text{Int}[(a + (b)*(x))^m*((c + (d)*(x))^n), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^3} dx &= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a + bx)^2} + \frac{B \int \frac{bc-ad}{g^2(a+bx)^3(c+dx)} dx}{2bg} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a + bx)^2} + \frac{(B(bc - ad)) \int \frac{1}{(a+bx)^3(c+dx)} dx}{2bg^3} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a + bx)^2} + \frac{(B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^3} - \frac{bd}{(bc-ad)^2(a+bx)^2} + \frac{bd^2}{(bc-ad)^3(a+bx)} - \frac{d^3}{(bc-ad)^3(c+dx)}\right) dx}{2bg^3} \\
&= -\frac{B}{4bg^3(a + bx)^2} + \frac{Bd}{2b(bc - ad)g^3(a + bx)} + \frac{Bd^2 \log(a + bx)}{2b(bc - ad)^2g^3} - \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{2bg^3(a + bx)^2} - \frac{Bd^2 \log(c + dx)}{2b(bc - ad)^3}
\end{aligned}$$

**Mathematica [A]** time = 0.134859, size = 110, normalized size = 0.76

$$\frac{2\left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right) + \frac{B(2d^2(a+bx)^2 \log(c+dx) + (bc-ad)(b(c-2dx) - 3ad) - 2d^2(a+bx)^2 \log(a+bx))}{(bc-ad)^2}}{4bg^3(a + bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))/(a\*g + b\*g\*x)^3,x]

[Out] -(2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])) + (B\*((b\*c - a\*d)\*(-3\*a\*d + b\*(c - 2\*d\*x)) - 2\*d^2\*(a + b\*x)^2\*Log[a + b\*x] + 2\*d^2\*(a + b\*x)^2\*Log[c + d\*x]))/(b\*c - a\*d)^2)/(4\*b\*g^3\*(a + b\*x)^2)

**Maple [B]** time = 0.049, size = 777, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)^3,x)

[Out] e\*d^2/(a\*d-b\*c)^3/g^3\*A/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)\*a-e\*d/(a\*d-b\*c)^3/g^3\*A/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)\*b\*c-1/2\*e^2\*d/(a\*d-b\*c)^3/g^3\*A\*b/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^2\*a+1/2\*e^2/(a\*d-b\*c)^3/g^3\*A\*b^2/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^2\*c+e\*d^2/(a\*d-b\*c)^3/g^3\*B/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*a-e\*d/(a\*d-b\*c)^3/g^3\*B/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*b\*c+e\*d^2/(a\*d-b\*c)^3/g^3\*B/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)\*a-e\*d/(a\*d-b\*c)^3/g^3\*B/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)\*b\*c-1/2\*e^2\*d/(a\*d-b\*c)^3/g^3\*B\*b/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*a+1/2\*e^2/(a\*d-b\*c)^3/g^3\*B\*b^2/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*c-1/4\*e^2\*d/(a\*d-b\*c)^3/g^3\*B\*b/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^2\*a+1/4\*e^2/(a\*d-b\*c)^3/g^3\*B\*b^2/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^2\*c

**Maxima [A]** time = 1.08486, size = 344, normalized size = 2.39

$$\frac{1}{4} B \left( \frac{2 b d x - b c + 3 a d}{(b^4 c - a b^3 d) g^3 x^2 + 2 (a b^3 c - a^2 b^2 d) g^3 x + (a^2 b^2 c - a^3 b d) g^3} - \frac{2 \log\left(\frac{b x}{d x + c} + \frac{a e}{d x + c}\right)}{b^3 g^3 x^2 + 2 a b^2 g^3 x + a^2 b g^3} + \frac{2 d^2 \log(b x + a)}{(b^3 c^2 - 2 a b^2 c d + a^2 b d^2)} \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)^3,x, algorithm="maxima")

[Out]  $\frac{1}{4}B \left( \frac{(2bdx - bc + 3ad)}{(b^4c - ab^3d)g^3x^2 + 2(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} - 2 \log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) \right) / (b^3g^3x^2 + 2a^2b^2g^3x + a^2bg^3) + 2d^2 \log(bx+a) / ((b^3c^2 - 2a^2b^2cd + a^2bd^2)g^3) - 2d^2 \log(dx+c) / ((b^3c^2 - 2a^2b^2cd + a^2bd^2)g^3) - \frac{1}{2} \frac{A}{(b^3g^3x^2 + 2a^2b^2g^3x + a^2bg^3)}$

**Fricas [A]** time = 1.00411, size = 455, normalized size = 3.16

$$\frac{(2A+B)b^2c^2 - 4(A+B)abcd + (2A+3B)a^2d^2 - 2(Bb^2cd - Babd^2)x - 2(Bb^2d^2x^2 + 2Babd^2x - Bb^2c^2 + 2Babcd)}{4((b^5c^2 - 2ab^4cd + a^2b^3d^2)g^3x^2 + 2(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2)g^3x + (a^2b^3c^2 - 2a^3b^2cd + a^4bd^2)g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)^3,x, algorithm="fricas")

[Out]  $-\frac{1}{4} \left( (2A+B)b^2c^2 - 4(A+B)abc d + (2A+3B)a^2d^2 - 2(Bb^2cd - Babd^2)x - 2(Bb^2d^2x^2 + 2Babd^2x - Bb^2c^2 + 2Babcd) \log\left(\frac{bex+ae}{dx+c}\right) \right) / ((b^5c^2 - 2a^2b^4cd + a^2b^3d^2)g^3x^2 + 2(a^2b^4c^2 - 2a^2b^3cd + a^3b^2d^2)g^3x + (a^2b^3c^2 - 2a^3b^2cd + a^4bd^2)g^3)$

**Sympy [B]** time = 4.51528, size = 422, normalized size = 2.93

$$\frac{B \log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2bg^3 + 4ab^2g^3x + 2b^3g^3x^2} - \frac{Bd^2 \log\left(x + \frac{-\frac{Ba^3d^5}{(ad-bc)^2} + \frac{3Ba^2bcd^4}{(ad-bc)^2} - \frac{3Bab^2c^2d^3}{(ad-bc)^2} + Bad^3 + \frac{Bb^3c^3d^2}{(ad-bc)^2} + Bbcd^2}{2Bbd^3}\right)}{2bg^3(ad-bc)^2} + \frac{Bd^2 \log\left(x + \frac{\frac{Ba^3d^5}{(ad-bc)^2} - \frac{3Ba^2bcd^4}{(ad-bc)^2}}{ad-bc}\right)}{2bg^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)\*\*3,x)

[Out]  $-B \log\left(\frac{e(a+bx)}{c+dx}\right) / (2a^2b^2g^3 + 4a^2b^2g^3x + 2b^3g^3x^2) - B d^2 \log\left(x + \frac{(-B a^3 d^5 / (a d - b c)^2 + 3 B a^2 b c d^4 / (a d - b c)^2 - 3 B a b^2 c^2 d^3 / (a d - b c)^2 + B a d^3 + B b^3 c^3 d^2 / (a d - b c)^2 + B b c d^2)}{2 B b d^3}\right) / (2 b^2 g^3 (a d - b c)^2) + B d^2 \log\left(x + \frac{(B a^3 d^5 / (a d - b c)^2 - 3 B a^2 b c d^4 / (a d - b c)^2 + 3 B a b^2 c^2 d^3 / (a d - b c)^2 + B a d^3 - B b^3 c^3 d^2 / (a d - b c)^2 + B b c d^2)}{2 B b d^3}\right) / (2 b^2 g^3 (a d - b c)^2) - (2 A a d - 2 A b c + 3 B a d - B b c + 2 B b d x) / (4 a^2 b^2 g^3 - 4 a^2 b^2 g^3 x + 2 b^3 g^3 x^2) + x (8 a^2 b^2 g^3 - 8 a b^2 g^3 c)$

**Giac [A]** time = 1.34628, size = 329, normalized size = 2.28

$$\frac{Bd^2 \log(bx+a)}{2(b^3c^2g^3 - 2ab^2cdg^3 + a^2bd^2g^3)} - \frac{Bd^2 \log(dx+c)}{2(b^3c^2g^3 - 2ab^2cdg^3 + a^2bd^2g^3)} - \frac{B \log\left(\frac{bx+a}{dx+c}\right)}{2(b^3g^3x^2 + 2ab^2g^3x + a^2bg^3)} + \frac{1}{4(b^4cg^3x^2 - 4ab^3cg^3x + a^2b^2g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] 1/2*B*d^2*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/
2*B*d^2*log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*
B*log((b*x + a)/(d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 1/4*
(2*B*b*d*x - 2*A*b*c - 3*B*b*c + 2*A*a*d + 5*B*a*d)/(b^4*c*g^3*x^2 - a*b^3*
d*g^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3*x + a^2*b^2*c*g^3 - a^3*b*d*g
^3)
```

$$3.95 \quad \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4} dx$$

**Optimal.** Leaf size=175

$$-\frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{3bg^4(a+bx)^3} - \frac{Bd^2}{3bg^4(a+bx)(bc-ad)^2} - \frac{Bd^3 \log(a+bx)}{3bg^4(bc-ad)^3} + \frac{Bd^3 \log(c+dx)}{3bg^4(bc-ad)^3} + \frac{Bd}{6bg^4(a+bx)^2(bc-ad)} - \frac{1}{9bg^4}$$

[Out]  $-B/(9*b*g^4*(a + b*x)^3) + (B*d)/(6*b*(b*c - a*d)*g^4*(a + b*x)^2) - (B*d^2)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) - (B*d^3*Log[a + b*x])/(3*b*(b*c - a*d)^3*g^4) - (A + B*Log[(e*(a + b*x))/(c + d*x])/(3*b*g^4*(a + b*x)^3) + (B*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4)$

**Rubi [A]** time = 0.129579, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{3bg^4(a+bx)^3} - \frac{Bd^2}{3bg^4(a+bx)(bc-ad)^2} - \frac{Bd^3 \log(a+bx)}{3bg^4(bc-ad)^3} + \frac{Bd^3 \log(c+dx)}{3bg^4(bc-ad)^3} + \frac{Bd}{6bg^4(a+bx)^2(bc-ad)} - \frac{1}{9bg^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Log}[(e*(a + b*x))/(c + d*x])]/(a*g + b*g*x)^4, x]$

[Out]  $-B/(9*b*g^4*(a + b*x)^3) + (B*d)/(6*b*(b*c - a*d)*g^4*(a + b*x)^2) - (B*d^2)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) - (B*d^3*Log[a + b*x])/(3*b*(b*c - a*d)^3*g^4) - (A + B*Log[(e*(a + b*x))/(c + d*x])/(3*b*g^4*(a + b*x)^3) + (B*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4)$

#### Rule 2525

$\text{Int}[(a + \text{Log}[c*\text{RFX}^p])*(b*x)^n*((d + e*x)^m), x\_Symbol] := \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^n/(e*(m+1)), x] - \text{Dist}[(b*n*p)/(e*(m+1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^{n-1}*D[\text{RFX}, x])/RFX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[(a)*(u), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b)\*(v)] /; FreeQ[b, x]

#### Rule 44

$\text{Int}[(a + (b*x)^m)*((c + d*x)^n), x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^4} dx &= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a + bx)^3} + \frac{B \int \frac{bc-ad}{g^3(a+bx)^4(c+dx)} dx}{3bg} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a + bx)^3} + \frac{(B(bc - ad)) \int \frac{1}{(a+bx)^4(c+dx)} dx}{3bg^4} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a + bx)^3} + \frac{(B(bc - ad)) \int \left( \frac{b}{(bc-ad)(a+bx)^4} - \frac{bd}{(bc-ad)^2(a+bx)^3} + \frac{bd^2}{(bc-ad)^3(a+bx)^2} - \frac{bd^3}{(bc-ad)^4(a+bx)} \right) dx}{3bg^4} \\
&= -\frac{B}{9bg^4(a + bx)^3} + \frac{Bd}{6b(bc - ad)g^4(a + bx)^2} - \frac{Bd^2}{3b(bc - ad)^2g^4(a + bx)} - \frac{Bd^3 \log(a + bx)}{3b(bc - ad)^3g^4} - \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{3bg^4(a + bx)^3}
\end{aligned}$$

**Mathematica [A]** time = 0.162892, size = 141, normalized size = 0.81

$$\frac{B((bc-ad)(11a^2d^2+abd(15dx-7c)+b^2(2c^2-3cdx+6d^2x^2))-6d^3(a+bx)^3 \log(c+dx)+6d^3(a+bx)^3 \log(a+bx))}{(bc-ad)^3} + 6 \left( B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)$$


---


$$\frac{18bg^4(a + bx)^3}{18bg^4(a + bx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))/(a\*g + b\*g\*x)^4,x]

[Out] -(6\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])) + (B\*((b\*c - a\*d)\*(11\*a^2\*d^2 + a\*b\*d\*(-7\*c + 15\*d\*x) + b^2\*(2\*c^2 - 3\*c\*d\*x + 6\*d^2\*x^2)) + 6\*d^3\*(a + b\*x)^3\*Log[a + b\*x] - 6\*d^3\*(a + b\*x)^3\*Log[c + d\*x]))/(b\*c - a\*d)^3/(18\*b\*g^4\*(a + b\*x)^3)

**Maple [B]** time = 0.052, size = 1191, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)^4,x)

[Out] e\*d^3/(a\*d-b\*c)^4/g^4\*A/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)\*a-e\*d^2/(a\*d-b\*c)^4/g^4\*A/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)\*b\*c-e^2\*d^2/(a\*d-b\*c)^4/g^4\*A\*b/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^2\*a+e^2\*d/(a\*d-b\*c)^4/g^4\*A\*b^2/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^2\*c+1/3\*e^3\*d/(a\*d-b\*c)^4/g^4\*A\*b^2/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^3\*a-1/3\*e^3/(a\*d-b\*c)^4/g^4\*A\*b^3/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^3\*c+e\*d^3/(a\*d-b\*c)^4/g^4\*B/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*a-e\*d^2/(a\*d-b\*c)^4/g^4\*B/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*b\*c+e\*d^3/(a\*d-b\*c)^4/g^4\*B/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)\*a-e\*d^2/(a\*d-b\*c)^4/g^4\*B/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)\*b\*c-e^2\*d^2/(a\*d-b\*c)^4/g^4\*B\*b/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*a+e^2\*d/(a\*d-b\*c)^4/g^4\*B\*b^2/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*c-1/2\*e^2\*d^2/(a\*d-b\*c)^4/g^4\*B\*b/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^2\*a+1/2\*e^2\*d/(a\*d-b\*c)^4/g^4\*B\*b^2/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^2\*c+1/3\*e^3\*d/(a\*d-b\*c)^4/g^4\*B\*b^2/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^3\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*a-1/3\*e^3/(a\*d-b\*c)^4/g^4\*B\*b^3/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^3\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*c+1/9\*e^3\*d/(a\*d-b\*c)^4/g^4\*B\*b^2/(b\*e/d+e/(d\*x+c)\*a-e/d/(d\*x+c)\*b\*c)^3\*a-1/9\*e^3/

$$(a*d-b*c)^4/g^4*B*b^3/(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c)^3*c$$

**Maxima [B]** time = 1.22489, size = 578, normalized size = 3.3

$$-\frac{1}{18} B \left( \frac{6b^2d^2x^2 + 2b^2c^2 - 7abcd + 11a^2d^2 - 3(b^2cd - 5abd^2)x}{(b^6c^2 - 2ab^5cd + a^2b^4d^2)g^4x^3 + 3(ab^5c^2 - 2a^2b^4cd + a^3b^3d^2)g^4x^2 + 3(a^2b^4c^2 - 2a^3b^3cd + a^4b^2d^2)g^4x + (a^3b^3c^2 - 2a^4b^2cd + a^5b^1d^2)g^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)^4,x, algorithm="maxima")

[Out] 
$$-1/18*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)$$

**Fricas [B]** time = 1.05122, size = 826, normalized size = 4.72

$$\frac{2(3A + B)b^3c^3 - 9(2A + B)ab^2c^2d + 18(A + B)a^2bcd^2 - (6A + 11B)a^3d^3 + 6(Bb^3cd^2 - Bab^2d^3)x^2 - 3(Bb^3c^2d - 6Ab^2cd^2)x + 3a^2b^3d^3}{18((b^7c^3 - 3ab^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3)g^4x^3 + 3(ab^6c^3 - 3a^2b^5c^2d + 3a^3b^4cd^2 - a^4b^3d^3)g^4x^2 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)^4,x, algorithm="fricas")

[Out] 
$$-1/18*(2*(3*A + B)*b^3*c^3 - 9*(2*A + B)*a*b^2*c^2*d + 18*(A + B)*a^2*b*c*d^2 - (6*A + 11*B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)$$

**Sympy [B]** time = 6.86449, size = 656, normalized size = 3.75

$$\frac{B \log\left(\frac{e(a+bx)}{c+dx}\right)}{3a^3bg^4 + 9a^2b^2g^4x + 9ab^3g^4x^2 + 3b^4g^4x^3} - \frac{Bd^3 \log\left(x + \frac{-\frac{Ba^4d^7}{(ad-bc)^3} + \frac{4Ba^3bcd^6}{(ad-bc)^3} - \frac{6Ba^2b^2c^2d^5}{(ad-bc)^3} + \frac{4Bab^3c^3d^4}{(ad-bc)^3} + Baad^4 - \frac{Bb^4c^4d^3}{(ad-bc)^3} + Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)\*\*4,x)

```
[Out] -B*log(e*(a + b*x)/(c + d*x))/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) - B*d**3*log(x + (-B*a**4*d**7/(a*d - b*c)**3 + 4*B*a**3*b*c*d**6/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 + 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + B*a*d**4 - B*b**4*c**4*d**3/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + B*d**3*log(x + (B*a**4*d**7/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6/(a*d - b*c)**3 + 6*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + B*a*d**4 + B*b**4*c**4*d**3/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) - (6*A*a**2*d**2 - 12*A*a*b*c*d + 6*A*b**2*c**2 + 11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**2*c**2 + 6*B*b**2*d**2*x**2 + x*(15*B*a*b*d**2 - 3*B*b**2*c*d))/(18*a**5*b*d**2*g**4 - 36*a**4*b**2*c*d*g**4 + 18*a**3*b**3*c**2*g**4 + x**3*(18*a**2*b**4*d**2*g**4 - 36*a*b**5*c*d*g**4 + 18*b**6*c**2*g**4) + x**2*(54*a**3*b**3*d**2*g**4 - 108*a**2*b**4*c*d*g**4 + 54*a*b**5*c**2*g**4) + x*(54*a**4*b**2*d**2*g**4 - 108*a**3*b**3*c*d*g**4 + 54*a**2*b**4*c**2*g**4))
```

**Giac [B]** time = 1.41307, size = 609, normalized size = 3.48

$$\frac{Ba^3 \log(bx + a)}{3(b^4c^3g^4 - 3ab^3c^2dg^4 + 3a^2b^2cd^2g^4 - a^3bd^3g^4)} + \frac{Bd^3 \log(dx + c)}{3(b^4c^3g^4 - 3ab^3c^2dg^4 + 3a^2b^2cd^2g^4 - a^3bd^3g^4)} - \frac{E}{3(b^4g^4x^3 + 3ab^3g^4x^2 + 3a^2b^2c^2g^4x + a^3b^3g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -1/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) + 1/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((b*x + a)/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*c^2*g^4*x + a^3*b^3*g^4) - 1/18*(6*B*b^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*B*a*b*d^2*x + 6*A*b^2*c^2 + 8*B*b^2*c^2 - 12*A*a*b*c*d - 19*B*a*b*c*d + 6*A*a^2*d^2 + 17*B*a^2*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d^2*g^4*x^3 + 3*a*b^5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g^4*x^2 + 3*a^2*b^4*c^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x + a^3*b^3*c^2*g^4 - 2*a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

$$3.96 \quad \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^5} dx$$

**Optimal.** Leaf size=206

$$-\frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{4bg^5(a+bx)^4} + \frac{Bd^3}{4bg^5(a+bx)(bc-ad)^3} - \frac{Bd^2}{8bg^5(a+bx)^2(bc-ad)^2} + \frac{Bd^4 \log(a+bx)}{4bg^5(bc-ad)^4} - \frac{Bd^4 \log(c+dx)}{4bg^5(bc-ad)^4} + \frac{12bd^4}{12bg^5(bc-ad)^4}$$

[Out]  $-B/(16*b*g^5*(a + b*x)^4) + (B*d)/(12*b*(b*c - a*d)*g^5*(a + b*x)^3) - (B*d^2)/(8*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (B*d^3)/(4*b*(b*c - a*d)^3*g^5*(a + b*x)) + (B*d^4*Log[a + b*x])/(4*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(4*b*g^5*(a + b*x)^4) - (B*d^4*Log[c + d*x])/(4*b*(b*c - a*d)^4*g^5)$

**Rubi [A]** time = 0.156583, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{4bg^5(a+bx)^4} + \frac{Bd^3}{4bg^5(a+bx)(bc-ad)^3} - \frac{Bd^2}{8bg^5(a+bx)^2(bc-ad)^2} + \frac{Bd^4 \log(a+bx)}{4bg^5(bc-ad)^4} - \frac{Bd^4 \log(c+dx)}{4bg^5(bc-ad)^4} + \frac{12bd^4}{12bg^5(bc-ad)^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)])/(a*g + b*g*x)^5, x]$

[Out]  $-B/(16*b*g^5*(a + b*x)^4) + (B*d)/(12*b*(b*c - a*d)*g^5*(a + b*x)^3) - (B*d^2)/(8*b*(b*c - a*d)^2*g^5*(a + b*x)^2) + (B*d^3)/(4*b*(b*c - a*d)^3*g^5*(a + b*x)) + (B*d^4*Log[a + b*x])/(4*b*(b*c - a*d)^4*g^5) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(4*b*g^5*(a + b*x)^4) - (B*d^4*Log[c + d*x])/(4*b*(b*c - a*d)^4*g^5)$

### Rule 2525

$\text{Int}[(a + \text{Log}[(c + \text{RFX})^p] * (b + (d + e*x)^m))] * (d + e*x)^n, x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1} * (a + b*\text{Log}[c*\text{RFX}^p])^n / (e^{m+1}), x] - \text{Dist}[(b*n*p) / (e^{m+1}), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1} * (a + b*\text{Log}[c*\text{RFX}^p])^{n-1} * D[\text{RFX}, x]) / \text{RFX}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x] \&\& \text{RationalFunctionQ}[\text{RFX}, x] \&\& \text{IGtQ}[n, 0] \&\& (\text{EqQ}[n, 1] \parallel \text{IntegerQ}[m]) \&\& \text{NeQ}[m, -1]$

### Rule 12

$\text{Int}[(a + (b + (c + d*x)^n))] * (u + v*x), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& \text{!MatchQ}[u, (b + (c + d*x)^n)] /; \text{FreeQ}[b, x]$

### Rule 44

$\text{Int}[(a + (b + (c + d*x)^n))] * (c + d*x)^m, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{ILtQ}[m, 0] \&\& \text{IntegerQ}[n] \&\& \text{!(IGtQ}[n, 0] \&\& \text{LtQ}[m + n + 2, 0])$

Rubi steps

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^5} dx = -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a + bx)^4} + \frac{B \int \frac{bc-ad}{g^4(a+bx)^5(c+dx)} dx}{4bg}$$

$$= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a + bx)^4} + \frac{(B(bc - ad)) \int \frac{1}{(a+bx)^5(c+dx)} dx}{4bg^5}$$

$$= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{4bg^5(a + bx)^4} + \frac{(B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^5} - \frac{bd}{(bc-ad)^2(a+bx)^4} + \frac{bd^2}{(bc-ad)^3(a+bx)^3} - \frac{bd^3}{(bc-ad)^4(a+bx)^2}\right) dx}{4bg^5}$$

$$= -\frac{B}{16bg^5(a + bx)^4} + \frac{Bd}{12b(bc - ad)g^5(a + bx)^3} - \frac{Bd^2}{8b(bc - ad)^2g^5(a + bx)^2} + \frac{Bd^3}{4b(bc - ad)^3g^5(a + bx)}$$

**Mathematica [A]** time = 0.228634, size = 158, normalized size = 0.77

$$\frac{B\left(\frac{12d^3(bc-ad)}{a+bx} - \frac{6d^2(bc-ad)^2}{(a+bx)^2} + \frac{4d(bc-ad)^3}{(a+bx)^3} - \frac{3(bc-ad)^4}{(a+bx)^4} + 12d^4 \log(a+bx) - 12d^4 \log(c+dx)\right)}{12(bc-ad)^4} - \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{(a+bx)^4}$$

$4bg^5$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x]))/(a*g + b*g*x)^5, x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x]))/(a + b*x)^4) + (B*((-3*(b*c - a*d)^4)/(a + b*x)^4 + (4*d*(b*c - a*d)^3)/(a + b*x)^3 - (6*d^2*(b*c - a*d)^2)/(a + b*x)^2 + (12*d^3*(b*c - a*d))/(a + b*x) + 12*d^4*Log[a + b*x] - 12*d^4*Log[c + d*x]))/(12*(b*c - a*d)^4))/(4*b*g^5)
```

**Maple [B]** time = 0.051, size = 1607, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5, x)
```

```
[Out] e*d^4/(a*d-b*c)^5/g^5*A/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*a-e*d^3/(a*d-b*c)^5/g^5*A/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*b*c-3/2*e^2*d^3/(a*d-b*c)^5/g^5*A*b/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*a+3/2*e^2*d^2/(a*d-b*c)^5/g^5*A*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*c+e^3*d^2/(a*d-b*c)^5/g^5*A*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*a-e^3*d/(a*d-b*c)^5/g^5*A*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*c-1/4*e^4*d/(a*d-b*c)^5/g^5*A*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*a+1/4*e^4/(a*d-b*c)^5/g^5*A*b^4/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*c+e*d^4/(a*d-b*c)^5/g^5*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a-e*d^3/(a*d-b*c)^5/g^5*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b*c+e*d^4/(a*d-b*c)^5/g^5*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*a-e*d^3/(a*d-b*c)^5/g^5*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*b*c-3/2*e^2*d^3/(a*d-b*c)^5/g^5*B*b/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a+3/2*e^2*d^2/(a*d-b*c)^5/g^5*B*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*c-3/4*e^2*d^3/(a*d-b*c)^5/g^5*B*b/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*a+3/4*e^2*d^2/(a*d-b*c)^5/g^5*B*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*c+e^3*d^2/(a*d-b*c)^5/g^5*B*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2
```



$$\begin{aligned} & d/(d*x+c)*b*c)^3*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a-e^3*d/(a*d-b*c)^5/g^5*B* \\ & b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*c \\ & +1/3*e^3*d^2/(a*d-b*c)^5/g^5*B*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*a- \\ & 1/3*e^3*d/(a*d-b*c)^5/g^5*B*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*c-1/4 \\ & *e^4*d/(a*d-b*c)^5/g^5*B*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*\ln(b*e/d \\ & +(a*d-b*c)*e/d/(d*x+c))*a+1/4*e^4/(a*d-b*c)^5/g^5*B*b^4/(b*e/d+e/(d*x+c)*a- \\ & e/d/(d*x+c)*b*c)^4*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*c-1/16*e^4*d/(a*d-b*c)^5 \\ & /g^5*B*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*a+1/16*e^4/(a*d-b*c)^5/g^5 \\ & *B*b^4/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*c \end{aligned}$$

**Maxima [B]** time = 1.39642, size = 873, normalized size = 4.24

$$\frac{1}{48} B \left( \frac{12 b^3 d^3 x^3 - 3 b^3 c^3 + 13 a b^2 c^2 d - 23 a^2 b c d^2 + 25 a^3 d^3 - 6 (b^3 c^3 d^2 - 7 a^2 b^2 c^2 d^3) x^2 + 4 (b^3 c^2 d^2 - 5 a^2 b^2 c^2 d^2 + 13 a^2 b^2 c^2 d^3) x}{(b^8 c^3 - 3 a b^7 c^2 d + 3 a^2 b^6 c d^2 - a^3 b^5 d^3) g^5 x^4 + 4 (a b^7 c^3 - 3 a^2 b^6 c^2 d + 3 a^3 b^5 c d^2 - a^4 b^4 d^3) g^5 x^3 + 6 (a^2 b^6 c^3 - 3 a^3 b^5 c^2 d + 3 a^4 b^4 c^2 d^2 - a^5 b^3 c^2 d^3) g^5 x^2 + 4 (a^3 b^5 c^3 - 3 a^4 b^4 c^2 d + 3 a^5 b^3 c^2 d^2 - a^6 b^2 c^2 d^3) g^5 x + (a^4 b^4 c^3 - 3 a^5 b^3 c^2 d + 3 a^6 b^2 c^2 d^2 - a^7 b^2 c^2 d^3) g^5} - 12 \log(b^5 g^5 x^4 + 4 a^2 b^3 g^5 x^2 + 4 a^3 b^2 g^5 x + a^4 b g^5) + 12 d^4 \log(b^5 c^4 - 4 a^2 b^4 c^3 d + 6 a^2 b^3 c^2 d^2 - 4 a^3 b^2 c^2 d^3 + a^4 b^2 c^2 d^4) g^5 - 12 d^4 \log(d^4 x + c) / ((b^5 c^4 - 4 a^2 b^4 c^3 d + 6 a^2 b^3 c^2 d^2 - 4 a^3 b^2 c^2 d^3 + a^4 b^2 c^2 d^4) g^5) - 1/4 A / (b^5 g^5 x^4 + 4 a^2 b^3 g^5 x^2 + 4 a^3 b^2 g^5 x + a^4 b g^5) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)^5,x, algorithm="maxima")

[Out] 1/48\*B\*((12\*b^3\*d^3\*x^3 - 3\*b^3\*c^3 + 13\*a\*b^2\*c^2\*d - 23\*a^2\*b\*c\*d^2 + 25\*a^3\*d^3 - 6\*(b^3\*c^3\*d^2 - 7\*a^2\*b^2\*c^2\*d^3)\*x^2 + 4\*(b^3\*c^2\*d^2 - 5\*a^2\*b^2\*c^2\*d^2 + 13\*a^2\*b^2\*c^2\*d^3)\*x)/(b^8\*c^3 - 3\*a\*b^7\*c^2\*d + 3\*a^2\*b^6\*c\*d^2 - a^3\*b^5\*d^3)\*g^5\*x^4 + 4\*(a\*b^7\*c^3 - 3\*a^2\*b^6\*c^2\*d + 3\*a^3\*b^5\*c\*d^2 - a^4\*b^4\*d^3)\*g^5\*x^3 + 6\*(a^2\*b^6\*c^3 - 3\*a^3\*b^5\*c^2\*d + 3\*a^4\*b^4\*c^2\*d^2 - a^5\*b^3\*d^3)\*g^5\*x^2 + 4\*(a^3\*b^5\*c^3 - 3\*a^4\*b^4\*c^2\*d + 3\*a^5\*b^3\*c^2\*d^2 - a^6\*b^2\*d^3)\*g^5\*x + (a^4\*b^4\*c^3 - 3\*a^5\*b^3\*c^2\*d + 3\*a^6\*b^2\*c^2\*d^2 - a^7\*b^2\*d^3)\*g^5) - 12\*log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c))/(b^5\*g^5\*x^4 + 4\*a\*b^4\*g^5\*x^3 + 6\*a^2\*b^3\*g^5\*x^2 + 4\*a^3\*b^2\*g^5\*x + a^4\*b\*g^5) + 12\*d^4\*log(b\*x + a)/((b^5\*c^4 - 4\*a\*b^4\*c^3\*d + 6\*a^2\*b^3\*c^2\*d^2 - 4\*a^3\*b^2\*c^2\*d^3 + a^4\*b^2\*d^4)\*g^5) - 12\*d^4\*log(d\*x + c)/((b^5\*c^4 - 4\*a\*b^4\*c^3\*d + 6\*a^2\*b^3\*c^2\*d^2 - 4\*a^3\*b^2\*c^2\*d^3 + a^4\*b^2\*d^4)\*g^5) - 1/4\*A/(b^5\*g^5\*x^4 + 4\*a\*b^4\*g^5\*x^3 + 6\*a^2\*b^3\*g^5\*x^2 + 4\*a^3\*b^2\*g^5\*x + a^4\*b\*g^5)

**Fricas [B]** time = 1.07202, size = 1284, normalized size = 6.23

$$\frac{3(4A+B)b^4c^4 - 16(3A+B)ab^3c^3d + 36(2A+B)a^2b^2c^2d^2 - 48(A+B)a^3bcd^3 + (12A+25B)a^4d^4 - 12(Bb^4cd^3 - 3B^2b^3cd^3 + 3B^2b^2cd^3 - 3B^2b^2cd^3)}{48((b^9c^4 - 4ab^8c^3d + 6a^2b^7c^2d^2 - 4a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4(ab^8c^4 - 4a^2b^7c^3d + 6a^3b^6c^2d^2 - 4a^4b^5cd^3 + a^5b^4d^4)g^5x^3 + 6(a^2b^7c^4 - 4a^3b^6c^3d + 6a^4b^5c^2d^2 - 4a^5b^4cd^3 + a^6b^3d^4)g^5x^2 + 4(a^3b^6c^4 - 4a^4b^5c^3d + 6a^5b^4c^2d^2 - 4a^6b^3cd^3 + a^7b^2d^4)g^5x + (a^4b^5c^4 - 4a^5b^4c^3d + 6a^6b^3c^2d^2 - 4a^7b^2cd^3 + a^8b^2d^4)g^5} - 12 \log(b^5 g^5 x^4 + 4 a^2 b^3 g^5 x^2 + 4 a^3 b^2 g^5 x + a^4 b g^5) + 12 d^4 \log(b^5 c^4 - 4 a^2 b^4 c^3 d + 6 a^2 b^3 c^2 d^2 - 4 a^3 b^2 c^2 d^3 + a^4 b^2 c^2 d^4) g^5 - 12 d^4 \log(d^4 x + c) / ((b^5 c^4 - 4 a^2 b^4 c^3 d + 6 a^2 b^3 c^2 d^2 - 4 a^3 b^2 c^2 d^3 + a^4 b^2 c^2 d^4) g^5) - 1/4 A / (b^5 g^5 x^4 + 4 a^2 b^3 g^5 x^2 + 4 a^3 b^2 g^5 x + a^4 b g^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)^5,x, algorithm="fricas")

[Out] -1/48\*(3\*(4\*A + B)\*b^4\*c^4 - 16\*(3\*A + B)\*a\*b^3\*c^3\*d + 36\*(2\*A + B)\*a^2\*b^2\*c^2\*d^2 - 48\*(A + B)\*a^3\*b\*c\*d^3 + (12\*A + 25\*B)\*a^4\*d^4 - 12\*(B\*b^4\*c\*d^3 - B\*a\*b^3\*d^4)\*x^3 + 6\*(B\*b^4\*c^2\*d^2 - 8\*B\*a\*b^3\*c\*d^3 + 7\*B\*a^2\*b^2\*d^4)\*x^2 - 4\*(B\*b^4\*c^3\*d - 6\*B\*a\*b^3\*c^2\*d^2 + 18\*B\*a^2\*b^2\*c\*d^3 - 13\*B\*a^3\*b\*d^4)\*x - 12\*(B\*b^4\*d^4\*x^4 + 4\*B\*a\*b^3\*d^4\*x^3 + 6\*B\*a^2\*b^2\*d^4\*x^2 + 4\*B\*a^3\*b\*d^4\*x - B\*b^4\*c^4 + 4\*B\*a\*b^3\*c^3\*d - 6\*B\*a^2\*b^2\*c^2\*d^2 + 4\*B\*a^3\*b\*c\*d^3)\*log((b\*e\*x + a\*e)/(d\*x + c)))/((b^9\*c^4 - 4\*a\*b^8\*c^3\*d + 6\*a^2\*b^7\*c^2\*d^2 - 4\*a^3\*b^6\*c^2\*d^3 + a^4\*b^5\*d^4)\*g^5\*x^4 + 4\*(a\*b^8\*c^4 - 4\*a^2\*b^7\*c^3\*d + 6\*a^3\*b^6\*c^2\*d^2 - 4\*a^4\*b^5\*c^2\*d^3 + a^5\*b^4\*d^4)\*g^5\*x^3 + 6\*(a^2\*b^7\*c^4 - 4\*a^3\*b^6\*c^3\*d + 6\*a^4\*b^5\*c^2\*d^2 - 4\*a^5\*b^4\*c^2\*d^3 + a^6\*b^3\*d^4)\*g^5) - 12\*d^4\*log(b^5 c^4 - 4 a^2 b^4 c^3 d + 6 a^2 b^3 c^2 d^2 - 4 a^3 b^2 c^2 d^3 + a^4 b^2 c^2 d^4) g^5 - 12\*d^4\*log(d^4 x + c) / ((b^5 c^4 - 4 a^2 b^4 c^3 d + 6 a^2 b^3 c^2 d^2 - 4 a^3 b^2 c^2 d^3 + a^4 b^2 c^2 d^4) g^5) - 1/4 A / (b^5 g^5 x^4 + 4 a^2 b^3 g^5 x^2 + 4 a^3 b^2 g^5 x + a^4 b g^5)

$$b^3d^4)g^5x^2 + 4(a^3b^6c^4 - 4a^4b^5c^3d + 6a^5b^4c^2d^2 - 4a^6b^3c^2d^3 + a^7b^2d^4)g^5x + (a^4b^5c^4 - 4a^5b^4c^3d + 6a^6b^3c^2d^2 - 4a^7b^2c^2d^3 + a^8b^2d^4)g^5)$$

**Sympy [B]** time = 11.107, size = 944, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)\*\*5,x)

[Out]  $-B \log(e(a + bx)/(c + dx)) / (4a^4b^5g^5 + 16a^3b^2g^5x + 24a^2b^3g^5x^2 + 16a^2b^4g^5x^3 + 4b^5g^5x^4) - B d^4 \log(x + (-B a^5 d^9 / (a d - b c)^4 + 5 B a^4 b c d^8 / (a d - b c)^4 - 10 B a^3 b^2 c^2 d^7 / (a d - b c)^4 + 10 B a^2 b^3 c^3 d^6 / (a d - b c)^4 - 5 B a b^4 c^4 d^5 / (a d - b c)^4 + B a d^5 + B b^5 c^5 d^4 / (a d - b c)^4 + B b^2 c^4 d^4) / (2 B b d^5)) / (4 b^5 g^5 (a d - b c)^4) + B d^4 \log(x + (B a^5 d^9 / (a d - b c)^4 - 5 B a^4 b c d^8 / (a d - b c)^4 + 10 B a^3 b^2 c^2 d^7 / (a d - b c)^4 - 10 B a^2 b^3 c^3 d^6 / (a d - b c)^4 + 5 B a b^4 c^4 d^5 / (a d - b c)^4 + B a d^5 - B b^5 c^5 d^4 / (a d - b c)^4 + B b^2 c^4 d^4) / (2 B b d^5)) / (4 b^5 g^5 (a d - b c)^4) - (12 A a^3 d^3 - 36 A a^2 b c d^2 + 36 A a b^2 c^2 d - 12 A b^3 c^3 + 25 B a^3 d^3 - 23 B a^2 b c d^2 + 13 B a b^2 c^2 d - 3 B b^3 c^3 + 12 B b^3 d^3 x^3 + x^2 (42 B a b^2 d^3 - 6 B b^3 c d^2) + x (52 B a^2 b d^3 - 20 B a b^2 c d^2 + 4 B b^3 c^2 d)) / (48 a^7 b d^3 g^5 - 144 a^6 b^2 c d^2 g^5 + 144 a^5 b^3 c^2 d g^5 - 48 a^4 b^4 c^3 g^5 + x^4 (48 a^3 b^5 d^3 g^5 - 144 a^2 b^6 c d^2 g^5 + 144 a b^7 c^2 d g^5 - 48 b^8 c^3 g^5) + x^3 (192 a^4 b^4 d^3 g^5 - 576 a^3 b^5 c d^2 g^5 + 576 a^2 b^6 c^2 d g^5 - 192 a b^7 c^3 g^5) + x^2 (288 a^5 b^3 d^3 g^5 - 864 a^4 b^4 c d^2 g^5 + 864 a^3 b^5 c^2 d g^5 - 288 a^2 b^6 c^3 g^5) + x (192 a^6 b^2 d^3 g^5 - 576 a^5 b^3 c d^2 g^5 + 576 a^4 b^4 c^2 d g^5 - 192 a^3 b^5 c^3 g^5))$

**Giac [B]** time = 1.37798, size = 967, normalized size = 4.69

$$\frac{Bd^4 \log(bx + a)}{4(b^5c^4g^5 - 4ab^4c^3dg^5 + 6a^2b^3c^2d^2g^5 - 4a^3b^2cd^3g^5 + a^4bd^4g^5)} - \frac{Bd^4 \log(dx + c)}{4(b^5c^4g^5 - 4ab^4c^3dg^5 + 6a^2b^3c^2d^2g^5 - 4a^3b^2cd^3g^5 + a^4bd^4g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(b\*g\*x+a\*g)^5,x, algorithm="giac")

[Out]  $1/4 B d^4 \log(bx + a) / (b^5 c^4 g^5 - 4 a b^4 c^3 d g^5 + 6 a^2 b^3 c^2 d^2 g^5 - 4 a^3 b^2 c d^3 g^5 + a^4 b d^4 g^5) - 1/4 B d^4 \log(dx + c) / (b^5 c^4 g^5 - 4 a b^4 c^3 d g^5 + 6 a^2 b^3 c^2 d^2 g^5 - 4 a^3 b^2 c d^3 g^5 + a^4 b d^4 g^5) - 1/4 B \log((bx + a)/(dx + c)) / (b^5 g^5 x^4 + 4 a b^4 g^5 x^3 + 6 a^2 b^3 g^5 x^2 + 4 a^3 b^2 g^5 x + a^4 b g^5) + 1/48 (12 B b^3 d^3 x^3 - 6 B b^3 c d^2 x^2 + 42 B a b^2 d^3 x^2 + 4 B b^3 c^2 d x - 20 B a b^2 c d^2 x + 52 B a^2 b d^3 x - 12 A b^3 c^3 - 15 B b^3 c^3 + 36 A a b^2 c^2 d + 49 B a b^2 c^2 d - 36 A a^2 b c d^2 - 59 B a^2 b c d^2 + 12 A a^3 d^3 + 37 B a^3 d^3) / (b^8 c^3 g^5 x^4 - 3 a b^7 c^2 d g^5 x^4 + 3 a^2 b^6 c d^2 g^5 x^4 - a^3 b^5 d^3 g^5 x^4 + 4 a b^7 c^3 g^5 x^3 - 12 a^2 b^6 c^2 d g^5 x^3 + 12 a^3 b^5 c d^2 g^5 x^3 - 4 a^4 b^4 d^3 g^5 x^3 + 6 a^2 b^6 c^3 g^5 x^3$

$$\begin{aligned} & x^2 - 18a^3b^5c^2d^2g^5x^2 + 18a^4b^4c^2d^2g^5x^2 - 6a^5b^3d^3g^5x^2 + 4a^3b^5c^3g^5x - 12a^4b^4c^2d^2g^5x + 12a^5b^3c^2d^2g^5x - 4a^6b^2d^3g^5x + a^4b^4c^3g^5 - 3a^5b^3c^2d^2g^5 + 3a^6b^2c^2d^2g^5 - a^7b^3d^3g^5) \end{aligned}$$

$$3.97 \quad \int (ag + bgx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx$$

**Optimal.** Leaf size=365

$$\frac{2B^2g^4(bc-ad)^5 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} + \frac{Bg^4(bc-ad)^5 \log\left(\frac{bc-ad}{b(c+dx)}\right) \left(12B \log\left(\frac{e(a+bx)}{c+dx}\right) + 12A + 25B\right)}{30bd^5} + \frac{Bg^4(a+bx)(bc-ad)}{5bd^3}$$

[Out]  $-(B*(b*c - a*d)*g^4*(a + b*x)^4*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(10*b*d) + (g^4*(a + b*x)^5*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)])^2)/(5*b) + (B*(b*c - a*d)^2*g^4*(a + b*x)^3*(4*A + B + 4*B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(30*b*d^2) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2*(12*A + 7*B + 12*B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(60*b*d^3) + (B*(b*c - a*d)^4*g^4*(a + b*x)*(12*A + 13*B + 12*B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(30*b*d^4) + (B*(b*c - a*d)^5*g^4*\text{Log}[(b*c - a*d)/(b*(c + d*x))]*(12*A + 25*B + 12*B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(30*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(5*b*d^5)$

**Rubi [A]** time = 0.854539, antiderivative size = 557, normalized size of antiderivative = 1.53, number of steps used = 28, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{2B^2g^4(bc-ad)^5 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{5bd^5} - \frac{2Bg^4(bc-ad)^5 \log(c+dx) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{5bd^5} - \frac{Bg^4(a+bx)^2(bc-ad)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{5bd^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^4*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)])^2, x]$

[Out]  $(2*A*B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (13*B^2*(b*c - a*d)^4*g^4*x)/(30*d^4) - (7*B^2*(b*c - a*d)^3*g^4*(a + b*x)^2)/(60*b*d^3) + (B^2*(b*c - a*d)^2*g^4*(a + b*x)^3)/(30*b*d^2) + (2*B^2*(b*c - a*d)^4*g^4*(a + b*x)*\text{Log}[(e*(a + b*x))/(c + d*x)])/(5*b*d^4) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(5*b*d^3) + (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(15*b*d^2) - (B*(b*c - a*d)*g^4*(a + b*x)^4*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(10*b*d) + (g^4*(a + b*x)^5*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)])^2)/(5*b) - (5*B^2*(b*c - a*d)^5*g^4*\text{Log}[c + d*x])/(6*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(5*b*d^5) - (2*B*(b*c - a*d)^5*g^4*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)])*\text{Log}[c + d*x])/(5*b*d^5) - (B^2*(b*c - a*d)^5*g^4*\text{Log}[c + d*x]^2)/(5*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(5*b*d^5)$

**Rule 2525**

$\text{Int}[(a_. + \text{Log}[c_.*(\text{RFx}_.)^{p_.}]*b_.)^{(n_.)*((d_. + (e_.)*(x_.))^{m_.}), x\_Symbol] :> \text{Simp}[(d + e*x)^{(m + 1)}*(a + b*\text{Log}[c*\text{RFx}^p])^n]/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{(m + 1)}*(a + b*\text{Log}[c*\text{RFx}^p])^{(n - 1)}*D[\text{RFx}, x])/(\text{RFx}, x)], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(RFX\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFX, x] && IntegerQ[p]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_) + (g_.)*(x_)^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int (ag + bgx)^4 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2 dx &= \frac{g^4(a + bx)^5 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{5b} - \frac{(2B) \int \frac{(bc - ad)g^5(a + bx)^4 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{c + dx} dx}{5bg} \\ &= \frac{g^4(a + bx)^5 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{5b} - \frac{(2B(bc - ad)g^4) \int \frac{(a + bx)^4 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{c + dx} dx}{5b} \\ &= \frac{g^4(a + bx)^5 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{5b} - \frac{(2B(bc - ad)g^4) \int \left( -\frac{b(bc - ad)^3 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{d^4} \right) dx}{5b} \\ &= \frac{g^4(a + bx)^5 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{5b} - \frac{(2B(bc - ad)g^4) \int (a + bx)^3 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right) dx}{5d} \\ &= \frac{2AB(bc - ad)^4 g^4 x}{5d^4} - \frac{B(bc - ad)^3 g^4 (a + bx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{5bd^3} + \frac{2B(bc - ad)^3 g^4 (a + bx)^2}{5bd^3} \\ &= \frac{2AB(bc - ad)^4 g^4 x}{5d^4} + \frac{2B^2(bc - ad)^4 g^4 (a + bx) \log \left( \frac{e(a + bx)}{c + dx} \right)}{5bd^4} - \frac{B(bc - ad)^3 g^4 (a + bx)^2}{5bd^3} \\ &= \frac{2AB(bc - ad)^4 g^4 x}{5d^4} + \frac{2B^2(bc - ad)^4 g^4 (a + bx) \log \left( \frac{e(a + bx)}{c + dx} \right)}{5bd^4} - \frac{B(bc - ad)^3 g^4 (a + bx)^2}{5bd^3} \\ &= \frac{2AB(bc - ad)^4 g^4 x}{5d^4} + \frac{13B^2(bc - ad)^4 g^4 x}{30d^4} - \frac{7B^2(bc - ad)^3 g^4 (a + bx)^2}{60bd^3} + \frac{B^2(bc - ad)^3 g^4 (a + bx)^2}{60bd^3} \\ &= \frac{2AB(bc - ad)^4 g^4 x}{5d^4} + \frac{13B^2(bc - ad)^4 g^4 x}{30d^4} - \frac{7B^2(bc - ad)^3 g^4 (a + bx)^2}{60bd^3} + \frac{B^2(bc - ad)^3 g^4 (a + bx)^2}{60bd^3} \\ &= \frac{2AB(bc - ad)^4 g^4 x}{5d^4} + \frac{13B^2(bc - ad)^4 g^4 x}{30d^4} - \frac{7B^2(bc - ad)^3 g^4 (a + bx)^2}{60bd^3} + \frac{B^2(bc - ad)^3 g^4 (a + bx)^2}{60bd^3} \end{aligned}$$

**Mathematica [A]** time = 0.497888, size = 511, normalized size = 1.4

$$g^4 \left( \frac{B(bc - ad) \left( 12B(bc - ad)^4 \left( 2 \text{PolyLog} \left( 2, \frac{b(c + dx)}{bc - ad} \right) + \log(c + dx) \right) \left( 2 \log \left( \frac{d(a + bx)}{ad - bc} \right) - \log(c + dx) \right) \right) - 12d^2(a + bx)^2(bc - ad)^2 \left( B \log \left( \frac{e(a + bx)}{c + dx} \right) + A \right) + 8d^3(a + bx)^3(bc - ad) \left( B \log \left( \frac{e(a + bx)}{c + dx} \right) + A \right)}{\dots} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (g^4*((a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(b*c - a*d)*
24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*x))
/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c
+ d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)
]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*c - a
*d)^4*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*
Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2
*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2
*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d
*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 12*B*(b*c
- a*d)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x
] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(12*d^5))/(5*b)
```

---

**Maple [F]** time = 2.536, size = 0, normalized size = 0.

$$\int (bgx + ag)^4 \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

---

**Maxima [B]** time = 1.83175, size = 3225, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] 1/5*A^2*b^4*g^4*x^5 + A^2*a*b^3*g^4*x^4 + 2*A^2*a^2*b^2*g^4*x^3 + 2*A^2*a^3
*b*g^4*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b -
c*log(d*x + c)/d)*A*B*a^4*g^4 + 4*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*
a^3*b*g^4 + 2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x +
a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b^2*g^4 + 1/3*(6*x^4*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^
3)*x)/(b^3*d^3))*A*B*a*b^3*g^4 + 1/30*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3
- a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*
b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*A*B*b^4*g^4 + A^2*a^4*g^4
*x - 1/30*((12*g^4*log(e) + 25*g^4)*b^4*c^5 - (60*g^4*log(e) + 113*g^4)*a*b
^3*c^4*d + 4*(30*g^4*log(e) + 49*g^4)*a^2*b^2*c^3*d^2 - 12*(10*g^4*log(e) +
13*g^4)*a^3*b*c^2*d^3 + 12*(5*g^4*log(e) + 4*g^4)*a^4*c*d^4)*B^2*log(d*x +
c)/d^5 - 2/5*(b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 1
```

$$0*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4 - a^5*d^5*g^4)*(\log(b*x + a)*\log((b*d*x + a*d)/(b*c - a*d) + 1) + \operatorname{dilog}(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^5) + 1/60*(12*B^2*b^5*d^5*g^4*x^5*\log(e)^2 - 6*(b^5*c*d^4*g^4*\log(e) - (10*g^4*\log(e)^2 + g^4*\log(e))*a*b^4*d^5)*B^2*x^4 + 2*((4*g^4*\log(e) + g^4)*b^5*c^2*d^3 - 2*(10*g^4*\log(e) + g^4)*a*b^4*c*d^4 + (60*g^4*\log(e)^2 + 16*g^4*\log(e) + g^4)*a^2*b^3*d^5)*B^2*x^3 - ((12*g^4*\log(e) + 7*g^4)*b^5*c^3*d^2 - 3*(20*g^4*\log(e) + 9*g^4)*a*b^4*c^2*d^3 + 3*(40*g^4*\log(e) + 11*g^4)*a^2*b^3*c*d^4 - (120*g^4*\log(e)^2 + 72*g^4*\log(e) + 13*g^4)*a^3*b^2*d^5)*B^2*x^2 + 2*((12*g^4*\log(e) + 13*g^4)*b^5*c^4*d - (60*g^4*\log(e) + 59*g^4)*a*b^4*c^3*d^2 + 6*(20*g^4*\log(e) + 17*g^4)*a^2*b^3*c^2*d^3 - (120*g^4*\log(e) + 79*g^4)*a^3*b^2*c*d^4 + (30*g^4*\log(e)^2 + 48*g^4*\log(e) + 23*g^4)*a^4*b*d^5)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + B^2*a^5*d^5*g^4)*\log(b*x + a)^2 + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + (b^5*c^5*g^4 - 5*a*b^4*c^4*d*g^4 + 10*a^2*b^3*c^3*d^2*g^4 - 10*a^3*b^2*c^2*d^3*g^4 + 5*a^4*b*c*d^4*g^4)*B^2)*\log(d*x + c)^2 + 2*(12*B^2*b^5*d^5*g^4*x^5*\log(e) - 3*(b^5*c*d^4*g^4 - (20*g^4*\log(e) + g^4)*a*b^4*d^5)*B^2*x^4 + 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 + 2*(15*g^4*\log(e) + 2*g^4)*a^2*b^3*d^5)*B^2*x^3 - 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 - 2*(10*g^4*\log(e) + 3*g^4)*a^3*b^2*d^5)*B^2*x^2 + 12*(b^5*c^4*d*g^4 - 5*a*b^4*c^3*d^2*g^4 + 10*a^2*b^3*c^2*d^3*g^4 - 10*a^3*b^2*c*d^4*g^4 + (5*g^4*\log(e) + 4*g^4)*a^4*b*d^5)*B^2*x + (12*a*b^4*c^4*d*g^4 - 54*a^2*b^3*c^3*d^2*g^4 + 94*a^3*b^2*c^2*d^3*g^4 - 77*a^4*b*c*d^4*g^4 + (12*g^4*\log(e) + 25*g^4)*a^5*d^5)*B^2)*\log(b*x + a) - 2*(12*B^2*b^5*d^5*g^4*x^5*\log(e) - 3*(b^5*c*d^4*g^4 - (20*g^4*\log(e) + g^4)*a*b^4*d^5)*B^2*x^4 + 4*(b^5*c^2*d^3*g^4 - 5*a*b^4*c*d^4*g^4 + 2*(15*g^4*\log(e) + 2*g^4)*a^2*b^3*d^5)*B^2*x^3 - 6*(b^5*c^3*d^2*g^4 - 5*a*b^4*c^2*d^3*g^4 + 10*a^2*b^3*c*d^4*g^4 - 2*(10*g^4*\log(e) + 3*g^4)*a^3*b^2*d^5)*B^2*x^2 + 12*(b^5*c^4*d*g^4 - 5*a*b^4*c^3*d^2*g^4 + 10*a^2*b^3*c^2*d^3*g^4 - 10*a^3*b^2*c*d^4*g^4 + (5*g^4*\log(e) + 4*g^4)*a^4*b*d^5)*B^2*x + 12*(B^2*b^5*d^5*g^4*x^5 + 5*B^2*a*b^4*d^5*g^4*x^4 + 10*B^2*a^2*b^3*d^5*g^4*x^3 + 10*B^2*a^3*b^2*d^5*g^4*x^2 + 5*B^2*a^4*b*d^5*g^4*x + B^2*a^5*d^5*g^4)*\log(b*x + a))*\log(d*x + c))/(b*d^5)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(A^2b^4g^4x^4 + 4A^2ab^3g^4x^3 + 6A^2a^2b^2g^4x^2 + 4A^2a^3bg^4x + A^2a^4g^4 + (B^2b^4g^4x^4 + 4B^2ab^3g^4x^3 + 6B^2a^2b^2g^4x^2 + \dots)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out] integral(A^2\*b^4\*g^4\*x^4 + 4\*A^2\*a\*b^3\*g^4\*x^3 + 6\*A^2\*a^2\*b^2\*g^4\*x^2 + 4\*A^2\*a^3\*b\*g^4\*x + A^2\*a^4\*g^4 + (B^2\*b^4\*g^4\*x^4 + 4\*B^2\*a\*b^3\*g^4\*x^3 + 6\*B^2\*a^2\*b^2\*g^4\*x^2 + 4\*B^2\*a^3\*b\*g^4\*x + B^2\*a^4\*g^4)\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*(A\*B\*b^4\*g^4\*x^4 + 4\*A\*B\*a\*b^3\*g^4\*x^3 + 6\*A\*B\*a^2\*b^2\*g^4\*x^2 + 4\*A\*B\*a^3\*b\*g^4\*x + A\*B\*a^4\*g^4)\*log((b\*e\*x + a\*e)/(d\*x + c)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^4 \left( B \log \left( \frac{(bx + a)e}{dx + c} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^4*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

$$3.98 \quad \int (ag + bgx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx$$

**Optimal.** Leaf size=309

$$\frac{B^2 g^3 (bc - ad)^4 \text{PolyLog} \left( 2, \frac{d(a+bx)}{b(c+dx)} \right)}{2bd^4} - \frac{Bg^3 (bc - ad)^4 \log \left( \frac{bc-ad}{b(c+dx)} \right) \left( 6B \log \left( \frac{e(a+bx)}{c+dx} \right) + 6A + 11B \right)}{12bd^4} - \frac{Bg^3 (a + bx)(bc - ad)}{12bd^4}$$

[Out]  $-(B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(6*b*d) + (g^3*(a + b*x)^4*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x]])^2)/(4*b) + (B*(b*c - a*d)^2*g^3*(a + b*x)^2*(3*A + B + 3*B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(12*b*d^2) - (B*(b*c - a*d)^3*g^3*(a + b*x)*(6*A + 5*B + 6*B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(12*b*d^3) - (B*(b*c - a*d)^4*g^3*\text{Log}[(b*c - a*d)/(b*(c + d*x))]*(6*A + 11*B + 6*B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(12*b*d^4) - (B^2*(b*c - a*d)^4*g^3*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b*d^4)$

**Rubi [A]** time = 0.64583, antiderivative size = 474, normalized size of antiderivative = 1.53, number of steps used = 24, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$-\frac{B^2 g^3 (bc - ad)^4 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right)}{2bd^4} + \frac{Bg^3 (bc - ad)^4 \log(c + dx) \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right)}{2bd^4} + \frac{Bg^3 (a + bx)^2 (bc - ad)^2 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right)}{4bd^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^3*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x]])^2, x]$

[Out]  $-(A*B*(b*c - a*d)^3*g^3*x)/(2*d^3) - (5*B^2*(b*c - a*d)^3*g^3*x)/(12*d^3) + (B^2*(b*c - a*d)^2*g^3*(a + b*x)^2)/(12*b*d^2) - (B^2*(b*c - a*d)^3*g^3*(a + b*x)*\text{Log}[(e*(a + b*x))/(c + d*x)]/(2*b*d^3) + (B*(b*c - a*d)^2*g^3*(a + b*x)^2*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(4*b*d^2) - (B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(6*b*d) + (g^3*(a + b*x)^4*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x]])^2)/(4*b) + (11*B^2*(b*c - a*d)^4*g^3*\text{Log}[c + d*x])/(12*b*d^4) - (B^2*(b*c - a*d)^4*g^3*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(2*b*d^4) + (B*(b*c - a*d)^4*g^3*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)])*\text{Log}[c + d*x])/(2*b*d^4) + (B^2*(b*c - a*d)^4*g^3*\text{Log}[c + d*x]^2)/(4*b*d^4) - (B^2*(b*c - a*d)^4*g^3*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(2*b*d^4)$

### Rule 2525

$\text{Int}[(a_.) + \text{Log}[(c_.)*(RfX_)^(p_.)]*(b_.)]^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^(m + 1)*(a + b*\text{Log}[c*RfX^p])^n/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^(m + 1)*(a + b*\text{Log}[c*RfX^p])^(n - 1)*D[RfX, x])/RfX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RfX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFX^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFX_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFX, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFX, x] && IntegerQ[p]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\int (ag + bgx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx = \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{4b} - \frac{B \int \frac{(bc-ad)g^4(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{c+dx} dx}{2bg}$$

$$= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{4b} - \frac{(B(bc-ad)g^3) \int \frac{(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{c+dx}}{2b}$$

$$= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{4b} - \frac{(B(bc-ad)g^3) \int \left( \frac{b(bc-ad)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{d^3} \right)}{2b}$$

$$= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{4b} - \frac{(B(bc-ad)g^3) \int (a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{2d}$$

$$= -\frac{AB(bc-ad)^3 g^3 x}{2d^3} + \frac{B(bc-ad)^2 g^3 (a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{4bd^2} - \frac{B(bc-ad)^2 g^3 (a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{2bd^3} + \frac{B(bc-ad)^2 g^3 (a+bx)^2 \log \left( \frac{e(a+bx)}{c+dx} \right)}{2bd^3}$$

$$= -\frac{AB(bc-ad)^3 g^3 x}{2d^3} - \frac{B^2(bc-ad)^3 g^3 (a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{2bd^3} + \frac{B(bc-ad)^2 g^3 (a+bx)^2 \log \left( \frac{e(a+bx)}{c+dx} \right)}{12bd^2} - \frac{B^2(bc-ad)^3 g^3 x}{12d^3} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{12bd^2} - \frac{B^2(bc-ad)^3 g^3 (a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{12bd^2}$$

$$= -\frac{AB(bc-ad)^3 g^3 x}{2d^3} - \frac{5B^2(bc-ad)^3 g^3 x}{12d^3} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{12bd^2} - \frac{B^2(bc-ad)^3 g^3 (a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{12bd^2} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2 \log \left( \frac{e(a+bx)}{c+dx} \right)}{12bd^2} - \frac{B^2(bc-ad)^3 g^3 x}{12d^3}$$

$$= -\frac{AB(bc-ad)^3 g^3 x}{2d^3} - \frac{5B^2(bc-ad)^3 g^3 x}{12d^3} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{12bd^2} - \frac{B^2(bc-ad)^3 g^3 (a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{12bd^2} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2 \log \left( \frac{e(a+bx)}{c+dx} \right)}{12bd^2} - \frac{B^2(bc-ad)^3 g^3 x}{12d^3}$$

**Mathematica [A]** time = 0.33544, size = 391, normalized size = 1.27

$$g^3 \left( (a+bx)^4 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right)^2 - \frac{B(bc-ad) \left( 3B(bc-ad)^3 \left( 2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) + 2d^3 (a+bx)^3 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right) \right)}{12bd^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*
6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(
c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c +
d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c
- a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x])
)*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(
b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) + a*d)*Log[
c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
+ d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4))
)/(4*b)
```

---

**Maple [F]** time = 2.177, size = 0, normalized size = 0.

$$\int (bgx + ag)^3 \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

---

**Maxima [B]** time = 1.75332, size = 2338, normalized size = 7.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(
b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B
*a^3*g^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b
^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3 + (2*x^3*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x
+ c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A
*B*a*b^2*g^3 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log
(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*
g^3 + A^2*a^3*g^3*x + 1/12*((6*g^3*log(e) + 11*g^3)*b^3*c^4 - 2*(12*g^3*log
(e) + 19*g^3)*a*b^2*c^3*d + 9*(4*g^3*log(e) + 5*g^3)*a^2*b*c^2*d^2 - 6*(4*g
^3*log(e) + 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*g^3 - 4*a
*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d^4*g^3)*(
log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 - 2*(b^4*c*d^3
*g^3*log(e) - (6*g^3*log(e)^2 + g^3*log(e))*a*b^3*d^4)*B^2*x^3 + ((3*g^3*lo
g(e) + g^3)*b^4*c^2*d^2 - 2*(6*g^3*log(e) + g^3)*a*b^3*c*d^3 + (18*g^3*log(
e)^2 + 9*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 - ((6*g^3*log(e) + 5*g^3)*b
^4*c^3*d - (24*g^3*log(e) + 17*g^3)*a*b^3*c^2*d^2 + (36*g^3*log(e) + 19*g^3
)*a^2*b^2*c*d^3 - (12*g^3*log(e)^2 + 18*g^3*log(e) + 7*g^3)*a^3*b*d^4)*B^2*
x + 3*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^
3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a)^2 + 3*(B^2*b^
4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2
```

```
*a^3*b*d^4*g^3*x - (b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3
- 4*a^3*b*c*d^3*g^3)*B^2*log(d*x + c)^2 + (6*B^2*b^4*d^4*g^3*x^4*log(e) -
2*(b^4*c*d^3*g^3 - (12*g^3*log(e) + g^3)*a*b^3*d^4)*B^2*x^3 + 3*(b^4*c^2*d
^2*g^3 - 4*a*b^3*c*d^3*g^3 + 3*(4*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 -
6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 - (4*g^3*log(e)
) + 3*g^3)*a^3*b*d^4)*B^2*x - (6*a*b^3*c^3*d*g^3 - 21*a^2*b^2*c^2*d^2*g^3 +
26*a^3*b*c*d^3*g^3 - (6*g^3*log(e) + 11*g^3)*a^4*d^4)*B^2*log(b*x + a) -
(6*B^2*b^4*d^4*g^3*x^4*log(e) - 2*(b^4*c*d^3*g^3 - (12*g^3*log(e) + g^3)*a^
b^3*d^4)*B^2*x^3 + 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 + 3*(4*g^3*log(e)
+ g^3)*a^2*b^2*d^4)*B^2*x^2 - 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a
^2*b^2*c*d^3*g^3 - (4*g^3*log(e) + 3*g^3)*a^3*b*d^4)*B^2*x + 6*(B^2*b^4*d^4
*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*
b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a))*log(d*x + c))/(b*d^4)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral  $\left( A^2 b^3 g^3 x^3 + 3 A^2 a b^2 g^3 x^2 + 3 A^2 a^2 b g^3 x + A^2 a^3 g^3 + (B^2 b^3 g^3 x^3 + 3 B^2 a b^2 g^3 x^2 + 3 B^2 a^2 b g^3 x + B^2 a^3 g^3) \log\left(\frac{bex}{dx}\right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^3 \left( B \log\left(\frac{(bx + a)e}{dx + c}\right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

$$3.99 \quad \int (ag + bgx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx$$

**Optimal.** Leaf size=253

$$\frac{2B^2g^2(bc-ad)^3 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} + \frac{Bg^2(bc-ad)^3 \log\left(\frac{bc-ad}{b(c+dx)}\right) \left(2B \log\left(\frac{e(a+bx)}{c+dx}\right) + 2A + 3B\right)}{3bd^3} + \frac{Bg^2(a+bx)(bc-a)}{3bd^3}$$

```
[Out] -(B*(b*c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*d) + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b) + (B*(b*c - a*d)^2*g^2*(a + b*x)*(2*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*d^2) + (B*(b*c - a*d)^3*g^2*Log[(b*c - a*d)/(b*(c + d*x))]*(2*A + 3*B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*d^3) + (2*B^2*(b*c - a*d)^3*g^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)
```

**Rubi [A]** time = 0.552817, antiderivative size = 389, normalized size of antiderivative = 1.54, number of steps used = 20, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{2B^2g^2(bc-ad)^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3bd^3} - \frac{2Bg^2(bc-ad)^3 \log(c+dx) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{3bd^3} + \frac{2ABg^2x(bc-ad)^2}{3d^2} - \frac{Bg^2(a+bx)(bc-a)}{3bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (2*A*B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (B^2*(b*c - a*d)^2*g^2*x)/(3*d^2) + (2*B^2*(b*c - a*d)^2*g^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)]/(3*b*d^2) - (B*(b*c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*d) + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b) - (B^2*(b*c - a*d)^3*g^2*Log[c + d*x]/(b*d^3) + (2*B^2*(b*c - a*d)^3*g^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*b*d^3) - (2*B*(b*c - a*d)^3*g^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x]/(3*b*d^3) - (B^2*(b*c - a*d)^3*g^2*Log[c + d*x]^2)/(3*b*d^3) + (2*B^2*(b*c - a*d)^3*g^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]/(3*b*d^3))
```

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
```

```
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

### Rule 31

```
Int[((a_.) + (b_.)*(x_))^(n_.), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2390



```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)
)*(x_.)^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qq[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rubi steps

$$\begin{aligned}
\int (ag + bgx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx &= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{3b} - \frac{(2B) \int \frac{(bc-ad)g^3(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{c+dx}}{3bg} \\
&= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{3b} - \frac{(2B(bc-ad)g^2) \int \frac{(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{c+dx}}{3b} \\
&= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{3b} - \frac{(2B(bc-ad)g^2) \int \left( -\frac{b(bc-ad) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{d^2} \right)}{3b} \\
&= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{3b} - \frac{(2B(bc-ad)g^2) \int (a+bx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{3d} \\
&= \frac{2AB(bc-ad)^2 g^2 x}{3d^2} - \frac{B(bc-ad)g^2(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{3bd} + \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{3b} \\
&= \frac{2AB(bc-ad)^2 g^2 x}{3d^2} + \frac{2B^2(bc-ad)^2 g^2(a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{3bd^2} - \frac{B(bc-ad)g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{3b} \\
&= \frac{2AB(bc-ad)^2 g^2 x}{3d^2} + \frac{2B^2(bc-ad)^2 g^2(a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{3bd^2} - \frac{B(bc-ad)g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{3b} \\
&= \frac{2AB(bc-ad)^2 g^2 x}{3d^2} + \frac{B^2(bc-ad)^2 g^2 x}{3d^2} + \frac{2B^2(bc-ad)^2 g^2(a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{3bd^2} \\
&= \frac{2AB(bc-ad)^2 g^2 x}{3d^2} + \frac{B^2(bc-ad)^2 g^2 x}{3d^2} + \frac{2B^2(bc-ad)^2 g^2(a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{3bd^2} \\
&= \frac{2AB(bc-ad)^2 g^2 x}{3d^2} + \frac{B^2(bc-ad)^2 g^2 x}{3d^2} + \frac{2B^2(bc-ad)^2 g^2(a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{3bd^2} \\
&= \frac{2AB(bc-ad)^2 g^2 x}{3d^2} + \frac{B^2(bc-ad)^2 g^2 x}{3d^2} + \frac{2B^2(bc-ad)^2 g^2(a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{3bd^2}
\end{aligned}$$

**Mathematica [A]** time = 0.220747, size = 287, normalized size = 1.13

$$g^2 \left( \frac{B(bc-ad) \left( B(bc-ad)^2 \left( 2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) - d^2(a+bx)^2 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right) - 2(bc-ad)^2 \log(c+dx) \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{d^3} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(b*c - a*d)*(
2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a
*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*L
og[c + d*x] + B*(b*c - a*d)*(b*d*x + (-b*c) + a*d)*Log[c + d*x]) + B*(b*c
- a*d)^2*((2*Log[(d*(a + b*x))/(-b*c) + a*d)] - Log[c + d*x])*Log[c + d*x]
+ 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/d^3)/(3*b)
```

**Maple [F]** time = 1.991, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

**Maxima [B]** time = 2.03334, size = 1573, normalized size = 6.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x
+ c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
- (b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(
d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a
*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 + A^2*a^2*g^2
*x - 1/3*((2*g^2*log(e) + 3*g^2)*b^2*c^3 - (6*g^2*log(e) + 7*g^2)*a*b*c^2*d
+ 2*(3*g^2*log(e) + 2*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(b^3*c^3*
g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*x + a)*lo
g((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(
b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 - (b^3*c*d^2*g^2*log(e) - (3*g^2
*log(e)^2 + g^2*log(e))*a*b^2*d^3)*B^2*x^2 + ((2*g^2*log(e) + g^2)*b^3*c^2*
d - 2*(3*g^2*log(e) + g^2)*a*b^2*c*d^2 + (3*g^2*log(e)^2 + 4*g^2*log(e) + g
^2)*a^2*b*d^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B
^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3
+ 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2
*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 + (2*B^2*b^3*d^3*g^2*x^
3*log(e) - (b^3*c*d^2*g^2 - (6*g^2*log(e) + g^2)*a*b^2*d^3)*B^2*x^2 + 2*(b^
3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 + (3*g^2*log(e) + 2*g^2)*a^2*b*d^3)*B^2*x +
(2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 + (2*g^2*log(e) + 3*g^2)*a^3*d^3)*B
^2)*log(b*x + a) - (2*B^2*b^3*d^3*g^2*x^3*log(e) - (b^3*c*d^2*g^2 - (6*g^2*
log(e) + g^2)*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 + (
3*g^2*log(e) + 2*g^2)*a^2*b*d^3)*B^2*x + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b
^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a))*log
(d*x + c))/(b*d^3)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(A^2b^2g^2x^2 + 2A^2abg^2x + A^2a^2g^2 + (B^2b^2g^2x^2 + 2B^2abg^2x + B^2a^2g^2)\log\left(\frac{bex + ae}{dx + c}\right)^2 + 2(ABb^2g^2x^2 + 2A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out] integral(A^2\*b^2\*g^2\*x^2 + 2\*A^2\*a\*b\*g^2\*x + A^2\*a^2\*g^2 + (B^2\*b^2\*g^2\*x^2 + 2\*B^2\*a\*b\*g^2\*x + B^2\*a^2\*g^2)\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*(A\*B\*b^2\*g^2\*x^2 + 2\*A\*B\*a\*b\*g^2\*x + A\*B\*a^2\*g^2)\*log((b\*e\*x + a\*e)/(d\*x + c)), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*2\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( B \log\left(\frac{(bx + a)e}{dx + c}\right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^2\*(B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2, x)

$$3.100 \quad \int (ag + bgx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx$$

**Optimal.** Leaf size=180

$$\frac{B^2 g(bc - ad)^2 \text{PolyLog} \left( 2, \frac{d(a+bx)}{b(c+dx)} \right)}{bd^2} - \frac{Bg(bc - ad)^2 \log \left( \frac{bc-ad}{b(c+dx)} \right) \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A + B \right)}{bd^2} - \frac{Bg(a + bx)(bc - ad) \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A + B \right)}{bd}$$

```
[Out] -((B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x]]))/(b*d) +
(g*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x]]^2)/(2*b) - (B*(b*c - a
*d)^2*g*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B + B*Log[(e*(a + b*x))/(c + d
*x]]))/(b*d^2) - (B^2*(b*c - a*d)^2*g*PolyLog[2, (d*(a + b*x))/(b*(c + d*x)
)])/(b*d^2)
```

**Rubi [A]** time = 0.455556, antiderivative size = 285, normalized size of antiderivative = 1.58, number of steps used = 16, number of rules used = 12, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{B^2 g(bc - ad)^2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right)}{bd^2} + \frac{Bg(bc - ad)^2 \log(c + dx) \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right)}{bd^2} + \frac{g(a + bx)^2 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right)^2}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x]])^2,x]
```

```
[Out] -((A*B*(b*c - a*d)*g*x)/d) - (B^2*(b*c - a*d)*g*(a + b*x)*Log[(e*(a + b*x)
)/(c + d*x]]/(b*d) + (g*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x]]^2)
/(2*b) + (B^2*(b*c - a*d)^2*g*Log[c + d*x]]/(b*d^2) - (B^2*(b*c - a*d)^2*g*
Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x]]/(b*d^2) + (B*(b*c - a*d)^2*
g*(A + B*Log[(e*(a + b*x))/(c + d*x]])*Log[c + d*x]]/(b*d^2) + (B^2*(b*c -
a*d)^2*g*Log[c + d*x]^2)/(2*b*d^2) - (B^2*(b*c - a*d)^2*g*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)]/(b*d^2)
```

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2524

```
Int[((a_) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFX^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFX_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFX, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFX, x] && IntegerQ[p]
```

### Rule 2394

```
Int[((a_) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2390

```
Int[((a_) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_) + Log[(c_.)*(x_))^(n_.)]*(b_.)/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rubi steps

$$\begin{aligned}
\int (ag + bgx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx &= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{2b} - \frac{B \int \frac{(bc-ad)g^2(a+bx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{c+dx} dx}{bg} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{2b} - \frac{(B(bc-ad)g) \int \frac{(a+bx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{c+dx} dx}{b} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{2b} - \frac{(B(bc-ad)g) \int \left( \frac{b \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{d} + \frac{(-bc)}{d} \right) dx}{b} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{2b} - \frac{(B(bc-ad)g) \int \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right) dx}{d} \\
&= -\frac{AB(bc-ad)gx}{d} + \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{2b} + \frac{B(bc-ad)^2 g \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{2b} \\
&= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{bd} + \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{2b} \\
&= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{bd} + \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{2b} \\
&= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{bd} + \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{2b} \\
&= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{bd} + \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{2b} \\
&= -\frac{AB(bc-ad)gx}{d} - \frac{B^2(bc-ad)g(a+bx) \log \left( \frac{e(a+bx)}{c+dx} \right)}{bd} + \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{2b}
\end{aligned}$$

**Mathematica [A]** time = 0.17703, size = 203, normalized size = 1.13

$$\frac{g \left( (a+bx)^2 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right)^2 - \frac{B(bc-ad) \left( B(bc-ad) \left( 2 \operatorname{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) - 2(bc-ad) \log(c+dx) \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right) \right)}{d^2} \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2,x]

[Out] (g\*((a + b\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2 - (B\*(b\*c - a\*d))\*(2\*A\*b\*d\*x + 2\*B\*d\*(a + b\*x)\*Log[(e\*(a + b\*x))/(c + d\*x]] - 2\*B\*(b\*c - a\*d)\*Log[c + d\*x] - 2\*(b\*c - a\*d)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))\*Log[c + d\*x] + B\*(b\*c - a\*d)\*((2\*Log[(d\*(a + b\*x))/(-b\*c) + a\*d]] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]))/d^2)/(2\*b)

**Maple [F]** time = 1.642, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)`

[Out] `int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)`

**Maxima [B]** time = 1.54517, size = 825, normalized size = 4.58

$$\frac{1}{2} A^2 b g x^2 + 2 \left( x \log \left( \frac{b e x}{d x + c} + \frac{a e}{d x + c} \right) + \frac{a \log(b x + a)}{b} - \frac{c \log(d x + c)}{d} \right) A B a g + \left( x^2 \log \left( \frac{b e x}{d x + c} + \frac{a e}{d x + c} \right) - \frac{a^2 \log}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")`

[Out] `1/2*A^2*b*g*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a*g + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*b*g + A^2*a*g*x + ((g*log(e) + g)*b*c^2 - (2*g*log(e) + g)*a*c*d)*B^2*log(d*x + c)/d^2 + (b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 - 2*(b^2*c*d*g*log(e) - (g*log(e)^2 + g*log(e))*a*b*d^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 + 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + ((g*log(e) + g)*a^2*d^2 - a*b*c*d*g)*B^2)*log(b*x + a) - 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c))/(b*d^2)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2 b g x + A^2 a g + (B^2 b g x + B^2 a g) \log \left( \frac{b e x + a e}{d x + c} \right)^2 + 2 (A B b g x + A B a g) \log \left( \frac{b e x + a e}{d x + c} \right), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")`

[Out] `integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)`

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag) \left( B \log \left( \frac{(bx + a)e}{dx + c} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)\*(B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2, x)



$$3.101 \quad \int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx} dx$$

**Optimal.** Leaf size=128

$$\frac{2B \text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{bg} + \frac{2B^2 \text{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right)}{bg} - \frac{\log\left(1 - \frac{b(c+dx)}{d(a+bx)}\right) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)^2}{bg}$$

[Out] -(((A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2\*Log[1 - (b\*(c + d\*x))/(d\*(a + b\*x) )])/(b\*g)) + (2\*B\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])\*PolyLog[2, (b\*(c + d\*x))/(d\*(a + b\*x))])/(b\*g) + (2\*B^2\*PolyLog[3, (b\*(c + d\*x))/(d\*(a + b\*x) )])/(b\*g)

**Rubi [B]** time = 3.41603, antiderivative size = 728, normalized size of antiderivative = 5.69, number of steps used = 46, number of rules used = 23, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.719$ , Rules used = {2524, 12, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 6742, 2500, 2440, 2434, 2433, 2375, 2317, 2374, 6589, 2499, 2302, 30, 2396}

$$\frac{2AB \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg} - \frac{2B^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right) \left(-\log\left(\frac{e(a+bx)}{c+dx}\right) + \log(a+bx) + \log\left(\frac{1}{c+dx}\right)\right)}{bg} - \frac{2B^2 \text{PolyLog}\left(3, -\frac{d(a+bx)}{bc-ad}\right)}{bg}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2/(a\*g + b\*g\*x), x]

[Out] -((A\*B\*Log[g\*(a + b\*x)]^2)/(b\*g)) + (B^2\*Log[g\*(a + b\*x)]^3)/(3\*b\*g) - (B^2 \*Log[a + b\*x]^2\*Log[-c - d\*x])/(b\*g) + (2\*B^2\*Log[a + b\*x]\*Log[g\*(a + b\*x)] \*Log[-c - d\*x])/(b\*g) - (B^2\*Log[g\*(a + b\*x)]^2\*Log[-c - d\*x])/(b\*g) + (B^2 \*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[(c + d\*x)^(-1)]^2)/(b\*g) - (B^2\*Log[ g\*(a + b\*x)]\*Log[(c + d\*x)^(-1)]^2)/(b\*g) + (B^2\*Log[a + b\*x]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g) + (B^2\*Log[g\*(a + b\*x)]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g) + ((A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2\*Log[a\*g + b\*g\*x]) / (b\*g) + (2\*A\*B\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*Log[a\*g + b\*g\*x])/(b\*g) - (2 \*B^2\*(Log[a + b\*x] + Log[(c + d\*x)^(-1)] - Log[(e\*(a + b\*x))/(c + d\*x)])\*Lo g[(b\*(c + d\*x))/(b\*c - a\*d)]\*Log[a\*g + b\*g\*x])/(b\*g) - (B^2\*Log[(e\*(a + b\*x) )/(c + d\*x)]\*Log[a\*g + b\*g\*x]^2)/(b\*g) - (B^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d) ]\*Log[a\*g + b\*g\*x]^2)/(b\*g) + (2\*A\*B\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d) )])/(b\*g) + (2\*B^2\*Log[a + b\*x]\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d) )])/( b\*g) - (2\*B^2\*(Log[a + b\*x] + Log[(c + d\*x)^(-1)] - Log[(e\*(a + b\*x))/(c + d\*x)])\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d) )])/(b\*g) - (2\*B^2\*Log[(c + d \*x)^(-1)]\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g) - (2\*B^2\*PolyLog[3, -((d\*(a + b\*x))/(b\*c - a\*d) )])/(b\*g) - (2\*B^2\*PolyLog[3, (b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g)

**Rule 2524**

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^n)/e, x] - Dist[(b\*n\*p)/e , Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]

**Rule 12**

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.)/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 6688

```
Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 2500

```

Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.))*((c_.) + (d_.)*(x_.))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_.))^(n_.)]*(t_.))/((j_.) + (k
_.)*(x_.)), x_Symbol] := Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]

```

#### Rule 2440

```

Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_.))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_.))^(r_.), x_Symbol] :=
Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]

```

#### Rule 2434

```

Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_.))^(m_.)]*(g_.)))/(x_), x_Symbol] := Simp[Log[x]*(a + b
*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[e*g*m, Int[(Lo
g[x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x), x], x] - Dist[b*j*n, Int[(Log[x
]*(f + g*Log[h*(i + j*x)^m]))/(i + j*x), x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, i, j, m, n}, x] && EqQ[e*i - d*j, 0]

```

#### Rule 2433

```

Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.))*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_.))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_.))^(r_.), x_Sym
bol] := Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]

```

#### Rule 2375

```

Int[(Log[(d_.)*((e_.) + (f_.)*(x_.))^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_.)^(n
_.)]*(b_.))^(p_.))/(x_), x_Symbol] := Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]

```

#### Rule 2317

```

Int[((a_.) + Log[(c_.)*(x_.)^(n_.)]*(b_.))^(p_.))/((d_.) + (e_.)*(x_.)), x_Symb
ol] := Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]

```

#### Rule 2374

```

Int[(Log[(d_.)*((e_.) + (f_.)*(x_.))^(m_.))]*((a_.) + Log[(c_.)*(x_.)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]

```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

### Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]*
((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.))/((j_.) + (k_.)*(x_)),
x_Symbol]
:> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/
(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)]/
(a + b*x), x], x] - Dist[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)]/
(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r}, x]
&& NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

### Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol]
:> Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]
```

### Rule 30

```
Int[(x_)^(m_.), x_Symbol]
:> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.))/((f_.) + (g_.)*(x_)),
x_Symbol]
:> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^p)/g, x] - Dist[
(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^(p - 1))/
(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag + bgx} dx &= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2} + \frac{be}{c+dx}\right)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(ag+bgx)}{e(a+bx)} dx}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2} + \frac{be}{c+dx}\right)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(ag+bgx)}{a+bx} dx}{beg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{(bc-ad)e\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(ag+bgx)}{(a+bx)(c+dx)} dx}{beg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B(bc-ad)) \int \frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(ag+bgx)}{(a+bx)(c+dx)} dx}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B(bc-ad)) \int \left(\frac{d\left(-A-B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(ag+bgx)}{(bc-ad)(c+dx)}\right) dx}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(ag+bgx)}{a+bx} dx}{g} - \frac{(2Bd) \int \frac{\log(ag+bgx)}{a+bx} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \left(\frac{A \log(ag+bgx)}{a+bx} + \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) \log(ag+bgx)}{a+bx}\right) dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2AB) \int \frac{\log(ag+bgx)}{a+bx} dx}{g} - \frac{(2B^2) \int \frac{\log\left(\frac{e(a+bx)}{c+dx}\right) \log(ag+bgx)}{a+bx} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(ag + bgx)}{bg} + \frac{2AB \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} - \frac{B^2 \log\left(\frac{e(a+bx)}{c+dx}\right) \log(ag+bgx)}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(ag + bgx)}{bg} + \frac{2AB \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} - \frac{2B^2 \left(\log(a+bx) \log(ag+bgx) + \log(a+bx) \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} \\
&= -\frac{AB \log^2(g(a+bx))}{bg} + \frac{2B^2 \log(a+bx) \log(g(a+bx)) \log(-c-dx)}{bg} - \frac{B^2 \log(g(a+bx)) \log\left(\frac{e(a+bx)}{c+dx}\right)}{bg} \\
&= -\frac{AB \log^2(g(a+bx))}{bg} + \frac{2B^2 \log(a+bx) \log(g(a+bx)) \log(-c-dx)}{bg} + \frac{B^2 \log\left(-\frac{d(a+bx)}{bc-ad}\right)}{bg} \\
&= -\frac{AB \log^2(g(a+bx))}{bg} + \frac{B^2 \log^3(g(a+bx))}{3bg} - \frac{B^2 \log^2(a+bx) \log(-c-dx)}{bg} + \frac{2B^2 \log(a+bx) \log(g(a+bx)) \log(-c-dx)}{bg} \\
&= -\frac{AB \log^2(g(a+bx))}{bg} + \frac{B^2 \log^3(g(a+bx))}{3bg} - \frac{B^2 \log^2(a+bx) \log(-c-dx)}{bg} + \frac{2B^2 \log(a+bx) \log(g(a+bx)) \log(-c-dx)}{bg} \\
&= -\frac{AB \log^2(g(a+bx))}{bg} + \frac{B^2 \log^3(g(a+bx))}{3bg} - \frac{B^2 \log^2(a+bx) \log(-c-dx)}{bg} + \frac{2B^2 \log(a+bx) \log(g(a+bx)) \log(-c-dx)}{bg} \\
&= -\frac{AB \log^2(g(a+bx))}{bg} + \frac{B^2 \log^3(g(a+bx))}{3bg} - \frac{B^2 \log^2(a+bx) \log(-c-dx)}{bg} + \frac{2B^2 \log(a+bx) \log(g(a+bx)) \log(-c-dx)}{bg}
\end{aligned}$$

**Mathematica [A]** time = 0.604136, size = 250, normalized size = 1.95

$$-2AB\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right) + 2B^2\text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) \log\left(\frac{e(a+bx)}{c+dx}\right) + 2B^2\text{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right) + A^2 \log(a+bx) + 2AB \log\left(\frac{e(a+bx)}{c+dx}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2/(a\*g + b\*g\*x), x]

[Out] (A\*B\*Log[a/b + x]^2 + A^2\*Log[a + b\*x] - 2\*A\*B\*Log[a/b + x]\*Log[a + b\*x] + 2\*A\*B\*Log[c/d + x]\*Log[a + b\*x] - 2\*A\*B\*Log[c/d + x]\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)] + 2\*A\*B\*Log[a + b\*x]\*Log[(e\*(a + b\*x))/(c + d\*x)] - B^2\*Log[(-(b\*c + a\*d)/(d\*(a + b\*x)))\*Log[(e\*(a + b\*x))/(c + d\*x)]^2 - 2\*A\*B\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)] + 2\*B^2\*Log[(e\*(a + b\*x))/(c + d\*x)]\*PolyLog[2, (b\*(c + d\*x))/(d\*(a + b\*x))] + 2\*B^2\*PolyLog[3, (b\*(c + d\*x))/(d\*(a + b\*x))])/ (b\*g)

**Maple [B]** time = 0.069, size = 1186, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2/(b\*g\*x+a\*g), x)

[Out] d/g/(a\*d-b\*c)\*A^2/b\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*a-1/g/(a\*d-b\*c)\*A^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*c-d/g/(a\*d-b\*c)\*A^2/b\*ln(d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))-b\*e)\*a+1/g/(a\*d-b\*c)\*A^2\*ln(d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))-b\*e)\*c+1/3\*d/g/(a\*d-b\*c)\*B^2/b\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))^3\*a-1/3/g/(a\*d-b\*c)\*B^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))^3\*c-d/g/(a\*d-b\*c)\*B^2/b\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))^2\*ln(1-1/b/e\*d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c)))\*a+1/g/(a\*d-b\*c)\*B^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))^2\*ln(1-1/b/e\*d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c)))\*c-2\*d/g/(a\*d-b\*c)\*B^2/b\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*polylog(2,1/b/e\*d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c)))\*a+2/g/(a\*d-b\*c)\*B^2\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*polylog(2,1/b/e\*d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c)))\*c+2\*d/g/(a\*d-b\*c)\*B^2/b\*polylog(3,1/b/e\*d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c)))\*a-2/g/(a\*d-b\*c)\*B^2\*polylog(3,1/b/e\*d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c)))\*c+d/g/(a\*d-b\*c)\*A\*B/b\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))^2\*a-1/g/(a\*d-b\*c)\*A\*B\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))^2\*c-2\*d/g/(a\*d-b\*c)\*A\*B/b\*dilog(-(d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))-b\*e)/b/e)\*a+2/g/(a\*d-b\*c)\*A\*B\*dilog(-(d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))-b\*e)/b/e)\*c-2\*d/g/(a\*d-b\*c)\*A\*B/b\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*ln(-(d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))-b\*e)/b/e)\*a+2/g/(a\*d-b\*c)\*A\*B\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))\*ln(-(d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))-b\*e)/b/e)\*c

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{B^2 \log(bx+a) \log(dx+c)^2}{bg} + \frac{A^2 \log(bgx+ag)}{bg} - \int -\frac{B^2bc \log(e)^2 + 2ABbc \log(e) + (B^2bdx + B^2bc) \log(bx+a)^2}{bg} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(b\*g\*x+a\*g), x, algorithm="maxima")

[Out]  $B^2 \log(bx + a) \log(dx + c)^2 / (bg) + A^2 \log(bgx + ag) / (bg) - \text{integrate}(- (B^2 b^2 c \log(e)^2 + 2ABb^2 c \log(e) + (B^2 b^2 d^2 x + B^2 b^2 c) \log(bx + a)^2 + (B^2 b^2 d \log(e)^2 + 2ABb^2 d \log(e)) x + 2(B^2 b^2 c \log(e) + ABb^2 c + (B^2 b^2 d \log(e) + ABb^2 d) x) \log(bx + a) - 2(B^2 b^2 c \log(e) + ABb^2 c + (B^2 b^2 d \log(e) + ABb^2 d) x + (2B^2 b^2 d^2 x + (b^2 c + a^2 d) B^2) \log(bx + a)) \log(dx + c) / (b^2 d^2 g x^2 + a b^2 c g + (b^2 c^2 g + a b^2 d^2 g) x), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2AB \log\left(\frac{bex+ae}{dx+c}\right) + A^2}{bgx + ag}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="fricas")`

[Out] `integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x + c)) + A^2)/(b*g*x + a*g), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{bgx + ag} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algorithm="giac")`

[Out] `integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g), x)`

$$3.102 \quad \int \frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2} dx$$

**Optimal.** Leaf size=126

$$\frac{2B(c+dx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2(a+bx)(bc-ad)} - \frac{(c+dx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{g^2(a+bx)(bc-ad)} - \frac{2B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

[Out]  $(-2*B^2*(c+d*x))/((b*c-a*d)*g^2*(a+b*x)) - (2*B*(c+d*x)*(A+B*\text{Log}[(e*(a+b*x))/(c+d*x)]))/((b*c-a*d)*g^2*(a+b*x)) - ((c+d*x)*(A+B*\text{Log}[(e*(a+b*x))/(c+d*x)]^2))/((b*c-a*d)*g^2*(a+b*x))$

**Rubi [C]** time = 0.771787, antiderivative size = 470, normalized size of antiderivative = 3.73, number of steps used = 26, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2B^2 d \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg^2(bc-ad)} - \frac{2B^2 d \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bg^2(bc-ad)} - \frac{2Bd \log(a+bx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg^2(bc-ad)} - \frac{2B\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A+B*\text{Log}[(e*(a+b*x))/(c+d*x)])^2/(a*g+b*g*x)^2, x]$

[Out]  $(-2*B^2)/(b*g^2*(a+b*x)) - (2*B^2*d*\text{Log}[a+b*x])/((b*(b*c-a*d)*g^2) + (B^2*d*\text{Log}[a+b*x]^2)/(b*(b*c-a*d)*g^2) - (2*B*(A+B*\text{Log}[(e*(a+b*x))/(c+d*x)]))/((b*g^2*(a+b*x)) - (2*B*d*\text{Log}[a+b*x]*(A+B*\text{Log}[(e*(a+b*x))/(c+d*x)]))/((b*(b*c-a*d)*g^2) - (A+B*\text{Log}[(e*(a+b*x))/(c+d*x)]^2)/(b*g^2*(a+b*x)) + (2*B^2*d*\text{Log}[c+d*x])/((b*(b*c-a*d)*g^2) - (2*B^2*d*\text{Log}[-((d*(a+b*x))/(b*c-a*d))]*\text{Log}[c+d*x])/((b*(b*c-a*d)*g^2) + (2*B*d*(A+B*\text{Log}[(e*(a+b*x))/(c+d*x)])*\text{Log}[c+d*x])/((b*(b*c-a*d)*g^2) + (B^2*d*\text{Log}[c+d*x]^2)/(b*(b*c-a*d)*g^2) - (2*B^2*d*\text{Log}[a+b*x]*\text{Log}[(b*(c+d*x))/(b*c-a*d)])/((b*(b*c-a*d)*g^2) - (2*B^2*d*\text{PolyLog}[2, -((d*(a+b*x))/(b*c-a*d))])/((b*(b*c-a*d)*g^2) - (2*B^2*d*\text{PolyLog}[2, (b*(c+d*x))/(b*c-a*d)])/((b*(b*c-a*d)*g^2)$

**Rule 2525**

$\text{Int}[(a_.) + \text{Log}[(c_.)*(\text{RFX}_.)^{(p_.)}]* (b_.)^{(n_.)} * ((d_.) + (e_.)*(x_))^{(m_.)}], x\_Symbol] :> \text{Simp}[(d+e*x)^{(m+1)}*(a+b*\text{Log}[c*\text{RFX}^p])^n/(e*(m+1)), x] - \text{Dist}[(b^n*p)/(e*(m+1)), \text{Int}[\text{SimplifyIntegrand}[(d+e*x)^{(m+1)}*(a+b*\text{Log}[c*\text{RFX}^p])^{(n-1)}*D[\text{RFX}, x])/RFX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

**Rule 12**

$\text{Int}[(a_)*(u_), x\_Symbol] :> \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 2528**

$\text{Int}[(a_.) + \text{Log}[(c_.)*(\text{RFX}_.)^{(p_.)}]* (b_.)^{(n_.)} * (\text{RGx}_)], x\_Symbol] :> \text{With}[\{u = \text{ExpandIntegrand}[(a+b*\text{Log}[c*\text{RFX}^p])^n, \text{RGx}, x]\}, \text{Int}[u, x] /;$  SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunc



onQ[RGx, x] && IGtQ[n, 0]

#### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 2524

Int[((a\_) + Log[(c\_)\*(RFX\_)^(p\_)])\*(b\_)^(n\_)/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_)^(p\_)\*(RFX\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFX, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_)^(p\_)\*((f\_) + (g\_)\*(x\_))^(q\_), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_) + Log[(c\_)\*(x\_)]^(n\_))\*((b\_)/(x\_)), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_)/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))])\*(b\_)/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)]^(n\_)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^2} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a + bx)} + \frac{(2B) \int \frac{(bc-ad)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(a+bx)^2(c+dx)} dx}{bg} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a + bx)} + \frac{(2B(bc - ad)) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^2(c+dx)} dx}{bg^2} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a + bx)} + \frac{(2B(bc - ad)) \int \left(\frac{b\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)^2} - \frac{bd\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2(a+bx)} + \frac{d^2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3}\right) dx}{bg^2} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a + bx)} + \frac{(2B) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^2} dx}{g^2} - \frac{(2Bd) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx} dx}{(bc - ad)g^2} + \frac{(2Bd^2) \int dx}{(bc - ad)^3 g^2} \\
&= -\frac{2B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a + bx)} - \frac{2Bd \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc - ad)g^2} - \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a + bx)} \\
&= -\frac{2B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a + bx)} - \frac{2Bd \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc - ad)g^2} - \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a + bx)} \\
&= -\frac{2B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a + bx)} - \frac{2Bd \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc - ad)g^2} - \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a + bx)} \\
&= -\frac{2B^2}{bg^2(a + bx)} - \frac{2B^2 d \log(a + bx)}{b(bc - ad)g^2} - \frac{2B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a + bx)} - \frac{2Bd \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc - ad)g^2} \\
&= -\frac{2B^2}{bg^2(a + bx)} - \frac{2B^2 d \log(a + bx)}{b(bc - ad)g^2} - \frac{2B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a + bx)} - \frac{2Bd \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc - ad)g^2} \\
&= -\frac{2B^2}{bg^2(a + bx)} - \frac{2B^2 d \log(a + bx)}{b(bc - ad)g^2} + \frac{B^2 d \log^2(a + bx)}{b(bc - ad)g^2} - \frac{2B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a + bx)} - \frac{2Bd \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc - ad)g^2} \\
&= -\frac{2B^2}{bg^2(a + bx)} - \frac{2B^2 d \log(a + bx)}{b(bc - ad)g^2} + \frac{B^2 d \log^2(a + bx)}{b(bc - ad)g^2} - \frac{2B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a + bx)} - \frac{2Bd \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc - ad)g^2}
\end{aligned}$$

**Mathematica [C]** time = 0.473057, size = 314, normalized size = 2.49

$$\frac{B\left(-Bd(a+bx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+Bd(a+bx)\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)+2(bc-ad)\log(c+dx)\right)}{b^2g^2(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2/(a\*g + b\*g\*x)^2,x]

[Out] -(((A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2 + (B\*(2\*(b\*c - a\*d)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]) + 2\*d\*(a + b\*x)\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]) - 2\*d\*(a + b\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])\*Log[c + d\*x] + 2\*B\*(b\*c - a\*d + d\*(a + b\*x)\*Log[a + b\*x] - d\*(a + b\*x)\*Log[c + d\*x]) - B\*d\*(a + b\*x)\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c) + a\*d]) + B\*d\*(a + b\*x)\*((2\*Log[(d\*(a + b\*x))/(-b\*c) + a\*d] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/(b\*c - a\*d)/(b\*g^2\*(a + b\*x))

---

**Maple [B]** time = 0.049, size = 828, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int ((A+B*\ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2, x)$

[Out] 
$$\begin{aligned} & e*d/(a*d-b*c)^2/g^2*A^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*a-e/(a*d-b*c)^2 \\ & /g^2*A^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*b*c+2*e*d/(a*d-b*c)^2/g^2*A*B/ \\ & (b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a-2*e/( \\ & a*d-b*c)^2/g^2*A*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*\ln(b*e/d+(a*d-b*c)*e \\ & /d/(d*x+c))*b*c+2*e*d/(a*d-b*c)^2/g^2*A*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b* \\ & c)*a-2*e/(a*d-b*c)^2/g^2*A*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*b*c+e*d/(a \\ & *d-b*c)^2/g^2*B^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*\ln(b*e/d+(a*d-b*c)*e/ \\ & d/(d*x+c))^2*a-e/(a*d-b*c)^2/g^2*B^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*\ln \\ & (b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*b*c+2*e*d/(a*d-b*c)^2/g^2*B^2/(b*e/d+e/(d*x \\ & +c)*a-e/d/(d*x+c)*b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a-2*e/(a*d-b*c)^2/g^ \\ & 2*B^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b \\ & *c+2*e*d/(a*d-b*c)^2/g^2*B^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*a-2*e/(a*d \\ & -b*c)^2/g^2*B^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*b*c \end{aligned}$$

---

**Maxima [B]** time = 1.28542, size = 562, normalized size = 4.46

$$-2 \left( \frac{1}{b^2 g^2 x + a b g^2} + \frac{d \log(bx + a)}{(b^2 c - a b d) g^2} - \frac{d \log(dx + c)}{(b^2 c - a b d) g^2} \right) \log \left( \frac{b e x}{d x + c} + \frac{a e}{d x + c} \right) - \frac{(b d x + a d) \log(bx + a)^2 + (b d x + a d) \log(dx + c)^2}{(b^2 c - a b d) g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((A+B*\log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2, x, \text{algorithm}="maxima")$

[Out] 
$$\begin{aligned} & -(2*(1/(b^2*g^2*x + a*b*g^2) + d*\log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*\log \\ & (d*x + c)/((b^2*c - a*b*d)*g^2))*\log(b*e*x/(d*x + c) + a*e/(d*x + c)) - ((b \\ & *d*x + a*d)*\log(b*x + a)^2 + (b*d*x + a*d)*\log(d*x + c)^2 - 2*b*c + 2*a*d - \\ & 2*(b*d*x + a*d)*\log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*\log(b*x + a) \\ & )*\log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))* \\ & B^2 - 2*A*B*(\log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1 \\ & /(b^2*g^2*x + a*b*g^2) + d*\log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*\log(d*x + \\ & c)/((b^2*c - a*b*d)*g^2)) - B^2*\log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^ \\ & 2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2) \end{aligned}$$

---

**Fricas [A]** time = 1.02027, size = 319, normalized size = 2.53

$$\frac{(A^2 + 2AB + 2B^2)bc - (A^2 + 2AB + 2B^2)ad + (B^2bdx + B^2bc) \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2((AB + B^2)bdx + (AB + B^2)bc) \log\left(\frac{bex+ae}{dx+c}\right)}{(b^3c - ab^2d)g^2x + (ab^2c - a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((A+B*\log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2, x, \text{algorithm}="fricas")$

[Out]  $-\left((A^2 + 2AB + 2B^2)bc - (A^2 + 2AB + 2B^2)ad + (B^2bdx + B^2bc)\log\left(\frac{bex + ae}{dx + c}\right)^2 + 2((AB + B^2)bdx + (AB + B^2)c)\log\left(\frac{bex + ae}{dx + c}\right)\right) / \left((b^3c - ab^2d)g^2x + (ab^2c - a^2bd)g^2\right)$

**Sympy [B]** time = 4.3354, size = 432, normalized size = 3.43

$$\frac{2Bd(A+B)\log\left(x + \frac{2ABad^2+2ABbcd+2B^2ad^2+2B^2bcd-\frac{2Ba^2d^3(A+B)}{ad-bc}+\frac{4Babcd^2(A+B)}{ad-bc}-\frac{2Bb^2c^2d(A+B)}{ad-bc}}{4ABbd^2+4B^2bd^2}\right)}{bg^2(ad-bc)} + \frac{2Bd(A+B)\log\left(x + \frac{2ABad^2+2ABbcd+2B^2ad^2+2B^2bcd-\frac{2Ba^2d^3(A+B)}{ad-bc}+\frac{4Babcd^2(A+B)}{ad-bc}-\frac{2Bb^2c^2d(A+B)}{ad-bc}}{4ABbd^2+4B^2bd^2}\right)}{bg^2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2/(b\*g\*x+a\*g)\*\*2,x)

[Out]  $-2Bd(A+B)\log(x + (2ABad^2 + 2ABbcd + 2B^2ad^2 + 2B^2bcd - 2B^2a^2d^3(A+B)/(ad-bc) + 4Babcd^2(A+B)/(ad-bc) - 2Bb^2c^2d(A+B)/(ad-bc)) / (4ABbd^2 + 4B^2bd^2)) / (bg^2(ad-bc)) + 2Bd(A+B)\log(x + (2ABad^2 + 2ABbcd + 2B^2ad^2 + 2B^2bcd + 2B^2a^2d^3(A+B)/(ad-bc) - 4Babcd^2(A+B)/(ad-bc) + 2Bb^2c^2d(A+B)/(ad-bc)) / (4ABbd^2 + 4B^2bd^2)) / (bg^2(ad-bc)) + (-2AB - 2B^2)\log(e(a+bx)/(c+dx)) / (abg^2 + b^2g^2x) + (B^2c + B^2dx)\log(e(a+bx)/(c+dx)) / (a^2d^2g^2 - abcg^2 + abd^2g^2x - b^2c^2g^2x) - (A^2 + 2AB + 2B^2) / (abg^2 + b^2g^2x)$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{bx+a}{dx+c}\right) + A\right)^2}{(bgx + ag)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(b\*g\*x+a\*g)^2,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2/(b\*g\*x + a\*g)^2, x)

$$3.103 \quad \int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3} dx$$

**Optimal.** Leaf size=268

$$\frac{bB(c+dx)^2 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{2g^3(a+bx)^2(bc-ad)^2} + \frac{2Bd(c+dx) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{g^3(a+bx)(bc-ad)^2} - \frac{b(c+dx)^2 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)^2}{2g^3(a+bx)^2(bc-ad)^2} + \frac{d(c+dx)}{g^3}$$

[Out]  $(2*B^2*d*(c+d*x))/((b*c-a*d)^2*g^3*(a+b*x)) - (b*B^2*(c+d*x)^2)/(4*(b*c-a*d)^2*g^3*(a+b*x)^2) + (2*B*d*(c+d*x)*(A+B*Log[(e*(a+b*x))/(c+d*x)]))/((b*c-a*d)^2*g^3*(a+b*x)) - (b*B*(c+d*x)^2*(A+B*Log[(e*(a+b*x))/(c+d*x)]))/(2*(b*c-a*d)^2*g^3*(a+b*x)^2) + (d*(c+d*x)*(A+B*Log[(e*(a+b*x))/(c+d*x)]^2))/((b*c-a*d)^2*g^3*(a+b*x)) - (b*(c+d*x)^2*(A+B*Log[(e*(a+b*x))/(c+d*x)]^2))/(2*(b*c-a*d)^2*g^3*(a+b*x)^2)$

**Rubi [C]** time = 0.90955, antiderivative size = 577, normalized size of antiderivative = 2.15, number of steps used = 30, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2 d^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg^3(bc-ad)^2} + \frac{B^2 d^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bg^3(bc-ad)^2} + \frac{Bd^2 \log(a+bx) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{bg^3(bc-ad)^2} - \frac{Bd^2 \log(c+dx) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{bg^3(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2/(a\*g + b\*g\*x)^3, x]

[Out]  $-B^2/(4*b*g^3*(a+b*x)^2) + (3*B^2*d)/(2*b*(b*c-a*d)*g^3*(a+b*x)) + (3*B^2*d^2*Log[a+b*x])/(2*b*(b*c-a*d)^2*g^3) - (B^2*d^2*Log[a+b*x]^2)/(2*b*(b*c-a*d)^2*g^3) - (B*(A+B*Log[(e*(a+b*x))/(c+d*x)]))/(2*b*g^3*(a+b*x)^2) + (B*d*(A+B*Log[(e*(a+b*x))/(c+d*x)]))/(b*(b*c-a*d)*g^3*(a+b*x)) + (B*d^2*Log[a+b*x]*(A+B*Log[(e*(a+b*x))/(c+d*x)]))/(b*(b*c-a*d)^2*g^3) - (A+B*Log[(e*(a+b*x))/(c+d*x)]^2)/(2*b*g^3*(a+b*x)^2) - (3*B^2*d^2*Log[c+d*x])/(2*b*(b*c-a*d)^2*g^3) + (B^2*d^2*Log[-((d*(a+b*x))/(b*c-a*d))*Log[c+d*x]])/(b*(b*c-a*d)^2*g^3) - (B*d^2*(A+B*Log[(e*(a+b*x))/(c+d*x)]*Log[c+d*x])/(b*(b*c-a*d)^2*g^3) - (B^2*d^2*Log[c+d*x]^2)/(2*b*(b*c-a*d)^2*g^3) + (B^2*d^2*Log[a+b*x]*Log[(b*(c+d*x))/(b*c-a*d)]/(b*(b*c-a*d)^2*g^3) + (B^2*d^2*PolyLog[2, -(d*(a+b*x))/(b*c-a*d)]/(b*(b*c-a*d)^2*g^3) + (B^2*d^2*PolyLog[2, (b*(c+d*x))/(b*c-a*d)]/(b*(b*c-a*d)^2*g^3)$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 44

```
Int[((a_) + (b.)*(x.))^(m.)*((c.) + (d.)*(x.))^(n.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x.)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x.))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x.))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x.))^(q.), x_Symbol] := Dist[1/e, Subst[Int[(f*x)/d]^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x.)^(n_.)]*(b_.))/(x.), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x.))^(n_.)]*(b_.))/((f_.) + (g_.)*(x.
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x.))]*(b_.))/((f_.) + (g_.)*(x.)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x.)^(n.))]/(x.), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

## Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^3} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a + bx)^2} + \frac{B \int \frac{(bc-ad)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g^2(a+bx)^3(c+dx)} dx}{bg} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a + bx)^2} + \frac{(B(bc - ad)) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^3(c+dx)} dx}{bg^3} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a + bx)^2} + \frac{(B(bc - ad)) \int \left(\frac{b\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)^3} - \frac{bd\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2(a+bx)^2} + \frac{bd^2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3}\right) dx}{bg^3} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a + bx)^2} + \frac{B \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^3} dx}{g^3} + \frac{(Bd^2) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx} dx}{(bc - ad)^2g^3} - \frac{(Bd^3) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} dx}{b} \\
&= -\frac{B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a + bx)^2} + \frac{Bd\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc - ad)g^3(a + bx)} + \frac{Bd^2 \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc - ad)^2g^3} \\
&= -\frac{B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a + bx)^2} + \frac{Bd\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc - ad)g^3(a + bx)} + \frac{Bd^2 \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc - ad)^2g^3} \\
&= -\frac{B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a + bx)^2} + \frac{Bd\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc - ad)g^3(a + bx)} + \frac{Bd^2 \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b(bc - ad)^2g^3} \\
&= -\frac{B^2}{4bg^3(a + bx)^2} + \frac{3B^2d}{2b(bc - ad)g^3(a + bx)} + \frac{3B^2d^2 \log(a + bx)}{2b(bc - ad)^2g^3} - \frac{B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a + bx)^2} \\
&= -\frac{B^2}{4bg^3(a + bx)^2} + \frac{3B^2d}{2b(bc - ad)g^3(a + bx)} + \frac{3B^2d^2 \log(a + bx)}{2b(bc - ad)^2g^3} - \frac{B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a + bx)^2} \\
&= -\frac{B^2}{4bg^3(a + bx)^2} + \frac{3B^2d}{2b(bc - ad)g^3(a + bx)} + \frac{3B^2d^2 \log(a + bx)}{2b(bc - ad)^2g^3} - \frac{B^2d^2 \log^2(a + bx)}{2b(bc - ad)^2g^3} - \frac{B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a + bx)^2} \\
&= -\frac{B^2}{4bg^3(a + bx)^2} + \frac{3B^2d}{2b(bc - ad)g^3(a + bx)} + \frac{3B^2d^2 \log(a + bx)}{2b(bc - ad)^2g^3} - \frac{B^2d^2 \log^2(a + bx)}{2b(bc - ad)^2g^3} - \frac{B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a + bx)^2}
\end{aligned}$$

**Mathematica [C]** time = 0.516738, size = 443, normalized size = 1.65

$$\frac{B\left(2Bd^2(a+bx)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)-2Bd^2(a+bx)^2\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)\right)}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2/(a\*g + b\*g\*x)^3,x]

[Out] -(2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2 + (B\*(2\*(b\*c - a\*d)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])) + 4\*d\*(-(b\*c) + a\*d)\*(a + b\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])) - 4\*d^2\*(a + b\*x)^2\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])) + 4\*d^2\*(a + b\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))\*Log[c + d\*x] - 4\*B\*d\*(a + b\*x)\*(b\*c - a\*d + d\*(a + b\*x))\*Log[a + b\*x] - d\*(a + b\*x)\*Log[c + d\*x]) + B\*((b\*c - a\*d)^2 + 2\*d\*(-(b\*c) + a\*d)\*(a + b\*x) - 2\*d^2\*(a + b\*x)^2\*Log[a + b\*x] + 2\*d^2\*(a + b\*x)^2\*Log[c + d\*x]) + 2\*B\*d^2\*(a +

$$b*x)^2*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-b*c + a*d)]) - 2*B*d^2*(a + b*x)^2*((2*\text{Log}[(d*(a + b*x))/(-b*c + a*d)] - \text{Log}[c + d*x])* \text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2/(4*b*g^3*(a + b*x)^2)$$

**Maple [B]** time = 0.049, size = 1715, normalized size = 6.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)`

[Out] 
$$\begin{aligned} & e^d \frac{d^2}{(a*d-b*c)^3} \frac{1}{g^3} \frac{A^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} * a-e*d/(a*d-b*c) \\ & \frac{c^3}{g^3} \frac{A^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} * b*c - 1/2 * e^2 * d / (a*d-b*c)^3 / \\ & \frac{g^3}{g^3} \frac{A^2 * b}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} ^{2*a+1/2} * e^2 / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{A^2 * b^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} ^{2*c+2} * e^d \frac{2}{(a*d-b*c)^3} / \frac{g^3}{g^3} \frac{A * B}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} * \ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)) * a-2 * e * d / \\ & (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{A * B}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} * \ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)) * b*c + 2 * e * d^2 / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{A * B}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} * a-2 * e * d / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{A * B}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} * b*c - \\ & e^2 * d / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{A * B * b}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} ^{2 * \ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))} * a + e^2 / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{A * B * b^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} ^{2 * \ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))} * c - 1/2 * e^2 * d / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{A * B * b^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} ^{2 * a + 1/2} * e^2 / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{A * B * b^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} ^{2 * c + e} * d^2 / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{B^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} * \ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)) ^{2 * a - e * d / (a*d-b*c)^3} / \frac{g^3}{g^3} \frac{B^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} * \ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)) ^{2 * b * c + 2 * e * d^2 / (a*d-b*c)^3} / \frac{g^3}{g^3} \frac{B^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} * \ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)) * a - 2 * e * d / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{B^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} * \ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)) * b * c + 2 * e * d^2 / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{B^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} * a - 2 * e * d / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{B^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} * b * c - 1/2 * e^2 * d / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{B^2 * b}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} ^{2 * \ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))} ^{2 * a + 1/2} * e^2 / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{B^2 * b^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} ^{2 * \ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))} ^{2 * c - 1/2} * e^2 * d / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{B^2 * b}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} ^{2 * \ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))} * a + 1/2 * e^2 / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{B^2 * b^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} ^{2 * \ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))} * c - 1/4 * e^2 * d / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{B^2 * b}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} ^{2 * a + 1/4} * e^2 / (a*d-b*c)^3 / \frac{g^3}{g^3} \frac{B^2 * b^2}{(b*e/d+e/(d*x+c))*a-e/d/(d*x+c)*b*c} ^{2 * c} \end{aligned}$$

**Maxima [B]** time = 1.46657, size = 1145, normalized size = 4.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & 1/4 * (2 * ((2 * b * d * x - b * c + 3 * a * d) / ((b^4 * c - a * b^3 * d) * g^3 * x^2 + 2 * (a * b^3 * c - a^2 * b^2 * d) * g^3 * x + (a^2 * b^2 * c - a^3 * b * d) * g^3) + 2 * d^2 * \log(b * x + a) / ((b^3 * c^2 - 2 * a * b^2 * c * d + a^2 * b * d^2) * g^3) - 2 * d^2 * \log(d * x + c) / ((b^3 * c^2 - 2 * a * b^2 * c \end{aligned}$$



$$\begin{aligned} & *d + a^2*b*d^2)*g^3))*\log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (b^2*c^2 - 8*a \\ & *b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(b*x + a)^2 \\ & + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(d*x + c)^2 - 6*(b^2*c*d - a* \\ & b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(b*x + a) + 2*(3*b^2* \\ & d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2) \\ & *\log(b*x + a)*\log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d \\ & ^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c \\ & ^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 + 1/2*A*B*((2*b*d*x - \\ & b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + \\ & (a^2*b^2*c - a^3*b*d)*g^3) - 2*\log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^3*g^ \\ & 3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*\log(b*x + a)/((b^3*c^2 - 2*a*b^2 \\ & *c*d + a^2*b*d^2)*g^3) - 2*d^2*\log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b \\ & *d^2)*g^3)) - 1/2*B^2*\log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^3*g^3*x^2 + \\ & 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b* \\ & g^3) \end{aligned}$$

**Fricas [A]** time = 1.0926, size = 767, normalized size = 2.86

$$\frac{(2A^2 + 2AB + B^2)b^2c^2 - 4(A^2 + 2AB + 2B^2)abcd + (2A^2 + 6AB + 7B^2)a^2d^2 - 2(B^2b^2d^2x^2 + 2B^2abd^2x - B^2b^2d^2)}{4((b^5c^2 - 2ab^4d^2x^2 + 2a^2b^3cd^2x - a^3b^2d^2x^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(b\*g\*x+a\*g)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/4*((2*A^2 + 2*A*B + B^2)*b^2*c^2 - 4*(A^2 + 2*A*B + 2*B^2)*a*b*c*d + (2* \\ & A^2 + 6*A*B + 7*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b \\ & ^2*c^2 + 2*B^2*a*b*c*d)*\log((b*e*x + a*e)/(d*x + c))^2 - 2*((2*A*B + 3*B^2) \\ & *b^2*c*d - (2*A*B + 3*B^2)*a*b*d^2)*x - 2*((2*A*B + 3*B^2)*b^2*d^2*x^2 - (2 \\ & *A*B + B^2)*b^2*c^2 + 4*(A*B + B^2)*a*b*c*d + 2*(B^2*b^2*c*d + 2*(A*B + B^2 \\ & )*a*b*d^2)*x)*\log((b*e*x + a*e)/(d*x + c)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b \\ & ^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2* \\ & b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3) \end{aligned}$$

**Sympy [B]** time = 7.64257, size = 892, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2/(b\*g\*x+a\*g)\*\*3,x)

[Out] 
$$\begin{aligned} & -B*d**2*(2*A + 3*B)*\log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 3*B**2*a*d**3 \\ & + 3*B**2*b*c*d**2 - B*a**3*d**5*(2*A + 3*B)/(a*d - b*c)**2 + 3*B*a**2*b*c*d \\ & **4*(2*A + 3*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3*(2*A + 3*B)/(a*d - b* \\ & c)**2 + B*b**3*c**3*d**2*(2*A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 6*B**2 \\ & *b*d**3))/(2*b*g**3*(a*d - b*c)**2) + B*d**2*(2*A + 3*B)*\log(x + (2*A*B*a*d \\ & **3 + 2*A*B*b*c*d**2 + 3*B**2*a*d**3 + 3*B**2*b*c*d**2 + B*a**3*d**5*(2*A + \\ & 3*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4*(2*A + 3*B)/(a*d - b*c)**2 + 3*B*a \\ & *b**2*c**2*d**3*(2*A + 3*B)/(a*d - b*c)**2 - B*b**3*c**3*d**2*(2*A + 3*B)/( \\ & a*d - b*c)**2)/(4*A*B*b*d**3 + 6*B**2*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + \\ & (2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*\log(e*(a \end{aligned}$$

```

+ b*x)/(c + d*x))**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*
g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*
g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**
3*x**2) + (-2*A*B*a*d + 2*A*B*b*c - 3*B**2*a*d + B**2*b*c - 2*B**2*b*d*x)*l
og(e*(a + b*x)/(c + d*x))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + 4*a**2*b*
*2*d*g**3*x - 4*a*b**3*c*g**3*x + 2*a*b**3*d*g**3*x**2 - 2*b**4*c*g**3*x**2
) - (2*A**2*a*d - 2*A**2*b*c + 6*A*B*a*d - 2*A*B*b*c + 7*B**2*a*d - B**2*b*
c + x*(4*A*B*b*d + 6*B**2*b*d))/(4*a**3*b*d*g**3 - 4*a**2*b**2*c*g**3 + x**
2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 - 8*a*b**3*c*g*
*3))

```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{bx+a}{dx+c}\right) + A\right)^2}{(bgx + ag)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g)^3, x)
```

$$3.104 \quad \int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4} dx$$

**Optimal.** Leaf size=418

$$\frac{b^2(c+dx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)^2}{3g^4(a+bx)^3(bc-ad)^3} - \frac{2b^2B(c+dx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{9g^4(a+bx)^3(bc-ad)^3} - \frac{d^2(c+dx) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)^2}{g^4(a+bx)(bc-ad)^3} - \frac{2Bd^2}{3g^4}$$

[Out]  $(-2*B^2*d^2*(c+d*x))/((b*c-a*d)^3*g^4*(a+b*x)) + (b*B^2*d*(c+d*x)^2)/((2*(b*c-a*d)^3*g^4*(a+b*x)^2) - (2*b^2*B^2*(c+d*x)^3)/(27*(b*c-a*d)^3*g^4*(a+b*x)^3) - (2*B*d^2*(c+d*x)*(A+B*Log[(e*(a+b*x))/(c+d*x]]))/((b*c-a*d)^3*g^4*(a+b*x)) + (b*B*d*(c+d*x)^2*(A+B*Log[(e*(a+b*x))/(c+d*x]])))/((b*c-a*d)^3*g^4*(a+b*x)^2) - (2*b^2*B*(c+d*x)^3*(A+B*Log[(e*(a+b*x))/(c+d*x]]))/((9*(b*c-a*d)^3*g^4*(a+b*x)^3) - (d^2*(c+d*x)*(A+B*Log[(e*(a+b*x))/(c+d*x]]))^2)/((b*c-a*d)^3*g^4*(a+b*x)) + (b*d*(c+d*x)^2*(A+B*Log[(e*(a+b*x))/(c+d*x]]))^2)/((b*c-a*d)^3*g^4*(a+b*x)^2) - (b^2*(c+d*x)^3*(A+B*Log[(e*(a+b*x))/(c+d*x]]))^2)/(3*(b*c-a*d)^3*g^4*(a+b*x)^3)$

**Rubi [C]** time = 1.05899, antiderivative size = 680, normalized size of antiderivative = 1.63, number of steps used = 34, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2B^2d^3 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{3bg^4(bc-ad)^3} - \frac{2B^2d^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3bg^4(bc-ad)^3} - \frac{2Bd^3 \log(a+bx) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{3bg^4(bc-ad)^3} + \frac{2Bd^3 \log(c+dx)}{3g^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2/(a\*g + b\*g\*x)^4, x]

[Out]  $(-2*B^2)/(27*b*g^4*(a+b*x)^3) + (5*B^2*d)/(18*b*(b*c-a*d)*g^4*(a+b*x)^2) - (11*B^2*d^2)/(9*b*(b*c-a*d)^2*g^4*(a+b*x)) - (11*B^2*d^3*Log[a+b*x])/(9*b*(b*c-a*d)^3*g^4) + (B^2*d^3*Log[a+b*x]^2)/(3*b*(b*c-a*d)^3*g^4) - (2*B*(A+B*Log[(e*(a+b*x))/(c+d*x]]))/(9*b*g^4*(a+b*x)^3) + (B*d*(A+B*Log[(e*(a+b*x))/(c+d*x]]))/(3*b*(b*c-a*d)*g^4*(a+b*x)^2) - (2*B*d^2*(A+B*Log[(e*(a+b*x))/(c+d*x]]))/(3*b*(b*c-a*d)^2*g^4*(a+b*x)) - (2*B*d^3*Log[a+b*x]*(A+B*Log[(e*(a+b*x))/(c+d*x]]))/(3*b*(b*c-a*d)^3*g^4) - (A+B*Log[(e*(a+b*x))/(c+d*x]]))^2/(3*b*g^4*(a+b*x)^3) + (11*B^2*d^3*Log[c+d*x])/(9*b*(b*c-a*d)^3*g^4) - (2*B^2*d^3*Log[-((d*(a+b*x))/(b*c-a*d))*Log[c+d*x]])/(3*b*(b*c-a*d)^3*g^4) + (2*B*d^3*(A+B*Log[(e*(a+b*x))/(c+d*x]])*Log[c+d*x])/(3*b*(b*c-a*d)^3*g^4) + (B^2*d^3*Log[c+d*x]^2)/(3*b*(b*c-a*d)^3*g^4) - (2*B^2*d^3*Log[a+b*x]*Log[(b*(c+d*x))/(b*c-a*d)])/(3*b*(b*c-a*d)^3*g^4) - (2*B^2*d^3*PolyLog[2, -((d*(a+b*x))/(b*c-a*d))])/(3*b*(b*c-a*d)^3*g^4) - (2*B^2*d^3*PolyLog[2, (b*(c+d*x))/(b*c-a*d)])/(3*b*(b*c-a*d)^3*g^4)$

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^n\*(d\_.) + (e\_.)\*(x\_)]^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 2528

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 2524

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]

Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(Rfx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :-Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^4} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg^4(a + bx)^3} + \frac{(2B) \int \frac{(bc-ad)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g^3(a+bx)^4(c+dx)} dx}{3bg} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg^4(a + bx)^3} + \frac{(2B(bc - ad)) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^4(c+dx)} dx}{3bg^4} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg^4(a + bx)^3} + \frac{(2B(bc - ad)) \int \left(\frac{b\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2(a+bx)^3} + \frac{bd^2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3(a+bx)^2} - \frac{bd^3\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4(a+bx)}\right) dx}{3b^2g^4} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg^4(a + bx)^3} + \frac{(2B) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^4} dx}{3g^4} - \frac{(2Bd^3) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx} dx}{3(bc - ad)^3g^4} + \frac{(2Bd^2) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^2} dx}{3(bc - ad)^2g^4} - \frac{(2Bd) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx} dx}{3(bc - ad)g^4} \\
&= -\frac{2B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9bg^4(a + bx)^3} + \frac{Bd\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc - ad)g^4(a + bx)^2} - \frac{2Bd^2\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc - ad)^2g^4(a + bx)} - \frac{2Bd^3\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc - ad)^3g^4(a + bx)} \\
&= -\frac{2B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9bg^4(a + bx)^3} + \frac{Bd\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc - ad)g^4(a + bx)^2} - \frac{2Bd^2\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc - ad)^2g^4(a + bx)} - \frac{2Bd^3\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc - ad)^3g^4(a + bx)} \\
&= -\frac{2B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9bg^4(a + bx)^3} + \frac{Bd\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc - ad)g^4(a + bx)^2} - \frac{2Bd^2\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc - ad)^2g^4(a + bx)} - \frac{2Bd^3\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b(bc - ad)^3g^4(a + bx)} \\
&= -\frac{2B^2}{27bg^4(a + bx)^3} + \frac{5B^2d}{18b(bc - ad)g^4(a + bx)^2} - \frac{11B^2d^2}{9b(bc - ad)^2g^4(a + bx)} - \frac{11B^2d^3 \log(a + bx)}{9b(bc - ad)^3g^4(a + bx)} \\
&= -\frac{2B^2}{27bg^4(a + bx)^3} + \frac{5B^2d}{18b(bc - ad)g^4(a + bx)^2} - \frac{11B^2d^2}{9b(bc - ad)^2g^4(a + bx)} - \frac{11B^2d^3 \log(a + bx)}{9b(bc - ad)^3g^4(a + bx)} \\
&= -\frac{2B^2}{27bg^4(a + bx)^3} + \frac{5B^2d}{18b(bc - ad)g^4(a + bx)^2} - \frac{11B^2d^2}{9b(bc - ad)^2g^4(a + bx)} - \frac{11B^2d^3 \log(a + bx)}{9b(bc - ad)^3g^4(a + bx)} \\
&= -\frac{2B^2}{27bg^4(a + bx)^3} + \frac{5B^2d}{18b(bc - ad)g^4(a + bx)^2} - \frac{11B^2d^2}{9b(bc - ad)^2g^4(a + bx)} - \frac{11B^2d^3 \log(a + bx)}{9b(bc - ad)^3g^4(a + bx)}
\end{aligned}$$

**Mathematica [C]** time = 0.766238, size = 585, normalized size = 1.4

$$\frac{B\left(-18Bd^3(a+bx)^3\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+18Bd^3(a+bx)^3\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)\right)}{27b^2g^4(a+bx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2/(a\*g + b\*g\*x)^4,x]

[Out] -(18\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2 + (B\*(12\*(b\*c - a\*d)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])) - 18\*d\*(b\*c - a\*d)^2\*(a + b\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])) + 11\*d^2\*(b\*c - a\*d)^2\*(a + b\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])) - 11\*d^3\*(b\*c - a\*d)\*log(a + b\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])))/27b^2g^4(a + b\*x)^3

$$\begin{aligned} & a + b*x)) / (c + d*x))] + 36*d^2*(b*c - a*d)*(a + b*x)^2*(A + B*\text{Log}[(e*(a + b \\ & *x)) / (c + d*x)]) + 36*d^3*(a + b*x)^3*\text{Log}[a + b*x]*(A + B*\text{Log}[(e*(a + b*x)) \\ & / (c + d*x)]) - 36*d^3*(a + b*x)^3*(A + B*\text{Log}[(e*(a + b*x)) / (c + d*x)])*\text{Log}[ \\ & c + d*x] + 36*B*d^2*(a + b*x)^2*(b*c - a*d + d*(a + b*x)*\text{Log}[a + b*x] - d*( \\ & a + b*x)*\text{Log}[c + d*x]) - 9*B*d*(a + b*x)*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d \\ & )*(a + b*x) - 2*d^2*(a + b*x)^2*\text{Log}[a + b*x] + 2*d^2*(a + b*x)^2*\text{Log}[c + d* \\ & x]) + 2*B*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d \\ & )*(a + b*x)^2 + 6*d^3*(a + b*x)^3*\text{Log}[a + b*x] - 6*d^3*(a + b*x)^3*\text{Log}[c + \\ & d*x]) - 18*B*d^3*(a + b*x)^3*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d* \\ & x)) / (b*c - a*d)])) - 2*\text{PolyLog}[2, (d*(a + b*x)) / (- (b*c) + a*d)] + 18*B*d^3* \\ & (a + b*x)^3*((2*\text{Log}[(d*(a + b*x)) / (- (b*c) + a*d)] - \text{Log}[c + d*x])*\text{Log}[c + d \\ & *x] + 2*\text{PolyLog}[2, (b*(c + d*x)) / (b*c - a*d)])) / (b*c - a*d)^3 / (54*b*g^4*( \\ & a + b*x)^3) \end{aligned}$$

**Maple [B]** time = 0.05, size = 2624, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((A+B*\ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4, x)$

[Out] 
$$\begin{aligned} & -2*e*d^2/(a*d-b*c)^4/g^4*A*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*\ln(b*e/d+( \\ & a*d-b*c)*e/d/(d*x+c)*b*c+2*e*d^3/(a*d-b*c)^4/g^4*A*B/(b*e/d+e/(d*x+c)*a-e/ \\ & d/(d*x+c)*b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a-e^2*d^2/(a*d-b*c)^4/g^4*A* \\ & B*b/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*a+e^2*d/(a*d-b*c)^4/g^4*A*B*b^2/( \\ & b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*c+2/9*e^3*d/(a*d-b*c)^4/g^4*A*B*b^2/(b \\ & *e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*a-2*e*d^2/(a*d-b*c)^4/g^4*A*B/(b*e/d+e/ \\ & (d*x+c)*a-e/d/(d*x+c)*b*c)*b*c+e*d^3/(a*d-b*c)^4/g^4*A^2/(b*e/d+e/(d*x+c)*a \\ & -e/d/(d*x+c)*b*c)*a+2*e*d^3/(a*d-b*c)^4/g^4*B^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x \\ & +c)*b*c)*a-2/27*e^3/(a*d-b*c)^4/g^4*B^2*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)* \\ & b*c)^3*c+e^2*d/(a*d-b*c)^4/g^4*A^2*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^ \\ & 2*c+1/3*e^3*d/(a*d-b*c)^4/g^4*A^2*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3 \\ & *a-2/9*e^3/(a*d-b*c)^4/g^4*B^2*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*\ln \\ & (b*e/d+(a*d-b*c)*e/d/(d*x+c))*c-1/3*e^3/(a*d-b*c)^4/g^4*A^2*b^3/(b*e/d+e/(d \\ & *x+c)*a-e/d/(d*x+c)*b*c)^3*c+2*e^2*d/(a*d-b*c)^4/g^4*A*B*b^2/(b*e/d+e/(d*x+ \\ & c)*a-e/d/(d*x+c)*b*c)^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*c+2/3*e^3*d/(a*d-b* \\ & c)^4/g^4*A*B*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*\ln(b*e/d+(a*d-b*c)*e \\ & /d/(d*x+c))*a-2*e^2*d^2/(a*d-b*c)^4/g^4*A*B*b/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c \\ & )*b*c)^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a-e*d^2/(a*d-b*c)^4/g^4*A^2/(b*e/d \\ & +e/(d*x+c)*a-e/d/(d*x+c)*b*c)*b*c-2/9*e^3/(a*d-b*c)^4/g^4*A*B*b^3/(b*e/d+e/ \\ & (d*x+c)*a-e/d/(d*x+c)*b*c)^3*c+2*e*d^3/(a*d-b*c)^4/g^4*B^2/(b*e/d+e/(d*x+c) \\ & *a-e/d/(d*x+c)*b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a+e*d^3/(a*d-b*c)^4/g^4 \\ & *B^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2* \\ & a-1/3*e^3/(a*d-b*c)^4/g^4*B^2*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*\ln( \\ & b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*c+1/2*e^2*d/(a*d-b*c)^4/g^4*B^2*b^2/(b*e/d+e \\ & / (d*x+c)*a-e/d/(d*x+c)*b*c)^2*c+2/27*e^3*d/(a*d-b*c)^4/g^4*B^2*b^2/(b*e/d+e \\ & / (d*x+c)*a-e/d/(d*x+c)*b*c)^3*a+2*e*d^3/(a*d-b*c)^4/g^4*A*B/(b*e/d+e/(d*x+c \\ & )*a-e/d/(d*x+c)*b*c)*a-1/2*e^2*d^2/(a*d-b*c)^4/g^4*B^2*b/(b*e/d+e/(d*x+c)*a \\ & -e/d/(d*x+c)*b*c)^2*a-e*d^2/(a*d-b*c)^4/g^4*B^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x \\ & +c)*b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*b*c+e^2*d/(a*d-b*c)^4/g^4*B^2*b^ \\ & 2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*c \\ & -e^2*d^2/(a*d-b*c)^4/g^4*B^2*b/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*\ln(b*e \\ & /d+(a*d-b*c)*e/d/(d*x+c))*a+e^2*d/(a*d-b*c)^4/g^4*B^2*b^2/(b*e/d+e/(d*x+c)* \\ & a-e/d/(d*x+c)*b*c)^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*c+1/3*e^3*d/(a*d-b*c)^ \\ & 4/g^4*B^2*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*\ln(b*e/d+(a*d-b*c)*e/d/ \\ & (d*x+c))^2*a+2/9*e^3*d/(a*d-b*c)^4/g^4*B^2*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+ \end{aligned}$$

$$c) * b * c)^3 * \ln(b * e / d + (a * d - b * c) * e / d / (d * x + c)) * a - e^{2 * d^2} / (a * d - b * c)^4 / g^4 * B^2 * b / (b * e / d + e / (d * x + c) * a - e / d / (d * x + c) * b * c)^2 * \ln(b * e / d + (a * d - b * c) * e / d / (d * x + c))^{2 * a - 2} / 3 * e^3 / (a * d - b * c)^4 / g^4 * A * B * b^3 / (b * e / d + e / (d * x + c) * a - e / d / (d * x + c) * b * c)^3 * \ln(b * e / d + (a * d - b * c) * e / d / (d * x + c)) * c - 2 * e * d^2 / (a * d - b * c)^4 / g^4 * B^2 / (b * e / d + e / (d * x + c) * a - e / d / (d * x + c) * b * c) * \ln(b * e / d + (a * d - b * c) * e / d / (d * x + c)) * b * c - 2 * e * d^2 / (a * d - b * c)^4 / g^4 * B^2 / (b * e / d + e / (d * x + c) * a - e / d / (d * x + c) * b * c) * b * c - e^{2 * d^2} / (a * d - b * c)^4 / g^4 * A^2 * b / (b * e / d + e / (d * x + c) * a - e / d / (d * x + c) * b * c)^2 * a$$

**Maxima [B]** time = 1.85789, size = 1916, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(b\*g\*x+a\*g)^4,x, algorithm="maxima")

[Out] 
$$-1/54 * (6 * ((6 * b^2 * d^2 * x^2 + 2 * b^2 * c^2 - 7 * a * b * c * d + 11 * a^2 * d^2 - 3 * (b^2 * c * d - 5 * a * b * d^2) * x) / ((b^6 * c^2 - 2 * a * b^5 * c * d + a^2 * b^4 * d^2) * g^4 * x^3 + 3 * (a * b^5 * c^2 - 2 * a^2 * b^4 * c * d + a^3 * b^3 * d^2) * g^4 * x^2 + 3 * (a^2 * b^4 * c^2 - 2 * a^3 * b^3 * c * d + a^4 * b^2 * d^2) * g^4 * x + (a^3 * b^3 * c^2 - 2 * a^4 * b^2 * c * d + a^5 * b * d^2) * g^4) + 6 * d^3 * \log(b * x + a) / ((b^4 * c^3 - 3 * a * b^3 * c^2 * d + 3 * a^2 * b^2 * c * d^2 - a^3 * b * d^3) * g^4) - 6 * d^3 * \log(d * x + c) / ((b^4 * c^3 - 3 * a * b^3 * c^2 * d + 3 * a^2 * b^2 * c * d^2 - a^3 * b * d^3) * g^4)) * \log(b * e * x / (d * x + c) + a * e / (d * x + c)) + (4 * b^3 * c^3 - 27 * a * b^2 * c^2 * d + 108 * a^2 * b * c * d^2 - 85 * a^3 * d^3 + 66 * (b^3 * c * d^2 - a * b^2 * d^3) * x^2 - 18 * (b^3 * d^3 * x^3 + 3 * a * b^2 * d^3 * x^2 + 3 * a^2 * b * d^3 * x + a^3 * d^3) * \log(b * x + a)^2 - 18 * (b^3 * d^3 * x^3 + 3 * a * b^2 * d^3 * x^2 + 3 * a^2 * b * d^3 * x + a^3 * d^3) * \log(d * x + c)^2 - 3 * (5 * b^3 * c^2 * d - 54 * a * b^2 * c * d^2 + 49 * a^2 * b * d^3) * x + 66 * (b^3 * d^3 * x^3 + 3 * a * b^2 * d^3 * x^2 + 3 * a^2 * b * d^3 * x + a^3 * d^3) * \log(b * x + a) - 6 * (11 * b^3 * d^3 * x^3 + 3 * a * b^2 * d^3 * x^2 + 33 * a^2 * b * d^3 * x + 11 * a^3 * d^3 - 6 * (b^3 * d^3 * x^3 + 3 * a * b^2 * d^3 * x^2 + 3 * a^2 * b * d^3 * x + a^3 * d^3) * \log(b * x + a)) * \log(d * x + c)) / (a^3 * b^4 * c^3 * g^4 - 3 * a^4 * b^3 * c^2 * d * g^4 + 3 * a^5 * b^2 * c * d^2 * g^4 - a^6 * b * d^3 * g^4 + (b^7 * c^3 * g^4 - 3 * a * b^6 * c^2 * d * g^4 + 3 * a^2 * b^5 * c * d^2 * g^4 - a^3 * b^4 * d^3 * g^4) * x^3 + 3 * (a * b^6 * c^3 * g^4 - 3 * a^2 * b^5 * c^2 * d * g^4 + 3 * a^3 * b^4 * c * d^2 * g^4 - a^4 * b^3 * d^3 * g^4) * x^2 + 3 * (a^2 * b^5 * c^3 * g^4 - 3 * a^3 * b^4 * c^2 * d * g^4 + 3 * a^4 * b^3 * c * d^2 * g^4 - a^5 * b^2 * d^3 * g^4) * x) * B^2 - 1/9 * A * B * ((6 * b^2 * d^2 * x^2 + 2 * b^2 * c^2 - 7 * a * b * c * d + 11 * a^2 * d^2 - 3 * (b^2 * c * d - 5 * a * b * d^2) * x) / ((b^6 * c^2 - 2 * a * b^5 * c * d + a^2 * b^4 * d^2) * g^4 * x^3 + 3 * (a * b^5 * c^2 - 2 * a^2 * b^4 * c * d + a^3 * b^3 * d^2) * g^4 * x^2 + 3 * (a^2 * b^4 * c^2 - 2 * a^3 * b^3 * c * d + a^4 * b^2 * d^2) * g^4 * x + (a^3 * b^3 * c^2 - 2 * a^4 * b^2 * c * d + a^5 * b * d^2) * g^4) + 6 * \log(b * e * x / (d * x + c) + a * e / (d * x + c)) / (b^4 * g^4 * x^3 + 3 * a * b^3 * g^4 * x^2 + 3 * a^2 * b^2 * g^4 * x + a^3 * b * g^4) + 6 * d^3 * \log(b * x + a) / ((b^4 * c^3 - 3 * a * b^3 * c^2 * d + 3 * a^2 * b^2 * c * d^2 - a^3 * b * d^3) * g^4) - 6 * d^3 * \log(d * x + c) / ((b^4 * c^3 - 3 * a * b^3 * c^2 * d + 3 * a^2 * b^2 * c * d^2 - a^3 * b * d^3) * g^4)) - 1/3 * B^2 * \log(b * e * x / (d * x + c) + a * e / (d * x + c))^2 / (b^4 * g^4 * x^3 + 3 * a * b^3 * g^4 * x^2 + 3 * a^2 * b^2 * g^4 * x + a^3 * b * g^4) - 1/3 * A^2 / (b^4 * g^4 * x^3 + 3 * a * b^3 * g^4 * x^2 + 3 * a^2 * b^2 * g^4 * x + a^3 * b * g^4)$$

**Fricas [A]** time = 1.10476, size = 1388, normalized size = 3.32

$$2(9A^2 + 6AB + 2B^2)b^3c^3 - 27(2A^2 + 2AB + B^2)ab^2c^2d + 54(A^2 + 2AB + 2B^2)a^2bcd^2 - (18A^2 + 66AB + 85B^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algorithm="fricas")
```

```
[Out] -1/54*(2*(9*A^2 + 6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 + 2*A*B + B^2)*a*b^2*c^2*d + 54*(A^2 + 2*A*B + 2*B^2)*a^2*b*c*d^2 - (18*A^2 + 66*A*B + 85*B^2)*a^3*d^3 + 6*((6*A*B + 11*B^2)*b^3*c*d^2 - (6*A*B + 11*B^2)*a*b^2*d^3)*x^2 + 18*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((b*e*x + a*e)/(d*x + c))^2 - 3*((6*A*B + 5*B^2)*b^3*c^2*d - 18*(2*A*B + 3*B^2)*a*b^2*c*d^2 + (30*A*B + 49*B^2)*a^2*b*d^3)*x + 6*((6*A*B + 11*B^2)*b^3*d^3*x^3 + 2*(3*A*B + B^2)*b^3*c^3 - 9*(2*A*B + B^2)*a*b^2*c^2*d + 18*(A*B + B^2)*a^2*b*c*d^2 + 3*(2*B^2*b^3*c*d^2 + 3*(2*A*B + 3*B^2)*a*b^2*d^3)*x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 6*(A*B + B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

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**Sympy [B]** time = 36.9113, size = 1544, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4,x)
```

```
[Out] -B*d**3*(6*A + 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 + 11*B**2*a*d**4 + 11*B**2*b*c*d**3 - B*a**4*d**7*(6*A + 11*B)/(a*d - b*c)**3 + 4*B*a**3*b*c*d**6*(6*A + 11*B)/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5*(6*A + 11*B)/(a*d - b*c)**3 + 4*B*a*b**3*c**3*d**4*(6*A + 11*B)/(a*d - b*c)**3 - B*b**4*c**4*d**3*(6*A + 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 + 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + B*d**3*(6*A + 11*B)*log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 + 11*B**2*a*d**4 + 11*B**2*b*c*d**3 + B*a**4*d**7*(6*A + 11*B)/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6*(6*A + 11*B)/(a*d - b*c)**3 + 6*B*a**2*b**2*c**2*d**5*(6*A + 11*B)/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4*(6*A + 11*B)/(a*d - b*c)**3 + B*b**4*c**4*d**3*(6*A + 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 + 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*log(e*(a + b*x)/(c + d*x))**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 - 11*B**2*a**2*d**2 + 7*B**2*a*b*c*d - 15*B**2*a*b*d**2*x - 2*B**2*b**2*c**2 + 3*B**2*b**2*c*d*x - 6*B**2*b**2*d**2*x**2)*log(e*(a + b*x)/(c + d*x))/(9*a**5*b*d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 27*a**4*b**2*d**2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*a**3*b**3*c*d*g**4*x + 27*a**3*b**3*d**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4*x - 54*a**2*b**4*c*d*g**4*x**2 + 9*a**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c**2*g**4*x**2 - 18*a*b**5*c*d*g**4*x**3 + 9*b**6*c**2*g**4*x**3) - (18*A**2*a**2*d**2 - 36*A**2*a*b*c*d + 18*A**2*b**2*c**2 + 66*A*B*a**2*d**2 - 42*A*B*a*b*c*d + 12*A*B*b**2*c**2 + 85*B**2*a**2*d**2 - 23*B**2*a*b*c*d + 4*B**2*b**2*c**2 + x**2*(36*A*B*b**2*d**2 + 66*B**2*b**2*d**2) + x*(90*A*B*a*b*d**2 - 18*A*B*b**2*c*d + 147*B**2*a*b*d**2 - 15*B**2*b**2*c*d))/(54*a**5*b*d**2*g**4 - 108*a**4*b**2*c*d*g**4 + 54*a**3*b**3*c**2*g**4 + x**3*(54*a**2*b**4*d**2*g**4 - 108*a*b**5*c*d*g**4 +
```



54\*b\*\*6\*c\*\*2\*g\*\*4) + x\*\*2\*(162\*a\*\*3\*b\*\*3\*d\*\*2\*g\*\*4 - 324\*a\*\*2\*b\*\*4\*c\*d\*g\*\*4 + 162\*a\*b\*\*5\*c\*\*2\*g\*\*4) + x\*(162\*a\*\*4\*b\*\*2\*d\*\*2\*g\*\*4 - 324\*a\*\*3\*b\*\*3\*c\*d\*g\*\*4 + 162\*a\*\*2\*b\*\*4\*c\*\*2\*g\*\*4))

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{(bgx+ag)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(b\*g\*x+a\*g)^4,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2/(b\*g\*x + a\*g)^4, x)

$$3.105 \quad \int \frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5} dx$$

**Optimal.** Leaf size=575

$$\frac{b^3(c+dx)^4 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)^2}{4g^5(a+bx)^4(bc-ad)^4} - \frac{b^3B(c+dx)^4 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{8g^5(a+bx)^4(bc-ad)^4} + \frac{b^2d(c+dx)^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)^2}{g^5(a+bx)^3(bc-ad)^4} + \frac{2b^2Bd(c+dx)^2 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{g^5(a+bx)^2(bc-ad)^4} + \frac{b^2d^2(c+dx) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{g^5(a+bx)(bc-ad)^4} + \frac{b^2d^3 \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{g^5(bc-ad)^4}$$

[Out]  $(2*B^2*d^3*(c+d*x))/((b*c-a*d)^4*g^5*(a+b*x)) - (3*b*B^2*d^2*(c+d*x)^2)/(4*(b*c-a*d)^4*g^5*(a+b*x)^2) + (2*b^2*B^2*d*(c+d*x)^3)/(9*(b*c-a*d)^4*g^5*(a+b*x)^3) - (b^3*B^2*(c+d*x)^4)/(32*(b*c-a*d)^4*g^5*(a+b*x)^4) + (2*B*d^3*(c+d*x)*(A+B*Log[(e*(a+b*x))/(c+d*x])))/((b*c-a*d)^4*g^5*(a+b*x)) - (3*b*B*d^2*(c+d*x)^2*(A+B*Log[(e*(a+b*x))/(c+d*x])))/(2*(b*c-a*d)^4*g^5*(a+b*x)^2) + (2*b^2*B*d*(c+d*x)^3*(A+B*Log[(e*(a+b*x))/(c+d*x])))/(3*(b*c-a*d)^4*g^5*(a+b*x)^3) - (b^3*B*(c+d*x)^4*(A+B*Log[(e*(a+b*x))/(c+d*x])))/(8*(b*c-a*d)^4*g^5*(a+b*x)^4) + (d^3*(c+d*x)*(A+B*Log[(e*(a+b*x))/(c+d*x]))^2)/((b*c-a*d)^4*g^5*(a+b*x)) - (3*b*d^2*(c+d*x)^2*(A+B*Log[(e*(a+b*x))/(c+d*x]))^2)/(2*(b*c-a*d)^4*g^5*(a+b*x)^2) + (b^2*d*(c+d*x)^3*(A+B*Log[(e*(a+b*x))/(c+d*x]))^2)/((b*c-a*d)^4*g^5*(a+b*x)^3) - (b^3*(c+d*x)^4*(A+B*Log[(e*(a+b*x))/(c+d*x]))^2)/(4*(b*c-a*d)^4*g^5*(a+b*x)^4)$

**Rubi [C]** time = 1.22722, antiderivative size = 763, normalized size of antiderivative = 1.33, number of steps used = 38, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2d^4 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{2bg^5(bc-ad)^4} + \frac{B^2d^4 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{2bg^5(bc-ad)^4} + \frac{Bd^4 \log(a+bx) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{2bg^5(bc-ad)^4} - \frac{Bd^4 \log(c+dx) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{2bg^5(bc-ad)^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2/(a\*g + b\*g\*x)^5, x]

[Out]  $-B^2/(32*b*g^5*(a+b*x)^4) + (7*B^2*d)/(72*b*(b*c-a*d)*g^5*(a+b*x)^3) - (13*B^2*d^2)/(48*b*(b*c-a*d)^2*g^5*(a+b*x)^2) + (25*B^2*d^3)/(24*b*(b*c-a*d)^3*g^5*(a+b*x)) + (25*B^2*d^4*Log[a+b*x])/(24*b*(b*c-a*d)^4*g^5) - (B^2*d^4*Log[a+b*x]^2)/(4*b*(b*c-a*d)^4*g^5) - (B*(A+B*Log[(e*(a+b*x))/(c+d*x])))/(8*b*g^5*(a+b*x)^4) + (B*d*(A+B*Log[(e*(a+b*x))/(c+d*x])))/(6*b*(b*c-a*d)*g^5*(a+b*x)^3) - (B*d^2*(A+B*Log[(e*(a+b*x))/(c+d*x])))/(4*b*(b*c-a*d)^2*g^5*(a+b*x)^2) + (B*d^3*(A+B*Log[(e*(a+b*x))/(c+d*x])))/(2*b*(b*c-a*d)^3*g^5*(a+b*x)) + (B*d^4*Log[a+b*x]*(A+B*Log[(e*(a+b*x))/(c+d*x])))/(2*b*(b*c-a*d)^4*g^5) - (A+B*Log[(e*(a+b*x))/(c+d*x]))^2/(4*b*g^5*(a+b*x)^4) - (25*B^2*d^4*Log[c+d*x])/(24*b*(b*c-a*d)^4*g^5) + (B^2*d^4*Log[-((d*(a+b*x))/(b*c-a*d))]*Log[c+d*x])/(2*b*(b*c-a*d)^4*g^5) - (B*d^4*(A+B*Log[(e*(a+b*x))/(c+d*x]))*Log[c+d*x])/(2*b*(b*c-a*d)^4*g^5) - (B^2*d^4*Log[c+d*x]^2)/(4*b*(b*c-a*d)^4*g^5) + (B^2*d^4*Log[a+b*x]*Log[(b*(c+d*x))/(b*c-a*d)])/(2*b*(b*c-a*d)^4*g^5) + (B^2*d^4*PolyLog[2, -((d*(a+b*x))/(b*c-a*d))])/(2*b*(b*c-a*d)^4*g^5) + (B^2*d^4*PolyLog[2, (b*(c+d*x))/(b*c-a*d)])/(2*b*(b*c-a*d)^4*g^5)$

Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

### Rule 44

```
Int[((a_) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_.))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_.)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_.)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))])*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^5} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B \int \frac{(bc-ad)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g^4(a+bx)^5(c+dx)} dx}{2bg} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad)) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^5(c+dx)} dx}{2bg^5} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad)) \int \left(\frac{b\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(a+bx)^5} - \frac{bd\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2(a+bx)^4} + \frac{bd^2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3(a+bx)^3}\right) dx}{2bg^5} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^5} dx}{2g^5} + \frac{(Bd^4) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx} dx}{2(bc-ad)^4g^5} - \frac{(Bd^5) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)^2} dx}{2b(bc-ad)^5g^5} \\
&= -\frac{B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8bg^5(a+bx)^4} + \frac{Bd\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b(bc-ad)^2g^5(a+bx)^2} + \frac{Bd^3\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b(bc-ad)^3g^5(a+bx)} \\
&= -\frac{B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8bg^5(a+bx)^4} + \frac{Bd\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b(bc-ad)^2g^5(a+bx)^2} + \frac{Bd^3\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b(bc-ad)^3g^5(a+bx)} \\
&= -\frac{B\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8bg^5(a+bx)^4} + \frac{Bd\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b(bc-ad)^2g^5(a+bx)^2} + \frac{Bd^3\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b(bc-ad)^3g^5(a+bx)} \\
&= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{25B^2d^3}{24b(bc-ad)^3g^5(a+bx)} \\
&= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{25B^2d^3}{24b(bc-ad)^3g^5(a+bx)} \\
&= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{25B^2d^3}{24b(bc-ad)^3g^5(a+bx)} \\
&= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{25B^2d^3}{24b(bc-ad)^3g^5(a+bx)}
\end{aligned}$$

**Mathematica [C]** time = 1.11006, size = 748, normalized size = 1.3

$$B\left(72Bd^4(a+bx)^4\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)-72Bd^4(a+bx)^4\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(a*g + b*g*x)^5,x]
```

```
[Out] -(72*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(36*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 48*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 72*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 144*d^3*(-(b*c) + a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 144*d^4*(a + b*x)^4*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 144*d^4*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 144*B*d^3*(a + b*x)^3*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + 36*B*d^2*(a + b*x)^2*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) - 8*B*d*(a + b*x)*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*Log[a + b*x] - 6*d^3*(a + b*x)^3*Log[c + d*x]) + 3*B*(3*(b*c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*Log[a + b*x] + 12*d^4*(a + b*x)^4*Log[c + d*x]) + 72*B*d^4*(a + b*x)^4*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 72*B*d^4*(a + b*x)^4*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^4/(288*b*g^5*(a + b*x)^4)
```

---

**Maple [B]** time = 0.049, size = 3538, normalized size = 6.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x)
```

```
[Out] e*d^4/(a*d-b*c)^5/g^5*A^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*a+1/4*e^4/(a*d-b*c)^5/g^5*A^2*b^4/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*c+1/32*e^4/(a*d-b*c)^5/g^5*B^2*b^4/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*c+2*e*d^4/(a*d-b*c)^5/g^5*B^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*a+2*e*d^4/(a*d-b*c)^5/g^5*A*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*a-1/2*e^4*d/(a*d-b*c)^5/g^5*A*B*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a-e*d^3/(a*d-b*c)^5/g^5*A^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*b*c+2/9*e^3*d^2/(a*d-b*c)^5/g^5*B^2*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*a-2/9*e^3*d/(a*d-b*c)^5/g^5*B^2*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*c-1/32*e^4*d/(a*d-b*c)^5/g^5*B^2*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*a-3/2*e^2*d^3/(a*d-b*c)^5/g^5*A^2*b/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*a+2*e*d^4/(a*d-b*c)^5/g^5*B^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a+3/2*e^2*d^2/(a*d-b*c)^5/g^5*A^2*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*c+e^3*d^2/(a*d-b*c)^5/g^5*A^2*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*a-e^3*d/(a*d-b*c)^5/g^5*A^2*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*c+3/4*e^2*d^2/(a*d-b*c)^5/g^5*B^2*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*c+1/8*e^4/(a*d-b*c)^5/g^5*A*B*b^4/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*c-1/4*e^4*d/(a*d-b*c)^5/g^5*A^2*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*a+1/4*e^4/(a*d-b*c)^5/g^5*B^2*b^4/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*c+1/8*e^4/(a*d-b*c)^5/g^5*B^2*b^4/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*c+e*d^4/(a*d-b*c)^5/g^5*B^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*a+2*e^3*d^2/(a*d-b*c)^5/g^5*A*B*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a-2*e*d^3/(a*d-b*c)^5/g^5*A*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b*c-2*e^3*d/(a
```

$$\begin{aligned}
& d-b*c)^5/g^5*A*B*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*\ln(b*e/d+(a*d-b* \\
& c)*e/d/(d*x+c))*c-3*e^2*d^3/(a*d-b*c)^5/g^5*A*B*b/(b*e/d+e/(d*x+c)*a-e/d/(d \\
& *x+c)*b*c)^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a+3*e^2*d^2/(a*d-b*c)^5/g^5*A* \\
& B*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)) \\
& *c-2*e*d^3/(a*d-b*c)^5/g^5*B^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*b*c-3/4* \\
& e^2*d^3/(a*d-b*c)^5/g^5*B^2*b/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*a-2/3*e \\
& ^3*d/(a*d-b*c)^5/g^5*A*B*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*c-1/8*e^ \\
& 4*d/(a*d-b*c)^5/g^5*A*B*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*a-2*e*d^3 \\
& /(a*d-b*c)^5/g^5*A*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*b*c+1/2*e^4/(a*d-b \\
& *c)^5/g^5*A*B*b^4/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*\ln(b*e/d+(a*d-b*c)* \\
& e/d/(d*x+c))*c-e*d^3/(a*d-b*c)^5/g^5*B^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c \\
& )*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*b*c-2*e*d^3/(a*d-b*c)^5/g^5*B^2/(b*e/d+ \\
& e/(d*x+c)*a-e/d/(d*x+c)*b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b*c+2*e*d^4/(a \\
& *d-b*c)^5/g^5*A*B/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)*\ln(b*e/d+(a*d-b*c)*e/ \\
& d/(d*x+c))*a+3/2*e^2*d^2/(a*d-b*c)^5/g^5*B^2*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d* \\
& x+c)*b*c)^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*c-3/2*e^2*d^3/(a*d-b*c)^5/g^5 \\
& *B^2*b/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c) \\
& )*a+2/3*e^3*d^2/(a*d-b*c)^5/g^5*B^2*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c) \\
& ^3*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a-2/3*e^3*d/(a*d-b*c)^5/g^5*B^2*b^3/(b*e \\
& /d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*c-1/4*e^4 \\
& *d/(a*d-b*c)^5/g^5*B^2*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^4*\ln(b*e/d+( \\
& a*d-b*c)*e/d/(d*x+c))^2*a-1/8*e^4*d/(a*d-b*c)^5/g^5*B^2*b^3/(b*e/d+e/(d*x+c) \\
& )*a-e/d/(d*x+c)*b*c)^4*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a+e^3*d^2/(a*d-b*c)^ \\
& 5/g^5*B^2*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*\ln(b*e/d+(a*d-b*c)*e/d/ \\
& (d*x+c))^2*a-e^3*d/(a*d-b*c)^5/g^5*B^2*b^3/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b \\
& *c)^3*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*c-3/2*e^2*d^3/(a*d-b*c)^5/g^5*B^2*b \\
& /(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*a- \\
& 3/2*e^2*d^3/(a*d-b*c)^5/g^5*A*B*b/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*a+3 \\
& /2*e^2*d^2/(a*d-b*c)^5/g^5*A*B*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2*c+ \\
& 2/3*e^3*d^2/(a*d-b*c)^5/g^5*A*B*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^3*a \\
& +3/2*e^2*d^2/(a*d-b*c)^5/g^5*B^2*b^2/(b*e/d+e/(d*x+c)*a-e/d/(d*x+c)*b*c)^2* \\
& \ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*c
\end{aligned}$$

**Maxima [B]** time = 2.18354, size = 2866, normalized size = 4.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(b\*g\*x+a\*g)^5,x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& 1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + \\
& 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 \\
& + 13*a^2*b*d^3)*x)/(b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d \\
& ^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^ \\
& 3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d \\
& ^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2* \\
& d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)* \\
& g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - \\
& 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4 \\
& *c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(b*e*x/( \\
& d*x + c) + a*e/(d*x + c)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d \\
& ^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(1 \\
& 3*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + \\
& 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) \\
& ^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x
\end{aligned}$$

$$\begin{aligned}
& + a^4 d^4 \log(dx + c)^2 - 4(7b^4 c^3 d - 60a^3 b^3 c^2 d^2 + 324a^2 b^2 c^3 d^3 - 271a^3 b^3 d^4) x - 300(b^4 d^4 x^4 + 4a^3 b^3 d^4 x^3 + 6a^2 b^2 d^4 x^2 + 4a^3 b^3 d^4 x + a^4 d^4) \log(bx + a) + 12(25b^4 d^4 x^4 + 100a^3 b^3 d^4 x^3 + 150a^2 b^2 d^4 x^2 + 100a^3 b^3 d^4 x + 25a^4 d^4 - 12(b^4 d^4 x^4 + 4a^3 b^3 d^4 x^3 + 6a^2 b^2 d^4 x^2 + 4a^3 b^3 d^4 x + a^4 d^4) \log(bx + a)) \log(dx + c) / (a^4 b^5 c^4 g^5 - 4a^5 b^4 c^3 d g^5 + 6a^6 b^3 c^2 d^2 g^5 - 4a^7 b^2 c^3 d^3 g^5 + a^8 b^3 d^4 g^5 + (b^9 c^4 g^5 - 4a^8 b^8 c^3 d g^5 + 6a^2 b^7 c^2 d^2 g^5 - 4a^3 b^6 c^3 d^3 g^5 + a^4 b^5 d^4 g^5) x^4 + 4(a^8 b^8 c^4 g^5 - 4a^2 b^7 c^3 d g^5 + 6a^3 b^6 c^2 d^2 g^5 - 4a^4 b^5 c^3 d^3 g^5 + a^5 b^4 d^4 g^5) x^3 + 6(a^2 b^7 c^4 g^5 - 4a^3 b^6 c^3 d g^5 + 6a^4 b^5 c^2 d^2 g^5 - 4a^5 b^4 c^3 d^3 g^5 + a^6 b^3 d^4 g^5) x^2 + 4(a^3 b^6 c^4 g^5 - 4a^4 b^5 c^3 d g^5 + 6a^5 b^4 c^2 d^2 g^5 - 4a^6 b^3 c^3 d^3 g^5 + a^7 b^2 d^4 g^5) x) B^2 + 1/24 A B ((12b^3 d^3 x^3 - 3b^3 c^3 + 13a^2 b^2 c^2 d - 23a^2 b^3 c^2 d^2 + 25a^3 d^3 - 6(b^3 c^3 d^2 - 7a^2 b^2 d^3) x^2 + 4(b^3 c^2 d - 5a^2 b^2 c^2 d^2 + 13a^2 b^3 d^3) x) / ((b^8 c^3 - 3a^2 b^7 c^2 d + 3a^2 b^6 c^2 d^2 - a^3 b^5 d^3) g^5 x^4 + 4(a^2 b^7 c^3 - 3a^2 b^6 c^2 d + 3a^3 b^5 c^2 d^2 - a^4 b^4 d^3) g^5 x^3 + 6(a^2 b^6 c^3 - 3a^3 b^5 c^2 d + 3a^4 b^4 c^2 d^2 - a^5 b^3 d^3) g^5 x^2 + 4(a^3 b^5 c^3 - 3a^4 b^4 c^2 d + 3a^5 b^3 c^2 d^2 - a^6 b^2 d^3) g^5 x + (a^4 b^4 c^3 - 3a^5 b^3 c^2 d + 3a^6 b^2 c^2 d^2 - a^7 b^2 d^3) g^5) - 12 \log(bex/(dx + c) + ae/(dx + c)) / (b^5 g^5 x^4 + 4a^2 b^4 g^5 x^3 + 6a^2 b^3 g^5 x^2 + 4a^3 b^2 g^5 x + a^4 b g^5) + 12 d^4 \log(bx + a) / ((b^5 c^4 - 4a^2 b^4 c^3 d + 6a^2 b^3 c^2 d^2 - 4a^3 b^2 c^2 d^3 + a^4 b^3 d^4) g^5) - 12 d^4 \log(dx + c) / ((b^5 c^4 - 4a^2 b^4 c^3 d + 6a^2 b^3 c^2 d^2 - 4a^3 b^2 c^2 d^3 + a^4 b^3 d^4) g^5) - 1/4 B^2 \log(bex/(dx + c) + ae/(dx + c))^2 / (b^5 g^5 x^4 + 4a^2 b^4 g^5 x^3 + 6a^2 b^3 g^5 x^2 + 4a^3 b^2 g^5 x + a^4 b g^5) - 1/4 A^2 / (b^5 g^5 x^4 + 4a^2 b^4 g^5 x^3 + 6a^2 b^3 g^5 x^2 + 4a^3 b^2 g^5 x + a^4 b g^5)
\end{aligned}$$

**Fricas [A]** time = 1.16413, size = 2152, normalized size = 3.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(b\*g\*x+a\*g)^5,x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& -1/288(9(8A^2 + 4AB + B^2)b^4c^4 - 32(9A^2 + 6AB + 2B^2)a^3b^3c^3d + 216(2A^2 + 2AB + B^2)a^2b^2c^2d^2 - 288(A^2 + 2AB + 2B^2)a^3b^3c^3d^3 + (72A^2 + 300AB + 415B^2)a^4d^4 - 12((12AB + 25B^2)b^4c^3d^3 - (12AB + 25B^2)a^3b^3d^4)x^3 + 6((12AB + 13B^2)b^4c^2d^2 - 16(6AB + 11B^2)a^3b^3c^3d^3 + (84AB + 163B^2)a^2b^2d^4)x^2 - 72(B^2b^4d^4x^4 + 4B^2a^3b^3d^4x^3 + 6B^2a^2b^2d^4x^2 + 4B^2a^3b^3d^4x - B^2b^4c^4 + 4B^2a^3b^3c^3d - 6B^2a^2b^2c^2d^2 + 4B^2a^3b^3c^3d^3) \log((bex + ae)/(dx + c))^2 - 4((12AB + 7B^2)b^4c^3d - 12(6AB + 5B^2)a^3b^3c^2d^2 + 108(2AB + 3B^2)a^2b^2c^3d^3 - (156AB + 271B^2)a^3b^3d^4)x - 12((12AB + 25B^2)b^4d^4x^4 - 3(4AB + B^2)b^4c^4 + 16(3AB + B^2)a^3b^3c^3d - 36(2AB + B^2)a^2b^2c^2d^2 + 48(AB + B^2)a^3b^3c^3d^3 + 4(3B^2b^4c^3d^3 + 2(6AB + 11B^2)a^3b^3d^4)x^3 - 6(B^2b^4c^2d^2 - 8B^2a^3b^3c^3d^3 - 6(2AB + 3B^2)a^2b^2d^4)x^2 + 4(B^2b^4c^3d - 6B^2a^3b^3c^2d^2 + 18B^2a^2b^2c^3d^3 + 12(AB + B^2)a^3b^3d^4)x) \log((bex + ae)/(dx + c))) / ((b^9c^4 - 4a^2b^8c^3d + 6a^2b^7c^2d^2 - 4a^3b^6c^3d^3 + a^4b^5d^4) g^5 x^4 + 4(a^8b^8c^4 - 4a^2b^7c^3d + 6a^3b^6c^2d^2 - 4a^4b^5c^3d^3 + a^5b^4d^4) g^5 x^3 + 6(a^2b^7c^4 - 4a^3b^6c^3d + 6a^4b^5c^2d^2 - 4a^5b^4c^3d^3 + a^6b^3d^4) g^5 x^2 + 4(a^3b^6c^3
\end{aligned}$$

$$4 - 4a^4b^5c^3d + 6a^5b^4c^2d^2 - 4a^6b^3c^2d^3 + a^7b^2d^4)g^5x + (a^4b^5c^4 - 4a^5b^4c^3d + 6a^6b^3c^2d^2 - 4a^7b^2c^2d^3 + a^8b^2d^4)g^5)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2/(b\*g\*x+a\*g)\*\*5,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{(bgx + ag)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(b\*g\*x+a\*g)^5,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2/(b\*g\*x + a\*g)^5, x)



$$3.106 \quad \int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx} dx$$

**Optimal.** Leaf size=28

$$\frac{\text{PolyLog}\left(2, \frac{bc-ad}{b(c+dx)}\right)}{df}$$

[Out] PolyLog[2, (b\*c - a\*d)/(b\*(c + d\*x))]/(d\*f)

**Rubi [A]** time = 0.0224578, antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 1, number of rules used = 1, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$ , Rules used = {2447}

$$\frac{\text{PolyLog}\left(2, 1 - \frac{d(a+bx)}{b(c+dx)}\right)}{df}$$

Antiderivative was successfully verified.

[In] Int[Log[(d\*(a + b\*x))/(b\*(c + d\*x))]/(c\*f + d\*f\*x), x]

[Out] PolyLog[2, 1 - (d\*(a + b\*x))/(b\*(c + d\*x))]/(d\*f)

#### Rule 2447

Int[Log[u]\*(Pq\_)^(m\_.), x\_Symbol] :> With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

#### Rubi steps

$$\int \frac{\log\left(\frac{d(a+bx)}{b(c+dx)}\right)}{cf+dfx} dx = \frac{\text{Li}_2\left(1 - \frac{d(a+bx)}{b(c+dx)}\right)}{df}$$

**Mathematica [B]** time = 0.050042, size = 114, normalized size = 4.07

$$\frac{\log\left(\frac{bc-ad}{bc+bdx}\right)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right) - 2\log\left(\frac{d(a+bx)}{b(c+dx)}\right) + \log\left(\frac{bc-ad}{bc+bdx}\right)\right) - 2\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{2df}$$

Antiderivative was successfully verified.

[In] Integrate[Log[(d\*(a + b\*x))/(b\*(c + d\*x))]/(c\*f + d\*f\*x), x]

[Out] (Log[(b\*c - a\*d)/(b\*c + b\*d\*x)]\*(2\*Log[(d\*(a + b\*x))/(- (b\*c) + a\*d)] - 2\*Log[(d\*(a + b\*x))/(b\*(c + d\*x))] + Log[(b\*c - a\*d)/(b\*c + b\*d\*x)]) - 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]/(2\*d\*f)

**Maple [A]** time = 0.048, size = 30, normalized size = 1.1

$$\frac{1}{df} \operatorname{dilog} \left( 1 + \frac{ad - bc}{b(dx + c)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x)`

[Out] `1/d/f*dilog(1+(a*d-b*c)/b/(d*x+c))`

**Maxima [B]** time = 1.23762, size = 213, normalized size = 7.61

$$\frac{b \left( \frac{\log(dx+c)^2}{bf} - \frac{2 \left( \log(bx+a) \log\left(\frac{bdx+ad}{bc-ad} + 1\right) + \operatorname{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right) \right)}{bf} \right)}{2d} - \frac{b \left( \frac{d \log(bx+a)}{b} - \frac{d \log(dx+c)}{b} \right) \log(dfx + cf)}{d^2 f} + \frac{\log(dfx + cf) \log\left(\frac{bx+a}{dx+c}\right)}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="maxima")`

[Out] `-1/2*b*(log(d*x + c)^2/(b*f) - 2*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*f))/d - b*(d*log(b*x + a)/b - d*log(d*x + c)/b)*log(d*f*x + c*f)/(d^2*f) + log(d*f*x + c*f)*log((b*x + a)*d/((d*x + c)*b))/(d*f)`

**Fricas [A]** time = 0.974562, size = 63, normalized size = 2.25

$$\frac{\operatorname{Li}_2 \left( -\frac{bdx+ad}{bdx+bc} + 1 \right)}{df}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="fricas")`

[Out] `dilog(-(b*d*x + a*d)/(b*d*x + b*c) + 1)/(d*f)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{(bx+a)d}{(dx+c)b}\right)}{dfx + cf} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(d*(b*x+a)/b/(d*x+c))/(d*f*x+c*f),x, algorithm="giac")
```

```
[Out] integrate(log((b*x + a)*d/((d*x + c)*b))/(d*f*x + c*f), x)
```

$$3.107 \quad \int \frac{\log\left(1 + \frac{1}{a+bx}\right)}{a+bx} dx$$

**Optimal.** Leaf size=15

$$\frac{\text{PolyLog}\left(2, -\frac{1}{a+bx}\right)}{b}$$

[Out] PolyLog[2, -(a + b\*x)^(-1)]/b

**Rubi [A]** time = 0.0140898, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$ , Rules used = {2447}

$$\frac{\text{PolyLog}\left(2, -\frac{1}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[Log[1 + (a + b\*x)^(-1)]/(a + b\*x), x]

[Out] PolyLog[2, -(a + b\*x)^(-1)]/b

#### Rule 2447

```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] := With[{C = FullSimplify[(Pq^m*(1 - u))
/D[u, x]]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
x][[2]], Expon[Pq, x]]
```

#### Rubi steps

$$\int \frac{\log\left(1 + \frac{1}{a+bx}\right)}{a+bx} dx = \frac{\text{Li}_2\left(-\frac{1}{a+bx}\right)}{b}$$

**Mathematica [B]** time = 0.0144037, size = 140, normalized size = 9.33

$$-\frac{\text{PolyLog}(2, -a - bx)}{b} + \frac{\log^2\left(\frac{ab-(a+1)b}{b(a+bx)}\right)}{2b} + \frac{\log\left(\frac{b(-a-bx-1)}{(-a-1)b+ab}\right)\log\left(\frac{ab-(a+1)b}{b(a+bx)}\right)}{b} - \frac{\log\left(\frac{a+bx+1}{a+bx}\right)\log\left(\frac{ab-(a+1)b}{b(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Log[1 + (a + b\*x)^(-1)]/(a + b\*x), x]

[Out] (Log[(b\*(-1 - a - b\*x))/((-1 - a)\*b + a\*b)]\*Log[(a\*b - (1 + a)\*b)/(b\*(a + b\*x))])/b + Log[(a\*b - (1 + a)\*b)/(b\*(a + b\*x))]^2/(2\*b) - (Log[(a\*b - (1 + a)\*b)/(b\*(a + b\*x))]\*Log[(1 + a + b\*x)/(a + b\*x)])/b - PolyLog[2, -a - b\*x]/b

**Maple [A]** time = 0.045, size = 15, normalized size = 1.

$$\frac{\operatorname{dilog}\left(1 + (bx + a)^{-1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln(1+1/(b\*x+a))/(b\*x+a), x)

[Out] 1/b\*dilog(1+1/(b\*x+a))

**Maxima [B]** time = 1.21988, size = 82, normalized size = 5.47

$$\frac{2 \log(bx + a + 1) \log(bx + a) - \log(bx + a)^2}{2b} - \frac{\log(bx + a + 1) \log(bx + a) + \operatorname{Li}_2(-bx - a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(1+1/(b\*x+a))/(b\*x+a), x, algorithm="maxima")

[Out] 1/2\*(2\*log(b\*x + a + 1)\*log(b\*x + a) - log(b\*x + a)^2)/b - (log(b\*x + a + 1)\*log(b\*x + a) + dilog(-b\*x - a))/b

**Fricas [A]** time = 0.952507, size = 53, normalized size = 3.53

$$\frac{\operatorname{Li}_2\left(-\frac{bx+a+1}{bx+a} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log(1+1/(b\*x+a))/(b\*x+a), x, algorithm="fricas")

[Out] dilog(-(b\*x + a + 1)/(b\*x + a) + 1)/b

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(1 + \frac{1}{a+bx}\right)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(ln(1+1/(b\*x+a))/(b\*x+a), x)

[Out] Integral(log(1 + 1/(a + b\*x))/(a + b\*x), x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(\frac{1}{bx+a} + 1\right)}{bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1+1/(b*x+a))/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(1/(b*x + a) + 1)/(b*x + a), x)
```

$$3.108 \quad \int \frac{\log\left(1 - \frac{1}{a+bx}\right)}{a+bx} dx$$

**Optimal.** Leaf size=13

$$\frac{\text{PolyLog}\left(2, \frac{1}{a+bx}\right)}{b}$$

[Out] PolyLog[2, (a + b\*x)^(-1)]/b

**Rubi [A]** time = 0.013087, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$ , Rules used = {2447}

$$\frac{\text{PolyLog}\left(2, \frac{1}{a+bx}\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[Log[1 - (a + b\*x)^(-1)]/(a + b\*x), x]

[Out] PolyLog[2, (a + b\*x)^(-1)]/b

**Rule 2447**

Int[Log[u\_]\*(Pq\_)^(m\_.), x\_Symbol] := With[{C = FullSimplify[(Pq^m\*(1 - u))/D[u, x]]}, Simp[C\*PolyLog[2, 1 - u], x] /; FreeQ[C, x] /; IntegerQ[m] && PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u, x][[2]], Expon[Pq, x]]

**Rubi steps**

$$\int \frac{\log\left(1 - \frac{1}{a+bx}\right)}{a+bx} dx = \frac{\text{Li}_2\left(\frac{1}{a+bx}\right)}{b}$$

**Mathematica [B]** time = 0.0144664, size = 133, normalized size = 10.23

$$\frac{\text{PolyLog}(2, a + bx)}{b} + \frac{\log^2\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{2b} + \frac{\log\left(\frac{b(a+bx-1)}{(a-1)b-ab}\right)\log\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{b} - \frac{\log\left(\frac{a+bx-1}{a+bx}\right)\log\left(\frac{ab-(a-1)b}{b(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Log[1 - (a + b\*x)^(-1)]/(a + b\*x), x]

[Out] (Log[(b\*(-1 + a + b\*x))/((-1 + a)\*b - a\*b)]\*Log[(-((-1 + a)\*b) + a\*b)/(b\*(a + b\*x))])/b + Log[(-((-1 + a)\*b) + a\*b)/(b\*(a + b\*x))]^2/(2\*b) - (Log[(-((-1 + a)\*b) + a\*b)/(b\*(a + b\*x))]\*Log[(-1 + a + b\*x)/(a + b\*x)])/b - PolyLog[2, a + b\*x]/b

**Maple [A]** time = 0.044, size = 17, normalized size = 1.3

$$\frac{1}{b} \operatorname{dilog}\left(1 - (bx + a)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(ln(1-1/(b*x+a))/(b*x+a),x)`

[Out] `1/b*dilog(1-1/(b*x+a))`

**Maxima [B]** time = 1.08632, size = 80, normalized size = 6.15

$$-\frac{\log(bx + a)^2 - 2 \log(bx + a) \log(bx + a - 1)}{2b} - \frac{\log(bx + a) \log(-bx - a + 1) + \operatorname{Li}_2(bx + a)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="maxima")`

[Out] `-1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(b*x + a - 1))/b - (log(b*x + a)*log(-b*x - a + 1) + dilog(b*x + a))/b`

**Fricas [A]** time = 0.980251, size = 53, normalized size = 4.08

$$\frac{\operatorname{Li}_2\left(-\frac{bx+a-1}{bx+a} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="fricas")`

[Out] `dilog(-(b*x + a - 1)/(b*x + a) + 1)/b`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(1 - \frac{1}{a+bx}\right)}{a + bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(ln(1-1/(b*x+a))/(b*x+a),x)`

[Out] `Integral(log(1 - 1/(a + b*x))/(a + b*x), x)`

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\log\left(-\frac{1}{bx+a} + 1\right)}{bx + a} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log(1-1/(b*x+a))/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(log(-1/(b*x + a) + 1)/(b*x + a), x)
```

$$3.109 \quad \int \frac{(ag+bgx)^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{(ag+bgx)^2}{B \log\left(\frac{e(a+bx)}{c+dx}\right)+A}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

**Rubi [A]** time = 0.20025, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(ag+bgx)^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

[Out] a^2\*g^2\*Defer[Int][(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]^(-1), x] + 2\*a\*b\*g^2\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x] + b^2\*g^2\*Defer[Int][x^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

**Rubi steps**

$$\begin{aligned} \int \frac{(ag+bgx)^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx &= \int \left( \frac{a^2g^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} + \frac{2abg^2x}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} + \frac{b^2g^2x^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} \right) dx \\ &= (a^2g^2) \int \frac{1}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx + (2abg^2) \int \frac{x}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx + (b^2g^2) \int \frac{x^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.594204, size = 0, normalized size = 0.

$$\int \frac{(ag+bgx)^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

[Out] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

**Maple [A]** time = 1.126, size = 0, normalized size = 0.

$$\int (bgx+ag)^2 \left( A + B \ln\left(\frac{e(bx+a)}{dx+c}\right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] `int((b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{B \log\left(\frac{bx+a}{dx+c}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")`

[Out] `integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2g^2x^2 + 2abg^2x + a^2g^2}{B \log\left(\frac{bex+ae}{dx+c}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")`

[Out] `integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{B \log\left(\frac{bx+a}{dx+c}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

$$3.110 \quad \int \frac{ag+bgx}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

**Optimal.** Leaf size=32

$$\text{Unintegrable}\left(\frac{ag+bgx}{B \log\left(\frac{e(a+bx)}{c+dx}\right)+A}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

**Rubi [A]** time = 0.100646, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{ag+bgx}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

[Out] a\*g\*Defer[Int][(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]<sup>-1</sup>), x] + b\*g\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

Rubi steps

$$\begin{aligned} \int \frac{ag+bgx}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx &= \int \left( \frac{ag}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} + \frac{bgx}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} \right) dx \\ &= (ag) \int \frac{1}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx + (bg) \int \frac{x}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.235532, size = 0, normalized size = 0.

$$\int \frac{ag+bgx}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

[Out] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

**Maple [A]** time = 0.961, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln\left(\frac{e(bx + a)}{dx + c}\right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] `int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bgx + ag}{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")`

[Out] `integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bgx + ag}{B \log\left(\frac{bex+ae}{dx+c}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")`

[Out] `integral((b*g*x + a*g)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bgx + ag}{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")`

[Out] `integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A), x)`

$$3.111 \quad \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{1}{(ag + bgx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}, x\right)$$

[Out] Unintegrable[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])), x]

**Rubi [A]** time = 0.0718844, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag + bgx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])),x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])), x]

Rubi steps

$$\int \frac{1}{(ag + bgx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)} dx = \int \frac{1}{(ag + bgx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)} dx$$

**Mathematica [A]** time = 0.225396, size = 0, normalized size = 0.

$$\int \frac{1}{(ag + bgx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])),x]

[Out] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])), x]

**Maple [A]** time = 1.183, size = 0, normalized size = 0.

$$\int \frac{1}{bgx + ag} \left(A + B \ln\left(\frac{e(bx + a)}{dx + c}\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(b\*x+a)/(d\*x+c))), x)

[Out] `int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")`

[Out] `integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{Abgx + Aag + (Bbgx + Bag) \log\left(\frac{bex+ae}{dx+c}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")`

[Out] `integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b*e*x + a*e)/(d*x + c))), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")`

[Out] `integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)`



$$3.112 \quad \int \frac{1}{(ag+bgx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

**Optimal.** Leaf size=50

$$\frac{ee^{A/B} \text{Ei}\left(-\frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{Bg^2(bc-ad)}$$

[Out] (e\*E^(A/B)\*ExpIntegralEi[-((A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))/B)])/ (B\*(b\*c - a\*d)\*g^2)

**Rubi [F]** time = 0.0900594, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag + bgx)^2 \left( A + B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))),x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))), x]

Rubi steps

$$\int \frac{1}{(ag + bgx)^2 \left( A + B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx = \int \frac{1}{(ag + bgx)^2 \left( A + B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

**Mathematica [A]** time = 0.102231, size = 52, normalized size = 1.04

$$\frac{ee^{A/B} \text{Ei}\left(-\frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{bBcg^2 - aBdg^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))),x]

[Out] (e\*E^(A/B)\*ExpIntegralEi[-((A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))/B)])/ (b\*B\*c\*g^2 - a\*B\*d\*g^2)

**Maple [F]** time = 1.187, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( A + B \ln\left(\frac{e(bx + a)}{dx + c}\right) \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] `int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(bx+a)e}{dx+c} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")`

[Out] `integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)`

**Fricas [A]** time = 1.03048, size = 108, normalized size = 2.16

$$\frac{e e^{\frac{A}{B}} \log\_integral \left( \frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae} \right)}{(Bbc - Bad)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")`

[Out] `e*e^(A/B)*log_integral((d*x + c)*e^(-A/B)/(b*e*x + a*e))/((B*b*c - B*a*d)*g^2)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(bx+a)e}{dx+c} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

$$3.113 \quad \int \frac{1}{(ag+bgx)^3 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

**Optimal.** Leaf size=107

$$\frac{be^2 e^{\frac{2A}{B}} \operatorname{Ei}\left(-\frac{2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{Bg^3(bc-ad)^2} - \frac{de e^{A/B} \operatorname{Ei}\left(-\frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{Bg^3(bc-ad)^2}$$

[Out]  $(b * e^{2 * E^{((2 * A) / B)} * \operatorname{ExpIntegralEi}[(-2 * (A + B * \operatorname{Log}[(e * (a + b * x)) / (c + d * x)])) / B]} / (B * (b * c - a * d)^{2 * g^3}) - (d * e * E^{(A / B)} * \operatorname{ExpIntegralEi}[(-(A + B * \operatorname{Log}[(e * (a + b * x)) / (c + d * x)] / B)]) / (B * (b * c - a * d)^{2 * g^3}))$

**Rubi [F]** time = 0.0729636, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag + bgx)^3 \left( A + B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

Verification is Not applicable to the result.

[In] `Int[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x]])),x]`

[Out] `Defer[Int][1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x]])), x]`

Rubi steps

$$\int \frac{1}{(ag + bgx)^3 \left( A + B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx = \int \frac{1}{(ag + bgx)^3 \left( A + B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

**Mathematica [A]** time = 0.165399, size = 89, normalized size = 0.83

$$\frac{e e^{A/B} \left( b e e^{A/B} \operatorname{Ei}\left(-\frac{2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right) - d \operatorname{Ei}\left(-\frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right) \right)}{Bg^3(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x]])),x]`

[Out]  $(e * E^{(A / B)} * (b * e * E^{(A / B)} * \operatorname{ExpIntegralEi}[(-2 * (A + B * \operatorname{Log}[(e * (a + b * x)) / (c + d * x)])) / B]} - d * \operatorname{ExpIntegralEi}[(-(A + B * \operatorname{Log}[(e * (a + b * x)) / (c + d * x)] / B)])) / (B * (b * c - a * d)^{2 * g^3})$

**Maple [F]** time = 1.283, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( A + B \ln\left(\frac{e(bx + a)}{dx + c}\right) \right)^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^3/(A+B\*ln(e\*(b\*x+a)/(d\*x+c))),x)

[Out] int(1/(b\*g\*x+a\*g)^3/(A+B\*ln(e\*(b\*x+a)/(d\*x+c))),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log \left( \frac{(bx+a)e}{dx+c} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="maxima")

[Out] integrate(1/((b\*g\*x + a\*g)^3\*(B\*log((b\*x + a)\*e/(d\*x + c)) + A)), x)

**Fricas [A]** time = 1.00978, size = 288, normalized size = 2.69

$$\frac{be^2 e^{\left(\frac{2A}{B}\right)} \log\_integral \left( \frac{(d^2x^2 + 2cdx + c^2)e^{\left(-\frac{2A}{B}\right)}}{b^2e^2x^2 + 2abe^2x + a^2e^2} \right) - dee^{\frac{A}{B}} \log\_integral \left( \frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae} \right)}{(Bb^2c^2 - 2Babcd + Ba^2d^2)g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="fricas")

[Out] (b\*e^2\*e^(2\*A/B)\*log\_integral((d^2\*x^2 + 2\*c\*d\*x + c^2)\*e^(-2\*A/B)/(b^2\*e^2\*x^2 + 2\*a\*b\*e^2\*x + a^2\*e^2)) - d\*e\*e^(A/B)\*log\_integral((d\*x + c)\*e^(-A/B)/(b\*e\*x + a\*e)))/((B\*b^2\*c^2 - 2\*B\*a\*b\*c\*d + B\*a^2\*d^2)\*g^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)\*\*3/(A+B\*ln(e\*(b\*x+a)/(d\*x+c))),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log \left( \frac{(bx+a)e}{dx+c} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)
```

$$3.114 \quad \int \frac{(ag+bgx)^2}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{(ag + bgx)^2}{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)^2}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2, x]

**Rubi [A]** time = 0.206123, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(ag + bgx)^2}{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2,x]

[Out] a^2\*g^2\*Defer[Int][(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^(-2), x] + 2\*a\*b\*g^2\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2, x] + b^2\*g^2\*Defer[Int][x^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{(ag + bgx)^2}{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx &= \int \left( \frac{a^2g^2}{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} + \frac{2abg^2x}{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} + \frac{b^2g^2x^2}{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} \right) dx \\ &= (a^2g^2) \int \frac{1}{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx + (2abg^2) \int \frac{x}{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx + (b^2g^2) \int \frac{x^2}{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 1.32327, size = 0, normalized size = 0.

$$\int \frac{(ag + bgx)^2}{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2,x]

[Out] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2, x]

**Maple [A]** time = 1.042, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

[Out] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{b^3 dg^2 x^4 + a^3 cg^2 + (b^3 cg^2 + 3 ab^2 dg^2)x^3 + 3(ab^2 cg^2 + a^2 bdg^2)x^2 + (3 a^2 bcg^2 + a^3 dg^2)x}{(bc - ad)B^2 \log(bx + a) - (bc - ad)B^2 \log(dx + c) + (bc - ad)AB + (bc \log(e) - ad \log(e))B^2} + \int \frac{4 b^3 dg^2 x^3 + 3 a^3 dg^2}{(bc - ad)B^2 \log(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="maxima")

[Out]  $-(b^3 d g^2 x^4 + a^3 c g^2 + (b^3 c g^2 + 3 a b^2 d g^2) x^3 + 3 (a b^2 c g^2 + a^2 b d g^2) x^2 + (3 a^2 b c g^2 + a^3 d g^2) x) / ((b c - a d) B^2 \log(b x + a) - (b c - a d) B^2 \log(d x + c) + (b c - a d) A B + (b c \log(e) - a d \log(e)) B^2) + \text{integrate}((4 b^3 d g^2 x^3 + 3 a^2 b c g^2 + a^3 d g^2 + 3 (b^3 c g^2 + 3 a b^2 d g^2) x^2 + 6 (a b^2 c g^2 + a^2 b d g^2) x) / ((b c - a d) B^2 \log(b x + a) - (b c - a d) B^2 \log(d x + c) + (b c - a d) A B + (b c \log(e) - a d \log(e)) B^2), x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b^2 g^2 x^2 + 2 a b g^2 x + a^2 g^2}{B^2 \log \left( \frac{b e x + a e}{d x + c} \right)^2 + 2 A B \log \left( \frac{b e x + a e}{d x + c} \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out] integral((b^2\*g^2\*x^2 + 2\*a\*b\*g^2\*x + a^2\*g^2)/(B^2\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*A\*B\*log((b\*e\*x + a\*e)/(d\*x + c)) + A^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*2/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2,x)



[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^2/(B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2, x)

$$3.115 \quad \int \frac{ag+bgx}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

**Optimal.** Leaf size=32

$$\text{Unintegrable}\left(\frac{ag+bgx}{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2, x]

**Rubi [A]** time = 0.106785, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{ag+bgx}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2, x]

[Out] a\*g\*Defer[Int][(A + B\*Log[(e\*(a + b\*x))/(c + d\*x))]^(-2), x] + b\*g\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x))]^2, x]

Rubi steps

$$\begin{aligned} \int \frac{ag+bgx}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx &= \int \left( \frac{ag}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} + \frac{bgx}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} \right) dx \\ &= (ag) \int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx + (bg) \int \frac{x}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.912937, size = 0, normalized size = 0.

$$\int \frac{ag+bgx}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2, x]

[Out] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2, x]

**Maple [A]** time = 1.043, size = 0, normalized size = 0.

$$\int (bgx+ag) \left( A + B \ln\left(\frac{e(bx+a)}{dx+c}\right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)`

[Out] `int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{b^2 d g x^3 + a^2 c g + (b^2 c g + 2 a b d g) x^2 + (2 a b c g + a^2 d g) x}{(b c - a d) B^2 \log(b x + a) - (b c - a d) B^2 \log(d x + c) + (b c - a d) A B + (b c \log(e) - a d \log(e)) B^2} + \int \frac{1}{(b c - a d) B^2 \log(b x + a) - (b c - a d) B^2 \log(d x + c) + (b c - a d) A B + (b c \log(e) - a d \log(e)) B^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")`

[Out] `-(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b^2*d*g*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b g x + a g}{B^2 \log\left(\frac{b e x + a e}{d x + c}\right)^2 + 2 A B \log\left(\frac{b e x + a e}{d x + c}\right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")`

[Out] `integral((b*g*x + a*g)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x + c)) + A^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b g x + a g}{\left(B \log\left(\frac{b x + a e}{d x + c}\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

$$3.116 \quad \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{1}{(ag+bgx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

**Rubi [A]** time = 0.0808696, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx = \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

**Mathematica [A]** time = 0.618128, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

[Out] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

**Maple [A]** time = 1.042, size = 0, normalized size = 0.

$$\int \frac{1}{bgx+ag}\left(A+B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

[Out] int(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$d \int \frac{1}{(bcg - adg)B^2 \log(bx + a) - (bcg - adg)B^2 \log(dx + c) + (bcg - adg)AB + (bcg \log(e) - adg \log(e))B^2} dx - \frac{1}{(bcg - adg)B^2 \log(bx + a) - (bcg - adg)B^2 \log(dx + c) + (bcg - adg)AB + (bcg \log(e) - adg \log(e))B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="maxima")

[Out] d\*integrate(1/((b\*c\*g - a\*d\*g)\*B^2\*log(b\*x + a) - (b\*c\*g - a\*d\*g)\*B^2\*log(d\*x + c) + (b\*c\*g - a\*d\*g)\*A\*B + (b\*c\*g\*log(e) - a\*d\*g\*log(e))\*B^2), x) - (d\*x + c)/((b\*c\*g - a\*d\*g)\*B^2\*log(b\*x + a) - (b\*c\*g - a\*d\*g)\*B^2\*log(d\*x + c) + (b\*c\*g - a\*d\*g)\*A\*B + (b\*c\*g\*log(e) - a\*d\*g\*log(e))\*B^2)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2bgx + A^2ag + (B^2bgx + B^2ag) \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2(ABbgx + ABag) \log\left(\frac{bex+ae}{dx+c}\right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out] integral(1/(A^2\*b\*g\*x + A^2\*a\*g + (B^2\*b\*g\*x + B^2\*a\*g)\*log((b\*e\*x + a\*e)/(d\*x + c)))^2 + 2\*(A\*B\*b\*g\*x + A\*B\*a\*g)\*log((b\*e\*x + a\*e)/(d\*x + c))), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag) \left( B \log\left(\frac{(bx+a)e}{dx+c}\right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

$$3.117 \quad \int \frac{1}{(ag+bgx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx$$

**Optimal.** Leaf size=103

$$\frac{ee^{A/B} \text{Ei}\left(-\frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2 g^2 (bc-ad)} - \frac{c+dx}{Bg^2(a+bx)(bc-ad) \left( B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}$$

[Out]  $-\left(\left(e \cdot E^{(A/B)} \cdot \text{ExpIntegralEi}\left[-\left(\frac{A + B \cdot \text{Log}\left[\frac{e \cdot (a + b \cdot x)}{c + d \cdot x}\right]}{B}\right)\right]\right) / \left(B^2 \cdot (b \cdot c - a \cdot d) \cdot g^2\right) - (c + d \cdot x) / \left(B \cdot (b \cdot c - a \cdot d) \cdot g^2 \cdot (a + b \cdot x) \cdot \left(A + B \cdot \text{Log}\left[\frac{e \cdot (a + b \cdot x)}{c + d \cdot x}\right]\right)\right)\right)$

**Rubi [F]** time = 0.094921, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx = \int \frac{1}{(ag+bgx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx$$

**Mathematica [A]** time = 0.187646, size = 87, normalized size = 0.84

$$\frac{ee^{A/B} \text{Ei}\left(-\frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right) + \frac{B(c+dx)}{(a+bx) \left( B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}}{B^2 g^2 (ad-bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

[Out]  $\left(e \cdot E^{(A/B)} \cdot \text{ExpIntegralEi}\left[-\left(\frac{A + B \cdot \text{Log}\left[\frac{e \cdot (a + b \cdot x)}{c + d \cdot x}\right]}{B}\right)\right] + \frac{B \cdot (c + d \cdot x)}{(a + b \cdot x) \cdot \left(A + B \cdot \text{Log}\left[\frac{e \cdot (a + b \cdot x)}{c + d \cdot x}\right]\right)}\right) / \left(B^2 \cdot (-b \cdot c + a \cdot d) \cdot g^2\right)$



**Maple [F]** time = 1.274, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2} \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

[Out] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{dx + \left( (abcg^2 - a^2dg^2)AB + (abcg^2 \log(e) - a^2dg^2 \log(e))B^2 + ((b^2cg^2 - abd^2)AB + (b^2cg^2 \log(e) - abd^2 \log(e))B^2) \right) x}{(abcg^2 - a^2dg^2)AB + (abcg^2 \log(e) - a^2dg^2 \log(e))B^2 + ((b^2cg^2 - abd^2)AB + (b^2cg^2 \log(e) - abd^2 \log(e))B^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="maxima")

[Out]  $-(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*\log(e) - a^2*d*g^2*\log(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*\log(e) - a*b*d*g^2*\log(e))*B^2)*x + ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*\log(b*x + a) - ((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*\log(d*x + c) + \text{integrate}(-1/(B^2*a^2*g^2*\log(e) + A*B*a^2*g^2 + (B^2*b^2*g^2*\log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*\log(e) + A*B*a*b*g^2)*x + (B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*\log(b*x + a) - (B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*\log(d*x + c)), x)$

**Fricas [A]** time = 1.02172, size = 423, normalized size = 4.11

$$\frac{Bdx + Bc + \left( (Bbex + Bae)e^{\frac{A}{B}} \log\left(\frac{bex+ae}{dx+c}\right) + (Abex + Aae)e^{\frac{A}{B}} \right) \log\_integral\left(\frac{(dx+c)e^{\left(-\frac{A}{B}\right)}}{bex+ae}\right)}{\left( AB^2b^2c - AB^2abd \right) g^2 x + \left( AB^2abc - AB^2a^2d \right) g^2 + \left( (B^3b^2c - B^3abd) g^2 x + (B^3abc - B^3a^2d) g^2 \right) \log\left(\frac{bex+ae}{dx+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out]  $-(B*d*x + B*c + ((B*b*e*x + B*a*e)*e^{(A/B)}*\log((b*e*x + a*e)/(d*x + c)) + (A*b*e*x + A*a*e)*e^{(A/B)})*\log\_integral(((d*x + c)*e^{(-A/B)/(b*e*x + a*e)}))/((A*B^2*b^2*c - A*B^2*a*b*d)*g^2*x + (A*B^2*a*b*c - A*B^2*a^2*d)*g^2 + ((B^3*b^2*c - B^3*a*b*d)*g^2*x + (B^3*a*b*c - B^3*a^2*d)*g^2)*\log((b*e*x + a*e)/(d*x + c))$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)\*\*2/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(bx+a)e}{dx+c} \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="giac")

[Out] integrate(1/((b\*g\*x + a\*g)^2\*(B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2), x)

$$3.118 \quad \int \frac{1}{(ag+bgx)^3 \left( A+B \log\left( \frac{e(a+bx)}{c+dx} \right) \right)^2} dx$$

**Optimal.** Leaf size=212

$$\frac{2be^2 e^{\frac{2A}{B}} \operatorname{Ei}\left(-\frac{2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right)}{B^2 g^3 (bc-ad)^2} + \frac{dee^{A/B} \operatorname{Ei}\left(-\frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right)}{B^2 g^3 (bc-ad)^2} - \frac{b(c+dx)^2}{Bg^3(a+bx)^2(bc-ad)^2 \left( B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)} + \frac{1}{Bg^3}$$

[Out]  $(-2*b*e^2*E^{\left(\frac{2*A}{B}\right)}*ExpIntegralEi\left[\frac{-2*(A+B*Log\left[\frac{e*(a+b*x)}{c+d*x}\right])}{B}\right])/(B^2*(b*c-a*d)^2*g^3) + (d*e*E^{A/B}*ExpIntegralEi\left[\frac{-\left(A+B*Log\left[\frac{e*(a+b*x)}{c+d*x}\right]\right)}{B}\right])/(B^2*(b*c-a*d)^2*g^3) + (d*(c+d*x))/(B*(b*c-a*d)^2*g^3*(a+b*x)*(A+B*Log\left[\frac{e*(a+b*x)}{c+d*x}\right])) - (b*(c+d*x)^2)/(B*(b*c-a*d)^2*g^3*(a+b*x)^2*(A+B*Log\left[\frac{e*(a+b*x)}{c+d*x}\right]))$

**Rubi [F]** time = 0.0837941, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)^3 \left( A+B \log\left( \frac{e(a+bx)}{c+dx} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)^3 \left( A+B \log\left( \frac{e(a+bx)}{c+dx} \right) \right)^2} dx = \int \frac{1}{(ag+bgx)^3 \left( A+B \log\left( \frac{e(a+bx)}{c+dx} \right) \right)^2} dx$$

**Mathematica [A]** time = 0.757335, size = 136, normalized size = 0.64

$$\frac{-2be^2 e^{\frac{2A}{B}} \operatorname{Ei}\left(-\frac{2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}\right) + dee^{A/B} \operatorname{Ei}\left(-\frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{B}\right) - \frac{B(c+dx)(bc-ad)}{(a+bx)^2 \left( B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}}{B^2 g^3 (bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

[Out]  $(-2*b*e^2*E^{\left(\frac{2*A}{B}\right)}*ExpIntegralEi\left[\frac{-2*(A+B*Log\left[\frac{e*(a+b*x)}{c+d*x}\right])}{B}\right]) + d*e*E^{A/B}*ExpIntegralEi\left[\frac{-\left(A+B*Log\left[\frac{e*(a+b*x)}{c+d*x}\right]\right)}{B}\right] - (B*(b*c-a*d)*(c+d*x))/((a+b*x)^2*(A+B*Log\left[\frac{e*(a+b*x)}{c+d*x}\right]))/(B^2*(b*c-a*d)^2*g^3)$

---

**Maple [F]** time = 1.43, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3} \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^3/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

[Out] int(1/(b\*g\*x+a\*g)^3/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="maxima")

[Out]  $-(dx + c)/((a^2bcg^3 - a^3d^2g^3)AB + (a^2bcg^3 \log(e) - a^3d^2g^3 \log(e))B^2 + ((b^3c^2g^3 - ab^2d^2g^3)AB + (b^3c^2g^3 \log(e) - ab^2d^2g^3 \log(e))B^2)x^2 + 2((ab^2c^2g^3 - a^2b^2d^2g^3)AB + (ab^2c^2g^3 \log(e) - a^2b^2d^2g^3 \log(e))B^2)x + ((b^3c^2g^3 - ab^2d^2g^3)B^2x^2 + 2(ab^2c^2g^3 - a^2b^2d^2g^3)B^2x + (a^2bcg^3 - a^3d^2g^3)B^2) \log(bx + a) - ((b^3c^2g^3 - ab^2d^2g^3)B^2x^2 + 2(ab^2c^2g^3 - a^2b^2d^2g^3)B^2x + (a^2bcg^3 - a^3d^2g^3)B^2) \log(dx + c)) - \text{integrate}((b^2dx + 2bc - ad)/((b^4c^2g^3 - ab^3d^2g^3)AB + (b^4c^2g^3 \log(e) - ab^3d^2g^3 \log(e))B^2)x^3 + (a^3bc^2g^3 - a^4d^2g^3)AB + (a^3bc^2g^3 \log(e) - a^4d^2g^3 \log(e))B^2 + 3((ab^3c^2g^3 - a^2b^2d^2g^3)AB + (ab^3c^2g^3 \log(e) - a^2b^2d^2g^3 \log(e))B^2)x^2 + 3((a^2b^2c^2g^3 - a^3b^2d^2g^3)AB + (a^2b^2c^2g^3 \log(e) - a^3b^2d^2g^3 \log(e))B^2)x + ((b^4c^2g^3 - ab^3d^2g^3)B^2x^3 + 3(ab^3c^2g^3 - a^2b^2d^2g^3)B^2x^2 + 3(a^2b^2c^2g^3 - a^3b^2d^2g^3)B^2x + (a^3bc^2g^3 - a^4d^2g^3)B^2) \log(bx + a) - ((b^4c^2g^3 - ab^3d^2g^3)B^2x^3 + 3(ab^3c^2g^3 - a^2b^2d^2g^3)B^2x^2 + 3(a^2b^2c^2g^3 - a^3b^2d^2g^3)B^2x + (a^3bc^2g^3 - a^4d^2g^3)B^2) \log(dx + c)), x)$

---

**Fricas [B]** time = 1.1091, size = 1187, normalized size = 5.6

$$Bbc^2 - Bacd + (Bbcd - Bad^2)x - \left( (Bb^2dex^2 + 2Babdex + Ba^2de) e^{\frac{A}{B}} \log\left(\frac{bex+ae}{dx+c}\right) + (Ab^2dex^2 + 2Aabdex + Aa^2de) e^{\frac{A}{B}} \right)$$

$$(AB^2b^4c^2 - 2AB^2ab^3cd + AB^2a^2b^2d^2)g^3x^2 + 2(AB^2ab^3c^2 - 2AB^2a^2b^2cd + AB^2a^3bd^2)g^3x + (AB^2a^2b^2c^2 - 2AB^2a^3bd^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out]  $-(B^2bc^2 - B^2acd + (B^2bcd - B^2ad^2))x - ((B^2b^2dex^2 + 2B^2abdex + B^2a^2de) e^{A/B} \log((bex + ae)/(dx + c)) + (A^2b^2dex^2 + 2A^2abdex + A^2a^2de) e^{A/B})$

```
*a*b*d*e*x + A*a^2*d*e)*e^(A/B))*log_integral((d*x + c)*e^(-A/B)/(b*e*x + a
*e)) + 2*((B*b^3*e^2*x^2 + 2*B*a*b^2*e^2*x + B*a^2*b*e^2)*e^(2*A/B))*log((b*
e*x + a*e)/(d*x + c)) + (A*b^3*e^2*x^2 + 2*A*a*b^2*e^2*x + A*a^2*b*e^2)*e^(
2*A/B))*log_integral((d^2*x^2 + 2*c*d*x + c^2)*e^(-2*A/B)/(b^2*e^2*x^2 + 2*
a*b*e^2*x + a^2*e^2)))/((A*B^2*b^4*c^2 - 2*A*B^2*a*b^3*c*d + A*B^2*a^2*b^2*
d^2)*g^3*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^2*c*d + A*B^2*a^3*b*d^2)*
g^3*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d + A*B^2*a^4*d^2)*g^3 + ((B^3
*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*g^3*x^2 + 2*(B^3*a*b^3*c^2 -
2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*g^3*x + (B^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c
*d + B^3*a^4*d^2)*g^3)*log((b*e*x + a*e)/(d*x + c)))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log\left(\frac{bx+a}{dx+c}\right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac
")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2), x)
```

$$3.119 \quad \int (ag + bgx)^4 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx$$

**Optimal.** Leaf size=182

$$\frac{g^4(a+bx)^5 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}{5b} + \frac{2Bg^4x(bc-ad)^4}{5d^4} - \frac{Bg^4(a+bx)^2(bc-ad)^3}{5bd^3} + \frac{2Bg^4(a+bx)^3(bc-ad)^2}{15bd^2} - \frac{2Bg^4(bc-ad)}{5bd}$$

[Out]  $(2*B*(b*c - a*d)^4*g^4*x)/(5*d^4) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(5*b*d^3) + (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*g^4*(a + b*x)^4)/(10*b*d) + (g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*b) - (2*B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5)$

**Rubi [A]** time = 0.109519, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 43}

$$\frac{g^4(a+bx)^5 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}{5b} + \frac{2Bg^4x(bc-ad)^4}{5d^4} - \frac{Bg^4(a+bx)^2(bc-ad)^3}{5bd^3} + \frac{2Bg^4(a+bx)^3(bc-ad)^2}{15bd^2} - \frac{2Bg^4(bc-ad)}{5bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]$

[Out]  $(2*B*(b*c - a*d)^4*g^4*x)/(5*d^4) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(5*b*d^3) + (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) - (B*(b*c - a*d)*g^4*(a + b*x)^4)/(10*b*d) + (g^4*(a + b*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*b) - (2*B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5)$

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol]
:> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned}
\int (ag + bgx)^4 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx &= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{5b} - \frac{B \int \frac{2(bc-ad)g^5(a+bx)^4}{c+dx} dx}{5bg} \\
&= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{5b} - \frac{(2B(bc-ad)g^4) \int \frac{(a+bx)^4}{c+dx} dx}{5b} \\
&= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{5b} - \frac{(2B(bc-ad)g^4) \int \left( -\frac{b(bc-ad)^3}{d^4} + \frac{b(bc-ad)^2}{d^3} \right) dx}{5b} \\
&= \frac{2B(bc-ad)^4 g^4 x}{5d^4} - \frac{B(bc-ad)^3 g^4 (a+bx)^2}{5bd^3} + \frac{2B(bc-ad)^2 g^4 (a+bx)^3}{15bd^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0956361, size = 144, normalized size = 0.79

$$\frac{g^4 \left( (a+bx)^5 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right) + \frac{B(bc-ad)(-6d^2(a+bx)^2(bc-ad)^2 + 4d^3(a+bx)^3(bc-ad) + 12bdx(bc-ad)^3 - 12(bc-ad)^4 \log(c+dx) - 3d^4(a+bx)^4)}{6d^5} \right)}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^4\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]),x]

[Out] (g^4\*((a + b\*x)^5\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) + (B\*(b\*c - a\*d) \* (12\*b\*d\*(b\*c - a\*d)^3\*x - 6\*d^2\*(b\*c - a\*d)^2\*(a + b\*x)^2 + 4\*d^3\*(b\*c - a\*d)\*(a + b\*x)^3 - 3\*d^4\*(a + b\*x)^4 - 12\*(b\*c - a\*d)^4\*Log[c + d\*x])))/(6\*d^5))/(5\*b)

**Maple [B]** time = 0.423, size = 1606, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^4\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2)),x)

[Out] 1/10\*B\*g^4\*b^3\*a\*x^4-113/30/d^4\*B\*g^4\*b^3\*a\*c^4+2/5/d^5\*B\*g^4\*b^4\*c^5\*ln(1/(d\*x+c))+8/d\*B\*g^4\*a^4\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*c+2/d\*B\*g^4\*a^4\*ln(1/(d\*x+c))\*c+1/d\*B\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*a^4\*c\*g^4+1/5/d^5\*B\*g^4\*b^4\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*c^5+8/5/d^5\*B\*g^4\*b^4\*c^5\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)+2\*B\*g^4\*b\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*a^3\*x^2-12\*B\*g^4/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a^5\*c+2\*B\*g^4\*b^2\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*a^2\*x^3+B\*g^4\*b^3\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*a\*x^4-1/10/d\*B\*g^4\*b^4\*c\*x^4+5/6/d^5\*B\*g^4\*b^4\*c^5+1/5/d^5\*A\*g^4\*b^4\*c^5+1/d\*A\*g^4\*a^4\*c+8/5/d\*B\*g^4\*a^4\*c+8/15\*B\*g^4\*b^2\*a^2\*x^3+1/5\*B\*g^4\*b^4\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*x^5+B\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*x\*a^4\*g^4-2/5\*B\*g^4/b\*a^5\*ln(1/(d\*x+c))-8/5\*B\*g^4/b\*a^5\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)+A\*g^4\*x^4\*a\*b^3+2\*A\*g^4\*x^3\*a^2\*b^2+2\*A\*g^4\*x^2\*a^3\*b+6/5\*B\*g^4\*b\*a^3\*x^2-1/d^4\*A\*g^4\*a\*b^3\*c^4+2/d^3\*A\*g^4\*a^2\*b^2\*c^3-2/d^2\*A\*g^4\*a^3\*b\*c^2-26/5/d^2\*B\*g^4\*b\*a^3\*c^2+98/15/d^3\*B\*g^4\*b^2\*a^2\*c^3+8/5\*B\*x\*a^4\*g^4+1/5\*A\*g^4\*x^5\*b^4+A\*g^4\*x\*a^4-12/d^4\*B\*g^4/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a\*c^5\*b^4-40/d^2\*B\*g^4\*b^2/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a^3\*c^3+30/d\*B\*g^4\*b/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a^4\*c^2+30/d^3\*B\*g^4\*b^3/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a^2\*c^4+2/15/d^2\*B\*g^4\*b^4\*c^2\*x^3-1/5/

$$d^3 B g^4 b^4 c^3 x^2 + 2/5 d^4 B g^4 b^4 c^4 x - 2/d^3 B g^4 b^3 a x c^3 - 4/d B g^4 b^3 a^3 x c - 1/d^4 B g^4 b^3 \ln(e^{(1/(d*x+c))*a*d-b*c/(d*x+c)+b})^2/d^2) * a c^4 + 2/d^3 B g^4 \ln(e^{(1/(d*x+c))*a*d-b*c/(d*x+c)+b})^2/d^2) * a^2 b^2 c^3 - 16/d^2 B g^4 b^3 a^3 \ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b) * c^2 + 16/d^3 B g^4 b^2 a^2 \ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b) * c^3 - 8/d^4 B g^4 b^3 a \ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b) * c^4 - 4/d^2 B g^4 b^3 a^3 \ln(1/(d*x+c)) * c^2 + 2*d B g^4/b/(a*d-b*c) * \ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b) * a^6 + 4/d^3 B g^4 b^2 a^2 \ln(1/(d*x+c)) * c^3 - 2/d^4 B g^4 b^3 a \ln(1/(d*x+c)) * c^4 + 2/d^5 B g^4/(a*d-b*c) * \ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b) * c^6 b^5 - 2/d^2 B g^4 \ln(e^{(1/(d*x+c))*a*d-b*c/(d*x+c)+b})^2/d^2) * a^3 b c^2 - 2/3/d B g^4 b^3 a x^3 c + 1/d^2 B g^4 b^3 a x^2 c^2 - 2/d B g^4 b^2 a^2 x^2 c + 4/d^2 B g^4 b^2 a^2 x c^2$$

**Maxima [B]** time = 1.38107, size = 1195, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="maxima")

[Out]  $1/5 A b^4 g^4 x^5 + A a b^3 g^4 x^4 + 2 A a^2 b^2 g^4 x^3 + 2 A a^3 b g^4 x^2 + (x \log(b^2 e x^2 / (d^2 x^2 + 2 c d x + c^2)) + 2 a b e x / (d^2 x^2 + 2 c d x + c^2) + a^2 e / (d^2 x^2 + 2 c d x + c^2)) + 2 a \log(b x + a) / b - 2 c \log(d x + c) / d) B a^4 g^4 + 2 (x^2 \log(b^2 e x^2 / (d^2 x^2 + 2 c d x + c^2)) + 2 a b e x / (d^2 x^2 + 2 c d x + c^2) + a^2 e / (d^2 x^2 + 2 c d x + c^2)) - 2 a^2 \log(b x + a) / b^2 + 2 c^2 \log(d x + c) / d^2 - 2 (b c - a d) x / (b d)) B a^3 b g^4 + 2 (x^3 \log(b^2 e x^2 / (d^2 x^2 + 2 c d x + c^2)) + 2 a b e x / (d^2 x^2 + 2 c d x + c^2) + a^2 e / (d^2 x^2 + 2 c d x + c^2)) + 2 a^3 \log(b x + a) / b^3 - 2 c^3 \log(d x + c) / d^3 - ((b^2 c d - a b d^2) x^2 - 2 (b^2 c^2 - a^2 d^2) x) / (b^2 d^2)) B a^2 b^2 g^4 + 1/3 (3 x^4 \log(b^2 e x^2 / (d^2 x^2 + 2 c d x + c^2)) + 2 a b e x / (d^2 x^2 + 2 c d x + c^2) + a^2 e / (d^2 x^2 + 2 c d x + c^2)) - 6 a^4 \log(b x + a) / b^4 + 6 c^4 \log(d x + c) / d^4 - (2 (b^3 c d^2 - a b^2 d^3) x^3 - 3 (b^3 c^2 d - a^2 b d^3) x^2 + 6 (b^3 c^3 - a^3 d^3) x) / (b^3 d^3)) B a b^3 g^4 + 1/30 (6 x^5 \log(b^2 e x^2 / (d^2 x^2 + 2 c d x + c^2)) + 2 a b e x / (d^2 x^2 + 2 c d x + c^2) + a^2 e / (d^2 x^2 + 2 c d x + c^2)) + 12 a^5 \log(b x + a) / b^5 - 12 c^5 \log(d x + c) / d^5 - (3 (b^4 c d^3 - a b^3 d^4) x^4 - 4 (b^4 c^2 d^2 - a^2 b^2 d^4) x^3 + 6 (b^4 c^3 d - a^3 b d^4) x^2 - 12 (b^4 c^4 - a^4 d^4) x) / (b^4 d^4)) B b^4 g^4 + A a^4 g^4 x$

**Fricas [B]** time = 1.30286, size = 952, normalized size = 5.23

$$6 A b^5 d^5 g^4 x^5 + 12 B a^5 d^5 g^4 \log(bx + a) - 3 (B b^5 c d^4 - (10 A + B) a b^4 d^5) g^4 x^4 + 4 (B b^5 c^2 d^3 - 5 B a b^4 c d^4 + (15 A + 4 B) a^2 b^3 d^5) g^4 x^3 - 6 (B b^5 c^3 d^2 - 5 B a b^4 c^2 d^3 + 10 B a^2 b^3 c d^4 - 2 (5 A + 3 B) a^3 b^2 d^5) g^4 x^2 + 6 (2 B b^5 c^4 d - 10$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="fricas")

[Out]  $1/30 (6 A b^5 d^5 g^4 x^5 + 12 B a^5 d^5 g^4 \log(bx + a) - 3 (B b^5 c d^4 - (10 A + B) a b^4 d^5) g^4 x^4 + 4 (B b^5 c^2 d^3 - 5 B a b^4 c d^4 + (15 A + 4 B) a^2 b^3 d^5) g^4 x^3 - 6 (B b^5 c^3 d^2 - 5 B a b^4 c^2 d^3 + 10 B a^2 b^3 c d^4 - 2 (5 A + 3 B) a^3 b^2 d^5) g^4 x^2 + 6 (2 B b^5 c^4 d - 10$



$$*B*a*b^4*c^3*d^2 + 20*B*a^2*b^3*c^2*d^3 - 20*B*a^3*b^2*c*d^4 + (5*A + 8*B)*a^4*b*d^5)*g^4*x - 12*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*B*a^4*b*c*d^4)*g^4*\log(d*x + c) + 6*(B*b^5*d^5*g^4*x^5 + 5*B*a*b^4*d^5*g^4*x^4 + 10*B*a^2*b^3*d^5*g^4*x^3 + 10*B*a^3*b^2*d^5*g^4*x^2 + 5*B*a^4*b*d^5*g^4*x)*\log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))/(b*d^5)$$

**Sympy [B]** time = 8.4322, size = 1018, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*4\*(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2)),x)

[Out]  $A*b**4*g**4*x**5/5 + 2*B*a**5*g**4*\log(x + (2*B*a**6*d**5*g**4/b + 10*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a**b**4*c**5*g**4)/(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*b) - 2*B*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)*\log(x + (12*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b**4*c**5*g**4 - 2*B*a*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4) + 2*B*b*c**2*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*d**5) + (B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x**3 + B*a*b**3*g**4*x**4 + B*b**4*g**4*x**5/5)*\log(e*(a + b*x)**2/(c + d*x)**2) + x**4*(10*A*a*b**3*d*g**4 + B*a*b**3*d*g**4 - B*b**4*c*g**4)/(10*d) + x**3*(30*A*a**2*b**2*d**2*g**4 + 8*B*a**2*b**2*d**2*g**4 - 10*B*a*b**3*c*d*g**4 + 2*B*b**4*c**2*g**4)/(15*d**2) + x**2*(10*A*a**3*b*d**3*g**4 + 6*B*a**3*b*d**3*g**4 - 10*B*a**2*b**2*c*d**2*g**4 + 5*B*a*b**3*c**2*d*g**4 - B*b**4*c**3*g**4)/(5*d**3) + x*(5*A*a**4*d**4*g**4 + 8*B*a**4*d**4*g**4 - 20*B*a**3*b*c*d**3*g**4 + 20*B*a**2*b**2*c**2*d**2*g**4 - 10*B*a*b**3*c**3*d*g**4 + 2*B*b**4*c**4*g**4)/(5*d**4)$

**Giac [B]** time = 162.17, size = 670, normalized size = 3.68

$$\frac{2Ba^5g^4 \log(bx+a)}{5b} + \frac{1}{5}(Ab^4g^4 + Bb^4g^4)x^5 - \frac{(Bb^4cg^4 - 10Aab^3dg^4 - 11Bab^3dg^4)x^4}{10d} + \frac{2(Bb^4c^2g^4 - 5Bab^3cdg^4 + 10Aa^2b^3dg^4 - 10Aa^2b^3dg^4 - 11Bab^3dg^4)x^3}{10d} + \frac{2(Bb^4c^2g^4 - 5Bab^3cdg^4 + 10Aa^2b^3dg^4 - 10Aa^2b^3dg^4 - 11Bab^3dg^4)x^2}{10d} + \frac{2(Bb^4c^2g^4 - 5Bab^3cdg^4 + 10Aa^2b^3dg^4 - 10Aa^2b^3dg^4 - 11Bab^3dg^4)x}{10d} + \frac{2(Bb^4c^2g^4 - 5Bab^3cdg^4 + 10Aa^2b^3dg^4 - 10Aa^2b^3dg^4 - 11Bab^3dg^4)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="giac")

[Out]  $2/5*B*a^5*g^4*\log(b*x + a)/b + 1/5*(A*b^4*g^4 + B*b^4*g^4)*x^5 - 1/10*(B*b^4*c*g^4 - 10*A*a*b^3*d*g^4 - 11*B*a*b^3*d*g^4)*x^4/d + 2/15*(B*b^4*c^2*g^4 - 5*B*a*b^3*c*d*g^4 + 15*A*a^2*b^2*d^2*g^4 + 19*B*a^2*b^2*d^2*g^4)*x^3/d^2 + 1/5*(B*b^4*g^4*x^5 + 5*B*a*b^3*g^4*x^4 + 10*B*a^2*b^2*g^4*x^3 + 10*B*a^3*b*g^4*x^2 + 5*B*a^4*g^4*x)*\log((b^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2)) - 1/5*(B*b^4*c^3*g^4 - 5*B*a*b^3*c^2*d*g^4 + 10*B*a^2*b^2*c*d^2*g^4 - 10*B*a^3*b*c*d^2*g^4 - 5*B*a^4*b*d^2*g^4 + 10*B*a^5*d^2*g^4)/(5*d^5)$

$$\begin{aligned}
& 4 - 10Aa^3bd^3g^4 - 16Ba^3bd^3g^4)x^2/d^3 + 1/5(2Bb^4c^4g^4 \\
& - 10Bab^3c^3d^3g^4 + 20Ba^2b^2c^2d^2g^4 - 20Ba^3b^3cd^3g^4 + \\
& 5Aa^4d^4g^4 + 13Ba^4d^4g^4)x/d^4 - 2/5(Bb^4c^5g^4 - 5Bab^3 \\
& c^4d^3g^4 + 10Ba^2b^2c^3d^2g^4 - 10Ba^3b^3c^2d^3g^4 + 5Ba^4c^3 \\
& d^4g^4)\log(-dx - c)/d^5
\end{aligned}$$

$$3.120 \quad \int (ag + bgx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx$$

**Optimal.** Leaf size=151

$$\frac{g^3(a+bx)^4 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}{4b} - \frac{Bg^3x(bc-ad)^3}{2d^3} + \frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd^2} + \frac{Bg^3(bc-ad)^4 \log(c+dx)}{2bd^4} - \frac{Bg^3(a+bx)^4 \log(c+dx)}{4bd^4}$$

[Out]  $-(B*(b*c - a*d)^3*g^3*x)/(2*d^3) + (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(4*b*d^2) - (B*(b*c - a*d)*g^3*(a + b*x)^3)/(6*b*d) + (g^3*(a + b*x)^4*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]))/(4*b) + (B*(b*c - a*d)^4*g^3*\text{Log}[c + d*x])/(2*b*d^4)$

**Rubi [A]** time = 0.0970548, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 43}

$$\frac{g^3(a+bx)^4 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}{4b} - \frac{Bg^3x(bc-ad)^3}{2d^3} + \frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd^2} + \frac{Bg^3(bc-ad)^4 \log(c+dx)}{2bd^4} - \frac{Bg^3(a+bx)^4 \log(c+dx)}{4bd^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^3*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]), x]$

[Out]  $-(B*(b*c - a*d)^3*g^3*x)/(2*d^3) + (B*(b*c - a*d)^2*g^3*(a + b*x)^2)/(4*b*d^2) - (B*(b*c - a*d)*g^3*(a + b*x)^3)/(6*b*d) + (g^3*(a + b*x)^4*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]))/(4*b) + (B*(b*c - a*d)^4*g^3*\text{Log}[c + d*x])/(2*b*d^4)$

#### Rule 2525

$\text{Int}[(a + \text{Log}[(c + d*x)^p])*(b + e*x)^n*(d + e*x)^m, x\_Symbol] := \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{Log}[c + d*x]^p)^n/(e^{m+1}), x] - \text{Dist}[(b*n*p)/(e^{m+1}), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1}*(a + b*\text{Log}[c + d*x]^p)^{n-1}*D[\text{Log}[c + d*x], x])/e, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Log[c + d\*x], x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[a*(u), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b + v)] /; FreeQ[b, x]

#### Rule 43

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int (ag + bgx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx &= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{4b} - \frac{B \int \frac{2(bc-ad)g^4(a+bx)^3}{c+dx} dx}{4bg} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{4b} - \frac{(B(bc-ad)g^3) \int \frac{(a+bx)^3}{c+dx} dx}{2b} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{4b} - \frac{(B(bc-ad)g^3) \int \left( \frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(a+bx)}{d^2} \right) dx}{2b} \\
&= -\frac{B(bc-ad)^3 g^3 x}{2d^3} + \frac{B(bc-ad)^2 g^3 (a+bx)^2}{4bd^2} - \frac{B(bc-ad)g^3 (a+bx)^3}{6bd} + \frac{g^3 (a+bx)^4}{4b}
\end{aligned}$$

**Mathematica [A]** time = 0.0896554, size = 122, normalized size = 0.81

$$\frac{g^3 \left( (a+bx)^4 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right) - \frac{B(bc-ad)(3d^2(a+bx)^2(ad-bc)+6bdx(bc-ad)^2-6(bc-ad)^3 \log(c+dx)+2d^3(a+bx)^3)}{3d^4} \right)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

[Out] (g^3\*((a + b\*x)^4\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) - (B\*(b\*c - a\*d)\*(6\*b\*d\*(b\*c - a\*d)^2\*x + 3\*d^2\*(-(b\*c) + a\*d)\*(a + b\*x)^2 + 2\*d^3\*(a + b\*x)^3 - 6\*(b\*c - a\*d)^3\*Log[c + d\*x]))/(3\*d^4))/(4\*b)

**Maple [B]** time = 0.24, size = 1249, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^3\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2)), x)

[Out] 1/4\*A\*g^3\*x^4\*b^3+A\*g^3\*x\*a^3+3/2\*B\*x\*a^3\*g^3+19/6/d^3\*B\*g^3\*b^2\*a\*c^3-3/2/d^2\*A\*g^3\*a^2\*b\*c^2+1/d^3\*A\*g^3\*a\*b^2\*c^3+B\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*x\*a^3\*g^3+1/4\*B\*g^3\*b^3\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*x^4+A\*g^3\*x^3\*a\*b^2+3/4\*B\*g^3\*b\*a^2\*x^2+3/2/d\*B\*a^3\*c\*g^3-11/12/d^4\*B\*g^3\*b^3\*c^4-1/4/d^4\*A\*g^3\*b^3\*c^4+1/d\*A\*g^3\*a^3\*c+3/2\*A\*g^3\*x^2\*a^2\*b-3/2\*B\*g^3/b\*a^4\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)-1/2\*B\*g^3/b\*a^4\*ln(1/(d\*x+c))-15/4/d^2\*B\*g^3\*b\*a^2\*c^2+1/6\*B\*g^3\*b^2\*a\*x^3-20/d^2\*B\*g^3\*b^2/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a^2\*c^3-1/4/d^4\*B\*g^3\*b^3\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*c^4+3/2\*B\*g^3\*b\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*a^2\*x^2+B\*g^3\*b^2\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*a\*x^3-10\*B\*g^3/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a^4\*c+20/d\*B\*g^3\*b/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a^3\*c^2+10/d^3\*B\*g^3/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a\*c^4\*b^3-1/6/d\*B\*g^3\*b^3\*c\*x^3+1/4/d^2\*B\*g^3\*b^3\*c^2\*x^2-1/2/d^3\*B\*g^3\*b^3\*c^3\*x-1/2/d^4\*B\*g^3\*b^3\*c^4\*ln(1/(d\*x+c))-3/2/d^4\*B\*g^3\*b^3\*c^4\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)+1/d\*B\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*a^3\*c\*g^3+6/d\*B\*g^3\*a^3\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*c^2/d\*B\*g^3\*a^3\*ln(1/(d\*x+c))\*c^2\*d\*B\*g^3/b/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a^5+1/d^3\*B\*g^3\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*a\*b^2\*c^3-3/2/d^2\*B\*g^3\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*a^2\*b\*c^2-1/d\*B\*g^3\*b^2\*a\*x^2\*c^2/d^2\*B\*g^3\*b^2\*a\*x\*c^2-3/d\*B\*g^3\*a^2\*b\*c\*x+2/d^3\*B\*g^3\*b^2\*a\*ln(1

$$\frac{1}{(d*x+c)} * c^3 - 2/d^4 * B * g^3 / (a*d-b*c) * \ln(1/(d*x+c) * a*d-b*c / (d*x+c)+b) * c^5 * b^4 + 6/d^3 * B * g^3 * b^2 * a * \ln(1/(d*x+c) * a*d-b*c / (d*x+c)+b) * c^3 - 3/d^2 * B * g^3 * b * a^2 * \ln(1/(d*x+c)) * c^2 - 9/d^2 * B * g^3 * b * a^2 * \ln(1/(d*x+c) * a*d-b*c / (d*x+c)+b) * c^2$$

**Maxima [B]** time = 1.36102, size = 873, normalized size = 5.78

$$\frac{1}{4} Ab^3 g^3 x^4 + Aab^2 g^3 x^3 + \frac{3}{2} Aa^2 b g^3 x^2 + \left( x \log \left( \frac{b^2 e x^2}{d^2 x^2 + 2 c d x + c^2} + \frac{2 a b e x}{d^2 x^2 + 2 c d x + c^2} + \frac{a^2 e}{d^2 x^2 + 2 c d x + c^2} \right) + \frac{2 a \log}{d^2 x^2 + 2 c d x + c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="maxima")

[Out] 1/4\*A\*b^3\*g^3\*x^4 + A\*a\*b^2\*g^3\*x^3 + 3/2\*A\*a^2\*b\*g^3\*x^2 + (x\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 2\*a\*log(b\*x + a)/b - 2\*c\*log(d\*x + c)/d)\*B\*a^3\*g^3 + 3/2\*(x^2\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) - 2\*a^2\*log(b\*x + a)/b^2 + 2\*c^2\*log(d\*x + c)/d^2 - 2\*(b\*c - a\*d)\*x/(b\*d))\*B\*a^2\*b\*g^3 + (x^3\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 2\*a^3\*log(b\*x + a)/b^3 - 2\*c^3\*log(d\*x + c)/d^3 - ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2))\*B\*a\*b^2\*g^3 + 1/12\*(3\*x^4\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) - 6\*a^4\*log(b\*x + a)/b^4 + 6\*c^4\*log(d\*x + c)/d^4 - (2\*(b^3\*c\*d^2 - a\*b^2\*d^3)\*x^3 - 3\*(b^3\*c^2\*d - a^2\*b\*d^3)\*x^2 + 6\*(b^3\*c^3 - a^3\*d^3)\*x)/(b^3\*d^3))\*B\*b^3\*g^3 + A\*a^3\*g^3\*x

**Fricas [B]** time = 1.16277, size = 709, normalized size = 4.7

$$3 Ab^4 d^4 g^3 x^4 + 6 Ba^4 d^4 g^3 \log(bx + a) - 2 (Bb^4 cd^3 - (6A + B)ab^3 d^4) g^3 x^3 + 3 (Bb^4 c^2 d^2 - 4 Bab^3 cd^3 + 3(2A + B)a^2 b^2 d^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="fricas")

[Out] 1/12\*(3\*A\*b^4\*d^4\*g^3\*x^4 + 6\*B\*a^4\*d^4\*g^3\*log(b\*x + a) - 2\*(B\*b^4\*c\*d^3 - (6\*A + B)\*a\*b^3\*d^4)\*g^3\*x^3 + 3\*(B\*b^4\*c^2\*d^2 - 4\*B\*a\*b^3\*c\*d^3 + 3\*(2\*A + B)\*a^2\*b^2\*d^4)\*g^3\*x^2 - 6\*(B\*b^4\*c^3\*d - 4\*B\*a\*b^3\*c^2\*d^2 + 6\*B\*a^2\*b^2\*c\*d^3 - (2\*A + 3\*B)\*a^3\*b\*d^4)\*g^3\*x + 6\*(B\*b^4\*c^4 - 4\*B\*a\*b^3\*c^3\*d + 6\*B\*a^2\*b^2\*c^2\*d^2 - 4\*B\*a^3\*b\*c\*d^3)\*g^3\*log(d\*x + c) + 3\*(B\*b^4\*d^4\*g^3\*x^4 + 4\*B\*a\*b^3\*d^4\*g^3\*x^3 + 6\*B\*a^2\*b^2\*d^4\*g^3\*x^2 + 4\*B\*a^3\*b\*d^4\*g^3\*x) \* log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)))/(b\*d^4)

**Sympy [B]** time = 5.73868, size = 722, normalized size = 4.78

$$\frac{Ab^3 g^3 x^4}{4} + \frac{Ba^4 g^3 \log \left( x + \frac{Ba^5 d^4 g^3 + 4Ba^4 cd^3 g^3 - 6Ba^3 bc^2 d^2 g^3 + 4Ba^2 b^2 c^3 dg^3 - Bab^3 c^4 g^3}{Ba^4 d^4 g^3 + 4Ba^3 bcd^3 g^3 - 6Ba^2 b^2 c^2 d^2 g^3 + 4Bab^3 c^3 dg^3 - Bb^4 c^4 g^3} \right)}{2b} - \frac{Bcg^3 (2ad - bc) (2a^2 d^2 - 2abcd + b^2 c^2)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*3\*(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2)),x)

[Out]  $A*b**3*g**3*x**4/4 + B*a**4*g**3*\log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*b) - B*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*\log(x + (5*B*a**4*c*d**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c**4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) + B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*d**4) + (B*a**3*g**3*x + 3*B*a**2*b*g**3*x**2/2 + B*a*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*\log(e*(a + b*x)**2/(c + d*x)**2) + x**3*(6*A*a*b**2*d*g**3 + B*a*b**2*d*g**3 - B*b**3*c*g**3)/(6*d) + x**2*(6*A*a**2*b*d**2*g**3 + 3*B*a**2*b*d**2*g**3 - 4*B*a*b**2*c*d*g**3 + B*b**3*c**2*g**3)/(4*d**2) + x*(2*A*a**3*d**3*g**3 + 3*B*a**3*d**3*g**3 - 6*B*a**2*b*c*d**2*g**3 + 4*B*a*b**2*c**2*d*g**3 - B*b**3*c**3*g**3)/(2*d**3)$

**Giac [B]** time = 27.5013, size = 487, normalized size = 3.23

$$\frac{Ba^4g^3 \log(bx + a)}{2b} + \frac{1}{4} (Ab^3g^3 + Bb^3g^3)x^4 - \frac{(Bb^3cg^3 - 6Aab^2dg^3 - 7Bab^2dg^3)x^3}{6d} + \frac{1}{4} (Bb^3g^3x^4 + 4Bab^2g^3x^3 + 6Ba^2bg^3x^2 + 4Aa^2b^2dg^3x + 4Aa^2b^2dg^3x^2 + 4Aa^2b^2dg^3x^3 + 4Aa^2b^2dg^3x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="giac")

[Out]  $1/2*B*a^4*g^3*\log(b*x + a)/b + 1/4*(A*b^3*g^3 + B*b^3*g^3)*x^4 - 1/6*(B*b^3*c*g^3 - 6*A*a*b^2*d*g^3 - 7*B*a*b^2*d*g^3)*x^3/d + 1/4*(B*b^3*g^3*x^4 + 4*B*a*b^2*g^3*x^3 + 6*B*a^2*b*g^3*x^2 + 4*B*a^3*g^3*x)*\log((b^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/4*(B*b^3*c^2*g^3 - 4*B*a*b^2*c*d*g^3 + 6*A*a^2*b*d^2*g^3 + 9*B*a^2*b*d^2*g^3)*x^2/d^2 - 1/2*(B*b^3*c^3*g^3 - 4*B*a*b^2*c^2*d*g^3 + 6*B*a^2*b*c*d^2*g^3 - 2*A*a^3*d^3*g^3 - 5*B*a^3*d^3*g^3)*x/d^3 + 1/2*(B*b^3*c^4*g^3 - 4*B*a*b^2*c^3*d*g^3 + 6*B*a^2*b*c^2*d^2*g^3 - 4*B*a^3*c^2*d^2*g^3)*\log(d*x + c)/d^4$

$$3.121 \quad \int (ag + bgx)^2 \left( A + B \log \left( \frac{e^{(a+bx)^2}}{(c+dx)^2} \right) \right) dx$$

**Optimal.** Leaf size=120

$$\frac{g^2(a+bx)^3 \left( B \log \left( \frac{e^{(a+bx)^2}}{(c+dx)^2} \right) + A \right)}{3b} + \frac{2Bg^2x(bc-ad)^2}{3d^2} - \frac{2Bg^2(bc-ad)^3 \log(c+dx)}{3bd^3} - \frac{Bg^2(a+bx)^2(bc-ad)}{3bd}$$

[Out]  $(2*B*(b*c - a*d)^2*g^2*x)/(3*d^2) - (B*(b*c - a*d)*g^2*(a + b*x)^2)/(3*b*d) + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b) - (2*B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3)$

**Rubi [A]** time = 0.0741202, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 43}

$$\frac{g^2(a+bx)^3 \left( B \log \left( \frac{e^{(a+bx)^2}}{(c+dx)^2} \right) + A \right)}{3b} + \frac{2Bg^2x(bc-ad)^2}{3d^2} - \frac{2Bg^2(bc-ad)^3 \log(c+dx)}{3bd^3} - \frac{Bg^2(a+bx)^2(bc-ad)}{3bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]$

[Out]  $(2*B*(b*c - a*d)^2*g^2*x)/(3*d^2) - (B*(b*c - a*d)*g^2*(a + b*x)^2)/(3*b*d) + (g^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b) - (2*B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3)$

#### Rule 2525

$\text{Int}[(a_. + \text{Log}[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^(m + 1)*(a + b*\text{Log}[c*RFX^p])^n/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^(m + 1)*(a + b*\text{Log}[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 43

$\text{Int}[(a_. + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned} \int (ag + bgx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx &= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{3b} - \frac{B \int \frac{2(bc-ad)g^3(a+bx)^2}{c+dx} dx}{3bg} \\ &= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{3b} - \frac{(2B(bc-ad)g^2) \int \frac{(a+bx)^2}{c+dx} dx}{3b} \\ &= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{3b} - \frac{(2B(bc-ad)g^2) \int \left( -\frac{b(bc-ad)}{d^2} + \frac{b(a+bx)}{d} \right) dx}{3b} \\ &= \frac{2B(bc-ad)^2 g^2 x}{3d^2} - \frac{B(bc-ad)g^2(a+bx)^2}{3bd} + \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{3b} \end{aligned}$$

**Mathematica [A]** time = 0.0524115, size = 98, normalized size = 0.82

$$\frac{g^2 \left( \frac{B(ad-bc)(d(a^2d+4abdx+b^2x(dx-2c))+2(bc-ad)^2 \log(c+dx))}{d^3} + (a+bx)^3 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right) \right)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) + (B*(-(b*c) + a*d)*(d*(a^2*d + 4*a*b*d*x + b^2*x*(-2*c + d*x)) + 2*(b*c - a*d)^2*Log[c + d*x]))/d^3)/(3*b)
```

**Maple [B]** time = 0.243, size = 915, normalized size = 7.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)), x)
```

```
[Out] -1/d^2*A*g^2*a*b*c^2-7/3/d^2*B*g^2*b*a*c^2+2/d^3*B*g^2/(a*d-b*c)*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*c^4*b^3+1/3*A*g^2*x^3*b^2+A*g^2*x*a^2+4/3*B*x*a^2*g^2+A*g^2*x^2*a*b+4/3/d^3*B*g^2*b^2*c^3*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+B*g^2*b*ln(e*(1/(d*x+c))*a*d-b*c/(d*x+c)+b)^2/d^2)*a*x^2+2/d*B*g^2*a^2*ln(1/(d*x+c))*c+4/d*B*g^2*a^2*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*c+1/d*B*ln(e*(1/(d*x+c))*a*d-b*c/(d*x+c)+b)^2/d^2)*a^2*c*g^2-8*B*g^2/(a*d-b*c)*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*a^3*c+1/3/d^3*B*g^2*b^2*ln(e*(1/(d*x+c))*a*d-b*c/(d*x+c)+b)^2/d^2)*c^3+2/3/d^3*B*g^2*b^2*c^3*ln(1/(d*x+c))+2/3/d^2*B*g^2*b^2*c^2*x-2/d^2*B*g^2*b*a*ln(1/(d*x+c))*c^2-1/d^2*B*g^2*ln(e*(1/(d*x+c))*a*d-b*c/(d*x+c)+b)^2/d^2)*a*b*c^2-4/d^2*B*g^2*b*a*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*c^2-2/d*B*g^2*b*a*x*c+2*d*B*g^2/b/(a*d-b*c)*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*a^4+12/d*B*g^2*b/(a*d-b*c)*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*a^2*c^2-8/d^2*B*g^2/(a*d-b*c)*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*a*c^3*b^2+1/3*B*g^2*b*a*x^2-2/3*B*g^2/b*a^3*ln(1/(d*x+c))+4/3/d*B*a^2*c*g^2+1/d^3*B*g^2*b^2*c^3+1/3/d^3*A*g^2*b^2*c^3+1/d*A*g^2*a^2*c-1/3/d*B*g^2*b^2*c*x^2-4/3*B*g^2/b*a^3*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)+1/3*B*g^2*b^2*ln(e*(1/(d*x+c))*a*d-b*c/(d*x+c)+b)^2/d^2)*x^3+B*ln(e*(1/(d*x+c))*a*d-b*c/(d*x+c)+b)^2/d^2)*x*a^2*g^2
```



**Maxima [B]** time = 1.28289, size = 590, normalized size = 4.92

$$\frac{1}{3} Ab^2 g^2 x^3 + Aabg^2 x^2 + \left( x \log \left( \frac{b^2 e x^2}{d^2 x^2 + 2cdx + c^2} + \frac{2abex}{d^2 x^2 + 2cdx + c^2} + \frac{a^2 e}{d^2 x^2 + 2cdx + c^2} \right) + \frac{2a \log(bx + a)}{b} - \frac{2c \log(dx + c)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="maxima")

[Out] 1/3\*A\*b^2\*g^2\*x^3 + A\*a\*b\*g^2\*x^2 + (x\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 2\*a\*log(b\*x + a)/b - 2\*c\*log(d\*x + c)/d)\*B\*a^2\*g^2 + (x^2\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) - 2\*a^2\*log(b\*x + a)/b^2 + 2\*c^2\*log(d\*x + c)/d^2 - 2\*(b\*c - a\*d)\*x/(b\*d))\*B\*a\*b\*g^2 + 1/3\*(x^3\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 2\*a^3\*log(b\*x + a)/b^3 - 2\*c^3\*log(d\*x + c)/d^3 - ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2))\*B\*b^2\*g^2 + A\*a^2\*g^2\*x

**Fricas [B]** time = 1.09778, size = 508, normalized size = 4.23

$$Ab^3 d^3 g^2 x^3 + 2Ba^3 d^3 g^2 \log(bx + a) - (Bb^3 cd^2 - (3A + B)ab^2 d^3)g^2 x^2 + (2Bb^3 c^2 d - 6Bab^2 cd^2 + (3A + 4B)a^2 bd^3)g^2 x$$

3

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="fricas")

[Out] 1/3\*(A\*b^3\*d^3\*g^2\*x^3 + 2\*B\*a^3\*d^3\*g^2\*log(b\*x + a) - (B\*b^3\*c\*d^2 - (3\*A + B)\*a\*b^2\*d^3)\*g^2\*x^2 + (2\*B\*b^3\*c^2\*d - 6\*B\*a\*b^2\*c\*d^2 + (3\*A + 4\*B)\*a^2\*b\*d^3)\*g^2\*x - 2\*(B\*b^3\*c^3 - 3\*B\*a\*b^2\*c^2\*d + 3\*B\*a^2\*b\*c\*d^2)\*g^2\*log(d\*x + c) + (B\*b^3\*d^3\*g^2\*x^3 + 3\*B\*a\*b^2\*d^3\*g^2\*x^2 + 3\*B\*a^2\*b\*d^3\*g^2\*x)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)))/(b\*d^3)

**Sympy [B]** time = 4.00468, size = 527, normalized size = 4.39

$$\frac{Ab^2 g^2 x^3}{3} + \frac{2Ba^3 g^2 \log \left( x + \frac{\frac{2Ba^4 d^3 g^2}{b} + 6Ba^3 cd^2 g^2 - 6Ba^2 bc^2 dg^2 + 2Bab^2 c^3 g^2}{2Ba^3 d^3 g^2 + 6Ba^2 bcd^2 g^2 - 6Bab^2 c^2 dg^2 + 2Bb^3 c^3 g^2} \right)}{3b} - \frac{2Bcg^2 (3a^2 d^2 - 3abcd + b^2 c^2) \log \left( x + \frac{8Ba^3 cd^2 g^2 - 6Ba^2 bc^2 dg^2 + 2Bab^2 c^3 g^2}{2Ba^3 d^3 g^2 + 6Ba^2 bcd^2 g^2 - 6Bab^2 c^2 dg^2 + 2Bb^3 c^3 g^2} \right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*2\*(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2)),x)

[Out] A\*b\*\*2\*g\*\*2\*x\*\*3/3 + 2\*B\*a\*\*3\*g\*\*2\*log(x + (2\*B\*a\*\*4\*d\*\*3\*g\*\*2/b + 6\*B\*a\*\*3\*c\*d\*\*2\*g\*\*2 - 6\*B\*a\*\*2\*b\*c\*\*2\*d\*g\*\*2 + 2\*B\*a\*b\*\*2\*c\*\*3\*g\*\*2)/(2\*B\*a\*\*3\*d\*\*3\*g\*\*2 + 6\*B\*a\*\*2\*b\*c\*d\*\*2\*g\*\*2 - 6\*B\*a\*b\*\*2\*c\*\*2\*d\*g\*\*2 + 2\*B\*b\*\*3\*c\*\*3\*g\*\*2))/(3\*b) - 2\*B\*c\*g\*\*2\*(3\*a\*\*2\*d\*\*2 - 3\*a\*b\*c\*d + b\*\*2\*c\*\*2)\*log(x + (8\*B\*a\*\*3\*c\*d\*\*2\*g\*\*2 - 6\*B\*a\*\*2\*b\*c\*\*2\*d\*g\*\*2 + 2\*B\*a\*b\*\*2\*c\*\*3\*g\*\*2 - 2\*B\*a\*c\*g\*\*2\*(3\*a\*\*2\*d\*\*2 - 3\*a\*b\*c\*d + b\*\*2\*c\*\*2) + 2\*B\*b\*c\*\*2\*g\*\*2\*(3\*a\*\*2\*d\*\*2 -

$$\frac{3*a*b*c*d + b**2*c**2)/d)/(2*B*a**3*d**3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g**2))/(3*d**3) + (B*a**2*g**2*x + B*a*b*g**2*x**2 + B*b**2*g**2*x**3/3)*\log(e*(a + b*x)**2/(c + d*x)**2) + x**2*(3*A*a*b*d*g**2 + B*a*b*d*g**2 - B*b**2*c*g**2)/(3*d) + x*(3*A*a**2*d**2*g**2 + 4*B*a**2*d**2*g**2 - 6*B*a*b*c*d*g**2 + 2*B*b**2*c**2*g**2)/(3*d**2)$$

**Giac [B]** time = 4.79757, size = 340, normalized size = 2.83

$$\frac{2Ba^3g^2 \log(bx + a)}{3b} + \frac{1}{3} (Ab^2g^2 + Bb^2g^2)x^3 - \frac{(Bb^2cg^2 - 3Aabdg^2 - 4Babdg^2)x^2}{3d} + \frac{1}{3} (Bb^2g^2x^3 + 3Babg^2x^2 + 3Ba^2g^2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="giac")

[Out] 2/3\*B\*a^3\*g^2\*log(b\*x + a)/b + 1/3\*(A\*b^2\*g^2 + B\*b^2\*g^2)\*x^3 - 1/3\*(B\*b^2\*c\*g^2 - 3\*A\*a\*b\*d\*g^2 - 4\*B\*a\*b\*d\*g^2)\*x^2/d + 1/3\*(B\*b^2\*g^2\*x^3 + 3\*B\*a\*b\*g^2\*x^2 + 3\*B\*a^2\*g^2\*x)\*log((b^2\*x^2 + 2\*a\*b\*x + a^2)/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 1/3\*(2\*B\*b^2\*c^2\*g^2 - 6\*B\*a\*b\*c\*d\*g^2 + 3\*A\*a^2\*d^2\*g^2 + 7\*B\*a^2\*d^2\*g^2)\*x/d^2 - 2/3\*(B\*b^2\*c^3\*g^2 - 3\*B\*a\*b\*c^2\*d\*g^2 + 3\*B\*a^2\*c\*d^2\*g^2)\*log(-d\*x - c)/d^3

$$3.122 \quad \int (ag + bgx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx$$

**Optimal.** Leaf size=78

$$\frac{g(a+bx)^2 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}{2b} + \frac{Bg(bc-ad)^2 \log(c+dx)}{bd^2} - \frac{Bgx(bc-ad)}{d}$$

[Out]  $-\left(\frac{B(b*c - a*d)*g*x}{d} + \frac{g*(a + b*x)^2*(A + B*\text{Log}[(e*(a + b*x)^2]/(c + d*x)^2])}{2*b} + \frac{B*(b*c - a*d)^2*g*\text{Log}[c + d*x]}{b*d^2}\right)$

**Rubi [A]** time = 0.0568225, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 43}

$$\frac{g(a+bx)^2 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}{2b} + \frac{Bg(bc-ad)^2 \log(c+dx)}{bd^2} - \frac{Bgx(bc-ad)}{d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)*(A + B*\text{Log}[(e*(a + b*x)^2]/(c + d*x)^2]),x]$

[Out]  $-\left(\frac{B(b*c - a*d)*g*x}{d} + \frac{g*(a + b*x)^2*(A + B*\text{Log}[(e*(a + b*x)^2]/(c + d*x)^2])}{2*b} + \frac{B*(b*c - a*d)^2*g*\text{Log}[c + d*x]}{b*d^2}\right)$

#### Rule 2525

$\text{Int}[(a + \text{Log}[c*(RFX)^p])*(b*x)^n*((d + e*x)^m), x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*RFX^p])^n/(e*(m+1)), x] - \text{Dist}[(b*n*p)/(e*(m+1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*RFX^p])^{n-1}*D[RFX, x])/RFX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[(a)*(u), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b)\*(v)] /; FreeQ[b, x]

#### Rule 43

$\text{Int}[(a + (b*x)^m)*((c + d*x)^n), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int (ag + bgx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx &= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{2b} - \frac{B \int \frac{2(bc-ad)g^2(a+bx)}{c+dx} dx}{2bg} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{2b} - \frac{(B(bc-ad)g) \int \frac{a+bx}{c+dx} dx}{b} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{2b} - \frac{(B(bc-ad)g) \int \left( \frac{b}{d} + \frac{-bc+ad}{d(c+dx)} \right) dx}{b} \\
&= -\frac{B(bc-ad)gx}{d} + \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{2b} + \frac{B(bc-ad)^2 g \log(c+dx)}{bd^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0360573, size = 72, normalized size = 0.92

$$\frac{g \left( (a+bx)^2 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right) + \frac{2B(ad-bc)((ad-bc) \log(c+dx)+bdx)}{d^2} \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

[Out] (g\*((a + b\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) + (2\*B\*(-(b\*c) + a\*d)\*(b\*d\*x + (-b\*c) + a\*d)\*Log[c + d\*x]))/d^2)/(2\*b)

**Maple [B]** time = 0.235, size = 560, normalized size = 7.2

$$-\frac{gBbcx}{d} - \frac{gBbc^2}{2d^2} \ln \left( \frac{e}{d^2} \left( \frac{ad}{dx+c} - \frac{bc}{dx+c} + b \right)^2 \right) + \frac{gBbx^2}{2} \ln \left( \frac{e}{d^2} \left( \frac{ad}{dx+c} - \frac{bc}{dx+c} + b \right)^2 \right) + B \ln \left( \frac{e}{d^2} \left( \frac{ad}{dx+c} - \frac{bc}{dx+c} + b \right)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2)), x)

[Out] -1/d\*g\*B\*b\*c\*x-1/2/d^2\*B\*g\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*b\*c^2+1/2\*B\*g\*b\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*x^2+B\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*x\*a\*g-1/d^2\*B\*g\*b\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*c^2-B\*g/b\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a^2+1/d\*B\*g\*a\*c-1/d^2\*B\*g\*c^2\*b-1/d^2\*B\*g\*ln(1/(d\*x+c))\*c^2\*b-B\*g/b\*ln(1/(d\*x+c))\*a^2+g\*B\*a\*x+g\*A\*a\*x-1/2/d^2\*A\*g\*c^2\*b+1/2\*A\*g\*x^2\*b+2/d\*B\*g\*ln(1/(d\*x+c))\*a\*c+2/d\*B\*g\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a\*c+1/d\*B\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*a\*c\*g+1/d\*A\*g\*a\*c+2\*d\*B\*g/b/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a^3-6\*B\*g/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a^2\*c+6/d\*B\*g/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a\*c^2\*b-2/d^2\*B\*g/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*c^3\*b^2

**Maxima [B]** time = 1.22796, size = 338, normalized size = 4.33

$$\frac{1}{2} Abgx^2 + \left( x \log \left( \frac{b^2 ex^2}{d^2 x^2 + 2cdx + c^2} + \frac{2 abex}{d^2 x^2 + 2cdx + c^2} + \frac{a^2 e}{d^2 x^2 + 2cdx + c^2} \right) + \frac{2 a \log (bx + a)}{b} - \frac{2 c \log (dx + c)}{d} \right) Bag$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="maxima")

[Out]  $\frac{1}{2}A*b*g*x^2 + (x*\log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*\log(b*x + a)/b - 2*c*\log(d*x + c)/d)*B*a*g + 1/2*(x^2*\log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*\log(b*x + a)/b^2 + 2*c^2*\log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*b*g + A*a*g*x$

**Fricas [A]** time = 1.08462, size = 329, normalized size = 4.22

$$\frac{Ab^2d^2gx^2 + 2Ba^2d^2g \log(bx + a) - 2(Bb^2cd - (A + B)abd^2)gx + 2(Bb^2c^2 - 2Babcd)g \log(dx + c) + (Bb^2d^2gx^2 + 2Babdgx + Ba^2d^2g)}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="fricas")

[Out]  $\frac{1}{2}*(A*b^2*d^2*g*x^2 + 2*B*a^2*d^2*g*\log(b*x + a) - 2*(B*b^2*c*d - (A + B)*a*b*d^2)*g*x + 2*(B*b^2*c^2 - 2*B*a*b*c*d)*g*\log(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*a*b*d^2*g*x)*\log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b*d^2)$

**Sympy [B]** time = 2.46477, size = 253, normalized size = 3.24

$$\frac{Abgx^2}{2} + \frac{Ba^2g \log\left(x + \frac{Ba^3d^2g + 2Ba^2cdg - Babc^2g}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{b} - \frac{Bcg(2ad - bc) \log\left(x + \frac{3Ba^2cdg - Babc^2g - Bacg(2ad - bc) + \frac{Bbc^2g(2ad - bc)}{d}}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{d^2} + (Bagx^2 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2)),x)

[Out]  $A*b*g*x**2/2 + B*a**2*g*\log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/b - B*c*g*(2*a*d - b*c)*\log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/d**2 + (B*a*g*x + B*b*g*x**2/2)*\log(e*(a + b*x)**2/(c + d*x)**2) + x*(A*a*d*g + B*a*d*g - B*b*c*g)/d$

**Giac [A]** time = 1.74159, size = 177, normalized size = 2.27

$$\frac{Ba^2g \log(bx + a)}{b} + \frac{1}{2}(Abg + Bbg)x^2 + \frac{1}{2}(Bbgx^2 + 2Bagx) \log\left(\frac{b^2x^2 + 2abx + a^2}{d^2x^2 + 2cdx + c^2}\right) - \frac{(Bbcg - Aadg - 2Badg)x}{d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="giac")

```
[Out] B*a^2*g*log(b*x + a)/b + 1/2*(A*b*g + B*b*g)*x^2 + 1/2*(B*b*g*x^2 + 2*B*a*g
*x)*log((b^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2)) - (B*b*c*g - A
*a*d*g - 2*B*a*d*g)*x/d + (B*b*c^2*g - 2*B*a*c*d*g)*log(d*x + c)/d^2
```

$$3.123 \quad \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag+bgx} dx$$

**Optimal.** Leaf size=83

$$\frac{2B \text{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right)}{bg} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{bg}$$

[Out] -((Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(b\*g)) + (2\*B\*PolyLog[2, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/(b\*g)

**Rubi [A]** time = 0.292001, antiderivative size = 122, normalized size of antiderivative = 1.47, number of steps used = 10, number of rules used = 8, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2524, 12, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2B \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg} + \frac{\log(ag + bgx) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{bg} + \frac{2B \log(ag + bgx) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{bg} - \frac{B \log^2(g(a + bx))}{bg}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(a\*g + b\*g\*x), x]

[Out] -((B\*Log[g\*(a + b\*x)]^2)/(b\*g)) + ((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) \* Log[a\*g + b\*g\*x])/(b\*g) + (2\*B\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] \* Log[a\*g + b\*g\*x])/(b\*g) + (2\*B\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g)

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_.))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[g\*c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{ag + bgx} dx &= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(ag + bgx)}{bg} - \frac{B \int \frac{(c+dx)^2 \left(-\frac{2de(a+bx)^2}{(c+dx)^3} + \frac{2be(a+bx)}{(c+dx)^2}\right) \log(ag+bgx)}{e(a+bx)^2} dx}{bg} \\
 &= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(ag + bgx)}{bg} - \frac{B \int \frac{(c+dx)^2 \left(-\frac{2de(a+bx)^2}{(c+dx)^3} + \frac{2be(a+bx)}{(c+dx)^2}\right) \log(ag+bgx)}{(a+bx)^2} dx}{beg} \\
 &= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(ag + bgx)}{bg} - \frac{B \int \left(\frac{2be \log(ag+bgx)}{a+bx} - \frac{2de \log(ag+bgx)}{c+dx}\right) dx}{beg} \\
 &= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{\log(ag+bgx)}{a+bx} dx}{g} + \frac{(2Bd) \int \frac{\log(ag+bgx)}{c+dx} dx}{bg} \\
 &= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(ag + bgx)}{bg} + \frac{2B \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} - (2B) \int \frac{\log\left(\frac{bg(c+dx)}{bcg-adg}\right)}{ag + bgx} \\
 &= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(ag + bgx)}{bg} + \frac{2B \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} - \frac{(2B) \text{Subst}\left(\int \frac{\log(x)}{x}\right)}{bg} \\
 &= -\frac{B \log^2(g(a + bx))}{bg} + \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(ag + bgx)}{bg} + \frac{2B \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg}
 \end{aligned}$$

**Mathematica [A]** time = 0.037118, size = 88, normalized size = 1.06

$$\frac{2BPolyLog\left(2, \frac{d(a+bx)}{ad-bc}\right) + \log(a + bx) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + 2B \log\left(\frac{b(c+dx)}{bc-ad}\right) - B \log(a + bx) + A\right)}{bg}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(a\*g + b\*g\*x), x]



[Out]  $(\text{Log}[a + b*x]*(A - B*\text{Log}[a + b*x] + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2] + 2*B*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) + 2*B*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]/(b*g)$

**Maple [B]** time = 0.375, size = 552, normalized size = 6.7

$$-\frac{A \ln((dx + c)^{-1})}{bg} + \frac{dAa}{bg(ad - bc)} \ln\left(\frac{ad}{dx + c} - \frac{bc}{dx + c} + b\right) - \frac{Ac}{g(ad - bc)} \ln\left(\frac{ad}{dx + c} - \frac{bc}{dx + c} + b\right) - \frac{B \ln((dx + c)^{-1})}{bg}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g), x)`

[Out]  $-1/g*A/b*\ln(1/(d*x+c))+d/g*A/b/(a*d-b*c)*\ln(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*a-1/g*A/(a*d-b*c)*\ln(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*c-1/g*B/b*\ln(1/(d*x+c))*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)+2*d/g*B/b*dilog((1/(d*x+c)*(a*d-b*c)+b)/b)/(a*d-b*c)*a-2/g*B*dilog((1/(d*x+c)*(a*d-b*c)+b)/b)/(a*d-b*c)*c+2*d/g*B/b*\ln(1/(d*x+c))*\ln((1/(d*x+c)*(a*d-b*c)+b)/b)/(a*d-b*c)*a-2/g*B*\ln(1/(d*x+c))*\ln((1/(d*x+c)*(a*d-b*c)+b)/b)/(a*d-b*c)*c+d/g*B/b*\ln(1/(d*x+c)*(a*d-b*c)+b)/(a*d-b*c)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*a-1/g*B*\ln(1/(d*x+c)*(a*d-b*c)+b)/(a*d-b*c)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*c-d/g*B/b/(a*d-b*c)*\ln(1/(d*x+c)*(a*d-b*c)+b)^2*a+1/g*B/(a*d-b*c)*\ln(1/(d*x+c)*(a*d-b*c)+b)^2*c$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-B\left(\frac{2 \log(bx + a) \log(dx + c)}{bg} - \int \frac{bdx \log(e) + bc \log(e) + 2(2bdx + bc + ad) \log(bx + a)}{b^2d^2x^2 + abcg + (b^2cg + abdg)x} dx\right) + \frac{A \log(bgx + ag)}{bg}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g), x, algorithm="maxima")`

[Out]  $-B*(2*\log(b*x + a)*\log(d*x + c)/(b*g) - \text{integrate}((b*d*x*\log(e) + b*c*\log(e) + 2*(2*b*d*x + b*c + a*d)*\log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)) + A*\log(b*g*x + a*g)/(b*g)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B \log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right) + A}{bgx + ag}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g), x, algorithm="fricas")`

[Out]  $\text{integral}((B*\log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A)/(b*g*x + a*g), x)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))/(b\*g\*x+a\*g), x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A}{bgx + ag} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(b\*g\*x+a\*g), x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)/(b\*g\*x + a\*g), x)

$$3.124 \quad \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^2} dx$$

**Optimal.** Leaf size=65

$$-\frac{(c+dx)\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}{g^2(a+bx)(bc-ad)} - \frac{2B}{bg^2(a+bx)}$$

[Out]  $(-2*B)/(b*g^2*(a+b*x)) - ((c+d*x)*(A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2]))/((b*c-a*d)*g^2*(a+b*x))$

**Rubi [A]** time = 0.0774241, antiderivative size = 105, normalized size of antiderivative = 1.62, number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}{bg^2(a+bx)} - \frac{2Bd \log(a+bx)}{bg^2(bc-ad)} + \frac{2Bd \log(c+dx)}{bg^2(bc-ad)} - \frac{2B}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(a\*g + b\*g\*x)^2,x]

[Out]  $(-2*B)/(b*g^2*(a+b*x)) - (2*B*d*Log[a+b*x])/(b*(b*c-a*d)*g^2) - (A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2])/(b*g^2*(a+b*x)) + (2*B*d*Log[c+d*x])/(b*(b*c-a*d)*g^2)$

#### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag + bgx)^2} dx &= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a + bx)} + \frac{B \int \frac{2(bc-ad)}{g(a+bx)^2(c+dx)} dx}{bg} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a + bx)} + \frac{(2B(bc - ad)) \int \frac{1}{(a+bx)^2(c+dx)} dx}{bg^2} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a + bx)} + \frac{(2B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)} + \frac{d^2}{(bc-ad)^2(c+dx)}\right) dx}{bg^2} \\
&= -\frac{2B}{bg^2(a + bx)} - \frac{2Bd \log(a + bx)}{b(bc - ad)g^2} - \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{bg^2(a + bx)} + \frac{2Bd \log(c + dx)}{b(bc - ad)g^2}
\end{aligned}$$

**Mathematica [A]** time = 0.058319, size = 111, normalized size = 1.71

$$\frac{2B(bc - ad) \left( -\frac{1}{(a+bx)(bc-ad)} - \frac{d \log(a+bx)}{(bc-ad)^2} + \frac{d \log(c+dx)}{(bc-ad)^2} \right)}{bg^2} - \frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{bg^2(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(a\*g + b\*g\*x)^2, x]

[Out] -((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(b\*g^2\*(a + b\*x))) + (2\*B\*(b\*c - a\*d)\*(-(1/((b\*c - a\*d)\*(a + b\*x))) - (d\*Log[a + b\*x])/(b\*c - a\*d)^2 + (d\*Log[c + d\*x])/(b\*c - a\*d)^2))/(b\*g^2)

**Maple [B]** time = 0.079, size = 157, normalized size = 2.4

$$\frac{dA}{g^2(ad - bc)} \left( \frac{ad}{dx + c} - \frac{bc}{dx + c} + b \right)^{-1} - 2 \frac{dB}{g^2 b(dx + c)} \left( \frac{ad}{dx + c} - \frac{bc}{dx + c} + b \right)^{-1} + \frac{dB}{g^2(ad - bc)} \ln \left( \frac{e}{d^2} \left( \frac{ad}{dx + c} - \frac{bc}{dx + c} + b \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))/(b\*g\*x+a\*g)^2, x)

[Out] d/g^2\*A/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)/(a\*d-b\*c)-2\*d/g^2/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*B/b/(d\*x+c)+d/g^2/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*B/(a\*d-b\*c)\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)

**Maxima [B]** time = 1.23987, size = 252, normalized size = 3.88

$$-B \left( \frac{\log\left(\frac{b^2 e x^2}{d^2 x^2 + 2 c d x + c^2} + \frac{2 a b e x}{d^2 x^2 + 2 c d x + c^2} + \frac{a^2 e}{d^2 x^2 + 2 c d x + c^2}\right)}{b^2 g^2 x + a b g^2} + \frac{2}{b^2 g^2 x + a b g^2} + \frac{2 d \log(bx + a)}{(b^2 c - a b d) g^2} - \frac{2 d \log(dx + c)}{(b^2 c - a b d) g^2} \right) - \frac{A}{b^2 g^2 x + a b g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(b\*g\*x+a\*g)^2, x, algorithm="maxima")

[Out]  $-B(\log(b^2 e x^2 / (d^2 x^2 + 2 c d x + c^2) + 2 a b e x / (d^2 x^2 + 2 c d x + c^2) + a^2 e / (d^2 x^2 + 2 c d x + c^2))) / (b^2 g^2 x + a b g^2) + 2 / (b^2 g^2 x + a b g^2) + 2 d \log(b x + a) / ((b^2 c - a b d) g^2) - 2 d \log(d x + c) / ((b^2 c - a b d) g^2) - A / (b^2 g^2 x + a b g^2)$

**Fricas [A]** time = 1.01482, size = 228, normalized size = 3.51

$$\frac{(A + 2B)bc - (A + 2B)ad + (Bbdx + Bbc) \log\left(\frac{b^2 ex^2 + 2 abex + a^2 e}{d^2 x^2 + 2 cdx + c^2}\right)}{(b^3 c - ab^2 d)g^2 x + (ab^2 c - a^2 bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="fricas")`

[Out]  $-((A + 2B) * b * c - (A + 2B) * a * d + (B * b * d * x + B * b * c) * \log((b^2 * e * x^2 + 2 * a * b * e * x + a^2 * e) / (d^2 * x^2 + 2 * c * d * x + c^2))) / ((b^3 * c - a * b^2 * d) * g^2 * x + (a * b^2 * c - a^2 * b * d) * g^2)$

**Sympy [B]** time = 1.91317, size = 253, normalized size = 3.89

$$\frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{abg^2 + b^2g^2x} - \frac{2Bd \log\left(x + \frac{\frac{2Ba^2d^3}{ad-bc} + \frac{4Babcd^2}{ad-bc} + 2Bad^2 - \frac{2Bb^2c^2d}{ad-bc} + 2Bbcd}{4Bbd^2}\right)}{bg^2(ad-bc)} + \frac{2Bd \log\left(x + \frac{\frac{2Ba^2d^3}{ad-bc} - \frac{4Babcd^2}{ad-bc} + 2Bad^2 + \frac{2Bb^2c^2d}{ad-bc} + 2Bbcd}{4Bbd^2}\right)}{bg^2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**2,x)`

[Out]  $-B \log(e * (a + b * x) ** 2 / (c + d * x) ** 2) / (a * b * g ** 2 + b ** 2 * g ** 2 * x) - 2 * B * d * \log(x + (-2 * B * a ** 2 * d ** 3 / (a * d - b * c) + 4 * B * a * b * c * d ** 2 / (a * d - b * c) + 2 * B * a * d ** 2 - 2 * B * b ** 2 * c ** 2 * d / (a * d - b * c) + 2 * B * b * c * d) / (4 * B * b * d ** 2)) / (b * g ** 2 * (a * d - b * c)) + 2 * B * d * \log(x + (2 * B * a ** 2 * d ** 3 / (a * d - b * c) - 4 * B * a * b * c * d ** 2 / (a * d - b * c) + 2 * B * a * d ** 2 + 2 * B * b ** 2 * c ** 2 * d / (a * d - b * c) + 2 * B * b * c * d) / (4 * B * b * d ** 2)) / (b * g ** 2 * (a * d - b * c)) - (A + 2 * B) / (a * b * g ** 2 + b ** 2 * g ** 2 * x)$

**Giac [B]** time = 1.39159, size = 254, normalized size = 3.91

$$\left(2(b^2 c g^2 - a b d g^2) \left( \frac{d \log\left(\left|\frac{bcg}{bgx+ag} - \frac{adg}{bgx+ag} + d\right|\right)}{b^4 c^2 g^4 - 2 a b^3 c d g^4 + a^2 b^2 d^2 g^4} - \frac{1}{(b^2 c g^2 - a b d g^2)(bgx + ag)bg} \right) - \frac{\log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right)}{(bgx + ag)bg} \right) B - \frac{A}{(bgx + ag)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^2,x, algorithm="giac")`

[Out]  $(2 * (b^2 * c * g^2 - a * b * d * g^2) * (d * \log(\text{abs}(b * c * g / (b * g * x + a * g) - a * d * g / (b * g * x + a * g) + d)) / (b^4 * c^2 * g^4 - 2 * a * b^3 * c * d * g^4 + a^2 * b^2 * d^2 * g^4) - 1 / ((b^2 * c * g^2 - a * b * d * g^2) * (b * g * x + a * g) * b * g)) - \log((b * x + a)^2 * e / (d * x + c)^2) / ((b * g * x + a * g) * b * g)) * B - A / ((b * g * x + a * g) * b * g)$

$$3.125 \quad \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^3} dx$$

**Optimal.** Leaf size=138

$$-\frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{2bg^3(a+bx)^2} + \frac{Bd^2 \log(a+bx)}{bg^3(bc-ad)^2} - \frac{Bd^2 \log(c+dx)}{bg^3(bc-ad)^2} + \frac{Bd}{bg^3(a+bx)(bc-ad)} - \frac{B}{2bg^3(a+bx)^2}$$

[Out]  $-B/(2*b*g^3*(a + b*x)^2) + (B*d)/(b*(b*c - a*d)*g^3*(a + b*x)) + (B*d^2*Log[a + b*x])/(b*(b*c - a*d)^2*g^3) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(2*b*g^3*(a + b*x)^2) - (B*d^2*Log[c + d*x])/(b*(b*c - a*d)^2*g^3)$

**Rubi [A]** time = 0.0934478, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{2bg^3(a+bx)^2} + \frac{Bd^2 \log(a+bx)}{bg^3(bc-ad)^2} - \frac{Bd^2 \log(c+dx)}{bg^3(bc-ad)^2} + \frac{Bd}{bg^3(a+bx)(bc-ad)} - \frac{B}{2bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])/(a*g + b*g*x)^3, x]$

[Out]  $-B/(2*b*g^3*(a + b*x)^2) + (B*d)/(b*(b*c - a*d)*g^3*(a + b*x)) + (B*d^2*Log[a + b*x])/(b*(b*c - a*d)^2*g^3) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(2*b*g^3*(a + b*x)^2) - (B*d^2*Log[c + d*x])/(b*(b*c - a*d)^2*g^3)$

### Rule 2525

$\text{Int}[(a + \text{Log}[c * \text{RFX}]^p) * (b + (d + e * x)^m)^n, x\_Symbol] :> \text{Simp}[(d + e * x)^{m+1} * (a + b * \text{Log}[c * \text{RFX}]^p)^n / (e * (m + 1)), x] - \text{Dist}[(b * n * p) / (e * (m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e * x)^{m+1} * (a + b * \text{Log}[c * \text{RFX}]^p)^{n-1} * D[\text{RFX}, x]) / \text{RFX}, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

$\text{Int}[(a + (b + (c + d * x)^m)^n) * (e + f * x)^p, x\_Symbol] :> \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b + (c + d \* x)^m)^n] /;

### Rule 44

$\text{Int}[(a + (b + (c + d * x)^m)^n) * (e + f * x)^p, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b * x)^m * (c + d * x)^n, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b \* c - a \* d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag + bgx)^3} dx &= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a + bx)^2} + \frac{B \int \frac{2(bc-ad)}{g^2(a+bx)^3(c+dx)} dx}{2bg} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a + bx)^2} + \frac{(B(bc - ad)) \int \frac{1}{(a+bx)^3(c+dx)} dx}{bg^3} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a + bx)^2} + \frac{(B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^3} - \frac{bd}{(bc-ad)^2(a+bx)^2} + \frac{bd^2}{(bc-ad)^3(a+bx)} - \frac{bd^3}{(bc-ad)^4}\right) dx}{bg^3} \\
&= -\frac{B}{2bg^3(a + bx)^2} + \frac{Bd}{b(bc - ad)g^3(a + bx)} + \frac{Bd^2 \log(a + bx)}{b(bc - ad)^2g^3} - \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2bg^3(a + bx)^2} - \frac{Bd^3}{b(bc - ad)^4}
\end{aligned}$$

**Mathematica [A]** time = 0.136493, size = 109, normalized size = 0.79

$$\frac{B(2d^2(a+bx)^2 \log(c+dx) + (bc-ad)(b(c-2dx) - 3ad) - 2d^2(a+bx)^2 \log(a+bx))}{(bc-ad)^2} + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{2bg^3(a + bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(a\*g + b\*g\*x)^3, x]

[Out] -(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2] + (B\*((b\*c - a\*d)\*(-3\*a\*d + b\*(c - 2\*d\*x)) - 2\*d^2\*(a + b\*x)^2\*Log[a + b\*x] + 2\*d^2\*(a + b\*x)^2\*Log[c + d\*x]))/(b\*c - a\*d)^2)/(2\*b\*g^3\*(a + b\*x)^2)

**Maple [B]** time = 0.108, size = 355, normalized size = 2.6

$$-\frac{d^2 Ab}{2g^3(ad - bc)^2} \left(\frac{ad}{dx + c} - \frac{bc}{dx + c} + b\right)^{-2} + \frac{d^2 A}{g^3(ad - bc)^2} \left(\frac{ad}{dx + c} - \frac{bc}{dx + c} + b\right)^{-1} - \frac{d^2 B}{g^3(ad - bc)(dx + c)} \left(\frac{ad}{dx + c} - \frac{bc}{dx + c} + b\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))/(b\*g\*x+a\*g)^3, x)

[Out] -1/2\*d^2/g^3\*A\*b/(a\*d-b\*c)^2/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2+d^2/g^3\*A/(a\*d-b\*c)^2/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)-d^2/g^3/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2\*B/(a\*d-b\*c)/(d\*x+c)-3/2\*d^2/g^3/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2\*B/b/(d\*x+c)^2+1/2\*d^2/g^3/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2\*b\*B/(a^2\*d^2-2\*a\*b\*c\*d+b^2\*c^2)\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)+d^2/g^3/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2\*B/(a\*d-b\*c)/(d\*x+c)\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)

**Maxima [B]** time = 1.29435, size = 414, normalized size = 3.

$$\frac{1}{2} B \left( \frac{2bdx - bc + 3ad}{(b^4c - ab^3d)g^3x^2 + 2(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} - \frac{\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2} + \frac{2abex}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right)}{b^3g^3x^2 + 2ab^2g^3x + a^2bg^3} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(b\*g\*x+a\*g)^3,x, algorithm="maxima")

[Out]  $\frac{1}{2}B \left( \frac{(2bdx - bc + 3ad)}{(b^4c - ab^3d)} g^3 x^2 + 2(ab^3c - a^2b^2d) g^3 x + (a^2b^2c - a^3bd) g^3 - \log\left(\frac{b^2 e x^2}{d^2 x^2 + 2cdx + c^2}\right) + 2 \frac{a b e x}{d^2 x^2 + 2cdx + c^2} + a^2 \frac{e}{d^2 x^2 + 2cdx + c^2} \right) / (b^3 g^3 x^2 + 2ab^2 g^3 x + a^2 b g^3) + 2d^2 \log(bx + a) / ((b^3 c^2 - 2ab^2 c d + a^2 b d^2) g^3) - 2d^2 \log(dx + c) / ((b^3 c^2 - 2ab^2 c d + a^2 b d^2) g^3) - \frac{1}{2}A / (b^3 g^3 x^2 + 2ab^2 g^3 x + a^2 b g^3)$

**Fricas [A]** time = 1.05097, size = 495, normalized size = 3.59

$$\frac{(A+B)b^2c^2 - 2(A+2B)abcd + (A+3B)a^2d^2 - 2(Bb^2cd - Babd^2)x - (Bb^2d^2x^2 + 2Babd^2x - Bb^2c^2 + 2Babcd) \log\left(\frac{b^2 e x^2}{d^2 x^2 + 2cdx + c^2}\right)}{2\left((b^5c^2 - 2ab^4cd + a^2b^3d^2)g^3x^2 + 2(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2)g^3x + (a^2b^3c^2 - 2a^3b^2cd + a^4bd^2)g^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(b\*g\*x+a\*g)^3,x, algorithm="fricas")

[Out]  $-\frac{1}{2} \left( (A+B)b^2c^2 - 2(A+2B)ab^2cd + (A+3B)a^2d^2 - 2(Bb^2cd - Babd^2)x - (Bb^2d^2x^2 + 2Babd^2x - Bb^2c^2 + 2Babcd) \log\left(\frac{b^2 e x^2}{d^2 x^2 + 2cdx + c^2}\right) \right) / (b^5c^2 - 2ab^4cd + a^2b^3d^2)g^3x^2 + 2(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2)g^3x + (a^2b^3c^2 - 2a^3b^2cd + a^4bd^2)g^3$

**Sympy [B]** time = 3.17597, size = 418, normalized size = 3.03

$$\frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2a^2bg^3 + 4ab^2g^3x + 2b^3g^3x^2} - \frac{Bd^2 \log\left(x + \frac{-\frac{Ba^3d^5}{(ad-bc)^2} + \frac{3Ba^2bcd^4}{(ad-bc)^2} - \frac{3Bab^2c^2d^3}{(ad-bc)^2} + \frac{Bad^3}{(ad-bc)^2} + \frac{Bb^3c^3d^2}{(ad-bc)^2} + \frac{Bbcd^2}{(ad-bc)^2}}{2Bbd^3}\right)}{bg^3(ad-bc)^2} + \frac{Bd^2 \log\left(x + \frac{\frac{Ba^3d^5}{(ad-bc)^2} - \frac{3Ba^2bcd^4}{(ad-bc)^2} + \frac{3Bab^2c^2d^3}{(ad-bc)^2} - \frac{Bad^3}{(ad-bc)^2} - \frac{Bb^3c^3d^2}{(ad-bc)^2} - \frac{Bbcd^2}{(ad-bc)^2}}{2Bbd^3}\right)}{bg^3(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))/(b\*g\*x+a\*g)\*\*3,x)

[Out]  $-B \log(e(a+bx)^2/(c+dx)^2) / (2a^2b^3g^3 + 4ab^2g^3x + 2b^3g^3x^2) - Bd^2 \log\left(x + \frac{-B a^3 d^5 / (a d - b c)^2 + 3 B a^2 b c d^4 / (a d - b c)^2 - 3 B a b^2 c^2 d^3 / (a d - b c)^2 + B a d^3 + B b^3 c^3 d^2 + B b c d^2}{2 B b d^3}\right) / (b g^3 (a d - b c)^2) + Bd^2 \log\left(x + \frac{B a^3 d^5 / (a d - b c)^2 - 3 B a^2 b c d^4 / (a d - b c)^2 + 3 B a b^2 c^2 d^3 / (a d - b c)^2 + B a d^3 - B b^3 c^3 d^2 + B b c d^2}{2 B b d^3}\right) / (b g^3 (a d - b c)^2) - (A a d - A b c + 3 B a d - B b c + 2 B b d x) / (2 a^2 b^3 d g^3 - 2 a^2 b^2 c g^3 + x^2 (2 a b^3 d g^3 - 2 b^4 c g^3) + x (4 a^2 b^2 d g^3 - 4 a b^3 c g^3))$



**Giac [A]** time = 1.31868, size = 356, normalized size = 2.58

$$\frac{Bd^2 \log(bx + a)}{b^3c^2g^3 - 2ab^2cdg^3 + a^2bd^2g^3} - \frac{Bd^2 \log(dx + c)}{b^3c^2g^3 - 2ab^2cdg^3 + a^2bd^2g^3} - \frac{B \log\left(\frac{b^2x^2 + 2abx + a^2}{d^2x^2 + 2cdx + c^2}\right)}{2(b^3g^3x^2 + 2ab^2g^3x + a^2bg^3)} + \frac{2B}{2(b^4cg^3x^2 - ab^3dg^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] B*d^2*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - B*d^2*log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2*B*log((b^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 1/2*(2*B*b*d*x - A*b*c - 2*B*b*c + A*a*d + 4*B*a*d)/(b^4*c*g^3*x^2 - a*b^3*d*g^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3*x + a^2*b^2*c*g^3 - a^3*b*d*g^3)
```

$$3.126 \quad \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^4} dx$$

**Optimal.** Leaf size=177

$$-\frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{3bg^4(a+bx)^3} - \frac{2Bd^2}{3bg^4(a+bx)(bc-ad)^2} - \frac{2Bd^3 \log(a+bx)}{3bg^4(bc-ad)^3} + \frac{2Bd^3 \log(c+dx)}{3bg^4(bc-ad)^3} + \frac{Bd}{3bg^4(a+bx)^2(bc-ad)} - \frac{1}{9bg^4}$$

[Out]  $(-2*B)/(9*b*g^4*(a+b*x)^3) + (B*d)/(3*b*(b*c-a*d)*g^4*(a+b*x)^2) - (2*B*d^2)/(3*b*(b*c-a*d)^2*g^4*(a+b*x)) - (2*B*d^3*Log[a+b*x])/(3*b*(b*c-a*d)^3*g^4) - (A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2])/(3*b*g^4*(a+b*x)^3) + (2*B*d^3*Log[c+d*x])/(3*b*(b*c-a*d)^3*g^4)$

**Rubi [A]** time = 0.114985, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{3bg^4(a+bx)^3} - \frac{2Bd^2}{3bg^4(a+bx)(bc-ad)^2} - \frac{2Bd^3 \log(a+bx)}{3bg^4(bc-ad)^3} + \frac{2Bd^3 \log(c+dx)}{3bg^4(bc-ad)^3} + \frac{Bd}{3bg^4(a+bx)^2(bc-ad)} - \frac{1}{9bg^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(a\*g + b\*g\*x)^4, x]

[Out]  $(-2*B)/(9*b*g^4*(a+b*x)^3) + (B*d)/(3*b*(b*c-a*d)*g^4*(a+b*x)^2) - (2*B*d^2)/(3*b*(b*c-a*d)^2*g^4*(a+b*x)) - (2*B*d^3*Log[a+b*x])/(3*b*(b*c-a*d)^3*g^4) - (A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2])/(3*b*g^4*(a+b*x)^3) + (2*B*d^3*Log[c+d*x])/(3*b*(b*c-a*d)^3*g^4)$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 44

Int[((a\_) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag + bgx)^4} dx &= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a + bx)^3} + \frac{B \int \frac{2(bc-ad)}{g^3(a+bx)^4(c+dx)} dx}{3bg} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a + bx)^3} + \frac{(2B(bc - ad)) \int \frac{1}{(a+bx)^4(c+dx)} dx}{3bg^4} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3bg^4(a + bx)^3} + \frac{(2B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^4} - \frac{bd}{(bc-ad)^2(a+bx)^3} + \frac{bd^2}{(bc-ad)^3(a+bx)^2} - \frac{bd^3}{(bc-ad)^4(a+bx)}\right) dx}{3bg^4} \\
&= -\frac{2B}{9bg^4(a + bx)^3} + \frac{Bd}{3b(bc - ad)g^4(a + bx)^2} - \frac{2Bd^2}{3b(bc - ad)^2g^4(a + bx)} - \frac{2Bd^3 \log(a + bx)}{3b(bc - ad)^3g^4}
\end{aligned}$$

**Mathematica [A]** time = 0.134791, size = 140, normalized size = 0.79

$$\frac{3 \left( B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A \right) + \frac{B(6d^2(a+bx)^2(bc-ad) - 6d^3(a+bx)^3 \log(c+dx) - 3d(a+bx)(bc-ad)^2 + 2(bc-ad)^3 + 6d^3(a+bx)^3 \log(a+bx))}{(bc-ad)^3}}{9bg^4(a + bx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(a\*g + b\*g\*x)^4, x]

[Out] -(3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) + (B\*(2\*(b\*c - a\*d)^3 - 3\*d\*(b\*c - a\*d)^2\*(a + b\*x) + 6\*d^2\*(b\*c - a\*d)\*(a + b\*x)^2 + 6\*d^3\*(a + b\*x)^3\*Log[a + b\*x] - 6\*d^3\*(a + b\*x)^3\*Log[c + d\*x]))/(b\*c - a\*d)^3)/(9\*b\*g^4\*(a + b\*x)^3)

**Maple [B]** time = 0.148, size = 579, normalized size = 3.3

$$-\frac{d^3 Ab}{g^4(ad - bc)^3} \left( \frac{ad}{dx + c} - \frac{bc}{dx + c} + b \right)^{-2} + \frac{d^3 A}{g^4(ad - bc)^3} \left( \frac{ad}{dx + c} - \frac{bc}{dx + c} + b \right)^{-1} + \frac{d^3 Ab^2}{3g^4(ad - bc)^3} \left( \frac{ad}{dx + c} - \frac{bc}{dx + c} + b \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))/(b\*g\*x+a\*g)^4, x)

[Out] -d^3/g^4\*A\*b/(a\*d-b\*c)^3/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2+d^3/g^4\*A/(a\*d-b\*c)^3/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)+1/3\*d^3/g^4\*A\*b^2/(a\*d-b\*c)^3/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^3-11/9\*d^3/g^4/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^3\*B/b/(d\*x+c)^3+1/3\*d^3/g^4/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^3\*b^2\*B/(a^3\*d^3-3\*a^2\*b\*c\*d^2+3\*a\*b^2\*c^2\*d-b^3\*c^3)\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)-2/3\*d^3/g^4/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^3\*B\*b/(a^2\*d^2-2\*a\*b\*c\*d+b^2\*c^2)/(d\*x+c)-5/3\*d^3/g^4/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^3\*B/(a\*d-b\*c)/(d\*x+c)^2+d^3/g^4/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^3\*B/(a\*d-b\*c)/(d\*x+c)^2\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)+d^3/g^4/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^3\*B\*b/(a^2\*d^2-2\*a\*b\*c\*d+b^2\*c^2)/(d\*x+c)\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)

**Maxima [B]** time = 1.23764, size = 648, normalized size = 3.66

$$-\frac{1}{9} B \left( \frac{6b^2d^2x^2 + 2b^2c^2 - 7abcd + 11a^2d^2 - 3(b^2cd - 5abd^2)x}{(b^6c^2 - 2ab^5cd + a^2b^4d^2)g^4x^3 + 3(ab^5c^2 - 2a^2b^4cd + a^3b^3d^2)g^4x^2 + 3(a^2b^4c^2 - 2a^3b^3cd + a^4b^2d^2)g^4x + (a^3b^3c^2 - 2a^4b^2cd + a^5b^1d^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="maxima")
```

```
[Out] -1/9*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

**Fricas [B]** time = 1.07211, size = 869, normalized size = 4.91

$$\frac{(3A + 2B)b^3c^3 - 9(A + B)ab^2c^2d + 9(A + 2B)a^2bcd^2 - (3A + 11B)a^3d^3 + 6(Bb^3cd^2 - Bab^2d^3)x^2 - 3(Bb^3c^2d - 6Bab^2cd^2 - 3Aab^2c^2d + 3a^2b^5cd^2 - a^3b^4d^3)g^4x^3 + 3(ab^6c^3 - 3a^2b^5c^2d + 3a^3b^4cd^2 - a^4b^3d^3)g^4x^2 + \dots}{9((b^7c^3 - 3ab^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3)g^4x^3 + 3(ab^6c^3 - 3a^2b^5c^2d + 3a^3b^4cd^2 - a^4b^3d^3)g^4x^2 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="fricas")
```

```
[Out] -1/9*((3*A + 2*B)*b^3*c^3 - 9*(A + B)*a*b^2*c^2*d + 9*(A + 2*B)*a^2*b*c*d^2 - (3*A + 11*B)*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 3*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

**Sympy [B]** time = 5.49235, size = 677, normalized size = 3.82

$$\frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3a^3bg^4 + 9a^2b^2g^4x + 9ab^3g^4x^2 + 3b^4g^4x^3} - \frac{2Bd^3 \log\left(x + \frac{-\frac{2Ba^4d^7}{(ad-bc)^3} + \frac{8Ba^3bcd^6}{(ad-bc)^3} - \frac{12Ba^2b^2c^2d^5}{(ad-bc)^3} + \frac{8Bab^3c^3d^4}{(ad-bc)^3} + 2Bad^4 - \frac{2Bb^4c^4d^3}{(ad-bc)^3} + 2Bbcd^3}{4Bbd^4}\right)}{3bg^4(ad-bc)^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(a + b*x)**2/(c + d*x)**2)/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) - 2*B*d**3*log(x + (-2*B*a**4*d**7/(a*d - b*c)**3 + 8*B*a**3*b*c*d**6/(a*d - b*c)**3 - 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 + 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 - 2*B*b**4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + 2*B*d**3*log(x + (2*B*a**4*d**7/(a*d - b*c)**3 - 8*B*a**3*b*c*d**6/(a*d - b*c)**3 + 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 - 2*B*a*d**4 + 2*B*b**4*c**4*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3)
```

$$\frac{6/(a*d - b*c)**3 + 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 + 2*B*b**4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) - (3*A*a**2*d**2 - 6*A*a*b*c*d + 3*A*b**2*c**2 + 11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**2*c**2 + 6*B*b**2*d**2*x**2 + x*(15*B*a*b*d**2 - 3*B*b**2*c*d))/(9*a**5*b*d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 9*a**3*b**3*c**2*g**4 + x**3*(9*a**2*b**4*d**2*g**4 - 18*a*b**5*c*d*g**4 + 9*b**6*c**2*g**4) + x**2*(27*a**3*b**3*d**2*g**4 - 54*a**2*b**4*c*d*g**4 + 27*a*b**5*c**2*g**4) + x*(27*a**4*b**2*d**2*g**4 - 54*a**3*b**3*c*d*g**4 + 27*a**2*b**4*c**2*g**4))$$

**Giac [B]** time = 1.33325, size = 639, normalized size = 3.61

$$\frac{2 B d^3 \log (b x+a)}{3\left(b^4 c^3 g^4-3 a b^3 c^2 d g^4+3 a^2 b^2 c d^2 g^4-a^3 b d^3 g^4\right)}+\frac{2 B d^3 \log (d x+c)}{3\left(b^4 c^3 g^4-3 a b^3 c^2 d g^4+3 a^2 b^2 c d^2 g^4-a^3 b d^3 g^4\right)}-\frac{B}{3\left(b^4 g^4 x^3+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] -2/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) + 2/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((b^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/9*(6*B*b^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*B*a*b*d^2*x + 3*A*b^2*c^2 + 5*B*b^2*c^2 - 6*A*a*b*c*d - 13*B*a*b*c*d + 3*A*a^2*d^2 + 14*B*a^2*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d^2*g^4*x^3 + 3*a*b^5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g^4*x^2 + 3*a^2*b^4*c^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x + a^3*b^3*c^2*g^4 - 2*a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

$$3.127 \quad \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag+bgx)^5} dx$$

**Optimal.** Leaf size=208

$$-\frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{4bg^5(a+bx)^4} + \frac{Bd^3}{2bg^5(a+bx)(bc-ad)^3} - \frac{Bd^2}{4bg^5(a+bx)^2(bc-ad)^2} + \frac{Bd^4 \log(a+bx)}{2bg^5(bc-ad)^4} - \frac{Bd^4 \log(c+dx)}{2bg^5(bc-ad)^4} + \frac{6bg^5(a+bx)^5}{6bg^5(a+bx)^5}$$

[Out]  $-B/(8*b*g^5*(a+b*x)^4) + (B*d)/(6*b*(b*c-a*d)*g^5*(a+b*x)^3) - (B*d^2)/(4*b*(b*c-a*d)^2*g^5*(a+b*x)^2) + (B*d^3)/(2*b*(b*c-a*d)^3*g^5*(a+b*x)) + (B*d^4*Log[a+b*x])/(2*b*(b*c-a*d)^4*g^5) - (A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2])/(4*b*g^5*(a+b*x)^4) - (B*d^4*Log[c+d*x])/(2*b*(b*c-a*d)^4*g^5)$

**Rubi [A]** time = 0.144397, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{4bg^5(a+bx)^4} + \frac{Bd^3}{2bg^5(a+bx)(bc-ad)^3} - \frac{Bd^2}{4bg^5(a+bx)^2(bc-ad)^2} + \frac{Bd^4 \log(a+bx)}{2bg^5(bc-ad)^4} - \frac{Bd^4 \log(c+dx)}{2bg^5(bc-ad)^4} + \frac{6bg^5(a+bx)^5}{6bg^5(a+bx)^5}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(a\*g + b\*g\*x)^5, x]

[Out]  $-B/(8*b*g^5*(a+b*x)^4) + (B*d)/(6*b*(b*c-a*d)*g^5*(a+b*x)^3) - (B*d^2)/(4*b*(b*c-a*d)^2*g^5*(a+b*x)^2) + (B*d^3)/(2*b*(b*c-a*d)^3*g^5*(a+b*x)) + (B*d^4*Log[a+b*x])/(2*b*(b*c-a*d)^4*g^5) - (A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2])/(4*b*g^5*(a+b*x)^4) - (B*d^4*Log[c+d*x])/(2*b*(b*c-a*d)^4*g^5)$

#### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(ag + bgx)^5} dx &= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a + bx)^4} + \frac{B \int \frac{2(bc-ad)}{g^4(a+bx)^5(c+dx)} dx}{4bg} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a + bx)^4} + \frac{(B(bc - ad)) \int \frac{1}{(a+bx)^5(c+dx)} dx}{2bg^5} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4bg^5(a + bx)^4} + \frac{(B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^5} - \frac{bd}{(bc-ad)^2(a+bx)^4} + \frac{bd^2}{(bc-ad)^3(a+bx)^3} - \frac{bd^3}{(bc-ad)^4(a+bx)^2} + \frac{bd^4}{(bc-ad)^5(a+bx)}\right) dx}{2bg^5} \\
&= -\frac{B}{8bg^5(a + bx)^4} + \frac{Bd}{6b(bc - ad)g^5(a + bx)^3} - \frac{Bd^2}{4b(bc - ad)^2g^5(a + bx)^2} + \frac{Bd^3}{2b(bc - ad)^3g^5(a + bx)} - \frac{Bd^4}{b(bc - ad)^4g^5(a + bx)}
\end{aligned}$$

**Mathematica [A]** time = 0.2014, size = 162, normalized size = 0.78

$$\frac{6 \left( B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A \right) + \frac{B(6d^2(a+bx)^2(bc-ad)^2 + 12d^3(a+bx)^3(ad-bc) + 12d^4(a+bx)^4 \log(c+dx) + 4d(a+bx)(ad-bc)^3 + 3(bc-ad)^4 - 12d^4(a+bx)^4 \log(a+bx))}{(bc-ad)^4}}{24bg^5(a + bx)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(a\*g + b\*g\*x)^5, x]

[Out] -(6\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) + (B\*(3\*(b\*c - a\*d)^4 + 4\*d\*(-(b\*c) + a\*d)^3\*(a + b\*x) + 6\*d^2\*(b\*c - a\*d)^2\*(a + b\*x)^2 + 12\*d^3\*(-(b\*c) + a\*d)\*(a + b\*x)^3 - 12\*d^4\*(a + b\*x)^4\*Log[a + b\*x] + 12\*d^4\*(a + b\*x)^4\*Log[c + d\*x]))/(b\*c - a\*d)^4)/(24\*b\*g^5\*(a + b\*x)^4)

**Maple [B]** time = 0.207, size = 833, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))/(b\*g\*x+a\*g)^5, x)

[Out] -3/2\*d^4/g^5\*A\*b/(a\*d-b\*c)^4/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2+d^4/g^5\*A/(a\*d-b\*c)^4/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)+d^4/g^5\*A\*b^2/(a\*d-b\*c)^4/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^3-1/4\*d^4/g^5\*A\*b^3/(a\*d-b\*c)^4/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^4-25/24\*d^4/g^5/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^4\*B/b/(d\*x+c)^4+1/4\*d^4/g^5/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^4\*b^3\*B/(a^4\*d^4-4\*a^3\*b\*c\*d^3+6\*a^2\*b^2\*c^2\*d^2-4\*a\*b^3\*c^3\*d+b^4\*c^4)\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)-1/2\*d^4/g^5/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^4\*B\*b^2/(a^3\*d^3-3\*a^2\*b\*c\*d^2+3\*a\*b^2\*c^2\*d-b^3\*c^3)/(d\*x+c)-7/4\*d^4/g^5/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^4\*B\*b/(a^2\*d^2-2\*a\*b\*c\*d+b^2\*c^2)/(d\*x+c)^2-13/6\*d^4/g^5/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^4\*B/(a\*d-b\*c)/(d\*x+c)^3+d^4/g^5/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^4\*B/(a\*d-b\*c)/(d\*x+c)^3\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)+3/2\*d^4/g^5/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^4\*b\*B/(a^2\*d^2-2\*a\*b\*c\*d+b^2\*c^2)/(d\*x+c)^2\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)+d^4/g^5/(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^4\*B\*b^2/(a^3\*d^3-3\*a^2\*b\*c\*d^2+3\*a\*b^2\*c^2\*d-b^3\*c^3)/(d\*x+c)\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)

**Maxima [B]** time = 1.3979, size = 944, normalized size = 4.54

$$\frac{1}{24} B \left( \frac{12 b^3 d^3 x^3 - 3 b^3 c^3 + 13 a b^2 c^2 d - 23 a^2 b c d^2 + 25 a^3 d^3}{(b^8 c^3 - 3 a b^7 c^2 d + 3 a^2 b^6 c d^2 - a^3 b^5 d^3)} g^5 x^4 + 4 (a b^7 c^3 - 3 a^2 b^6 c^2 d + 3 a^3 b^5 c d^2 - a^4 b^4 d^3) g^5 x^3 + 6 (a^2 b^6 c^3 - 3 a^3 b^5 c^2 d + 3 a^4 b^4 c^2 d^2 - 3 a^5 b^3 c^2 d^3 + 3 a^6 b^2 c^2 d^3 - 3 a^7 b^2 c^2 d^3 + 3 a^8 b^2 c^2 d^3) g^5 x^2 + 4 (a^3 b^5 c^3 - 3 a^4 b^4 c^2 d + 3 a^5 b^3 c^2 d^2 - a^6 b^2 c^2 d^3) g^5 x + (a^4 b^4 c^3 - 3 a^5 b^3 c^2 d + 3 a^6 b^2 c^2 d^2 - a^7 b^2 c^2 d^3) g^5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(b\*g\*x+a\*g)^5,x, algorithm="maxima")

[Out] 1/24\*B\*((12\*b^3\*d^3\*x^3 - 3\*b^3\*c^3 + 13\*a\*b^2\*c^2\*d - 23\*a^2\*b\*c\*d^2 + 25\*a^3\*d^3 - 6\*(b^3\*c\*d^2 - 7\*a\*b^2\*d^3)\*x^2 + 4\*(b^3\*c^2\*d - 5\*a\*b^2\*c\*d^2 + 13\*a^2\*b\*d^3)\*x)/((b^8\*c^3 - 3\*a\*b^7\*c^2\*d + 3\*a^2\*b^6\*c\*d^2 - a^3\*b^5\*d^3)\*g^5\*x^4 + 4\*(a\*b^7\*c^3 - 3\*a^2\*b^6\*c^2\*d + 3\*a^3\*b^5\*c\*d^2 - a^4\*b^4\*d^3)\*g^5\*x^3 + 6\*(a^2\*b^6\*c^3 - 3\*a^3\*b^5\*c^2\*d + 3\*a^4\*b^4\*c\*d^2 - a^5\*b^3\*d^3)\*g^5\*x^2 + 4\*(a^3\*b^5\*c^3 - 3\*a^4\*b^4\*c^2\*d + 3\*a^5\*b^3\*c\*d^2 - a^6\*b^2\*d^3)\*g^5\*x + (a^4\*b^4\*c^3 - 3\*a^5\*b^3\*c^2\*d + 3\*a^6\*b^2\*c\*d^2 - a^7\*b\*d^3)\*g^5) - 6\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2))/(b^5\*g^5\*x^4 + 4\*a\*b^4\*g^5\*x^3 + 6\*a^2\*b^3\*g^5\*x^2 + 4\*a^3\*b^2\*g^5\*x + a^4\*b\*g^5) + 12\*d^4\*log(b\*x + a)/((b^5\*c^4 - 4\*a\*b^4\*c^3\*d + 6\*a^2\*b^3\*c^2\*d^2 - 4\*a^3\*b^2\*c\*d^3 + a^4\*b\*d^4)\*g^5) - 12\*d^4\*log(d\*x + c)/((b^5\*c^4 - 4\*a\*b^4\*c^3\*d + 6\*a^2\*b^3\*c^2\*d^2 - 4\*a^3\*b^2\*c\*d^3 + a^4\*b\*d^4)\*g^5) - 1/4\*A/(b^5\*g^5\*x^4 + 4\*a\*b^4\*g^5\*x^3 + 6\*a^2\*b^3\*g^5\*x^2 + 4\*a^3\*b^2\*g^5\*x + a^4\*b\*g^5)

**Fricas [B]** time = 1.08479, size = 1328, normalized size = 6.38

$$\frac{3(2A+B)b^4c^4 - 8(3A+2B)ab^3c^3d + 36(A+B)a^2b^2c^2d^2 - 24(A+2B)a^3bcd^3 + (6A+25B)a^4d^4 - 12(Bb^4cd^3 - Ba^4cd^3)}{24((b^9c^4 - 4ab^8c^3d + 6a^2b^7c^2d^2 - 4a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4(ab^8c^4 - 4a^2b^7c^3d + 6a^3b^6c^2d^2 - 4a^4b^5c^2d^3 + 4a^5b^4c^2d^3 + 4a^6b^3c^2d^3 - 4a^7b^2c^2d^3 + a^8b^2c^2d^3)g^5x^3 + 6(a^2b^7c^4 - 4a^3b^6c^3d + 6a^4b^5c^2d^2 - 4a^5b^4c^2d^3 + a^6b^3c^2d^3)g^5x^2 + 4(a^3b^6c^4 - 4a^4b^5c^3d + 6a^5b^4c^2d^2 - 4a^6b^3c^2d^3 + a^7b^2c^2d^4)g^5x + (a^4b^5c^4 - 4a^5b^4c^3d + 6a^6b^3c^2d^2 - 4a^7b^2c^2d^3 + a^8b^2c^2d^4)g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(b\*g\*x+a\*g)^5,x, algorithm="fricas")

[Out] -1/24\*(3\*(2\*A + B)\*b^4\*c^4 - 8\*(3\*A + 2\*B)\*a\*b^3\*c^3\*d + 36\*(A + B)\*a^2\*b^2\*c^2\*d^2 - 24\*(A + 2\*B)\*a^3\*b\*c\*d^3 + (6\*A + 25\*B)\*a^4\*d^4 - 12\*(B\*b^4\*c\*d^3 - B\*a\*b^3\*d^4)\*x^3 + 6\*(B\*b^4\*c^2\*d^2 - 8\*B\*a\*b^3\*c\*d^3 + 7\*B\*a^2\*b^2\*d^4)\*x^2 - 4\*(B\*b^4\*c^3\*d - 6\*B\*a\*b^3\*c^2\*d^2 + 18\*B\*a^2\*b^2\*c\*d^3 - 13\*B\*a^3\*b\*d^4)\*x - 6\*(B\*b^4\*d^4\*x^4 + 4\*B\*a\*b^3\*d^4\*x^3 + 6\*B\*a^2\*b^2\*d^4\*x^2 + 4\*B\*a^3\*b\*d^4\*x - B\*b^4\*c^4 + 4\*B\*a\*b^3\*c^3\*d - 6\*B\*a^2\*b^2\*c^2\*d^2 + 4\*B\*a^3\*b\*c\*d^3)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)))/((b^9\*c^4 - 4\*a\*b^8\*c^3\*d + 6\*a^2\*b^7\*c^2\*d^2 - 4\*a^3\*b^6\*c\*d^3 + a^4\*b^5\*d^4)\*g^5\*x^4 + 4\*(a\*b^8\*c^4 - 4\*a^2\*b^7\*c^3\*d + 6\*a^3\*b^6\*c^2\*d^2 - 4\*a^4\*b^5\*c\*d^3 + a^5\*b^4\*d^4)\*g^5\*x^3 + 6\*(a^2\*b^7\*c^4 - 4\*a^3\*b^6\*c^3\*d + 6\*a^4\*b^5\*c^2\*d^2 - 4\*a^5\*b^4\*c\*d^3 + a^6\*b^3\*d^4)\*g^5\*x^2 + 4\*(a^3\*b^6\*c^4 - 4\*a^4\*b^5\*c^3\*d + 6\*a^5\*b^4\*c^2\*d^2 - 4\*a^6\*b^3\*c\*d^3 + a^7\*b^2\*d^4)\*g^5\*x + (a^4\*b^5\*c^4 - 4\*a^5\*b^4\*c^3\*d + 6\*a^6\*b^3\*c^2\*d^2 - 4\*a^7\*b^2\*c\*d^3 + a^8\*b^2\*d^4)\*g^5)

**Sympy [B]** time = 8.26937, size = 947, normalized size = 4.55

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))/(b\*g\*x+a\*g)\*\*5,x)

[Out] 
$$-B \log\left(\frac{e^{(a+bx)^2/(c+dx)^2}}{(4a^4bg^5 + 16a^3b^2g^5x + 24a^2b^3g^5x^2 + 16ab^4g^5x^3 + 4b^5g^5x^4) - B d^4 \log\left(x + \frac{-B a^5 d^9/(a d - b c)^4 + 5 B a^4 b c d^8/(a d - b c)^4 - 10 B a^3 b^2 c^2 d^7/(a d - b c)^4 + 10 B a^2 b^3 c^3 d^6/(a d - b c)^4 - 5 B a b^4 c^4 d^5/(a d - b c)^4 + B a d^5 + B b^5 c^5 d^4/(a d - b c)^4 + B b c d^4}{(2 B b d^5)}\right)}\right) / (2 b g^5 (a d - b c)^4) + B d^4 \log\left(x + \frac{B a^5 d^9/(a d - b c)^4 - 5 B a^4 b c d^8/(a d - b c)^4 + 10 B a^3 b^2 c^2 d^7/(a d - b c)^4 - 10 B a^2 b^3 c^3 d^6/(a d - b c)^4 + 5 B a b^4 c^4 d^5/(a d - b c)^4 + B a d^5 - B b^5 c^5 d^4/(a d - b c)^4 + B b c d^4}{(2 B b d^5)}\right) / (2 b g^5 (a d - b c)^4) - (6 A a^3 d^3 - 18 A a^2 b c d^2 + 18 A a b^2 c^2 d - 6 A b^3 c^3 + 25 B a^3 d^3 - 23 B a^2 b c d^2 + 13 B a b^2 c^2 d - 3 B b^3 c^3 + 12 B b^3 d^3 x^3 + x^2 (42 B a b^2 d^3 - 6 B b^3 c d^2) + x (52 B a^2 b d^3 - 20 B a b^2 c d^2 + 4 B b^3 c^2 d)) / (24 a^7 b d^3 g^5 - 72 a^6 b^2 c d^2 g^5 + 72 a^5 b^3 c^2 d g^5 - 24 a^4 b^4 c^3 g^5 + x^4 (24 a^3 b^5 d^3 g^5 - 72 a^2 b^6 c d^2 g^5 + 72 a b^7 c^2 d g^5 - 24 b^8 c^3 g^5) + x^3 (96 a^4 b^4 d^3 g^5 - 288 a^3 b^5 c d^2 g^5 + 288 a^2 b^6 c^2 d g^5 - 96 a b^7 c^3 g^5) + x^2 (144 a^5 b^3 d^3 g^5 - 432 a^4 b^4 c d^2 g^5 + 432 a^3 b^5 c^2 d g^5 - 144 a^2 b^6 c^3 g^5) + x (96 a^6 b^2 d^3 g^5 - 288 a^5 b^3 c d^2 g^5 + 288 a^4 b^4 c^2 d g^5 - 96 a^3 b^5 c^3 g^5))$$

**Giac [B]** time = 1.44465, size = 566, normalized size = 2.72

$$\frac{Bd^4 \log\left(-\frac{bcg}{bgx+ag} + \frac{adg}{bgx+ag} - d\right)}{2(b^5c^4g^5 - 4ab^4c^3dg^5 + 6a^2b^3c^2d^2g^5 - 4a^3b^2cd^3g^5 + a^4bd^4g^5)} + \frac{Bd^3}{2(b^3c^3g^3 - 3ab^2c^2dg^3 + 3a^2bcd^2g^3 - a^3d^3g^3)(bgx+ag)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(b\*g\*x+a\*g)^5,x, algorithm="giac")

[Out] 
$$-1/2 B d^4 \log(-b c g / (b g x + a g) + a d g / (b g x + a g) - d) / (b^5 c^4 g^5 - 4 a b^4 c^3 d g^5 + 6 a^2 b^3 c^2 d^2 g^5 - 4 a^3 b^2 c d^3 g^5 + a^4 b d^4 g^5) + 1/2 B d^3 / ((b^3 c^3 g^3 - 3 a b^2 c^2 d g^3 + 3 a^2 b c d^2 g^3 - a^3 d^3 g^3) * (b g x + a g) * b g) - 1/4 B d^2 / ((b^2 c^2 g - 2 a b c d g + a^2 d^2 g) * (b g x + a g)^2 * b g^2) - 1/4 B \log(b^2 / (b^2 c^2 g^2 / (b g x + a g)^2 - 2 a b c d g^2 / (b g x + a g)^2 + a^2 d^2 g^2 / (b g x + a g)^2 + 2 b c d g / (b g x + a g) - 2 a d^2 g / (b g x + a g) + d^2)) / ((b g x + a g)^4 * b g) + 1/6 B d / ((b g x + a g)^3 * (b c - a d) * b g^2) - 1/8 * (2 A b^3 g^3 + 3 B b^3 g^3) / ((b g x + a g)^4 * b^4 g^4)$$

$$3.128 \quad \int (ag + bgx)^4 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=377

$$\frac{8B^2g^4(bc-ad)^5 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} + \frac{2Bg^4(bc-ad)^5 \log\left(\frac{bc-ad}{b(c+dx)}\right) \left(6B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + 6A + 25B\right)}{15bd^5} + \frac{2Bg^4(a+bx)(bc-ad)^3 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{5bd^3}$$

[Out]  $-(B*(b*c - a*d)*g^4*(a + b*x)^4*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*b*d) + (g^4*(a + b*x)^5*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(5*b) + (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3*(2*A + B + 2*B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]))/(15*b*d^2) - (B*(b*c - a*d)^3*g^4*(a + b*x)^2*(6*A + 7*B + 6*B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]))/(15*b*d^3) + (2*B*(b*c - a*d)^4*g^4*(a + b*x)*(6*A + 13*B + 6*B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]))/(15*b*d^4) + (2*B*(b*c - a*d)^5*g^4*(6*A + 25*B + 6*B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c - a*d)/(b*(c + d*x))])/(15*b*d^5) + (8*B^2*(b*c - a*d)^5*g^4*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(5*b*d^5)$

**Rubi [A]** time = 0.871967, antiderivative size = 569, normalized size of antiderivative = 1.51, number of steps used = 28, number of rules used = 13, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{8B^2g^4(bc-ad)^5 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{5bd^5} - \frac{4Bg^4(bc-ad)^5 \log(c+dx) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{5bd^5} - \frac{2Bg^4(a+bx)^2(bc-ad)^3 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{5bd^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^4*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]$

[Out]  $(4*A*B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (26*B^2*(b*c - a*d)^4*g^4*x)/(15*d^4) - (7*B^2*(b*c - a*d)^3*g^4*(a + b*x)^2)/(15*b*d^3) + (2*B^2*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) + (4*B^2*(b*c - a*d)^4*g^4*(a + b*x)*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])/(5*b*d^4) - (2*B*(b*c - a*d)^3*g^4*(a + b*x)^2*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*b*d^3) + (4*B*(b*c - a*d)^2*g^4*(a + b*x)^3*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]))/(15*b*d^2) - (B*(b*c - a*d)*g^4*(a + b*x)^4*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*b*d) + (g^4*(a + b*x)^5*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(5*b) - (10*B^2*(b*c - a*d)^5*g^4*\text{Log}[c + d*x])/(3*b*d^5) + (8*B^2*(b*c - a*d)^5*g^4*\text{Log}[-(d*(a + b*x))/(b*c - a*d)])*\text{Log}[c + d*x])/(5*b*d^5) - (4*B*(b*c - a*d)^5*g^4*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(5*b*d^5) - (4*B^2*(b*c - a*d)^5*g^4*\text{Log}[c + d*x]^2)/(5*b*d^5) + (8*B^2*(b*c - a*d)^5*g^4*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(5*b*d^5)$

### Rule 2525

$\text{Int}[(a_. + \text{Log}[c_.*(\text{RFx}_.)^{(p_.)}]*(b_.))^{(n_.)}*((d_.) + (e_.)*(x_.))^{(m_.)}, x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{(m+1)}*(a + b*\text{Log}[c*\text{RFx}^p])^n/(e*(m+1)), x] - \text{Dist}[(b*n*p)/(e*(m+1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{(m+1)}*(a + b*\text{Log}[c*\text{RFx}^p])^{(n-1)}*D[\text{RFx}, x])/(\text{RFx}, x), x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(RFX\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFX, x] && IntegerQ[p]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))^(p_.)*((f_) + (g_.)*(x_)^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
 \int (ag + bgx)^4 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2 dx &= \frac{g^4(a + bx)^5 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{5b} - \frac{(2B) \int \frac{2(bc - ad)g^5(a + bx)^4 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{c + dx}}{5bg} \\
 &= \frac{g^4(a + bx)^5 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{5b} - \frac{(4B(bc - ad)g^4) \int \frac{(a + bx)^4 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{c + dx}}{5b} \\
 &= \frac{g^4(a + bx)^5 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{5b} - \frac{(4B(bc - ad)g^4) \int \left( -\frac{b(bc - ad)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{d^4} \right)}{5b} \\
 &= \frac{g^4(a + bx)^5 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{5b} - \frac{(4B(bc - ad)g^4) \int (a + bx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{5d} \\
 &= \frac{4AB(bc - ad)^4 g^4 x}{5d^4} - \frac{2B(bc - ad)^3 g^4 (a + bx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{5bd^3} + \frac{4B^2(bc - ad)^4 g^4 (a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{5bd^4} - \frac{2B(bc - ad)^3 g^4 (a + bx)^2}{15bd^3} + \frac{2B^2(bc - ad)^4 g^4 (a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{15bd^4} \\
 &= \frac{4AB(bc - ad)^4 g^4 x}{5d^4} + \frac{4B^2(bc - ad)^4 g^4 (a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{5bd^4} - \frac{2B(bc - ad)^3 g^4 (a + bx)^2}{15bd^3} + \frac{2B^2(bc - ad)^4 g^4 (a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{15bd^4} \\
 &= \frac{4AB(bc - ad)^4 g^4 x}{5d^4} + \frac{26B^2(bc - ad)^4 g^4 x}{15d^4} - \frac{7B^2(bc - ad)^3 g^4 (a + bx)^2}{15bd^3} + \frac{2B^2(bc - ad)^4 g^4 (a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{15bd^4} \\
 &= \frac{4AB(bc - ad)^4 g^4 x}{5d^4} + \frac{26B^2(bc - ad)^4 g^4 x}{15d^4} - \frac{7B^2(bc - ad)^3 g^4 (a + bx)^2}{15bd^3} + \frac{2B^2(bc - ad)^4 g^4 (a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{15bd^4} \\
 &= \frac{4AB(bc - ad)^4 g^4 x}{5d^4} + \frac{26B^2(bc - ad)^4 g^4 x}{15d^4} - \frac{7B^2(bc - ad)^3 g^4 (a + bx)^2}{15bd^3} + \frac{2B^2(bc - ad)^4 g^4 (a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{15bd^4}
 \end{aligned}$$

**Mathematica [A]** time = 0.465664, size = 523, normalized size = 1.39

$$g^4 \left( \frac{B(bc-ad) \left( 12B(bc-ad)^4 \left( 2 \operatorname{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) - 6d^2(a+bx)^2(bc-ad)^2 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right) + 4d^3(a+bx)^3(bc-ad) \right)}{\dots} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^4\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2,x]

[Out]  $(g^4 * ((a + b*x)^5 * (A + B * \operatorname{Log}[(e * (a + b*x)^2] / (c + d*x)^2])^2 + (B * (b*c - a*d) * (12 * A * b*d * (b*c - a*d)^3 * x + 12 * B * d * (b*c - a*d)^3 * (a + b*x) * \operatorname{Log}[(e * (a + b*x)^2] / (c + d*x)^2] - 6 * d^2 * (b*c - a*d)^2 * (a + b*x)^2 * (A + B * \operatorname{Log}[(e * (a + b*x)^2] / (c + d*x)^2]) + 4 * d^3 * (b*c - a*d) * (a + b*x)^3 * (A + B * \operatorname{Log}[(e * (a + b*x)^2] / (c + d*x)^2]) - 3 * d^4 * (a + b*x)^4 * (A + B * \operatorname{Log}[(e * (a + b*x)^2] / (c + d*x)^2]) - 24 * B * (b*c - a*d)^4 * \operatorname{Log}[c + d*x] - 12 * (b*c - a*d)^4 * (A + B * \operatorname{Log}[(e * (a + b*x)^2] / (c + d*x)^2]) * \operatorname{Log}[c + d*x] + 4 * B * (b*c - a*d)^2 * (2 * b*d * (b*c - a*d) * x - d^2 * (a + b*x)^2 - 2 * (b*c - a*d)^2 * \operatorname{Log}[c + d*x]) + B * (b*c - a*d) * (6 * b*d * (b*c - a*d)^2 * x + 3 * d^2 * (- (b*c) + a*d) * (a + b*x)^2 + 2 * d^3 * (a + b*x)^3 - 6 * (b*c - a*d)^3 * \operatorname{Log}[c + d*x]) + 12 * B * (b*c - a*d)^3 * (b*d * x + (- (b*c) + a*d) * \operatorname{Log}[c + d*x]) + 12 * B * (b*c - a*d)^4 * ((2 * \operatorname{Log}[(d * (a + b*x)) / (- (b*c) + a*d)]) - \operatorname{Log}[c + d*x]) * \operatorname{Log}[c + d*x] + 2 * \operatorname{PolyLog}[2, (b * (c + d*x)) / (b*c - a*d)])) / (3 * d^5)) / (5 * b)$

**Maple [F]** time = 1.7, size = 0, normalized size = 0.

$$\int (bgx + ag)^4 \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^4\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int((b\*g\*x+a\*g)^4\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [B]** time = 2.08981, size = 3578, normalized size = 9.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out]  $1/5 * A^2 * b^4 * g^4 * x^5 + A^2 * a * b^3 * g^4 * x^4 + 2 * A^2 * a^2 * b^2 * g^4 * x^3 + 2 * A^2 * a^3 * b * g^4 * x^2 + 2 * (x * \log(b^2 * e * x^2 / (d^2 * x^2 + 2 * c * d * x + c^2)) + 2 * a * b * e * x / (d^2 * x^2 + 2 * c * d * x + c^2) + a^2 * e / (d^2 * x^2 + 2 * c * d * x + c^2)) + 2 * a * \log(b * x + a) / b - 2 * c * \log(d * x + c) / d * A * B * a^4 * g^4 + 4 * (x^2 * \log(b^2 * e * x^2 / (d^2 * x^2 + 2 * c * d * x + c^2)) + 2 * a * b * e * x / (d^2 * x^2 + 2 * c * d * x + c^2) + a^2 * e / (d^2 * x^2 + 2 * c * d * x + c^2)) - 2 * a^2 * \log(b * x + a) / b^2 + 2 * c^2 * \log(d * x + c) / d^2 - 2 * (b * c - a * d) * x / (b * d) * A * B * a^3 * b * g^4 + 4 * (x^3 * \log(b^2 * e * x^2 / (d^2 * x^2 + 2 * c * d * x + c^2)) + 2 * a * b * e * x / (d^2 * x^2 + 2 * c * d * x + c^2) + a^2 * e / (d^2 * x^2 + 2 * c * d * x + c^2)) + 2 * a^3 * \log(b * x + a) / b^3 - 2 * c^3 * \log(d * x + c) / d^3 - ((b^2 * c * d - a * b * d^2) * x^2 - 2 *$

$$\begin{aligned}
& (b^2c^2 - a^2d^2)x/(b^2d^2)) * A * B * a^2 * b^2 * g^4 + 2/3 * (3x^4 * \log(b^2 * e * x^2 / (d^2 * x^2 + 2 * c * d * x + c^2)) + 2 * a * b * e * x / (d^2 * x^2 + 2 * c * d * x + c^2) + a^2 * e / (d^2 * x^2 + 2 * c * d * x + c^2)) - 6 * a^4 * \log(b * x + a) / b^4 + 6 * c^4 * \log(d * x + c) / d^4 \\
& - (2 * (b^3 * c * d^2 - a * b^2 * d^3) * x^3 - 3 * (b^3 * c^2 * d - a^2 * b * d^3) * x^2 + 6 * (b^3 * c^3 - a^3 * d^3) * x) / (b^3 * d^3)) * A * B * a * b^3 * g^4 + 1/15 * (6 * x^5 * \log(b^2 * e * x^2 / (d^2 * x^2 + 2 * c * d * x + c^2)) + 2 * a * b * e * x / (d^2 * x^2 + 2 * c * d * x + c^2) + a^2 * e / (d^2 * x^2 + 2 * c * d * x + c^2)) + 12 * a^5 * \log(b * x + a) / b^5 - 12 * c^5 * \log(d * x + c) / d^5 - (3 * (b^4 * c * d^3 - a * b^3 * d^4) * x^4 - 4 * (b^4 * c^2 * d^2 - a^2 * b^2 * d^4) * x^3 + 6 * (b^4 * c^3 * d - a^3 * b * d^4) * x^2 - 12 * (b^4 * c^4 - a^4 * d^4) * x) / (b^4 * d^4)) * A * B * b^4 * g^4 + \\
& A^2 * a^4 * g^4 * x - 2/15 * ((6 * g^4 * \log(e) + 25 * g^4) * b^4 * c^5 - (30 * g^4 * \log(e) + 113 * g^4) * a * b^3 * c^4 * d + 4 * (15 * g^4 * \log(e) + 49 * g^4) * a^2 * b^2 * c^3 * d^2 - 12 * (5 * g^4 * \log(e) + 13 * g^4) * a^3 * b * c^2 * d^3 + 6 * (5 * g^4 * \log(e) + 8 * g^4) * a^4 * c * d^4) * B^2 * \log(d * x + c) / d^5 - 8/5 * (b^5 * c^5 * g^4 - 5 * a * b^4 * c^4 * d * g^4 + 10 * a^2 * b^3 * c^3 * d^2 * g^4 - 10 * a^3 * b^2 * c^2 * d^3 * g^4 + 5 * a^4 * b * c * d^4 * g^4 - a^5 * d^5 * g^4) * (\log(b * x + a) * \log((b * d * x + a * d) / (b * c - a * d)) + 1) + \operatorname{dilog}(-(b * d * x + a * d) / (b * c - a * d)) * B^2 / (b * d^5) + 1/15 * (3 * B^2 * b^5 * d^5 * g^4 * x^5 * \log(e)^2 - 3 * (b^5 * c * d^4 * g^4 * \log(e) - (5 * g^4 * \log(e)^2 + g^4 * \log(e)) * a * b^4 * d^5) * B^2 * x^4 + 2 * ((2 * g^4 * \log(e) + g^4) * b^5 * c^2 * d^3 - 2 * (5 * g^4 * \log(e) + g^4) * a * b^4 * c * d^4 + (15 * g^4 * \log(e)^2 + 8 * g^4 * \log(e) + g^4) * a^2 * b^3 * d^5) * B^2 * x^3 - ((6 * g^4 * \log(e) + 7 * g^4) * b^5 * c^3 * d^2 - 3 * (10 * g^4 * \log(e) + 9 * g^4) * a * b^4 * c^2 * d^3 + 3 * (20 * g^4 * \log(e) + 11 * g^4) * a^2 * b^3 * c * d^4 - (30 * g^4 * \log(e)^2 + 36 * g^4 * \log(e) + 13 * g^4) * a^3 * b^2 * d^5) * B^2 * x^2 + (2 * (6 * g^4 * \log(e) + 13 * g^4) * b^5 * c^4 * d - 2 * (30 * g^4 * \log(e) + 59 * g^4) * a * b^4 * c^3 * d^2 + 12 * (10 * g^4 * \log(e) + 17 * g^4) * a^2 * b^3 * c^2 * d^3 - 2 * (60 * g^4 * \log(e) + 79 * g^4) * a^3 * b^2 * c * d^4 + (15 * g^4 * \log(e)^2 + 48 * g^4 * \log(e) + 46 * g^4) * a^4 * b * d^5) * B^2 * x + 12 * (B^2 * b^5 * d^5 * g^4 * x^5 + 5 * B^2 * a * b^4 * d^5 * g^4 * x^4 + 10 * B^2 * a^2 * b^3 * d^5 * g^4 * x^3 + 10 * B^2 * a^3 * b^2 * d^5 * g^4 * x^2 + 5 * B^2 * a^4 * b * d^5 * g^4 * x + B^2 * a^5 * d^5 * g^4) * \log(b * x + a)^2 + 12 * (B^2 * b^5 * d^5 * g^4 * x^5 + 5 * B^2 * a * b^4 * d^5 * g^4 * x^4 + 10 * B^2 * a^2 * b^3 * d^5 * g^4 * x^3 + 10 * B^2 * a^3 * b^2 * d^5 * g^4 * x^2 + 5 * B^2 * a^4 * b * d^5 * g^4 * x + (b^5 * c^5 * g^4 - 5 * a * b^4 * c^4 * d * g^4 + 10 * a^2 * b^3 * c^3 * d^2 * g^4 - 10 * a^3 * b^2 * c^2 * d^3 * g^4 + 5 * a^4 * b * c * d^4 * g^4) * B^2) * \log(d * x + c)^2 + 2 * (6 * B^2 * b^5 * d^5 * g^4 * x^5 * \log(e) - 3 * (b^5 * c * d^4 * g^4 - (10 * g^4 * \log(e) + g^4) * a * b^4 * d^5) * B^2 * x^4 + 4 * (b^5 * c^2 * d^3 * g^4 - 5 * a * b^4 * c * d^4 * g^4 + (15 * g^4 * \log(e) + 4 * g^4) * a^2 * b^3 * d^5) * B^2 * x^3 - 6 * (b^5 * c^3 * d^2 * g^4 - 5 * a * b^4 * c^2 * d^3 * g^4 + 10 * a^2 * b^3 * c * d^4 * g^4 - 2 * (5 * g^4 * \log(e) + 3 * g^4) * a^3 * b^2 * d^5) * B^2 * x^2 + 6 * (2 * b^5 * c^4 * d * g^4 - 10 * a * b^4 * c^3 * d^2 * g^4 + 20 * a^2 * b^3 * c^2 * d^3 * g^4 - 20 * a^3 * b^2 * c * d^4 * g^4 + (5 * g^4 * \log(e) + 8 * g^4) * a^4 * b * d^5) * B^2 * x + (12 * a * b^4 * c^4 * d * g^4 - 54 * a^2 * b^3 * c^3 * d^2 * g^4 + 94 * a^3 * b^2 * c^2 * d^3 * g^4 - 77 * a^4 * b * c * d^4 * g^4 + (6 * g^4 * \log(e) + 25 * g^4) * a^5 * d^5) * B^2) * \log(b * x + a) - 2 * (6 * B^2 * b^5 * d^5 * g^4 * x^5 * \log(e) - 3 * (b^5 * c * d^4 * g^4 - (10 * g^4 * \log(e) + g^4) * a * b^4 * d^5) * B^2 * x^4 + 4 * (b^5 * c^2 * d^3 * g^4 - 5 * a * b^4 * c * d^4 * g^4 + (15 * g^4 * \log(e) + 4 * g^4) * a^2 * b^3 * d^5) * B^2 * x^3 - 6 * (b^5 * c^3 * d^2 * g^4 - 5 * a * b^4 * c^2 * d^3 * g^4 + 10 * a^2 * b^3 * c * d^4 * g^4 - 2 * (5 * g^4 * \log(e) + 3 * g^4) * a^3 * b^2 * d^5) * B^2 * x^2 + 6 * (2 * b^5 * c^4 * d * g^4 - 10 * a * b^4 * c^3 * d^2 * g^4 + 20 * a^2 * b^3 * c^2 * d^3 * g^4 - 20 * a^3 * b^2 * c * d^4 * g^4 + (5 * g^4 * \log(e) + 8 * g^4) * a^4 * b * d^5) * B^2 * x + 12 * (B^2 * b^5 * d^5 * g^4 * x^5 + 5 * B^2 * a * b^4 * d^5 * g^4 * x^4 + 10 * B^2 * a^2 * b^3 * d^5 * g^4 * x^3 + 10 * B^2 * a^3 * b^2 * d^5 * g^4 * x^2 + 5 * B^2 * a^4 * b * d^5 * g^4 * x + B^2 * a^5 * d^5 * g^4) * \log(b * x + a)) * \log(d * x + c)) / (b * d^5)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( A^2 b^4 g^4 x^4 + 4 A^2 a b^3 g^4 x^3 + 6 A^2 a^2 b^2 g^4 x^2 + 4 A^2 a^3 b g^4 x + A^2 a^4 g^4 + (B^2 b^4 g^4 x^4 + 4 B^2 a b^3 g^4 x^3 + 6 B^2 a^2 b^2 g^4 x^2 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((b^2*e*x^2 + 2*a
*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a
*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log
((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^4 \left( B \log \left( \frac{(bx + a)^2 e}{(dx + c)^2} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^4*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

$$3.129 \quad \int (ag + bgx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=319

$$\frac{2B^2g^3(bc-ad)^4 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right) - Bg^3(bc-ad)^4 \log\left(\frac{bc-ad}{b(c+dx)}\right) \left(3B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + 3A + 11B\right) - Bg^3(a+bx)(bc-ad)^2}{bd^4}$$

[Out]  $-(B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b*d) + (g^3*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*b) + (B*(b*c - a*d)^2*g^3*(a + b*x)^2*(3*A + 2*B + 3*B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(6*b*d^2) - (B*(b*c - a*d)^3*g^3*(a + b*x)*(3*A + 5*B + 3*B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b*d^3) - (B*(b*c - a*d)^4*g^3*(3*A + 11*B + 3*B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c - a*d)/(b*(c + d*x))])/(3*b*d^4) - (2*B^2*(b*c - a*d)^4*g^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^4)$

**Rubi [A]** time = 0.759814, antiderivative size = 470, normalized size of antiderivative = 1.47, number of steps used = 24, number of rules used = 13, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$-\frac{2B^2g^3(bc-ad)^4 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bd^4} + \frac{Bg^3(bc-ad)^4 \log(c+dx) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{bd^4} + \frac{Bg^3(a+bx)^2(bc-ad)^2 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{2bd^2}$$

Antiderivative was successfully verified.

[In] Int[(a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

[Out]  $-((A*B*(b*c - a*d)^3*g^3*x)/d^3) - (5*B^2*(b*c - a*d)^3*g^3*x)/(3*d^3) + (B^2*(b*c - a*d)^2*g^3*(a + b*x)^2)/(3*b*d^2) - (B^2*(b*c - a*d)^3*g^3*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b*d^3) + (B*(b*c - a*d)^2*g^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*b*d^2) - (B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b*d) + (g^3*(a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*b) + (11*B^2*(b*c - a*d)^4*g^3*Log[c + d*x])/(3*b*d^4) - (2*B^2*(b*c - a*d)^4*g^3*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d^4) + (B*(b*c - a*d)^4*g^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(b*d^4) + (B^2*(b*c - a*d)^4*g^3*Log[c + d*x]^2)/(b*d^4) - (2*B^2*(b*c - a*d)^4*g^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*d^4)$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]



Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]

Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(RFX\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFX, x] && IntegerQ[p]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\int (ag + bgx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx &= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{4b} - \frac{B \int \frac{2(bc-ad)g^4(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{c+dx}}{2bg} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{4b} - \frac{(B(bc-ad)g^3) \int \frac{(a+bx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{c+dx}}{b} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{4b} - \frac{(B(bc-ad)g^3) \int \left( \frac{b(bc-ad)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{d^3} \right)}{d} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{4b} - \frac{(B(bc-ad)g^3) \int (a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{d} \\
&= -\frac{AB(bc-ad)^3 g^3 x}{d^3} + \frac{B(bc-ad)^2 g^3 (a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{2bd^2} - \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{2bd^2} \\
&= -\frac{AB(bc-ad)^3 g^3 x}{d^3} - \frac{B^2(bc-ad)^3 g^3 (a+bx) \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{bd^3} + \frac{B(bc-ad)^2 g^3 (a+bx)^2}{bd^3} \\
&= -\frac{AB(bc-ad)^3 g^3 x}{d^3} - \frac{B^2(bc-ad)^3 g^3 (a+bx) \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{bd^3} + \frac{B(bc-ad)^2 g^3 (a+bx)^2}{bd^3} \\
&= -\frac{AB(bc-ad)^3 g^3 x}{d^3} - \frac{5B^2(bc-ad)^3 g^3 x}{3d^3} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{3bd^2} - \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{3bd^2} \\
&= -\frac{AB(bc-ad)^3 g^3 x}{d^3} - \frac{5B^2(bc-ad)^3 g^3 x}{3d^3} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{3bd^2} - \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{3bd^2} \\
&= -\frac{AB(bc-ad)^3 g^3 x}{d^3} - \frac{5B^2(bc-ad)^3 g^3 x}{3d^3} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{3bd^2} - \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{3bd^2} \\
&= -\frac{AB(bc-ad)^3 g^3 x}{d^3} - \frac{5B^2(bc-ad)^3 g^3 x}{3d^3} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{3bd^2} - \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{3bd^2}
\end{aligned}$$

**Mathematica [A]** time = 0.327971, size = 402, normalized size = 1.26

$$g^3 \left( (a+bx)^4 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)^2 - \frac{2B(bc-ad) \left( 6B(bc-ad)^3 \left( 2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \right) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) + 2d^3(a+bx)^3 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}{3bd^2} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 - (2*B*(b*c - a*d)*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - 12*B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x] + 2*B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 6*B*(b*c - a*d)^2*(b*d*x + (-b*c) + a*d)*Log[c + d*x]) + 6*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-b*c) + a*d] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4))/(4*b)
```

**Maple [F]** time = 1.577, size = 0, normalized size = 0.

$$\int (bgx + ag)^3 \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

**Maxima [B]** time = 1.99019, size = 2630, normalized size = 8.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*a^3*g^3 + 3*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3 + 2*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b^2*g^3 + 1/6*(3*x^4*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*g^3 + A^2*a^3*g^3*x + 1/3*((3*g^3*log(e) + 11*g^3)*b^3*c^4 - 2*(6*g^3*log(e) + 19*g^3)*a*b^2*c^3*d + 9*(2*g^3*log(e) + 5*g^3)*a^2*b*c^2*d^2 - 6*(2*g^3*log(e) + 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 2*(b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 - 4*(
```

$$\begin{aligned}
& b^4 c d^3 g^3 \log(e) - (3g^3 \log(e)^2 + g^3 \log(e)) a b^3 d^4 B^2 x^3 + 2 \\
& * ((3g^3 \log(e) + 2g^3) b^4 c^2 d^2 - 4(3g^3 \log(e) + g^3) a b^3 c d^3 + \\
& (9g^3 \log(e)^2 + 9g^3 \log(e) + 2g^3) a^2 b^2 d^4) B^2 x^2 - 4((3g^3 \log(e) + 5g^3) b^4 c^3 d - \\
& (12g^3 \log(e) + 17g^3) a b^3 c^2 d^2 + (18g^3 \log(e) + 19g^3) a^2 b^2 c d^3 - (3g^3 \log(e)^2 + 9g^3 \log(e) + 7g^3) a \\
& ^3 b d^4) B^2 x + 12(B^2 b^4 d^4 g^3 x^4 + 4B^2 a b^3 d^4 g^3 x^3 + 6B^2 a^2 b^2 d^4 g^3 x^2 + 4B^2 a^3 b d^4 g^3 x + B^2 a^4 d^4 g^3) \log(bx + a) \\
& ^2 + 12(B^2 b^4 d^4 g^3 x^4 + 4B^2 a b^3 d^4 g^3 x^3 + 6B^2 a^2 b^2 d^4 g^3 x^2 + 4B^2 a^3 b d^4 g^3 x - (b^4 c^4 g^3 - 4a b^3 c^3 d g^3 + 6a^2 b^2 c^2 d^2 g^3 - \\
& 4a^3 b c d^3 g^3) B^2) \log(dx + c)^2 + 4(3B^2 b^4 d^4 g^3 x^4 \log(e) - 2(b^4 c d^3 g^3 - (6g^3 \log(e) + g^3) a b^3 d^4) B^2 x \\
& ^3 + 3(b^4 c^2 d^2 g^3 - 4a b^3 c d^3 g^3 + 3(2g^3 \log(e) + g^3) a^2 b^2 d^4) B^2 x^2 - 6(b^4 c^3 d g^3 - 4a b^3 c^2 d^2 g^3 + 6a^2 b^2 c d^3 g^3 - \\
& (2g^3 \log(e) + 3g^3) a^3 b d^4) B^2 x - (6a b^3 c^3 d g^3 - 21a^2 b^2 c^2 d^2 g^3 + 26a^3 b c d^3 g^3 - (3g^3 \log(e) + 11g^3) a^4 d^4) B^2 \\
& ) \log(bx + a) - 4(3B^2 b^4 d^4 g^3 x^4 \log(e) - 2(b^4 c d^3 g^3 - (6g^3 \log(e) + g^3) a b^3 d^4) B^2 x^3 + 3(b^4 c^2 d^2 g^3 - 4a b^3 c d^3 g^3 \\
& + 3(2g^3 \log(e) + g^3) a^2 b^2 d^4) B^2 x^2 - 6(b^4 c^3 d g^3 - 4a b^3 c^2 d^2 g^3 + 6a^2 b^2 c d^3 g^3 - (2g^3 \log(e) + 3g^3) a^3 b d^4) B^2 x \\
& + 6(B^2 b^4 d^4 g^3 x^4 + 4B^2 a b^3 d^4 g^3 x^3 + 6B^2 a^2 b^2 d^4 g^3 x^2 + 4B^2 a^3 b d^4 g^3 x + B^2 a^4 d^4 g^3) \log(bx + a) \log(dx + c) \\
& ) / (b d^4)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral  $\left( A^2 b^3 g^3 x^3 + 3 A^2 a b^2 g^3 x^2 + 3 A^2 a^2 b g^3 x + A^2 a^3 g^3 + (B^2 b^3 g^3 x^3 + 3 B^2 a b^2 g^3 x^2 + 3 B^2 a^2 b g^3 x + B^2 a^3 g^3) \log\left(\frac{b^2 e}{d^2}\right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(A^2\*b^3\*g^3\*x^3 + 3\*A^2\*a\*b^2\*g^3\*x^2 + 3\*A^2\*a^2\*b\*g^3\*x + A^2\*a^3\*g^3 + (B^2\*b^3\*g^3\*x^3 + 3\*B^2\*a\*b^2\*g^3\*x^2 + 3\*B^2\*a^2\*b\*g^3\*x + B^2\*a^3\*g^3)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*(A\*B\*b^3\*g^3\*x^3 + 3\*A\*B\*a\*b^2\*g^3\*x^2 + 3\*A\*B\*a^2\*b\*g^3\*x + A\*B\*a^3\*g^3)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*3\*(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^3 \left( B \log\left(\frac{(bx + a)^2 e}{(dx + c)^2}\right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

$$3.130 \quad \int (ag + bgx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=255

$$\frac{8B^2g^2(bc-ad)^3 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} + \frac{4Bg^2(bc-ad)^3 \log\left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A + 3B\right)}{3bd^3} + \frac{4Bg^2(a+bx)(bc-ad)}{3bd^3}$$

[Out] (-2\*B\*(b\*c - a\*d)\*g^2\*(a + b\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/(3\*b\*d) + (g^2\*(a + b\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2)/(3\*b) + (4\*B\*(b\*c - a\*d)^2\*g^2\*(a + b\*x)\*(A + B + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/(3\*b\*d^2) + (4\*B\*(b\*c - a\*d)^3\*g^2\*(A + 3\*B + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[(b\*c - a\*d)/(b\*(c + d\*x))])/(3\*b\*d^3) + (8\*B^2\*(b\*c - a\*d)^3\*g^2\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(3\*b\*d^3)

**Rubi [A]** time = 0.621454, antiderivative size = 397, normalized size of antiderivative = 1.56, number of steps used = 20, number of rules used = 13, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{8B^2g^2(bc-ad)^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3bd^3} - \frac{4Bg^2(bc-ad)^3 \log(c+dx) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{3bd^3} + \frac{4ABg^2x(bc-ad)^2}{3d^2} - \frac{2Bg^2(a+bx)(bc-ad)}{3bd^3}$$

Antiderivative was successfully verified.

[In] Int[(a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2,x]

[Out] (4\*A\*B\*(b\*c - a\*d)^2\*g^2\*x)/(3\*d^2) + (4\*B^2\*(b\*c - a\*d)^2\*g^2\*x)/(3\*d^2) + (4\*B^2\*(b\*c - a\*d)^2\*g^2\*(a + b\*x)\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(3\*b\*d^2) - (2\*B\*(b\*c - a\*d)\*g^2\*(a + b\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/(3\*b\*d) + (g^2\*(a + b\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2)/(3\*b) - (4\*B^2\*(b\*c - a\*d)^3\*g^2\*Log[c + d\*x])/(b\*d^3) + (8\*B^2\*(b\*c - a\*d)^3\*g^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x])/(3\*b\*d^3) - (4\*B\*(b\*c - a\*d)^3\*g^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[c + d\*x])/(3\*b\*d^3) - (4\*B^2\*(b\*c - a\*d)^3\*g^2\*Log[c + d\*x]^2)/(3\*b\*d^3) + (8\*B^2\*(b\*c - a\*d)^3\*g^2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/(3\*b\*d^3)

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*Rfx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] :> With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u

```
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_.))^(n_.), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_.)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_.)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned} \int (ag + bgx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2 dx &= \frac{g^2(a + bx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{3b} - \frac{(2B) \int \frac{2(bc - ad)g^3(a + bx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{c + dx}}{3bg} \\ &= \frac{g^2(a + bx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{3b} - \frac{(4B(bc - ad)g^2) \int \frac{(a + bx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{c + dx}}{3b} \\ &= \frac{g^2(a + bx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{3b} - \frac{(4B(bc - ad)g^2) \int \left( -\frac{b(bc - ad) \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{d^2} \right)}{3b} \\ &= \frac{g^2(a + bx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{3b} - \frac{(4B(bc - ad)g^2) \int (a + bx) \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{3d} \\ &= \frac{4AB(bc - ad)^2 g^2 x}{3d^2} - \frac{2B(bc - ad)g^2(a + bx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{3bd} + \frac{g^2(a + bx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{3b} \\ &= \frac{4AB(bc - ad)^2 g^2 x}{3d^2} + \frac{4B^2(bc - ad)^2 g^2(a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{3bd^2} - \frac{2B(bc - ad)g^2(a + bx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{3bd} \\ &= \frac{4AB(bc - ad)^2 g^2 x}{3d^2} + \frac{4B^2(bc - ad)^2 g^2(a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{3bd^2} - \frac{2B(bc - ad)g^2(a + bx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{3bd} \\ &= \frac{4AB(bc - ad)^2 g^2 x}{3d^2} + \frac{4B^2(bc - ad)^2 g^2 x}{3d^2} + \frac{4B^2(bc - ad)^2 g^2(a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{3bd^2} \\ &= \frac{4AB(bc - ad)^2 g^2 x}{3d^2} + \frac{4B^2(bc - ad)^2 g^2 x}{3d^2} + \frac{4B^2(bc - ad)^2 g^2(a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{3bd^2} \\ &= \frac{4AB(bc - ad)^2 g^2 x}{3d^2} + \frac{4B^2(bc - ad)^2 g^2 x}{3d^2} + \frac{4B^2(bc - ad)^2 g^2(a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{3bd^2} \\ &= \frac{4AB(bc - ad)^2 g^2 x}{3d^2} + \frac{4B^2(bc - ad)^2 g^2 x}{3d^2} + \frac{4B^2(bc - ad)^2 g^2(a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{3bd^2} \end{aligned}$$

**Mathematica [A]** time = 0.220749, size = 298, normalized size = 1.17

$$g^2 \left( \frac{2B(bc - ad) \left( 2B(bc - ad)^2 \left( 2 \text{PolyLog} \left( 2, \frac{b(c + dx)}{bc - ad} \right) + \log(c + dx) \left( 2 \log \left( \frac{d(a + bx)}{ad - bc} \right) - \log(c + dx) \right) \right) - d^2(a + bx)^2 \left( B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) + A \right) - 2(bc - ad)^2 \log(c + dx) \left( B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{d^3} \right)$$

Antiderivative was successfully verified.



[In] Integrate[(a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2,x]

[Out] (g^2\*((a + b\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2 + (2\*B\*(b\*c - a\*d)\*(2\*A\*b\*d\*(b\*c - a\*d)\*x + 2\*B\*d\*(b\*c - a\*d)\*(a + b\*x)\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2] - d^2\*(a + b\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) - 4\*B\*(b\*c - a\*d)^2\*Log[c + d\*x] - 2\*(b\*c - a\*d)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[c + d\*x] + 2\*B\*(b\*c - a\*d)\*(b\*d\*x + (-b\*c) + a\*d)\*Log[c + d\*x]) + 2\*B\*(b\*c - a\*d)^2\*((2\*Log[(d\*(a + b\*x))/(-b\*c) + a\*d]) - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/d^3)/(3\*b)

**Maple [F]** time = 1.503, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int((b\*g\*x+a\*g)^2\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [B]** time = 1.71885, size = 1790, normalized size = 7.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out] 1/3\*A^2\*b^2\*g^2\*x^3 + A^2\*a\*b\*g^2\*x^2 + 2\*(x\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 2\*a\*log(b\*x + a)/b - 2\*c\*log(d\*x + c)/d)\*A\*B\*a^2\*g^2 + 2\*(x^2\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) - 2\*a^2\*log(b\*x + a)/b^2 + 2\*c^2\*log(d\*x + c)/d^2 - 2\*(b\*c - a\*d)\*x/(b\*d))\*A\*B\*a\*b\*g^2 + 2/3\*(x^3\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 2\*a^3\*log(b\*x + a)/b^3 - 2\*c^3\*log(d\*x + c)/d^3 - ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2))\*A\*B\*b^2\*g^2 + A^2\*a^2\*g^2\*x - 4/3\*((g^2\*log(e) + 3\*g^2)\*b^2\*c^3 - (3\*g^2\*log(e) + 7\*g^2)\*a\*b\*c^2\*d + (3\*g^2\*log(e) + 4\*g^2)\*a^2\*c\*d^2)\*B^2\*log(d\*x + c)/d^3 - 8/3\*(b^3\*c^3\*g^2 - 3\*a\*b^2\*c^2\*d\*g^2 + 3\*a^2\*b\*c\*d^2\*g^2 - a^3\*d^3\*g^2)\*(log(b\*x + a)\*log((b\*d\*x + a\*d)/(b\*c - a\*d) + 1) + dilog(-(b\*d\*x + a\*d)/(b\*c - a\*d)))\*B^2/(b\*d^3) + 1/3\*(B^2\*b^3\*d^3\*g^2\*x^3\*log(e)^2 - (2\*b^3\*c\*d^2\*g^2\*log(e) - (3\*g^2\*log(e)^2 + 2\*g^2\*log(e))\*a\*b^2\*d^3)\*B^2\*x^2 + (4\*(g^2\*log(e) + g^2)\*b^3\*c^2\*d - 4\*(3\*g^2\*log(e) + 2\*g^2)\*a\*b^2\*c\*d^2 + (3\*g^2\*log(e)^2 + 8\*g^2\*log(e) + 4\*g^2)\*a^2\*b\*d^3)\*B^2\*x + 4\*(B^2\*b^3\*d^3\*g^2\*x^3 + 3\*B^2\*a\*b^2\*d^3\*g^2\*x^2 + 3\*B^2\*a^2\*b\*d^3\*g^2\*x + (b^3\*c^3\*g^2 - 3\*a\*b^2\*c^2\*d\*g^2 + 3\*a^2\*b\*c\*d^2\*g^2)\*B^2)\*log(d\*x + c)^2 + 4\*(B^2\*b^3\*d^3\*g^2\*x^3\*log(e) - (b^3\*c\*d^2\*g^2 - (3\*g^2\*log(e) + g^2)\*a\*b^2\*d^3)\*B^2\*x^2 + (2\*b^3\*c^2\*d\*g^2 - 6\*a\*b^2\*c\*d^2\*g^2 + (3\*g^2\*log(e) + 4\*g^2)\*a^2\*b\*d^3)\*B^2\*x + (2\*a\*b^2\*c^2\*d\*g^2 - 5\*a^2\*b\*c\*d^2\*g^2 + (g^2\*log(e) +

$$3g^2a^3d^3B^2 \log(bx+a) - 4(B^2b^3d^3g^2x^3 \log(e) - (b^3cd^2g^2 - (3g^2 \log(e) + g^2)ab^2d^3)B^2x^2 + (2b^3c^2dg^2 - 6ab^2cd^2g^2 + (3g^2 \log(e) + 4g^2)a^2bd^3)B^2x + 2(B^2b^3d^3g^2x^3 + 3B^2ab^2d^3g^2x^2 + 3B^2a^2bd^3g^2x + B^2a^3d^3g^2) \log(bx+a)) \log(dx+c) / (bd^3)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2b^2g^2x^2 + 2A^2abg^2x + A^2a^2g^2 + (B^2b^2g^2x^2 + 2B^2abg^2x + B^2a^2g^2) \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right)^2 + 2(ABb^2g^2x^2 + 2A^2abg^2x + A^2a^2g^2) \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(A^2\*b^2\*g^2\*x^2 + 2\*A^2\*a\*b\*g^2\*x + A^2\*a^2\*g^2 + (B^2\*b^2\*g^2\*x^2 + 2\*B^2\*a\*b\*g^2\*x + B^2\*a^2\*g^2)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*(A\*B\*b^2\*g^2\*x^2 + 2\*A\*B\*a\*b\*g^2\*x + A\*B\*a^2\*g^2)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*2\*(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^2\*(B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2, x)

$$3.131 \quad \int (ag + bgx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=188

$$\frac{4B^2g(bc-ad)^2 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} - \frac{2Bg(bc-ad)^2 \log\left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A + 2B\right)}{bd^2} - \frac{2Bg(a+bx)(bc-ad)}{bd^2}$$

[Out]  $(-2*B*(b*c - a*d)*g*(a + b*x)*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]))/(b*d) + (g*(a + b*x)^2*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*b) - (2*B*(b*c - a*d)^2*g*(A + 2*B + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])* \text{Log}[(b*c - a*d)/(b*(c + d*x))])/(b*d^2) - (4*B^2*(b*c - a*d)^2*g*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2)$

**Rubi [A]** time = 0.492536, antiderivative size = 291, normalized size of antiderivative = 1.55, number of steps used = 16, number of rules used = 12, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{4B^2g(bc-ad)^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bd^2} + \frac{2Bg(bc-ad)^2 \log(c+dx) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{bd^2} + \frac{g(a+bx)^2 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{2b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])^2, x]$

[Out]  $(-2*A*B*(b*c - a*d)*g*x)/d - (2*B^2*(b*c - a*d)*g*(a + b*x)*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])/(b*d) + (g*(a + b*x)^2*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*b) + (4*B^2*(b*c - a*d)^2*g*\text{Log}[c + d*x])/(b*d^2) - (4*B^2*(b*c - a*d)^2*g*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(b*d^2) + (2*B*(b*c - a*d)^2*g*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])* \text{Log}[c + d*x])/(b*d^2) + (2*B^2*(b*c - a*d)^2*g*\text{Log}[c + d*x]^2)/(b*d^2) - (4*B^2*(b*c - a*d)^2*g*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(b*d^2)$

#### Rule 2525

$\text{Int}[(a + \text{Log}[(c + \text{RFX})^p])*(b)^n*((d + e*x)^m + (e + x)^m), x\_Symbol] := \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^n/(e*(m+1)), x] - \text{Dist}[(b^n*p)/(e*(m+1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^{n-1}*D[\text{RFX}, x])/ \text{RFX}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x \&\& \text{RationalFunctionQ}[\text{RFX}, x] \&\& \text{IGtQ}[n, 0] \&\& (\text{EqQ}[n, 1] || \text{IntegerQ}[m]) \&\& \text{NeQ}[m, -1]$

#### Rule 12

$\text{Int}[(a)*(u), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& !\text{MatchQ}[u, (b)*(v)] /; \text{FreeQ}[b, x]$

#### Rule 2528

$\text{Int}[(a + \text{Log}[(c + \text{RFX})^p])*(b)^n*(\text{RGx}), x\_Symbol] := \text{With}[\{u = \text{ExpandIntegrand}[(a + b*\text{Log}[c*\text{RFX}^p])^n, \text{RGx}, x]\}, \text{Int}[u, x] /; \text{SumQ}[u]] /; \text{FreeQ}\{a, b, c, p\}, x \&\& \text{RationalFunctionQ}[\text{RFX}, x] \&\& \text{RationalFunctionQ}[\text{RGx}, x] \&\& \text{IGtQ}[n, 0]$

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFX^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFX_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFX, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFX, x] && IntegerQ[p]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
]^n))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_))^(n_.)]*(b_.)/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rubi steps

$$\begin{aligned}
\int (ag + bgx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx &= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{2b} - \frac{B \int \frac{2(bc-ad)g^2(a+bx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{c+dx}}{bg} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{2b} - \frac{(2B(bc-ad)g) \int \frac{(a+bx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{c+dx}}{b} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{2b} - \frac{(2B(bc-ad)g) \int \left( \frac{b \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{d} \right)}{b} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{2b} - \frac{(2B(bc-ad)g) \int \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{d} \\
&= -\frac{2AB(bc-ad)gx}{d} + \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{2b} + \frac{2B(bc-ad)^2g}{d} \\
&= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx) \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{bd} + \frac{g(a+bx)^2}{d} \\
&= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx) \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{bd} + \frac{g(a+bx)^2}{d} \\
&= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx) \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{bd} + \frac{g(a+bx)^2}{d} \\
&= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx) \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{bd} + \frac{g(a+bx)^2}{d} \\
&= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx) \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{bd} + \frac{g(a+bx)^2}{d} \\
&= -\frac{2AB(bc-ad)gx}{d} - \frac{2B^2(bc-ad)g(a+bx) \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{bd} + \frac{g(a+bx)^2}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.173694, size = 207, normalized size = 1.1

$$\frac{g \left( (a+bx)^2 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)^2 - \frac{4B(bc-ad) \left( B(bc-ad) \left( 2 \operatorname{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) - (bc-ad) \log(c+dx) \right)}{d^2}}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2,x]

[Out] (g\*((a + b\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))^2 - (4\*B\*(b\*c - a\*d)\*(A\*b\*d\*x + B\*d\*(a + b\*x)\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2] - 2\*B\*(b\*c - a\*d)\*Log[c + d\*x] - (b\*c - a\*d)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[c + d\*x] + B\*(b\*c - a\*d)\*((2\*Log[(d\*(a + b\*x))/(-b\*c) + a\*d]) - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]))/d^2)/(2\*b)

**Maple [F]** time = 1.273, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int((b\*g\*x+a\*g)\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [B]** time = 1.58367, size = 981, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out]  $\frac{1}{2}A^2b^2g^2x^2 + 2*(x*\log(b^2*ex^2/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*b*ex/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*\log(b*x + a)/b - 2*c*\log(d*x + c)/d)*A*B*a*g + (x^2*\log(b^2*ex^2/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*b*ex/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*\log(b*x + a)/b^2 + 2*c^2*\log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*A*B*b*g + A^2*a*g*x + 2*((g*\log(e) + 2*g)*b*c^2 - 2*(g*\log(e) + g)*a*c*d)*B^2*\log(d*x + c)/d^2 + 4*(b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*\log(e)^2 - 2*(2*b^2*c*d*g*\log(e) - (g*\log(e)^2 + 2*g*\log(e))*a*b*d^2)*B^2*x + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*\log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*\log(d*x + c)^2 + 4*(B^2*b^2*d^2*g*x^2*\log(e) + 2*((g*\log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + ((g*\log(e) + 2*g)*a^2*d^2 - 2*a*b*c*d*g)*B^2)*\log(b*x + a) - 4*(B^2*b^2*d^2*g*x^2*\log(e) + 2*((g*\log(e) + g)*a*b*d^2 - b^2*c*d*g)*B^2*x + 2*(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*\log(b*x + a))*\log(d*x + c))/(b*d^2)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2bgx + A^2ag + (B^2bgx + B^2ag) \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right)^2 + 2(ABbgx + ABag) \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(A^2\*b\*g\*x + A^2\*a\*g + (B^2\*b\*g\*x + B^2\*a\*g)\*log((b^2\*ex^2 + 2\*a\*b\*ex + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*(A\*B\*b\*g\*x + A\*B\*a\*g)\*log((b^2\*ex^2 + 2\*a\*b\*ex + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag) \left( B \log \left( \frac{(bx + a)^2 e}{(dx + c)^2} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)\*(B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2, x)

$$3.132 \quad \int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag+bgx} dx$$

**Optimal.** Leaf size=132

$$\frac{4B \operatorname{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{bg} + \frac{8B^2 \operatorname{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right) \log\left(1 - \frac{b(c+dx)}{d(a+bx)}\right) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}{bg}$$

[Out] -(((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2\*Log[1 - (b\*(c + d\*x))/(d\*(a + b\*x))])/(b\*g)) + (4\*B\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*PolyLog[2, (b\*(c + d\*x))/(d\*(a + b\*x))])/(b\*g) + (8\*B^2\*PolyLog[3, (b\*(c + d\*x))/(d\*(a + b\*x))])/(b\*g)

**Rubi [B]** time = 4.14106, antiderivative size = 749, normalized size of antiderivative = 5.67, number of steps used = 46, number of rules used = 23, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.676$ , Rules used = {2524, 12, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 6742, 2500, 2440, 2434, 2433, 2375, 2317, 2374, 6589, 2499, 2302, 30, 2396}

$$\frac{4AB \operatorname{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg} - \frac{4B^2 \operatorname{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right) \left(-\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + \log((a+bx)^2) + \log\left(\frac{1}{(c+dx)^2}\right)\right)}{bg} - 8B^2 \operatorname{PolyLog}\left(3, -\frac{d(a+bx)}{bc-ad}\right)$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(a\*g + b\*g\*x), x]

[Out] (-2\*A\*B\*Log[g\*(a + b\*x)]^2)/(b\*g) + (4\*B^2\*Log[g\*(a + b\*x)]^3)/(3\*b\*g) - (4\*B^2\*Log[g\*(a + b\*x)]^2\*Log[-c - d\*x])/(b\*g) + (4\*B^2\*Log[g\*(a + b\*x)]\*Log[(a + b\*x)^2]\*Log[-c - d\*x])/(b\*g) - (B^2\*Log[(a + b\*x)^2]^2\*Log[-c - d\*x])/(b\*g) + (B^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[(c + d\*x)^(-2)]^2)/(b\*g) - (B^2\*Log[g\*(a + b\*x)]\*Log[(c + d\*x)^(-2)]^2)/(b\*g) + (4\*B^2\*Log[g\*(a + b\*x)]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g) + (B^2\*Log[(a + b\*x)^2]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g) + ((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2\*Log[a\*g + b\*g\*x])/(b\*g) + (4\*A\*B\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*Log[a\*g + b\*g\*x])/(b\*g) - (4\*B^2\*(Log[(a + b\*x)^2] + Log[(c + d\*x)^(-2)] - Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*Log[a\*g + b\*g\*x])/(b\*g) - (2\*B^2\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]\*Log[a\*g + b\*g\*x]^2)/(b\*g) - (4\*B^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*Log[a\*g + b\*g\*x]^2)/(b\*g) + (4\*A\*B\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g) + (4\*B^2\*Log[(a + b\*x)^2]\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g) - (4\*B^2\*(Log[(a + b\*x)^2] + Log[(c + d\*x)^(-2)] - Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g) - (4\*B^2\*Log[(c + d\*x)^(-2)]\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g) - (8\*B^2\*PolyLog[3, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g) - (8\*B^2\*PolyLog[3, (b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g)

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

#### Rule 12



Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(Rfx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2500

```

Int[(Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))/((j_.) + (k
_.)*(x_)), x_Symbol] := Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]

```

#### Rule 2440

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_) + (l_.)*(x_))^(r_.), x_Symbol] :=
Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]

```

#### Rule 2434

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_))^(m_.)]*(g_.)))/(x_), x_Symbol] := Simp[Log[x]*(a + b
*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[e*g*m, Int[(Lo
g[x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x), x], x] - Dist[b*j*n, Int[(Log[x
]*(f + g*Log[h*(i + j*x)^m]))/(i + j*x), x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, i, j, m, n}, x] && EqQ[e*i - d*j, 0]

```

#### Rule 2433

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] := Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
(e*i - d*j)/e + (j*x)/e)^m]), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]

```

#### Rule 2375

```

Int[(Log[(d_.)*((e_) + (f_.)*(x_))^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_))^(n_
.))*((b_.))^(p_.))/(x_), x_Symbol] := Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]

```

#### Rule 2317

```

Int[((a_.) + Log[(c_.)*(x_))^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] := Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]

```

#### Rule 2374

```

Int[(Log[(d_.)*((e_) + (f_.)*(x_))^(m_.)]*((a_.) + Log[(c_.)*(x_))^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]

```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x]
&& EqQ[b*d, a*e]
```

#### Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)]^(r_.)]*
((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.)]/((j_.) + (k_.)*(x_)),
x_Symbol]
:> Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/(k*n*t*(m + 1)), x] +
(-Dist[(b*p*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] -
Dist[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x), x], x]) /;
FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r}, x]
&& NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

#### Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol]
:> Dist[1/(b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}, x]
```

#### Rule 30

```
Int[(x_)^(m_.), x_Symbol]
:> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

#### Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)/((f_.) + (g_.)*(x_)),
x_Symbol]
:> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^p)/g, x] -
Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /;
FreeQ[{a, b, c, d, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{ag + bgx} dx &= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{(c+dx)^2 \left(-\frac{2de(a+bx)^2}{(c+dx)^3} + \frac{2be(a+bx)}{(c+dx)^2}\right) \left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(ag + bgx)}{e(a+bx)^2} dx}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{(c+dx)^2 \left(-\frac{2de(a+bx)^2}{(c+dx)^3} + \frac{2be(a+bx)}{(c+dx)^2}\right) \left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(ag + bgx)}{(a+bx)^2} dx}{beg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{2(bc-ad)e \left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(ag+bgx)}{(a+bx)(c+dx)} dx}{beg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(4B(bc-ad)) \int \frac{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(ag+bgx)}{(a+bx)(c+dx)} dx}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(4B(bc-ad)) \int \left(\frac{d \left(-A-B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(ag+bgx)}{(bc-ad)(c+dx)} + \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) \log(ag+bgx)}{a+bx}\right) dx}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(4B) \int \frac{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(ag+bgx)}{a+bx} dx}{g} - \frac{(4Bd) \int \frac{\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) \log(ag+bgx)}{a+bx} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(4B) \int \left(\frac{A \log(ag+bgx)}{a+bx} + \frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) \log(ag+bgx)}{a+bx}\right) dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(4AB) \int \frac{\log(ag+bgx)}{a+bx} dx}{g} - \frac{(4B^2) \int \frac{\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) \log(ag+bgx)}{a+bx} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(ag + bgx)}{bg} + \frac{4AB \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} - \frac{2B^2 \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) \log(ag+bgx)}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(ag + bgx)}{bg} + \frac{4AB \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} - \frac{4B^2 \left(\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) \log(ag+bgx)\right)}{g} \\
&= -\frac{2AB \log^2(g(a+bx))}{bg} + \frac{4B^2 \log(g(a+bx)) \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) \log(-c-dx)}{bg} - \frac{B^2 \log(g(a+bx)) \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) \log(-c-dx)}{g} \\
&= -\frac{2AB \log^2(g(a+bx))}{bg} + \frac{4B^2 \log(g(a+bx)) \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) \log(-c-dx)}{bg} + \frac{B^2 \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) \log(-c-dx)}{g} \\
&= -\frac{2AB \log^2(g(a+bx))}{bg} + \frac{4B^2 \log^3(g(a+bx))}{3bg} - \frac{4B^2 \log^2(g(a+bx)) \log(-c-dx)}{bg} + \frac{4B^2 \log(g(a+bx)) \log(-c-dx)}{g} \\
&= -\frac{2AB \log^2(g(a+bx))}{bg} + \frac{4B^2 \log^3(g(a+bx))}{3bg} - \frac{4B^2 \log^2(g(a+bx)) \log(-c-dx)}{bg} + \frac{4B^2 \log(g(a+bx)) \log(-c-dx)}{g} \\
&= -\frac{2AB \log^2(g(a+bx))}{bg} + \frac{4B^2 \log^3(g(a+bx))}{3bg} - \frac{4B^2 \log^2(g(a+bx)) \log(-c-dx)}{bg} + \frac{4B^2 \log(g(a+bx)) \log(-c-dx)}{g} \\
&= -\frac{2AB \log^2(g(a+bx))}{bg} + \frac{4B^2 \log^3(g(a+bx))}{3bg} - \frac{4B^2 \log^2(g(a+bx)) \log(-c-dx)}{bg} + \frac{4B^2 \log(g(a+bx)) \log(-c-dx)}{g}
\end{aligned}$$

**Mathematica [A]** time = 0.327194, size = 257, normalized size = 1.95

$$-4AB\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right) + 4B^2\text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + 8B^2\text{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right) + A^2 \log(a+bx) + 2A$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(a\*g + b\*g\*x), x]

[Out] (2\*A\*B\*Log[a/b + x]^2 + A^2\*Log[a + b\*x] - 4\*A\*B\*Log[a/b + x]\*Log[a + b\*x] + 4\*A\*B\*Log[c/d + x]\*Log[a + b\*x] - 4\*A\*B\*Log[c/d + x]\*Log[(d\*(a + b\*x))/(-(b\*c) + a\*d)] + 2\*A\*B\*Log[a + b\*x]\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2] - B^2\*Log[-(b\*c) + a\*d]/(d\*(a + b\*x)))\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]^2 - 4\*A\*B\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)] + 4\*B^2\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]\*PolyLog[2, (b\*(c + d\*x))/(d\*(a + b\*x))] + 8\*B^2\*PolyLog[3, (b\*(c + d\*x))/(d\*(a + b\*x))]/(b\*g)

**Maple [F]** time = 1.374, size = 0, normalized size = 0.

$$\int \frac{1}{bgx + ag} \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g), x)

[Out] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{4B^2 \log(bx + a) \log(dx + c)^2}{bg} + \frac{A^2 \log(bgx + ag)}{bg} - \int \frac{B^2bc \log(e)^2 + 2ABbc \log(e) + 4(B^2bdx + B^2bc) \log(bx + a)}{bgx + ag} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g), x, algorithm="maxima")

[Out] 4\*B^2\*log(b\*x + a)\*log(d\*x + c)^2/(b\*g) + A^2\*log(b\*g\*x + a\*g)/(b\*g) - integrate(-(B^2\*b\*c\*log(e)^2 + 2\*A\*B\*b\*c\*log(e) + 4\*(B^2\*b\*d\*x + B^2\*b\*c)\*log(b\*x + a)^2 + (B^2\*b\*d\*log(e)^2 + 2\*A\*B\*b\*d\*log(e))\*x + 4\*(B^2\*b\*c\*log(e) + A\*B\*b\*c + (B^2\*b\*d\*log(e) + A\*B\*b\*d)\*x)\*log(b\*x + a) - 4\*(B^2\*b\*c\*log(e) + A\*B\*b\*c + (B^2\*b\*d\*log(e) + A\*B\*b\*d)\*x + 2\*(2\*B^2\*b\*d\*x + (b\*c + a\*d)\*B^2)\*log(b\*x + a))\*log(d\*x + c))/(b^2\*d\*g\*x^2 + a\*b\*c\*g + (b^2\*c\*g + a\*b\*d\*g)\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B^2 \log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)^2 + 2AB \log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right) + A^2}{bgx + ag}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A^2)/(b*g*x + a*g), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2/(b*g*x+a*g),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log\left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)^2}{bgx + ag} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(b*g*x+a*g),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2/(b*g*x + a*g), x)
```

$$3.133 \quad \int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^2} dx$$

**Optimal.** Leaf size=130

$$-\frac{4B(c+dx)\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{g^2(a+bx)(bc-ad)} - \frac{(c+dx)\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}{g^2(a+bx)(bc-ad)} - \frac{8B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

[Out]  $(-8*B^2*(c+d*x))/((b*c-a*d)*g^2*(a+b*x)) - (4*B*(c+d*x)*(A+B*\text{Log}[(e*(a+b*x)^2)/(c+d*x)^2]))/((b*c-a*d)*g^2*(a+b*x)) - ((c+d*x)*(A+B*\text{Log}[(e*(a+b*x)^2)/(c+d*x)^2])^2)/((b*c-a*d)*g^2*(a+b*x))$

**Rubi [C]** time = 0.888483, antiderivative size = 480, normalized size of antiderivative = 3.69, number of steps used = 26, number of rules used = 11, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{8B^2 d \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg^2(bc-ad)} - \frac{8B^2 d \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bg^2(bc-ad)} - \frac{4Bd \log(a+bx)\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{bg^2(bc-ad)} - \frac{4B\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}{bg^2(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])^2/(a*g + b*g*x)^2, x]$

[Out]  $(-8*B^2)/(b*g^2*(a+b*x)) - (8*B^2*d*\text{Log}[a+b*x])/(b*(b*c-a*d)*g^2) + (4*B^2*d*\text{Log}[a+b*x]^2)/(b*(b*c-a*d)*g^2) - (4*B*(A+B*\text{Log}[(e*(a+b*x)^2)/(c+d*x)^2]))/(b*g^2*(a+b*x)) - (4*B*d*\text{Log}[a+b*x]*(A+B*\text{Log}[(e*(a+b*x)^2)/(c+d*x)^2]))/(b*(b*c-a*d)*g^2) - (A+B*\text{Log}[(e*(a+b*x)^2)/(c+d*x)^2])^2/(b*g^2*(a+b*x)) + (8*B^2*d*\text{Log}[c+d*x])/(b*(b*c-a*d)*g^2) - (8*B^2*d*\text{Log}[-((d*(a+b*x))/(b*c-a*d))]*\text{Log}[c+d*x])/(b*(b*c-a*d)*g^2) + (4*B*d*(A+B*\text{Log}[(e*(a+b*x)^2)/(c+d*x)^2])*\text{Log}[c+d*x])/(b*(b*c-a*d)*g^2) + (4*B^2*d*\text{Log}[c+d*x]^2)/(b*(b*c-a*d)*g^2) - (8*B^2*d*\text{Log}[a+b*x]*\text{Log}[(b*(c+d*x))/(b*c-a*d)])/(b*(b*c-a*d)*g^2) - (8*B^2*d*\text{PolyLog}[2, -((d*(a+b*x))/(b*c-a*d))])/(b*(b*c-a*d)*g^2) - (8*B^2*d*\text{PolyLog}[2, (b*(c+d*x))/(b*c-a*d)])/(b*(b*c-a*d)*g^2)$

#### Rule 2525

$\text{Int}[(a + \text{Log}[(c + \text{RFX})^{(p)}])*(b)^{(n)}*((d + e*x)^{(m)}), x\_Symbol] := \text{Simp}[(d + e*x)^{(m+1)}*(a + b*\text{Log}[c*\text{RFX}^p])^n/(e*(m+1)), x] - \text{Dist}[(b^n*p)/(e*(m+1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{(m+1)}*(a + b*\text{Log}[c*\text{RFX}^p])^{(n-1)}*D[\text{RFX}, x]/\text{RFX}, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x] \&\& \text{RationalFunctionQ}[\text{RFX}, x] \&\& \text{IGtQ}[n, 0] \&\& (\text{EqQ}[n, 1] || \text{IntegerQ}[m]) \&\& \text{NeQ}[m, -1]$

#### Rule 12

$\text{Int}[(a)*(u), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& !\text{MatchQ}[u, (b)*(v)] /; \text{FreeQ}[b, x]$

#### Rule 2528

$\text{Int}[(a + \text{Log}[(c + \text{RFX})^{(p)}])*(b)^{(n)}*(\text{RGx}), x\_Symbol] := \text{With}[\{u = \text{ExpandIntegrand}[(a + b*\text{Log}[c*\text{RFX}^p])^n, \text{RGx}, x]\}, \text{Int}[u, x] /; \text{SumQ}[u]] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{RationalFunctionQ}[\text{RFX}, x] \&\& \text{RationalFunctionQ}[\text{RGx}, x]$

onQ[RGx, x] && IGtQ[n, 0]

#### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 2524

Int[((a\_) + Log[(c\_)\*(Rfx\_)^(p\_)])\*(b\_))^(n\_)/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_))^(p\_)\*(Rfx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_))^(p\_)\*((f\_) + (g\_)\*(x\_))^(q\_), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_) + Log[(c\_)\*(x\_)]^(n\_))\*((b\_))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_))/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))])\*(b\_))/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps



$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag + bgx)^2} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{bg^2(a + bx)} + \frac{(2B) \int \frac{2(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(a+bx)^2(c+dx)} dx}{bg} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{bg^2(a + bx)} + \frac{(4B(bc - ad)) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^2(c+dx)} dx}{bg^2} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{bg^2(a + bx)} + \frac{(4B(bc - ad)) \int \left(\frac{b\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(a+bx)^2} - \frac{bd\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2(a+bx)}\right) dx}{bg^2} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{bg^2(a + bx)} + \frac{(4B) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^2} dx}{g^2} - \frac{(4Bd) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx} dx}{(bc - ad)g^2} \\
&= -\frac{4B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a + bx)} - \frac{4Bd \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc - ad)g^2} - \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a + bx)} \\
&= -\frac{4B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a + bx)} - \frac{4Bd \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc - ad)g^2} - \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a + bx)} \\
&= -\frac{4B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a + bx)} - \frac{4Bd \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc - ad)g^2} - \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a + bx)} \\
&= -\frac{8B^2}{bg^2(a + bx)} - \frac{8B^2d \log(a + bx)}{b(bc - ad)g^2} - \frac{4B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a + bx)} - \frac{4Bd \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc - ad)g^2} \\
&= -\frac{8B^2}{bg^2(a + bx)} - \frac{8B^2d \log(a + bx)}{b(bc - ad)g^2} - \frac{4B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a + bx)} - \frac{4Bd \log(a + bx)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc - ad)g^2} \\
&= -\frac{8B^2}{bg^2(a + bx)} - \frac{8B^2d \log(a + bx)}{b(bc - ad)g^2} + \frac{4B^2d \log^2(a + bx)}{b(bc - ad)g^2} - \frac{4B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a + bx)} \\
&= -\frac{8B^2}{bg^2(a + bx)} - \frac{8B^2d \log(a + bx)}{b(bc - ad)g^2} + \frac{4B^2d \log^2(a + bx)}{b(bc - ad)g^2} - \frac{4B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^2(a + bx)}
\end{aligned}$$

**Mathematica [C]** time = 0.450889, size = 321, normalized size = 2.47

$$\frac{4B\left(-Bd(a+bx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+Bd(a+bx)\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)\right)}{g^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(a\*g + b\*g\*x)^2,x]

[Out] -(((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2 + (4\*B\*((b\*c - a\*d)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) + d\*(a + b\*x)\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) - d\*(a + b\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) \*Log[c + d\*x] + 2\*B\*(b\*c - a\*d + d\*(a + b\*x)\*Log[a + b\*x] - d\*(a + b\*x)\*Log[c + d\*x]) - B\*d\*(a + b\*x)\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c + a\*d)]) + B\*d\*(a

$$+ b*x)*((2*\text{Log}[(d*(a + b*x))/(-b*c) + a*d]) - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)/(b*g^2*(a + b*x))$$

**Maple [B]** time = 0.09, size = 357, normalized size = 2.8

$$\frac{dA^2}{g^2(ad - bc)} \left( \frac{ad}{dx + c} - \frac{bc}{dx + c} + b \right)^{-1} - 8 \frac{dB^2}{bg^2(dx + c)} \left( \frac{ad}{dx + c} - \frac{bc}{dx + c} + b \right)^{-1} + 4 \frac{dB^2}{g^2(ad - bc)} \ln \left( \frac{e}{d^2} \left( \frac{ad}{dx + c} - \frac{bc}{dx + c} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^2,x)

[Out]  $d/g^2A^2/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)/(a*d-b*c)-8*d/g^2/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*B^2/b/(d*x+c)+4*d/g^2/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*B^2/(a*d-b*c)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)+d/g^2/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*B^2/(a*d-b*c)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)^2-4*d/g^2/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*A*B/b/(d*x+c)+2*d/g^2/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*A*B/(a*d-b*c)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)$

**Maxima [B]** time = 1.4819, size = 775, normalized size = 5.96

$$-4 \left( \left( \frac{1}{b^2g^2x + abg^2} + \frac{d \log(bx + a)}{(b^2c - abd)g^2} - \frac{d \log(dx + c)}{(b^2c - abd)g^2} \right) \log \left( \frac{b^2ex^2}{d^2x^2 + 2cdx + c^2} + \frac{2abex}{d^2x^2 + 2cdx + c^2} + \frac{a^2e}{d^2x^2 + 2cdx + c^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^2,x, algorithm="maxima")

[Out]  $-4*((1/(b^2*g^2*x + a*b*g^2) + d*\log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*\log(d*x + c)/((b^2*c - a*b*d)*g^2))*\log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - ((b*d*x + a*d)*\log(b*x + a)^2 + (b*d*x + a*d)*\log(d*x + c)^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*\log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*\log(b*x + a))*\log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x))*B^2 - 2*A*B*(\log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^2*g^2*x + a*b*g^2) + 2/(b^2*g^2*x + a*b*g^2) + 2*d*\log(b*x + a)/((b^2*c - a*b*d)*g^2) - 2*d*\log(d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*\log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)$

**Fricas [A]** time = 0.998379, size = 416, normalized size = 3.2

$$\frac{(A^2 + 4AB + 8B^2)bc - (A^2 + 4AB + 8B^2)ad + (B^2bdx + B^2bc) \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right)^2 + 2((AB + 2B^2)bdx + (AB + 2B^2)bc)}{(b^3c - ab^2d)g^2x + (ab^2c - a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^2,x, algorithm="fricas")

[Out] -((A^2 + 4\*A\*B + 8\*B^2)\*b\*c - (A^2 + 4\*A\*B + 8\*B^2)\*a\*d + (B^2\*b\*d\*x + B^2\*b\*c)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*((A\*B + 2\*B^2)\*b\*d\*x + (A\*B + 2\*B^2)\*b\*c)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)))/((b^3\*c - a\*b^2\*d)\*g^2\*x + (a\*b^2\*c - a^2\*b\*d)\*g^2)

**Sympy [B]** time = 3.79672, size = 452, normalized size = 3.48

$$\frac{4Bd(A+2B)\log\left(x + \frac{4ABad^2+4ABbcd+8B^2ad^2+8B^2bcd-\frac{4Ba^2d^3(A+2B)}{ad-bc}+\frac{8Babcd(A+2B)}{ad-bc}-\frac{4Bb^2c^2d(A+2B)}{ad-bc}}{8ABbd^2+16B^2bd^2}\right)}{bg^2(ad-bc)} + \frac{4Bd(A+2B)\log\left(x + \frac{4ABad^2+4ABbcd+8B^2ad^2+8B^2bcd-\frac{4Ba^2d^3(A+2B)}{ad-bc}+\frac{8Babcd(A+2B)}{ad-bc}-\frac{4Bb^2c^2d(A+2B)}{ad-bc}}{8ABbd^2+16B^2bd^2}\right)}{bg^2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2/(b\*g\*x+a\*g)\*\*2,x)

[Out] -4\*B\*d\*(A + 2\*B)\*log(x + (4\*A\*B\*a\*d\*\*2 + 4\*A\*B\*b\*c\*d + 8\*B\*\*2\*a\*d\*\*2 + 8\*B\*\*2\*b\*c\*d - 4\*B\*a\*\*2\*d\*\*3\*(A + 2\*B)/(a\*d - b\*c) + 8\*B\*a\*b\*c\*d\*\*2\*(A + 2\*B)/(a\*d - b\*c) - 4\*B\*b\*\*2\*c\*\*2\*d\*(A + 2\*B)/(a\*d - b\*c))/(8\*A\*B\*b\*d\*\*2 + 16\*B\*\*2\*b\*d\*\*2))/(b\*g\*\*2\*(a\*d - b\*c)) + 4\*B\*d\*(A + 2\*B)\*log(x + (4\*A\*B\*a\*d\*\*2 + 4\*A\*B\*b\*c\*d + 8\*B\*\*2\*a\*d\*\*2 + 8\*B\*\*2\*b\*c\*d + 4\*B\*a\*\*2\*d\*\*3\*(A + 2\*B)/(a\*d - b\*c) - 8\*B\*a\*b\*c\*d\*\*2\*(A + 2\*B)/(a\*d - b\*c) + 4\*B\*b\*\*2\*c\*\*2\*d\*(A + 2\*B)/(a\*d - b\*c))/(8\*A\*B\*b\*d\*\*2 + 16\*B\*\*2\*b\*d\*\*2))/(b\*g\*\*2\*(a\*d - b\*c)) + (-2\*A\*B - 4\*B\*\*2)\*log(e\*(a + b\*x)\*\*2/(c + d\*x)\*\*2)/(a\*b\*g\*\*2 + b\*\*2\*g\*\*2\*x) + (B\*\*2\*c + B\*\*2\*d\*x)\*log(e\*(a + b\*x)\*\*2/(c + d\*x)\*\*2)\*\*2/(a\*\*2\*d\*g\*\*2 - a\*b\*c\*g\*\*2 + a\*b\*d\*g\*\*2\*x - b\*\*2\*c\*g\*\*2\*x) - (A\*\*2 + 4\*A\*B + 8\*B\*\*2)/(a\*b\*g\*\*2 + b\*\*2\*g\*\*2\*x)

**Giac [B]** time = 1.47634, size = 510, normalized size = 3.92

$$-\left(\frac{B^2d}{b^2cg^2 - abdg^2} + \frac{B^2}{(bgx + ag)bg}\right)\log\left(\frac{b^2}{\frac{b^2c^2g^2}{(bgx+ag)^2} - \frac{2abcdg^2}{(bgx+ag)^2} + \frac{a^2d^2g^2}{(bgx+ag)^2} + \frac{2bcdg}{bgx+ag} - \frac{2ad^2g}{bgx+ag} + d^2}\right)^2 + \frac{4(ABd + 3B^2d)\log\left(\frac{b^2}{\frac{b^2c^2g^2}{(bgx+ag)^2} - \frac{2abcdg^2}{(bgx+ag)^2} + \frac{a^2d^2g^2}{(bgx+ag)^2} + \frac{2bcdg}{bgx+ag} - \frac{2ad^2g}{bgx+ag} + d^2}\right)}{b^2cg^2 - abdg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^2,x, algorithm="giac")

[Out] -(B^2\*d/(b^2\*c\*g^2 - a\*b\*d\*g^2) + B^2/((b\*g\*x + a\*g)\*b\*g))\*log(b^2/(b^2\*c^2\*g^2/(b\*g\*x + a\*g)^2 - 2\*a\*b\*c\*d\*g^2/(b\*g\*x + a\*g)^2 + a^2\*d^2\*g^2/(b\*g\*x + a\*g)^2 + 2\*b\*c\*d\*g/(b\*g\*x + a\*g) - 2\*a\*d^2\*g/(b\*g\*x + a\*g) + d^2))^2 + 4\*(A\*B\*d + 3\*B^2\*d)\*log(b\*c\*g/(b\*g\*x + a\*g) - a\*d\*g/(b\*g\*x + a\*g) + d)/(b^2\*c\*g^2 - a\*b\*d\*g^2) - 2\*(A\*B + 3\*B^2)\*log(b^2/(b^2\*c^2\*g^2/(b\*g\*x + a\*g)^2 - 2\*a\*b\*c\*d\*g^2/(b\*g\*x + a\*g)^2 + a^2\*d^2\*g^2/(b\*g\*x + a\*g)^2 + 2\*b\*c\*d\*g/(b\*g\*x + a\*g) - 2\*a\*d^2\*g/(b\*g\*x + a\*g) + d^2))/((b\*g\*x + a\*g)\*b\*g) - (A^2 + 6\*A\*B + 13\*B^2)/((b\*g\*x + a\*g)\*b\*g)

$$3.134 \quad \int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^3} dx$$

**Optimal.** Leaf size=272

$$-\frac{bB(c+dx)^2 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{g^3(a+bx)^2(bc-ad)^2} + \frac{4Bd(c+dx) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{g^3(a+bx)(bc-ad)^2} - \frac{b(c+dx)^2 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}{2g^3(a+bx)^2(bc-ad)^2} + \frac{d(c+dx)}{g^3}$$

[Out]  $(8*B^2*d*(c+d*x))/((b*c-a*d)^2*g^3*(a+b*x)) - (b*B^2*(c+d*x)^2)/((b*c-a*d)^2*g^3*(a+b*x)^2) + (4*B*d*(c+d*x)*(A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2]))/((b*c-a*d)^2*g^3*(a+b*x)) - (b*B*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2]))/((b*c-a*d)^2*g^3*(a+b*x)^2) + (d*(c+d*x)*(A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2])^2)/((b*c-a*d)^2*g^3*(a+b*x)) - (b*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2])^2)/(2*(b*c-a*d)^2*g^3*(a+b*x)^2)$

**Rubi [C]** time = 1.04621, antiderivative size = 579, normalized size of antiderivative = 2.13, number of steps used = 30, number of rules used = 11, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{4B^2d^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg^3(bc-ad)^2} + \frac{4B^2d^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bg^3(bc-ad)^2} + \frac{2Bd^2 \log(a+bx) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{bg^3(bc-ad)^2} - \frac{2Bd^2 \log(c+dx)}{bg^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(a\*g + b\*g\*x)^3, x]

[Out]  $-(B^2/(b*g^3*(a+b*x)^2)) + (6*B^2*d)/(b*(b*c-a*d)*g^3*(a+b*x)) + (6*B^2*d^2*Log[a+b*x])/(b*(b*c-a*d)^2*g^3) - (2*B^2*d^2*Log[a+b*x]^2)/(b*(b*c-a*d)^2*g^3) - (B*(A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2]))/(b*g^3*(a+b*x)^2) + (2*B*d*(A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2]))/(b*(b*c-a*d)*g^3*(a+b*x)) + (2*B*d^2*Log[a+b*x]*(A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2]))/(b*(b*c-a*d)^2*g^3) - (A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2])^2/(2*b*g^3*(a+b*x)^2) - (6*B^2*d^2*Log[c+d*x])/(b*(b*c-a*d)^2*g^3) + (4*B^2*d^2*Log[-((d*(a+b*x))/(b*c-a*d))]*Log[c+d*x])/(b*(b*c-a*d)^2*g^3) - (2*B*d^2*(A+B*Log[(e*(a+b*x)^2)/(c+d*x)^2])*Log[c+d*x])/(b*(b*c-a*d)^2*g^3) - (2*B^2*d^2*Log[c+d*x]^2)/(b*(b*c-a*d)^2*g^3) + (4*B^2*d^2*Log[a+b*x]*Log[(b*(c+d*x))/(b*c-a*d)])/(b*(b*c-a*d)^2*g^3) + (4*B^2*d^2*PolyLog[2, -((d*(a+b*x))/(b*c-a*d))])/(b*(b*c-a*d)^2*g^3) + (4*B^2*d^2*PolyLog[2, (b*(c+d*x))/(b*c-a*d)])/(b*(b*c-a*d)^2*g^3)$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFX^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.)/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

## Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag + b gx)^3} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2bg^3(a + bx)^2} + \frac{B \int \frac{2(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g^2(a+bx)^3(c+dx)} dx}{bg} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2bg^3(a + bx)^2} + \frac{(2B(bc - ad)) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^3(c+dx)} dx}{bg^3} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2bg^3(a + bx)^2} + \frac{(2B(bc - ad)) \int \left(\frac{b\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(a+bx)^3} - \frac{bd\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2(a+bx)^2} + \frac{bd^2}{(bc-ad)^3}\right) dx}{bg^3} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2bg^3(a + bx)^2} + \frac{(2B) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^3} dx}{g^3} + \frac{(2Bd^2) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx} dx}{(bc - ad)^2g^3} - \frac{2Bd^2 \log(a + bx) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc - ad)^2g^3} \\
&= -\frac{B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^3(a + bx)^2} + \frac{2Bd\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc - ad)g^3(a + bx)} + \frac{2Bd^2 \log(a + bx) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc - ad)^2g^3} \\
&= -\frac{B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^3(a + bx)^2} + \frac{2Bd\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc - ad)g^3(a + bx)} + \frac{2Bd^2 \log(a + bx) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc - ad)^2g^3} \\
&= -\frac{B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^3(a + bx)^2} + \frac{2Bd\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc - ad)g^3(a + bx)} + \frac{2Bd^2 \log(a + bx) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{b(bc - ad)^2g^3} \\
&= -\frac{B^2}{bg^3(a + bx)^2} + \frac{6B^2d}{b(bc - ad)g^3(a + bx)} + \frac{6B^2d^2 \log(a + bx)}{b(bc - ad)^2g^3} - \frac{B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^3(a + bx)^2} + \\
&= -\frac{B^2}{bg^3(a + bx)^2} + \frac{6B^2d}{b(bc - ad)g^3(a + bx)} + \frac{6B^2d^2 \log(a + bx)}{b(bc - ad)^2g^3} - \frac{B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^3(a + bx)^2} + \\
&= -\frac{B^2}{bg^3(a + bx)^2} + \frac{6B^2d}{b(bc - ad)g^3(a + bx)} + \frac{6B^2d^2 \log(a + bx)}{b(bc - ad)^2g^3} - \frac{2B^2d^2 \log^2(a + bx)}{b(bc - ad)^2g^3} - \frac{B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^3(a + bx)^2} \\
&= -\frac{B^2}{bg^3(a + bx)^2} + \frac{6B^2d}{b(bc - ad)g^3(a + bx)} + \frac{6B^2d^2 \log(a + bx)}{b(bc - ad)^2g^3} - \frac{2B^2d^2 \log^2(a + bx)}{b(bc - ad)^2g^3} - \frac{B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{bg^3(a + bx)^2}
\end{aligned}$$

**Mathematica [C]** time = 0.490212, size = 451, normalized size = 1.66

$$\frac{2B\left(2Bd^2(a+bx)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)-2Bd^2(a+bx)^2\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)}{g^3(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(a\*g + b\*g\*x)^3,x]

[Out] -((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2 + (2\*B\*((b\*c - a\*d)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) + 2\*d\*(-(b\*c) + a\*d)\*(a + b\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) - 2\*d^2\*(a + b\*x)^2\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) + 2\*d^2\*(a + b\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) - 2\*B\*d^2\*log^2(a + bx))/(b\*(bc - ad)^2\*g^3) - (B\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/g^3(a + bx)^2

$$\begin{aligned} &/ (c + d*x)^2] * \text{Log}[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a + b*x)*\text{Log}[ \\ &a + b*x] - d*(a + b*x)*\text{Log}[c + d*x]) + B*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d \\ &)*(a + b*x) - 2*d^2*(a + b*x)^2*\text{Log}[a + b*x] + 2*d^2*(a + b*x)^2*\text{Log}[c + d* \\ &x]) + 2*B*d^2*(a + b*x)^2*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x)) \\ &/ (b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*(a + \\ &b*x)^2*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \text{Log}[c + d*x])* \text{Log}[c + d*x] \\ &+ 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2/(2*b*g^3*(a + b \\ &*x)^2) \end{aligned}$$

**Maple [B]** time = 0.139, size = 815, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^3,x)

[Out] 
$$\begin{aligned} &-1/2*d^2/g^3*A^2*b/(a*d-b*c)^2/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2+d^2/g^3*A^2/ \\ &(a*d-b*c)^2/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)-7*d^2/g^3/(1/(d*x+c)*a*d-b*c/(d*x \\ &+c)+b)^2/b*B^2/(d*x+c)^2+3*d^2/g^3/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2*b*B^2/(a \\ &^2*d^2-2*a*b*c*d+b^2*c^2)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)-6*d^2/g \\ &^3/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2*B^2/(a*d-b*c)/(d*x+c)+4*d^2/g^3/(1/(d*x+ \\ &c)*a*d-b*c/(d*x+c)+b)^2*B^2/(a*d-b*c)/(d*x+c)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+ \\ &c)+b)^2/d^2)+d^2/g^3/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2*B^2/(a*d-b*c)/(d*x+c)* \\ &\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)^2+1/2*d^2/g^3/(1/(d*x+c)*a*d-b*c/ \\ &(d*x+c)+b)^2*b*B^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x \\ &+c)+b)^2/d^2)^2-3*d^2/g^3/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2*A*B/b/(d*x+c)^2+d \\ &^2/g^3/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2*b*A*B/(a^2*d^2-2*a*b*c*d+b^2*c^2)*\ln \\ &(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)-2*d^2/g^3/(1/(d*x+c)*a*d-b*c/(d*x+c \\ &)+b)^2*A*B/(a*d-b*c)/(d*x+c)+2*d^2/g^3/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2*A*B/ \\ &(a*d-b*c)/(d*x+c)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2) \end{aligned}$$

**Maxima [B]** time = 1.85735, size = 1351, normalized size = 4.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^3,x, algorithm="maxima")

[Out] 
$$\begin{aligned} &(((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2 \\ &*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*\log(b*x + a)/((b^3*c^2 - 2*a \\ &*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*\log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a \\ &^2*b*d^2)*g^3))*\log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^ \\ &2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - (b^2*c^2 - 8*a*b*c* \\ &d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(b*x + a)^2 + 2* \\ &(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2 \\ &)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(b*x + a) + 2*(3*b^2*d^2*x \\ &^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log( \\ &b*x + a))*\log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^ \\ &3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^ \\ &3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 + A*B*((2*b*d*x - b*c + 3* \\ &a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2* \end{aligned}$$

$$c - a^3 b d) g^3) - \log(b^2 e x^2 / (d^2 x^2 + 2 c d x + c^2) + 2 a b e x / (d^2 x^2 + 2 c d x + c^2) + a^2 e / (d^2 x^2 + 2 c d x + c^2)) / (b^3 g^3 x^2 + 2 a b^2 g^3 x + a^2 b g^3) + 2 d^2 \log(b x + a) / ((b^3 c^2 - 2 a b^2 c d + a^2 b d^2) g^3) - 2 d^2 \log(d x + c) / ((b^3 c^2 - 2 a b^2 c d + a^2 b d^2) g^3) - 1/2 B^2 \log(b^2 e x^2 / (d^2 x^2 + 2 c d x + c^2) + 2 a b e x / (d^2 x^2 + 2 c d x + c^2) + a^2 e / (d^2 x^2 + 2 c d x + c^2))^2 / (b^3 g^3 x^2 + 2 a b^2 g^3 x + a^2 b g^3) - 1/2 A^2 / (b^3 g^3 x^2 + 2 a b^2 g^3 x + a^2 b g^3)$$

**Fricas [A]** time = 1.04559, size = 846, normalized size = 3.11

$$\frac{(A^2 + 2AB + 2B^2)b^2c^2 - 2(A^2 + 4AB + 8B^2)abcd + (A^2 + 6AB + 14B^2)a^2d^2 - (B^2b^2d^2x^2 + 2B^2abd^2x - B^2b^2c^2 + 2A^2b^2c^2 + 2A^2b^2d^2x^2)}{2((b^5c^2 - 2abd^2x^2 + 2abd^2x - B^2b^2c^2 + 2A^2b^2c^2 + 2A^2b^2d^2x^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^3,x, algorithm="fricas")

[Out]  $-1/2*((A^2 + 2AB + 2B^2)*b^2*c^2 - 2*(A^2 + 4AB + 8B^2)*a*b*c*d + (A^2 + 6AB + 14B^2)*a^2*d^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b^2*c^2 + 2*B^2*a*b*c*d)*\log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 - 4*((A*B + 3*B^2)*b^2*c*d - (A*B + 3*B^2)*a*b*d^2)*x - 2*((A*B + 3*B^2)*b^2*d^2*x^2 - (A*B + B^2)*b^2*c^2 + 2*(A*B + 2*B^2)*a*b*c*d + 2*(B^2*b^2*c*d + (A*B + 2*B^2)*a*b*d^2)*x)*\log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)$

**Sympy [B]** time = 6.76981, size = 877, normalized size = 3.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2/(b\*g\*x+a\*g)\*\*3,x)

[Out]  $-2*B*d**2*(A + 3*B)*\log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 6*B**2*a*d**3 + 6*B**2*b*c*d**2 - 2*B*a**3*d**5*(A + 3*B))/(a*d - b*c)**2 + 6*B*a**2*b*c*d**4*(A + 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A + 3*B)/(a*d - b*c)**2 + 2*B*b**3*c**3*d**2*(A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + 2*B*d**2*(A + 3*B)*\log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 + 6*B**2*a*d**3 + 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A + 3*B))/(a*d - b*c)**2 - 6*B*a**2*b*c*d**4*(A + 3*B)/(a*d - b*c)**2 + 6*B*a*b**2*c**2*d**3*(A + 3*B)/(a*d - b*c)**2 - 2*B*b**3*c**3*d**2*(A + 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 + 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + (2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*\log(e*(a + b*x)**2/(c + d*x)**2)/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**3*x**2) + (-A*B*a*d + A*B*b*c - 3*B**2*a*d + B**2*b*c - 2*B**2*b*d*x)*\log(e*(a + b*x)**2/(c + d*x)**2)/(a**3*b*d*g**3 - a**2*b**2*c*g**3 + 2*a**2*b**2*d*g**3*x - 2*a*b**3*c*g**3*x + a*b**3*d*g**3*x**2 - b**4*c*g**3*x**2) - (A**2*a*d - A**2*b*c + 6*A*B*a*d - 2*A*B*b*c + 14*B**2*a*d - 2*B**2*b*c + x*(4*A$



$B*b*d + 12*B**2*b*d)/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + x**2*(2*a*b**3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4*a*b**3*c*g**3))$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A\right)^2}{(bgx + ag)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^3,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2/(b\*g\*x + a\*g)^3, x)

$$3.135 \quad \int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^4} dx$$

**Optimal.** Leaf size=429

$$\frac{b^2(c+dx)^3 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}{3g^4(a+bx)^3(bc-ad)^3} - \frac{4b^2B(c+dx)^3 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{9g^4(a+bx)^3(bc-ad)^3} - \frac{d^2(c+dx) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}{g^4(a+bx)(bc-ad)^3} - \frac{4Bd^2(c+dx)}{g^4(a+bx)(bc-ad)^3}$$

[Out]  $(-8B^2d^2(c+dx))/((b^2c - a^2d)^3g^4(a+bx)) + (2b^2B^2d^2(c+dx)^2)/((b^2c - a^2d)^3g^4(a+bx)^2) - (8b^2B^2d^2(c+dx)^3)/(27(b^2c - a^2d)^3g^4(a+bx)^3) - (4B^2d^2(c+dx)(A + B \log[(e(a+bx)^2)/(c+dx)^2]))/((b^2c - a^2d)^3g^4(a+bx)) + (2b^2B^2d^2(c+dx)^2(A + B \log[(e(a+bx)^2)/(c+dx)^2]))/((b^2c - a^2d)^3g^4(a+bx)^2) - (4b^2B^2d^2(c+dx)^3(A + B \log[(e(a+bx)^2)/(c+dx)^2]))/(9(b^2c - a^2d)^3g^4(a+bx)^3) - (d^2(c+dx)(A + B \log[(e(a+bx)^2)/(c+dx)^2]))^2/((b^2c - a^2d)^3g^4(a+bx)) + (b^2d^2(c+dx)^2(A + B \log[(e(a+bx)^2)/(c+dx)^2]))^2/((b^2c - a^2d)^3g^4(a+bx)^2) - (b^2d^2(c+dx)^3(A + B \log[(e(a+bx)^2)/(c+dx)^2]))^2/(3(b^2c - a^2d)^3g^4(a+bx)^3)$

**Rubi [C]** time = 1.22523, antiderivative size = 692, normalized size of antiderivative = 1.61, number of steps used = 34, number of rules used = 11, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{8B^2d^3 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{3bg^4(bc-ad)^3} - \frac{8B^2d^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3bg^4(bc-ad)^3} - \frac{4Bd^3 \log(a+bx) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{3bg^4(bc-ad)^3} + \frac{4Bd^3 \log(c+dx)}{3bg^4(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(a\*g + b\*g\*x)^4, x]

[Out]  $(-8B^2)/(27b^2g^4(a+bx)^3) + (10B^2d)/(9b^2(b^2c - a^2d)g^4(a+bx)^2) - (44B^2d^2)/(9b^2(b^2c - a^2d)^2g^4(a+bx)) - (44B^2d^3 \text{Log}[a+bx])/(9b^2(b^2c - a^2d)^3g^4) + (4B^2d^3 \text{Log}[a+bx]^2)/(3b^2(b^2c - a^2d)^3g^4) - (4B^2(A + B \log[(e(a+bx)^2)/(c+dx)^2]))/(9b^2g^4(a+bx)^3) + (2B^2d(A + B \log[(e(a+bx)^2)/(c+dx)^2]))/(3b^2(b^2c - a^2d)g^4(a+bx)^2) - (4B^2d^2(A + B \log[(e(a+bx)^2)/(c+dx)^2]))/(3b^2(b^2c - a^2d)^2g^4(a+bx)) - (4B^2d^3 \text{Log}[a+bx](A + B \log[(e(a+bx)^2)/(c+dx)^2]))/(3b^2(b^2c - a^2d)^3g^4) - (A + B \log[(e(a+bx)^2)/(c+dx)^2])^2/(3b^2g^4(a+bx)^3) + (44B^2d^3 \text{Log}[c+dx])/(9b^2(b^2c - a^2d)^3g^4) - (8B^2d^3 \text{Log}[-((d(a+bx))/(b^2c - a^2d))] * \text{Log}[c+dx])/(3b^2(b^2c - a^2d)^3g^4) + (4B^2d^3(A + B \log[(e(a+bx)^2)/(c+dx)^2]) * \text{Log}[c+dx])/(3b^2(b^2c - a^2d)^3g^4) + (4B^2d^3 \text{Log}[c+dx]^2)/(3b^2(b^2c - a^2d)^3g^4) - (8B^2d^3 \text{Log}[a+bx] * \text{Log}[(b(c+dx))/(b^2c - a^2d)])/(3b^2(b^2c - a^2d)^3g^4) - (8B^2d^3 \text{PolyLog}[2, -((d(a+bx))/(b^2c - a^2d))])/(3b^2(b^2c - a^2d)^3g^4) - (8B^2d^3 \text{PolyLog}[2, (b(c+dx))/(b^2c - a^2d)])/(3b^2(b^2c - a^2d)^3g^4)$

**Rule 2525**

Int[(a\_. + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*Rfx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||

IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]

### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(Rfx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c

$(e*f - d*g), 0]$

### Rule 2391

$\text{Int}[\text{Log}[(c\_.)*((d\_.) + (e\_.)*(x\_.)^{(n\_.)})]/(x\_.), x\_Symbol] \text{ :> } -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}[\{c, d, e, n\}, x] \ \&\& \ \text{EqQ}[c*d, 1]$

### Rubi steps

$$\begin{aligned}
 \int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag + bgx)^4} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(2B) \int \frac{2(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g^3(a+bx)^4(c+dx)} dx}{3bg} \\
 &= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B(bc-ad)) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^4(c+dx)} dx}{3bg^4} \\
 &= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B(bc-ad)) \int \left(\frac{b\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2(a+bx)^3} + \frac{bd^2}{(bc-ad)^3}\right) dx}{3} \\
 &= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3bg^4(a+bx)^3} + \frac{(4B) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^4} dx}{3g^4} - \frac{(4Bd^3) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx} dx}{3(bc-ad)^3g^4} + \frac{(4Bd^3) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx} dx}{3(bc-ad)^3g^4} \\
 &= -\frac{4B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{9bg^4(a+bx)^3} + \frac{2Bd\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{4Bd^2\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)^2g^4(a+bx)} \\
 &= -\frac{4B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{9bg^4(a+bx)^3} + \frac{2Bd\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{4Bd^2\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)^2g^4(a+bx)} \\
 &= -\frac{4B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{9bg^4(a+bx)^3} + \frac{2Bd\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^4(a+bx)^2} - \frac{4Bd^2\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)^2g^4(a+bx)} \\
 &= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{44B^2d^3 \log(a+bx)}{9b(bc-ad)^3g^4} \\
 &= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{44B^2d^3 \log(a+bx)}{9b(bc-ad)^3g^4} \\
 &= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{44B^2d^3 \log(a+bx)}{9b(bc-ad)^3g^4} \\
 &= -\frac{8B^2}{27bg^4(a+bx)^3} + \frac{10B^2d}{9b(bc-ad)g^4(a+bx)^2} - \frac{44B^2d^2}{9b(bc-ad)^2g^4(a+bx)} - \frac{44B^2d^3 \log(a+bx)}{9b(bc-ad)^3g^4}
 \end{aligned}$$

**Mathematica [C]** time = 0.713385, size = 598, normalized size = 1.39

$$\frac{2B\left(-18Bd^3(a+bx)^3\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+18Bd^3(a+bx)^3\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)\right)}{27g^4(a+bx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(a\*g + b\*g\*x)^4,x]

[Out]  $-(9*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (2*B*(6*(b*c - a*d)^3*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]) - 9*d*(b*c - a*d)^2*(a + b*x)*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]) + 18*d^2*(b*c - a*d)*(a + b*x)^2*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]) + 18*d^3*(a + b*x)^3*\text{Log}[a + b*x]*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]) - 18*d^3*(a + b*x)^3*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])*\text{Log}[c + d*x] + 36*B*d^2*(a + b*x)^2*(b*c - a*d + d*(a + b*x)*\text{Log}[a + b*x] - d*(a + b*x)*\text{Log}[c + d*x]) - 9*B*d*(a + b*x)*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*\text{Log}[a + b*x] + 2*d^2*(a + b*x)^2*\text{Log}[c + d*x]) + 2*B*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*\text{Log}[a + b*x] - 6*d^3*(a + b*x)^3*\text{Log}[c + d*x]) - 18*B*d^3*(a + b*x)^3*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 18*B*d^3*(a + b*x)^3*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^3)/(27*b*g^4*(a + b*x)^3)$

**Maple [B]** time = 0.204, size = 1343, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^4,x)

[Out]  $-d^3/g^4*A^2*b/(a*d-b*c)^3/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2+d^3/g^4*A^2/(a*d-b*c)^3/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)+1/3*d^3/g^4*A^2*b^2/(a*d-b*c)^3/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3-170/27*d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*B^2/b/(d*x+c)^3+22/9*d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*b^2*B^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)-44/9*d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*b*B^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)-98/9*d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*B^2/(a*d-b*c)/(d*x+c)^2+4*d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*B^2/(a*d-b*c)/(d*x+c)^2*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)+1/3*d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*B^2*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)^2+d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*B^2/(a*d-b*c)/(d*x+c)^2*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)^2+6*d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*B^2*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)+d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*b*B^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)^2-22/9*d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*A*B/b/(d*x+c)^3+2/3*d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*b^2*A*B/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)-4/3*d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*A*B*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)-10/3*d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*A*B/(a*d-b*c)/(d*x+c)^2+2*d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*A*B/(a*d-b*c)/(d*x+c)^2*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)+2*d^3/g^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3*A*B*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)$

**Maxima [B]** time = 2.81718, size = 2126, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^4,x, algorithm="maxima")

[Out] 
$$-2/27*(3*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4))*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + (4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*b^6*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*x^2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^5*b^2*d^3*g^4)*x))*B^2 - 2/9*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/3*B^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) - 1/3*A^2/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)$$

**Fricas [A]** time = 1.1358, size = 1474, normalized size = 3.44

$$(9A^2 + 12AB + 8B^2)b^3c^3 - 27(A^2 + 2AB + 2B^2)ab^2c^2d + 27(A^2 + 4AB + 8B^2)a^2bcd^2 - (9A^2 + 66AB + 170B^2)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^4,x, algorithm="fricas")

[Out] 
$$-1/27*((9*A^2 + 12*A*B + 8*B^2)*b^3*c^3 - 27*(A^2 + 2*A*B + 2*B^2)*a*b^2*c^2*d + 27*(A^2 + 4*A*B + 8*B^2)*a^2*b*c*d^2 - (9*A^2 + 66*A*B + 170*B^2)*a^3*d^3 + 12*((3*A*B + 11*B^2)*b^3*c*d^2 - (3*A*B + 11*B^2)*a*b^2*d^3)*x^2 + 9*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 - 6*((3*A*B + 5*B^2)*b^3*c^2*d - 18*(A*B + 3*B^2)*a*b^2*c*d^2 + (15*A*B + 49*B^2)*a^2*b*d^3)*x + 6*((3*A*B + 11*B^2)*b^3*d^3*x^3 + (3*A*B + 2*B^2)*b^3*c^3 - 9*(A*B + B^2)*a*b^2*c^2*d + 9*(A*B + 2$$

$$\begin{aligned} & *B^2)*a^2*b*c*d^2 + 3*(2*B^2*b^3*c*d^2 + 3*(A*B + 3*B^2)*a*b^2*d^3)*x^2 - 3 \\ & *(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 3*(A*B + 2*B^2)*a^2*b*d^3)*x)*\log((b^2 \\ & *e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^7*c^3 - 3*a*b^6 \\ & *c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5 \\ & *c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4 \\ & *c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c \\ & ^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4) \end{aligned}$$

**Sympy [B]** time = 35.2154, size = 1561, normalized size = 3.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2/(b\*g\*x+a\*g)\*\*4,x)

[Out] 
$$\begin{aligned} & -4*B*d**3*(3*A + 11*B)*\log(x + (12*A*B*a*d**4 + 12*A*B*b*c*d**3 + 44*B**2*a \\ & *d**4 + 44*B**2*b*c*d**3 - 4*B*a**4*d**7*(3*A + 11*B)/(a*d - b*c)**3 + 16*B \\ & *a**3*b*c*d**6*(3*A + 11*B)/(a*d - b*c)**3 - 24*B*a**2*b**2*c**2*d**5*(3*A \\ & + 11*B)/(a*d - b*c)**3 + 16*B*a*b**3*c**3*d**4*(3*A + 11*B)/(a*d - b*c)**3 \\ & - 4*B*b**4*c**4*d**3*(3*A + 11*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 + 88*B**2* \\ & b*d**4)/(9*b*g**4*(a*d - b*c)**3) + 4*B*d**3*(3*A + 11*B)*\log(x + (12*A*B* \\ & a*d**4 + 12*A*B*b*c*d**3 + 44*B**2*a*d**4 + 44*B**2*b*c*d**3 + 4*B*a**4*d** \\ & 7*(3*A + 11*B)/(a*d - b*c)**3 - 16*B*a**3*b*c*d**6*(3*A + 11*B)/(a*d - b*c) \\ & **3 + 24*B*a**2*b**2*c**2*d**5*(3*A + 11*B)/(a*d - b*c)**3 - 16*B*a*b**3*c* \\ & *3*d**4*(3*A + 11*B)/(a*d - b*c)**3 + 4*B*b**4*c**4*d**3*(3*A + 11*B)/(a*d \\ & - b*c)**3)/(24*A*B*b*d**4 + 88*B**2*b*d**4)/(9*b*g**4*(a*d - b*c)**3) + (3 \\ & *B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d** \\ & 3*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*\log(e*(a + b*x)**2/(c + d*x) \\ & **2)**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9 \\ & *a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4 \\ & *x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c \\ & *d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27 \\ & *a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3* \\ & g**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a* \\ & *2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 - 22*B**2*a**2*d**2 + 14*B**2*a* \\ & b*c*d - 30*B**2*a*b*d**2*x - 4*B**2*b**2*c**2 + 6*B**2*b**2*c*d*x - 12*B**2 \\ & *b**2*d**2*x**2)*\log(e*(a + b*x)**2/(c + d*x)**2)/(9*a**5*b*d**2*g**4 - 18* \\ & a**4*b**2*c*d*g**4 + 27*a**4*b**2*d**2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54* \\ & a**3*b**3*c*d*g**4*x + 27*a**3*b**3*d**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4 \\ & *x - 54*a**2*b**4*c*d*g**4*x**2 + 9*a**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c* \\ & *2*g**4*x**2 - 18*a*b**5*c*d*g**4*x**3 + 9*b**6*c**2*g**4*x**3) - (9*A**2*a \\ & **2*d**2 - 18*A**2*a*b*c*d + 9*A**2*b**2*c**2 + 66*A*B*a**2*d**2 - 42*A*B*a \\ & *b*c*d + 12*A*B*b**2*c**2 + 170*B**2*a**2*d**2 - 46*B**2*a*b*c*d + 8*B**2*b \\ & **2*c**2 + x**2*(36*A*B*b**2*d**2 + 132*B**2*b**2*d**2) + x*(90*A*B*a*b*d** \\ & 2 - 18*A*B*b**2*c*d + 294*B**2*a*b*d**2 - 30*B**2*b**2*c*d))/(27*a**5*b*d** \\ & 2*g**4 - 54*a**4*b**2*c*d*g**4 + 27*a**3*b**3*c**2*g**4 + x**3*(27*a**2*b** \\ & 4*d**2*g**4 - 54*a*b**5*c*d*g**4 + 27*b**6*c**2*g**4) + x**2*(81*a**3*b**3* \\ & d**2*g**4 - 162*a**2*b**4*c*d*g**4 + 81*a*b**5*c**2*g**4) + x*(81*a**4*b**2 \\ & *d**2*g**4 - 162*a**3*b**3*c*d*g**4 + 81*a**2*b**4*c**2*g**4) \end{aligned}$$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)^2}{(bgx + ag)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^4,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2/(b\*g\*x + a\*g)^4, x)



$$3.136 \quad \int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag+bgx)^5} dx$$

**Optimal.** Leaf size=587

$$\frac{b^3(c+dx)^4 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}{4g^5(a+bx)^4(bc-ad)^4} - \frac{b^3B(c+dx)^4 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{4g^5(a+bx)^4(bc-ad)^4} + \frac{b^2d(c+dx)^3 \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}{g^5(a+bx)^3(bc-ad)^4} + \dots$$

[Out]  $(8B^2d^3(c+dx))/((b^2c-ad)^4g^5(a+bx)) - (3b^2B^2d^2(c+dx)^2)/((b^2c-ad)^4g^5(a+bx)^2) + (8b^2B^2d^2(c+dx)^3)/(9(b^2c-ad)^4g^5(a+bx)^3) - (b^3B^2d^2(c+dx)^4)/(8(b^2c-ad)^4g^5(a+bx)^4) + (4B^2d^3(c+dx)(A+B \log[(e(a+bx)^2)/(c+dx)^2]))/((b^2c-ad)^4g^5(a+bx)) - (3b^2B^2d^2(c+dx)^2(A+B \log[(e(a+bx)^2)/(c+dx)^2]))/((b^2c-ad)^4g^5(a+bx)^2) + (4b^2B^2d^2(c+dx)^3(A+B \log[(e(a+bx)^2)/(c+dx)^2]))/(3(b^2c-ad)^4g^5(a+bx)^3) - (b^3B^2d^2(c+dx)^4(A+B \log[(e(a+bx)^2)/(c+dx)^2]))/(4(b^2c-ad)^4g^5(a+bx)^4) + (d^3(c+dx)(A+B \log[(e(a+bx)^2)/(c+dx)^2]))^2/((b^2c-ad)^4g^5(a+bx)) - (3b^2d^2(c+dx)^2(A+B \log[(e(a+bx)^2)/(c+dx)^2]))^2/((b^2c-ad)^4g^5(a+bx)^2) + (b^2d^2(c+dx)^3(A+B \log[(e(a+bx)^2)/(c+dx)^2]))^2/((b^2c-ad)^4g^5(a+bx)^3) - (b^3(c+dx)^4(A+B \log[(e(a+bx)^2)/(c+dx)^2]))^2/(4(b^2c-ad)^4g^5(a+bx)^4)$

**Rubi [C]** time = 1.39387, antiderivative size = 757, normalized size of antiderivative = 1.29, number of steps used = 38, number of rules used = 11, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2B^2d^4 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg^5(bc-ad)^4} + \frac{2B^2d^4 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bg^5(bc-ad)^4} + \frac{Bd^4 \log(a+bx) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{bg^5(bc-ad)^4} - \frac{Bd^4 \log(c+dx)}{bg^5}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(a\*g + b\*g\*x)^5, x]

[Out]  $-B^2/(8b^2g^5(a+bx)^4) + (7B^2d)/(18b(b^2c-ad)g^5(a+bx)^3) - (13B^2d^2)/(12b(b^2c-ad)^2g^5(a+bx)^2) + (25B^2d^3)/(6b(b^2c-ad)^3g^5(a+bx)) + (25B^2d^4 \log[a+bx])/(6b(b^2c-ad)^4g^5) - (B^2d^4 \log[a+bx]^2)/(b(b^2c-ad)^4g^5) - (B^2(A+B \log[(e(a+bx)^2)/(c+dx)^2]))/(4b^2g^5(a+bx)^4) + (B^2d(A+B \log[(e(a+bx)^2)/(c+dx)^2]))/(3b(b^2c-ad)g^5(a+bx)^3) - (B^2d^2(A+B \log[(e(a+bx)^2)/(c+dx)^2]))/(2b(b^2c-ad)^2g^5(a+bx)^2) + (B^2d^3(A+B \log[(e(a+bx)^2)/(c+dx)^2]))/(b(b^2c-ad)^3g^5(a+bx)) + (B^2d^4 \log[a+bx](A+B \log[(e(a+bx)^2)/(c+dx)^2]))/(b(b^2c-ad)^4g^5) - (A+B \log[(e(a+bx)^2)/(c+dx)^2])^2/(4b^2g^5(a+bx)^4) - (25B^2d^4 \log[c+dx])/(6b(b^2c-ad)^4g^5) + (2B^2d^4 \log[-((d(a+bx))/(b^2c-ad))]*\log[c+dx])/(b(b^2c-ad)^4g^5) - (B^2d^4(A+B \log[(e(a+bx)^2)/(c+dx)^2])*\log[c+dx])/(b(b^2c-ad)^4g^5) - (B^2d^4 \log[c+dx]^2)/(b(b^2c-ad)^4g^5) + (2B^2d^4 \log[a+bx]*\log[(b(c+dx))/(b^2c-ad)])/(b(b^2c-ad)^4g^5) + (2B^2d^4 \text{PolyLog}[2, -((d(a+bx))/(b^2c-ad))])/(b(b^2c-ad)^4g^5) + (2B^2d^4 \text{PolyLog}[2, (b(c+dx))/(b^2c-ad)])/(b(b^2c-ad)^4g^5)$

**Rule 2525**

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

### Rule 44

```
Int[((a_) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFX^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*(RFX_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFX, x] && IntegerQ[p]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.)*(x_.))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_.)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_.)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_.)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.)^n)]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(ag + bgx)^5} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B \int \frac{2(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g^4(a+bx)^5(c+dx)} dx}{2bg} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad)) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^5(c+dx)} dx}{bg^5} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad)) \int \left(\frac{b\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(a+bx)^5} - \frac{bd\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)^2(a+bx)^4}\right) dx}{bg^5} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^5} dx}{g^5} + \frac{(Bd^4) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx} dx}{(bc-ad)^4g^5} - \frac{(Bbd^4) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^4} dx}{g^5} \\
&= -\frac{B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4bg^5(a+bx)^4} + \frac{Bd\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b(bc-ad)^2g^5(a+bx)^2} + \frac{Bbd^4\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g^5} \\
&= -\frac{B\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{4bg^5(a+bx)^4} + \frac{Bd\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3b(bc-ad)g^5(a+bx)^3} - \frac{Bd^2\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2b(bc-ad)^2g^5(a+bx)^2} + \frac{Bbd^4\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g^5} \\
&= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \frac{25B^2d^3}{6b(bc-ad)^3g^5} \\
&= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \frac{25B^2d^3}{6b(bc-ad)^3g^5} \\
&= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \frac{25B^2d^3}{6b(bc-ad)^3g^5} \\
&= -\frac{B^2}{8bg^5(a+bx)^4} + \frac{7B^2d}{18b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{12b(bc-ad)^2g^5(a+bx)^2} + \frac{25B^2d^3}{6b(bc-ad)^3g^5}
\end{aligned}$$

**Mathematica [C]** time = 1.02916, size = 762, normalized size = 1.3

$$B \left( 72 B d^4 (a+bx)^4 \left( \log(a+bx) \left( \log(a+bx) - 2 \log\left(\frac{b(c+dx)}{bc-ad}\right) \right) - 2 \text{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right) \right) - 72 B d^4 (a+bx)^4 \left( 2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right) + \log(c+dx) \left( 2 \log\left(\frac{d(a+bx)}{ad-bc}\right) - \log(c+dx) \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(a\*g + b\*g\*x)^5, x]

[Out]  $-(18*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (B*(18*(b*c - a*d)^4*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]) + 24*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]) + 36*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]) + 72*d^3*(-(b*c) + a*d)*(a + b*x)^3*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]) - 72*d^4*(a + b*x)^4*\text{Log}[a + b*x]*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]) + 72*d^4*(a + b*x)^4*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])*\text{Log}[c + d*x] - 144*B*d^3*(a + b*x)^3*(b*c - a*d + d*(a + b*x)*\text{Log}[a + b*x] - d*(a + b*x)*\text{Log}[c + d*x]) + 36*B*d^2*(a + b*x)^2*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*\text{Log}[a + b*x] + 2*d^2*(a + b*x)^2*\text{Log}[c + d*x]) - 8*B*d*(a + b*x)*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*\text{Log}[a + b*x] - 6*d^3*(a + b*x)^3*\text{Log}[c + d*x]) + 3*B*(3*(b*c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*\text{Log}[a + b*x] + 12*d^4*(a + b*x)^4*\text{Log}[c + d*x]) + 72*B*d^4*(a + b*x)^4*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 72*B*d^4*(a + b*x)^4*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^4)/(72*b*g^5*(a + b*x)^4)$

**Maple [B]** time = 0.297, size = 1943, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^5, x)

[Out]  $-3/2*d^4/g^5*A^2*b/(a*d-b*c)^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2+d^4/g^5*A^2/(a*d-b*c)^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)+d^4/g^5*A^2*b^2/(a*d-b*c)^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^3-1/4*d^4/g^5*A^2*b^3/(a*d-b*c)^4/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^4-415/72*d^4/g^5/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^4*B^2/b/(d*x+c)^4+25/12*d^4/g^5/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^4*b^3*B^2/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)-25/6*d^4/g^5/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^4*B^2*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)-163/12*d^4/g^5/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^4*b*B^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^2-271/18*d^4/g^5/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^4*B^2/(a*d-b*c)/(d*x+c)^3+4*d^4/g^5/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^4*B^2/(a*d-b*c)/(d*x+c)^3*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)+1/4*d^4/g^5/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^4*B^2*b^3/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)^2+d^4/g^5/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^4*B^2/(a*d-b*c)/(d*x+c)^3*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)^2+9*d^4/g^5/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^4*B^2*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(d*x+c)^2*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)+22/3*d^4/g^5/(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^4*B^2*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(d*x+c)*\ln(e*$

$$\frac{1}{(dx+c)*a*d-b*c/(dx+c)+b}^2/d^2+d^4/g^5/(1/(dx+c)*a*d-b*c/(dx+c)+b)^4*B^2*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(dx+c)*\ln(e*(1/(dx+c)*a*d-b*c/(dx+c)+b)^2/d^2)^2+3/2*d^4/g^5/(1/(dx+c)*a*d-b*c/(dx+c)+b)^4*b*B^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(dx+c)^2*\ln(e*(1/(dx+c)*a*d-b*c/(dx+c)+b)^2/d^2)^2-d^4/g^5/(1/(dx+c)*a*d-b*c/(dx+c)+b)^4*A*B*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(dx+c)-25/12*d^4/g^5/(1/(dx+c)*a*d-b*c/(dx+c)+b)^4*A*B/b/(dx+c)^4+1/2*d^4/g^5/(1/(dx+c)*a*d-b*c/(dx+c)+b)^4*b^3*A*B/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*\ln(e*(1/(dx+c)*a*d-b*c/(dx+c)+b)^2/d^2)-7/2*d^4/g^5/(1/(dx+c)*a*d-b*c/(dx+c)+b)^4*A*B*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(dx+c)^2-13/3*d^4/g^5/(1/(dx+c)*a*d-b*c/(dx+c)+b)^4*A*B/(a*d-b*c)/(dx+c)^3+2*d^4/g^5/(1/(dx+c)*a*d-b*c/(dx+c)+b)^4*A*B/(a*d-b*c)/(dx+c)^3*\ln(e*(1/(dx+c)*a*d-b*c/(dx+c)+b)^2/d^2)+3*d^4/g^5/(1/(dx+c)*a*d-b*c/(dx+c)+b)^4*A*B*b/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(dx+c)^2*\ln(e*(1/(dx+c)*a*d-b*c/(dx+c)+b)^2/d^2)+2*d^4/g^5/(1/(dx+c)*a*d-b*c/(dx+c)+b)^4*A*B*b^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(dx+c)*\ln(e*(1/(dx+c)*a*d-b*c/(dx+c)+b)^2/d^2)$$

**Maxima [B]** time = 2.25964, size = 3077, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^5,x, algorithm="maxima")

[Out]  $\frac{1}{72} * (6 * ((12 * b^3 * d^3 * x^3 - 3 * b^3 * c^3 + 13 * a * b^2 * c^2 * d - 23 * a^2 * b * c * d^2 + 25 * a^3 * d^3 - 6 * (b^3 * c * d^2 - 7 * a * b^2 * d^3)) * x^2 + 4 * (b^3 * c^2 * d - 5 * a * b^2 * c * d^2 + 13 * a^2 * b * d^3) * x) / ((b^8 * c^3 - 3 * a * b^7 * c^2 * d + 3 * a^2 * b^6 * c * d^2 - a^3 * b^5 * d^3) * g^5 * x^4 + 4 * (a * b^7 * c^3 - 3 * a^2 * b^6 * c^2 * d + 3 * a^3 * b^5 * c * d^2 - a^4 * b^4 * d^3) * g^5 * x^3 + 6 * (a^2 * b^6 * c^3 - 3 * a^3 * b^5 * c^2 * d + 3 * a^4 * b^4 * c * d^2 - a^5 * b^3 * d^3) * g^5 * x^2 + 4 * (a^3 * b^5 * c^3 - 3 * a^4 * b^4 * c^2 * d + 3 * a^5 * b^3 * c * d^2 - a^6 * b^2 * d^3) * g^5 * x + (a^4 * b^4 * c^3 - 3 * a^5 * b^3 * c^2 * d + 3 * a^6 * b^2 * c * d^2 - a^7 * b * d^3) * g^5 + 12 * d^4 * \log(b * x + a) / ((b^5 * c^4 - 4 * a * b^4 * c^3 * d + 6 * a^2 * b^3 * c^2 * d^2 - 4 * a^3 * b^2 * c * d^3 + a^4 * b * d^4) * g^5) - 12 * d^4 * \log(d * x + c) / ((b^5 * c^4 - 4 * a * b^4 * c^3 * d + 6 * a^2 * b^3 * c^2 * d^2 - 4 * a^3 * b^2 * c * d^3 + a^4 * b * d^4) * g^5)) * \log(b^2 * e * x^2 / (d^2 * x^2 + 2 * c * d * x + c^2)) + 2 * a * b * e * x / (d^2 * x^2 + 2 * c * d * x + c^2) + a^2 * e / (d^2 * x^2 + 2 * c * d * x + c^2)) - (9 * b^4 * c^4 - 64 * a * b^3 * c^3 * d + 216 * a^2 * b^2 * c^2 * d^2 - 576 * a^3 * b * c * d^3 + 415 * a^4 * d^4 - 300 * (b^4 * c * d^3 - a * b^3 * d^4)) * x^3 + 6 * (13 * b^4 * c^2 * d^2 - 176 * a * b^3 * c * d^3 + 163 * a^2 * b^2 * d^4) * x^2 + 72 * (b^4 * d^4 * x^4 + 4 * a * b^3 * d^4 * x^3 + 6 * a^2 * b^2 * d^4 * x^2 + 4 * a^3 * b * d^4 * x + a^4 * d^4) * \log(b * x + a)^2 + 72 * (b^4 * d^4 * x^4 + 4 * a * b^3 * d^4 * x^3 + 6 * a^2 * b^2 * d^4 * x^2 + 4 * a^3 * b * d^4 * x + a^4 * d^4) * \log(d * x + c)^2 - 4 * (7 * b^4 * c^3 * d - 60 * a * b^3 * c^2 * d^2 + 324 * a^2 * b^2 * c * d^3 - 271 * a^3 * b * d^4) * x - 300 * (b^4 * d^4 * x^4 + 4 * a * b^3 * d^4 * x^3 + 6 * a^2 * b^2 * d^4 * x^2 + 4 * a^3 * b * d^4 * x + a^4 * d^4) * \log(b * x + a) + 12 * (25 * b^4 * d^4 * x^4 + 100 * a * b^3 * d^4 * x^3 + 150 * a^2 * b^2 * d^4 * x^2 + 100 * a^3 * b * d^4 * x + 25 * a^4 * d^4 - 12 * (b^4 * d^4 * x^4 + 4 * a * b^3 * d^4 * x^3 + 6 * a^2 * b^2 * d^4 * x^2 + 4 * a^3 * b * d^4 * x + a^4 * d^4) * \log(b * x + a)) * \log(d * x + c) / (a^4 * b^5 * c^4 * g^5 - 4 * a^5 * b^4 * c^3 * d * g^5 + 6 * a^6 * b^3 * c^2 * d^2 * g^5 - 4 * a^7 * b^2 * c * d^3 * g^5 + a^8 * b * d^4 * g^5 + (b^9 * c^4 * g^5 - 4 * a * b^8 * c^3 * d * g^5 + 6 * a^2 * b^7 * c^2 * d^2 * g^5 - 4 * a^3 * b^6 * c * d^3 * g^5 + a^4 * b^5 * d^4 * g^5) * x^4 + 4 * (a * b^8 * c^4 * g^5 - 4 * a^2 * b^7 * c^3 * d * g^5 + 6 * a^3 * b^6 * c^2 * d^2 * g^5 - 4 * a^4 * b^5 * c * d^3 * g^5 + a^5 * b^4 * d^4 * g^5) * x^3 + 6 * (a^2 * b^7 * c^4 * g^5 - 4 * a^3 * b^6 * c^3 * d * g^5 + 6 * a^4 * b^5 * c^2 * d^2 * g^5 - 4 * a^5 * b^4 * c * d^3 * g^5 + a^6 * b^3 * d^4 * g^5) * x^2 + 4 * (a^3 * b^6 * c^4 * g^5 - 4 * a^4 * b^5 * c^3 * d * g^5 + 6 * a^5 * b^4 * c^2 * d^2 * g^5 - 4 * a^6 * b^3 * c * d^3 * g^5 + a^7 * b^2 * d^4 * g^5) * x) * B^2 + 1/12 * A * B * ((12 * b^3 * d^3 * x^3 - 3 * b^3 * c^3 + 13 * a * b^2 * c^2 * d - 23 * a^2 * b * c * d^2 + 25 * a^3 * d^3 - 6 * (b^3 * c * d^2 - 7 * a * b^2 * d^3)) * x^2 + 4 * (b^3 * c^2 * d - 5 * a * b^2 * c * d^2 + 13 * a^2 * b * d^3) * x) / ((b^8 * c^3 - 3 * a * b^7 * c^2 * d + 3 * a^2 * b^6 * c * d^2 - a^3 * b^5 * d^3) * g^5 * x^4 + 4 * (a * b^7 * c^3 - 3 * a^2 * b^6 * c^2 * d + 3 * a^3 * b^5 * c * d^2 - a^4 * b^4 * d^3) * g^5 * x^3 + 6 * (a^2 * b^6 * c^3 - 3 * a^3 * b^5 * c^2 * d + 3 * a^4 * b^4 * c * d^2 - a^5 * b^3 * d^3) * g^5 * x^2 + 4 * (a^3 * b^5 * c^3 - 3 * a^4 * b^4 * c^2 * d + 3 * a^5 * b^3 * c * d^2 - a^6 * b^2 * d^3) * g^5 * x + (a^4 * b^4 * c^3 - 3 * a^5 * b^3 * c^2 * d + 3 * a^6 * b^2 * c * d^2 - a^7 * b * d^3) * g^5 + 12 * d^4 * \log(b * x + a) / ((b^5 * c^4 - 4 * a * b^4 * c^3 * d + 6 * a^2 * b^3 * c^2 * d^2 - 4 * a^3 * b^2 * c * d^3 + a^4 * b * d^4) * g^5) - 12 * d^4 * \log(d * x + c) / ((b^5 * c^4 - 4 * a * b^4 * c^3 * d + 6 * a^2 * b^3 * c^2 * d^2 - 4 * a^3 * b^2 * c * d^3 + a^4 * b * d^4) * g^5)) * \log(b^2 * e * x^2 / (d^2 * x^2 + 2 * c * d * x + c^2)) + 2 * a * b * e * x / (d^2 * x^2 + 2 * c * d * x + c^2) + a^2 * e / (d^2 * x^2 + 2 * c * d * x + c^2))$

$$\begin{aligned}
& - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - \\
& 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - \\
& 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - \\
& 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3 \\
& *a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) - 6*\log(b^2*e*x^2/(d^2*x \\
& ^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 \\
& + 2*c*d*x + c^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^ \\
& 3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*\log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + \\
& 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*\log(d*x + c) \\
& /((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^ \\
& 4)*g^5)) - 1/4*B^2*\log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2 \\
& *x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))^2/(b^5*g^5*x^4 + 4 \\
& *a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^2 \\
& /(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4 \\
& *b*g^5)
\end{aligned}$$

**Fricas [A]** time = 1.20757, size = 2229, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^5,x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& -1/72*(9*(2*A^2 + 2*A*B + B^2)*b^4*c^4 - 8*(9*A^2 + 12*A*B + 8*B^2)*a*b^3*c \\
& ^3*d + 108*(A^2 + 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 72*(A^2 + 4*A*B + 8*B^2) \\
& *a^3*b*c*d^3 + (18*A^2 + 150*A*B + 415*B^2)*a^4*d^4 - 12*((6*A*B + 25*B^2)* \\
& b^4*c*d^3 - (6*A*B + 25*B^2)*a*b^3*d^4)*x^3 + 6*((6*A*B + 13*B^2)*b^4*c^2*d \\
& ^2 - 16*(3*A*B + 11*B^2)*a*b^3*c*d^3 + (42*A*B + 163*B^2)*a^2*b^2*d^4)*x^2 \\
& - 18*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2 \\
& *a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4* \\
& B^2*a^3*b*c*d^3)*\log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c \\
& ^2))^2 - 4*((6*A*B + 7*B^2)*b^4*c^3*d - 12*(3*A*B + 5*B^2)*a*b^3*c^2*d^2 + \\
& 108*(A*B + 3*B^2)*a^2*b^2*c*d^3 - (78*A*B + 271*B^2)*a^3*b*d^4)*x - 6*((6*A \\
& *B + 25*B^2)*b^4*d^4*x^4 - 3*(2*A*B + B^2)*b^4*c^4 + 8*(3*A*B + 2*B^2)*a*b^ \\
& 3*c^3*d - 36*(A*B + B^2)*a^2*b^2*c^2*d^2 + 24*(A*B + 2*B^2)*a^3*b*c*d^3 + 4 \\
& *(3*B^2*b^4*c*d^3 + 2*(3*A*B + 11*B^2)*a*b^3*d^4)*x^3 - 6*(B^2*b^4*c^2*d^2 \\
& - 8*B^2*a*b^3*c*d^3 - 6*(A*B + 3*B^2)*a^2*b^2*d^4)*x^2 + 4*(B^2*b^4*c^3*d - \\
& 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 + 6*(A*B + 2*B^2)*a^3*b*d^4)*x) \\
& *\log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/((b^9*c^4 \\
& - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^ \\
& 4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + \\
& a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 \\
& - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3* \\
& d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 \\
& - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2/(b\*g\*x+a\*g)\*\*5,x)

[Out] Timed out

**Giac [A]** time = 1.64779, size = 1180, normalized size = 2.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(b\*g\*x+a\*g)^5,x, algorithm="giac")

[Out] 
$$\begin{aligned} & \frac{1}{4} * (B^2 * d^4 / (b^5 * c^4 * g^5 - 4 * a * b^4 * c^3 * d * g^5 + 6 * a^2 * b^3 * c^2 * d^2 * g^5 - 4 * a^3 * b^2 * c * d^3 * g^5 + a^4 * b * d^4 * g^5) - B^2 / ((b * g * x + a * g)^4 * b * g)) * \log(b^2 / (b^2 * c^2 * g^2 / (b * g * x + a * g)^2 - 2 * a * b * c * d * g^2 / (b * g * x + a * g)^2 + a^2 * d^2 * g^2 / (b * g * x + a * g)^2 + 2 * b * c * d * g / (b * g * x + a * g) - 2 * a * d^2 * g / (b * g * x + a * g) + d^2)) ^2 + \\ & \frac{1}{12} * (12 * B^2 * d^3 / ((b^3 * c^3 * g^3 - 3 * a * b^2 * c^2 * d * g^3 + 3 * a^2 * b * c * d^2 * g^3 - a^3 * d^3 * g^3) * (b * g * x + a * g) * b * g) - 6 * B^2 * d^2 / ((b^2 * c^2 * g - 2 * a * b * c * d * g + a^2 * d^2 * g) * (b * g * x + a * g)^2 * b * g^2) + 4 * B^2 * d / ((b * g * x + a * g)^3 * (b * c - a * d) * b * g^2) - \\ & 3 * (2 * A * B * b^3 * g^3 + 3 * B^2 * b^3 * g^3) / ((b * g * x + a * g)^4 * b^4 * g^4)) * \log(b^2 / (b^2 * c^2 * g^2 / (b * g * x + a * g)^2 - 2 * a * b * c * d * g^2 / (b * g * x + a * g)^2 + a^2 * d^2 * g^2 / (b * g * x + a * g)^2 + 2 * b * c * d * g / (b * g * x + a * g) - 2 * a * d^2 * g / (b * g * x + a * g) + d^2)) - \\ & \frac{1}{6} * (6 * A * B * d^4 + 31 * B^2 * d^4) * \log(-b * c * g / (b * g * x + a * g) + a * d * g / (b * g * x + a * g) - d) / (b^5 * c^4 * g^5 - 4 * a * b^4 * c^3 * d * g^5 + 6 * a^2 * b^3 * c^2 * d^2 * g^5 - 4 * a^3 * b^2 * c * d^3 * g^5 + a^4 * b * d^4 * g^5) + \\ & \frac{1}{6} * (6 * A * B * d^3 + 31 * B^2 * d^3) / ((b^3 * c^3 * g^3 - 3 * a * b^2 * c^2 * d * g^3 + 3 * a^2 * b * c * d^2 * g^3 - a^3 * d^3 * g^3) * (b * g * x + a * g) * b * g) - \\ & \frac{1}{12} * (6 * A * B * b * d^2 + 19 * B^2 * b * d^2) / ((b^2 * c^2 * g - 2 * a * b * c * d * g + a^2 * d^2 * g) * (b * g * x + a * g)^2 * b^2 * g^2) + \\ & \frac{1}{18} * (6 * A * B * b^2 * d * g + 13 * B^2 * b^2 * d * g) / ((b * g * x + a * g)^3 * (b * c - a * d) * b^3 * g^3) - \\ & \frac{1}{8} * (2 * A^2 * b^3 * g^3 + 6 * A * B * b^3 * g^3 + 5 * B^2 * b^3 * g^3) / ((b * g * x + a * g)^4 * b^4 * g^4) \end{aligned}$$

$$3.137 \quad \int \frac{(ag+bgx)^2}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable}\left(\frac{(ag + bgx)^2}{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

**Rubi [A]** time = 0.196614, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(ag + bgx)^2}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

[Out] a^2\*g^2\*Defer[Int][(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-1), x] + 2\*a\*b\*g^2\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x] + b^2\*g^2\*Defer[Int][x^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

Rubi steps

$$\begin{aligned} \int \frac{(ag + bgx)^2}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx &= \int \left( \frac{a^2g^2}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} + \frac{2abg^2x}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} + \frac{b^2g^2x^2}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} \right) dx \\ &= (a^2g^2) \int \frac{1}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx + (2abg^2) \int \frac{x}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx + (b^2g^2) \int \frac{x^2}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.158243, size = 0, normalized size = 0.

$$\int \frac{(ag + bgx)^2}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

[Out] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]



**Maple [A]** time = 0.986, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2)),x)

[Out] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="maxima")

[Out] integrate((b\*g\*x + a\*g)^2/(B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b^2 g^2 x^2 + 2 a b g^2 x + a^2 g^2}{B \log \left( \frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2} \right) + A}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="fricas")

[Out] integral((b^2\*g^2\*x^2 + 2\*a\*b\*g^2\*x + a^2\*g^2)/(B\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + A), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*2/(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^2/(B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A), x)

$$3.138 \quad \int \frac{ag+bgx}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{ag+bgx}{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

**Rubi [A]** time = 0.101449, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{ag+bgx}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

[Out] a\*g\*Defer[Int][(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-1), x] + b\*g\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

Rubi steps

$$\begin{aligned} \int \frac{ag+bgx}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx &= \int \left( \frac{ag}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} + \frac{bgx}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} \right) dx \\ &= (ag) \int \frac{1}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx + (bg) \int \frac{x}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.115246, size = 0, normalized size = 0.

$$\int \frac{ag+bgx}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

[Out] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

**Maple [A]** time = 0.857, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

[Out] `int((b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bgx + ag}{B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")`

[Out] `integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bgx + ag}{B \log\left(\frac{b^2 ex^2 + 2 abex + a^2 e}{d^2 x^2 + 2 cdx + c^2}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")`

[Out] `integral((b*g*x + a*g)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bgx + ag}{B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

$$3.139 \quad \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable}\left(\frac{1}{(ag+bgx)\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)}, x\right)$$

[Out] Unintegrable[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

**Rubi [A]** time = 0.0731074, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} dx = \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} dx$$

**Mathematica [A]** time = 0.0704607, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

[Out] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

**Maple [A]** time = 1.065, size = 0, normalized size = 0.

$$\int \frac{1}{bgx+ag}\left(A+B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

[Out] `int(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)\left(B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")`

[Out] `integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{Abgx + Aag + (Bbgx + Bag) \log\left(\frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")`

[Out] `integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)\left(B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```



$$3.140 \quad \int \frac{1}{(ag+bgx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

**Optimal.** Leaf size=91

$$\frac{e^{\frac{A}{2B}}(c+dx)\sqrt{\frac{e(a+bx)^2}{(c+dx)^2}} \operatorname{Ei} \left( -\frac{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{2B} \right)}{2Bg^2(a+bx)(bc-ad)}$$

[Out] (E^(A/(2\*B))\*Sqrt[(e\*(a + b\*x)^2)/(c + d\*x)^2]\*(c + d\*x)\*ExpIntegralEi[-(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(2\*B)])/(2\*B\*(b\*c - a\*d)\*g^2\*(a + b\*x))

**Rubi [F]** time = 0.0867152, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag + bgx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

Rubi steps

$$\int \frac{1}{(ag + bgx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx = \int \frac{1}{(ag + bgx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

**Mathematica [F]** time = 0.0738722, size = 0, normalized size = 0.

$$\int \frac{1}{(ag + bgx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

[Out] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

**Maple [F]** time = 1.091, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

[Out] `int(1/(b*g*x+a*g)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")`

[Out] `integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{Ab^2g^2x^2 + 2Aabg^2x + Aa^2g^2 + (Bb^2g^2x^2 + 2Babg^2x + Ba^2g^2) \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")`

[Out] `integral(1/(A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B*a*b*g^2*x + B*a^2*g^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

$$3.141 \quad \int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

**Optimal.** Leaf size=149

$$\frac{bee^{A/B} \operatorname{Ei} \left( -\frac{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{B} \right)}{2Bg^3(bc-ad)^2} - \frac{de^{\frac{A}{2B}}(c+dx) \sqrt{\frac{e(a+bx)^2}{(c+dx)^2}} \operatorname{Ei} \left( -\frac{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{2B} \right)}{2Bg^3(a+bx)(bc-ad)^2}$$

[Out] (b\*e\*E^(A/B)\*ExpIntegralEi[-((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/B)])/ (2\*B\*(b\*c - a\*d)^2\*g^3) - (d\*E^(A/(2\*B))\*Sqrt[(e\*(a + b\*x)^2)/(c + d\*x)^2]\*(c + d\*x)\*ExpIntegralEi[-(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]/(2\*B))])/ (2\*B\*(b\*c - a\*d)^2\*g^3\*(a + b\*x))

**Rubi [F]** time = 0.0738272, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag + bgx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])),x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

Rubi steps

$$\int \frac{1}{(ag + bgx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx = \int \frac{1}{(ag + bgx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

**Mathematica [F]** time = 0.0797001, size = 0, normalized size = 0.

$$\int \frac{1}{(ag + bgx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])),x]

[Out] Integrate[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

**Maple [F]** time = 1.153, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

[Out] `int(1/(b*g*x+a*g)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")`

[Out] `integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{Ab^3g^3x^3 + 3Aab^2g^3x^2 + 3Aa^2bg^3x + Aa^3g^3 + (Bb^3g^3x^3 + 3Bab^2g^3x^2 + 3Ba^2bg^3x + Ba^3g^3) \log \left( \frac{b^2ex^2 + 2abx + a^2}{d^2x^2 + 2cdx + c^2} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")`

[Out] `integral(1/(A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="gi  
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

$$3.142 \quad \int \frac{(ag+bgx)^2}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable}\left(\frac{(ag + bgx)^2}{\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

**Rubi [A]** time = 0.215104, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(ag + bgx)^2}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

[Out] a^2\*g^2\*Defer[Int][(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-2), x] + 2\*a\*b\*g^2\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x] + b^2\*g^2\*Defer[Int][x^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{(ag + bgx)^2}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx &= \int \left( \frac{a^2g^2}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} + \frac{2abg^2x}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} + \frac{b^2g^2x^2}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} \right) dx \\ &= (a^2g^2) \int \frac{1}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx + (2abg^2) \int \frac{x}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx + (b^2g^2) \int \frac{x^2}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.467073, size = 0, normalized size = 0.

$$\int \frac{(ag + bgx)^2}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

[Out] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

**Maple [A]** time = 0.933, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{b^3 dg^2 x^4 + a^3 cg^2 + (b^3 cg^2 + 3ab^2 dg^2)x^3 + 3(ab^2 cg^2 + a^2 bdg^2)x^2 + (3a^2 bcg^2 + a^3 dg^2)x}{2(2(bc - ad)B^2 \log(bx + a) - 2(bc - ad)B^2 \log(dx + c) + (bc - ad)AB + (bc \log(e) - ad \log(e))B^2)} + \int \frac{4b}{2(2(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out] -1/2\*(b^3\*d\*g^2\*x^4 + a^3\*c\*g^2 + (b^3\*c\*g^2 + 3\*a\*b^2\*d\*g^2)\*x^3 + 3\*(a\*b^2\*c\*g^2 + a^2\*b\*d\*g^2)\*x^2 + (3\*a^2\*b\*c\*g^2 + a^3\*d\*g^2)\*x)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) + (b\*c - a\*d)\*A\*B + (b\*c\*log(e) - a\*d\*log(e))\*B^2) + integrate(1/2\*(4\*b^3\*d\*g^2\*x^3 + 3\*a^2\*b\*c\*g^2 + a^3\*d\*g^2 + 3\*(b^3\*c\*g^2 + 3\*a\*b^2\*d\*g^2)\*x^2 + 6\*(a\*b^2\*c\*g^2 + a^2\*b\*d\*g^2)\*x)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) + (b\*c - a\*d)\*A\*B + (b\*c\*log(e) - a\*d\*log(e))\*B^2), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b^2 g^2 x^2 + 2 a b g^2 x + a^2 g^2}{B^2 \log \left( \frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2} \right)^2 + 2 A B \log \left( \frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2} \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral((b^2\*g^2\*x^2 + 2\*a\*b\*g^2\*x + a^2\*g^2)/(B^2\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*A\*B\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + A^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{\left(B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2, x)
```

$$3.143 \quad \int \frac{ag+bgx}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{ag + bgx}{\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

**Rubi [A]** time = 0.114215, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{ag + bgx}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2,x]

[Out] a\*g\*Defer[Int][(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-2), x] + b\*g\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{ag + bgx}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx &= \int \left( \frac{ag}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} + \frac{bgx}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} \right) dx \\ &= (ag) \int \frac{1}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx + (bg) \int \frac{x}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.344706, size = 0, normalized size = 0.

$$\int \frac{ag + bgx}{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2,x]

[Out] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

**Maple [A]** time = 0.935, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int((b\*g\*x+a\*g)/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{b^2 d g x^3 + a^2 c g + (b^2 c g + 2 a b d g) x^2 + (2 a b c g + a^2 d g) x}{2 (2 (b c - a d) B^2 \log (b x + a) - 2 (b c - a d) B^2 \log (d x + c) + (b c - a d) A B + (b c \log (e) - a d \log (e)) B^2)} + \int \frac{1}{2 (2 (b c -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out] -1/2\*(b^2\*d\*g\*x^3 + a^2\*c\*g + (b^2\*c\*g + 2\*a\*b\*d\*g)\*x^2 + (2\*a\*b\*c\*g + a^2\*d\*g)\*x)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) + (b\*c - a\*d)\*A\*B + (b\*c\*log(e) - a\*d\*log(e))\*B^2) + integrate(1/2\*(3\*b^2\*d\*g\*x^2 + 2\*a\*b\*c\*g + a^2\*d\*g + 2\*(b^2\*c\*g + 2\*a\*b\*d\*g)\*x)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) + (b\*c - a\*d)\*A\*B + (b\*c\*log(e) - a\*d\*log(e))\*B^2), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{bgx + ag}{B^2 \log \left( \frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2} \right)^2 + 2 A B \log \left( \frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2} \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral((b\*g\*x + a\*g)/(B^2\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*A\*B\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + A^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bgx + ag}{\left(B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)/(B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2, x)

$$3.144 \quad \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable}\left(\frac{1}{(ag+bgx)\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))^2, x]

**Rubi [A]** time = 0.0814711, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))^2, x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))^2, x]

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx = \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

**Mathematica [A]** time = 0.140253, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))^2, x]

[Out] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))^2, x]

**Maple [A]** time = 0.902, size = 0, normalized size = 0.

$$\int \frac{1}{bgx+ag}\left(A+B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$d \int \frac{1}{2 \left( (bcg - adg)B^2 \log(bx + a) - 2(bcg - adg)B^2 \log(dx + c) + (bcg - adg)AB + (bcg \log(e) - adg \log(e))B^2 \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out] d\*integrate(1/2/(2\*(b\*c\*g - a\*d\*g)\*B^2\*log(b\*x + a) - 2\*(b\*c\*g - a\*d\*g)\*B^2\*log(d\*x + c) + (b\*c\*g - a\*d\*g)\*A\*B + (b\*c\*g\*log(e) - a\*d\*g\*log(e))\*B^2), x) - 1/2\*(d\*x + c)/(2\*(b\*c\*g - a\*d\*g)\*B^2\*log(b\*x + a) - 2\*(b\*c\*g - a\*d\*g)\*B^2\*log(d\*x + c) + (b\*c\*g - a\*d\*g)\*A\*B + (b\*c\*g\*log(e) - a\*d\*g\*log(e))\*B^2)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2bgx + A^2ag + (B^2bgx + B^2ag) \log\left(\frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2}\right)^2 + 2(ABbgx + ABag) \log\left(\frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2}\right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(1/(A^2\*b\*g\*x + A^2\*a\*g + (B^2\*b\*g\*x + B^2\*a\*g)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*(A\*B\*b\*g\*x + A\*B\*a\*g)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag) \left( B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

$$3.145 \quad \int \frac{1}{(ag+bgx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

**Optimal.** Leaf size=147

$$\frac{e^{\frac{A}{2B}}(c+dx) \sqrt{\frac{e(a+bx)^2}{(c+dx)^2}} \operatorname{Ei} \left( -\frac{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{2B} \right)}{4B^2 g^2 (a+bx)(bc-ad)} - \frac{c+dx}{2Bg^2(a+bx)(bc-ad) \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}$$

[Out]  $-(E^{(A/(2*B))} * \operatorname{Sqrt}[(e*(a+b*x)^2)/(c+d*x)^2] * (c+d*x) * \operatorname{ExpIntegralEi}[-(A+B*\operatorname{Log}[(e*(a+b*x)^2)/(c+d*x)^2])/(2*B)]) / (4*B^2*(b*c-a*d)*g^2*(a+b*x)) - (c+d*x)/(2*B*(b*c-a*d)*g^2*(a+b*x)*(A+B*\operatorname{Log}[(e*(a+b*x)^2)/(c+d*x)^2]))$

**Rubi [F]** time = 0.0935896, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[1/((a*g+b*g*x)^2*(A+B*\operatorname{Log}[(e*(a+b*x)^2)/(c+d*x)^2])^2),x]$

[Out]  $\operatorname{Defer}[\operatorname{Int}[1/((a*g+b*g*x)^2*(A+B*\operatorname{Log}[(e*(a+b*x)^2)/(c+d*x)^2])^2),x]$

Rubi steps

$$\int \frac{1}{(ag+bgx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx = \int \frac{1}{(ag+bgx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

**Mathematica [F]** time = 0.178894, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Integrate}[1/((a*g+b*g*x)^2*(A+B*\operatorname{Log}[(e*(a+b*x)^2)/(c+d*x)^2])^2),x]$

[Out]  $\operatorname{Integrate}[1/((a*g+b*g*x)^2*(A+B*\operatorname{Log}[(e*(a+b*x)^2)/(c+d*x)^2])^2),x]$



**Maple [F]** time = 1.224, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2} \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2 \left( (abcg^2 - a^2dg^2)AB + (abcg^2 \log(e) - a^2dg^2 \log(e))B^2 + ((b^2cg^2 - abdg^2)AB + (b^2cg^2 \log(e) - abdg^2 \log(e))B^2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out] 
$$-1/2*(d*x + c)/((a*b*c*g^2 - a^2*d*g^2)*A*B + (a*b*c*g^2*\log(e) - a^2*d*g^2*\log(e))*B^2 + ((b^2*c*g^2 - a*b*d*g^2)*A*B + (b^2*c*g^2*\log(e) - a*b*d*g^2*\log(e))*B^2)*x + 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*\log(b*x + a) - 2*((b^2*c*g^2 - a*b*d*g^2)*B^2*x + (a*b*c*g^2 - a^2*d*g^2)*B^2)*\log(d*x + c) + \text{integrate}(-1/2/(B^2*a^2*g^2*\log(e) + A*B*a^2*g^2 + (B^2*b^2*g^2*\log(e) + A*B*b^2*g^2)*x^2 + 2*(B^2*a*b*g^2*\log(e) + A*B*a*b*g^2)*x + 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*\log(b*x + a) - 2*(B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*\log(d*x + c)), x)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2b^2g^2x^2 + 2A^2abg^2x + A^2a^2g^2 + (B^2b^2g^2x^2 + 2B^2abg^2x + B^2a^2g^2) \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right)^2} + 2(ABb^2g^2x^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] 
$$\text{integral}(1/(A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*\log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*\log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

$$3.146 \quad \int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

**Optimal.** Leaf size=263

$$\frac{de^{\frac{A}{2B}}(c+dx) \sqrt{\frac{e(a+bx)^2}{(c+dx)^2}} \operatorname{Ei} \left( -\frac{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{2B} \right)}{4B^2 g^3 (a+bx)(bc-ad)^2} - \frac{bee^{A/B} \operatorname{Ei} \left( -\frac{A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{B} \right)}{2B^2 g^3 (bc-ad)^2} - \frac{b(c+dx)^2}{2Bg^3 (a+bx)^2 (bc-ad)^2 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}$$

[Out]  $-(b * e * E^{(A/B)} * \operatorname{ExpIntegralEi}[-((A + B * \operatorname{Log}[(e * (a + b * x)^2] / (c + d * x)^2]) / B)]) / (2 * B^2 * (b * c - a * d)^2 * g^3) + (d * E^{(A / (2 * B))} * \operatorname{Sqrt}[(e * (a + b * x)^2] / (c + d * x)^2] * (c + d * x) * \operatorname{ExpIntegralEi}[-(A + B * \operatorname{Log}[(e * (a + b * x)^2] / (c + d * x)^2]) / (2 * B)]) / (4 * B^2 * (b * c - a * d)^2 * g^3 * (a + b * x)) + (d * (c + d * x)) / (2 * B * (b * c - a * d)^2 * g^3 * (a + b * x) * (A + B * \operatorname{Log}[(e * (a + b * x)^2] / (c + d * x)^2])) - (b * (c + d * x)^2) / (2 * B * (b * c - a * d)^2 * g^3 * (a + b * x)^2 * (A + B * \operatorname{Log}[(e * (a + b * x)^2] / (c + d * x)^2]))$

**Rubi [F]** time = 0.0824103, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx = \int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

**Mathematica [F]** time = 0.347693, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

[Out] Integrate[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

**Maple [F]** time = 1.471, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3} \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^3/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int(1/(b\*g\*x+a\*g)^3/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out] 
$$-1/2*(d*x + c)/((a^2*b*c*g^3 - a^3*d*g^3)*A*B + (a^2*b*c*g^3*\log(e) - a^3*d*g^3*\log(e))*B^2 + ((b^3*c*g^3 - a*b^2*d*g^3)*A*B + (b^3*c*g^3*\log(e) - a*b^2*d*g^3*\log(e))*B^2)*x^2 + 2*((a*b^2*c*g^3 - a^2*b*d*g^3)*A*B + (a*b^2*c*g^3*\log(e) - a^2*b*d*g^3*\log(e))*B^2)*x + 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*\log(b*x + a) - 2*((b^3*c*g^3 - a*b^2*d*g^3)*B^2*x^2 + 2*(a*b^2*c*g^3 - a^2*b*d*g^3)*B^2*x + (a^2*b*c*g^3 - a^3*d*g^3)*B^2)*\log(d*x + c) - \text{integrate}(1/2*(b*d*x + 2*b*c - a*d)/((b^4*c*g^3 - a*b^3*d*g^3)*A*B + (b^4*c*g^3*\log(e) - a*b^3*d*g^3*\log(e))*B^2)*x^3 + (a^3*b*c*g^3 - a^4*d*g^3)*A*B + (a^3*b*c*g^3*\log(e) - a^4*d*g^3*\log(e))*B^2 + 3*((a*b^3*c*g^3 - a^2*b^2*d*g^3)*A*B + (a*b^3*c*g^3*\log(e) - a^2*b^2*d*g^3*\log(e))*B^2)*x^2 + 3*((a^2*b^2*c*g^3 - a^3*b*d*g^3)*A*B + (a^2*b^2*c*g^3*\log(e) - a^3*b*d*g^3*\log(e))*B^2)*x + 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*\log(b*x + a) - 2*((b^4*c*g^3 - a*b^3*d*g^3)*B^2*x^3 + 3*(a*b^3*c*g^3 - a^2*b^2*d*g^3)*B^2*x^2 + 3*(a^2*b^2*c*g^3 - a^3*b*d*g^3)*B^2*x + (a^3*b*c*g^3 - a^4*d*g^3)*B^2)*\log(d*x + c)), x$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2 b^3 g^3 x^3 + 3 A^2 a b^2 g^3 x^2 + 3 A^2 a^2 b g^3 x + A^2 a^3 g^3 + (B^2 b^3 g^3 x^3 + 3 B^2 a b^2 g^3 x^2 + 3 B^2 a^2 b g^3 x + B^2 a^3 g^3) \log \left( \frac{b^2 e x}{d^2 x} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(1/(A^2\*b^3\*g^3\*x^3 + 3\*A^2\*a\*b^2\*g^3\*x^2 + 3\*A^2\*a^2\*b\*g^3\*x + A^2\*a^3\*g^3 + (B^2\*b^3\*g^3\*x^3 + 3\*B^2\*a\*b^2\*g^3\*x^2 + 3\*B^2\*a^2\*b\*g^3\*x + B^2

```
*a^3*g^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2
+ 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^
3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="
giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

### 3.147 $\int (a+bx)^4 (A + B \log(e(a+bx)^n(c+dx)^{-n})) dx$

**Optimal.** Leaf size=171

$$\frac{(a+bx)^5 (B \log(e(a+bx)^n(c+dx)^{-n}) + A)}{5b} + \frac{Bnx(bc-ad)^4}{5d^4} - \frac{Bn(a+bx)^2(bc-ad)^3}{10bd^3} + \frac{Bn(a+bx)^3(bc-ad)^2}{15bd^2} - \frac{Bn(bc-ad)}{5bd}$$

[Out]  $(B*(b*c - a*d)^{4*n*x})/(5*d^4) - (B*(b*c - a*d)^{3*n*(a + b*x)^2})/(10*b*d^3) + (B*(b*c - a*d)^{2*n*(a + b*x)^3})/(15*b*d^2) - (B*(b*c - a*d)*n*(a + b*x)^4)/(20*b*d) - (B*(b*c - a*d)^{5*n*\text{Log}[c + d*x]})/(5*b*d^5) + ((a + b*x)^5*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]))/(5*b)$

**Rubi [A]** time = 0.183445, antiderivative size = 183, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6742, 2492, 43}

$$\frac{A(a+bx)^5}{5b} + \frac{Bnx(bc-ad)^4}{5d^4} - \frac{Bn(a+bx)^2(bc-ad)^3}{10bd^3} + \frac{Bn(a+bx)^3(bc-ad)^2}{15bd^2} - \frac{Bn(bc-ad)^5 \log(c+dx)}{5bd^5} + \frac{B(a+bx)^5}{5bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x)^4*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]), x]$

[Out]  $(B*(b*c - a*d)^{4*n*x})/(5*d^4) - (B*(b*c - a*d)^{3*n*(a + b*x)^2})/(10*b*d^3) + (B*(b*c - a*d)^{2*n*(a + b*x)^3})/(15*b*d^2) - (B*(b*c - a*d)*n*(a + b*x)^4)/(20*b*d) + (A*(a + b*x)^5)/(5*b) - (B*(b*c - a*d)^{5*n*\text{Log}[c + d*x]})/(5*b*d^5) + (B*(a + b*x)^5*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(5*b)$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \text{ :> With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

#### Rule 2492

$\text{Int}[\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^{(p_.)*((c_.) + (d_.)*(x_.))^{(q_.)})^{(r_.)})^{(s_.)*((g_.) + (h_.)*(x_.))^{(m_.)}], x\_Symbol] \text{ :> Simp}[\text{((g + h*x)^{(m + 1)}*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)^s)/(h*(m + 1))}, x] - \text{Dist}[(p*r*s*(b*c - a*d))/(h*(m + 1)), \text{Int}[\text{((g + h*x)^{(m + 1)}*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r)^{(s - 1)})/((a + b*x)*(c + d*x))}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, m, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{NeQ}[m, -1]$

#### Rule 43

$\text{Int}[\text{((a_.) + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}}, x\_Symbol] \text{ :> Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (\text{!IntegerQ}[n] \text{ || } (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0]) \text{ || } \text{LtQ}[9*m + 5*(n + 1), 0] \text{ || } \text{GtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned}
\int (a+bx)^4 (A+B \log(e(a+bx)^n(c+dx)^{-n})) dx &= \int (A(a+bx)^4 + B(a+bx)^4 \log(e(a+bx)^n(c+dx)^{-n})) dx \\
&= \frac{A(a+bx)^5}{5b} + B \int (a+bx)^4 \log(e(a+bx)^n(c+dx)^{-n}) dx \\
&= \frac{A(a+bx)^5}{5b} + \frac{B(a+bx)^5 \log(e(a+bx)^n(c+dx)^{-n})}{5b} - \frac{(B(bc-ad))}{5b} \\
&= \frac{A(a+bx)^5}{5b} + \frac{B(a+bx)^5 \log(e(a+bx)^n(c+dx)^{-n})}{5b} - \frac{(B(bc-ad))}{5b} \\
&= \frac{B(bc-ad)^4 nx}{5d^4} - \frac{B(bc-ad)^3 n(a+bx)^2}{10bd^3} + \frac{B(bc-ad)^2 n(a+bx)^3}{15bd^2}
\end{aligned}$$

**Mathematica [B]** time = 0.784216, size = 364, normalized size = 2.13

$$bdx \left( 4a^2 b^2 d^2 (30Ad^2 x^2 + Bn(30c^2 - 15cdx + 4d^2 x^2)) + 12a^3 b d^3 (10Adx - 10Bcn + 3Bdnx) + 12a^4 d^4 (5A + 4Bn) + ab \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^4\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]),x]

[Out] (b\*d\*x\*(12\*a^4\*d^4\*(5\*A + 4\*B\*n) + 12\*a^3\*b\*d^3\*(-10\*B\*c\*n + 10\*A\*d\*x + 3\*B\*d\*n\*x) + 4\*a^2\*b^2\*d^2\*(30\*A\*d^2\*x^2 + B\*n\*(30\*c^2 - 15\*c\*d\*x + 4\*d^2\*x^2)) + b^4\*(12\*A\*d^4\*x^4 + B\*c\*n\*(12\*c^3 - 6\*c^2\*d\*x + 4\*c\*d^2\*x^2 - 3\*d^3\*x^3)) + a\*b^3\*d\*(60\*A\*d^3\*x^3 + B\*n\*(-60\*c^3 + 30\*c^2\*d\*x - 20\*c\*d^2\*x^2 + 3\*d^3\*x^3))) - 48\*a^5\*B\*d^5\*n\*Log[a + b\*x] - 12\*B\*(b^5\*c^5 - 5\*a\*b^4\*c^4\*d + 10\*a^2\*b^3\*c^3\*d^2 - 10\*a^3\*b^2\*c^2\*d^3 + 5\*a^4\*b\*c\*d^4 - 5\*a^5\*d^5)\*n\*Log[c + d\*x] + 12\*B\*d^5\*(5\*a^5 + 5\*a^4\*b\*x + 10\*a^3\*b^2\*x^2 + 10\*a^2\*b^3\*x^3 + 5\*a\*b^4\*x^4 + b^5\*x^5)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]/(60\*b\*d^5)

**Maple [C]** time = 0.833, size = 2374, normalized size = 13.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^4\*(A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n))),x)

[Out] 1/5\*B\*a^5\*n/b\*ln(-b\*x-a)+1/5\*b^4\*A\*x^5+1/5\*b^4\*B\*ln(e)\*x^5+1/5\*b^4\*B\*x^5\*ln((b\*x+a)^n)+B\*ln(e)\*a^4\*x-1/2\*I\*b^3\*B\*Pi\*a\*x^4\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3-I\*b^2\*B\*Pi\*a^2\*x^3\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-I\*b^2\*B\*Pi\*a^2\*x^3\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3-I\*b\*B\*Pi\*a^3\*x^2\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-I\*b\*B\*Pi\*a^3\*x^2\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3+1/2\*I\*B\*Pi\*a^4\*x\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+1/2\*I\*B\*Pi\*a^4\*x\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+1/2\*I\*B\*Pi\*a^4\*x\*csgn(I\*e\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+1/2\*I\*B\*Pi\*a^4\*x\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+B\*a^4\*x\*ln((b\*x+a)^n)-1/5\*(b\*x+a)^5\*B/b\*ln((d\*x+c)^n)-1/3\*b^3/d\*B\*a\*c\*n\*x^3-b^2/d\*B\*a^2\*c\*n\*x^2+1/2\*b^3/d^2\*B\*a\*c^2\*n\*x^2-2\*b/d\*B\*a^3\*c\*n\*x+b^3\*A\*a\*x^4+2\*b^2\*A\*a^2\*x^3+2\*b\*A\*a^3\*x^2+A\*a^4\*x+1/5/b\*B\*ln(d\*x+c)\*a^5+n+b^3\*B\*ln(e)\*a\*x^4+b^3\*B\*a\*x^4\*ln((b\*x+a)^n)+2\*b^2\*B\*ln(e)\*a^2\*x^3+2\*b^2\*B\*a^2\*x^3\*ln((b\*x+a)^n)+2\*b\*B\*ln(e)\*a^3\*x^2+2\*b\*B\*a^3\*x^2\*ln((b\*x+a)^n)+3/5\*b\*B\*a^3\*n\*x^2-1/10\*b^4/d^3\*B\*c^3\*n\*x^2+4/5\*B\*a^4\*n\*x+1/5\*b^4/d^4\*B\*c^4\*n\*x-1/d\*B\*ln(d\*x+c)\*a^4\*c\*n-1/5\*b^4/d^5\*B\*ln(d\*x+c)\*c^

$$\begin{aligned}
& 5^n - 1/10 * I * b^4 * B * \text{Pi} * x^5 * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n))^3 - 1/10 * I * b^4 * B * \text{Pi} * x^5 \\
& * \text{csgn}(I * e / ((d * x + c)^n) * (b * x + a)^n)^3 - 1/2 * I * B * \text{Pi} * a^4 * x * \text{csgn}(I * (b * x + a)^n / ((d * x + \\
& c)^n))^3 - 1/2 * I * B * \text{Pi} * a^4 * x * \text{csgn}(I * e / ((d * x + c)^n) * (b * x + a)^n)^3 + I * b * B * \text{Pi} * a^3 * x^2 * \text{csgn}(I \\
& * e) * \text{csgn}(I * (b * x + a)^n) * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n))^2 + I * b * B * \text{Pi} * a^3 * x^2 * \text{csgn}(I \\
& * e) * \text{csgn}(I * e / ((d * x + c)^n) * (b * x + a)^n)^2 + I * b * B * \text{Pi} * a^3 * x^2 * \text{csgn}(I * (b * x + a)^n / ((d \\
& * x + c)^n)) * \text{csgn}(I * e / ((d * x + c)^n) * (b * x + a)^n)^2 - 1/10 * I * b^4 * B * \text{Pi} * x^5 * \text{csgn}(I * (b * x \\
& + a)^n) * \text{csgn}(I / ((d * x + c)^n)) * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n)) - 1/10 * I * b^4 * B * \text{Pi} * x^5 \\
& * \text{csgn}(I * e) * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n)) * \text{csgn}(I * e / ((d * x + c)^n) * (b * x + a)^n) + 1 \\
& / 2 * I * b^3 * B * \text{Pi} * a * x^4 * \text{csgn}(I / ((d * x + c)^n)) * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n))^2 + 1/2 \\
& * I * b^3 * B * \text{Pi} * a * x^4 * \text{csgn}(I * (b * x + a)^n) * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n))^2 + 1/2 * I * b \\
& ^3 * B * \text{Pi} * a * x^4 * \text{csgn}(I * e) * \text{csgn}(I * e / ((d * x + c)^n) * (b * x + a)^n)^2 + 1/2 * I * b^3 * B * \text{Pi} * a * \\
& x^4 * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n)) * \text{csgn}(I * e / ((d * x + c)^n) * (b * x + a)^n)^2 - 1/2 * I * B \\
& * \text{Pi} * a^4 * x * \text{csgn}(I * (b * x + a)^n) * \text{csgn}(I / ((d * x + c)^n)) * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n \\
& )) - 1/2 * I * B * \text{Pi} * a^4 * x * \text{csgn}(I * e) * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n)) * \text{csgn}(I * e / ((d * x + \\
& c)^n) * (b * x + a)^n) + 1/20 * b^3 * B * a * n * x^4 - 1/20 * b^4 / d * B * c * n * x^4 + 4/15 * b^2 * B * a^2 * n * x \\
& ^3 + 1/15 * b^4 / d^2 * B * c^2 * n * x^3 + I * b^2 * B * \text{Pi} * a^2 * x^3 * \text{csgn}(I / ((d * x + c)^n)) * \text{csgn}(I * ( \\
& b * x + a)^n / ((d * x + c)^n))^2 + I * b^2 * B * \text{Pi} * a^2 * x^3 * \text{csgn}(I * (b * x + a)^n) * \text{csgn}(I * (b * x + a) \\
& ^n / ((d * x + c)^n))^2 + I * b^2 * B * \text{Pi} * a^2 * x^3 * \text{csgn}(I * e) * \text{csgn}(I * e / ((d * x + c)^n) * (b * x + a) \\
& ^n)^2 + I * b^2 * B * \text{Pi} * a^2 * x^3 * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n)) * \text{csgn}(I * e / ((d * x + c)^n) \\
& * (b * x + a)^n)^2 + I * b * B * \text{Pi} * a^3 * x^2 * \text{csgn}(I / ((d * x + c)^n)) * \text{csgn}(I * (b * x + a)^n / ((d * x + c \\
& )^n))^2 + 2 * b^2 / d^2 * B * a^2 * c^2 * n * x - b^3 / d^3 * B * a * c^3 * n * x + 2 * b / d^2 * B * \ln(d * x + c) * a^3 \\
& * c^2 * n - 2 * b^2 / d^3 * B * \ln(d * x + c) * a^2 * c^3 * n + b^3 / d^4 * B * \ln(d * x + c) * a * c^4 * n + 1/10 * I * b \\
& ^4 * B * \text{Pi} * x^5 * \text{csgn}(I / ((d * x + c)^n)) * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n))^2 + 1/10 * I * b^4 * \\
& B * \text{Pi} * x^5 * \text{csgn}(I * (b * x + a)^n) * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n))^2 + 1/10 * I * b^4 * B * \text{Pi} * \\
& x^5 * \text{csgn}(I * e) * \text{csgn}(I * e / ((d * x + c)^n) * (b * x + a)^n)^2 + 1/10 * I * b^4 * B * \text{Pi} * x^5 * \text{csgn}(I * \\
& (b * x + a)^n / ((d * x + c)^n)) * \text{csgn}(I * e / ((d * x + c)^n) * (b * x + a)^n)^2 - 1/2 * I * b^3 * B * \text{Pi} * a * x \\
& ^4 * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n))^3 - 1/2 * I * b^3 * B * \text{Pi} * a * x^4 * \text{csgn}(I * (b * x + a)^n) * \text{c} \\
& \text{sgn}(I / ((d * x + c)^n)) * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n)) - 1/2 * I * b^3 * B * \text{Pi} * a * x^4 * \text{csgn}( \\
& I * e) * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n)) * \text{csgn}(I * e / ((d * x + c)^n) * (b * x + a)^n) - I * b^2 * B * \\
& \text{Pi} * a^2 * x^3 * \text{csgn}(I * (b * x + a)^n) * \text{csgn}(I / ((d * x + c)^n)) * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n \\
& )) - I * b^2 * B * \text{Pi} * a^2 * x^3 * \text{csgn}(I * e) * \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n)) * \text{csgn}(I * e / ((d \\
& * x + c)^n) * (b * x + a)^n) - I * b * B * \text{Pi} * a^3 * x^2 * \text{csgn}(I * (b * x + a)^n) * \text{csgn}(I / ((d * x + c)^n)) * \\
& \text{csgn}(I * (b * x + a)^n / ((d * x + c)^n)) - I * b * B * \text{Pi} * a^3 * x^2 * \text{csgn}(I * e) * \text{csgn}(I * (b * x + a)^n / ( \\
& (d * x + c)^n)) * \text{csgn}(I * e / ((d * x + c)^n) * (b * x + a)^n)
\end{aligned}$$

**Maxima [B]** time = 1.29535, size = 906, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^4\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="maxima")

[Out]  $1/5 * B * b^4 * x^5 * \log((b * x + a)^n * e / (d * x + c)^n) + 1/5 * A * b^4 * x^5 + B * a * b^3 * x^4 * \log((b * x + a)^n * e / (d * x + c)^n) + A * a * b^3 * x^4 + 2 * B * a^2 * b^2 * x^3 * \log((b * x + a)^n * e / (d * x + c)^n) + 2 * A * a^2 * b^2 * x^3 + 2 * B * a^3 * b * x^2 * \log((b * x + a)^n * e / (d * x + c)^n) + 2 * A * a^3 * b * x^2 + B * a^4 * x * \log((b * x + a)^n * e / (d * x + c)^n) + A * a^4 * x + (a * e * n * \log(b * x + a) / b - c * e * n * \log(d * x + c) / d) * B * a^4 / e - 2 * (a^2 * e * n * \log(b * x + a) / b^2 - c^2 * e * n * \log(d * x + c) / d^2 + (b * c * e * n - a * d * e * n) * x / (b * d)) * B * a^3 * b / e + (2 * a^3 * e * n * \log(b * x + a) / b^3 - 2 * c^3 * e * n * \log(d * x + c) / d^3 - ((b^2 * c * d * e * n - a * b * d^2 * e * n) * x^2 - 2 * (b^2 * c^2 * e * n - a^2 * d^2 * e * n) * x) / (b^2 * d^2)) * B * a^2 * b^2 / e - 1/6 * (6 * a^4 * e * n * \log(b * x + a) / b^4 - 6 * c^4 * e * n * \log(d * x + c) / d^4 + (2 * (b^3 * c * d^2 * e * n - a * b^2 * d^3 * e * n) * x^3 - 3 * (b^3 * c^2 * d * e * n - a^2 * b * d^3 * e * n) * x^2 + 6 * (b^3 * c^3 * e * n - a^3 * d^3 * e * n) * x) / (b^3 * d^3)) * B * a * b^3 / e + 1/60 * (12 * a^5 * e * n * \log(b * x + a) / b^5 - 12 * c^5 * e * n * \log(d * x + c) / d^5 - (3 * (b^4 * c * d^3 * e * n - a * b^3 * d^4 * e * n) * x^4 - 4 * (b^4 * c^2 * d^2 * e * n - a^2 * b^2 * d^4 * e * n) * x^3 + 6 * (b^4 * c^3 * d * e * n$



$$n - a^3 b^4 d^4 e^n) x^2 - 12 (b^4 c^4 e^n - a^4 d^4 e^n) x) / (b^4 d^4) * B * b^4 / e$$

**Fricas [B]** time = 1.0928, size = 1188, normalized size = 6.95

$$12 A b^5 d^5 x^5 + 3 (20 A a b^4 d^5 - (B b^5 c d^4 - B a b^4 d^5) n) x^4 + 4 (30 A a^2 b^3 d^5 + (B b^5 c^2 d^3 - 5 B a b^4 c d^4 + 4 B a^2 b^3 d^5) n) x^3 + 6 ($$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^4\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="fricas")

[Out] 1/60\*(12\*A\*b^5\*d^5\*x^5 + 3\*(20\*A\*a\*b^4\*d^5 - (B\*b^5\*c\*d^4 - B\*a\*b^4\*d^5)\*n)\*x^4 + 4\*(30\*A\*a^2\*b^3\*d^5 + (B\*b^5\*c^2\*d^3 - 5\*B\*a\*b^4\*c\*d^4 + 4\*B\*a^2\*b^3\*d^5)\*n)\*x^3 + 6\*(20\*A\*a^3\*b^2\*d^5 - (B\*b^5\*c^3\*d^2 - 5\*B\*a\*b^4\*c^2\*d^3 + 10\*B\*a^2\*b^3\*c\*d^4 - 6\*B\*a^3\*b^2\*d^5)\*n)\*x^2 + 12\*(5\*A\*a^4\*b\*d^5 + (B\*b^5\*c^4\*d - 5\*B\*a\*b^4\*c^3\*d^2 + 10\*B\*a^2\*b^3\*c^2\*d^3 - 10\*B\*a^3\*b^2\*c\*d^4 + 4\*B\*a^4\*b\*d^5)\*n)\*x + 12\*(B\*b^5\*d^5\*n\*x^5 + 5\*B\*a\*b^4\*d^5\*n\*x^4 + 10\*B\*a^2\*b^3\*d^5\*n\*x^3 + 10\*B\*a^3\*b^2\*d^5\*n\*x^2 + 5\*B\*a^4\*b\*d^5\*n\*x + B\*a^5\*d^5\*n)\*log(b\*x + a) - 12\*(B\*b^5\*d^5\*n\*x^5 + 5\*B\*a\*b^4\*d^5\*n\*x^4 + 10\*B\*a^2\*b^3\*d^5\*n\*x^3 + 10\*B\*a^3\*b^2\*d^5\*n\*x^2 + 5\*B\*a^4\*b\*d^5\*n\*x + (B\*b^5\*c^5 - 5\*B\*a\*b^4\*c^4\*d + 10\*B\*a^2\*b^3\*c^3\*d^2 - 10\*B\*a^3\*b^2\*c^2\*d^3 + 5\*B\*a^4\*b\*c\*d^4)\*n)\*log(d\*x + c) + 12\*(B\*b^5\*d^5\*x^5 + 5\*B\*a\*b^4\*d^5\*x^4 + 10\*B\*a^2\*b^3\*d^5\*x^3 + 10\*B\*a^3\*b^2\*d^5\*x^2 + 5\*B\*a^4\*b\*d^5\*x)\*log(e))/(b\*d^5)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*4\*(A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n))),x)

[Out] Timed out

**Giac [B]** time = 13.9964, size = 671, normalized size = 3.92

$$\frac{B a^5 n \log(b x + a)}{5 b} + \frac{1}{5} (A b^4 + B b^4) x^5 - \frac{(B b^4 c n - B a b^3 d n - 20 A a b^3 d - 20 B a b^3 d) x^4}{20 d} + \frac{(B b^4 c^2 n - 5 B a b^3 c d n + 4 B a^2 b^3 c d n)}{20 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^4\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="giac")

[Out] 1/5\*B\*a^5\*n\*log(b\*x + a)/b + 1/5\*(A\*b^4 + B\*b^4)\*x^5 - 1/20\*(B\*b^4\*c\*n - B\*a\*b^3\*d\*n - 20\*A\*a\*b^3\*d - 20\*B\*a\*b^3\*d)\*x^4/d + 1/15\*(B\*b^4\*c^2\*n - 5\*B\*a\*b^3\*c\*d\*n + 4\*B\*a^2\*b^2\*d^2\*n + 30\*A\*a^2\*b^2\*d^2 + 30\*B\*a^2\*b^2\*d^2)\*x^3/d^2 + 1/5\*(B\*b^4\*n\*x^5 + 5\*B\*a\*b^3\*n\*x^4 + 10\*B\*a^2\*b^2\*n\*x^3 + 10\*B\*a^3\*b\*n\*x^2 + 5\*B\*a^4\*n\*x)\*log(b\*x + a) - 1/5\*(B\*b^4\*n\*x^5 + 5\*B\*a\*b^3\*n\*x^4 + 10\*B\*a^2\*b^2\*n\*x^3 + 10\*B\*a^3\*b\*n\*x^2 + 5\*B\*a^4\*n\*x)\*log(d\*x + c) - 1/10\*(B\*b^4\*c^3\*n - 5\*B\*a\*b^3\*c^2\*d\*n + 10\*B\*a^2\*b^2\*c\*d^2\*n - 6\*B\*a^3\*b\*d^3\*n - 20\*A\*

$$\begin{aligned}
& a^3 b d^3 - 20 B a^3 b d^3) x^2 / d^3 + 1/5 (B b^4 c^4 n - 5 B a b^3 c^3 d n \\
& + 10 B a^2 b^2 c^2 d^2 n - 10 B a^3 b c d^3 n + 4 B a^4 d^4 n + 5 A a^4 d^4 \\
& + 5 B a^4 d^4) x / d^4 - 1/5 (B b^4 c^5 n - 5 B a b^3 c^4 d n + 10 B a^2 b^2 \\
& c^3 d^2 n - 10 B a^3 b c^2 d^3 n + 5 B a^4 c d^4 n) \log(-d x - c) / d^5
\end{aligned}$$

### 3.148 $\int (a+bx)^3 (A + B \log (e(a + bx)^n (c + dx)^{-n})) dx$

**Optimal.** Leaf size=142

$$\frac{(a + bx)^4 (B \log (e(a + bx)^n (c + dx)^{-n}) + A)}{4b} - \frac{Bnx(bc - ad)^3}{4d^3} + \frac{Bn(a + bx)^2(bc - ad)^2}{8bd^2} + \frac{Bn(bc - ad)^4 \log(c + dx)}{4bd^4} - \dots$$

```
[Out] -(B*(b*c - a*d)^3*n*x)/(4*d^3) + (B*(b*c - a*d)^2*n*(a + b*x)^2)/(8*b*d^2)
- (B*(b*c - a*d)*n*(a + b*x)^3)/(12*b*d) + (B*(b*c - a*d)^4*n*Log[c + d*x])
/(4*b*d^4) + ((a + b*x)^4*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(4*b)
```

**Rubi [A]** time = 0.135311, antiderivative size = 154, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6742, 2492, 43}

$$\frac{A(a + bx)^4}{4b} - \frac{Bnx(bc - ad)^3}{4d^3} + \frac{Bn(a + bx)^2(bc - ad)^2}{8bd^2} + \frac{Bn(bc - ad)^4 \log(c + dx)}{4bd^4} + \frac{B(a + bx)^4 \log(e(a + bx)^n (c + dx)^{-n})}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]),x]
```

```
[Out] -(B*(b*c - a*d)^3*n*x)/(4*d^3) + (B*(b*c - a*d)^2*n*(a + b*x)^2)/(8*b*d^2)
- (B*(b*c - a*d)*n*(a + b*x)^3)/(12*b*d) + (A*(a + b*x)^4)/(4*b) + (B*(b*c
- a*d)^4*n*Log[c + d*x])/(4*b*d^4) + (B*(a + b*x)^4*Log[(e*(a + b*x)^n)/(c
+ d*x)^n])/(4*b)
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q]^r]^(s - 1))/(a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
0] && NeQ[m, -1]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
\int (a + bx)^3 (A + B \log(e(a + bx)^n(c + dx)^{-n})) dx &= \int (A(a + bx)^3 + B(a + bx)^3 \log(e(a + bx)^n(c + dx)^{-n})) dx \\
&= \frac{A(a + bx)^4}{4b} + B \int (a + bx)^3 \log(e(a + bx)^n(c + dx)^{-n}) dx \\
&= \frac{A(a + bx)^4}{4b} + \frac{B(a + bx)^4 \log(e(a + bx)^n(c + dx)^{-n})}{4b} - \frac{(B(bc - ad)n)}{4b} \\
&= \frac{A(a + bx)^4}{4b} + \frac{B(a + bx)^4 \log(e(a + bx)^n(c + dx)^{-n})}{4b} - \frac{(B(bc - ad)n)}{4b} \\
&= -\frac{B(bc - ad)^3 nx}{4d^3} + \frac{B(bc - ad)^2 n(a + bx)^2}{8bd^2} - \frac{B(bc - ad)n(a + bx)^3}{12bd} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.472615, size = 273, normalized size = 1.92

$$bdx(9a^2bd^2(4Adx - 4Bcn + Bdnx) + 6a^3d^3(4A + 3Bn) + 2ab^2d(12Ad^2x^2 + Bn(12c^2 - 6cdx + d^2x^2))) + b^3(6Ad^3x^3 + B$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]), x]

[Out] (b\*d\*x\*(6\*a^3\*d^3\*(4\*A + 3\*B\*n) + 9\*a^2\*b\*d^2\*(-4\*B\*c\*n + 4\*A\*d\*x + B\*d\*n\*x) + b^3\*(6\*A\*d^3\*x^3 + B\*c\*n\*(-6\*c^2 + 3\*c\*d\*x - 2\*d^2\*x^2)) + 2\*a\*b^2\*d\*(1\*2\*A\*d^2\*x^2 + B\*n\*(12\*c^2 - 6\*c\*d\*x + d^2\*x^2))) - 18\*a^4\*B\*d^4\*n\*Log[a + b\*x] + 6\*B\*(b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + 4\*a^4\*d^4)\*n\*Log[c + d\*x] + 6\*B\*d^4\*(4\*a^4 + 4\*a^3\*b\*x + 6\*a^2\*b^2\*x^2 + 4\*a\*b^3\*x^3 + b^4\*x^4)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]/(24\*b\*d^4)

**Maple [C]** time = 0.564, size = 1840, normalized size = 13.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^3\*(A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n))), x)

[Out] 3/2\*b/d^2\*B\*ln(d\*x+c)\*a^2\*c^2\*n-b^2/d^3\*B\*ln(d\*x+c)\*a\*c^3\*n+1/8\*I\*b^3\*B\*Pi\*x^4\*csgn(I\*e)\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+1/8\*I\*b^3\*B\*Pi\*x^4\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+1/8\*I\*b^3\*B\*Pi\*x^4\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+1/8\*I\*b^3\*B\*Pi\*x^4\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+1/4\*B\*a^4\*n/b\*ln(-b\*x-a)+B\*a^3\*x\*ln((b\*x+a)^n)-1/4\*(b\*x+a)^4\*B/b\*ln((d\*x+c)^n)+1/4\*b^3\*A\*x^4+1/4\*b^3\*B\*x^4\*ln((b\*x+a)^n)+1/4\*b^3\*B\*ln(e)\*x^4+B\*ln(e)\*a^3\*x-1/2\*I\*b^2\*B\*Pi\*a\*x^3\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-1/2\*I\*b^2\*B\*Pi\*a\*x^3\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3-3/4\*I\*b\*B\*Pi\*a^2\*x^2\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-3/4\*I\*b\*B\*Pi\*a^2\*x^2\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3+1/2\*I\*B\*Pi\*a^3\*x\*csgn(I\*e)\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+b^2\*A\*a\*x^3+3/2\*b\*A\*a^2\*x^2+A\*a^3\*x+b^2\*B\*a\*x^3\*ln((b\*x+a)^n)+b^2\*B\*ln(e)\*a\*x^3+3/2\*b\*B\*a^2\*x^2\*ln((b\*x+a)^n)+3/2\*b\*B\*ln(e)\*a^2\*x^2+1/4/b\*B\*ln(d\*x+c)\*a^4\*n+1/12\*b^2\*B\*a\*n\*x^3-1/12\*b^3/d\*B\*c\*n\*x^3-1/2\*I\*B\*Pi\*a^3\*x\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3-1/8\*I\*b^3\*B\*Pi\*x^4\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-1/8\*I\*b^3\*B\*Pi\*x^4\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3-1/8\*I\*b^3\*B\*Pi\*x^4\*csgn(I\*e)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)-1/8\*I\*b^3\*B\*Pi\*x^4\*csgn(I\*(b\*x+a)^n)\*csgn(I/((d\*x+c)^n))

```

)*csgn(I*(b*x+a)^n/((d*x+c)^n))+1/2*I*b^2*B*Pi*a*x^3*csgn(I*e)*csgn(I*e/((d
*x+c)^n)*(b*x+a)^n)^2+1/2*I*b^2*B*Pi*a*x^3*csgn(I/((d*x+c)^n))*csgn(I*(b*x+
a)^n/((d*x+c)^n))^2+1/2*I*b^2*B*Pi*a*x^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n
/((d*x+c)^n))^2+1/2*I*b^2*B*Pi*a*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e
/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*b^2*B*Pi*a*x^3*csgn(I*e)*csgn(I*(b*x+a)^n/(
(d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*b^2*B*Pi*a*x^3*csgn(I*(b*
x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-3/4*I*b*B*Pi*a^2*
x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)
-3/4*I*b*B*Pi*a^2*x^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^
n/((d*x+c)^n))-1/2*b^2/d*B*a*c*n*x^2+3/4*I*b*B*Pi*a^2*x^2*csgn(I*e)*csgn(I*
e/((d*x+c)^n)*(b*x+a)^n)^2+3/4*I*b*B*Pi*a^2*x^2*csgn(I/((d*x+c)^n))*csgn(I*
(b*x+a)^n/((d*x+c)^n))^2+3/4*I*b*B*Pi*a^2*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x
+a)^n/((d*x+c)^n))^2+3/4*I*b*B*Pi*a^2*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csg
n(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*B*Pi*a^3*x*csgn(I*e)*csgn(I*(b*x+a)^n/
((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*B*Pi*a^3*x*csgn(I*(b*x+a
)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-3/2*b/d*B*a^2*c*n*x+
b^2/d^2*B*a*c^2*n*x+3/8*b*B*a^2*n*x^2+1/8*b^3/d^2*B*c^2*n*x^2+3/4*B*a^3*n*x
-1/4*b^3/d^3*B*c^3*n*x-1/d*B*ln(d*x+c)*a^3*c*n+1/4*b^3/d^4*B*ln(d*x+c)*c^4*
n-1/2*I*B*Pi*a^3*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+1/2*I*B*Pi*a^3*x*csgn(I/
((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*a^3*x*csgn(I*(b*x+a
)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*a^3*x*csgn(I*(b*x+a)^n/((d*
x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2

```

---

**Maxima [B]** time = 1.24692, size = 630, normalized size = 4.44

$$\frac{1}{4} B b^3 x^4 \log\left(\frac{(b x + a)^n e}{(d x + c)^n}\right) + \frac{1}{4} A b^3 x^4 + B a b^2 x^3 \log\left(\frac{(b x + a)^n e}{(d x + c)^n}\right) + A a b^2 x^3 + \frac{3}{2} B a^2 b x^2 \log\left(\frac{(b x + a)^n e}{(d x + c)^n}\right) + \frac{3}{2} A a^2 b x^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima
")

```

```

[Out] 1/4*B*b^3*x^4*log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A*b^3*x^4 + B*a*b^2*x^3*
log((b*x + a)^n*e/(d*x + c)^n) + A*a*b^2*x^3 + 3/2*B*a^2*b*x^2*log((b*x + a
)^n*e/(d*x + c)^n) + 3/2*A*a^2*b*x^2 + B*a^3*x*log((b*x + a)^n*e/(d*x + c)^
n) + A*a^3*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^3/e - 3/2*
(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*
x/(b*d))*B*a^2*b/e + 1/2*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x +
c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x
)/(b^2*d^2))*B*a*b^2/e - 1/24*(6*a^4*e*n*log(b*x + a)/b^4 - 6*c^4*e*n*log(d
*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a
^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*b^3/e

```

---

**Fricas [B]** time = 1.06623, size = 871, normalized size = 6.13

$$\frac{6 A b^4 d^4 x^4 + 2 (12 A a b^3 d^4 - (B b^4 c d^3 - B a b^3 d^4) n) x^3 + 3 (12 A a^2 b^2 d^4 + (B b^4 c^2 d^2 - 4 B a b^3 c d^3 + 3 B a^2 b^2 d^4) n) x^2 + 6 (4$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas
")

```

```
[Out] 1/24*(6*A*b^4*d^4*x^4 + 2*(12*A*a*b^3*d^4 - (B*b^4*c*d^3 - B*a*b^3*d^4)*n)*
x^3 + 3*(12*A*a^2*b^2*d^4 + (B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + 3*B*a^2*b^2*
d^4)*n)*x^2 + 6*(4*A*a^3*b*d^4 - (B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*
b^2*c*d^3 - 3*B*a^3*b*d^4)*n)*x + 6*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3
+ 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x + B*a^4*d^4*n)*log(b*x + a) -
6*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*
b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*
c*d^3)*n)*log(d*x + c) + 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2
*d^4*x^2 + 4*B*a^3*b*d^4*x)*log(e))/(b*d^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Timed out
```

**Giac [B]** time = 5.42466, size = 479, normalized size = 3.37

$$\frac{Ba^4n \log(bx + a)}{4b} + \frac{1}{4}(Ab^3 + Bb^3)x^4 - \frac{(Bb^3cn - Bab^2dn - 12Aab^2d - 12Bab^2d)x^3}{12d} + \frac{1}{4}(Bb^3nx^4 + 4Bab^2nx^3 + 6Ba^2bnx^2 + 4Aa^3bx^2 + 4Aa^2b^2x + 4Aa^3b)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] 1/4*B*a^4*n*log(b*x + a)/b + 1/4*(A*b^3 + B*b^3)*x^4 - 1/12*(B*b^3*c*n - B*
a*b^2*d*n - 12*A*a*b^2*d - 12*B*a*b^2*d)*x^3/d + 1/4*(B*b^3*n*x^4 + 4*B*a*b
^2*n*x^3 + 6*B*a^2*b*n*x^2 + 4*B*a^3*n*x)*log(b*x + a) - 1/4*(B*b^3*n*x^4 +
4*B*a*b^2*n*x^3 + 6*B*a^2*b*n*x^2 + 4*B*a^3*n*x)*log(d*x + c) + 1/8*(B*b^3
*c^2*n - 4*B*a*b^2*c*d*n + 3*B*a^2*b*d^2*n + 12*A*a^2*b*d^2 + 12*B*a^2*b*d^
2)*x^2/d^2 - 1/4*(B*b^3*c^3*n - 4*B*a*b^2*c^2*d*n + 6*B*a^2*b*c*d^2*n - 3*B
*a^3*d^3*n - 4*A*a^3*d^3 - 4*B*a^3*d^3)*x/d^3 + 1/4*(B*b^3*c^4*n - 4*B*a*b^
2*c^3*d*n + 6*B*a^2*b*c^2*d^2*n - 4*B*a^3*c*d^3*n)*log(d*x + c)/d^4
```

### 3.149 $\int (a+bx)^2 (A + B \log (e(a + bx)^n (c + dx)^{-n})) dx$

**Optimal.** Leaf size=113

$$\frac{(a + bx)^3 (B \log (e(a + bx)^n (c + dx)^{-n}) + A)}{3b} + \frac{Bnx(bc - ad)^2}{3d^2} - \frac{Bn(bc - ad)^3 \log(c + dx)}{3bd^3} - \frac{Bn(a + bx)^2 (bc - ad)}{6bd}$$

[Out] (B\*(b\*c - a\*d)^2\*n\*x)/(3\*d^2) - (B\*(b\*c - a\*d)\*n\*(a + b\*x)^2)/(6\*b\*d) - (B\*(b\*c - a\*d)^3\*n\*Log[c + d\*x])/(3\*b\*d^3) + ((a + b\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^n]/(c + d\*x)^n]))/(3\*b)

**Rubi [A]** time = 0.120999, antiderivative size = 125, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6742, 2492, 43}

$$\frac{A(a + bx)^3}{3b} + \frac{Bnx(bc - ad)^2}{3d^2} - \frac{Bn(bc - ad)^3 \log(c + dx)}{3bd^3} + \frac{B(a + bx)^3 \log (e(a + bx)^n (c + dx)^{-n})}{3b} - \frac{Bn(a + bx)^2 (bc - ad)}{6bd}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^n]/(c + d\*x)^n]),x]

[Out] (B\*(b\*c - a\*d)^2\*n\*x)/(3\*d^2) - (B\*(b\*c - a\*d)\*n\*(a + b\*x)^2)/(6\*b\*d) + (A\*(a + b\*x)^3)/(3\*b) - (B\*(b\*c - a\*d)^3\*n\*Log[c + d\*x])/(3\*b\*d^3) + (B\*(a + b\*x)^3\*Log[(e\*(a + b\*x)^n]/(c + d\*x)^n]))/(3\*b)

#### Rule 6742

Int[u\_, x\_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2492

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*((g\_.) + (h\_.)\*(x\_))^(m\_.), x\_Symbol] :=> Simp[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(h\*(m + 1)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(h\*(m + 1)), Int[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0] && NeQ[m, -1]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :=> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int (a+bx)^2 (A+B \log(e(a+bx)^n(c+dx)^{-n})) dx &= \int (A(a+bx)^2 + B(a+bx)^2 \log(e(a+bx)^n(c+dx)^{-n})) dx \\
&= \frac{A(a+bx)^3}{3b} + B \int (a+bx)^2 \log(e(a+bx)^n(c+dx)^{-n}) dx \\
&= \frac{A(a+bx)^3}{3b} + \frac{B(a+bx)^3 \log(e(a+bx)^n(c+dx)^{-n})}{3b} - \frac{(B(bc-ad)n)}{3b} \\
&= \frac{A(a+bx)^3}{3b} + \frac{B(a+bx)^3 \log(e(a+bx)^n(c+dx)^{-n})}{3b} - \frac{(B(bc-ad)n)}{3b} \\
&= \frac{B(bc-ad)^2 nx}{3d^2} - \frac{B(bc-ad)n(a+bx)^2}{6bd} + \frac{A(a+bx)^3}{3b} - \frac{B(bc-ad)^3 n}{3bd}
\end{aligned}$$

**Mathematica [A]** time = 0.281757, size = 194, normalized size = 1.72

$$\frac{bdx(2a^2d^2(3A+2Bn) + abd(6Adx - 6Bcn + Bdnx) + b^2(2Ad^2x^2 + Bcn(2c-dx))) - 2Bn(3a^2bcd^2 - 3a^3d^3 - 3ab^2c^2d + 6bd^3)}{6bd^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]), x]

[Out] (b\*d\*x\*(2\*a^2\*d^2\*(3\*A + 2\*B\*n) + a\*b\*d\*(-6\*B\*c\*n + 6\*A\*d\*x + B\*d\*n\*x) + b^2\*(2\*A\*d^2\*x^2 + B\*c\*n\*(2\*c - d\*x))) - 4\*a^3\*B\*d^3\*n\*Log[a + b\*x] - 2\*B\*(b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - 3\*a^3\*d^3)\*n\*Log[c + d\*x] + 2\*B\*d^3\*(3\*a^3 + 3\*a^2\*b\*x + 3\*a\*b^2\*x^2 + b^3\*x^3)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(6\*b\*d^3)

**Maple [C]** time = 0.527, size = 1325, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^2\*(A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n))), x)

[Out] -1/3\*(b\*x+a)^3\*B/b\*ln((d\*x+c)^n)+B\*a^2\*x\*ln((b\*x+a)^n)+1/3\*B\*a^3\*n/b\*ln(-b\*x-a)+1/3\*b^2\*A\*x^3+1/3\*b^2\*B\*ln(e)\*x^3+1/3\*b^2\*B\*x^3\*ln((b\*x+a)^n)+B\*ln(e)\*a^2\*x+b\*A\*a\*x^2+A\*a^2\*x+b\*B\*ln(e)\*a\*x^2+b\*B\*a\*x^2\*ln((b\*x+a)^n)+1/3/b\*B\*ln(d\*x+c)\*a^3\*n+1/6\*I\*b^2\*B\*Pi\*x^3\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+1/6\*I\*b^2\*B\*Pi\*x^3\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n))\*(b\*x+a)^n)^2+1/6\*I\*b^2\*B\*Pi\*x^3\*csgn(I\*e)\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+1/2\*I\*B\*Pi\*a^2\*x\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+1/2\*I\*B\*Pi\*a^2\*x\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+1/2\*I\*B\*Pi\*a^2\*x\*csgn(I\*e)\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2-1/2\*I\*b\*B\*Pi\*a\*x^2\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-1/2\*I\*b\*B\*Pi\*a\*x^2\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3+1/6\*b\*B\*a\*n\*x^2-1/6\*b^2/d\*B\*c\*n\*x^2+2/3\*B\*a^2\*n\*x+1/3\*b^2/d^2\*B\*c^2\*n\*x-1/3\*b^2/d^3\*B\*ln(d\*x+c)\*c^3\*n-1/d\*B\*ln(d\*x+c)\*a^2\*c\*n-1/6\*I\*b^2\*B\*Pi\*x^3\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-1/6\*I\*b^2\*B\*Pi\*x^3\*csgn(I\*e/((d\*x+c)^n))\*(b\*x+a)^n)^3-1/2\*I\*B\*Pi\*a^2\*x\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-1/2\*I\*B\*Pi\*a^2\*x\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3-1/2\*I\*b\*B\*Pi\*a\*x^2\*csgn(I\*e)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)-1/2\*I\*b\*B\*Pi\*a\*x^2\*csgn(I\*(b\*x+a)^n)\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))-b/d\*B\*a



```
*c*n*x+b/d^2*B*ln(d*x+c)*a*c^2*n+1/6*I*b^2*B*Pi*x^3*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*b*B*Pi*a*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*b*B*Pi*a*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*b*B*Pi*a*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*B*Pi*a^2*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*B*Pi*a^2*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/6*I*b^2*B*Pi*x^3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/6*I*b^2*B*Pi*x^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+1/2*I*b*B*Pi*a*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2
```

**Maxima [B]** time = 1.28175, size = 397, normalized size = 3.51

$$\frac{1}{3} B b^2 x^3 \log\left(\frac{(b x + a)^n e}{(d x + c)^n}\right) + \frac{1}{3} A b^2 x^3 + B a b x^2 \log\left(\frac{(b x + a)^n e}{(d x + c)^n}\right) + A a b x^2 + B a^2 x \log\left(\frac{(b x + a)^n e}{(d x + c)^n}\right) + A a^2 x + \frac{\left(\frac{a e n \log(b x + a)}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima")
```

```
[Out] 1/3*B*b^2*x^3*log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A*b^2*x^3 + B*a*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + A*a*b*x^2 + B*a^2*x*log((b*x + a)^n*e/(d*x + c)^n) + A*a^2*x + (a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*B*a^2/e - (a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*B*a*b/e + 1/6*(2*a^3*e*n*log(b*x + a)/b^3 - 2*c^3*e*n*log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*B*b^2/e
```

**Fricas [B]** time = 1.04023, size = 599, normalized size = 5.3

$$\frac{2 A b^3 d^3 x^3 + (6 A a b^2 d^3 - (B b^3 c d^2 - B a b^2 d^3) n) x^2 + 2 (3 A a^2 b d^3 + (B b^3 c^2 d - 3 B a b^2 c d^2 + 2 B a^2 b d^3) n) x + 2 (B b^3 d^3 n x^3 + 3 B a b^2 d^3 n x^2 + 3 B a^2 b d^3 n x + B a^3 d^3 n) \log(b x + a) - 2 (B b^3 d^3 n x^3 + 3 B a b^2 d^3 n x^2 + 3 B a^2 b d^3 n x + (B b^3 c^2 - 3 B a b^2 c^2 d + 3 B a^2 b c d^2) n) \log(d x + c) + 2 (B b^3 d^3 x^3 + 3 B a b^2 d^3 x^2 + 3 B a^2 b d^3 x) \log(e)}{(b d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*x^3 + (6*A*a*b^2*d^3 - (B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2 + 2*(3*A*a^2*b*d^3 + (B*b^3*c^2*d - 3*B*a*b^2*c*d^2 + 2*B*a^2*b*d^3)*n)*x + 2*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + B*a^3*d^3*n)*log(b*x + a) - 2*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + (B*b^3*c^2 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(d*x + c) + 2*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x)*log(e))/(b*d^3)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*(A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n))),x)

[Out] Timed out

**Giac [B]** time = 2.29535, size = 317, normalized size = 2.81

$$\frac{Ba^3n \log(bx + a)}{3b} + \frac{1}{3}(Ab^2 + Bb^2)x^3 - \frac{(Bb^2cn - Babdn - 6Aabd - 6Babd)x^2}{6d} + \frac{1}{3}(Bb^2nx^3 + 3Babnx^2 + 3Ba^2nx) \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="giac")

[Out]  $\frac{1}{3}B^3a^3n \log(bx + a)/b + \frac{1}{3}(A^2b^2 + B^2b^2)x^3 - \frac{1}{6}(B^2b^2c^2n - B^2a^2b^2d^2n - 6A^2a^2b^2d - 6B^2a^2b^2d)x^2/d + \frac{1}{3}(B^2b^2n^2x^3 + 3B^2a^2b^2n^2x^2 + 3B^2a^2n^2x) \log(bx + a) - \frac{1}{3}(B^2b^2n^2x^3 + 3B^2a^2b^2n^2x^2 + 3B^2a^2n^2x) \log(dx + c) + \frac{1}{3}(B^2b^2c^2n^2 - 3B^2a^2b^2c^2d^2n + 2B^2a^2d^2n + 3A^2a^2d^2 + 3B^2a^2d^2)x/d^2 - \frac{1}{3}(B^2b^2c^3n - 3B^2a^2b^2c^2d^2n + 3B^2a^2c^2d^2n) \log(-dx - c)/d^3$

### 3.150 $\int (a + bx) (A + B \log (e(a + bx)^n (c + dx)^{-n})) dx$

**Optimal.** Leaf size=84

$$\frac{(a + bx)^2 (B \log (e(a + bx)^n (c + dx)^{-n}) + A)}{2b} + \frac{Bn(bc - ad)^2 \log(c + dx)}{2bd^2} - \frac{Bnx(bc - ad)}{2d}$$

[Out]  $-(B*(b*c - a*d)*n*x)/(2*d) + (B*(b*c - a*d)^2*n*\text{Log}[c + d*x])/(2*b*d^2) + (a + b*x)^2*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b)$

**Rubi [A]** time = 0.0908031, antiderivative size = 96, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {6742, 2492, 43}

$$\frac{A(a + bx)^2}{2b} + \frac{Bn(bc - ad)^2 \log(c + dx)}{2bd^2} + \frac{B(a + bx)^2 \log(e(a + bx)^n (c + dx)^{-n})}{2b} - \frac{Bnx(bc - ad)}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x)*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]), x]$

[Out]  $-(B*(b*c - a*d)*n*x)/(2*d) + (A*(a + b*x)^2)/(2*b) + (B*(b*c - a*d)^2*n*\text{Log}[c + d*x])/(2*b*d^2) + (B*(a + b*x)^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b)$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \text{ :> With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] \text{ /; SumQ}[v]]$

#### Rule 2492

$\text{Int}[\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.))*((c_.) + (d_.)*(x_.))^(q_.))^(r_.)]^(s_.)*((g_.) + (h_.)*(x_.))^(m_.), x\_Symbol] \text{ :> Simp}[(g + h*x)^(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - \text{Dist}[(p*r*s*(b*c - a*d))/(h*(m + 1)), \text{Int}[(g + h*x)^(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(a + b*x)*(c + d*x), x], x] \text{ /; FreeQ}\{a, b, c, d, e, f, g, h, m, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{NeQ}[m, -1]$

#### Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x\_Symbol] \text{ :> Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] \text{ /; FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{LeQ}[7*m + 4*n + 4, 0])) || \text{LtQ}[9*m + 5*(n + 1), 0] || \text{GtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned}
\int (a + bx)(A + B \log(e(a + bx)^n(c + dx)^{-n})) dx &= \int (A(a + bx) + B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})) dx \\
&= \frac{A(a + bx)^2}{2b} + B \int (a + bx) \log(e(a + bx)^n(c + dx)^{-n}) dx \\
&= \frac{A(a + bx)^2}{2b} + \frac{B(a + bx)^2 \log(e(a + bx)^n(c + dx)^{-n})}{2b} - \frac{(B(bc - ad)n)}{2b} \int \frac{1}{c + dx} dx \\
&= \frac{A(a + bx)^2}{2b} + \frac{B(a + bx)^2 \log(e(a + bx)^n(c + dx)^{-n})}{2b} - \frac{(B(bc - ad)n)}{2b} \log(c + dx) \\
&= -\frac{B(bc - ad)nx}{2d} + \frac{A(a + bx)^2}{2b} + \frac{B(bc - ad)^2n \log(c + dx)}{2bd^2} + \frac{B(a + bx)^2}{2b}
\end{aligned}$$

**Mathematica [A]** time = 0.146117, size = 126, normalized size = 1.5

$$\frac{d \left( Bd \left( 2a^2 + 2abx + b^2x^2 \right) \log \left( e(a + bx)^n(c + dx)^{-n} \right) + bx(2aAd + aBdn + Abdx - bBcn) \right) + Bn \left( 2a^2d^2 - 2abcd + b^2c^2 \right) \log(c + dx)}{2bd^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]),x]

[Out]  $(-a^2Bd^2n \operatorname{Log}[a + b*x]) + B(b^2c^2 - 2a*b*c*d + 2a^2d^2)n \operatorname{Log}[c + d*x] + d(b*x*(2a*A*d - b*B*c*n + a*B*d*n + A*b*d*x) + B*d*(2a^2 + 2a*b*x + b^2*x^2) \operatorname{Log}[(e*(a + b*x)^n)/(c + d*x)^n]) / (2*b*d^2)$

**Maple [C]** time = 0.51, size = 817, normalized size = 9.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*(A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n))),x)

[Out]  $\frac{1}{2}A*b*x^2 + A*a*x + \frac{1}{2}B*a^2*n/b \ln(-b*x-a) + \frac{1}{2}b*B*x^2 \ln((b*x+a)^n) + \frac{1}{2}B \ln(e)*b*x^2 + B \ln(e)*a*x - \frac{1}{2}B*x*(b*x+2*a) \ln((d*x+c)^n) + \frac{1}{2}B*n*a*x + B*a*x \ln((b*x+a)^n) - \frac{1}{2}I*B*Pi*a*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n) - \frac{1}{2}I*B*Pi*a*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n)) + \frac{1}{4}I*b*B*Pi*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2 + \frac{1}{4}I*b*B*Pi*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2 + \frac{1}{4}I*b*B*Pi*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2 + \frac{1}{2}I*B*Pi*a*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2 + \frac{1}{2}I*B*Pi*a*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2 + \frac{1}{2}I*B*Pi*a*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2 - \frac{1}{2}b/d*B*c*n*x - \frac{1}{d}B \ln(d*x+c)*a*c*n + \frac{1}{2}b/d^2*B \ln(d*x+c)*c^2*n - \frac{1}{4}I*b*B*Pi*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3 - \frac{1}{4}I*b*B*Pi*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3 - \frac{1}{2}I*B*Pi*a*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3 - \frac{1}{4}I*b*B*Pi*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n) - \frac{1}{4}I*b*B*Pi*x^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))$

**Maxima [A]** time = 1.15689, size = 208, normalized size = 2.48

$$\frac{1}{2} Bbx^2 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{2} Abx^2 + Bax \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Aax + \frac{\left(\frac{aen \log(bx+a)}{b} - \frac{cen \log(dx+c)}{d}\right) Ba}{e} - \frac{\left(\frac{a^2 en \log(bx+a)}{b^2} - \frac{c^2 en \log(dx+c)}{d^2}\right) B}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="maxima")

[Out] 1/2\*B\*b\*x^2\*log((b\*x + a)^n\*e/(d\*x + c)^n) + 1/2\*A\*b\*x^2 + B\*a\*x\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A\*a\*x + (a\*e\*n\*log(b\*x + a)/b - c\*e\*n\*log(d\*x + c)/d)\*B\*a/e - 1/2\*(a^2\*e\*n\*log(b\*x + a)/b^2 - c^2\*e\*n\*log(d\*x + c)/d^2 + (b\*c\*e\*n - a\*d\*e\*n)\*x/(b\*d))\*B\*b/e

**Fricas [B]** time = 1.07428, size = 354, normalized size = 4.21

$$\frac{Ab^2 d^2 x^2 + (2 Aabd^2 - (Bb^2 cd - Babd^2)n)x + (Bb^2 d^2 nx^2 + 2 Babd^2 nx + Ba^2 d^2 n) \log(bx + a) - (Bb^2 d^2 nx^2 + 2 Babd^2 nx + Ba^2 d^2 n) \log(dx + c)}{2 bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="fricas")

[Out] 1/2\*(A\*b^2\*d^2\*x^2 + (2\*A\*a\*b\*d^2 - (B\*b^2\*c\*d - B\*a\*b\*d^2)\*n)\*x + (B\*b^2\*d^2\*n\*x^2 + 2\*B\*a\*b\*d^2\*n\*x + B\*a^2\*d^2\*n)\*log(b\*x + a) - (B\*b^2\*d^2\*n\*x^2 + 2\*B\*a\*b\*d^2\*n\*x - (B\*b^2\*c^2 - 2\*B\*a\*b\*c\*d)\*n)\*log(d\*x + c) + (B\*b^2\*d^2\*x^2 + 2\*B\*a\*b\*d^2\*x)\*log(e))/(b\*d^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*(A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n))),x)

[Out] Timed out

**Giac [A]** time = 1.70434, size = 171, normalized size = 2.04

$$\frac{Ba^2 n \log(bx + a)}{2b} + \frac{1}{2} (Ab + Bb)x^2 + \frac{1}{2} (Bbnx^2 + 2Banx) \log(bx + a) - \frac{1}{2} (Bbnx^2 + 2Banx) \log(dx + c) - \frac{(Bbcn - B^2)}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="giac")

[Out] 1/2\*B\*a^2\*n\*log(b\*x + a)/b + 1/2\*(A\*b + B\*b)\*x^2 + 1/2\*(B\*b\*n\*x^2 + 2\*B\*a\*n\*x)\*log(b\*x + a) - 1/2\*(B\*b\*n\*x^2 + 2\*B\*a\*n\*x)\*log(d\*x + c) - 1/2\*(B\*b\*c\*n - B\*a\*d\*n - 2\*A\*a\*d - 2\*B\*a\*d)\*x/d + 1/2\*(B\*b\*c^2\*n - 2\*B\*a\*c\*d\*n)\*log(d\*x + c)/d^2

$$3.151 \quad \int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{a+bx} dx$$

**Optimal.** Leaf size=79

$$\frac{Bn \operatorname{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right)}{b} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right) (B \log(e(a+bx)^n(c+dx)^{-n}) + A)}{b}$$

[Out] -((Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]))/b) + (B\*n\*PolyLog[2, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/b

**Rubi [A]** time = 0.26782, antiderivative size = 87, normalized size of antiderivative = 1.1, number of steps used = 7, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {6742, 2488, 2411, 2343, 2333, 2315}

$$\frac{Bn \operatorname{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right)}{b} + \frac{A \log(a+bx)}{b} - \frac{B \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a+bx)^n(c+dx)^{-n})}{b}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(a + b\*x), x]

[Out] (A\*Log[a + b\*x])/b - (B\*Log[-((b\*c - a\*d)/(d\*(a + b\*x))]) \* Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/b + (B\*n\*PolyLog[2, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/b

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2488

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := -Simp[(Log[-((b\*c - a\*d)/(d\*(a + b\*x))]) \* Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d))/h, Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x))]) \* Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

#### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

#### Rule 2343

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_)])\*(b\_.)/((x\_)\*((d\_.) + (e\_.)\*(x\_)^(r\_.))), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*Log[c\*x^n])/(x\*(d + e\*x^(r/n))), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]

#### Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] := Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

**Rule 2315**

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

**Rubi steps**

$$\begin{aligned} \int \frac{A + B \log(e(a + bx)^n(c + dx)^{-n})}{a + bx} dx &= \int \left( \frac{A}{a + bx} + \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{a + bx} \right) dx \\ &= \frac{A \log(a + bx)}{b} + B \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{a + bx} dx \\ &= \frac{A \log(a + bx)}{b} - \frac{B \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{(B(bc - ad)n)}{b} \\ &= \frac{A \log(a + bx)}{b} - \frac{B \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{(B(bc - ad)n)}{b} \\ &= \frac{A \log(a + bx)}{b} - \frac{B \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{(B(bc - ad)n)}{b} \\ &= \frac{A \log(a + bx)}{b} - \frac{B \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{(B(bc - ad)n)}{b} \\ &= \frac{A \log(a + bx)}{b} - \frac{B \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{Bn \operatorname{Li}_2\left(\frac{b(c+dx)}{d(a+bx)}\right)}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0972473, size = 129, normalized size = 1.63

$$\frac{2Bn \operatorname{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right) + 2A \log(a + bx) - 2B \log\left(\frac{ad-bc}{d(a+bx)}\right) \left(\log(e(a + bx)^n(c + dx)^{-n}) + n \log\left(\frac{b(c+dx)}{bc-ad}\right)\right) - Bn \log^2\left(\frac{b(c+dx)}{bc-ad}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x), x]
```

```
[Out] (- (B*n*Log[(-(b*c) + a*d)/(d*(a + b*x))])^2 + 2*A*Log[a + b*x] - 2*B*Log[(-(
(b*c) + a*d)/(d*(a + b*x))]*(n*Log[(b*(c + d*x))/(b*c - a*d)] + Log[(e*(a +
b*x)^n)/(c + d*x)^n]) + 2*B*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])/(2
*b)
```

**Maple [C]** time = 1.534, size = 523, normalized size = 6.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a), x)
```

```
[Out] -B/b*ln(b*x+a)*ln((d*x+c)^n)+1/b*B*n*dilog((-a*d+b*c+d*(b*x+a))/(-a*d+b*c))
+1/b*B*n*ln(b*x+a)*ln((-a*d+b*c+d*(b*x+a))/(-a*d+b*c))+1/2*I/b*B*ln(b*x+a)*
Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/b*B*ln(b*x+a)*Pi*csgn(
I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/b*B*ln(b*x
+a)*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I/b*B*ln(b*x+a
)*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+A*ln(b*x+a)/b+1/b*
B*ln(b*x+a)*ln(e)+1/2/b*B/n*ln((b*x+a)^n)^2-1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*e
/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*(b*x+a)^n/((d*x+c)^
n))^3-1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I
*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I/b*B*ln(b*x+a)*Pi*csgn(I*(b*x+a)^n)*csgn(I/(
(d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$B \left( \frac{\log(bx+a) \log((bx+a)^n) - \log(bx+a) \log((dx+c)^n)}{b} + \int \frac{bdx \log(e) + bc \log(e) - (bcn - adn) \log(bx+a)}{b^2 dx^2 + abc + (b^2c + abd)x} dx \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a), x, algorithm="maxima")
```

```
[Out] B*((log(b*x + a)*log((b*x + a)^n) - log(b*x + a)*log((d*x + c)^n))/b + inte
grate((b*d*x*log(e) + b*c*log(e) - (b*c*n - a*d*n)*log(b*x + a))/(b^2*d*x^2
+ a*b*c + (b^2*c + a*b*d)*x), x) + A*log(b*x + a)/b
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A}{bx+a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a), x, algorithm="fricas")
```

```
[Out] integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*x + a), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a), x)
```

```
[Out] Timed out
```



**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*x + a), x)
```

$$3.152 \quad \int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^2} dx$$

**Optimal.** Leaf size=97

$$\frac{B \log(e(a+bx)^n(c+dx)^{-n}) + A}{b(a+bx)} - \frac{Bdn \log(a+bx)}{b(bc-ad)} + \frac{Bdn \log(c+dx)}{b(bc-ad)} - \frac{Bn}{b(a+bx)}$$

[Out] -((B\*n)/(b\*(a + b\*x))) - (B\*d\*n\*Log[a + b\*x])/(b\*(b\*c - a\*d)) + (B\*d\*n\*Log[c + d\*x])/(b\*(b\*c - a\*d)) - (A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(b\*(a + b\*x))

**Rubi [A]** time = 0.0835585, antiderivative size = 72, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6742, 2490, 32}

$$-\frac{A}{b(a+bx)} - \frac{B(c+dx) \log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)(bc-ad)} - \frac{Bn}{b(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(a + b\*x)^2, x]

[Out] -(A/(b\*(a + b\*x))) - (B\*n)/(b\*(a + b\*x)) - (B\*(c + d\*x)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/((b\*c - a\*d)\*(a + b\*x))

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2490

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_))^2, x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/((b\*g - a\*h)\*(g + h\*x)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(b\*g - a\*h), Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x)\*(g + h\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && NeQ[b\*g - a\*h, 0] && IGtQ[s, 0]

#### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{A + B \log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^2} dx &= \int \left( \frac{A}{(a+bx)^2} + \frac{B \log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^2} \right) dx \\ &= -\frac{A}{b(a+bx)} + B \int \frac{\log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^2} dx \\ &= -\frac{A}{b(a+bx)} - \frac{B(c+dx) \log(e(a+bx)^n(c+dx)^{-n})}{(bc-ad)(a+bx)} + (Bn) \int \frac{1}{(a+bx)^2} dx \\ &= -\frac{A}{b(a+bx)} - \frac{Bn}{b(a+bx)} - \frac{B(c+dx) \log(e(a+bx)^n(c+dx)^{-n})}{(bc-ad)(a+bx)} \end{aligned}$$

**Mathematica [A]** time = 0.0895557, size = 89, normalized size = 0.92

$$\frac{-(bc - ad) (B \log(e(a + bx)^n(c + dx)^{-n}) + A + Bn) + Bdn(a + bx) \log(c + dx) - Bdn(a + bx) \log(a + bx)}{b(a + bx)(bc - ad)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(a + b\*x)^2,x]

[Out]  $(-(B*d*n*(a + b*x)*Log[a + b*x]) + B*d*n*(a + b*x)*Log[c + d*x] - (b*c - a*d)*(A + B*n + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*(b*c - a*d)*(a + b*x))$

**Maple [C]** time = 0.4, size = 823, normalized size = 8.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))/(b\*x+a)^2,x)

[Out]  $B/b/(b*x+a)*ln((d*x+c)^n)-1/2*(-2*A*a*d-2*B*a*d*n+2*B*b*c*n+2*A*b*c-2*B*ln(d*x+c)*a*d*n+2*B*ln(-b*x-a)*a*d*n+2*B*b*c*ln((b*x+a)^n)-2*B*a*d*ln((b*x+a)^n)-I*B*Pi*b*c*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*B*Pi*b*c*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+2*B*ln(e)*b*c-2*B*ln(e)*a*d+I*B*Pi*a*d*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*a*d*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-I*B*Pi*a*d*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*a*d*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-I*B*Pi*a*d*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*B*Pi*a*d*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*b*c*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*b*c*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*a*d*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+I*B*Pi*b*c*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-2*B*ln(d*x+c)*b*d*n*x+2*B*ln(-b*x-a)*b*d*n*x-I*B*Pi*b*c*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+I*B*Pi*a*d*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I*B*Pi*b*c*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+I*B*Pi*b*c*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2)/(b*x+a)/b/(-a*d+b*c)$

**Maxima [A]** time = 1.18632, size = 157, normalized size = 1.62

$$\frac{\left(\frac{den \log(bx+a)}{b^2c-abd} - \frac{den \log(dx+c)}{b^2c-abd} + \frac{en}{b^2x+ab}\right)B - B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) - \frac{A}{b^2x+ab}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(b\*x+a)^2,x, algorithm="maxima")

[Out]  $-(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b^2*x + a*b))*B/e - B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b) - A/(b^2*x + a*b)$

---

**Fricas [A]** time = 1.06594, size = 242, normalized size = 2.49

$$\frac{Abc - Aad + (Bbc - Bad)n + (Bbdnx + Bbcn) \log(bx + a) - (Bbdnx + Bbcn) \log(dx + c) + (Bbc - Bad) \log(e)}{ab^2c - a^2bd + (b^3c - ab^2d)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(b\*x+a)^2,x, algorithm="fricas")

[Out] -(A\*b\*c - A\*a\*d + (B\*b\*c - B\*a\*d)\*n + (B\*b\*d\*n\*x + B\*b\*c\*n)\*log(b\*x + a) - (B\*b\*d\*n\*x + B\*b\*c\*n)\*log(d\*x + c) + (B\*b\*c - B\*a\*d)\*log(e))/(a\*b^2\*c - a^2\*b\*d + (b^3\*c - a\*b^2\*d)\*x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))/(b\*x+a)\*\*2,x)

[Out] Timed out

---

**Giac [A]** time = 1.26185, size = 146, normalized size = 1.51

$$-\frac{Bdn \log(bx + a)}{b^2c - abd} + \frac{Bdn \log(dx + c)}{b^2c - abd} - \frac{Bn \log(bx + a)}{b^2x + ab} + \frac{Bn \log(dx + c)}{b^2x + ab} - \frac{Bn + A + B}{b^2x + ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(b\*x+a)^2,x, algorithm="giac")

[Out] -B\*d\*n\*log(b\*x + a)/(b^2\*c - a\*b\*d) + B\*d\*n\*log(d\*x + c)/(b^2\*c - a\*b\*d) - B\*n\*log(b\*x + a)/(b^2\*x + a\*b) + B\*n\*log(d\*x + c)/(b^2\*x + a\*b) - (B\*n + A + B)/(b^2\*x + a\*b)

$$3.153 \quad \int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^3} dx$$

**Optimal.** Leaf size=137

$$-\frac{B \log(e(a+bx)^n(c+dx)^{-n}) + A}{2b(a+bx)^2} + \frac{Bd^2n \log(a+bx)}{2b(bc-ad)^2} - \frac{Bd^2n \log(c+dx)}{2b(bc-ad)^2} + \frac{Bdn}{2b(a+bx)(bc-ad)} - \frac{Bn}{4b(a+bx)^2}$$

[Out]  $-(B*n)/(4*b*(a+b*x)^2) + (B*d*n)/(2*b*(b*c-a*d)*(a+b*x)) + (B*d^2*n*Log[a+b*x])/(2*b*(b*c-a*d)^2) - (B*d^2*n*Log[c+d*x])/(2*b*(b*c-a*d)^2) - (A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(2*b*(a+b*x)^2)$

**Rubi [A]** time = 0.154141, antiderivative size = 149, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6742, 2492, 44}

$$-\frac{A}{2b(a+bx)^2} + \frac{Bd^2n \log(a+bx)}{2b(bc-ad)^2} - \frac{Bd^2n \log(c+dx)}{2b(bc-ad)^2} - \frac{B \log(e(a+bx)^n(c+dx)^{-n})}{2b(a+bx)^2} + \frac{Bdn}{2b(a+bx)(bc-ad)} - \frac{Bn}{4b(a+bx)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(a + b*x)^3, x]$

[Out]  $-A/(2*b*(a+b*x)^2) - (B*n)/(4*b*(a+b*x)^2) + (B*d*n)/(2*b*(b*c-a*d)*(a+b*x)) + (B*d^2*n*Log[a+b*x])/(2*b*(b*c-a*d)^2) - (B*d^2*n*Log[c+d*x])/(2*b*(b*c-a*d)^2) - (B*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(2*b*(a+b*x)^2)$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

#### Rule 2492

$\text{Int}[\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))^(r_.)]^(s_.)*((g_.) + (h_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(g + h*x)^(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s/(h*(m + 1)), x] - \text{Dist}[(p*r*s*(b*c - a*d))/(h*(m + 1)), \text{Int}[(g + h*x)^(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s/(a + b*x)*(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{NeQ}[m, -1]$

#### Rule 44

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{ILtQ}[m, 0] \&\& \text{IntegerQ}[n] \&\& !(\text{IGtQ}[n, 0] \&\& \text{LtQ}[m + n + 2, 0])$

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^3} dx &= \int \left( \frac{A}{(a + bx)^3} + \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^3} \right) dx \\
&= -\frac{A}{2b(a + bx)^2} + B \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^3} dx \\
&= -\frac{A}{2b(a + bx)^2} - \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{2b(a + bx)^2} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)^3(c+dx)} dx}{2b} \\
&= -\frac{A}{2b(a + bx)^2} - \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{2b(a + bx)^2} + \frac{(B(bc - ad)n) \int \left( \frac{b}{(bc-ad)(a+bx)^3} - \frac{1}{(a+bx)^3} \right) dx}{2b} \\
&= -\frac{A}{2b(a + bx)^2} - \frac{Bn}{4b(a + bx)^2} + \frac{Bdn}{2b(bc - ad)(a + bx)} + \frac{Bd^2n \log(a + bx)}{2b(bc - ad)^2} - \frac{Bd^2n}{2b}
\end{aligned}$$

**Mathematica [A]** time = 0.338144, size = 121, normalized size = 0.88

$$\frac{\frac{2A}{(a+bx)^2} + Bn \left( -\frac{2d^2 \log(a+bx)}{(bc-ad)^2} + \frac{2d^2 \log(c+dx)}{(bc-ad)^2} + \frac{2d(a+bx)}{ad-bc} + 1 \right) + \frac{2B \log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^2}}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(a + b\*x)^3, x]

[Out] -((2\*A)/(a + b\*x)^2 + B\*n\*((1 + (2\*d\*(a + b\*x))/(-b\*c) + a\*d))/(a + b\*x)^2 - (2\*d^2\*Log[a + b\*x])/(b\*c - a\*d)^2 + (2\*d^2\*Log[c + d\*x])/(b\*c - a\*d)^2 + (2\*B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(a + b\*x)^2)/(4\*b)

**Maple [C]** time = 0.425, size = 1379, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))/(b\*x+a)^3, x)

[Out] 1/2\*B/b/(b\*x+a)^2\*ln((d\*x+c)^n)-1/4\*(2\*I\*B\*Pi\*a\*b\*c\*d\*csgn(I\*e)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)+I\*B\*Pi\*a^2\*d^2\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2-I\*B\*Pi\*a^2\*d^2\*csgn(I\*(b\*x+a)^n)\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))+2\*I\*B\*Pi\*a\*b\*c\*d\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3+2\*I\*B\*Pi\*a\*b\*c\*d\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3-I\*B\*Pi\*b^2\*c^2\*csgn(I\*e)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)-4\*A\*a\*b\*c\*d-I\*B\*Pi\*b^2\*c^2\*csgn(I\*(b\*x+a)^n)\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))+2\*B\*ln(e)\*a^2\*d^2+2\*B\*ln(e)\*b^2\*c^2+2\*A\*a^2\*d^2+2\*A\*b^2\*c^2+2\*B\*ln(d\*x+c)\*a^2\*d^2\*n+3\*B\*a^2\*d^2\*n+B\*c^2\*n\*b^2-2\*B\*a^2\*n\*ln(-b\*x-a)\*d^2+I\*B\*Pi\*b^2\*c^2\*csgn(I\*e)\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+I\*B\*Pi\*b^2\*c^2\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+I\*B\*Pi\*b^2\*c^2\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2-I\*B\*Pi\*a^2\*d^2\*csgn(I\*e)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)+2\*B\*b^2\*c^2\*ln((b\*x+a)^n)+2\*B\*a^2\*d^2\*ln((b\*x+a)^n)-2\*I\*B\*Pi\*a\*b\*c\*d\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2-2\*I\*B\*Pi\*a\*b\*c\*d\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2-2\*B\*b^2\*c\*d\*n\*x+2\*B\*a\*b\*d^2\*n\*x-4\*B\*a\*c\*d\*n\*b-4\*B\*ln(e)\*a\*b\*c\*d-I\*B\*Pi\*a^2\*d^2\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3-I\*B\*Pi\*b^2\*c^2\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-I\*B\*Pi\*b^2\*c^2\*csgn(I\*e/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n)^3-2\*I\*B\*Pi\*a\*b\*c\*d\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d



[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))/(b\*x+a)\*\*3,x)

[Out] Timed out

**Giac [A]** time = 1.27317, size = 323, normalized size = 2.36

$$\frac{Bd^2n \log(bx + a)}{2(b^3c^2 - 2ab^2cd + a^2bd^2)} - \frac{Bd^2n \log(dx + c)}{2(b^3c^2 - 2ab^2cd + a^2bd^2)} - \frac{Bn \log(bx + a)}{2(b^3x^2 + 2ab^2x + a^2b)} + \frac{Bn \log(dx + c)}{2(b^3x^2 + 2ab^2x + a^2b)} + \frac{2Bbdnx}{4(b^4cx^2 + 2b^3cdx + a^2bd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(b\*x+a)^3,x, algorithm="giac")

[Out]  $\frac{1}{2}B*d^{2*n}*\log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - \frac{1}{2}B*d^{2*n}*\log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - \frac{1}{2}B*n*\log(b*x + a)/(b^3*x^2 + 2*a*b^2*x + a^2*b) + \frac{1}{2}B*n*\log(d*x + c)/(b^3*x^2 + 2*a*b^2*x + a^2*b) + \frac{1}{4}*(2*B*b*d*n*x - B*b*c*n + 3*B*a*d*n - 2*A*b*c - 2*B*b*c + 2*A*a*d + 2*B*a*d)/(b^4*c*x^2 - a*b^3*d*x^2 + 2*a*b^3*c*x - 2*a^2*b^2*d*x + a^2*b^2*c - a^3*b*d)$



$$3.154 \quad \int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^4} dx$$

**Optimal.** Leaf size=166

$$\frac{B \log(e(a+bx)^n(c+dx)^{-n}) + A}{3b(a+bx)^3} - \frac{Bd^2n}{3b(a+bx)(bc-ad)^2} - \frac{Bd^3n \log(a+bx)}{3b(bc-ad)^3} + \frac{Bd^3n \log(c+dx)}{3b(bc-ad)^3} + \frac{Bdn}{6b(a+bx)^2(bc-ad)}$$

[Out]  $-(B*n)/(9*b*(a+b*x)^3) + (B*d*n)/(6*b*(b*c-a*d)*(a+b*x)^2) - (B*d^2*n)/(3*b*(b*c-a*d)^2*(a+b*x)) - (B*d^3*n*Log[a+b*x])/(3*b*(b*c-a*d)^3) + (B*d^3*n*Log[c+d*x])/(3*b*(b*c-a*d)^3) - (A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n)]/(3*b*(a+b*x)^3)$

**Rubi [A]** time = 0.169881, antiderivative size = 178, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6742, 2492, 44}

$$\frac{A}{3b(a+bx)^3} - \frac{Bd^2n}{3b(a+bx)(bc-ad)^2} - \frac{Bd^3n \log(a+bx)}{3b(bc-ad)^3} + \frac{Bd^3n \log(c+dx)}{3b(bc-ad)^3} - \frac{B \log(e(a+bx)^n(c+dx)^{-n})}{3b(a+bx)^3} + \frac{Bdn}{6b(a+bx)^2(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n)]/(a + b\*x)^4, x]

[Out]  $-A/(3*b*(a+b*x)^3) - (B*n)/(9*b*(a+b*x)^3) + (B*d*n)/(6*b*(b*c-a*d)*(a+b*x)^2) - (B*d^2*n)/(3*b*(b*c-a*d)^2*(a+b*x)) - (B*d^3*n*Log[a+b*x])/(3*b*(b*c-a*d)^3) + (B*d^3*n*Log[c+d*x])/(3*b*(b*c-a*d)^3) - (B*Log[(e*(a+b*x)^n)/(c+d*x]^n)]/(3*b*(a+b*x)^3)$

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2492

Int[Log[(e\_.)\*((f\_.)\*(a\_.) + (b\_.)\*(x\_.))^p\_.)\*((c\_.) + (d\_.)\*(x\_.))^q\_.]^r\_.]^s\_.)\*((g\_.) + (h\_.)\*(x\_.))^m\_.], x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(h\*(m + 1)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(h\*(m + 1)), Int[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0] && NeQ[m, -1]

#### Rule 44

Int[((a\_.) + (b\_.)\*(x\_.))^m\_.)\*((c\_.) + (d\_.)\*(x\_.))^n\_.], x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^4} dx &= \int \left( \frac{A}{(a + bx)^4} + \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^4} \right) dx \\
&= -\frac{A}{3b(a + bx)^3} + B \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^4} dx \\
&= -\frac{A}{3b(a + bx)^3} - \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{3b(a + bx)^3} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)^4(c+dx)} dx}{3b} \\
&= -\frac{A}{3b(a + bx)^3} - \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{3b(a + bx)^3} + \frac{(B(bc - ad)n) \int \left( \frac{b}{(bc-ad)(a+bx)^4} - \frac{1}{(a+bx)^4(c+dx)} \right) dx}{3b} \\
&= -\frac{A}{3b(a + bx)^3} - \frac{Bn}{9b(a + bx)^3} + \frac{Bdn}{6b(bc - ad)(a + bx)^2} - \frac{Bd^2n}{3b(bc - ad)^2(a + bx)} - \frac{B}{3b}
\end{aligned}$$

**Mathematica [A]** time = 0.400264, size = 143, normalized size = 0.86

$$\frac{\frac{6A}{(a+bx)^3} + Bn \left( \frac{\frac{6d^2(a+bx)^2 + 3d(a+bx)}{(bc-ad)^2} + 2}{(a+bx)^3} + \frac{6d^3 \log(a+bx)}{(bc-ad)^3} - \frac{6d^3 \log(c+dx)}{(bc-ad)^3} \right) + \frac{6B \log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^3}}{18b}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n)]/(a + b\*x)^4, x]

[Out] -((6\*A)/(a + b\*x)^3 + B\*n\*((2 + (3\*d\*(a + b\*x)))/(-b\*c) + a\*d) + (6\*d^2\*(a + b\*x)^2)/(b\*c - a\*d)^2)/(a + b\*x)^3 + (6\*d^3\*Log[a + b\*x])/(b\*c - a\*d)^3 - (6\*d^3\*Log[c + d\*x])/(b\*c - a\*d)^3) + (6\*B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n)]/(a + b\*x)^3)/(18\*b)

**Maple [C]** time = 0.475, size = 1976, normalized size = 11.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))/(b\*x+a)^4, x)

[Out] 1/3\*B/b/(b\*x+a)^3\*ln((d\*x+c)^n)-1/18\*(-6\*B\*a^3\*d^3\*ln((b\*x+a)^n)+6\*B\*b^3\*c^3\*ln((b\*x+a)^n)+18\*A\*a^2\*b\*c\*d^2-18\*A\*a\*b^2\*c^2\*d-11\*B\*a^3\*d^3\*n-6\*B\*ln(d\*x+c)\*a^3\*d^3\*n+6\*B\*a^3\*n\*ln(-b\*x-a)\*d^3+6\*A\*b^3\*c^3-6\*A\*a^3\*d^3+18\*B\*ln(e)\*a^2\*b\*c\*d^2-18\*B\*ln(e)\*a\*b^2\*c^2\*d-6\*B\*ln(e)\*a^3\*d^3+6\*B\*ln(e)\*b^3\*c^3-6\*B\*ln(d\*x+c)\*b^3\*d^3\*n\*x^3+6\*B\*ln(-b\*x-a)\*b^3\*d^3\*n\*x^3+9\*I\*B\*Pi\*a^2\*b\*c\*d^2\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2-9\*I\*B\*Pi\*a\*b^2\*c^2\*d\*csgn(I\*e)\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2-9\*I\*B\*Pi\*a\*b^2\*c^2\*d\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2-9\*I\*B\*Pi\*a\*b^2\*c^2\*d\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2-9\*I\*B\*Pi\*a\*b^2\*c^2\*d\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+2\*B\*b^3\*c^3\*n-9\*B\*a\*c^2\*d\*n\*b^2+18\*B\*a\*b^2\*c\*d^2\*n\*x-9\*I\*B\*Pi\*a^2\*b\*c\*d^2\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-9\*I\*B\*Pi\*a^2\*b\*c\*d^2\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3+9\*I\*B\*Pi\*a\*b^2\*c^2\*d\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3+3\*I\*B\*Pi\*a^3\*d^3\*csgn(I\*e)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)+3\*I\*B\*Pi\*a^3\*d^3\*csgn(I\*(b\*x+a)^n)\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))-3\*I\*B\*Pi\*b^3\*c^3\*csgn(I\*e)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)-3\*I\*B\*Pi\*b^3

$$\begin{aligned}
& *c^3 \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn}(I/((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n/((d*x+c)^n)) + 9 * \\
& I * B * \Pi * a^2 * b * c * d^2 * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * e / ((d*x+c)^n) * (b*x+a)^n)^2 + 9 * I * B * \Pi * a^2 * \\
& b * c * d^2 * \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn}(I*(b*x+a)^n/((d*x+c)^n))^2 + 9 * I * B * \Pi * a^2 * b * c * \\
& d^2 * \operatorname{csgn}(I/((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n/((d*x+c)^n))^2 - 18 * B * \ln(d*x+c) * a * b \\
& ^2 * d^3 * n * x^2 + 18 * B * \ln(-b*x-a) * a * b^2 * d^3 * n * x^2 - 18 * B * \ln(d*x+c) * a^2 * b * d^3 * n * x + 1 \\
& 8 * B * \ln(-b*x-a) * a^2 * b * d^3 * n * x - 3 * I * B * \Pi * b^3 * c^3 * \operatorname{csgn}(I*(b*x+a)^n/((d*x+c)^n)) \\
& ^3 - 3 * I * B * \Pi * b^3 * c^3 * \operatorname{csgn}(I * e / ((d*x+c)^n) * (b*x+a)^n)^3 + 3 * I * B * \Pi * b^3 * c^3 * \operatorname{csgn} \\
& (I*(b*x+a)^n/((d*x+c)^n)) * \operatorname{csgn}(I * e / ((d*x+c)^n) * (b*x+a)^n)^2 - 15 * B * a^2 * b * d^3 * \\
& n * x - 3 * B * b^3 * c^2 * d * n * x - 3 * I * B * \Pi * a^3 * d^3 * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * e / ((d*x+c)^n) * (b*x+ \\
& a)^n)^2 + 18 * B * a^2 * b * c * d^2 * n + 18 * B * a^2 * b * c * d^2 * \ln((b*x+a)^n) - 18 * B * a * b^2 * c^2 * d * \\
& \ln((b*x+a)^n) - 6 * B * a * b^2 * d^3 * n * x^2 + 6 * B * b^3 * c * d^2 * n * x^2 - 9 * I * B * \Pi * a^2 * b * c * d^2 * \\
& \operatorname{csgn}(I * e) * \operatorname{csgn}(I*(b*x+a)^n/((d*x+c)^n)) * \operatorname{csgn}(I * e / ((d*x+c)^n) * (b*x+a)^n) - 9 * I \\
& * B * \Pi * a^2 * b * c * d^2 * \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn}(I/((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n/(( \\
& d*x+c)^n)) + 3 * I * B * \Pi * a^3 * d^3 * \operatorname{csgn}(I*(b*x+a)^n/((d*x+c)^n))^3 + 3 * I * B * \Pi * a^3 * d^3 * \\
& 3 * \operatorname{csgn}(I * e / ((d*x+c)^n) * (b*x+a)^n)^3 - 3 * I * B * \Pi * a^3 * d^3 * \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn} \\
& (I*(b*x+a)^n/((d*x+c)^n))^2 - 3 * I * B * \Pi * a^3 * d^3 * \operatorname{csgn}(I/((d*x+c)^n)) * \operatorname{csgn}(I*(b * \\
& x+a)^n/((d*x+c)^n))^2 - 3 * I * B * \Pi * a^3 * d^3 * \operatorname{csgn}(I*(b*x+a)^n/((d*x+c)^n)) * \operatorname{csgn}(I \\
& * e / ((d*x+c)^n) * (b*x+a)^n)^2 + 3 * I * B * \Pi * b^3 * c^3 * \operatorname{csgn}(I * e) * \operatorname{csgn}(I * e / ((d*x+c)^n) \\
& * (b*x+a)^n)^2 + 3 * I * B * \Pi * b^3 * c^3 * \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn}(I*(b*x+a)^n/((d*x+c)^ \\
& n))^2 + 3 * I * B * \Pi * b^3 * c^3 * \operatorname{csgn}(I/((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n/((d*x+c)^n))^2 + \\
& 9 * I * B * \Pi * a * b^2 * c^2 * d * \operatorname{csgn}(I * e) * \operatorname{csgn}(I*(b*x+a)^n/((d*x+c)^n)) * \operatorname{csgn}(I * e / ((d*x \\
& +c)^n) * (b*x+a)^n) + 9 * I * B * \Pi * a * b^2 * c^2 * d * \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn}(I/((d*x+c)^n) \\
& ) * \operatorname{csgn}(I*(b*x+a)^n/((d*x+c)^n)) / (b*x+a)^3 / (a^2 * d^2 - 2 * a * b * c * d + b^2 * c^2) / (-a * \\
& d + b * c) / b
\end{aligned}$$

**Maxima [B]** time = 1.25239, size = 540, normalized size = 3.25

$$\frac{\left( \frac{6d^3 en \log(bx+a)}{b^4 c^3 - 3ab^3 c^2 d + 3a^2 b^2 c d^2 - a^3 b d^3} - \frac{6d^3 en \log(dx+c)}{b^4 c^3 - 3ab^3 c^2 d + 3a^2 b^2 c d^2 - a^3 b d^3} + \frac{6b^2 d^2 en x^2 + 2b^2 c^2 en - 7abcden + 11a^2 d^2 en - 3(b^2 cden - 5b^2 c^2 d)}{a^3 b^3 c^2 - 2a^4 b^2 cd + a^5 b d^2 + (b^6 c^2 - 2ab^5 cd + a^2 b^4 d^2)x^3 + 3(ab^5 c^2 - 2a^2 b^4 cd + a^3 b^3 d^2)} \right)}{18e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(b\*x+a)^4,x, algorithm="maxima")

[Out]  $-1/18 * (6 * d^3 * e * n * \log(b * x + a) / (b^4 * c^3 - 3 * a * b^3 * c^2 * d + 3 * a^2 * b^2 * c * d^2 - a^3 * b * d^3) - 6 * d^3 * e * n * \log(d * x + c) / (b^4 * c^3 - 3 * a * b^3 * c^2 * d + 3 * a^2 * b^2 * c * d^2 - a^3 * b * d^3) + (6 * b^2 * d^2 * e * n * x^2 + 2 * b^2 * c^2 * e * n - 7 * a * b * c * d * e * n + 11 * a^2 * d^2 * e * n - 3 * (b^2 * c * d * e * n - 5 * a * b * d^2 * e * n) * x) / (a^3 * b^3 * c^2 - 2 * a^4 * b^2 * c * d + a^5 * b * d^2 + (b^6 * c^2 - 2 * a * b^5 * c * d + a^2 * b^4 * d^2) * x^3 + 3 * (a * b^5 * c^2 - 2 * a^2 * b^4 * c * d + a^3 * b^3 * d^2) * x^2 + 3 * (a^2 * b^4 * c^2 - 2 * a^3 * b^3 * c * d + a^4 * b^2 * d^2) * x)) * B / e - 1/3 * B * \log((b * x + a)^n * e / (d * x + c)^n) / (b^4 * x^3 + 3 * a * b^3 * x^2 + 3 * a^2 * b^2 * x + a^3 * b) - 1/3 * A / (b^4 * x^3 + 3 * a * b^3 * x^2 + 3 * a^2 * b^2 * x + a^3 * b)$

**Fricas [B]** time = 1.13249, size = 1123, normalized size = 6.77

$$\frac{6Ab^3c^3 - 18Aab^2c^2d + 18Aa^2bcd^2 - 6Aa^3d^3 + 6(Bb^3cd^2 - Bab^2d^3)nx^2 - 3(Bb^3c^2d - 6Bab^2cd^2 + 5Ba^2bd^3)nx + \dots}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(b\*x+a)^4,x, algorithm="fricas")

```
[Out] -1/18*(6*A*b^3*c^3 - 18*A*a*b^2*c^2*d + 18*A*a^2*b*c*d^2 - 6*A*a^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*n*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*n*x + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 - 11*B*a^3*d^3)*n + 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(b*x + a) - 6*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + (B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*n)*log(d*x + c) + 6*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 - B*a^3*d^3)*log(e))/(a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)**4,x)
```

```
[Out] Timed out
```

**Giac [B]** time = 1.29256, size = 605, normalized size = 3.64

$$\frac{Bd^3n \log(bx + a)}{3(b^4c^3 - 3ab^3c^2d + 3a^2b^2cd^2 - a^3bd^3)} + \frac{Bd^3n \log(dx + c)}{3(b^4c^3 - 3ab^3c^2d + 3a^2b^2cd^2 - a^3bd^3)} - \frac{Bn \log(bx + a)}{3(b^4x^3 + 3ab^3x^2 + 3a^2b^2x + a^3b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)^4,x, algorithm="giac")
```

```
[Out] -1/3*B*d^3*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + 1/3*B*d^3*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 1/3*B*n*log(b*x + a)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b) + 1/3*B*n*log(d*x + c)/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b) - 1/18*(6*B*b^2*d^2*n*x^2 - 3*B*b^2*c*d*n*x + 15*B*a*b*d^2*n*x + 2*B*b^2*c^2*n - 7*B*a*b*c*d*n + 11*B*a^2*d^2*n + 6*A*b^2*c^2 + 6*B*b^2*c^2 - 12*A*a*b*c*d - 12*B*a*b*c*d + 6*A*a^2*d^2 + 6*B*a^2*d^2)/(b^6*c^2*x^3 - 2*a*b^5*c*d*x^3 + a^2*b^4*d^2*x^3 + 3*a*b^5*c^2*x^2 - 6*a^2*b^4*c*d*x^2 + 3*a^3*b^3*d^2*x^2 + 3*a^2*b^4*c^2*x - 6*a^3*b^3*c*d*x + 3*a^4*b^2*d^2*x + a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)
```

$$3.155 \quad \int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^5} dx$$

**Optimal.** Leaf size=195

$$-\frac{B \log(e(a+bx)^n(c+dx)^{-n}) + A}{4b(a+bx)^4} + \frac{Bd^3n}{4b(a+bx)(bc-ad)^3} - \frac{Bd^2n}{8b(a+bx)^2(bc-ad)^2} + \frac{Bd^4n \log(a+bx)}{4b(bc-ad)^4} - \frac{Bd^4n \log(c+dx)}{4b(bc-ad)^4}$$

[Out]  $-(B*n)/(16*b*(a+b*x)^4) + (B*d*n)/(12*b*(b*c-a*d)*(a+b*x)^3) - (B*d^2*n)/(8*b*(b*c-a*d)^2*(a+b*x)^2) + (B*d^3*n)/(4*b*(b*c-a*d)^3*(a+b*x)) + (B*d^4*n*Log[a+b*x])/(4*b*(b*c-a*d)^4) - (B*d^4*n*Log[c+d*x])/(4*b*(b*c-a*d)^4) - (A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(4*b*(a+b*x)^4)$

**Rubi [A]** time = 0.192558, antiderivative size = 207, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6742, 2492, 44}

$$-\frac{A}{4b(a+bx)^4} + \frac{Bd^3n}{4b(a+bx)(bc-ad)^3} - \frac{Bd^2n}{8b(a+bx)^2(bc-ad)^2} + \frac{Bd^4n \log(a+bx)}{4b(bc-ad)^4} - \frac{Bd^4n \log(c+dx)}{4b(bc-ad)^4} - \frac{B \log(e(a+bx)^n(c+dx)^{-n})}{4b(a+bx)^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(a + b\*x)^5,x]

[Out]  $-A/(4*b*(a+b*x)^4) - (B*n)/(16*b*(a+b*x)^4) + (B*d*n)/(12*b*(b*c-a*d)*(a+b*x)^3) - (B*d^2*n)/(8*b*(b*c-a*d)^2*(a+b*x)^2) + (B*d^3*n)/(4*b*(b*c-a*d)^3*(a+b*x)) + (B*d^4*n*Log[a+b*x])/(4*b*(b*c-a*d)^4) - (B*d^4*n*Log[c+d*x])/(4*b*(b*c-a*d)^4) - (B*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(4*b*(a+b*x)^4)$

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2492

Int[Log[(e\_.)\*((f\_.)\*(a\_.) + (b\_.)\*(x\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_.))^(q\_.)]^(r\_.)]^(s\_.)\*((g\_.) + (h\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(h\*(m + 1)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(h\*(m + 1)), Int[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0] && NeQ[m, -1]

#### Rule 44

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^5} dx &= \int \left( \frac{A}{(a + bx)^5} + \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^5} \right) dx \\
&= -\frac{A}{4b(a + bx)^4} + B \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^5} dx \\
&= -\frac{A}{4b(a + bx)^4} - \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{4b(a + bx)^4} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)^5(c+dx)} dx}{4b} \\
&= -\frac{A}{4b(a + bx)^4} - \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{4b(a + bx)^4} + \frac{(B(bc - ad)n) \int \left( \frac{b}{(bc-ad)(a+bx)^5} - \right)}{4b} \\
&= -\frac{A}{4b(a + bx)^4} - \frac{Bn}{16b(a + bx)^4} + \frac{Bdn}{12b(bc - ad)(a + bx)^3} - \frac{Bd^2n}{8b(bc - ad)^2(a + bx)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.395787, size = 165, normalized size = 0.85

$$\frac{\frac{12A}{(a+bx)^4} + Bn \left( \frac{-\frac{12d^3(a+bx)^3}{(bc-ad)^3} + \frac{6d^2(a+bx)^2}{(bc-ad)^2} + \frac{4d(a+bx)}{ad-bc} + 3}{(a+bx)^4} - \frac{12d^4 \log(a+bx)}{(bc-ad)^4} + \frac{12d^4 \log(c+dx)}{(bc-ad)^4} \right) + \frac{12B \log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^4}}{48b}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(a + b\*x)^5, x]

[Out] -((12\*A)/(a + b\*x)^4 + B\*n\*((3 + (4\*d\*(a + b\*x)))/(-b\*c) + a\*d) + (6\*d^2\*(a + b\*x)^2)/(b\*c - a\*d)^2 - (12\*d^3\*(a + b\*x)^3)/(b\*c - a\*d)^3)/(a + b\*x)^4 - (12\*d^4\*Log[a + b\*x])/(b\*c - a\*d)^4 + (12\*d^4\*Log[c + d\*x])/(b\*c - a\*d)^4) + (12\*B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(a + b\*x)^4)/(48\*b)

**Maple [C]** time = 0.509, size = 2583, normalized size = 13.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))/(b\*x+a)^5, x)

[Out] 1/4\*B/b/(b\*x+a)^4\*ln((d\*x+c)^n)+1/48\*(24\*I\*B\*Pi\*a^3\*b\*c\*d^3\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+24\*I\*B\*Pi\*a^3\*b\*c\*d^3\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2-12\*B\*ln(d\*x+c)\*a^4\*d^4\*n+12\*B\*a^4\*n\*ln(-b\*x-a)\*d^4-12\*B\*a^4\*d^4\*ln((b\*x+a)^n)-12\*B\*b^4\*c^4\*ln((b\*x+a)^n)-3\*B\*b^4\*c^4\*n-12\*A\*a^4\*d^4-25\*B\*a^4\*d^4\*n-12\*B\*ln(e)\*a^4\*d^4-12\*B\*ln(e)\*b^4\*c^4-12\*A\*b^4\*c^4-6\*I\*B\*Pi\*b^4\*c^4\*csgn(I\*e)\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2-6\*I\*B\*Pi\*b^4\*c^4\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2-6\*I\*B\*Pi\*b^4\*c^4\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+48\*B\*ln(e)\*a^3\*b\*c\*d^3-72\*B\*ln(e)\*a^2\*b^2\*c^2\*d^2+48\*B\*ln(e)\*a\*b^3\*c^3\*d+36\*I\*B\*Pi\*a^2\*b^2\*c^2\*d^2\*csgn(I\*(b\*x+a)^n)\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))-24\*I\*B\*Pi\*a\*b^3\*c^3\*d\*csgn(I\*e)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)-24\*I\*B\*Pi\*a\*b^3\*c^3\*d\*csgn(I\*(b\*x+a)^n)\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))-24\*I\*B\*Pi\*a\*b^3\*c^3\*d\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3+48\*A\*a^3\*b\*c\*d^3-72\*A\*a^2\*b^2\*c^2\*d^2+48\*A\*a\*b^3\*c^3\*d-6\*I\*B\*Pi\*b^4\*c^4\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+48\*B\*a^3\*b\*c\*d^3\*n-36\*B\*a^2\*b^2\*c^2\*d^2\*n+16\*B\*a\*b^3\*c^3\*d\*n-12\*B\*a\*b^3\*d^4\*n\*x^3+12\*B\*b^4\*c\*d^3\*n\*x^3-24\*I\*B\*Pi\*a^3\*b\*c\*d^3\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)

```

^n)^3+36*I*B*Pi*a^2*b^2*c^2*d^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+36*I*B*Pi*a
^2*b^2*c^2*d^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-24*I*B*Pi*a*b^3*c^3*d*csgn
(I*(b*x+a)^n/((d*x+c)^n))^3+6*I*B*Pi*a^4*d^4*csgn(I*e)*csgn(I*(b*x+a)^n/((d
*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+6*I*B*Pi*a^4*d^4*csgn(I*(b*x+a)^n
)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-24*I*B*Pi*a^3*b*c*d^3*c
sgn(I*(b*x+a)^n/((d*x+c)^n))^3-24*I*B*Pi*a^3*b*c*d^3*csgn(I*(b*x+a)^n)*csgn
(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+36*I*B*Pi*a^2*b^2*c^2*d^2*csg
n(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-6*I*B*
Pi*a^4*d^4*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-24*I*B*Pi*a^3*b*c*d^
3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+4
8*B*a*b^3*c*d^3*n*x^2+72*B*a^2*b^2*c*d^3*n*x-24*B*a*b^3*c^2*d^2*n*x+24*I*B*
Pi*a^3*b*c*d^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^2-36*I*B*Pi*a^2*b^2*c^2*d^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-3
6*I*B*Pi*a^2*b^2*c^2*d^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+
24*I*B*Pi*a*b^3*c^3*d*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b
*x+a)^n)^2+24*I*B*Pi*a^3*b*c*d^3*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^
2-36*I*B*Pi*a^2*b^2*c^2*d^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^
n))^2-36*I*B*Pi*a^2*b^2*c^2*d^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x
+c)^n)*(b*x+a)^n)^2-48*B*ln(d*x+c)*a*b^3*d^4*n*x^3+48*B*ln(-b*x-a)*a*b^3*d^
4*n*x^3-72*B*ln(d*x+c)*a^2*b^2*d^4*n*x^2+72*B*ln(-b*x-a)*a^2*b^2*d^4*n*x^2-
48*B*ln(d*x+c)*a^3*b*d^4*n*x+48*B*ln(-b*x-a)*a^3*b*d^4*n*x-12*B*ln(d*x+c)*b
^4*d^4*n*x^4+12*B*ln(-b*x-a)*b^4*d^4*n*x^4+6*I*B*Pi*a^4*d^4*csgn(I*(b*x+a)^
n/((d*x+c)^n))^3+6*I*B*Pi*a^4*d^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+6*I*B*P
i*b^4*c^4*csgn(I*(b*x+a)^n/((d*x+c)^n))^3+6*I*B*Pi*b^4*c^4*csgn(I*e/((d*x+c
)^n)*(b*x+a)^n)^3-6*I*B*Pi*a^4*d^4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x
+c)^n))^2-6*I*B*Pi*a^4*d^4*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n)
)^2-6*I*B*Pi*a^4*d^4*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*
x+a)^n)^2+6*I*B*Pi*b^4*c^4*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e
/((d*x+c)^n)*(b*x+a)^n)+6*I*B*Pi*b^4*c^4*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^
n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+48*B*a^3*b*c*d^3*ln((b*x+a)^n)-72*B*a^2*b
^2*c^2*d^2*ln((b*x+a)^n)+48*B*a*b^3*c^3*d*ln((b*x+a)^n)+24*I*B*Pi*a*b^3*c^3
*d*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+24*I*B*Pi*a*b^3*c^3*d*csgn(I
*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+24*I*B*Pi*a*b^3*c^3*d*csgn(I/((
d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-42*B*a^2*b^2*d^4*n*x^2-6*B*b^4*c
^2*d^2*n*x^2-52*B*a^3*b*d^4*n*x+4*B*b^4*c^3*d*n*x)/(b*x+a)^4/(-a^3*d^3+3*a^
2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/(-a*d+b*c)/b

```

**Maxima [B]** time = 1.7803, size = 834, normalized size = 4.28

$$\left( \frac{12d^4en \log(bx+a)}{b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4} - \frac{12d^4en \log(dx+c)}{b^5c^4-4ab^4c^3d+6a^2b^3c^2d^2-4a^3b^2cd^3+a^4bd^4} + \frac{12b^3d^3enx^3-3b^3}{a^4b^4c^3-3a^5b^3c^2d+3a^6b^2cd^2-a^7bd^3+(b^8c^3-3ab^7c^2d+3a^2b^6c^2d^2-3a^3b^5cd^3+3a^4b^4c^2d^2-3a^5b^3cd^2+3a^6b^2c^2d+3a^7b^2c^2d^2-3a^8b^2c^2d^2+3a^9b^2c^2d^2)} \right) / (-a^3d^3+3a^2b^3c^2d+b^3c^3) / (-a*d+b*c) / b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(b\*x+a)^5,x, algorithm="maxima")

```

[Out] 1/48*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
- 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4
*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n
*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3
*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*a*b^2
*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^
2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*
d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*
x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2

```

$$+ 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x)) * B / e - 1/4*B*log((b*x + a)^n * e / (d*x + c)^n) / (b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*A / (b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)$$

**Fricas [B]** time = 1.1799, size = 1697, normalized size = 8.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(b\*x+a)^5,x, algorithm="fricas")

[Out] 
$$-1/48*(12*A*b^4*c^4 - 48*A*a*b^3*c^3*d + 72*A*a^2*b^2*c^2*d^2 - 48*A*a^3*b*c*d^3 + 12*A*a^4*d^4 - 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*n*x^3 + 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*n*x^2 - 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*n*x + (3*B*b^4*c^4 - 16*B*a*b^3*c^3*d + 36*B*a^2*b^2*c^2*d^2 - 48*B*a^3*b*c*d^3 + 25*B*a^4*d^4)*n - 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*n)*log(b*x + a) + 12*(B*b^4*d^4*n*x^4 + 4*B*a*b^3*d^4*n*x^3 + 6*B*a^2*b^2*d^4*n*x^2 + 4*B*a^3*b*d^4*n*x - (B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*n)*log(d*x + c) + 12*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B*a^4*d^4)*log(e))/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))/(b\*x+a)\*\*5,x)

[Out] Timed out

**Giac [B]** time = 1.37101, size = 959, normalized size = 4.92

$$\frac{Bd^4n \log(bx + a)}{4(b^5c^4 - 4ab^4c^3d + 6a^2b^3c^2d^2 - 4a^3b^2cd^3 + a^4bd^4)} - \frac{Bd^4n \log(dx + c)}{4(b^5c^4 - 4ab^4c^3d + 6a^2b^3c^2d^2 - 4a^3b^2cd^3 + a^4bd^4)} - \frac{1}{4(b^5x^4 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(b\*x+a)^5,x, algorithm="giac")



```
[Out] 1/4*B*d^4*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*B*d^4*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 1/4*B*n*log(b*x + a)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/4*B*n*log(d*x + c)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/48*(12*B*b^3*d^3*n*x^3 - 6*B*b^3*c*d^2*n*x^2 + 42*B*a*b^2*d^3*n*x^2 + 4*B*b^3*c^2*d*n*x - 20*B*a*b^2*c*d^2*n*x + 52*B*a^2*b*d^3*n*x - 3*B*b^3*c^3*n + 13*B*a*b^2*c^2*d*n - 23*B*a^2*b*c*d^2*n + 25*B*a^3*d^3*n - 12*A*b^3*c^3 - 12*B*b^3*c^3 + 36*A*a*b^2*c^2*d + 36*B*a*b^2*c^2*d - 36*A*a^2*b*c*d^2 - 36*B*a^2*b*c*d^2 + 12*A*a^3*d^3 + 12*B*a^3*d^3)/(b^8*c^3*x^4 - 3*a*b^7*c^2*d*x^4 + 3*a^2*b^6*c*d^2*x^4 - a^3*b^5*d^3*x^4 + 4*a*b^7*c^3*x^3 - 12*a^2*b^6*c^2*d*x^3 + 12*a^3*b^5*c*d^2*x^3 - 4*a^4*b^4*d^3*x^3 + 6*a^2*b^6*c^3*x^2 - 18*a^3*b^5*c^2*d*x^2 + 18*a^4*b^4*c*d^2*x^2 - 6*a^5*b^3*d^3*x^2 + 4*a^3*b^5*c^3*x - 12*a^4*b^4*c^2*d*x + 12*a^5*b^3*c*d^2*x - 4*a^6*b^2*d^3*x + a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
```

### 3.156 $\int (a+bx)^3 (A + B \log(e(a + bx)^n(c + dx)^{-n}))^2 dx$

**Optimal.** Leaf size=322

$$\frac{B^2 n^2 (bc - ad)^4 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4} - \frac{Bn(bc - ad)^4 \log\left(\frac{bc-ad}{b(c+dx)}\right) (6B \log(e(a + bx)^n(c + dx)^{-n}) + 6A + 11Bn)}{12bd^4} - \frac{Bn(a + bx)^4}{2bd^4}$$

[Out]  $-(B*(b*c - a*d)*n*(a + b*x)^3*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]))/(6*b*d) + ((a + b*x)^4*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(4*b) + (B*(b*c - a*d)^2*n*(a + b*x)^2*(3*A + B*n + 3*B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]))/(12*b*d^2) - (B*(b*c - a*d)^3*n*(a + b*x)*(6*A + 5*B*n + 6*B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]))/(12*b*d^3) - (B*(b*c - a*d)^4*n*\text{Log}[(b*c - a*d)/(b*(c + d*x))])*(6*A + 11*B*n + 6*B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(12*b*d^4) - (B^2*(b*c - a*d)^4*n^2*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b*d^4)$

**Rubi [A]** time = 0.771862, antiderivative size = 542, normalized size of antiderivative = 1.68, number of steps used = 21, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 2492, 43, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315}

$$-\frac{B^2 n^2 (bc - ad)^4 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{2bd^4} + \frac{A^2(a + bx)^4}{4b} - \frac{ABnx(bc - ad)^3}{2d^3} + \frac{ABn(a + bx)^2(bc - ad)^2}{4bd^2} + \frac{ABn(bc - ad)^4 \log\left(\frac{bc-ad}{b(c+dx)}\right)}{2bd^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2,x]

[Out]  $-(A*B*(b*c - a*d)^3*n*x)/(2*d^3) - (5*B^2*(b*c - a*d)^3*n^2*x)/(12*d^3) + (A*B*(b*c - a*d)^2*n*(a + b*x)^2)/(4*b*d^2) + (B^2*(b*c - a*d)^2*n^2*(a + b*x)^2)/(12*b*d^2) - (A*B*(b*c - a*d)*n*(a + b*x)^3)/(6*b*d) + (A^2*(a + b*x)^4)/(4*b) + (A*B*(b*c - a*d)^4*n*\text{Log}[c + d*x])/(2*b*d^4) + (11*B^2*(b*c - a*d)^4*n^2*\text{Log}[c + d*x])/(12*b*d^4) - (B^2*(b*c - a*d)^3*n*(a + b*x)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b*d^3) + (B^2*(b*c - a*d)^2*n*(a + b*x)^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(4*b*d^2) - (B^2*(b*c - a*d)*n*(a + b*x)^3*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(6*b*d) + (A*B*(a + b*x)^4*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b) - (B^2*(b*c - a*d)^4*n*\text{Log}[(b*c - a*d)/(b*(c + d*x))])*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b*d^4) + (B^2*(a + b*x)^4*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(4*b) - (B^2*(b*c - a*d)^4*n^2*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b*d^4)$

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2492

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.)\*((g\_.) + (h\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(h\*(m + 1)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(h\*(m + 1)), Int[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0] && NeQ[m, -1]

Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2514

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*(RFX\_), x\_Symbol] := With[{u = ExpandIntegrand[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]

Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s, x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2488

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := -Simp[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d))/h, Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*(e\*h - d\*i)/e + (i\*x)/e]^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

Rule 2343

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/((x\_)\*((d\_) + (e\_.)\*(x\_))^(r\_.)), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*Log[c\*x^n])/(x\*(d + e\*x^(r/n))), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]

Rule 2333

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)/(x\_))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[(e + d\*x)^q\*(a + b\*Log[c\*x^n])^p, x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]

Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rubi steps

$$\begin{aligned}
 \int (a + bx)^3 (A + B \log(e(a + bx)^n(c + dx)^{-n}))^2 dx &= \int (A^2(a + bx)^3 + 2AB(a + bx)^3 \log(e(a + bx)^n(c + dx)^{-n}) + B^2(a + bx)^3) dx \\
 &= \frac{A^2(a + bx)^4}{4b} + (2AB) \int (a + bx)^3 \log(e(a + bx)^n(c + dx)^{-n}) dx + B^2 \int (a + bx)^3 dx \\
 &= \frac{A^2(a + bx)^4}{4b} + \frac{AB(a + bx)^4 \log(e(a + bx)^n(c + dx)^{-n})}{2b} + \frac{B^2(a + bx)^4}{4b} \\
 &= \frac{A^2(a + bx)^4}{4b} + \frac{AB(a + bx)^4 \log(e(a + bx)^n(c + dx)^{-n})}{2b} + \frac{B^2(a + bx)^4}{4b} \\
 &= -\frac{AB(bc - ad)^3 nx}{2d^3} + \frac{AB(bc - ad)^2 n(a + bx)^2}{4bd^2} - \frac{AB(bc - ad)n(a + bx)}{6bd} \\
 &= -\frac{AB(bc - ad)^3 nx}{2d^3} + \frac{AB(bc - ad)^2 n(a + bx)^2}{4bd^2} - \frac{AB(bc - ad)n(a + bx)}{6bd} \\
 &= -\frac{AB(bc - ad)^3 nx}{2d^3} + \frac{AB(bc - ad)^2 n(a + bx)^2}{4bd^2} - \frac{AB(bc - ad)n(a + bx)}{6bd} \\
 &= -\frac{AB(bc - ad)^3 nx}{2d^3} - \frac{5B^2(bc - ad)^3 n^2 x}{12d^3} + \frac{AB(bc - ad)^2 n(a + bx)^2}{4bd^2} + \dots \\
 &= -\frac{AB(bc - ad)^3 nx}{2d^3} - \frac{5B^2(bc - ad)^3 n^2 x}{12d^3} + \frac{AB(bc - ad)^2 n(a + bx)^2}{4bd^2} + \dots \\
 &= -\frac{AB(bc - ad)^3 nx}{2d^3} - \frac{5B^2(bc - ad)^3 n^2 x}{12d^3} + \frac{AB(bc - ad)^2 n(a + bx)^2}{4bd^2} + \dots
 \end{aligned}$$

**Mathematica [B]** time = 1.551, size = 1709, normalized size = 5.31

$$3A^2d^4x^4b^4 - 2ABcd^3nx^3b^4 + B^2c^2d^2n^2x^2b^4 + 3ABC^2d^2nx^2b^4 + 3B^2c^4n^2 \log^2(c + dx)b^4 + 3B^2d^4x^4 \log^2(e(a + bx)^n(c + dx))$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n)]^2,x]

[Out] (-24\*a^4\*A\*B\*d^4\*n + 6\*a\*b^3\*B^2\*c^3\*d\*n^2 - 24\*a^2\*b^2\*B^2\*c^2\*d^2\*n^2 + 3\*6\*a^3\*b\*B^2\*c\*d^3\*n^2 - 24\*a^4\*B^2\*d^4\*n^2 + 12\*a^3\*A^2\*b\*d^4\*x - 6\*A\*b^4\*B\*c^3\*d\*n\*x + 24\*a\*A\*b^3\*B\*c^2\*d^2\*n\*x - 36\*a^2\*A\*b^2\*B\*c\*d^3\*n\*x + 18\*a^3\*A\*b\*B\*d^4\*n\*x - 5\*b^4\*B^2\*c^3\*d\*n^2\*x + 17\*a\*b^3\*B^2\*c^2\*d^2\*n^2\*x - 19\*a^2\*b^2\*B^2\*c\*d^3\*n^2\*x + 7\*a^3\*b\*B^2\*d^4\*n^2\*x + 18\*a^2\*A^2\*b^2\*d^4\*x^2 + 3\*A\*b^4\*B\*c^2\*d^2\*n\*x^2 - 12\*a\*A\*b^3\*B\*c\*d^3\*n\*x^2 + 9\*a^2\*A\*b^2\*B\*d^4\*n\*x^2 + b^4\*B^2\*c^2\*d^2\*n^2\*x^2 - 2\*a\*b^3\*B^2\*c\*d^3\*n^2\*x^2 + a^2\*b^2\*B^2\*d^4\*n^2\*x^2 + 12\*a\*A^2\*b^3\*d^4\*x^3 - 2\*A\*b^4\*B\*c\*d^3\*n\*x^3 + 2\*a\*A\*b^3\*B\*d^4\*n\*x^3 + 3\*A^2\*b^4\*d^4\*x^4 - 3\*a^4\*B^2\*d^4\*n^2\*Log[a + b\*x]^2 + 6\*A\*b^4\*B\*c^4\*n\*Log[c + d\*x] - 24\*a\*A\*b^3\*B\*c^3\*d\*n\*Log[c + d\*x] + 36\*a^2\*A\*b^2\*B\*c^2\*d^2\*n\*Log[c + d\*x] - 24\*a^3\*A\*b\*B\*c\*d^3\*n\*Log[c + d\*x] + 11\*b^4\*B^2\*c^4\*n^2\*Log[c + d\*x] - 38\*a\*b^3\*B^2\*c^3\*d\*n^2\*Log[c + d\*x] + 45\*a^2\*b^2\*B^2\*c^2\*d^2\*n^2\*Log[c + d\*x] - 18\*a^3\*b\*B^2\*c\*d^3\*n^2\*Log[c + d\*x] - 24\*a^4\*B^2\*d^4\*n^2\*Log[c

$$\begin{aligned}
& + d*x] + 3*b^4*B^2*c^4*n^2*Log[c + d*x]^2 - 12*a*b^3*B^2*c^3*d*n^2*Log[c + \\
& d*x]^2 + 18*a^2*b^2*B^2*c^2*d^2*n^2*Log[c + d*x]^2 - 12*a^3*b*B^2*c*d^3*n^2 \\
& *Log[c + d*x]^2 - 24*a^4*B^2*d^4*n*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 24*a \\
& ^3*A*b*B*d^4*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 6*b^4*B^2*c^3*d*n*x*Log[( \\
& e*(a + b*x)^n)/(c + d*x)^n] + 24*a*b^3*B^2*c^2*d^2*n*x*Log[(e*(a + b*x)^n)/ \\
& (c + d*x)^n] - 36*a^2*b^2*B^2*c*d^3*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + \\
& 18*a^3*b*B^2*d^4*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 36*a^2*A*b^2*B*d^4* \\
& x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 3*b^4*B^2*c^2*d^2*n*x^2*Log[(e*(a + \\
& b*x)^n)/(c + d*x)^n] - 12*a*b^3*B^2*c*d^3*n*x^2*Log[(e*(a + b*x)^n)/(c + d* \\
& x)^n] + 9*a^2*b^2*B^2*d^4*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 24*a*A*b \\
& ^3*B*d^4*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b^4*B^2*c*d^3*n*x^3*Log[( \\
& e*(a + b*x)^n)/(c + d*x)^n] + 2*a*b^3*B^2*d^4*n*x^3*Log[(e*(a + b*x)^n)/(c \\
& + d*x)^n] + 6*A*b^4*B*d^4*x^4*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 6*b^4*B^2* \\
& c^4*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 24*a*b^3*B^2*c^3*d*n* \\
& Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 36*a^2*b^2*B^2*c^2*d^2*n*Lo \\
& g[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 24*a^3*b*B^2*c*d^3*n*Log[c + \\
& d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*a^3*b*B^2*d^4*x*Log[(e*(a + b*x) \\
& ^n)/(c + d*x)^n]^2 + 18*a^2*b^2*B^2*d^4*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n \\
& ]^2 + 12*a*b^3*B^2*d^4*x^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 3*b^4*B^2*d \\
& ^4*x^4*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + B*n*Log[a + b*x]*(-6*b*B*c*(b^3 \\
& *c^3 - 4*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 4*a^3*d^3)*n*Log[c + d*x] + 6*B*(b*c \\
& - a*d)^4*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(-6*b^3*B*c^3*n + 21*a*b^2 \\
& *B*c^2*d*n - 26*a^2*b*B*c*d^2*n + a^3*d^3*(6*A + 35*B*n) + 6*a^3*B*d^3*Log[ \\
& (e*(a + b*x)^n)/(c + d*x)^n])) + 6*B^2*(b*c - a*d)^4*n^2*PolyLog[2, (d*(a + \\
& b*x))/(-b*c) + a*d)]/(12*b*d^4)
\end{aligned}$$

**Maple [C]** time = 2.524, size = 26948, normalized size = 83.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^3\*(A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x)

[Out] result too large to display

**Maxima [B]** time = 3.86044, size = 2526, normalized size = 7.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^3\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x, algorithm="maxima")

[Out]  $\frac{1}{2}A*B*b^3*x^4*\log((b*x + a)^n*e/(d*x + c)^n) + \frac{1}{4}A^2*b^3*x^4 + 2*A*B*a*b^2*x^3*\log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*b^2*x^3 + 3*A*B*a^2*b*x^2*\log((b*x + a)^n*e/(d*x + c)^n) + \frac{3}{2}A^2*a^2*b*x^2 + 2*A*B*a^3*x*\log((b*x + a)^n*e/(d*x + c)^n) + A^2*a^3*x + 2*(a*e*n*\log(b*x + a)/b - c*e*n*\log(d*x + c)/d)*A*B*a^3/e - 3*(a^2*e*n*\log(b*x + a)/b^2 - c^2*e*n*\log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*a^2*b/e + (2*a^3*e*n*\log(b*x + a)/b^3 - 2*c^3*e*n*\log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*a*b^2/e - 1/12*(6*a^4*e*n*\log(b*x + a)/b^4 - 6*c^4*e*n*\log(d*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 -$

$$3*(b^3*c^2*d*e^n - a^2*b*d^3*e^n)*x^2 + 6*(b^3*c^3*e^n - a^3*d^3*e^n)*x)/(b^3*d^3)) * A*B*b^3/e + 1/12*((11*n^2 + 6*n*log(e))*b^3*c^4 - 2*(19*n^2 + 12*n*log(e))*a*b^2*c^3*d + 9*(5*n^2 + 4*n*log(e))*a^2*b*c^2*d^2 - 6*(3*n^2 + 4*n*log(e))*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*n^2 - 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3*b*c*d^3*n^2 + a^4*d^4*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d))) *B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*x^4*log(e)^2 - 3*B^2*a^4*d^4*n^2*log(b*x + a)^2 - 2*(b^4*c*d^3*n*log(e) - (n*log(e) + 6*log(e)^2)*a*b^3*d^4)*B^2*x^3 + ((n^2 + 3*n*log(e))*b^4*c^2*d^2 - 2*(n^2 + 6*n*log(e))*a*b^3*c*d^3 + (n^2 + 9*n*log(e) + 18*log(e)^2)*a^2*b^2*d^4)*B^2*x^2 - 6*(b^4*c^4*n^2 - 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3*b*c*d^3*n^2)*B^2*log(b*x + a) *log(d*x + c) + 3*(b^4*c^4*n^2 - 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3*b*c*d^3*n^2)*B^2*log(d*x + c)^2 - ((5*n^2 + 6*n*log(e))*b^4*c^3*d - (17*n^2 + 24*n*log(e))*a*b^3*c^2*d^2 + (19*n^2 + 36*n*log(e))*a^2*b^2*c*d^3 - (7*n^2 + 18*n*log(e) + 12*log(e)^2)*a^3*b*d^4)*B^2*x - (6*a*b^3*c^3*d*n^2 - 21*a^2*b^2*c^2*d^2*n^2 + 26*a^3*b*c*d^3*n^2 - (11*n^2 + 6*n*log(e))*a^4*d^4)*B^2*log(b*x + a) + 3*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2*a^3*b*d^4*x) *log((b*x + a)^n)^2 + 3*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2*a^3*b*d^4*x) *log((d*x + c)^n)^2 + (6*B^2*b^4*d^4*x^4*log(e) + 6*B^2*a^4*d^4*n*log(b*x + a) + 2*(a*b^3*d^4*(n + 12*log(e)) - b^4*c*d^3*n)*B^2*x^3 + 3*(3*a^2*b^2*d^4*(n + 4*log(e)) + b^4*c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^2*x^2 + 6*(a^3*b*d^4*(3*n + 4*log(e)) - b^4*c^3*d*n + 4*a*b^3*c^2*d^2*n - 6*a^2*b^2*c*d^3*n)*B^2*x + 6*(b^4*c^4*n - 4*a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n - 4*a^3*b*c*d^3*n)*B^2*log(d*x + c)) *log((b*x + a)^n) - (6*B^2*b^4*d^4*x^4*log(e) + 6*B^2*a^4*d^4*n*log(b*x + a) + 2*(a*b^3*d^4*(n + 12*log(e)) - b^4*c*d^3*n)*B^2*x^3 + 3*(3*a^2*b^2*d^4*(n + 4*log(e)) + b^4*c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^2*x^2 + 6*(a^3*b*d^4*(3*n + 4*log(e)) - b^4*c^3*d*n + 4*a*b^3*c^2*d^2*n - 6*a^2*b^2*c*d^3*n)*B^2*x + 6*(b^4*c^4*n - 4*a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n - 4*a^3*b*c*d^3*n)*B^2*log(d*x + c) + 6*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2*a^3*b*d^4*x) *log((b*x + a)^n)) *log((d*x + c)^n))/(b*d^4)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2 b^3 x^3 + 3 A^2 a b^2 x^2 + 3 A^2 a^2 b x + A^2 a^3 + (B^2 b^3 x^3 + 3 B^2 a b^2 x^2 + 3 B^2 a^2 b x + B^2 a^3) \log \left( \frac{(b x + a)^n e}{(d x + c)^n} \right)^2 + 2 (A B b^3 x^3 + 3 A^2 a b^2 x^2 + 3 A B a^2 b x + A^2 a^3) \log \left( \frac{(b x + a)^n e}{(d x + c)^n} \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^3\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x, algorithm="fricas")

[Out] integral(A^2\*b^3\*x^3 + 3\*A^2\*a\*b^2\*x^2 + 3\*A^2\*a^2\*b\*x + A^2\*a^3 + (B^2\*b^3\*x^3 + 3\*B^2\*a\*b^2\*x^2 + 3\*B^2\*a^2\*b\*x + B^2\*a^3)\*log((b\*x + a)^n\*e/(d\*x + c)^n)^2 + 2\*(A\*B\*b^3\*x^3 + 3\*A\*B\*a^2\*b\*x + A\*B\*a^3)\*log((b\*x + a)^n\*e/(d\*x + c)^n), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*3\*(A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^3 \left( B \log \left( \frac{(bx + a)^n e}{(dx + c)^n} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^3\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x, algorithm="giac")

[Out] integrate((b\*x + a)^3\*(B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^2, x)

### 3.157 $\int (a+bx)^2 (A + B \log(e(a + bx)^n(c + dx)^{-n}))^2 dx$

**Optimal.** Leaf size=263

$$\frac{2B^2n^2(bc - ad)^3 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} + \frac{Bn(bc - ad)^3 \log\left(\frac{bc-ad}{b(c+dx)}\right) (2B \log(e(a + bx)^n(c + dx)^{-n}) + 2A + 3Bn)}{3bd^3} + \frac{Bn(a + bx)^3}{3bd^3}$$

[Out]  $-(B*(b*c - a*d)*n*(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*b*d) + ((a + b*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(3*b) + (B*(b*c - a*d)^2*n*(a + b*x)*(2*A + B*n + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*b*d^2) + (B*(b*c - a*d)^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*(2*A + 3*B*n + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*b*d^3) + (2*B^2*(b*c - a*d)^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)$

**Rubi [A]** time = 0.625638, antiderivative size = 427, normalized size of antiderivative = 1.62, number of steps used = 18, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 2492, 43, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315}

$$\frac{2B^2n^2(bc - ad)^3 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} + \frac{A^2(a + bx)^3}{3b} + \frac{2ABnx(bc - ad)^2}{3d^2} - \frac{2ABn(bc - ad)^3 \log(c + dx)}{3bd^3} + \frac{2AB(a + bx)^3}{3bd^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2, x]$

[Out]  $(2*A*B*(b*c - a*d)^2*n*x)/(3*d^2) + (B^2*(b*c - a*d)^2*n^2*x)/(3*d^2) - (A*B*(b*c - a*d)*n*(a + b*x)^2)/(3*b*d) + (A^2*(a + b*x)^3)/(3*b) - (2*A*B*(b*c - a*d)^3*n*Log[c + d*x])/(3*b*d^3) - (B^2*(b*c - a*d)^3*n^2*Log[c + d*x])/(b*d^3) + (2*B^2*(b*c - a*d)^2*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b*d^2) - (B^2*(b*c - a*d)*n*(a + b*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b*d) + (2*A*B*(a + b*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b) + (2*B^2*(b*c - a*d)^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b*d^3) + (B^2*(a + b*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(3*b) + (2*B^2*(b*c - a*d)^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

#### Rule 2492

$\text{Int}[\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.))*((c_.) + (d_.)*(x_.))^(q_.))]^(r_.)]^(s_.)*((g_.) + (h_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[\text{((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)^s)/(h*(m + 1)), x] - \text{Dist}[(p*r*s*(b*c - a*d))/(h*(m + 1)), \text{Int}[\text{((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r)^s - 1)/(a + b*x*(c + d*x)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{NeQ}[m, -1]$

#### Rule 43

$\text{Int}[\text{((a_.) + (b_.)*(x_.))^(m_.))*((c_.) + (d_.)*(x_.))^(n_.), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[m, 0] \&\& (!\text{IntegerQ}[n] || (\text{EqQ}[c, 0] \&\& \text{Le}$



$Q[7*m + 4*n + 4, 0] \mid \mid LtQ[9*m + 5*(n + 1), 0] \mid \mid GtQ[m + n + 2, 0]$

#### Rule 2514

$Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.)*(RFX_), x\_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]$

#### Rule 2486

$Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.), x\_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]$

#### Rule 31

$Int[((a_) + (b_.)*(x_))^{-1}, x\_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]$

#### Rule 2488

$Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x\_Symbol] := -Simp[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/h, x] + Dist[(p*r*s*(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ[b*g - a*h, 0] && IGtQ[s, 0]$

#### Rule 2411

$Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x\_Symbol] := Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]$

#### Rule 2343

$Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))), x\_Symbol] := Dist[1/n, Subst[Int[(a + b*Log[c*x])/x*(d + e*x^(r/n))], x], x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]$

#### Rule 2333

$Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(x_)^(m_.), x\_Symbol] := Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]$

#### Rule 2315

$Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]$

#### Rubi steps



$$d^{2n} \text{Log}[c + dx] \text{Log}[(e(a + bx)^n)/(c + dx)^n] + 3a^2 b B^2 d^{3x} \text{Log}[(e(a + bx)^n)/(c + dx)^n]^2 + 3ab^2 B^2 d^{3x} \text{Log}[(e(a + bx)^n)/(c + dx)^n]^2 + b^3 B^2 d^{3x} \text{Log}[(e(a + bx)^n)/(c + dx)^n]^2 + B \text{Log}[a + bx] * (2bBc(b^2c^2 - 3a*b*c*d + 3a^2d^2) * n \text{Log}[c + dx] - 2B(b*c - a*d)^3 * n \text{Log}[(b(c + dx))/(b*c - a*d)] + a*d(2b^2Bc^2n - 5a*b*B*c*d*n + a^2d^2(2A + 9B*n) + 2a^2B*d^2 \text{Log}[(e(a + bx)^n)/(c + dx)^n])) - 2B^2(b*c - a*d)^3 * n^2 \text{PolyLog}[2, (d(a + bx))/(-b*c + a*d)] / (3*b*d^3)$$


---

**Maple [C]** time = 2.135, size = 19969, normalized size = 75.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^2\*(A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x)

[Out] result too large to display

---

**Maxima [B]** time = 3.73536, size = 1733, normalized size = 6.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x, algorithm="maxima")

[Out]  $\frac{2}{3}A*B*b^2*x^3*\log((b*x + a)^n*e/(d*x + c)^n) + \frac{1}{3}A^2*b^2*x^3 + 2*A*B*a*b*x^2*\log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*b*x^2 + 2*A*B*a^2*x*\log((b*x + a)^n*e/(d*x + c)^n) + A^2*a^2*x + 2*(a*e*n*\log(b*x + a)/b - c*e*n*\log(d*x + c)/d)*A*B*a^2/e - 2*(a^2*e*n*\log(b*x + a)/b^2 - c^2*e*n*\log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*a*b/e + \frac{1}{3}*(2*a^3*e*n*\log(b*x + a)/b^3 - 2*c^3*e*n*\log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A*B*b^2/e - \frac{1}{3}*((3*n^2 + 2*n*\log(e))*b^2*c^3 - (7*n^2 + 6*n*\log(e))*a*b*c^2*d + 2*(2*n^2 + 3*n*\log(e))*a^2*c*d^2)*B^2*\log(d*x + c)/d^3 - \frac{2}{3}*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2 + 3*a^2*b*c*d^2*n^2 - a^3*d^3*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + \text{dilog}(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^3) + \frac{1}{3}*(B^2*b^3*d^3*x^3*\log(e)^2 - B^2*a^3*d^3*n^2*\log(b*x + a)^2 - (b^3*c*d^2*n*\log(e) - (n*\log(e) + 3*\log(e)^2)*a*b^2*d^3)*B^2*x^2 + 2*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2 + 3*a^2*b*c*d^2*n^2)*B^2*\log(b*x + a)*log(d*x + c) - (b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2 + 3*a^2*b*c*d^2*n^2)*B^2*\log(d*x + c)^2 + ((n^2 + 2*n*\log(e))*b^3*c^2*d - 2*(n^2 + 3*n*\log(e))*a*b^2*c*d^2 + (n^2 + 4*n*\log(e) + 3*\log(e)^2)*a^2*b*d^3)*B^2*x + (2*a*b^2*c^2*d*n^2 - 5*a^2*b*c*d^2*n^2 + (3*n^2 + 2*n*\log(e))*a^3*d^3)*B^2*\log(b*x + a) + (B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x)*log((b*x + a)^n)^2 + (B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x)*log((d*x + c)^n)^2 + (2*B^2*b^3*d^3*x^3*\log(e) + 2*B^2*a^3*d^3*n*\log(b*x + a) + (a*b^2*d^3*(n + 6*\log(e)) - b^3*c*d^2*n)*B^2*x^2 + 2*(a^2*b*d^3*(2*n + 3*\log(e)) + b^3*c^2*d*n - 3*a*b^2*c*d^2*n)*B^2*x - 2*(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n)*B^2*x - 2*(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n)*B^2*x - 2*(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n)*B^2*x$

$c*d^{2*n}*B^2*\log(d*x + c) + 2*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x)*\log((b*x + a)^n)*\log((d*x + c)^n)/(b*d^3)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral  $\left( A^2 b^2 x^2 + 2 A^2 a b x + A^2 a^2 + (B^2 b^2 x^2 + 2 B^2 a b x + B^2 a^2) \log\left(\frac{(b x + a)^n e}{(d x + c)^n}\right)^2 + 2 (A B b^2 x^2 + 2 A B a b x + A B a^2) \log\left(\frac{(b x + a)^n e}{(d x + c)^n}\right) \right) dx$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x, algorithm="fricas")

[Out] integral(A^2\*b^2\*x^2 + 2\*A^2\*a\*b\*x + A^2\*a^2 + (B^2\*b^2\*x^2 + 2\*B^2\*a\*b\*x + B^2\*a^2)\*log((b\*x + a)^n\*e/(d\*x + c)^n)^2 + 2\*(A\*B\*b^2\*x^2 + 2\*A\*B\*a\*b\*x + A\*B\*a^2)\*log((b\*x + a)^n\*e/(d\*x + c)^n), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*(A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^2 \left( B \log\left(\frac{(bx + a)^n e}{(dx + c)^n}\right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x, algorithm="giac")

[Out] integrate((b\*x + a)^2\*(B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^2, x)

### 3.158 $\int (a+bx) (A + B \log (e(a + bx)^n (c + dx)^{-n}))^2 dx$

**Optimal.** Leaf size=195

$$\frac{B^2 n^2 (bc - ad)^2 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} - \frac{Bn(bc - ad)^2 \log\left(\frac{bc-ad}{b(c+dx)}\right) (B \log (e(a + bx)^n (c + dx)^{-n}) + A + Bn)}{bd^2} - \frac{Bn(a + b)}{bd^2}$$

```
[Out] -((B*(b*c - a*d)*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n]/(c + d*x)^n)))/(b*d
)) + ((a + b*x)^2*(A + B*Log[(e*(a + b*x)^n]/(c + d*x)^n])^2)/(2*b) - (B*(b
*c - a*d)^2*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*n + B*Log[(e*(a + b*x)^
n]/(c + d*x)^n)))/(b*d^2) - (B^2*(b*c - a*d)^2*n^2*PolyLog[2, (d*(a + b*x))
/(b*(c + d*x))])/(b*d^2)
```

**Rubi [A]** time = 0.485903, antiderivative size = 308, normalized size of antiderivative = 1.58, number of steps used = 15, number of rules used = 11, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$ , Rules used = {6742, 2492, 43, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315}

$$\frac{B^2 n^2 (bc - ad)^2 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} + \frac{A^2 (a + bx)^2}{2b} + \frac{ABn(bc - ad)^2 \log(c + dx)}{bd^2} + \frac{AB(a + bx)^2 \log(e(a + bx)^n (c + dx)^{-n})}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(A + B*Log[(e*(a + b*x)^n]/(c + d*x)^n])^2, x]
```

```
[Out] -((A*B*(b*c - a*d)*n*x)/d) + (A^2*(a + b*x)^2)/(2*b) + (A*B*(b*c - a*d)^2*n
*Log[c + d*x])/(b*d^2) + (B^2*(b*c - a*d)^2*n^2*Log[c + d*x])/(b*d^2) - (B^
2*(b*c - a*d)*n*(a + b*x)*Log[(e*(a + b*x)^n]/(c + d*x)^n])/(b*d) + (A*B*(a
+ b*x)^2*Log[(e*(a + b*x)^n]/(c + d*x)^n])/b - (B^2*(b*c - a*d)^2*n*Log[(b
*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n]/(c + d*x)^n])/(b*d^2) + (B^2*(
a + b*x)^2*Log[(e*(a + b*x)^n]/(c + d*x)^n]^2)/(2*b) - (B^2*(b*c - a*d)^2*n
^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2)
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_.))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
0] && NeQ[m, -1]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFX_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]
```

### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s, x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q]^r]^s)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] := Dist[1/e, Subst[Int
[(g*x)/e]^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x]^n)^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

### Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] := Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

### Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] := Int[(e + d*x)^q*(a + b*Log[c*x]^n)^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rubi steps

$$\begin{aligned}
\int (a + bx) (A + B \log(e(a + bx)^n(c + dx)^{-n}))^2 dx &= \int (A^2(a + bx) + 2AB(a + bx) \log(e(a + bx)^n(c + dx)^{-n}) + B^2(a + bx) \log^2(e(a + bx)^n(c + dx)^{-n})) dx \\
&= \frac{A^2(a + bx)^2}{2b} + (2AB) \int (a + bx) \log(e(a + bx)^n(c + dx)^{-n}) dx + \frac{B^2}{2} \int (a + bx) \log^2(e(a + bx)^n(c + dx)^{-n}) dx \\
&= \frac{A^2(a + bx)^2}{2b} + \frac{AB(a + bx)^2 \log(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{B^2(a + bx)^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{2b} \\
&= \frac{A^2(a + bx)^2}{2b} + \frac{AB(a + bx)^2 \log(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{B^2(a + bx)^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{2b} \\
&= -\frac{AB(bc - ad)nx}{d} + \frac{A^2(a + bx)^2}{2b} + \frac{AB(bc - ad)^2 n \log(c + dx)}{bd^2} + \frac{B^2(a + bx)^2 \log^2(c + dx)}{2bd^2} \\
&= -\frac{AB(bc - ad)nx}{d} + \frac{A^2(a + bx)^2}{2b} + \frac{AB(bc - ad)^2 n \log(c + dx)}{bd^2} + \frac{B^2(a + bx)^2 \log^2(c + dx)}{2bd^2} \\
&= -\frac{AB(bc - ad)nx}{d} + \frac{A^2(a + bx)^2}{2b} + \frac{AB(bc - ad)^2 n \log(c + dx)}{bd^2} + \frac{B^2(a + bx)^2 \log^2(c + dx)}{2bd^2} \\
&= -\frac{AB(bc - ad)nx}{d} + \frac{A^2(a + bx)^2}{2b} + \frac{AB(bc - ad)^2 n \log(c + dx)}{bd^2} + \frac{B^2(a + bx)^2 \log^2(c + dx)}{2bd^2} \\
&= -\frac{AB(bc - ad)nx}{d} + \frac{A^2(a + bx)^2}{2b} + \frac{AB(bc - ad)^2 n \log(c + dx)}{bd^2} + \frac{B^2(a + bx)^2 \log^2(c + dx)}{2bd^2} \\
&= -\frac{AB(bc - ad)nx}{d} + \frac{A^2(a + bx)^2}{2b} + \frac{AB(bc - ad)^2 n \log(c + dx)}{bd^2} + \frac{B^2(a + bx)^2 \log^2(c + dx)}{2bd^2} \\
&= -\frac{AB(bc - ad)nx}{d} + \frac{A^2(a + bx)^2}{2b} + \frac{AB(bc - ad)^2 n \log(c + dx)}{bd^2} + \frac{B^2(a + bx)^2 \log^2(c + dx)}{2bd^2}
\end{aligned}$$

**Mathematica [B]** time = 0.708813, size = 656, normalized size = 3.36

$$\frac{B^2 n^2 (bc - ad)^2 \text{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right)}{bd^2} - \frac{2a^2 ABn}{b} - \frac{2a^2 B^2 n \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{2a^2 B^2 n^2 \log(c + dx)}{b} - \frac{a^2 B^2 n^2}{b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2, x]

[Out]  $(-2*a^2*A*B*n)/b - (2*a^2*B^2*n^2)/b + (a*B^2*c*n^2)/d + a*A^2*x + a*A*B*n*x - (A*b*B*c*n*x)/d + (A^2*b*x^2)/2 - (a^2*B^2*n^2*\text{Log}[a + b*x]^2)/(2*b) + (A*b*B*c^2*n*\text{Log}[c + d*x])/d^2 - (2*a*A*B*c*n*\text{Log}[c + d*x])/d - (2*a^2*B^2*n^2*\text{Log}[c + d*x])/b + (b*B^2*c^2*n^2*\text{Log}[c + d*x])/d^2 - (a*B^2*c*n^2*\text{Log}[c + d*x])/d + (b*B^2*c^2*n^2*\text{Log}[c + d*x]^2)/(2*d^2) - (a*B^2*c*n^2*\text{Log}[c + d*x]^2)/d - (2*a^2*B^2*n*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/b + 2*a*A*B*x*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + a*B^2*n*x*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] - (b*B^2*c*n*x*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/d + A*b*B*x^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + (b*B^2*c^2*n*\text{Log}[c + d*x]*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/d^2 - (2*a*B^2*c*n*\text{Log}[c + d*x]*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/d + a*B^2*x*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2 + (b*B^2*x^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2)/2 + (B*n*\text{Log}[a + b*x]*(b*B*c*(-(b*c) + 2*a*d)*n*\text{Log}[c + d*x] + B*(b*c - a*d)^2*n*\text{Log}[(b*(c + d*x))/(b*c - a*d)] + a*d*(-(b*B*c*n) + a*d*(A + 3*B*n) + a*B*d*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*d^2) + (B^2*(b*c - a*d)^2*n^2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]/(b*d^2)$

**Maple [C]** time = 1.651, size = 10210, normalized size = 52.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x+a)*(A+B*\ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)$

[Out] result too large to display

**Maxima [B]** time = 3.611, size = 1052, normalized size = 5.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x+a)*(A+B*\log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, \text{algorithm}="maxima")$

[Out]  $A*B*b*x^2*\log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^2*b*x^2 + 2*A*B*a*x*\log((b*x + a)^n*e/(d*x + c)^n) + A^2*a*x + 2*(a*e*n*\log(b*x + a)/b - c*e*n*\log(d*x + c)/d)*A*B*a/e - (a^2*e*n*\log(b*x + a)/b^2 - c^2*e*n*\log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*b/e + ((n^2 + n*\log(e))*b*c^2 - (n^2 + 2*n*\log(e))*a*c*d)*B^2*\log(d*x + c)/d^2 + (b^2*c^2*n^2 - 2*a*b*c*d*n^2 + a^2*d^2*n^2)*(\log(b*x + a)*\log((b*d*x + a*d)/(b*c - a*d) + 1) + \text{dilog}(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) - 1/2*(B^2*a^2*d^2*n^2*\log(b*x + a)^2 - B^2*b^2*d^2*x^2*\log(e)^2 + 2*(b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^2*\log(b*x + a)*\log(d*x + c) - (b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^2*\log(d*x + c)^2 + 2*(b^2*c*d*n*\log(e) - (n*\log(e) + \log(e)^2)*a*b*d^2)*B^2*x + 2*(a*b*c*d*n^2 - (n^2 + n*\log(e))*a^2*d^2)*B^2*\log(b*x + a) - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*\log((b*x + a)^n)^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*\log((d*x + c)^n)^2 - 2*(B^2*b^2*d^2*x^2*\log(e) + B^2*a^2*d^2*n*\log(b*x + a) + (a*b*d^2*(n + 2*\log(e)) - b^2*c*d*n)*B^2*x + (b^2*c^2*n - 2*a*b*c*d*n)*B^2*\log(d*x + c))*\log((b*x + a)^n) + 2*(B^2*b^2*d^2*x^2*\log(e) + B^2*a^2*d^2*n*\log(b*x + a) + (a*b*d^2*(n + 2*\log(e)) - b^2*c*d*n)*B^2*x + (b^2*c^2*n - 2*a*b*c*d*n)*B^2*\log(d*x + c) + (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x)*\log((b*x + a)^n))*\log((d*x + c)^n)/(b*d^2)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(A^2bx + A^2a + (B^2bx + B^2a)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 2(ABbx + ABA)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x+a)*(A+B*\log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, \text{algorithm}="fricas")$

[Out]  $\text{integral}(A^2*b*x + A^2*a + (B^2*b*x + B^2*a)*\log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*(A*B*b*x + A*B*a)*\log((b*x + a)^n*e/(d*x + c)^n), x)$



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*(A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a) \left( B \log \left( \frac{(bx + a)^n e}{(dx + c)^n} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x, algorithm="giac")

[Out] integrate((b\*x + a)\*(B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^2, x)

$$3.159 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2}{a+bx} dx$$

**Optimal.** Leaf size=131

$$\frac{2Bn \operatorname{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) (B \log(e(a+bx)^n(c+dx)^{-n}) + A)}{b} + \frac{2B^2 n^2 \operatorname{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right) \log\left(1 - \frac{b(c+dx)}{d(a+bx)}\right) (B \log(e(a+bx)^n(c+dx)^{-n}) + A)}{b}$$

[Out] -(((A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2\*Log[1 - (b\*(c + d\*x))/(d\*(a + b\*x)]))/b) + (2\*B\*n\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])\*PolyLog[2, (b\*(c + d\*x))/(d\*(a + b\*x)]))/b + (2\*B^2\*n^2\*PolyLog[3, (b\*(c + d\*x))/(d\*(a + b\*x)]))/b

**Rubi [A]** time = 0.504468, antiderivative size = 227, normalized size of antiderivative = 1.73, number of steps used = 10, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {6742, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

$$\frac{2ABn \operatorname{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right)}{b} + \frac{2B^2 n \operatorname{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right) \log(e(a+bx)^n(c+dx)^{-n})}{b} + \frac{2B^2 n^2 \operatorname{PolyLog}\left(3, \frac{bc-ad}{d(a+bx)} + 1\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2/(a + b\*x), x]

[Out] (A^2\*Log[a + b\*x])/b - (2\*A\*B\*Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/b - (B^2\*Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]^2)/b + (2\*A\*B\*n\*PolyLog[2, 1 + (b\*c - a\*d)/(d\*(a + b\*x)]))/b + (2\*B^2\*n\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]\*PolyLog[2, 1 + (b\*c - a\*d)/(d\*(a + b\*x)]))/b + (2\*B^2\*n^2\*PolyLog[3, 1 + (b\*c - a\*d)/(d\*(a + b\*x)]))/b

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2488

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := -Simp[(Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d))/h, Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

#### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

#### Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

### Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2506

```
Int[Log[v_] * Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_
))^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^2}{a + bx} dx &= \int \left( \frac{A^2}{a + bx} + \frac{2AB \log(e(a + bx)^n(c + dx)^{-n})}{a + bx} + \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{a + bx} \right) dx \\
&= \frac{A^2 \log(a + bx)}{b} + (2AB) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{a + bx} dx + B^2 \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{a + bx} dx \\
&= \frac{A^2 \log(a + bx)}{b} - \frac{2AB \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{B^2 \log^2\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} \\
&= \frac{A^2 \log(a + bx)}{b} - \frac{2AB \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{B^2 \log^2\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} \\
&= \frac{A^2 \log(a + bx)}{b} - \frac{2AB \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{B^2 \log^2\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} \\
&= \frac{A^2 \log(a + bx)}{b} - \frac{2AB \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{B^2 \log^2\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b}
\end{aligned}$$

**Mathematica [B]** time = 0.180241, size = 269, normalized size = 2.05

$$2ABn\text{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right) + 2B^2n\text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) \log(e(a+bx)^n(c+dx)^{-n}) + 2B^2n^2\text{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right) + A^2 \log$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2/(a + b\*x), x]

[Out] 
$$\begin{aligned} & -(A*B*n*\text{Log}[(-b*c) + a*d]/(d*(a + b*x)))^2 + A^2*\text{Log}[a + b*x] - 2*A*B*n* \\ & \text{Log}[(-b*c) + a*d]/(d*(a + b*x))*\text{Log}[(b*(c + d*x))/(b*c - a*d)] - 2*A*B*Lo \\ & \text{g}[(-b*c) + a*d]/(d*(a + b*x))*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] - B^2*\text{Log}[ \\ & (-b*c) + a*d]/(d*(a + b*x))*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*A*B*n* \\ & \text{PolyLog}[2, (d*(a + b*x))/(-b*c) + a*d] + 2*B^2*n*\text{Log}[(e*(a + b*x)^n)/(c + \\ & d*x)^n]*\text{PolyLog}[2, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*n^2*\text{PolyLog}[3, (b* \\ & (c + d*x))/(d*(a + b*x))]/b \end{aligned}$$

**Maple [F]** time = 2.042, size = 0, normalized size = 0.

$$\int \frac{1}{bx+a} \left( A + B \ln \left( \frac{e(bx+a)^n}{(dx+c)^n} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a), x)

[Out] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{B^2 \log(bx+a) \log((dx+c)^n)^2}{b} + \frac{A^2 \log(bx+a)}{b} - \int -\frac{B^2bc \log(e)^2 + 2ABbc \log(e) + (B^2bdx + B^2bc) \log((bx+a)^n)^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a), x, algorithm="maxima")

[Out] 
$$\begin{aligned} & B^2*\log(b*x + a)*\log((d*x + c)^n)^2/b + A^2*\log(b*x + a)/b - \text{integrate}(-(B^ \\ & 2*b*c*\log(e)^2 + 2*A*B*b*c*\log(e) + (B^2*b*d*x + B^2*b*c)*\log((b*x + a)^n)^ \\ & 2 + (B^2*b*d*\log(e)^2 + 2*A*B*b*d*\log(e))*x + 2*(B^2*b*c*\log(e) + A*B*b*c + \\ & (B^2*b*d*\log(e) + A*B*b*d)*x)*\log((b*x + a)^n) - 2*(B^2*b*c*\log(e) + A*B*b \\ & *c + (B^2*b*d*\log(e) + A*B*b*d)*x + (B^2*b*d*n*x + B^2*a*d*n)*\log(b*x + a) \\ & + (B^2*b*d*x + B^2*b*c)*\log((b*x + a)^n))*\log((d*x + c)^n))/(b^2*d*x^2 + a* \\ & b*c + (b^2*c + a*b*d)*x), x \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B^2 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + 2AB \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^2}{bx+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(d*x + c)^n) + A^2)/(b*x + a), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log\left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)^2}{bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a), x)
```

$$3.160 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^2} dx$$

**Optimal.** Leaf size=129

$$-\frac{2Bn(c+dx)(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{(a+bx)(bc-ad)} - \frac{(c+dx)(B \log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)(bc-ad)} - \frac{2B^2n^2(c+dx)}{(a+bx)(bc-ad)}$$

[Out]  $(-2*B^2*n^2*(c+d*x))/((b*c-a*d)*(a+b*x)) - (2*B*n*(c+d*x)*(A+B*Log[(e*(a+b*x)^n]/(c+d*x)^n]))/((b*c-a*d)*(a+b*x)) - ((c+d*x)*(A+B*Log[(e*(a+b*x)^n]/(c+d*x)^n])^2)/((b*c-a*d)*(a+b*x))$

**Rubi [A]** time = 0.182181, antiderivative size = 189, normalized size of antiderivative = 1.47, number of steps used = 7, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {6742, 2490, 32}

$$-\frac{A^2}{b(a+bx)} - \frac{2AB(c+dx) \log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)(bc-ad)} - \frac{2ABn}{b(a+bx)} - \frac{B^2(c+dx) \log^2(e(a+bx)^n(c+dx)^{-n})}{(a+bx)(bc-ad)} - \frac{2B^2n(c+dx)}{(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n]/(c + d\*x)^n])^2/(a + b\*x)^2, x]

[Out]  $-(A^2/(b*(a+b*x))) - (2*A*B*n)/(b*(a+b*x)) - (2*B^2*n^2)/(b*(a+b*x)) - (2*A*B*(c+d*x)*Log[(e*(a+b*x)^n]/(c+d*x)^n])/((b*c-a*d)*(a+b*x)) - (2*B^2*n*(c+d*x)*Log[(e*(a+b*x)^n]/(c+d*x)^n])/((b*c-a*d)*(a+b*x)) - (B^2*(c+d*x)*Log[(e*(a+b*x)^n]/(c+d*x)^n]^2)/((b*c-a*d)*(a+b*x))$

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 2490

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_))^2, x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/((b\*g - a\*h)\*(g + h\*x)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(b\*g - a\*h), Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x)\*(g + h\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && NeQ[b\*g - a\*h, 0] && IGtQ[s, 0]

### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^2}{(a + bx)^2} dx &= \int \left( \frac{A^2}{(a + bx)^2} + \frac{2AB \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^2} + \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^2} \right) dx \\
&= -\frac{A^2}{b(a + bx)} + (2AB) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^2} dx + B^2 \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^2} dx \\
&= -\frac{A^2}{b(a + bx)} - \frac{2AB(c + dx) \log(e(a + bx)^n(c + dx)^{-n})}{(bc - ad)(a + bx)} - \frac{B^2(c + dx) \log^2(e(a + bx)^n(c + dx)^{-n})}{(bc - ad)(a + bx)} \\
&= -\frac{A^2}{b(a + bx)} - \frac{2ABn}{b(a + bx)} - \frac{2AB(c + dx) \log(e(a + bx)^n(c + dx)^{-n})}{(bc - ad)(a + bx)} - \frac{2B^2n^2}{(bc - ad)(a + bx)} \\
&= -\frac{A^2}{b(a + bx)} - \frac{2ABn}{b(a + bx)} - \frac{2B^2n^2}{b(a + bx)} - \frac{2AB(c + dx) \log(e(a + bx)^n(c + dx)^{-n})}{(bc - ad)(a + bx)}
\end{aligned}$$

**Mathematica [A]** time = 0.370513, size = 236, normalized size = 1.83

$$\frac{-(bc - ad) \left( 2B(A + Bn) \log(e(a + bx)^n(c + dx)^{-n}) + B^2 \log^2(e(a + bx)^n(c + dx)^{-n}) + A^2 + 2ABn + 2B^2n^2 \right) - 2Bdn(a + bx)}{(a + bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2/(a + b\*x)^2,x]

[Out] (B^2\*d\*n^2\*(a + b\*x)\*Log[a + b\*x]^2 + B^2\*d\*n^2\*(a + b\*x)\*Log[c + d\*x]^2 + 2\*B\*d\*n\*(a + b\*x)\*Log[c + d\*x]\*(A + B\*n + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]) - 2\*B\*d\*n\*(a + b\*x)\*Log[a + b\*x]\*(A + B\*n + B\*n\*Log[c + d\*x] + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]) - (b\*c - a\*d)\*(A^2 + 2\*A\*B\*n + 2\*B^2\*n^2 + 2\*B\*(A + B\*n)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n] + B^2\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]^2))/(b\*(b\*c - a\*d)\*(a + b\*x))

**Maple [C]** time = 1.365, size = 10098, normalized size = 78.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a)^2,x)

[Out] result too large to display

**Maxima [B]** time = 1.25491, size = 606, normalized size = 4.7

$$-B^2 \left( \frac{2 \left( \frac{den \log(bx+a)}{b^2c-abd} - \frac{den \log(dx+c)}{b^2c-abd} + \frac{en}{b^2x+ab} \right) \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + \frac{2bce^2n^2 - 2ade^2n^2 - (bde^2n^2x + ade^2n^2) \log(bx+a)^2}{e}}{e} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a)^2,x, algorithm="maxima")

```
[Out] -B^2*(2*(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a
*b*d) + e*n/(b^2*x + a*b))*log((b*x + a)^n*e/(d*x + c)^n)/e + (2*b*c*e^2*n^
2 - 2*a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a)^2 - (b*d*e^2
*n^2*x + a*d*e^2*n^2)*log(d*x + c)^2 + 2*(b*d*e^2*n^2*x + a*d*e^2*n^2)*log(
b*x + a) - 2*(b*d*e^2*n^2*x + a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*l
og(b*x + a))*log(d*x + c))/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)*e^2))
- B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^2*x + a*b) - 2*(d*e*n*log(b*x +
a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b^2*x + a*b)
)*A*B/e - 2*A*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b) - A^2/(b^2*x +
a*b)
```

**Fricas [B]** time = 1.11469, size = 755, normalized size = 5.85

$$A^2bc - A^2ad + 2(B^2bc - B^2ad)n^2 + (B^2bdn^2x + B^2bcn^2)\log(bx + a)^2 + (B^2bdn^2x + B^2bcn^2)\log(dx + c)^2 + (B^2bc - B^2ad)n^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="fric
as")
```

```
[Out] -(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*d*n^2*x + B^2*b*c*
n^2)*log(b*x + a)^2 + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log(d*x + c)^2 + (B^2*b
*c - B^2*a*d)*log(e)^2 + 2*(A*B*b*c - A*B*a*d)*n + 2*(B^2*b*c*n^2 + A*B*b*c
*n + (B^2*b*d*n^2 + A*B*b*d*n)*x + (B^2*b*d*n*x + B^2*b*c*n)*log(e))*log(b*
x + a) - 2*(B^2*b*c*n^2 + A*B*b*c*n + (B^2*b*d*n^2 + A*B*b*d*n)*x + (B^2*b*
d*n^2*x + B^2*b*c*n^2)*log(b*x + a) + (B^2*b*d*n*x + B^2*b*c*n)*log(e))*log
(d*x + c) + 2*(A*B*b*c - A*B*a*d + (B^2*b*c - B^2*a*d)*n)*log(e))/(a*b^2*c
- a^2*b*d + (b^3*c - a*b^2*d)*x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^2, x)
```



$$3.161 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^3} dx$$

**Optimal.** Leaf size=274

$$\frac{bBn(c+dx)^2(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{2(a+bx)^2(bc-ad)^2} + \frac{2Bdn(c+dx)(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{(a+bx)(bc-ad)^2} - \frac{b(c+dx)^2(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{2(a+bx)^2(bc-ad)^2}$$

```
[Out] (2*B^2*d*n^2*(c+d*x))/((b*c-a*d)^2*(a+b*x)) - (b*B^2*n^2*(c+d*x)^2)
/(4*(b*c-a*d)^2*(a+b*x)^2) + (2*B*d*n*(c+d*x)*(A+B*Log[(e*(a+b*x)
^n)/(c+d*x)^n]))/((b*c-a*d)^2*(a+b*x)) - (b*B*n*(c+d*x)^2*(A+B*Lo
g[(e*(a+b*x)^n)/(c+d*x)^n]))/(2*(b*c-a*d)^2*(a+b*x)^2) + (d*(c+d*
x)*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n])^2)/((b*c-a*d)^2*(a+b*x)) -
(b*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n])^2)/(2*(b*c-a*d)^2
*(a+b*x)^2)
```

**Rubi [A]** time = 0.423289, antiderivative size = 411, normalized size of antiderivative = 1.5, number of steps used = 12, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {6742, 2492, 44, 2491, 2490, 32, 2509, 37}

$$-\frac{A^2}{2b(a+bx)^2} + \frac{ABd^2n \log(a+bx)}{b(bc-ad)^2} - \frac{ABd^2n \log(c+dx)}{b(bc-ad)^2} - \frac{AB \log(e(a+bx)^n(c+dx)^{-n})}{b(a+bx)^2} + \frac{ABdn}{b(a+bx)(bc-ad)} - \frac{b(c+dx)^2(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{2(a+bx)^2(bc-ad)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a + b*x)^3,x]
```

```
[Out] -A^2/(2*b*(a + b*x)^2) - (A*B*n)/(2*b*(a + b*x)^2) + (A*B*d*n)/(b*(b*c - a*
d)*(a + b*x)) + (2*B^2*d*n^2)/(b*(b*c - a*d)*(a + b*x)) - (b*B^2*n^2*(c + d
*x)^2)/(4*(b*c - a*d)^2*(a + b*x)^2) + (A*B*d^2*n*Log[a + b*x])/(b*(b*c - a
*d)^2) - (A*B*d^2*n*Log[c + d*x])/(b*(b*c - a*d)^2) - (A*B*Log[(e*(a + b*x)
^n)/(c + d*x)^n])/(b*(a + b*x)^2) + (2*B^2*d*n*(c + d*x)*Log[(e*(a + b*x)^n
)/(c + d*x)^n])/((b*c - a*d)^2*(a + b*x)) - (b*B^2*n*(c + d*x)^2*Log[(e*(a
+ b*x)^n)/(c + d*x)^n])/(2*(b*c - a*d)^2*(a + b*x)^2) + (B^2*d*(c + d*x)*Lo
g[(e*(a + b*x)^n)/(c + d*x)^n]^2)/((b*c - a*d)^2*(a + b*x)) - (b*B^2*(c + d
*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*(b*c - a*d)^2*(a + b*x)^2)
```

**Rule 6742**

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

**Rule 2492**

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.)
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_.))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
0] && NeQ[m, -1]
```

**Rule 44**

```
Int[((a_) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
```

+ n + 2, 0])

### Rule 2491

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_))^3, x_Symbol] := Dist[d/(d*g - c*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/(g + h*x)^2, x], x] - Dist[h/(d*g - c*h), Int[((c + d*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(g + h*x)^3, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 0]
```

### Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_))^2, x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1)/(c + d*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0]
```

### Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

### Rule 2509

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/((m + 1)*(b*c - a*d)), x] - Dist[(p*r*s*(b*c - a*d))/(m + 1)*(b*c - a*d), Int[(a + b*x)^m*(c + d*x)^n*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^(s - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1] && IGtQ[s, 0]
```

### Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^2}{(a + bx)^3} dx &= \int \left( \frac{A^2}{(a + bx)^3} + \frac{2AB \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^3} + \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^3} \right) dx \\
&= -\frac{A^2}{2b(a + bx)^2} + (2AB) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^3} dx + B^2 \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^3} dx \\
&= -\frac{A^2}{2b(a + bx)^2} - \frac{AB \log(e(a + bx)^n(c + dx)^{-n})}{b(a + bx)^2} + \frac{(bB^2) \int \frac{(c+dx) \log^2(e(a+bx)^n(c+dx)^{-n})}{(a+bx)^3} dx}{bc - ad} \\
&= -\frac{A^2}{2b(a + bx)^2} - \frac{AB \log(e(a + bx)^n(c + dx)^{-n})}{b(a + bx)^2} + \frac{B^2 d(c + dx) \log^2(e(a + bx)^n(c + dx)^{-n})}{(bc - ad)^2(a + bx)} \\
&= -\frac{A^2}{2b(a + bx)^2} - \frac{ABn}{2b(a + bx)^2} + \frac{ABdn}{b(bc - ad)(a + bx)} + \frac{ABd^2n \log(a + bx)}{b(bc - ad)^2} \\
&= -\frac{A^2}{2b(a + bx)^2} - \frac{ABn}{2b(a + bx)^2} + \frac{ABdn}{b(bc - ad)(a + bx)} + \frac{2B^2dn^2}{b(bc - ad)(a + bx)}
\end{aligned}$$

**Mathematica [A]** time = 0.530402, size = 332, normalized size = 1.21

$$\frac{(bc - ad) \left( 2A^2(bc - ad) + 2B(2A(bc - ad) + Bn(-3ad + bc - 2bdx)) \log(e(a + bx)^n(c + dx)^{-n}) + 2ABn(-3ad + bc - 2bdx) \right)}{(a + bx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]^2)/(a + b\*x)^3,x]

[Out]  $-(2*B^2*d^2*n^2*(a + b*x)^2*Log[a + b*x]^2 + 2*B^2*d^2*n^2*(a + b*x)^2*Log[c + d*x]^2 + 2*B*d^2*n*(a + b*x)^2*Log[c + d*x]*(2*A + 3*B*n + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)] - 2*B*d^2*n*(a + b*x)^2*Log[a + b*x]*(2*A + 3*B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)] + (b*c - a*d)*(2*A^2*(b*c - a*d) + B^2*n^2*(b*c - 7*a*d - 6*b*d*x) + 2*A*B*n*(b*c - 3*a*d - 2*b*d*x) + 2*B*(2*A*(b*c - a*d) + B*n*(b*c - 3*a*d - 2*b*d*x))*Log[(e*(a + b*x)^n)/(c + d*x)] + 2*B^2*(b*c - a*d)*Log[(e*(a + b*x)^n)/(c + d*x)]^2))/(4*b*(b*c - a*d)^2*(a + b*x)^2)$

**Maple [C]** time = 1.968, size = 17300, normalized size = 63.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a)^3,x)

[Out] result too large to display

**Maxima [B]** time = 1.47313, size = 1214, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a)^3,x, algorithm="maxima")

[Out]  $\frac{1}{4}B^2(2(2d^2e^n \log(bx+a)/(b^3c^2 - 2ab^2cd + a^2bd^2) - 2d^2e^n \log(dx+c)/(b^3c^2 - 2ab^2cd + a^2bd^2) + (2b^2d^2e^n x - bc^2e^n + 3ad^2e^n)/(a^2b^2c - a^3bd + (b^4c - ab^3d)x^2 + 2(ab^3c - a^2b^2d)x)) \log((bx+a)^n e/(dx+c)^n)/e - (b^2c^2e^{2n} - 8ab^2cd^2e^{2n} + 7a^2d^2e^{2n} + 2(b^2d^2e^{2n}x^2 + 2ab^2d^2e^{2n}x + a^2d^2e^{2n}) \log(bx+a)^2 + 2(b^2d^2e^{2n}x^2 + 2ab^2d^2e^{2n}x + a^2d^2e^{2n}) \log(dx+c)^2 - 6(b^2cd^2e^{2n} - ab^2d^2e^{2n})x - 6(b^2d^2e^{2n}x^2 + 2ab^2d^2e^{2n}x + a^2d^2e^{2n}) \log(bx+a) + 2(3b^2d^2e^{2n}x^2 + 6ab^2d^2e^{2n}x + 3a^2d^2e^{2n}) \log(bx+a)) \log(dx+c))/(a^2b^3c^2 - 2a^3b^2cd + a^4bd^2 + (b^5c^2 - 2ab^4cd + a^2b^3d^2)x^2 + 2(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2)x)e^2) - \frac{1}{2}B^2 \log((bx+a)^n e/(dx+c)^n)^2/(b^3x^2 + 2ab^2x + a^2b) + \frac{1}{2}(2d^2e^n \log(bx+a)/(b^3c^2 - 2ab^2cd + a^2bd^2) - 2d^2e^n \log(dx+c)/(b^3c^2 - 2ab^2cd + a^2bd^2) + (2b^2d^2e^n x - bc^2e^n + 3ad^2e^n)/(a^2b^2c - a^3bd + (b^4c - ab^3d)x^2 + 2(ab^3c - a^2b^2d)x)) \frac{A}{B}e - \frac{A}{B} \log((bx+a)^n e/(dx+c)^n)/(b^3x^2 + 2ab^2x + a^2b) - \frac{1}{2}A^2/(b^3x^2 + 2ab^2x + a^2b)$

**Fricas [B]** time = 1.21899, size = 1918, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a)^3,x, algorithm="fricas")

[Out]  $-1/4(2A^2b^2c^2 - 4A^2abc^2d + 2A^2a^2d^2 + (B^2b^2c^2 - 8B^2abc^2d + 7B^2a^2d^2)n^2 - 2(B^2b^2d^2n^2x^2 + 2B^2ab^2d^2n^2x - (B^2b^2c^2 - 2B^2abc^2d)n^2) \log(bx+a)^2 - 2(B^2b^2d^2n^2x^2 + 2B^2ab^2d^2n^2x - (B^2b^2c^2 - 2B^2abc^2d)n^2) \log(dx+c)^2 + 2(B^2b^2c^2 - 2B^2abc^2d + B^2a^2d^2) \log(e)^2 + 2(ABb^2c^2 - 4ABabc^2d + 3ABa^2d^2)n - 2(3(B^2b^2c^2d - B^2ab^2d^2)n^2 + 2(ABb^2c^2d - ABa^2bd^2)n)x + 2((B^2b^2c^2 - 4B^2abc^2d)n^2 - (3B^2b^2d^2n^2 + 2ABb^2d^2n)x^2 + 2(ABb^2c^2 - 2ABabc^2d)n - 2(2ABa^2bd^2n + (B^2b^2c^2d + 2B^2ab^2d^2)n^2)x - 2(B^2b^2d^2n^2x^2 + 2B^2ab^2d^2n^2x - (B^2b^2c^2 - 2B^2abc^2d)n) \log(e)) \log(bx+a) - 2((B^2b^2c^2 - 4B^2abc^2d)n^2 - (3B^2b^2d^2n^2 + 2ABb^2d^2n)x^2 + 2(ABb^2c^2 - 2ABabc^2d)n - 2(2ABa^2bd^2n + (B^2b^2c^2d + 2B^2ab^2d^2)n^2)x - 2(B^2b^2d^2n^2x^2 + 2B^2ab^2d^2n^2x - (B^2b^2c^2 - 2B^2abc^2d)n) \log(e)) \log(dx+c) + 2(2ABb^2c^2 - 4ABabc^2d + 2ABa^2d^2 - 2(B^2b^2c^2d - B^2ab^2d^2)n^2)x + (B^2b^2c^2 - 4B^2abc^2d + 3B^2a^2d^2)n) \log(e))/(a^2b^3c^2 - 2a^3b^2cd + a^4bd^2 + (b^5c^2 - 2ab^4cd + a^2b^3d^2)x^2 + 2(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2)x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log\left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)^2}{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*x + a)^3, x)
```

$$3.162 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^4} dx$$

**Optimal.** Leaf size=427

$$\frac{b^2(c+dx)^3(B \log(e(a+bx)^n(c+dx)^{-n})+A)^2}{3(a+bx)^3(bc-ad)^3} - \frac{2b^2Bn(c+dx)^3(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{9(a+bx)^3(bc-ad)^3} - \frac{d^2(c+dx)(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{3b(bc-ad)^2}$$

[Out]  $(-2B^2d^2n^2(c+dx))/((b^3c-a^3d)^3(a+bx)) + (b^3B^2d^2n^2(c+dx)^2)/(2(b^3c-a^3d)^3(a+bx)^2) - (2b^2B^2n^2(c+dx)^3)/(27(b^3c-a^3d)^3(a+bx)^3) - (2B^2d^2n^2(c+dx)(A+B \log(e(a+bx)^n(c+dx)^{-n}))/((b^3c-a^3d)^3(a+bx)) + (b^3B^2d^2n^2(c+dx)^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))/((b^3c-a^3d)^3(a+bx)^2) - (2b^2B^2n^2(c+dx)^3(A+B \log(e(a+bx)^n(c+dx)^{-n}))/((b^3c-a^3d)^3(a+bx)^3) - (d^2(c+dx)(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2)/((b^3c-a^3d)^3(a+bx)) + (b^3d^2(c+dx)^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2)/((b^3c-a^3d)^3(a+bx)^2) - (b^2d^2(c+dx)^3(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2)/((b^3c-a^3d)^3(a+bx)^3)$

**Rubi [C]** time = 1.21235, antiderivative size = 730, normalized size of antiderivative = 1.71, number of steps used = 26, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 2492, 44, 2514, 2490, 32, 2488, 2411, 2343, 2333, 2315}

$$\frac{2B^2d^3n^2 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3b(bc-ad)^3} - \frac{2B^2d^3n^2 \text{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right)}{3b(bc-ad)^3} - \frac{A^2}{3b(a+bx)^3} - \frac{2ABd^2n}{3b(a+bx)(bc-ad)^2} - \frac{2ABd^3n \log(e(a+bx)^n(c+dx)^{-n})}{3b(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2/(a + b\*x)^4, x]

[Out]  $-A^2/(3b^3(a+bx)^3) - (2AB^2n)/(9b^3(a+bx)^3) - (2B^2n^2)/(27b^3(a+bx)^3) + (AB^2d^2n)/(3b^3(b^3c-a^3d)(a+bx)^2) + (5B^2d^2n^2)/(18b^3(b^3c-a^3d)(a+bx)^2) - (2AB^2d^2n)/(3b^3(b^3c-a^3d)^2(a+bx)) - (11B^2d^2n^2)/(9b^3(b^3c-a^3d)^2(a+bx)) - (2AB^2d^3n \text{Log}[a+bx])/((3b^3(b^3c-a^3d)^3) - (5B^2d^3n^2 \text{Log}[a+bx]))/(9b^3(b^3c-a^3d)^3) + (2AB^2d^3n \text{Log}[c+dx])/((3b^3(b^3c-a^3d)^3) + (5B^2d^3n^2 \text{Log}[c+dx]))/(9b^3(b^3c-a^3d)^3) - (2AB^2 \text{Log}[(e(a+bx)^n)/(c+dx)^n])/((3b^3(a+bx)^3) - (2B^2n \text{Log}[(e(a+bx)^n)/(c+dx)^n]))/(9b^3(a+bx)^3) + (B^2d^2n \text{Log}[(e(a+bx)^n)/(c+dx)^n])/((3b^3(b^3c-a^3d)(a+bx)^2) - (2B^2d^2n^2(c+dx) \text{Log}[(e(a+bx)^n)/(c+dx)^n]))/(3(b^3c-a^3d)^3(a+bx)) + (2B^2d^3n \text{Log}[-((b^3c-a^3d)/(d(a+bx)))] \text{Log}[(e(a+bx)^n)/(c+dx)^n])/((3b^3(b^3c-a^3d)^3) - (2B^2d^3n \text{Log}[(b^3c-a^3d)/(b^3(c+dx))]) \text{Log}[(e(a+bx)^n)/(c+dx)^n])/((3b^3(b^3c-a^3d)^3) - (B^2 \text{Log}[(e(a+bx)^n)/(c+dx)^n]^2)/((3b^3(a+bx)^3) - (2B^2d^3n^2 \text{PolyLog}[2, (d(a+bx))/(b^3(c+dx))]))/(3b^3(b^3c-a^3d)^3) - (2B^2d^3n^2 \text{PolyLog}[2, 1 + (b^3c-a^3d)/(d(a+bx))]))/(3b^3(b^3c-a^3d)^3)$

**Rule 6742**

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

**Rule 2492**

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*((g\_.) + (h\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((g + h\*x)^(m +

1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s)/(h\*(m + 1)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(h\*(m + 1)), Int[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0] && NeQ[m, -1]

#### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 2514

Int[Log[(e\_)\*((f\_)\*((a\_) + (b\_)\*(x\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(q\_))^(r\_)]^(s\_)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s, RFx, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]

#### Rule 2490

Int[Log[(e\_)\*((f\_)\*((a\_) + (b\_)\*(x\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(q\_))^(r\_)]^(s\_)/((g\_) + (h\_)\*(x\_))^2, x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s)/((b\*g - a\*h)\*(g + h\*x)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(b\*g - a\*h), Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^(s - 1)/((c + d\*x)\*(g + h\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && NeQ[b\*g - a\*h, 0] && IGtQ[s, 0]

#### Rule 32

Int[((a\_) + (b\_)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rule 2488

Int[Log[(e\_)\*((f\_)\*((a\_) + (b\_)\*(x\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(q\_))^(r\_)]^(s\_)/((g\_) + (h\_)\*(x\_)), x\_Symbol] := -Simp[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d))/h, Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

#### Rule 2411

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_))^(p\_)\*((f\_) + (g\_)\*(x\_))^(q\_)\*((h\_) + (i\_)\*(x\_))^(r\_), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

#### Rule 2343

Int[((a\_) + Log[(c\_)\*(x\_)^n])\*(b\_))/((x\_)\*((d\_) + (e\_)\*(x\_)^r)), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*Log[c\*x^n])/(x\*(d + e\*x^r/n)), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]

Rule 2333

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)/(x\_))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] :> Int[(e + d\*x)^q\*(a + b\*Log[c\*x^n])^p, x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]

Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^2}{(a + bx)^4} dx &= \int \left( \frac{A^2}{(a + bx)^4} + \frac{2AB \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^4} + \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^4} \right) dx \\
 &= -\frac{A^2}{3b(a + bx)^3} + (2AB) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^4} dx + B^2 \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^4} dx \\
 &= -\frac{A^2}{3b(a + bx)^3} - \frac{2AB \log(e(a + bx)^n(c + dx)^{-n})}{3b(a + bx)^3} - \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{3b(a + bx)^3} \\
 &= -\frac{A^2}{3b(a + bx)^3} - \frac{2AB \log(e(a + bx)^n(c + dx)^{-n})}{3b(a + bx)^3} - \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{3b(a + bx)^3} \\
 &= -\frac{A^2}{3b(a + bx)^3} - \frac{2ABn}{9b(a + bx)^3} + \frac{ABdn}{3b(bc - ad)(a + bx)^2} - \frac{2ABd^2n}{3b(bc - ad)^2(a + bx)} \\
 &= -\frac{A^2}{3b(a + bx)^3} - \frac{2ABn}{9b(a + bx)^3} + \frac{ABdn}{3b(bc - ad)(a + bx)^2} - \frac{2ABd^2n}{3b(bc - ad)^2(a + bx)} \\
 &= -\frac{A^2}{3b(a + bx)^3} - \frac{2ABn}{9b(a + bx)^3} + \frac{ABdn}{3b(bc - ad)(a + bx)^2} - \frac{2ABd^2n}{3b(bc - ad)^2(a + bx)} \\
 &= -\frac{A^2}{3b(a + bx)^3} - \frac{2ABn}{9b(a + bx)^3} - \frac{2B^2n^2}{27b(a + bx)^3} + \frac{ABdn}{3b(bc - ad)(a + bx)^2} + \frac{18b(bc - ad)^2n}{18b(bc - ad)^2(a + bx)} \\
 &= -\frac{A^2}{3b(a + bx)^3} - \frac{2ABn}{9b(a + bx)^3} - \frac{2B^2n^2}{27b(a + bx)^3} + \frac{ABdn}{3b(bc - ad)(a + bx)^2} + \frac{18b(bc - ad)^2n}{18b(bc - ad)^2(a + bx)} \\
 &= -\frac{A^2}{3b(a + bx)^3} - \frac{2ABn}{9b(a + bx)^3} - \frac{2B^2n^2}{27b(a + bx)^3} + \frac{ABdn}{3b(bc - ad)(a + bx)^2} + \frac{18b(bc - ad)^2n}{18b(bc - ad)^2(a + bx)}
 \end{aligned}$$

**Mathematica [A]** time = 0.743327, size = 432, normalized size = 1.01

$$\frac{-(bc - ad) \left( 6B \left( Bn \left( 11a^2d^2 + abd(15dx - 7c) + b^2 \left( 2c^2 - 3cdx + 6d^2x^2 \right) \right) + 6A(bc - ad)^2 \right) \log(e(a + bx)^n(c + dx)^{-n}) + 6A \right)}{18b(bc - ad)^2(a + bx)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2/(a + b\*x)^4, x]

[Out] (18\*B^2\*d^3\*n^2\*(a + b\*x)^3\*Log[a + b\*x]^2 + 18\*B^2\*d^3\*n^2\*(a + b\*x)^3\*Log[c + d\*x]^2 + 6\*B\*d^3\*n\*(a + b\*x)^3\*Log[c + d\*x]\*(6\*A + 11\*B\*n + 6\*B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]) - 6\*B\*d^3\*n\*(a + b\*x)^3\*Log[a + b\*x]\*(6\*A + 11\*B\*Log[c + d\*x]^2 + 6\*B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]))/(18\*b\*(bc - ad)^2\*(a + b\*x))



$$B^n + 6*B^n*\text{Log}[c + d*x] + 6*B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] - (b*c - a*d)*(18*A^2*(b*c - a*d)^2 + 6*A*B^n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2)) + B^2*n^2*(85*a^2*d^2 + a*b*d*(-23*c + 14*7*d*x) + b^2*(4*c^2 - 15*c*d*x + 66*d^2*x^2)) + 6*B*(6*A*(b*c - a*d)^2 + B^n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - 3*c*d*x + 6*d^2*x^2))) * \text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*(b*c - a*d)^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(54*b*(b*c - a*d)^3*(a + b*x)^3)$$

**Maple [C]** time = 2.658, size = 25057, normalized size = 58.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a)^4,x)

[Out] result too large to display

**Maxima [B]** time = 1.7076, size = 2025, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a)^4,x, algorithm="maxima")

[Out] 
$$-1/54*B^2*(6*(6*d^3*e*n*\log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3*e*n*\log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*x))*\log((b*x + a)^n*e/(d*x + c)^n)/e + (4*b^3*c^3*e^2*n^2 - 2*7*a*b^2*c^2*d*e^2*n^2 + 108*a^2*b*c*d^2*e^2*n^2 - 85*a^3*d^3*e^2*n^2 + 66*(b^3*c*d^2*e^2*n^2 - a*b^2*d^3*e^2*n^2)*x^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*\log(b*x + a)^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*\log(d*x + c)^2 - 3*(5*b^3*c^2*d*e^2*n^2 - 54*a*b^2*c*d^2*e^2*n^2 + 49*a^2*b*d^3*e^2*n^2)*x + 66*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*\log(b*x + a) - 6*(11*b^3*d^3*e^2*n^2*x^3 + 33*a*b^2*d^3*e^2*n^2*x^2 + 33*a^2*b*d^3*e^2*n^2*x + 11*a^3*d^3*e^2*n^2 - 6*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*\log(b*x + a))*\log(d*x + c))/((a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x)*e^2) - 1/3*B^2*\log((b*x + a)^n*e/(d*x + c)^n)^2/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b) - 1/9*(6*d^3*e*n*\log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3*e*n*\log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a$$

$$\begin{aligned} &^3b^3d^2)x^2 + 3(a^2b^4c^2 - 2a^3b^3cd + a^4b^2d^2)x) * A * B / e - \\ &2/3 * A * B * \log((b * x + a)^n * e / (d * x + c)^n) / (b^4 * x^3 + 3 * a * b^3 * x^2 + 3 * a^2 * b^2 * \\ &x + a^3 * b) - 1/3 * A^2 / (b^4 * x^3 + 3 * a * b^3 * x^2 + 3 * a^2 * b^2 * x + a^3 * b) \end{aligned}$$

**Fricas [B]** time = 1.40228, size = 3351, normalized size = 7.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a)^4,x, algorithm="fricas")

[Out] 
$$\begin{aligned} &-1/54 * (18 * A^2 * b^3 * c^3 - 54 * A^2 * a * b^2 * c^2 * d + 54 * A^2 * a^2 * b * c * d^2 - 18 * A^2 * a^3 * d^3 + (4 * B^2 * b^3 * c^3 - 27 * B^2 * a * b^2 * c^2 * d + 108 * B^2 * a^2 * b * c * d^2 - 85 * B^2 * a^3 * d^3) * n^2 + 6 * (11 * (B^2 * b^3 * c * d^2 - B^2 * a * b^2 * d^3) * n^2 + 6 * (A * B * b^3 * c * d^2 - A * B * a * b^2 * d^3) * n) * x^2 + 18 * (B^2 * b^3 * d^3 * n^2 * x^3 + 3 * B^2 * a * b^2 * d^3 * n^2 * x^2 + 3 * B^2 * a^2 * b * d^3 * n^2 * x + (B^2 * b^3 * c^3 - 3 * B^2 * a * b^2 * c^2 * d + 3 * B^2 * a^2 * b * c * d^2) * n^2) * \log(b * x + a)^2 + 18 * (B^2 * b^3 * d^3 * n^2 * x^3 + 3 * B^2 * a * b^2 * d^3 * n^2 * x^2 + 3 * B^2 * a^2 * b * d^3 * n^2 * x + (B^2 * b^3 * c^3 - 3 * B^2 * a * b^2 * c^2 * d + 3 * B^2 * a^2 * b * c * d^2) * n^2) * \log(d * x + c)^2 + 18 * (B^2 * b^3 * c^3 - 3 * B^2 * a * b^2 * c^2 * d + 3 * B^2 * a^2 * b * c * d^2 - B^2 * a^3 * d^3) * \log(e)^2 + 6 * (2 * A * B * b^3 * c^3 - 9 * A * B * a * b^2 * c^2 * d + 18 * A * B * a^2 * b * c * d^2 - 11 * A * B * a^3 * d^3) * n - 3 * ((5 * B^2 * b^3 * c^2 * d - 54 * B^2 * a * b^2 * c * d^2 + 49 * B^2 * a^2 * b * d^3) * n^2 + 6 * (A * B * b^3 * c^2 * d - 6 * A * B * a * b^2 * c * d^2 + 5 * A * B * a^2 * b * d^3) * n) * x + 6 * ((11 * B^2 * b^3 * d^3 * n^2 + 6 * A * B * b^3 * d^3 * n) * x^3 + (2 * B^2 * b^3 * c^3 - 9 * B^2 * a * b^2 * c^2 * d + 18 * B^2 * a^2 * b * c * d^2) * n^2 + 3 * (6 * A * B * a * b^2 * d^3 * n + (2 * B^2 * b^3 * c * d^2 + 9 * B^2 * a * b^2 * d^3) * n^2) * x^2 + 6 * (A * B * b^3 * c^3 - 3 * A * B * a * b^2 * c^2 * d + 3 * A * B * a^2 * b * c * d^2) * n + 3 * (6 * A * B * a^2 * b * d^3 * n - (B^2 * b^3 * c^2 * d - 6 * B^2 * a * b^2 * c * d^2 - 6 * B^2 * a^2 * b * d^3) * n^2) * x + 6 * (B^2 * b^3 * d^3 * n * x^3 + 3 * B^2 * a * b^2 * d^3 * n * x^2 + 3 * B^2 * a^2 * b * d^3 * n * x + (B^2 * b^3 * c^3 - 3 * B^2 * a * b^2 * c^2 * d + 3 * B^2 * a^2 * b * c * d^2) * n) * \log(e)) * \log(b * x + a) - 6 * ((11 * B^2 * b^3 * d^3 * n^2 + 6 * A * B * b^3 * d^3 * n) * x^3 + (2 * B^2 * b^3 * c^3 - 9 * B^2 * a * b^2 * c^2 * d + 18 * B^2 * a^2 * b * c * d^2) * n^2 + 3 * (6 * A * B * a * b^2 * d^3 * n + (2 * B^2 * b^3 * c * d^2 + 9 * B^2 * a * b^2 * d^3) * n^2) * x^2 + 6 * (A * B * b^3 * c^3 - 3 * A * B * a * b^2 * c^2 * d + 3 * A * B * a^2 * b * c * d^2) * n + 3 * (6 * A * B * a^2 * b * d^3 * n - (B^2 * b^3 * c^2 * d - 6 * B^2 * a * b^2 * c * d^2 - 6 * B^2 * a^2 * b * d^3) * n^2) * x + 6 * (B^2 * b^3 * d^3 * n^2 * x^3 + 3 * B^2 * a * b^2 * d^3 * n^2 * x^2 + 3 * B^2 * a^2 * b * d^3 * n^2 * x + (B^2 * b^3 * c^3 - 3 * B^2 * a * b^2 * c^2 * d + 3 * B^2 * a^2 * b * c * d^2) * n^2) * \log(b * x + a) + 6 * (B^2 * b^3 * d^3 * n * x^3 + 3 * B^2 * a * b^2 * d^3 * n * x^2 + 3 * B^2 * a^2 * b * d^3 * n * x + (B^2 * b^3 * c^3 - 3 * B^2 * a * b^2 * c^2 * d + 3 * B^2 * a^2 * b * c * d^2) * n) * \log(e)) * \log(d * x + c) + 6 * (6 * A * B * b^3 * c^3 - 18 * A * B * a * b^2 * c^2 * d + 18 * A * B * a^2 * b * c * d^2 - 6 * A * B * a^3 * d^3 + 6 * (B^2 * b^3 * c * d^2 - B^2 * a * b^2 * d^3) * n * x^2 - 3 * (B^2 * b^3 * c^2 * d - 6 * B^2 * a * b^2 * c * d^2 + 5 * B^2 * a^2 * b * d^3) * n * x + (2 * B^2 * b^3 * c^3 - 9 * B^2 * a * b^2 * c^2 * d + 18 * B^2 * a^2 * b * c * d^2 - 11 * B^2 * a^3 * d^3) * n) * \log(e)) / (a^3 * b^4 * c^3 - 3 * a^4 * b^3 * c^2 * d + 3 * a^5 * b^2 * c * d^2 - a^6 * b * d^3 + (b^7 * c^3 - 3 * a * b^6 * c^2 * d + 3 * a^2 * b^5 * c * d^2 - a^3 * b^4 * d^3) * x^3 + 3 * (a * b^6 * c^3 - 3 * a^2 * b^5 * c^2 * d + 3 * a^3 * b^4 * c * d^2 - a^4 * b^3 * d^3) * x^2 + 3 * (a^2 * b^5 * c^3 - 3 * a^3 * b^4 * c^2 * d + 3 * a^4 * b^3 * c * d^2 - a^5 * b^2 * d^3) * x) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*2/(b\*x+a)\*\*4,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log\left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)^2}{(bx+a)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a)^4,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^2/(b\*x + a)^4, x)

$$3.163 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)^5} dx$$

**Optimal.** Leaf size=587

$$\frac{b^3(c+dx)^4(B \log(e(a+bx)^n(c+dx)^{-n})+A)^2}{4(a+bx)^4(bc-ad)^4} - \frac{b^3Bn(c+dx)^4(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{8(a+bx)^4(bc-ad)^4} + \frac{b^2d(c+dx)^3(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{8(a+bx)^4(bc-ad)^4}$$

[Out]  $(2*B^2*d^3*n^2*(c+d*x))/((b*c-a*d)^4*(a+b*x)) - (3*b*B^2*d^2*n^2*(c+d*x)^2)/(4*(b*c-a*d)^4*(a+b*x)^2) + (2*b^2*B^2*d*n^2*(c+d*x)^3)/(9*(b*c-a*d)^4*(a+b*x)^3) - (b^3*B^2*n^2*(c+d*x)^4)/(32*(b*c-a*d)^4*(a+b*x)^4) + (2*B*d^3*n*(c+d*x)*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n]))/((b*c-a*d)^4*(a+b*x)) - (3*b*B*d^2*n*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n]))/(2*(b*c-a*d)^4*(a+b*x)^2) + (2*b^2*B*d*n*(c+d*x)^3*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n]))/(3*(b*c-a*d)^4*(a+b*x)^3) - (b^3*B*n*(c+d*x)^4*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n]))/(8*(b*c-a*d)^4*(a+b*x)^4) + (d^3*(c+d*x)*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n]))^2/((b*c-a*d)^4*(a+b*x)) - (3*b*d^2*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n]))^2/(2*(b*c-a*d)^4*(a+b*x)^2) + (b^2*d*(c+d*x)^3*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n]))^2/((b*c-a*d)^4*(a+b*x)^3) - (b^3*(c+d*x)^4*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n]))^2/(4*(b*c-a*d)^4*(a+b*x)^4)$

**Rubi [C]** time = 1.40797, antiderivative size = 843, normalized size of antiderivative = 1.44, number of steps used = 29, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 2492, 44, 2514, 2490, 32, 2488, 2411, 2343, 2333, 2315}

$$\frac{13B^2n^2 \log(a+bx)d^4}{24b(bc-ad)^4} + \frac{ABn \log(a+bx)d^4}{2b(bc-ad)^4} - \frac{13B^2n^2 \log(c+dx)d^4}{24b(bc-ad)^4} - \frac{ABn \log(c+dx)d^4}{2b(bc-ad)^4} - \frac{B^2n \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a+bx)^n(c+dx)^{-n})}{2b(bc-ad)^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n)]^2/(a + b\*x)^5, x]

[Out]  $-A^2/(4*b*(a+b*x)^4) - (A*B*n)/(8*b*(a+b*x)^4) - (B^2*n^2)/(32*b*(a+b*x)^4) + (A*B*d*n)/(6*b*(b*c-a*d)*(a+b*x)^3) + (7*B^2*d*n^2)/(72*b*(b*c-a*d)*(a+b*x)^3) - (A*B*d^2*n)/(4*b*(b*c-a*d)^2*(a+b*x)^2) - (13*B^2*d^2*n^2)/(48*b*(b*c-a*d)^2*(a+b*x)^2) + (A*B*d^3*n)/(2*b*(b*c-a*d)^3*(a+b*x)) + (25*B^2*d^3*n^2)/(24*b*(b*c-a*d)^3*(a+b*x)) + (A*B*d^4*n*Log[a+b*x])/(2*b*(b*c-a*d)^4) + (13*B^2*d^4*n^2*Log[a+b*x])/(24*b*(b*c-a*d)^4) - (A*B*d^4*n*Log[c+d*x])/(2*b*(b*c-a*d)^4) - (13*B^2*d^4*n^2*Log[c+d*x])/(24*b*(b*c-a*d)^4) - (A*B*Log[(e*(a+b*x)^n)/(c+d*x]^n]))/(2*b*(a+b*x)^4) - (B^2*n*Log[(e*(a+b*x)^n)/(c+d*x]^n]))/(8*b*(a+b*x)^4) + (B^2*d*n*Log[(e*(a+b*x)^n)/(c+d*x]^n]))/(6*b*(b*c-a*d)*(a+b*x)^3) - (B^2*d^2*n*Log[(e*(a+b*x)^n)/(c+d*x]^n]))/(4*b*(b*c-a*d)^2*(a+b*x)^2) + (B^2*d^3*n*(c+d*x)*Log[(e*(a+b*x)^n)/(c+d*x]^n]))/(2*(b*c-a*d)^4*(a+b*x)) - (B^2*d^4*n*Log[-((b*c-a*d)/(d*(a+b*x)))]*Log[(e*(a+b*x)^n)/(c+d*x]^n]))/(2*b*(b*c-a*d)^4) + (B^2*d^4*n*Log[(b*c-a*d)/(b*(c+d*x))]*Log[(e*(a+b*x)^n)/(c+d*x]^n]))/(2*b*(b*c-a*d)^4) - (B^2*Log[(e*(a+b*x)^n)/(c+d*x]^n)]^2)/(4*b*(a+b*x)^4) + (B^2*d^4*n^2*PolyLog[2, (d*(a+b*x))/(b*(c+d*x))])/(2*b*(b*c-a*d)^4) + (B^2*d^4*n^2*PolyLog[2, 1 + (b*c-a*d)/(d*(a+b*x))])/(2*b*(b*c-a*d)^4)$

**Rule 6742**

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 2492

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_.))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
0] && NeQ[m, -1]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

#### Rule 2514

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))
^(r_.)]^(s_.)*(RFX_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]
```

#### Rule 2490

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_.))^2, x_Symbol] := Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q]^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/((
c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0]
]
```

#### Rule 32

```
Int[((a_.) + (b_.)*(x_.))^(m_.), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

#### Rule 2488

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_.)), x_Symbol] := -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/h, x] + Dist[(p*r*s*(
b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

#### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_.))^(q_.)*((h_.) + (i_.)*(x_.))^(r_.), x_Symbol] := Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
```

\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

### Rule 2343

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/((x\_)\*((d\_) + (e\_.)\*(x\_)^(r\_.))),  
x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*Log[c\*x])/(x\*(d + e\*x^(r/n))), x],  
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]

### Rule 2333

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)/(x\_))^(q\_.)\*  
x\_)^(m\_.), x\_Symbol] := Int[(e + d\*x)^q\*(a + b\*Log[c\*x^n])^p, x] /; FreeQ[{  
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 -  
c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^2}{(a + bx)^5} dx &= \int \left( \frac{A^2}{(a + bx)^5} + \frac{2AB \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^5} + \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^5} \right) dx \\
 &= -\frac{A^2}{4b(a + bx)^4} + (2AB) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^5} dx + B^2 \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^5} dx \\
 &= -\frac{A^2}{4b(a + bx)^4} - \frac{AB \log(e(a + bx)^n(c + dx)^{-n})}{2b(a + bx)^4} - \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{4b(a + bx)^4} \\
 &= -\frac{A^2}{4b(a + bx)^4} - \frac{AB \log(e(a + bx)^n(c + dx)^{-n})}{2b(a + bx)^4} - \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{4b(a + bx)^4} \\
 &= -\frac{A^2}{4b(a + bx)^4} - \frac{ABn}{8b(a + bx)^4} + \frac{ABdn}{6b(bc - ad)(a + bx)^3} - \frac{ABd^2n}{4b(bc - ad)^2(a + bx)^2} \\
 &= -\frac{A^2}{4b(a + bx)^4} - \frac{ABn}{8b(a + bx)^4} + \frac{ABdn}{6b(bc - ad)(a + bx)^3} - \frac{ABd^2n}{4b(bc - ad)^2(a + bx)^2} \\
 &= -\frac{A^2}{4b(a + bx)^4} - \frac{ABn}{8b(a + bx)^4} + \frac{ABdn}{6b(bc - ad)(a + bx)^3} - \frac{ABd^2n}{4b(bc - ad)^2(a + bx)^2} \\
 &= -\frac{A^2}{4b(a + bx)^4} - \frac{ABn}{8b(a + bx)^4} - \frac{B^2n^2}{32b(a + bx)^4} + \frac{ABdn}{6b(bc - ad)(a + bx)^3} + \frac{ABd^2n}{72b(bc - ad)^2(a + bx)^2} \\
 &= -\frac{A^2}{4b(a + bx)^4} - \frac{ABn}{8b(a + bx)^4} - \frac{B^2n^2}{32b(a + bx)^4} + \frac{ABdn}{6b(bc - ad)(a + bx)^3} + \frac{ABd^2n}{72b(bc - ad)^2(a + bx)^2} \\
 &= -\frac{A^2}{4b(a + bx)^4} - \frac{ABn}{8b(a + bx)^4} - \frac{B^2n^2}{32b(a + bx)^4} + \frac{ABdn}{6b(bc - ad)(a + bx)^3} + \frac{ABd^2n}{72b(bc - ad)^2(a + bx)^2}
 \end{aligned}$$

**Mathematica [A]** time = 1.07058, size = 1011, normalized size = 1.72

$$\frac{9(8A^2 + 4BnA + 16B(-n \log(a + bx) + n \log(c + dx) + \log(e(a + bx)^n(c + dx)^{-n}))A + B^2n^2 + 8B^2(-n \log(a + bx) + n \log(c + dx) + \log(e(a + bx)^n(c + dx)^{-n})))}{72b^2(bc - ad)^2(a + bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2/(a + b\*x)^5,x]

[Out] 
$$-(72*b*B^2*n^2*(-4*a^3*d^3*(c + d*x) + 6*a^2*b*d^2*(c^2 - d^2*x^2) - 4*a*b^2*d*(c^3 + d^3*x^3) + b^3*(c^4 - d^4*x^4))*Log[a + b*x]^2 + 72*b*B^2*n^2*(-4*a^3*d^3*(c + d*x) + 6*a^2*b*d^2*(c^2 - d^2*x^2) - 4*a*b^2*d*(c^3 + d^3*x^3) + b^3*(c^4 - d^4*x^4))*Log[c + d*x]^2 - 4*B*d*(b*c - a*d)^3*n*(a + b*x)*(12*A + 7*B*n + 12*B*(-(n*Log[a + b*x])) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 6*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2*(12*A + 13*B*n + 12*B*(-(n*Log[a + b*x])) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]) - 12*B*d^3*(b*c - a*d)*n*(a + b*x)^3*(12*A + 25*B*n + 12*B*(-(n*Log[a + b*x])) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]) - 12*B*d^4*n*(a + b*x)^4*Log[a + b*x]*(12*A + 25*B*n + 12*B*(-(n*Log[a + b*x])) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 12*B*d^4*n*(a + b*x)^4*Log[c + d*x]*(12*A + 25*B*n + 12*B*(-(n*Log[a + b*x])) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 9*(b*c - a*d)^4*(8*A^2 + 4*A*B*n + B^2*n^2 + 16*A*B*(-(n*Log[a + b*x])) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 4*B^2*n*(-(n*Log[a + b*x])) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]) + 8*B^2*(-(n*Log[a + b*x])) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])^2 - 12*B*(b*c - a*d)*n*Log[a + b*x]*(4*B*d*(b*c - a*d)^2*n*(a + b*x) + 6*B*d^2*(-(b*c) + a*d)*n*(a + b*x)^2 + 12*B*d^3*n*(a + b*x)^3 - 3*(b*c - a*d)^3*(4*A + B*n + 4*B*(-(n*Log[a + b*x])) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])) + 12*B*n*Log[c + d*x]*(4*B*d*(b*c - a*d)^3*n*(a + b*x) - 6*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2 + 12*B*d^3*(b*c - a*d)*n*(a + b*x)^3 - 12*B*(b*c - a*d)^4*n*Log[a + b*x] + 12*B*d^4*n*(a + b*x)^4*Log[a + b*x] - 3*(b*c - a*d)^4*(4*A + B*n + 4*B*(-(n*Log[a + b*x])) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n])))/(288*b*(b*c - a*d)^4*(a + b*x)^4)$$

**Maple [C]** time = 3.333, size = 33370, normalized size = 56.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a)^5,x)

[Out] result too large to display

**Maxima [B]** time = 2.05568, size = 3021, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a)^5,x, algorithm="maxima")

[Out] 
$$1/288*B^2*(12*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n$$

$$\begin{aligned}
& - 5*a*b^2*c*d^2*e^n + 13*a^2*b*d^3*e^n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + \\
& 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - \\
& a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4* \\
& b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3* \\
& d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^ \\
& 3)*x))*log((b*x + a)^n*e/(d*x + c)^n)/e - (9*b^4*c^4*e^2*n^2 - 64*a*b^3*c^3 \\
& *d*e^2*n^2 + 216*a^2*b^2*c^2*d^2*e^2*n^2 - 576*a^3*b*c*d^3*e^2*n^2 + 415*a^ \\
& 4*d^4*e^2*n^2 - 300*(b^4*c*d^3*e^2*n^2 - a*b^3*d^4*e^2*n^2)*x^3 + 6*(13*b^4 \\
& *c^2*d^2*e^2*n^2 - 176*a*b^3*c*d^3*e^2*n^2 + 163*a^2*b^2*d^4*e^2*n^2)*x^2 + \\
& 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2* \\
& x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a)^2 + 72*(b^4*d^4 \\
& *e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3* \\
& b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(d*x + c)^2 - 4*(7*b^4*c^3*d*e^2*n^2 \\
& - 60*a*b^3*c^2*d^2*e^2*n^2 + 324*a^2*b^2*c*d^3*e^2*n^2 - 271*a^3*b*d^4*e^2* \\
& n^2)*x - 300*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4 \\
& *e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a) + 12*( \\
& 25*b^4*d^4*e^2*n^2*x^4 + 100*a*b^3*d^4*e^2*n^2*x^3 + 150*a^2*b^2*d^4*e^2*n^ \\
& 2*x^2 + 100*a^3*b*d^4*e^2*n^2*x + 25*a^4*d^4*e^2*n^2 - 12*(b^4*d^4*e^2*n^2* \\
& x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2 \\
& *n^2*x + a^4*d^4*e^2*n^2)*log(b*x + a))*log(d*x + c))/((a^4*b^5*c^4 - 4*a^5 \\
& *b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4 \\
& *a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*( \\
& a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4 \\
& *d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^ \\
& 4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c \\
& ^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*x)*e^2)) - 1/4*B^2*log((b*x + a)^n* \\
& e/(d*x + c)^n)^2/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4 \\
& *b) + 1/24*(12*d^4*e*n*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^ \\
& 2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4 \\
& *a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d \\
& ^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a \\
& ^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5 \\
& *a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e*n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3* \\
& a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^ \\
& 3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4 \\
& *d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^ \\
& 3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)* \\
& x))*A*B/e - 1/2*A*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^5*x^4 + 4*a*b^4*x^3 + \\
& 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) - 1/4*A^2/(b^5*x^4 + 4*a*b^4*x^3 + 6* \\
& a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b)
\end{aligned}$$

**Fricas [B]** time = 1.69859, size = 5029, normalized size = 8.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a)^5,x, algorithm="fricas")

[Out]  $-1/288*(72*A^2*b^4*c^4 - 288*A^2*a*b^3*c^3*d + 432*A^2*a^2*b^2*c^2*d^2 - 288*A^2*a^3*b*c*d^3 + 72*A^2*a^4*d^4 - 12*(25*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*n^2 + 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*n)*x^3 + (9*B^2*b^4*c^4 - 64*B^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 - 576*B^2*a^3*b*c*d^3 + 415*B^2*a^4*d^4)*n^2 + 6*((13*B^2*b^4*c^2*d^2 - 176*B^2*a*b^3*c*d^3 + 163*B^2*a^2*b^2*d^4)*n^2 + 12*(A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)*n)*x^2 - 72*(B^2*b^4*d^4*n^2*x^4 + 4*B^2*a*b^3*d^4*n^2*x^3 + 6*B^2*a^2*b^2*d^4*n$



$$\begin{aligned}
& ^2x^2 + 4B^2a^3bd^4n^2x - (B^2b^4c^4 - 4B^2ab^3c^3d + 6B^2a \\
& ^2b^2c^2d^2 - 4B^2a^3b^3cd^3)n^2) \log(bx + a)^2 - 72(B^2b^4d^4n \\
& ^2x^4 + 4B^2ab^3d^4n^2x^3 + 6B^2a^2b^2d^4n^2x^2 + 4B^2a^3b^3 \\
& d^4n^2x - (B^2b^4c^4 - 4B^2ab^3c^3d + 6B^2a^2b^2c^2d^2 - 4B^2 \\
& a^3b^3cd^3)n^2) \log(dx + c)^2 + 72(B^2b^4c^4 - 4B^2ab^3c^3d + \\
& 6B^2a^2b^2c^2d^2 - 4B^2a^3b^3cd^3 + B^2a^4d^4) \log(e)^2 + 12(3A \\
& *Bb^4c^4 - 16ABab^3c^3d + 36ABa^2b^2c^2d^2 - 48ABa^3b^3cd \\
& ^3 + 25ABa^4d^4)n - 4((7B^2b^4c^3d - 60B^2ab^3c^2d^2 + 324B \\
& ^2a^2b^2cd^3 - 271B^2a^3bd^4)n^2 + 12(ABb^4c^3d - 6ABab^3 \\
& c^2d^2 + 18ABa^2b^2cd^3 - 13ABa^3bd^4)n)x - 12((25B^2b^4* \\
& d^4n^2 + 12ABb^4d^4n)x^4 + 4(12ABab^3d^4n + (3B^2b^4cd^3 \\
& + 22B^2ab^3d^4)n^2)x^3 - (3B^2b^4c^4 - 16B^2ab^3c^3d + 36B^2 \\
& a^2b^2c^2d^2 - 48B^2a^3b^3cd^3)n^2 + 6(12ABa^2b^2d^4n - (B^2 \\
& b^4c^2d^2 - 8B^2ab^3cd^3 - 18B^2a^2b^2d^4)n^2)x^2 - 12(ABb \\
& ^4c^4 - 4ABab^3c^3d + 6ABa^2b^2c^2d^2 - 4ABa^3b^3cd^3)n + \\
& 4(12ABa^3bd^4n + (B^2b^4c^3d - 6B^2ab^3c^2d^2 + 18B^2a^2* \\
& b^2cd^3 + 12B^2a^3bd^4)n^2)x + 12(B^2b^4d^4nx^4 + 4B^2ab^3* \\
& d^4nx^3 + 6B^2a^2b^2d^4nx^2 + 4B^2a^3bd^4nx - (B^2b^4c^4 - \\
& 4B^2ab^3c^3d + 6B^2a^2b^2c^2d^2 - 4B^2a^3b^3cd^3)n) \log(e)) * \\
& \log(bx + a) + 12((25B^2b^4d^4n^2 + 12ABb^4d^4n)x^4 + 4(12ABa \\
& b^3d^4n + (3B^2b^4cd^3 + 22B^2ab^3d^4)n^2)x^3 - (3B^2b^4c^4 \\
& - 16B^2ab^3c^3d + 36B^2a^2b^2c^2d^2 - 48B^2a^3b^3cd^3)n^2 + \\
& 6(12ABa^2b^2d^4n - (B^2b^4c^2d^2 - 8B^2ab^3cd^3 - 18B^2a^2 \\
& b^2d^4)n^2)x^2 - 12(ABb^4c^4 - 4ABab^3c^3d + 6ABa^2b^2c^2 \\
& d^2 - 4ABa^3b^3cd^3)n + 4(12ABa^3bd^4n + (B^2b^4c^3d - 6B \\
& ^2ab^3c^2d^2 + 18B^2a^2b^2cd^3 + 12B^2a^3bd^4)n^2)x + 12(B^2 \\
& b^4d^4n^2x^4 + 4B^2ab^3d^4n^2x^3 + 6B^2a^2b^2d^4n^2x^2 + 4 \\
& B^2a^3bd^4n^2x - (B^2b^4c^4 - 4B^2ab^3c^3d + 6B^2a^2b^2c^2 \\
& d^2 - 4B^2a^3b^3cd^3)n^2) \log(bx + a) + 12(B^2b^4d^4nx^4 + 4B^2 \\
& ab^3d^4nx^3 + 6B^2a^2b^2d^4nx^2 + 4B^2a^3bd^4nx - (B^2b^4 \\
& c^4 - 4B^2ab^3c^3d + 6B^2a^2b^2c^2d^2 - 4B^2a^3b^3cd^3)n) \log \\
& (e)) \log(dx + c) + 12(12ABb^4c^4 - 48ABab^3c^3d + 72ABa^2b \\
& ^2c^2d^2 - 48ABa^3b^3cd^3 + 12ABa^4d^4 - 12(B^2b^4cd^3 - B^2 \\
& ab^3d^4)nx^3 + 6(B^2b^4c^2d^2 - 8B^2ab^3cd^3 + 7B^2a^2b^2d \\
& ^4)nx^2 - 4(B^2b^4c^3d - 6B^2ab^3c^2d^2 + 18B^2a^2b^2cd^3 - \\
& 13B^2a^3bd^4)nx + (3B^2b^4c^4 - 16B^2ab^3c^3d + 36B^2a^2b \\
& ^2c^2d^2 - 48B^2a^3b^3cd^3 + 25B^2a^4d^4)n) \log(e)) / (a^4b^5c^4 - \\
& 4a^5b^4c^3d + 6a^6b^3c^2d^2 - 4a^7b^2cd^3 + a^8bd^4 + (b^9c \\
& ^4 - 4ab^8c^3d + 6a^2b^7c^2d^2 - 4a^3b^6cd^3 + a^4b^5d^4)x^4 \\
& + 4(a^8c^4 - 4a^2b^7c^3d + 6a^3b^6c^2d^2 - 4a^4b^5cd^3 + a \\
& ^5b^4d^4)x^3 + 6(a^2b^7c^4 - 4a^3b^6c^3d + 6a^4b^5c^2d^2 - 4 \\
& a^5b^4cd^3 + a^6b^3d^4)x^2 + 4(a^3b^6c^4 - 4a^4b^5c^3d + 6a^5 \\
& b^4c^2d^2 - 4a^6b^3cd^3 + a^7b^2d^4)x)
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n)/((d\*x+c)\*\*n))\*\*2/(b\*x+a)\*\*5,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)^2}{(bx+a)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(b\*x+a)^5,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^2/(b\*x + a)^5, x)

### 3.164 $\int (a+bx)^3 (A + B \log (e(a + bx)^n (c + dx)^{-n}))^3 dx$

**Optimal.** Leaf size=809

$$\frac{3Bn \log\left(\frac{bc-ad}{b(c+dx)}\right) (A + B \log (e(a + bx)^n (c + dx)^{-n}))^2 (bc - ad)^4}{4bd^4} - \frac{B^3 n^3 \log\left(\frac{a+bx}{c+dx}\right) (bc - ad)^4}{4bd^4} + \frac{3B^3 n^3 \log(c + dx)(bc - ad)^4}{2bd^4}$$

[Out]  $-(B^3(b^3c - a^3d)^3 n^3 x)/(4d^3) - (B^3(b^3c - a^3d)^4 n^3 \text{Log}[(a + bx)/(c + dx)])/(4b^3 d^4) + (3B^3(b^3c - a^3d)^4 n^3 \text{Log}[c + dx])/(2b^3 d^4) - (7B^2(b^3c - a^3d)^3 n^2 (a + bx)(A + B \text{Log}[(e(a + bx)^n)/(c + dx)^n]))/(4b^3 d^3) + (b^2 B^2(b^3c - a^3d)^2 n^2 (c + dx)^2 (A + B \text{Log}[(e(a + bx)^n)/(c + dx)^n]))/(4d^4) - (9B^2(b^3c - a^3d)^4 n^2 \text{Log}[(b^3c - a^3d)/(b(c + dx)]) * (A + B \text{Log}[(e(a + bx)^n)/(c + dx)^n]))/(2b^3 d^4) - (9B(b^3c - a^3d)^3 n (a + bx)(A + B \text{Log}[(e(a + bx)^n)/(c + dx)^n])^2)/(4b^3 d^3) + (9b^2 B(b^3c - a^3d)^2 n (c + dx)^2 (A + B \text{Log}[(e(a + bx)^n)/(c + dx)^n])^2)/(8d^4) - (b^2 B(b^3c - a^3d) n (c + dx)^3 (A + B \text{Log}[(e(a + bx)^n)/(c + dx)^n])^2)/(4d^4) - (3B(b^3c - a^3d)^4 n \text{Log}[(b^3c - a^3d)/(b(c + dx)]) * (A + B \text{Log}[(e(a + bx)^n)/(c + dx)^n])^2)/(4b^3 d^4) + ((a + bx)^4 (A + B \text{Log}[(e(a + bx)^n)/(c + dx)^n])^3)/(4b) + (7B^2(b^3c - a^3d)^4 n^2 (A + B \text{Log}[(e(a + bx)^n)/(c + dx)^n]) * \text{Log}[1 - (b(c + dx))/(d(a + bx))])/(4b^3 d^4) - (9B^3(b^3c - a^3d)^4 n^3 \text{PolyLog}[2, (d(a + bx))/(b(c + dx))])/(2b^3 d^4) - (3B^2(b^3c - a^3d)^4 n^2 (A + B \text{Log}[(e(a + bx)^n)/(c + dx)^n]) * \text{PolyLog}[2, (d(a + bx))/(b(c + dx))])/(2b^3 d^4) - (7B^3(b^3c - a^3d)^4 n^3 \text{PolyLog}[2, (b(c + dx))/(d(a + bx))])/(4b^3 d^4) + (3B^3(b^3c - a^3d)^4 n^3 \text{PolyLog}[3, (d(a + bx))/(b(c + dx))])/(2b^3 d^4)$

**Rubi [A]** time = 2.40354, antiderivative size = 1203, normalized size of antiderivative = 1.49, number of steps used = 56, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {6742, 2492, 43, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

$$\frac{3B^3 n \log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2 (e(a + bx)^n (c + dx)^{-n}) (bc - ad)^4}{4bd^4} + \frac{3B^3 n^3 \log(c + dx)(bc - ad)^4}{2bd^4} + \frac{11AB^2 n^2 \log(c + dx)(bc - ad)^4}{4bd^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^3,x]

[Out]  $(-3A^2 B(b^3c - a^3d)^3 n x)/(4d^3) - (5A^2 B^2(b^3c - a^3d)^3 n^2 x)/(4d^3) - (B^3(b^3c - a^3d)^3 n^3 x)/(4d^3) + (3A^2 B(b^3c - a^3d)^2 n (a + bx)^2)/(8b^3 d^2) + (A^2 B^2(b^3c - a^3d)^2 n^2 (a + bx)^2)/(4b^3 d^2) - (A^2 B(b^3c - a^3d) n (a + bx)^3)/(4b^3 d) + (A^3 (a + bx)^4)/(4b) + (3A^2 B(b^3c - a^3d)^4 n \text{Log}[c + dx])/(4b^3 d^4) + (11A^2 B^2(b^3c - a^3d)^4 n^2 \text{Log}[c + dx])/(4b^3 d^4) + (3B^3(b^3c - a^3d)^4 n^3 \text{Log}[c + dx])/(2b^3 d^4) - (3A^2 B^2(b^3c - a^3d)^3 n (a + bx) * \text{Log}[(e(a + bx)^n)/(c + dx)^n])/(2b^3 d^3) - (5B^3(b^3c - a^3d)^3 n^2 (a + bx) * \text{Log}[(e(a + bx)^n)/(c + dx)^n])/(4b^3 d^3) + (3A^2 B^2(b^3c - a^3d)^2 n (a + bx)^2 * \text{Log}[(e(a + bx)^n)/(c + dx)^n])/(4b^3 d^2) + (B^3(b^3c - a^3d)^2 n^2 (a + bx)^2 * \text{Log}[(e(a + bx)^n)/(c + dx)^n])/(4b^3 d^2) - (A^2 B^2(b^3c - a^3d) n (a + bx)^3 * \text{Log}[(e(a + bx)^n)/(c + dx)^n])/(2b^3 d) + (3A^2 B(a + bx)^4 * \text{Log}[(e(a + bx)^n)/(c + dx)^n])/(4b) - (3A^2 B^2(b^3c - a^3d)^4 n * \text{Log}[(b^3c - a^3d)/(b(c + dx)]) * \text{Log}[(e(a + bx)^n)/(c + dx)^n])/(2b^3 d^4) - (11B^3(b^3c - a^3d)^4 n^2 * \text{Log}[(b^3c - a^3d)/(b(c + dx)]) * \text{Log}[(e(a + bx)^n)/(c + dx)^n])/(4b^3 d^4) - (3B^3(b^3c - a^3d)^3 n (a + bx) * \text{Log}[(e(a + bx)^n)/(c + dx)^n]^2)/(4b^3 d^3) + (3B^3(b^3c - a^3d)^2 n (a + bx)^2 * \text{Log}[(e(a + bx)^n)/(c + dx)^n]^2)/(8b^3 d^2) - (B^3(b^3c - a^3d) n (a + bx)^3 * \text{Log}[(e(a + bx)^n)/(c + dx)^n]^2)/(4b^3 d)$

$$\begin{aligned}
& + (3AB^2(a+bx)^4 \log[(e(a+bx)^n)/(c+dx)^n]^2)/(4b) - (3B^3(b^2c-ad)^4n \log[(b^2c-ad)/(b(c+dx))] \log[(e(a+bx)^n)/(c+dx)^n]^2)/(4b^2d^4) \\
& + (B^3(a+bx)^4 \log[(e(a+bx)^n)/(c+dx)^n]^3)/(4b) - (3AB^2(b^2c-ad)^4n^2 \text{PolyLog}[2, (d(a+bx))/(b(c+dx))])/(2b^2d^4) \\
& - (11B^3(b^2c-ad)^4n^3 \text{PolyLog}[2, (d(a+bx))/(b(c+dx))])/(4b^2d^4) - (3B^3(b^2c-ad)^4n^2 \log[(e(a+bx)^n)/(c+dx)^n] \text{PolyLog}[2, 1 - (b^2c-ad)/(b(c+dx))])/(2b^2d^4) \\
& + (3B^3(b^2c-ad)^4n^3 \text{PolyLog}[3, 1 - (b^2c-ad)/(b(c+dx))])/(2b^2d^4)
\end{aligned}$$
Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)]^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0] && NeQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)]^(r_.)]^(s_.)*(RFX_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]
```

Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)]^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)]^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/h, x] + Dist[(p*r*s*(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
```

$[b*g - a*h, 0] \&\& \text{IGtQ}[s, 0]$

#### Rule 2411

$\text{Int}[(a_.) + \text{Log}[(c_.) * ((d_.) + (e_.) * (x_.)^{(n_.)}) * (b_.)]^{(p_.)} * ((f_.) + (g_.) * (x_.)^{(q_.)}) * ((h_.) + (i_.) * (x_.)^{(r_.)}), x\_Symbol] \rightarrow \text{Dist}[1/e, \text{Subst}[\text{Int}[(g*x)/e]^q * ((e*h - d*i)/e + (i*x)/e)^r * (a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, i, n, p, q, r\}, x] \&\& \text{EqQ}[e*f - d*g, 0] \&\& (\text{IGtQ}[p, 0] \mid \mid \text{IGtQ}[r, 0]) \&\& \text{IntegerQ}[2*r]$

#### Rule 2343

$\text{Int}[(a_.) + \text{Log}[(c_.) * (x_.)^{(n_.)}] * (b_.)] / ((x_.) * ((d_.) + (e_.) * (x_.)^{(r_.)})), x\_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[(a + b*\text{Log}[c*x]) / (x*(d + e*x^{(r/n)}))], x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, e, n, r\}, x] \&\& \text{IntegerQ}[r/n]$

#### Rule 2333

$\text{Int}[(a_.) + \text{Log}[(c_.) * (x_.)^{(n_.)}] * (b_.)]^{(p_.)} * ((d_.) + (e_.) / (x_.)^{(q_.)}) * (x_.)^{(m_.)}, x\_Symbol] \rightarrow \text{Int}[(e + d*x)^q * (a + b*\text{Log}[c*x^n])^p, x] /; \text{FreeQ}[\{a, b, c, d, e, m, n, p\}, x] \&\& \text{EqQ}[m, q] \&\& \text{IntegerQ}[q]$

#### Rule 2315

$\text{Int}[\text{Log}[(c_.) * (x_.)] / ((d_.) + (e_.) * (x_.)], x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}[\{c, d, e\}, x] \&\& \text{EqQ}[e + c*d, 0]$

#### Rule 2506

$\text{Int}[\text{Log}[v_] * \text{Log}[(e_.) * ((f_.) * (a_.) + (b_.) * (x_.)^{(p_.)}) * ((c_.) + (d_.) * (x_.)^{(q_.)})^{(r_.)}]^{(s_.)} * (u_.), x\_Symbol] \rightarrow \text{With}[\{g = \text{Simplify}[(v - 1) * (c + d*x) / (a + b*x)], h = \text{Simplify}[u * (a + b*x) * (c + d*x)]\}, -\text{Simp}[(h * \text{PolyLog}[2, 1 - v] * \text{Log}[e * (f * (a + b*x)^p * (c + d*x)^q]^r]^s) / (b*c - a*d), x] + \text{Dist}[h * p * r * s, \text{Int}[(\text{PolyLog}[2, 1 - v] * \text{Log}[e * (f * (a + b*x)^p * (c + d*x)^q]^r]^s - 1) / ((a + b*x) * (c + d*x)), x], x] /; \text{FreeQ}[\{g, h\}, x] /; \text{FreeQ}[\{a, b, c, d, e, f, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{EqQ}[p + q, 0]$

#### Rule 6610

$\text{Int}[(u_) * \text{PolyLog}[n_, v_], x\_Symbol] \rightarrow \text{With}[\{w = \text{DerivativeDivides}[v, u*v, x]\}, \text{Simp}[w * \text{PolyLog}[n + 1, v], x] /; \text{!FalseQ}[w] /; \text{FreeQ}[n, x]$

#### Rubi steps

$$\begin{aligned}
\int (a + bx)^3 (A + B \log(e(a + bx)^n(c + dx)^{-n}))^3 dx &= \int (A^3(a + bx)^3 + 3A^2B(a + bx)^3 \log(e(a + bx)^n(c + dx)^{-n}) + 3AB^2(a + bx)^3 \log^2(e(a + bx)^n(c + dx)^{-n}) + B^3(a + bx)^3 \log^3(e(a + bx)^n(c + dx)^{-n})) dx \\
&= \frac{A^3(a + bx)^4}{4b} + (3A^2B) \int (a + bx)^3 \log(e(a + bx)^n(c + dx)^{-n}) dx + \frac{3AB^2}{4b} \int (a + bx)^3 \log^2(e(a + bx)^n(c + dx)^{-n}) dx + \frac{B^3}{4b} \int (a + bx)^3 \log^3(e(a + bx)^n(c + dx)^{-n}) dx \\
&= \frac{A^3(a + bx)^4}{4b} + \frac{3A^2B(a + bx)^4 \log(e(a + bx)^n(c + dx)^{-n})}{4b} + \frac{3AB^2(a + bx)^4 \log^2(e(a + bx)^n(c + dx)^{-n})}{4b} + \frac{B^3(a + bx)^4 \log^3(e(a + bx)^n(c + dx)^{-n})}{4b} \\
&= \frac{A^3(a + bx)^4}{4b} + \frac{3A^2B(a + bx)^4 \log(e(a + bx)^n(c + dx)^{-n})}{4b} + \frac{3AB^2(a + bx)^4 \log^2(e(a + bx)^n(c + dx)^{-n})}{4b} + \frac{B^3(a + bx)^4 \log^3(e(a + bx)^n(c + dx)^{-n})}{4b} \\
&= -\frac{3A^2B(bc - ad)^3 nx}{4d^3} + \frac{3A^2B(bc - ad)^2 n(a + bx)^2}{8bd^2} - \frac{A^2B(bc - ad)n(a + bx)}{4bd} \\
&= -\frac{3A^2B(bc - ad)^3 nx}{4d^3} + \frac{3A^2B(bc - ad)^2 n(a + bx)^2}{8bd^2} - \frac{A^2B(bc - ad)n(a + bx)}{4bd} \\
&= -\frac{3A^2B(bc - ad)^3 nx}{4d^3} + \frac{3A^2B(bc - ad)^2 n(a + bx)^2}{8bd^2} - \frac{A^2B(bc - ad)n(a + bx)}{4bd} \\
&= -\frac{3A^2B(bc - ad)^3 nx}{4d^3} - \frac{5AB^2(bc - ad)^3 n^2 x}{4d^3} + \frac{3A^2B(bc - ad)^2 n(a + bx)}{8bd^2} \\
&= -\frac{3A^2B(bc - ad)^3 nx}{4d^3} - \frac{5AB^2(bc - ad)^3 n^2 x}{4d^3} + \frac{3A^2B(bc - ad)^2 n(a + bx)}{8bd^2} \\
&= -\frac{3A^2B(bc - ad)^3 nx}{4d^3} - \frac{5AB^2(bc - ad)^3 n^2 x}{4d^3} + \frac{3A^2B(bc - ad)^2 n(a + bx)}{8bd^2} \\
&= -\frac{3A^2B(bc - ad)^3 nx}{4d^3} - \frac{5AB^2(bc - ad)^3 n^2 x}{4d^3} - \frac{B^3(bc - ad)^3 n^3 x}{4d^3} + \frac{3A^2B(bc - ad)^2 n(a + bx)}{8bd^2} \\
&= -\frac{3A^2B(bc - ad)^3 nx}{4d^3} - \frac{5AB^2(bc - ad)^3 n^2 x}{4d^3} - \frac{B^3(bc - ad)^3 n^3 x}{4d^3} + \frac{3A^2B(bc - ad)^2 n(a + bx)}{8bd^2} \\
&= -\frac{3A^2B(bc - ad)^3 nx}{4d^3} - \frac{5AB^2(bc - ad)^3 n^2 x}{4d^3} - \frac{B^3(bc - ad)^3 n^3 x}{4d^3} + \frac{3A^2B(bc - ad)^2 n(a + bx)}{8bd^2}
\end{aligned}$$

**Mathematica [B]** time = 9.69639, size = 9054, normalized size = 11.19

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^3,x]

[Out] Result too large to show

**Maple [F]** time = 5.133, size = 0, normalized size = 0.

$$\int (bx + a)^3 \left( A + B \ln \left( \frac{e(bx + a)^n}{(dx + c)^n} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x+a)^3*(A+B*\ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)$

[Out]  $\text{int}((b*x+a)^3*(A+B*\ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x+a)^3*(A+B*\log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, \text{algorithm}="maxima")$

[Out] 
$$\begin{aligned} & 3/4*A^2*B*b^3*x^4*\log((b*x + a)^n*e/(d*x + c)^n) + 1/4*A^3*b^3*x^4 + 3*A^2*B*a*b^2*x^3*\log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*b^2*x^3 + 9/2*A^2*B*a^2*b*x^2*\log((b*x + a)^n*e/(d*x + c)^n) + 3/2*A^3*a^2*b*x^2 + 3*A^2*B*a^3*x*\log((b*x + a)^n*e/(d*x + c)^n) + A^3*a^3*x + 3*(a*e*n*\log(b*x + a)/b - c*e*n*\log(d*x + c)/d)*A^2*B*a^3/e - 9/2*(a^2*e*n*\log(b*x + a)/b^2 - c^2*e*n*\log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*a^2*b/e + 3/2*(2*a^3*e*n*\log(b*x + a)/b^3 - 2*c^3*e*n*\log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*a*b^2/e - 1/8*(6*a^4*e*n*\log(b*x + a)/b^4 - 6*c^4*e*n*\log(d*x + c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*A^2*B*b^3/e - 1/8*(2*(B^3*b^4*d^4*x^4 + 4*B^3*a*b^3*d^4*x^3 + 6*B^3*a^2*b^2*d^4*x^2 + 4*B^3*a^3*b*d^4*x)*\log((d*x + c)^n)^3 - (6*B^3*a^4*d^4*n*\log(b*x + a) + 6*(B^3*b^4*d^4*\log(e) + A*B^2*b^4*d^4)*x^4 + 6*(b^4*c^4*n - 4*a*b^3*c^3*d*n + 6*a^2*b^2*c^2*d^2*n - 4*a^3*b*c*d^3*n)*B^3*\log(d*x + c) + 2*(12*A*B^2*a*b^3*d^4 + (a*b^3*d^4*(n + 12*\log(e)) - b^4*c*d^3*n)*B^3)*x^3 + 3*(12*A*B^2*a^2*b^2*d^4 + (3*a^2*b^2*d^4*(n + 4*\log(e)) + b^4*c^2*d^2*n - 4*a*b^3*c*d^3*n)*B^3)*x^2 + 6*(4*A*B^2*a^3*b*d^4 + (a^3*b*d^4*(3*n + 4*\log(e)) - b^4*c^3*d*n + 4*a*b^3*c^2*d^2*n - 6*a^2*b^2*c*d^3*n)*B^3)*x + 6*(B^3*b^4*d^4*x^4 + 4*B^3*a*b^3*d^4*x^3 + 6*B^3*a^2*b^2*d^4*x^2 + 4*B^3*a^3*b*d^4*x)*\log((b*x + a)^n)*\log((d*x + c)^n)^2)/(b*d^4) - \text{integrate}(-1/4*(4*B^3*a^3*b*c*d^3*\log(e)^3 + 12*A*B^2*a^3*b*c*d^3*\log(e)^2 + 4*(B^3*b^4*d^4*\log(e)^3 + 3*A*B^2*b^4*d^4*\log(e)^2)*x^4 + 4*(3*(b^4*c*d^3*\log(e)^2 + 3*a*b^3*d^4*\log(e)^2)*A*B^2 + (b^4*c*d^3*\log(e)^3 + 3*a*b^3*d^4*\log(e)^3)*B^3)*x^3 + 4*(B^3*b^4*d^4*x^4 + B^3*a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*B^3*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*B^3*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*B^3*x)*\log((b*x + a)^n)^3 + 12*(3*(a*b^3*c*d^3*\log(e)^2 + a^2*b^2*d^4*\log(e)^2)*A*B^2 + (a*b^3*c*d^3*\log(e)^3 + a^2*b^2*d^4*\log(e)^3)*B^3)*x^2 + 12*(B^3*a^3*b*c*d^3*\log(e) + A*B^2*a^3*b*c*d^3 + (B^3*b^4*d^4*\log(e) + A*B^2*b^4*d^4)*x^4 + ((b^4*c*d^3 + 3*a*b^3*d^4)*A*B^2 + (b^4*c*d^3*\log(e) + 3*a*b^3*d^4*\log(e))*B^3)*x^3 + 3*((a*b^3*c*d^3 + a^2*b^2*d^4)*A*B^2 + (a*b^3*c*d^3*\log(e) + a^2*b^2*d^4*\log(e))*B^3)*x^2 + ((3*a^2*b^2*c*d^3 + a^3*b*d^4)*A*B^2 + (3*a^2*b^2*c*d^3*\log(e) + a^3*b*d^4*\log(e))*B^3)*x)*\log((b*x + a)^n)^2 + 4*(3*(3*a^2*b^2*c*d^3*\log(e)^2 + a^3*b*d^4*\log(e)^2)*A*B^2 + (3*a^2*b^2*c*d^3*\log(e)^3 + a^3*b*d^4*\log(e)^3)*B^3)*x + 12*(B^3*a^3*b*c*d^3*\log(e)^2 + 2*A*B^2*a^3*b*c*d^3*\log(e) + (B^3*b^4*d^4*\log(e)^2 + 2*A*B^2*b^4*d^4*\log(e))*x^4 + (2*(b^4*c*d^3*\log(e) + 3*a*b^3*d^4*\log(e))*A*B^2 + (b^4*c*d^3*\log(e)^2 + 3*a*b^3*d^4*\log(e)^2)*B^3)*x^3 + 3*(2*(a*b^3*c*d^3*\log(e) + a^2*b^2*d^4*\log(e))*A*B^2 + (a*b^3*c*d^3*\log(e)^2 + a^2*b^2*d^4*\log(e)^2)*B^3)*x^2 + (2*(3*a^2*b^2*c*d^3*\log(e) + a^3*b*d^4*\log(e))*A*B^2 + (3*a^2*b^2*c*d^3*\log(e)^2 + a^3*b*d^4*\log(e)^2)*B^3)*x)*\log((b*x + a)^n) - (6*B^3*a^4*d^4*n^2*\log(b*x + a) + 12*B^3*a^3*b*c*d^3*\log(e)^2 + 24*A*B^2*a^3*b*c*d^3*\log(e) + 6*((n*\log(e) + 2*\log(e)^2)*B^3*b^4*d^4 + A*B^2*b^4*d^4* \end{aligned}$$

```
(n + 4*log(e))*x^4 + 6*(b^4*c^4*n^2 - 4*a*b^3*c^3*d*n^2 + 6*a^2*b^2*c^2*d^2*n^2 - 4*a^3*b*c*d^3*n^2)*B^3*log(dx + c) + 2*(12*(a*b^3*d^4*(n + 3*log(e)) + b^4*c*d^3*log(e))*A*B^2 - ((n^2 - 6*log(e)^2)*b^4*c*d^3 - (n^2 + 12*n*log(e) + 18*log(e)^2)*a*b^3*d^4)*B^3)*x^3 + 3*(12*(a^2*b^2*d^4*(n + 2*log(e)) + 2*a*b^3*c*d^3*log(e))*A*B^2 + (b^4*c^2*d^2*n^2 - 4*(n^2 - 3*log(e)^2)*a*b^3*c*d^3 + 3*(n^2 + 4*n*log(e) + 4*log(e)^2)*a^2*b^2*d^4)*B^3)*x^2 + 12*(B^3*b^4*d^4*x^4 + B^3*a^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*B^3*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*B^3*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*B^3*x)*log((b*x + a)^n)^2 + 6*(4*(a^3*b*d^4*(n + log(e)) + 3*a^2*b^2*c*d^3*log(e))*A*B^2 - (b^4*c^3*d*n^2 - 4*a*b^3*c^2*d^2*n^2 + 6*(n^2 - log(e)^2)*a^2*b^2*c*d^3 - (3*n^2 + 4*n*log(e) + 2*log(e)^2)*a^3*b*d^4)*B^3)*x + 6*(4*B^3*a^3*b*c*d^3*log(e) + 4*A*B^2*a^3*b*c*d^3 + (B^3*b^4*d^4*(n + 4*log(e)) + 4*A*B^2*b^4*d^4)*x^4 + 4*((b^4*c*d^3 + 3*a*b^3*d^4)*A*B^2 + (a*b^3*d^4*(n + 3*log(e)) + b^4*c*d^3*log(e))*B^3)*x^3 + 6*(2*(a*b^3*c*d^3 + a^2*b^2*d^4)*A*B^2 + (a^2*b^2*d^4*(n + 2*log(e)) + 2*a*b^3*c*d^3*log(e))*B^3)*x^2 + 4*((3*a^2*b^2*c*d^3 + a^3*b*d^4)*A*B^2 + (a^3*b*d^4*(n + log(e)) + 3*a^2*b^2*c*d^3*log(e))*B^3)*x)*log((b*x + a)^n)*log((dx + c)^n)/(b*d^4*x + b*c*d^3), x
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^3 b^3 x^3 + 3 A^3 a b^2 x^2 + 3 A^3 a^2 b x + A^3 a^3 + \left( B^3 b^3 x^3 + 3 B^3 a b^2 x^2 + 3 B^3 a^2 b x + B^3 a^3 \right) \log \left( \frac{(b x + a)^n e}{(d x + c)^n} \right)^3 + 3 \left( A B^3 \right) \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas")
```

```
[Out] integral(A^3*b^3*x^3 + 3*A^3*a*b^2*x^2 + 3*A^3*a^2*b*x + A^3*a^3 + (B^3*b^3*x^3 + 3*B^3*a*b^2*x^2 + 3*B^3*a^2*b*x + B^3*a^3)*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*(A*B^2*b^3*x^3 + 3*A*B^2*a*b^2*x^2 + 3*A*B^2*a^2*b*x + A*B^2*a^3)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b^3*x^3 + 3*A^2*B*a*b^2*x^2 + 3*A^2*B*a^2*b*x + A^2*B*a^3)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a)^3 \left( B \log \left( \frac{(bx + a)^n e}{(dx + c)^n} \right) + A \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((b*x+a)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((b*x + a)^3*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

### 3.165 $\int (a+bx)^2 (A + B \log (e(a + bx)^n(c + dx)^{-n}))^3 dx$

**Optimal.** Leaf size=614

$$\frac{2B^2n^2(bc - ad)^3 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right) (B \log (e(a + bx)^n(c + dx)^{-n}) + A)}{bd^3} + \frac{4B^3n^3(bc - ad)^3 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd^3} + \frac{B^3n^3}{bd^3}$$

```
[Out] -((B^3*(b*c - a*d)^3*n^3*Log[c + d*x])/(b*d^3)) + (B^2*(b*c - a*d)^2*n^2*(a
+ b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x]^n)))/(b*d^2) + (4*B^2*(b*c - a
*d)^3*n^2*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*
x)^n]))/(b*d^3) + (2*B*(b*c - a*d)^2*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n
/(c + d*x)^n])^2)/(b*d^2) - (b*B*(b*c - a*d)*n*(c + d*x)^2*(A + B*Log[(e*(a
+ b*x)^n)/(c + d*x)^n])^2)/(2*d^3) + (B*(b*c - a*d)^3*n*Log[(b*c - a*d)/(b
*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(b*d^3) + ((a + b*
x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(3*b) - (B^2*(b*c - a*d)^3
*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/(b*d^3) + (4*B^3*(b*c - a*d)^3*n^3*PolyLog[2, (d*(a + b*x))/(b*(c
+ d*x))])/(b*d^3) + (2*B^2*(b*c - a*d)^3*n^2*(A + B*Log[(e*(a + b*x)^n)/(c
+ d*x)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^3) + (B^3*(b*c - a
*d)^3*n^3*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b*d^3) - (2*B^3*(b*c -
a*d)^3*n^3*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(b*d^3)
```

**Rubi [A]** time = 1.7369, antiderivative size = 915, normalized size of antiderivative = 1.49, number of steps used = 40, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {6742, 2492, 43, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

$$\frac{(a + bx)^3 \log^3 (e(a + bx)^n(c + dx)^{-n}) B^3}{3b} - \frac{(bc - ad)n(a + bx)^2 \log^2 (e(a + bx)^n(c + dx)^{-n}) B^3}{2bd} + \frac{(bc - ad)^2 n(a + bx) \log^2 (e(a + bx)^n(c + dx)^{-n}) B^3}{bd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x]^n])^3,x]
```

```
[Out] (A^2*B*(b*c - a*d)^2*n*x)/d^2 + (A*B^2*(b*c - a*d)^2*n^2*x)/d^2 - (A^2*B*(b
*c - a*d)*n*(a + b*x)^2)/(2*b*d) + (A^3*(a + b*x)^3)/(3*b) - (A^2*B*(b*c -
a*d)^3*n*Log[c + d*x])/(b*d^3) - (3*A*B^2*(b*c - a*d)^3*n^2*Log[c + d*x])/(
b*d^3) - (B^3*(b*c - a*d)^3*n^3*Log[c + d*x])/(b*d^3) + (2*A*B^2*(b*c - a*d
)^2*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x]^n])/(b*d^2) + (B^3*(b*c - a*d
)^2*n^2*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x]^n])/(b*d^2) - (A*B^2*(b*c -
a*d)*n*(a + b*x)^2*Log[(e*(a + b*x)^n)/(c + d*x]^n])/(b*d) + (A^2*B*(a + b
*x)^3*Log[(e*(a + b*x)^n)/(c + d*x]^n])/b + (2*A*B^2*(b*c - a*d)^3*n*Log[(b
*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x]^n])/(b*d^3) + (3*B^3
*(b*c - a*d)^3*n^2*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c +
d*x]^n])/(b*d^3) + (B^3*(b*c - a*d)^2*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c +
d*x]^n])^2)/(b*d^2) - (B^3*(b*c - a*d)*n*(a + b*x)^2*Log[(e*(a + b*x)^n)/(c
+ d*x]^n])^2)/(2*b*d) + (A*B^2*(a + b*x)^3*Log[(e*(a + b*x)^n)/(c + d*x]^n])
^2)/b + (B^3*(b*c - a*d)^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)
^n)/(c + d*x]^n])^2)/(b*d^3) + (B^3*(a + b*x)^3*Log[(e*(a + b*x)^n)/(c + d*x
]^n])^3)/(3*b) + (2*A*B^2*(b*c - a*d)^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c +
d*x))])/(b*d^3) + (3*B^3*(b*c - a*d)^3*n^3*PolyLog[2, (d*(a + b*x))/(b*(c +
d*x))])/(b*d^3) + (2*B^3*(b*c - a*d)^3*n^2*Log[(e*(a + b*x)^n)/(c + d*x]^n
]*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*d^3) - (2*B^3*(b*c - a*d)^
3*n^3*PolyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*d^3)
```

**Rule 6742**

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 2492

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
0] && NeQ[m, -1]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 2514

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFX_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]
```

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_.), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rule 2488

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

#### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] := Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

#### Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] := Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

### Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] := Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

### Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] := With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned}
\int (a + bx)^2 (A + B \log(e(a + bx)^n(c + dx)^{-n}))^3 dx &= \int (A^3(a + bx)^2 + 3A^2B(a + bx)^2 \log(e(a + bx)^n(c + dx)^{-n}) + 3AB^2(a + bx)^2 \log^2(e(a + bx)^n(c + dx)^{-n}) + B^3(a + bx)^2 \log^3(e(a + bx)^n(c + dx)^{-n})) dx \\
&= \frac{A^3(a + bx)^3}{3b} + (3A^2B) \int (a + bx)^2 \log(e(a + bx)^n(c + dx)^{-n}) dx \\
&= \frac{A^3(a + bx)^3}{3b} + \frac{A^2B(a + bx)^3 \log(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{AB^2(a + bx)^3 \log^2(e(a + bx)^n(c + dx)^{-n})}{2b} + \frac{B^3(a + bx)^3 \log^3(e(a + bx)^n(c + dx)^{-n})}{3b} \\
&= \frac{A^3(a + bx)^3}{3b} + \frac{A^2B(a + bx)^3 \log(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{AB^2(a + bx)^3 \log^2(e(a + bx)^n(c + dx)^{-n})}{2b} + \frac{B^3(a + bx)^3 \log^3(e(a + bx)^n(c + dx)^{-n})}{3b} \\
&= \frac{A^2B(bc - ad)^2 nx}{d^2} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} + \frac{A^3(a + bx)^3}{3b} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} + \frac{AB^2(bc - ad)^2 n^2 x}{d^2} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} \\
&= \frac{A^2B(bc - ad)^2 nx}{d^2} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} + \frac{A^3(a + bx)^3}{3b} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} + \frac{AB^2(bc - ad)^2 n^2 x}{d^2} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} \\
&= \frac{A^2B(bc - ad)^2 nx}{d^2} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} + \frac{A^3(a + bx)^3}{3b} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} + \frac{AB^2(bc - ad)^2 n^2 x}{d^2} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} \\
&= \frac{A^2B(bc - ad)^2 nx}{d^2} + \frac{AB^2(bc - ad)^2 n^2 x}{d^2} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} \\
&= \frac{A^2B(bc - ad)^2 nx}{d^2} + \frac{AB^2(bc - ad)^2 n^2 x}{d^2} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} \\
&= \frac{A^2B(bc - ad)^2 nx}{d^2} + \frac{AB^2(bc - ad)^2 n^2 x}{d^2} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} \\
&= \frac{A^2B(bc - ad)^2 nx}{d^2} + \frac{AB^2(bc - ad)^2 n^2 x}{d^2} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} \\
&= \frac{A^2B(bc - ad)^2 nx}{d^2} + \frac{AB^2(bc - ad)^2 n^2 x}{d^2} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} \\
&= \frac{A^2B(bc - ad)^2 nx}{d^2} + \frac{AB^2(bc - ad)^2 n^2 x}{d^2} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd} \\
&= \frac{A^2B(bc - ad)^2 nx}{d^2} + \frac{AB^2(bc - ad)^2 n^2 x}{d^2} - \frac{A^2B(bc - ad)n(a + bx)^2}{2bd}
\end{aligned}$$

**Mathematica [B]** time = 4.11063, size = 5668, normalized size = 9.23

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n)]^3,x]

[Out] Result too large to show

**Maple [F]** time = 4.456, size = 0, normalized size = 0.

$$\int (bx + a)^2 \left( A + B \ln \left( \frac{e(bx + a)^n}{(dx + c)^n} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x+a)^2*(A+B*\ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)$

[Out]  $\text{int}((b*x+a)^2*(A+B*\ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x+a)^2*(A+B*\log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, \text{algorithm}="maxima")$

[Out]  $A^2*B*b^2*x^3*\log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^3*b^2*x^3 + 3*A^2*B*a*b*x^2*\log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*b*x^2 + 3*A^2*B*a^2*x*\log((b*x + a)^n*e/(d*x + c)^n) + A^3*a^2*x + 3*(a*e*n*\log(b*x + a)/b - c*e*n*\log(d*x + c)/d)*A^2*B*a^2/e - 3*(a^2*e*n*\log(b*x + a)/b^2 - c^2*e*n*\log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*a*b/e + 1/2*(2*a^3*e*n*\log(b*x + a)/b^3 - 2*c^3*e*n*\log(d*x + c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*A^2*B*b^2/e - 1/6*(2*(B^3*b^3*d^3*x^3 + 3*B^3*a*b^2*d^3*x^2 + 3*B^3*a^2*b*d^3*x)*\log((d*x + c)^n)^3 - 3*(2*B^3*a^3*d^3*n*\log(b*x + a) - 2*(b^3*c^3*n - 3*a*b^2*c^2*d*n + 3*a^2*b*c*d^2*n)*B^3*\log(d*x + c) + 2*(B^3*b^3*d^3*\log(e) + A*B^2*b^3*d^3)*x^3 + (6*A*B^2*a*b^2*d^3 + (a*b^2*d^3*(n + 6*\log(e)) - b^3*c*d^2*n)*B^3)*x^2 + 2*(3*A*B^2*a^2*b*d^3 + (a^2*b*d^3*(2*n + 3*\log(e)) + b^3*c^2*d*n - 3*a*b^2*c*d^2*n)*B^3)*x + 2*(B^3*b^3*d^3*x^3 + 3*B^3*a*b^2*d^3*x^2 + 3*B^3*a^2*b*d^3*x)*\log((b*x + a)^n))*\log((d*x + c)^n)^2)/(b*d^3) - \text{integrate}(-(B^3*a^2*b*c*d^2*\log(e))^3 + 3*A*B^2*a^2*b*c*d^2*\log(e)^2 + (B^3*b^3*d^3*\log(e))^3 + 3*A*B^2*b^3*d^3*\log(e)^2)*x^3 + (B^3*b^3*d^3*x^3 + B^3*a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*B^3*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*B^3*x)*\log((b*x + a)^n)^3 + (3*(b^3*c*d^2*\log(e)^2 + 2*a*b^2*d^3*\log(e)^2)*A*B^2 + (b^3*c*d^2*\log(e)^3 + 2*a*b^2*d^3*\log(e)^3)*B^3)*x^2 + 3*(B^3*a^2*b*c*d^2*\log(e) + A*B^2*a^2*b*c*d^2 + (B^3*b^3*d^3*\log(e) + A*B^2*b^3*d^3)*x^3 + ((b^3*c*d^2 + 2*a*b^2*d^3)*A*B^2 + (b^3*c*d^2*\log(e) + 2*a*b^2*d^3*\log(e))*B^3)*x^2 + ((2*a*b^2*c*d^2 + a^2*b*d^3)*A*B^2 + (2*a*b^2*c*d^2*\log(e) + a^2*b*d^3*\log(e))*B^3)*x)*\log((b*x + a)^n)^2 + (3*(2*a*b^2*c*d^2*\log(e)^2 + a^2*b*d^3*\log(e)^2)*A*B^2 + (2*a*b^2*c*d^2*\log(e)^3 + a^2*b*d^3*\log(e)^3)*B^3)*x + 3*(B^3*a^2*b*c*d^2*\log(e)^2 + 2*A*B^2*a^2*b*c*d^2*\log(e) + (B^3*b^3*d^3*\log(e))^2 + 2*A*B^2*b^3*d^3*\log(e))*x^3 + (2*(b^3*c*d^2*\log(e) + 2*a*b^2*d^3*\log(e))*A*B^2 + (b^3*c*d^2*\log(e)^2 + 2*a*b^2*d^3*\log(e)^2)*B^3)*x^2 + (2*(2*a*b^2*c*d^2*\log(e) + a^2*b*d^3*\log(e))*A*B^2 + (2*a*b^2*c*d^2*\log(e)^2 + a^2*b*d^3*\log(e)^2)*B^3)*x)*\log((b*x + a)^n) - (2*B^3*a^3*d^3*n^2*\log(b*x + a) + 3*B^3*a^2*b*c*d^2*\log(e)^2 + 6*A*B^2*a^2*b*c*d^2*\log(e) - 2*(b^3*c^3*n^2 - 3*a*b^2*c^2*d*n^2 + 3*a^2*b*c*d^2*n^2)*B^3*\log(d*x + c) + ((2*n*\log(e) + 3*\log(e)^2)*B^3*b^3*d^3 + 2*A*B^2*b^3*d^3*(n + 3*\log(e)))*x^3 + (6*(a*b^2*d^3*(n + 2*\log(e)) + b^3*c*d^2*\log(e))*A*B^2 - ((n^2 - 3*\log(e)^2)*b^3*c*d^2 - (n^2 + 6*n*\log(e) + 6*\log(e)^2)*a*b^2*d^3)*B^3)*x^2 + 3*(B^3*b^3*d^3*x^3 + B^3*a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*B^3*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*B^3*x)*\log((b*x + a)^n)^2 + (6*(a^2*b*d^3*(n + \log(e)) + 2*a*b^2*c*d^2*\log(e))*A*B^2 + (2*b^3*c^2*d*n^2 - 6*(n^2 - \log(e)^2)*a*b^2*c*d^2 + (4*n^2 + 6*n*\log(e) + 3*\log(e)^2)*a^2*b*d^3)*B^3)*x + 2*(3*B^3*a^2*b*c*d^2*\log(e) + 3*A*B^2*a^2*b*c*d^2 + (B^3*b^3*d^3*(n + 3*\log(e)) + 3*A*B^2*b^3*d^3)*x^3 + 3*((b^3*c*d^2 + 2*a*b^2*d^3)*A*B^2 + (a*b^2*d^3*(n + 2*\log(e)) + b^3*c*d^2*\log(e))*B^3)*x^2 + 3*((2*a*b^2*c*d^2 + a^2*b*d^3)*A*B^2 + (a^2*b*d^3*(n + \log(e)) + 2*a*b^2*c*d^2*\log(e))*B^3)*x)*\log((b*x + a)^n))*\log((d*x + c)^n)/(b*d^3*x +$

$b*c*d^2), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

integral  $\left( A^3 b^2 x^2 + 2 A^3 a b x + A^3 a^2 + (B^3 b^2 x^2 + 2 B^3 a b x + B^3 a^2) \log \left( \frac{(b x + a)^n e}{(d x + c)^n} \right) + 3 (A B^2 b^2 x^2 + 2 A B^2 a b x + A B^2 a^2) \log \left( \frac{(b x + a)^n e}{(d x + c)^n} \right) \right)^3, x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3,x, algorithm="fricas")

[Out] integral(A^3\*b^2\*x^2 + 2\*A^3\*a\*b\*x + A^3\*a^2 + (B^3\*b^2\*x^2 + 2\*B^3\*a\*b\*x + B^3\*a^2)\*log((b\*x + a)^n\*e/(d\*x + c)^n)^3 + 3\*(A\*B^2\*b^2\*x^2 + 2\*A\*B^2\*a\*b\*x + A\*B^2\*a^2)\*log((b\*x + a)^n\*e/(d\*x + c)^n)^2 + 3\*(A^2\*B\*b^2\*x^2 + 2\*A^2\*B\*a\*b\*x + A^2\*B\*a^2)\*log((b\*x + a)^n\*e/(d\*x + c)^n), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*2\*(A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b x + a)^2 \left( B \log \left( \frac{(b x + a)^n e}{(d x + c)^n} \right) + A \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)^2\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3,x, algorithm="giac")

[Out] integrate((b\*x + a)^2\*(B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^3, x)

### 3.166 $\int (a+bx) (A + B \log (e(a + bx)^n(c + dx)^{-n}))^3 dx$

**Optimal.** Leaf size=376

$$\frac{3B^2n^2(bc - ad)^2 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right) (B \log (e(a + bx)^n(c + dx)^{-n}) + A)}{bd^2} - \frac{3B^3n^3(bc - ad)^2 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} + \frac{3B^3n^3(bc - ad)^2 \text{PolyLog}\left(3, \frac{d(a+bx)}{b(c+dx)}\right)}{bd^2}$$

[Out]  $(-3*B^2*(b*c - a*d)^2*n^2*\text{Log}[(b*c - a*d)/(b*(c + d*x))]*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*d^2) - (3*B*(b*c - a*d)*n*(a + b*x)*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*b*d) - (3*B*(b*c - a*d)^2*n*\text{Log}[(b*c - a*d)/(b*(c + d*x))]*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*b*d^2) + ((a + b*x)^2*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(2*b) - (3*B^3*(b*c - a*d)^2*n^3*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2) - (3*B^2*(b*c - a*d)^2*n^2*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2) + (3*B^3*(b*c - a*d)^2*n^3*\text{PolyLog}[3, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2)$

**Rubi [A]** time = 1.1825, antiderivative size = 700, normalized size of antiderivative = 1.86, number of steps used = 27, number of rules used = 13, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$ , Rules used = {6742, 2492, 43, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

$$\frac{3AB^2n^2(bc - ad)^2 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} - \frac{3B^3n^2(bc - ad)^2 \text{PolyLog}\left(2, 1 - \frac{bc-ad}{b(c+dx)}\right) \log (e(a + bx)^n(c + dx)^{-n})}{bd^2} - \frac{3B^3n^3(bc - ad)^2 \text{PolyLog}\left(3, 1 - \frac{bc-ad}{b(c+dx)}\right)}{bd^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x)*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^3, x]$

[Out]  $(-3*A^2*B*(b*c - a*d)*n*x)/(2*d) + (A^3*(a + b*x)^2)/(2*b) + (3*A^2*B*(b*c - a*d)^2*n*\text{Log}[c + d*x])/(2*b*d^2) + (3*A*B^2*(b*c - a*d)^2*n^2*\text{Log}[c + d*x])/(b*d^2) - (3*A*B^2*(b*c - a*d)*n*(a + b*x)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d) + (3*A^2*B*(a + b*x)^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(2*b) - (3*A*B^2*(b*c - a*d)^2*n*\text{Log}[(b*c - a*d)/(b*(c + d*x))]*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d^2) - (3*B^3*(b*c - a*d)^2*n^2*\text{Log}[(b*c - a*d)/(b*(c + d*x))]*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d^2) - (3*B^3*(b*c - a*d)*n*(a + b*x)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*b*d) + (3*A*B^2*(a + b*x)^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*b) - (3*B^3*(b*c - a*d)^2*n*\text{Log}[(b*c - a*d)/(b*(c + d*x))]*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(2*b*d^2) + (B^3*(a + b*x)^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^3)/(2*b) - (3*A*B^2*(b*c - a*d)^2*n^2*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2) - (3*B^3*(b*c - a*d)^2*n^3*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2) - (3*B^3*(b*c - a*d)^2*n^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]*\text{PolyLog}[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*d^2) + (3*B^3*(b*c - a*d)^2*n^3*\text{PolyLog}[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*d^2)$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

#### Rule 2492

$\text{Int}[\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.))*((c_.) + (d_.)*(x_.))^(q_.))]^(r_.)]^(s_.)*((g_.) + (h_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(g + h*x)^(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s]/(h*(m + 1)), x] - \text{Dist}[(p*r*s*($



$b*c - a*d$ ))/((h\*(m + 1)), Int[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0] && NeQ[m, -1]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rule 2514

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s, RFx, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFx, x] && IGtQ[s, 0]

### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 2488

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := -Simp[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d))/h, Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

### Rule 2411

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

### Rule 2343

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/((x\_)\*((d\_) + (e\_.)\*(x\_)^(r\_.))), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*Log[c\*x^n])/(x\*(d + e\*x^(r/n))), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]

### Rule 2333

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)/(x\_))^(q\_.)\*

$x_)^{\wedge}(m\_), x\_Symbol] := \text{Int}[(e + d*x)^{\wedge}q*(a + b*\text{Log}[c*x^{\wedge}n])^{\wedge}p, x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \ \&\& \ \text{EqQ}[m, q] \ \&\& \ \text{IntegerQ}[q]$

### Rule 2315

$\text{Int}[\text{Log}[(c\_)*(x\_)]/((d\_)+(e\_)*(x\_)), x\_Symbol] := -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x] \ \&\& \ \text{EqQ}[e + c*d, 0]$

### Rule 2506

$\text{Int}[\text{Log}[v\_]*\text{Log}[(e\_)*((f\_)*((a\_)+(b\_)*(x\_))^{\wedge}(p\_)*((c\_)+(d\_)*(x\_))^{\wedge}(q\_))^{\wedge}(r\_)]^{\wedge}(s\_)*(u\_), x\_Symbol] := \text{With}\{g = \text{Simplify}[(v - 1)*(c + d*x)/(a + b*x)], h = \text{Simplify}[u*(a + b*x)*(c + d*x)]\}, -\text{Simp}[(h*\text{PolyLog}[2, 1 - v]*\text{Log}[e*(f*(a + b*x)^{\wedge}p*(c + d*x)^{\wedge}q]^{\wedge}r]^{\wedge}s)/(b*c - a*d), x] + \text{Dist}[h*p*r*s, \text{Int}[(\text{PolyLog}[2, 1 - v]*\text{Log}[e*(f*(a + b*x)^{\wedge}p*(c + d*x)^{\wedge}q]^{\wedge}r]^{\wedge}(s - 1))/(a + b*x)*(c + d*x)], x], x] /; \text{FreeQ}\{g, h\}, x] /; \text{FreeQ}\{a, b, c, d, e, f, p, q, r, s\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[s, 0] \ \&\& \ \text{EqQ}[p + q, 0]$

### Rule 6610

$\text{Int}[(u\_)*\text{PolyLog}[n, v\_], x\_Symbol] := \text{With}\{w = \text{DerivativeDivides}[v, u*v, x]\}, \text{Simp}[w*\text{PolyLog}[n + 1, v], x] /; \text{!FalseQ}[w] /; \text{FreeQ}[n, x]$

### Rubi steps

$$\begin{aligned} \int (a + bx)(A + B \log(e(a + bx)^n(c + dx)^{-n}))^3 dx &= \int (A^3(a + bx) + 3A^2B(a + bx) \log(e(a + bx)^n(c + dx)^{-n}) + 3AB^2(a + bx) \log^2(e(a + bx)^n(c + dx)^{-n})) dx \\ &= \frac{A^3(a + bx)^2}{2b} + (3A^2B) \int (a + bx) \log(e(a + bx)^n(c + dx)^{-n}) dx + (3AB^2) \int (a + bx) \log^2(e(a + bx)^n(c + dx)^{-n}) dx \\ &= \frac{A^3(a + bx)^2}{2b} + \frac{3A^2B(a + bx)^2 \log(e(a + bx)^n(c + dx)^{-n})}{2b} + \frac{3AB^2(a + bx)^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{2b} \\ &= \frac{A^3(a + bx)^2}{2b} + \frac{3A^2B(a + bx)^2 \log(e(a + bx)^n(c + dx)^{-n})}{2b} + \frac{3AB^2(a + bx)^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{2b} \\ &= -\frac{3A^2B(bc - ad)nx}{2d} + \frac{A^3(a + bx)^2}{2b} + \frac{3A^2B(bc - ad)^2n \log(c + dx)}{2bd^2} + \frac{3AB^2(bc - ad)^2n \log^2(c + dx)}{2bd^2} \\ &= -\frac{3A^2B(bc - ad)nx}{2d} + \frac{A^3(a + bx)^2}{2b} + \frac{3A^2B(bc - ad)^2n \log(c + dx)}{2bd^2} + \frac{3AB^2(bc - ad)^2n \log^2(c + dx)}{2bd^2} \\ &= -\frac{3A^2B(bc - ad)nx}{2d} + \frac{A^3(a + bx)^2}{2b} + \frac{3A^2B(bc - ad)^2n \log(c + dx)}{2bd^2} + \frac{3AB^2(bc - ad)^2n \log^2(c + dx)}{2bd^2} \\ &= -\frac{3A^2B(bc - ad)nx}{2d} + \frac{A^3(a + bx)^2}{2b} + \frac{3A^2B(bc - ad)^2n \log(c + dx)}{2bd^2} + \frac{3AB^2(bc - ad)^2n \log^2(c + dx)}{2bd^2} \\ &= -\frac{3A^2B(bc - ad)nx}{2d} + \frac{A^3(a + bx)^2}{2b} + \frac{3A^2B(bc - ad)^2n \log(c + dx)}{2bd^2} + \frac{3AB^2(bc - ad)^2n \log^2(c + dx)}{2bd^2} \end{aligned}$$

**Mathematica [B]** time = 2.97117, size = 3813, normalized size = 10.14

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^3,x]

[Out]  $(4a^2B^3d^2n^3\text{Log}[a + bx]^3 - 6a^2B^2d^2n^2\text{Log}[a + bx]^2(2A - Bn + 2Bn\text{Log}[c + dx] + 2B\text{Log}[(e(a + bx)^n)/(c + dx)^n]) + 6a^2Bd^2n\text{Log}[a + bx](2A^2 - 2ABn + B^2n^2 + 2B^2n^2\text{Log}[c + dx]^2 - 2B(-2A + Bn)\text{Log}[(e(a + bx)^n)/(c + dx)^n] + 2B^2\text{Log}[(e(a + bx)^n)/(c + dx)^n]^2 + 2Bn\text{Log}[c + dx](2A - Bn + 2B\text{Log}[(e(a + bx)^n)/(c + dx)^n])) + b(4B^3c(b^2c - 2ad)n^3\text{Log}[c + dx]^3 + 6B^2d^2n^2x(2a + b)x\text{Log}[c + dx]^2(2A - Bn + 2B\text{Log}[(e(a + bx)^n)/(c + dx)^n]) + 6Bd^2n^2x(2a + b)x\text{Log}[c + dx](2A^2 - 2ABn + B^2n^2 - 2B(-2A + Bn)\text{Log}[(e(a + bx)^n)/(c + dx)^n] + 2B^2\text{Log}[(e(a + bx)^n)/(c + dx)^n]^2) + d^2x(2a + b)x(4A^3 - 6A^2Bn + 6AB^2n^2 - 3B^3n^3 + 6B(2A^2 - 2ABn + B^2n^2)\text{Log}[(e(a + bx)^n)/(c + dx)^n] - 6B^2(-2A + Bn)\text{Log}[(e(a + bx)^n)/(c + dx)^n]^2 + 4B^3\text{Log}[(e(a + bx)^n)/(c + dx)^n]^3)))/(8b^2d^2) - (3Bn(-8aAbBc^2dn + 16a^2ABd^2n - 8b^2B^2c^2n^2 + 16aAbB^2c^2dn^2 - 8a^2B^2d^2n^2 + 4A^2b^2c^2dx - 8aA^2b^2d^2x + 4aAbB^2d^2nx - 2aAbB^2d^2n^2x - 2A^2b^2d^2x^2 + 2Ab^2Bd^2nx^2 - b^2B^2d^2n^2x^2 + 8aAbBc^2dn\text{Log}[a + bx] - 12a^2ABd^2n\text{Log}[a + bx] + 8aAbB^2c^2dn^2\text{Log}[a + bx] - 14a^2B^2d^2n^2\text{Log}[a + bx] - 4aAbB^2c^2dn^2\text{Log}[a + bx]^2 + 6a^2B^2d^2n^2\text{Log}[a + bx]^2 - 4A^2b^2c^2\text{Log}[c + dx] + 8aA^2b^2c^2d\text{Log}[c + dx] - 8Ab^2B^2c^2n\text{Log}[c + dx] + 8aAbB^2c^2dn\text{Log}[c + dx] - 8aAbB^2c^2dn^2\text{Log}[c + dx] + 16a^2B^2d^2n^2\text{Log}[c + dx] + 8aA^2b^2d^2x\text{Log}[c + dx] - 8aAbB^2d^2nx\text{Log}[c + dx] + 4aAbB^2d^2n^2x\text{Log}[c + dx] + 4A^2b^2d^2x^2\text{Log}[c + dx] - 4Ab^2B^2d^2nx^2\text{Log}[c + dx] + 2b^2B^2d^2n^2x^2\text{Log}[c + dx] + 8Ab^2B^2c^2n\text{Log}[a + bx]\text{Log}[c + dx] - 16aAbB^2c^2dn\text{Log}[a + bx]\text{Log}[c + dx] + 8a^2ABd^2n\text{Log}[a + bx]\text{Log}[c + dx] - 4b^2B^2c^2n^2\text{Log}[a + bx]\text{Log}[c + dx] + 16aAbB^2c^2dn^2\text{Log}[a + bx]\text{Log}[c + dx] - 16a^2B^2d^2n^2\text{Log}[a + bx]\text{Log}[c + dx] - 4b^2B^2c^2n^2\text{Log}[a + bx]^2\text{Log}[c + dx] + 8aAbB^2c^2dn^2\text{Log}[a + bx]^2\text{Log}[c + dx] - 4a^2B^2d^2n^2\text{Log}[a + bx]^2\text{Log}[c + dx] + 12b^2B^2c^2n^2\text{Log}[(d(a + bx))/(-(b^2c) + ad)]\text{Log}[c + dx] - 24aAbB^2c^2dn^2\text{Log}[(d(a + bx))/(-(b^2c) + ad)]\text{Log}[c + dx] + 12a^2B^2d^2n^2\text{Log}[(d(a + bx))/(-(b^2c) + ad)]\text{Log}[c + dx] - 4Ab^2B^2c^2n\text{Log}[c + dx]^2 + 8aAbB^2c^2dn\text{Log}[c + dx]^2 - 4b^2B^2c^2n^2\text{Log}[c + dx]^2 + 4aAbB^2c^2dn^2\text{Log}[c + dx]^2 + 8aAbB^2c^2dn^2\text{Log}[c + dx]^2 - 4aAbB^2d^2n^2x\text{Log}[c + dx]^2 + 4Ab^2B^2d^2nx\text{Log}[c + dx]^2 - 2b^2B^2d^2n^2x^2\text{Log}[c + dx]^2 + 8b^2B^2c^2n^2\text{Log}[a + bx]\text{Log}[c + dx]^2 - 16aAbB^2c^2dn^2\text{Log}[a + bx]\text{Log}[c + dx]^2 + 8a^2B^2d^2n^2\text{Log}[a + bx]\text{Log}[c + dx]^2 - 4b^2B^2c^2n^2\text{Log}[(d(a + bx))/(-(b^2c) + ad)]\text{Log}[c + dx]^2 + 8aAbB^2c^2dn^2\text{Log}[(d(a + bx))/(-(b^2c) + ad)]\text{Log}[c + dx]^2 - 4a^2B^2d^2n^2\text{Log}[(d(a + bx))/(-(b^2c) + ad)]\text{Log}[c + dx]^2 - 8Ab^2B^2c^2n\text{Log}[a + bx]\text{Log}[(b(c + dx))/(b^2c - ad)] + 16aAbB^2c^2dn\text{Log}[a + bx]\text{Log}[(b(c + dx))/(b^2c - ad)] - 8a^2ABd^2n\text{Log}[a + bx]\text{Log}[(b(c + dx))/(b^2c - ad)] + 4b^2B^2c^2n^2\text{Log}[a + bx]\text{Log}[(b(c + dx))/(b^2c - ad)] - 8aAbB^2c^2dn^2\text{Log}[a + bx]\text{Log}[(b(c + dx))/(b^2c - ad)] + 4a^2B^2d^2n^2\text{Log}[a + bx]\text{Log}[(b(c + dx))/(b^2c - ad)] + 4b^2B^2c^2n^2\text{Log}[a + bx]^2\text{Log}[(b(c + dx))/(b^2c - ad)] - 8aAbB^2c^2dn^2\text{Log}[a + bx]^2\text{Log}[(b(c + dx))/(b^2c - ad)] + 4a^2B^2d^2n^2\text{Log}[a + bx]^2\text{Log}[(b(c + dx))/(b^2c - ad)] - 8b^2B^2c^2n^2\text{Log}[a + bx]\text{Log}[c + dx]\text{Log}[(b(c + dx))/(b^2c - ad)] + 16aAbB^2c^2dn^2\text{Log}[a + bx]\text{Log}[c + dx]\text{Log}[(b(c + dx))/(b^2c - ad)] - 8a^2B^2d^2n^2\text{Log}[a + bx]\text{Log}[c + dx]\text{Log}[(b(c + dx))/(b^2c - ad)]$

$$\begin{aligned}
& + d*x)) / (b*c - a*d)] - 8*a*b*B^2*c*d*n*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 1 \\
& 6*a^2*B^2*d^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*A*b^2*B*c*d*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 16*a*A*b*B*d^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] \\
& + 4*a*b*B^2*d^2*n*x*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 4*A*b^2*B*d^2*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*b^2*B^2*d^2*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] \\
& + 8*a*b*B^2*c*d*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 12*a^2*B^2*d^2*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 8*A*b^2*B*c^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] \\
& + 16*a*A*b*B*c*d*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 8*b^2*B^2*c^2*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*a*b*B^2*c*d*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] \\
& + 16*a*A*b*B*d^2*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 8*a*b*B^2*d^2*n*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*A*b^2*B*d^2*x^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] \\
& - 4*b^2*B^2*d^2*n*x^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*b^2*B^2*c^2*n*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 16*a*b*B^2*c*d*n*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] \\
& + 8*a^2*B^2*d^2*n*Log[a + b*x]*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 4*b^2*B^2*c^2*n*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 8*a*b*B^2*c*d*n*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] \\
& + 8*a*b*B^2*d^2*n*x*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 4*b^2*B^2*d^2*n*x^2*Log[c + d*x]^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 8*b^2*B^2*c^2*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] \\
& + 16*a*b*B^2*c*d*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 8*a^2*B^2*d^2*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 4*b^2*B^2*c*d*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 \\
& - 8*a*b*B^2*d^2*x*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 2*b^2*B^2*d^2*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 4*b^2*B^2*c^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 \\
& + 8*a*b*B^2*c*d*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 8*a*b*B^2*d^2*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 4*b^2*B^2*d^2*x^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 4*B*(b*c - a*d)^2*n*(2*A - B*n + 2*B*n*Log[c + d*x] + 2*B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 4*B^2*(b*c - a*d)^2*n^2*(-3 + 2*Log[c + d*x])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 8*b^2*B^2*c^2*n^2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 16*a*b*B^2*c*d*n^2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] + 8*a^2*B^2*d^2*n^2*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] + 8*b^2*B^2*c^2*n^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] - 16*a*b*B^2*c*d*n^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] + 8*a^2*B^2*d^2*n^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)))/(8*b*d^2)
\end{aligned}$$

---

**Maple [F]** time = 6.112, size = 0, normalized size = 0.

$$\int (bx + a) \left( A + B \ln \left( \frac{e(bx + a)^n}{(dx + c)^n} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)\*(A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3,x)

[Out] int((b\*x+a)\*(A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxima")
```

```
[Out] 3/2*A^2*B*b*x^2*log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^3*b*x^2 + 3*A^2*B*a*x*log((b*x + a)^n*e/(d*x + c)^n) + A^3*a*x + 3*(a*e*n*log(b*x + a)/b - c*e*n*log(d*x + c)/d)*A^2*B*a/e - 3/2*(a^2*e*n*log(b*x + a)/b^2 - c^2*e*n*log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A^2*B*b/e - 1/2*((B^3*b^2*d^2*x^2 + 2*B^3*a*b*d^2*x)*log((d*x + c)^n)^3 - 3*(B^3*a^2*d^2*n*log(b*x + a) + (b^2*c^2*n - 2*a*b*c*d*n)*B^3*log(d*x + c) + (B^3*b^2*d^2*log(e) + A*B^2*b^2*d^2)*x^2 + (2*A*B^2*a*b*d^2 + (a*b*d^2*(n + 2*log(e)) - b^2*c*d*n)*B^3)*x + (B^3*b^2*d^2*x^2 + 2*B^3*a*b*d^2*x)*log((b*x + a)^n))*log((d*x + c)^n)^2)/(b*d^2) - integrate(-(B^3*a*b*c*d*log(e)^3 + 3*A*B^2*a*b*c*d*log(e)^2 + (B^3*b^2*d^2*x^2 + B^3*a*b*c*d + (b^2*c*d + a*b*d^2)*B^3*x)*log((b*x + a)^n)^3 + (B^3*b^2*d^2*log(e)^3 + 3*A*B^2*b^2*d^2*log(e)^2)*x^2 + 3*(B^3*a*b*c*d*log(e) + A*B^2*a*b*c*d + (B^3*b^2*d^2*log(e) + A*B^2*b^2*d^2)*x^2 + ((b^2*c*d + a*b*d^2)*A*B^2 + (b^2*c*d*log(e) + a*b*d^2*log(e))*B^3)*x)*log((b*x + a)^n)^2 + (3*(b^2*c*d*log(e)^2 + a*b*d^2*log(e)^2)*A*B^2 + (b^2*c*d*log(e)^3 + a*b*d^2*log(e)^3)*B^3)*x + 3*(B^3*a*b*c*d*log(e)^2 + 2*A*B^2*a*b*c*d*log(e) + (B^3*b^2*d^2*log(e)^2 + 2*A*B^2*b^2*d^2*log(e))*x^2 + (2*(b^2*c*d*log(e) + a*b*d^2*log(e))*A*B^2 + (b^2*c*d*log(e)^2 + a*b*d^2*log(e)^2)*B^3)*x)*log((b*x + a)^n) - 3*(B^3*a^2*d^2*n^2*log(b*x + a) + B^3*a*b*c*d*log(e)^2 + 2*A*B^2*a*b*c*d*log(e) + (b^2*c^2*n^2 - 2*a*b*c*d*n^2)*B^3*log(d*x + c) + ((n*log(e) + log(e)^2)*B^3*b^2*d^2 + A*B^2*b^2*d^2*(n + 2*log(e))))*x^2 + (B^3*b^2*d^2*x^2 + B^3*a*b*c*d + (b^2*c*d + a*b*d^2)*B^3*x)*log((b*x + a)^n)^2 + (2*(a*b*d^2*(n + log(e)) + b^2*c*d*log(e))*A*B^2 - ((n^2 - log(e)^2)*b^2*c*d - (n^2 + 2*n*log(e) + log(e)^2)*a*b*d^2)*B^3)*x + (2*B^3*a*b*c*d*log(e) + 2*A*B^2*a*b*c*d + (B^3*b^2*d^2*(n + 2*log(e)) + 2*A*B^2*b^2*d^2)*x^2 + 2*((b^2*c*d + a*b*d^2)*A*B^2 + (a*b*d^2*(n + log(e)) + b^2*c*d*log(e))*B^3)*x)*log((b*x + a)^n))*log((d*x + c)^n)/(b*d^2*x + b*c*d), x)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(A^3bx + A^3a + (B^3bx + B^3a)\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^3 + 3(AB^2bx + AB^2a)\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^2 + 3(A^2Bbx + A^2Ba)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas")
```

```
[Out] integral(A^3*b*x + A^3*a + (B^3*b*x + B^3*a)*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*(A*B^2*b*x + A*B^2*a)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*(A^2*B*b*x + A^2*B*a)*log((b*x + a)^n*e/(d*x + c)^n), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))*3,x)
```

```
[Out] Timed out
```

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bx + a) \left( B \log \left( \frac{(bx + a)^n e}{(dx + c)^n} \right) + A \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3,x, algorithm="giac")

[Out] integrate((b\*x + a)\*(B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^3, x)

$$3.167 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^3}{a+bx} dx$$

**Optimal.** Leaf size=186

$$\frac{6B^2n^2 \text{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right) (B \log(e(a+bx)^n(c+dx)^{-n}) + A)}{b} + \frac{3Bn \text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) (B \log(e(a+bx)^n(c+dx)^{-n}))}{b}$$

[Out] -(((A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]^3\*Log[1 - (b\*(c + d\*x))/(d\*(a + b\*x))])/b) + (3\*B\*n\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]^2\*PolyLog[2, (b\*(c + d\*x))/(d\*(a + b\*x))])/b + (6\*B^2\*n^2\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]\*PolyLog[3, (b\*(c + d\*x))/(d\*(a + b\*x))])/b + (6\*B^3\*n^3\*PolyLog[4, (b\*(c + d\*x))/(d\*(a + b\*x))])/b

**Rubi [B]** time = 0.850658, antiderivative size = 424, normalized size of antiderivative = 2.28, number of steps used = 14, number of rules used = 9, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6742, 2488, 2411, 2343, 2333, 2315, 2506, 6610, 2508}

$$\frac{3A^2Bn \text{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right)}{b} + \frac{6AB^2n \text{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right) \log(e(a+bx)^n(c+dx)^{-n})}{b} + \frac{6AB^2n^2 \text{PolyLog}\left(3, \frac{bc-ad}{d(a+bx)} + 1\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]^3)/(a + b\*x), x]

[Out] (A^3\*Log[a + b\*x])/b - (3\*A^2\*B\*Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]/b - (3\*A\*B^2\*Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]^2)/b - (B^3\*Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]^3)/b + (3\*A^2\*B\*n\*PolyLog[2, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/b + (6\*A\*B^2\*n\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]\*PolyLog[2, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/b + (3\*B^3\*n\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]^2\*PolyLog[2, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/b + (6\*A\*B^2\*n^2\*PolyLog[3, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/b + (6\*B^3\*n^2\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]\*PolyLog[3, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/b + (6\*B^3\*n^3\*PolyLog[4, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/b

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2488

Int[Log[(e\_.)\*((f\_.)\*(a\_.) + (b\_.)\*(x\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_.))^(q\_.)]^(r\_.)]^(s\_.)/(g\_.) + (h\_.)\*(x\_.), x\_Symbol] := -Simp[(Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d)/h, Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1))/(a + b\*x)\*(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

#### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_.))^(q\_.)\*((h\_.) + (i\_.)\*(x\_.))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e

\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

#### Rule 2343

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_)])\*(b\_.)/((x\_)\*((d\_) + (e\_.)\*(x\_)^(r\_))), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*Log[c\*x])/(x\*(d + e\*x^(r/n))), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]

#### Rule 2333

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_)])\*(b\_.))^(p\_.)\*((d\_) + (e\_.)/(x\_))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[(e + d\*x)^q\*(a + b\*Log[c\*x^n])^p, x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 2506

Int[Log[v\_]\*Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*(u\_), x\_Symbol] := With[{g = Simplify[((v - 1)\*(c + d\*x))/(a + b\*x)], h = Simplify[u\*(a + b\*x)\*(c + d\*x)]}, -Simp[(h\*PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(b\*c - a\*d), x] + Dist[h\*p\*r\*s, Int[(PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(a + b\*x)\*(c + d\*x), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

#### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]

#### Rule 2508

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{g = Simplify[(v\*(c + d\*x))/(a + b\*x)], h = Simplify[u\*(a + b\*x)\*(c + d\*x)]}, Simp[(h\*PolyLog[n + 1, v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(b\*c - a\*d), x] - Dist[h\*p\*r\*s, Int[(PolyLog[n + 1, v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(a + b\*x)\*(c + d\*x), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, n, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

#### Rubi steps



$$\begin{aligned}
\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^3}{a + bx} dx &= \int \left( \frac{A^3}{a + bx} + \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{a + bx} + \frac{3AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{a + bx} \right) dx \\
&= \frac{A^3 \log(a + bx)}{b} + (3A^2B) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{a + bx} dx + (3AB^2) \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{a + bx} dx \\
&= \frac{A^3 \log(a + bx)}{b} - \frac{3A^2B \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{3AB^2 \log^2\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} \\
&= \frac{A^3 \log(a + bx)}{b} - \frac{3A^2B \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{3AB^2 \log^2\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} \\
&= \frac{A^3 \log(a + bx)}{b} - \frac{3A^2B \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{3AB^2 \log^2\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} \\
&= \frac{A^3 \log(a + bx)}{b} - \frac{3A^2B \log\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{3AB^2 \log^2\left(-\frac{bc-ad}{d(a+bx)}\right) \log(e(a + bx)^n(c + dx)^{-n})}{b}
\end{aligned}$$

**Mathematica [B]** time = 1.02327, size = 2513, normalized size = 13.51

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^3/(a + b\*x), x]

[Out] (4\*A^3\*Log[a + b\*x] - 6\*A^2\*B\*n\*Log[a + b\*x]^2 + 4\*A\*B^2\*n^2\*Log[a + b\*x]^3 - B^3\*n^3\*Log[a + b\*x]^4 + B^3\*n^3\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]^4 - 4\*B^3\*n^3\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]^3\*Log[-((d\*(a + b\*x))/(b\*(c + d\*x)))] + 6\*B^3\*n^3\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]^2\*Log[-((d\*(a + b\*x))/(b\*(c + d\*x)))]^2 - 4\*B^3\*n^3\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]\*Log[-((d\*(a + b\*x))/(b\*(c + d\*x)))]^3 + B^3\*n^3\*Log[-((d\*(a + b\*x))/(b\*(c + d\*x)))]^4 - 12\*A\*B^2\*n^2\*Log[a + b\*x]\*Log[c + d\*x]^2 + 12\*B^3\*n^3\*Log[a + b\*x]^2\*Log[c + d\*x]^2 + 12\*A\*B^2\*n^2\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]\*Log[c + d\*x]^2 - 12\*B^3\*n^3\*Log[a + b\*x]\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]\*Log[c + d\*x]^2 - 8\*B^3\*n^3\*Log[a + b\*x]\*Log[c + d\*x]^3 + 8\*B^3\*n^3\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]\*Log[c + d\*x]^3 + 12\*A^2\*B\*n\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] - 12\*A\*B^2\*n^2\*Log[a + b\*x]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] + 4\*B^3\*n^3\*Log[a + b\*x]^3\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] + 8\*B^3\*n^3\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]^3\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] - 12\*B^3\*n^3\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]^2\*Log[-((d\*(a + b\*x))/(b\*(c + d\*x)))]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] + 24\*A\*B^2\*n^2\*Log[a + b\*x]\*Log[c + d\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] - 24\*B^3\*n^3\*Log[a + b\*x]^2\*Log[c + d\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] + 12\*B^3\*n^3\*Log[a + b\*x]\*Log[c + d\*x]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] + 6\*B^3\*n^3\*Log[a + b\*x]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]^2 + 12\*B^3\*n^3\*Log[a + b\*x]\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]^2 - 18\*B^3\*n^3\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]^2 + 12\*A^2\*B\*Log[a + b\*x]\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n] - 12\*A\*B^2\*n\*Log[a + b\*x]^2\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n] + 4\*B^3\*n^2\*Log[a + b\*x]^3\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n] - 12\*B^3\*n^2\*Log[a + b\*x]\*Log[c + d\*x]^2\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n] + 12\*B^3\*n^2\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)]\*Log[c + d\*x]^2\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n] + 24\*A

```

*B^2*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c +
d*x)^n] - 12*B^3*n^2*Log[a + b*x]^2*Log[(b*(c + d*x))/(b*c - a*d)]*Log[(e*
(a + b*x)^n)/(c + d*x)^n] + 24*B^3*n^2*Log[a + b*x]*Log[c + d*x]*Log[(b*(c
+ d*x))/(b*c - a*d)]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 12*A*B^2*Log[a + b*
x]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 6*B^3*n*Log[a + b*x]^2*Log[(e*(a +
b*x)^n)/(c + d*x)^n]^2 + 12*B^3*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d
)]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 4*B^3*Log[a + b*x]*Log[(e*(a + b*x)
^n)/(c + d*x)^n]^3 - 4*B^3*n^3*Log[-((d*(a + b*x))/(b*(c + d*x)))]^3*Log[(b
*c - a*d)/(b*c + b*d*x)] + 12*B*n*(A^2 + B^2*n^2*Log[(d*(a + b*x))/(-b*c)
+ a*d])^2 + B^2*n^2*Log[c + d*x]^2 + 2*B^2*n^2*Log[a + b*x]*Log[(b*(c + d*x
))/(b*c - a*d)] - 2*B^2*n^2*Log[(d*(a + b*x))/(-b*c) + a*d]*(Log[-((d*(a
+ b*x))/(b*(c + d*x))]) + Log[(b*(c + d*x))/(b*c - a*d)]) + 2*A*B*Log[(e*(a
+ b*x)^n)/(c + d*x)^n] + B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*B*n*Lo
g[c + d*x]*(A - B*n*Log[a + b*x] + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Pol
yLog[2, (d*(a + b*x))/(-b*c) + a*d] - 12*B^3*n^3*Log[-((d*(a + b*x))/(b*(
c + d*x)))]^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] + 12*B^3*n^3*Log[(d*(
a + b*x))/(-b*c) + a*d]^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*
n^3*Log[(d*(a + b*x))/(-b*c) + a*d]*Log[-((d*(a + b*x))/(b*(c + d*x)))]*P
olyLog[2, (b*(c + d*x))/(b*c - a*d)] + 12*B^3*n^3*Log[-((d*(a + b*x))/(b*(c
+ d*x)))]^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 24*A*B^2*n^2*Log[c + d
*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^3*Log[a + b*x]*Log[c +
d*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 12*B^3*n^3*Log[c + d*x]^2*Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)] + 24*B^3*n^3*Log[a + b*x]*Log[(b*(c + d*
x))/(b*c - a*d)]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^3*Log[(d*(
a + b*x))/(-b*c) + a*d]*Log[(b*(c + d*x))/(b*c - a*d)]*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)] + 24*B^3*n^2*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x
)^n]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 24*A*B^2*n^2*PolyLog[3, (d*(a
+ b*x))/(-b*c) + a*d] + 24*B^3*n^3*Log[-((d*(a + b*x))/(b*(c + d*x)))]*Po
lyLog[3, (d*(a + b*x))/(-b*c) + a*d] - 24*B^3*n^2*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]*PolyLog[3, (d*(a + b*x))/(-b*c) + a*d] + 24*B^3*n^3*Log[-((d*(a
+ b*x))/(b*(c + d*x)))]*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] - 24*A*B^2
*n^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] + 24*B^3*n^3*Log[-((d*(a + b*x))
/(b*(c + d*x)))]*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^2*Log[(e*
(a + b*x)^n)/(c + d*x)^n]*PolyLog[3, (b*(c + d*x))/(b*c - a*d)] - 24*B^3*n^
3*PolyLog[4, (d*(a + b*x))/(b*(c + d*x))]/(4*b)

```

**Maple [F]** time = 2.592, size = 0, normalized size = 0.

$$\int \frac{1}{bx+a} \left( A + B \ln \left( \frac{e^{(bx+a)^n}}{(dx+c)^n} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a), x)

[Out] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{B^3 \log(bx+a) \log((dx+c)^n)^3}{b} + \frac{A^3 \log(bx+a)}{b} + \int \frac{B^3 bc \log(e)^3 + 3 AB^2 bc \log(e)^2 + 3 A^2 Bbc \log(e) + (B^3 bdx + B^3 bdx + B^3 bdx)}{b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a),x, algorithm="maxima")

[Out]  $-B^3 \log(bx + a) \log((dx + c)^n)^3/b + A^3 \log(bx + a)/b + \text{integrate}((B^3 b^3 c^3 \log(e)^3 + 3 A^2 B^2 b^2 c^2 \log(e)^2 + 3 A^2 B^2 b^2 c^2 \log(e) + (B^3 b^3 d^3 x + B^3 b^3 c^3) \log((bx + a)^n)^3 + 3 (B^3 b^3 c^3 \log(e) + A B^2 b^2 c^2 + (B^3 b^3 d^3 \log(e) + A B^2 b^2 d^2) x) \log((bx + a)^n)^2 + 3 (B^3 b^3 c^3 \log(e) + A B^2 b^2 c^2 + (B^3 b^3 d^3 \log(e) + A B^2 b^2 d^2) x + (B^3 b^3 d^3 n x + B^3 a^3 d^3 n) \log(bx + a) + (B^3 b^3 d^3 x + B^3 b^3 c^3) \log((bx + a)^n)) \log((dx + c)^n)^2 + (B^3 b^3 d^3 \log(e)^3 + 3 A^2 B^2 b^2 d^2 \log(e)^2 + 3 A^2 B^2 b^2 d^2 \log(e)) x + 3 (B^3 b^3 c^3 \log(e)^2 + 2 A B^2 b^2 c^2 \log(e) + A^2 B^2 b^2 c^2) \log((bx + a)^n) - 3 (B^3 b^3 c^3 \log(e)^2 + 2 A B^2 b^2 c^2 \log(e) + A^2 B^2 b^2 c^2 + (B^3 b^3 d^3 x + B^3 b^3 c^3) \log((bx + a)^n)^2 + (B^3 b^3 d^3 \log(e)^2 + 2 A B^2 b^2 d^2 \log(e) + A^2 B^2 b^2 d^2) x) \log((bx + a)^n) \log((dx + c)^n) / (b^2 d^2 x^2 + a b^2 c + (b^2 c + a b^2 d) x), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^3 \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right)^3 + 3 A B^2 \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right)^2 + 3 A^2 B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A^3}{bx + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a),x, algorithm="fricas")

[Out]  $\text{integral}((B^3 \log((bx + a)^n e / (dx + c)^n)^3 + 3 A^2 B^2 \log((bx + a)^n e / (dx + c)^n)^2 + 3 A^2 B^2 \log((bx + a)^n e / (dx + c)^n) + A^3) / (bx + a), x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*3/(b\*x+a),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)^3}{bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a),x, algorithm="giac")

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*x + a), x)
```

$$3.168 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^2} dx$$

**Optimal.** Leaf size=184

$$\frac{6B^2n^2(c+dx)(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{(a+bx)(bc-ad)} - \frac{3Bn(c+dx)(B \log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)(bc-ad)} - \frac{(c+dx)(B \log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)(bc-ad)}$$

[Out]  $(-6*B^3*n^3*(c+d*x))/((b*c-a*d)*(a+b*x)) - (6*B^2*n^2*(c+d*x)*(A+B*\text{Log}[(e*(a+b*x)^n]/(c+d*x)^n]))/((b*c-a*d)*(a+b*x)) - (3*B*n*(c+d*x)*(A+B*\text{Log}[(e*(a+b*x)^n]/(c+d*x)^n])^2)/((b*c-a*d)*(a+b*x)) - ((c+d*x)*(A+B*\text{Log}[(e*(a+b*x)^n]/(c+d*x)^n])^3)/((b*c-a*d)*(a+b*x))$

**Rubi [A]** time = 0.314674, antiderivative size = 360, normalized size of antiderivative = 1.96, number of steps used = 11, number of rules used = 3, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {6742, 2490, 32}

$$\frac{3A^2B(c+dx) \log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)(bc-ad)} - \frac{3A^2Bn}{b(a+bx)} - \frac{A^3}{b(a+bx)} - \frac{3AB^2(c+dx) \log^2(e(a+bx)^n(c+dx)^{-n})}{(a+bx)(bc-ad)} - \frac{6AB^3 \log(e(a+bx)^n(c+dx)^{-n})}{(a+bx)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n]/(c + d\*x)^n])^3/(a + b\*x)^2, x]

[Out]  $-(A^3/(b*(a+b*x))) - (3*A^2*B*n)/(b*(a+b*x)) - (6*A*B^2*n^2)/(b*(a+b*x)) - (6*B^3*n^3)/(b*(a+b*x)) - (3*A^2*B*(c+d*x)*\text{Log}[(e*(a+b*x)^n]/(c+d*x)^n])/((b*c-a*d)*(a+b*x)) - (6*A*B^2*n*(c+d*x)*\text{Log}[(e*(a+b*x)^n]/(c+d*x)^n])/((b*c-a*d)*(a+b*x)) - (6*B^3*n^2*(c+d*x)*\text{Log}[(e*(a+b*x)^n]/(c+d*x)^n])/((b*c-a*d)*(a+b*x)) - (3*A*B^2*(c+d*x)*\text{Log}[(e*(a+b*x)^n]/(c+d*x)^n]^2)/((b*c-a*d)*(a+b*x)) - (3*B^3*n*(c+d*x)*\text{Log}[(e*(a+b*x)^n]/(c+d*x)^n]^2)/((b*c-a*d)*(a+b*x)) - (B^3*(c+d*x)*\text{Log}[(e*(a+b*x)^n]/(c+d*x)^n]^3)/((b*c-a*d)*(a+b*x))$

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2490

Int[Log[(e\_.)\*((f\_.)\*(a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.)/(g\_.) + (h\_.)\*(x\_)^2, x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/((b\*g - a\*h)\*(g + h\*x)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(b\*g - a\*h), Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x)\*(g + h\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && NeQ[b\*g - a\*h, 0] && IGtQ[s, 0]

#### Rule 32

Int[((a\_.) + (b\_.)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^2} dx &= \int \left( \frac{A^3}{(a + bx)^2} + \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^2} + \frac{3AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^2} \right) dx \\
&= -\frac{A^3}{b(a + bx)} + (3A^2B) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^2} dx + (3AB^2) \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^2} dx \\
&= -\frac{A^3}{b(a + bx)} - \frac{3A^2B(c + dx) \log(e(a + bx)^n(c + dx)^{-n})}{(bc - ad)(a + bx)} - \frac{3AB^2(c + dx) \log^2(e(a + bx)^n(c + dx)^{-n})}{(bc - ad)(a + bx)} \\
&= -\frac{A^3}{b(a + bx)} - \frac{3A^2Bn}{b(a + bx)} - \frac{3A^2B(c + dx) \log(e(a + bx)^n(c + dx)^{-n})}{(bc - ad)(a + bx)} - \frac{6AB^2n \log(e(a + bx)^n(c + dx)^{-n})}{(bc - ad)(a + bx)} \\
&= -\frac{A^3}{b(a + bx)} - \frac{3A^2Bn}{b(a + bx)} - \frac{6AB^2n^2}{b(a + bx)} - \frac{3A^2B(c + dx) \log(e(a + bx)^n(c + dx)^{-n})}{(bc - ad)(a + bx)} \\
&= -\frac{A^3}{b(a + bx)} - \frac{3A^2Bn}{b(a + bx)} - \frac{6AB^2n^2}{b(a + bx)} - \frac{6B^3n^3}{b(a + bx)} - \frac{3A^2B(c + dx) \log(e(a + bx)^n(c + dx)^{-n})}{(bc - ad)(a + bx)}
\end{aligned}$$

**Mathematica [B]** time = 0.756876, size = 524, normalized size = 2.85

---


$$-3Bdn(a + bx) \log(a + bx) \left( 2B(A + Bn) \log(e(a + bx)^n(c + dx)^{-n}) + 2Bn \log(c + dx) (B \log(e(a + bx)^n(c + dx)^{-n}) + A + \dots) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^3/(a + b\*x)^2,x]

[Out] 
$$(-B^3d^n^3(a + b*x)*\text{Log}[a + b*x]^3) + B^3d^n^3(a + b*x)*\text{Log}[c + d*x]^3 + 3*B^2*d^n^2*(a + b*x)*\text{Log}[c + d*x]^2*(A + B*n + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]) + 3*B^2*d^n^2*(a + b*x)*\text{Log}[a + b*x]^2*(A + B*n + B*n*\text{Log}[c + d*x] + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]) + 3*B*d^n*(a + b*x)*\text{Log}[c + d*x]*(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*B*(A + B*n)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + B^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2) - (b*c - a*d)*(A^3 + 3*A^2*B*n + 6*A*B^2*n^2 + 6*B^3*n^3 + 3*B*(A^2 + 2*A*B*n + 2*B^2*n^2)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + 3*B^2*(A + B*n)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2 + B^3*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 3*B*d^n*(a + b*x)*\text{Log}[a + b*x]*(A^2 + 2*A*B*n + 2*B^2*n^2 + B^2*n^2*\text{Log}[c + d*x]^2 + 2*B*(A + B*n)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + B^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 2*B*n*\text{Log}[c + d*x]*(A + B*n + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]))/(b*(b*c - a*d)*(a + b*x))$$

---

**Maple [C]** time = 11.812, size = 69354, normalized size = 376.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^2,x)

[Out] result too large to display

---

**Maxima [B]** time = 1.50655, size = 1524, normalized size = 8.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -B^3*log((b*x + a)^n*e/(d*x + c)^n)^3/(b^2*x + a*b) - (3*(d*e*n*log(b*x + a)
)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b^2*x + a*b))
*log((b*x + a)^n*e/(d*x + c)^n)^2/e + (3*(2*b*c*e^2*n^2 - 2*a*d*e^2*n^2 - (
b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a)^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)
*log(d*x + c)^2 + 2*(b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a) - 2*(b*d*e^2
*n^2*x + a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a))*log(d*x
+ c))*log((b*x + a)^n*e/(d*x + c)^n)/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d
)*x)*e) + (6*b*c*e^3*n^3 - 6*a*d*e^3*n^3 + (b*d*e^3*n^3*x + a*d*e^3*n^3)*lo
g(b*x + a)^3 - (b*d*e^3*n^3*x + a*d*e^3*n^3)*log(d*x + c)^3 - 3*(b*d*e^3*n^
3*x + a*d*e^3*n^3)*log(b*x + a)^2 - 3*(b*d*e^3*n^3*x + a*d*e^3*n^3 - (b*d*e
^3*n^3*x + a*d*e^3*n^3)*log(b*x + a))*log(d*x + c)^2 + 6*(b*d*e^3*n^3*x + a
*d*e^3*n^3)*log(b*x + a) - 3*(2*b*d*e^3*n^3*x + 2*a*d*e^3*n^3 + (b*d*e^3*n^
3*x + a*d*e^3*n^3)*log(b*x + a)^2 - 2*(b*d*e^3*n^3*x + a*d*e^3*n^3)*log(b*x
+ a))*log(d*x + c))/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)*e^2))/e)*B^
3 - 3*A*B^2*(2*(d*e*n*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^
2*c - a*b*d) + e*n/(b^2*x + a*b))*log((b*x + a)^n*e/(d*x + c)^n)/e + (2*b*c
*e^2*n^2 - 2*a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2*n^2)*log(b*x + a)^2 - (
b*d*e^2*n^2*x + a*d*e^2*n^2)*log(d*x + c)^2 + 2*(b*d*e^2*n^2*x + a*d*e^2*n^
2)*log(b*x + a) - 2*(b*d*e^2*n^2*x + a*d*e^2*n^2 - (b*d*e^2*n^2*x + a*d*e^2
*n^2)*log(b*x + a))*log(d*x + c))/((a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x
)*e^2)) - 3*A*B^2*log((b*x + a)^n*e/(d*x + c)^n)^2/(b^2*x + a*b) - 3*(d*e*n
*log(b*x + a)/(b^2*c - a*b*d) - d*e*n*log(d*x + c)/(b^2*c - a*b*d) + e*n/(b
^2*x + a*b))*A^2*B/e - 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n)/(b^2*x + a*b)
- A^3/(b^2*x + a*b)
```

---

**Fricas [B]** time = 1.23035, size = 1808, normalized size = 9.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(A^3*b*c - A^3*a*d + 6*(B^3*b*c - B^3*a*d)*n^3 + (B^3*b*d*n^3*x + B^3*b*c*
n^3)*log(b*x + a)^3 - (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(d*x + c)^3 + (B^3*b
*c - B^3*a*d)*log(e)^3 + 6*(A*B^2*b*c - A*B^2*a*d)*n^2 + 3*(B^3*b*c*n^3 + A
*B^2*b*c*n^2 + (B^3*b*d*n^3 + A*B^2*b*d*n^2)*x + (B^3*b*d*n^2*x + B^3*b*c*n
^2)*log(e))*log(b*x + a)^2 + 3*(B^3*b*c*n^3 + A*B^2*b*c*n^2 + (B^3*b*d*n^3
+ A*B^2*b*d*n^2)*x + (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(b*x + a) + (B^3*b*d*
n^2*x + B^3*b*c*n^2)*log(e))*log(d*x + c)^2 + 3*(A*B^2*b*c - A*B^2*a*d + (B
^3*b*c - B^3*a*d)*n)*log(e)^2 + 3*(A^2*B*b*c - A^2*B*a*d)*n + 3*(2*B^3*b*c*
n^3 + 2*A*B^2*b*c*n^2 + A^2*B*b*c*n + (B^3*b*d*n*x + B^3*b*c*n)*log(e)^2 +
(2*B^3*b*d*n^3 + 2*A*B^2*b*d*n^2 + A^2*B*b*d*n)*x + 2*(B^3*b*c*n^2 + A*B^2*
b*c*n + (B^3*b*d*n^2 + A*B^2*b*d*n)*x)*log(e))*log(b*x + a) - 3*(2*B^3*b*c*
n^3 + 2*A*B^2*b*c*n^2 + A^2*B*b*c*n + (B^3*b*d*n^3*x + B^3*b*c*n^3)*log(b*x
+ a)^2 + (B^3*b*d*n*x + B^3*b*c*n)*log(e)^2 + (2*B^3*b*d*n^3 + 2*A*B^2*b*d
*n^2 + A^2*B*b*d*n)*x + 2*(B^3*b*c*n^3 + A*B^2*b*c*n^2 + (B^3*b*d*n^3 + A*B
^2*b*d*n^2)*x + (B^3*b*d*n^2*x + B^3*b*c*n^2)*log(e))*log(b*x + a) + 2*(B^3
*b*c*n^2 + A*B^2*b*c*n + (B^3*b*d*n^2 + A*B^2*b*d*n)*x)*log(e))*log(d*x + c
) + 3*(A^2*B*b*c - A^2*B*a*d + 2*(B^3*b*c - B^3*a*d)*n^2 + 2*(A*B^2*b*c - A
```

$*B^2*a*d)^n)*\log(e))/(a*b^2*c - a^2*b*d + (b^3*c - a*b^2*d)*x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*3/(b\*x+a)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{(bx+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^2,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^3/(b\*x + a)^2, x)



$$3.169 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^3} dx$$

**Optimal.** Leaf size=390

$$\frac{3bB^2n^2(c+dx)^2(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{4(a+bx)^2(bc-ad)^2} + \frac{6B^2dn^2(c+dx)(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{(a+bx)(bc-ad)^2} - \frac{3bBn(c+dx)}{(a+bx)^2}$$

[Out]  $(6*B^3*d*n^3*(c+d*x))/((b*c-a*d)^2*(a+b*x)) - (3*b*B^3*n^3*(c+d*x)^2)/(8*(b*c-a*d)^2*(a+b*x)^2) + (6*B^2*d*n^2*(c+d*x)*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n]))/((b*c-a*d)^2*(a+b*x)) - (3*b*B^2*n^2*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n]))/(4*(b*c-a*d)^2*(a+b*x)^2) + (3*B*d*n*(c+d*x)*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n])^2)/((b*c-a*d)^2*(a+b*x)) - (3*b*B*n*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n])^2)/(4*(b*c-a*d)^2*(a+b*x)^2) + (d*(c+d*x)*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n])^3)/((b*c-a*d)^2*(a+b*x)) - (b*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n])^3)/(2*(b*c-a*d)^2*(a+b*x)^2)$

**Rubi [B]** time = 0.803893, antiderivative size = 811, normalized size of antiderivative = 2.08, number of steps used = 21, number of rules used = 8, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$ , Rules used = {6742, 2492, 44, 2491, 2490, 32, 2509, 37}

$$-\frac{A^3}{2b(a+bx)^2} + \frac{3Bd^2n \log(a+bx)A^2}{2b(bc-ad)^2} - \frac{3Bd^2n \log(c+dx)A^2}{2b(bc-ad)^2} - \frac{3B \log(e(a+bx)^n(c+dx)^{-n})A^2}{2b(a+bx)^2} + \frac{3BdnA^2}{2b(bc-ad)(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^3/(a + b\*x)^3,x]

[Out]  $-A^3/(2*b*(a+b*x)^2) - (3*A^2*B*n)/(4*b*(a+b*x)^2) + (3*A^2*B*d*n)/(2*b*(b*c-a*d)*(a+b*x)) + (6*A*B^2*d*n^2)/(b*(b*c-a*d)*(a+b*x)) + (6*B^3*d*n^3)/(b*(b*c-a*d)*(a+b*x)) - (3*A*b*B^2*n^2*(c+d*x)^2)/(4*(b*c-a*d)^2*(a+b*x)^2) - (3*b*B^3*n^3*(c+d*x)^2)/(8*(b*c-a*d)^2*(a+b*x)^2) + (3*A^2*B*d^2*n*Log[a+b*x])/(2*b*(b*c-a*d)^2) - (3*A^2*B*d^2*n*Log[c+d*x])/(2*b*(b*c-a*d)^2) - (3*A^2*B*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(2*b*(a+b*x)^2) + (6*A*B^2*d*n*(c+d*x)*Log[(e*(a+b*x)^n)/(c+d*x)^n])/((b*c-a*d)^2*(a+b*x)) + (6*B^3*d*n^2*(c+d*x)*Log[(e*(a+b*x)^n)/(c+d*x)^n])/((b*c-a*d)^2*(a+b*x)) - (3*A*b*B^2*n*(c+d*x)^2*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(2*(b*c-a*d)^2*(a+b*x)^2) - (3*b*B^3*n^2*(c+d*x)^2*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(4*(b*c-a*d)^2*(a+b*x)^2) + (3*A*B^2*d*(c+d*x)*Log[(e*(a+b*x)^n)/(c+d*x)^n]^2)/((b*c-a*d)^2*(a+b*x)) + (3*B^3*d*n*(c+d*x)*Log[(e*(a+b*x)^n)/(c+d*x)^n]^2)/((b*c-a*d)^2*(a+b*x)) - (3*A*b*B^2*(c+d*x)^2*Log[(e*(a+b*x)^n)/(c+d*x)^n]^2)/(2*(b*c-a*d)^2*(a+b*x)^2) - (3*b*B^3*n*(c+d*x)^2*Log[(e*(a+b*x)^n)/(c+d*x)^n]^2)/(4*(b*c-a*d)^2*(a+b*x)^2) + (B^3*d*(c+d*x)*Log[(e*(a+b*x)^n)/(c+d*x)^n]^3)/((b*c-a*d)^2*(a+b*x)) - (b*B^3*(c+d*x)^2*Log[(e*(a+b*x)^n)/(c+d*x)^n]^3)/(2*(b*c-a*d)^2*(a+b*x)^2)$

**Rule 6742**

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

**Rule 2492**

Int[Log[(e\_.)\*((f\_.)\*(a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.)\*((g\_.) + (h\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[(g + h\*x)^(m +

1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s/(h\*(m + 1)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(h\*(m + 1)), Int[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0] && NeQ[m, -1]

#### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 2491

Int[Log[(e\_)\*((f\_)\*((a\_) + (b\_)\*(x\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(q\_))^(r\_)]^(s\_)/((g\_) + (h\_)\*(x\_))^(3), x\_Symbol] := Dist[d/(d\*g - c\*h), Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s/(g + h\*x)^2, x], x] - Dist[h/(d\*g - c\*h), Int[((c + d\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s)/(g + h\*x)^3, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && NeQ[d\*g - c\*h, 0] && IGtQ[s, 0]

#### Rule 2490

Int[Log[(e\_)\*((f\_)\*((a\_) + (b\_)\*(x\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(q\_))^(r\_)]^(s\_)/((g\_) + (h\_)\*(x\_))^(2), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s)/((b\*g - a\*h)\*(g + h\*x)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(b\*g - a\*h), Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^(s - 1)/((c + d\*x)\*(g + h\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && NeQ[b\*g - a\*h, 0] && IGtQ[s, 0]

#### Rule 32

Int[((a\_) + (b\_)\*(x\_))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rule 2509

Int[Log[(e\_)\*((f\_)\*((a\_) + (b\_)\*(x\_))^(p\_)\*((c\_) + (d\_)\*(x\_))^(q\_))^(r\_)]^(s\_)\*((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s)/((m + 1)\*(b\*c - a\*d)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/((m + 1)\*(b\*c - a\*d)), Int[(a + b\*x)^m\*(c + d\*x)^n\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^(s - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1] && IGtQ[s, 0]

#### Rule 37

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b\*c - a\*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^3} dx &= \int \left( \frac{A^3}{(a + bx)^3} + \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^3} + \frac{3AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^3} \right) dx \\
&= -\frac{A^3}{2b(a + bx)^2} + (3A^2B) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^3} dx + (3AB^2) \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^3} dx \\
&= -\frac{A^3}{2b(a + bx)^2} - \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{2b(a + bx)^2} + \frac{(3AB^2) \int \frac{(c+dx) \log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^3} dx}{bc - ad} \\
&= -\frac{A^3}{2b(a + bx)^2} - \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{2b(a + bx)^2} + \frac{3AB^2 d(c + dx) \log^2(e(a + bx)^n(c + dx)^{-n})}{(bc - ad)^2} \\
&= -\frac{A^3}{2b(a + bx)^2} - \frac{3A^2Bn}{4b(a + bx)^2} + \frac{3A^2Bdn}{2b(bc - ad)(a + bx)} + \frac{3A^2Bd^2n \log(a + bx)}{2b(bc - ad)^2} \\
&= -\frac{A^3}{2b(a + bx)^2} - \frac{3A^2Bn}{4b(a + bx)^2} + \frac{3A^2Bdn}{2b(bc - ad)(a + bx)} + \frac{6AB^2dn^2}{b(bc - ad)(a + bx)} \\
&= -\frac{A^3}{2b(a + bx)^2} - \frac{3A^2Bn}{4b(a + bx)^2} + \frac{3A^2Bdn}{2b(bc - ad)(a + bx)} + \frac{6AB^2dn^2}{b(bc - ad)(a + bx)}
\end{aligned}$$

**Mathematica [A]** time = 1.15434, size = 693, normalized size = 1.78

$$\frac{-6Bd^2n(a + bx)^2 \log(a + bx) (2B(2A + 3Bn) \log(e(a + bx)^n(c + dx)^{-n}) + 2Bn \log(c + dx) (2B \log(e(a + bx)^n(c + dx)^{-n})))}{(a + bx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n)]^3/(a + b\*x)^3, x]

[Out] -(-4\*B^3\*d^2\*n^3\*(a + b\*x)^2\*Log[a + b\*x]^3 + 4\*B^3\*d^2\*n^3\*(a + b\*x)^2\*Log[c + d\*x]^3 + 6\*B^2\*d^2\*n^2\*(a + b\*x)^2\*Log[c + d\*x]^2\*(2\*A + 3\*B\*n + 2\*B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]) + 6\*B^2\*d^2\*n^2\*(a + b\*x)^2\*Log[a + b\*x]^2\*(2\*A + 3\*B\*n + 2\*B\*n\*Log[c + d\*x] + 2\*B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]) + 6\*B\*d^2\*n\*(a + b\*x)^2\*Log[c + d\*x]\*(2\*A^2 + 6\*A\*B\*n + 7\*B^2\*n^2 + 2\*B\*(2\*A + 3\*B\*n)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)] + 2\*B^2\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]^2) + (b\*c - a\*d)\*(4\*A^3\*(b\*c - a\*d) + 3\*B^3\*n^3\*(-15\*a\*d + b\*(c - 14\*d\*x)) + 6\*A\*B^2\*n^2\*(-7\*a\*d + b\*(c - 6\*d\*x)) + 6\*A^2\*B\*n\*(-3\*a\*d + b\*(c - 2\*d\*x)) + 6\*B\*(2\*A^2\*(b\*c - a\*d) + B^2\*n^2\*(-7\*a\*d + b\*(c - 6\*d\*x)) + 2\*A\*B\*n\*(-3\*a\*d + b\*(c - 2\*d\*x))) \* Log[(e\*(a + b\*x)^n)/(c + d\*x)] + 6\*B^2\*(2\*A\*(b\*c - a\*d) + B\*n\*(-3\*a\*d + b\*(c - 2\*d\*x))) \* Log[(e\*(a + b\*x)^n)/(c + d\*x)]^2 + 4\*B^3\*(b\*c - a\*d) \* Log[(e\*(a + b\*x)^n)/(c + d\*x)]^3 - 6\*B\*d^2\*n\*(a + b\*x)^2\*Log[a + b\*x]\*(2\*A^2 + 6\*A\*B\*n + 7\*B^2\*n^2 + 2\*B^2\*n^2\*Log[c + d\*x]^2 + 2\*B\*(2\*A + 3\*B\*n)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)] + 2\*B^2\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]^2 + 2\*B\*n\*Log[c + d\*x]\*(2\*A + 3\*B\*n + 2\*B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)])))/(8\*b\*(b\*c - a\*d)^2\*(a + b\*x)^2)

**Maple [C]** time = 18.137, size = 120138, normalized size = 308.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^3, x)

[Out] result too large to display

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**Maxima [B]** time = 1.97299, size = 3032, normalized size = 7.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^3,x, algorithm="maxima")

[Out] 
$$-1/2*B^3*\log((b*x + a)^n*e/(d*x + c)^n)^3/(b^3*x^2 + 2*a*b^2*x + a^2*b) + 1/8*(6*(2*d^2*e*n*\log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*e*n*\log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b*c*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c - a^2*b^2*d)*x))*\log((b*x + a)^n*e/(d*x + c)^n)^2/e - (6*(b^2*c^2*e^2*n^2 - 8*a*b*c*d*e^2*n^2 + 7*a^2*d^2*e^2*n^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*\log(b*x + a)^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*\log(d*x + c)^2 - 6*(b^2*c*d*e^2*n^2 - a*b*d^2*e^2*n^2)*x - 6*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*\log(b*x + a) + 2*(3*b^2*d^2*e^2*n^2*x^2 + 6*a*b*d^2*e^2*n^2*x + 3*a^2*d^2*e^2*n^2 - 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*\log(b*x + a))*\log(d*x + c))*\log((b*x + a)^n*e/(d*x + c)^n)/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)*e) + (3*b^2*c^2*e^3*n^3 - 48*a*b*c*d*e^3*n^3 + 45*a^2*d^2*e^3*n^3 - 4*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*\log(b*x + a)^3 + 4*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*\log(d*x + c)^3 + 18*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*\log(b*x + a)^2 + 6*(3*b^2*d^2*e^3*n^3*x^2 + 6*a*b*d^2*e^3*n^3*x + 3*a^2*d^2*e^3*n^3 - 2*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*\log(b*x + a))*\log(d*x + c)^2 - 42*(b^2*c*d*e^3*n^3 - a*b*d^2*e^3*n^3)*x - 42*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*\log(b*x + a) + 6*(7*b^2*d^2*e^3*n^3*x^2 + 14*a*b*d^2*e^3*n^3*x + 7*a^2*d^2*e^3*n^3 + 2*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*\log(b*x + a)^2 - 6*(b^2*d^2*e^3*n^3*x^2 + 2*a*b*d^2*e^3*n^3*x + a^2*d^2*e^3*n^3)*\log(d*x + c))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)*e^2))/e)*B^3 + 3/4*A*B^2*(2*(2*d^2*e*n*\log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*e*n*\log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b*c*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c - a^2*b^2*d)*x))*\log((b*x + a)^n*e/(d*x + c)^n)/e - (b^2*c^2*e^2*n^2 - 8*a*b*c*d*e^2*n^2 + 7*a^2*d^2*e^2*n^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*\log(b*x + a)^2 + 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*\log(d*x + c)^2 - 6*(b^2*c*d*e^2*n^2 - a*b*d^2*e^2*n^2)*x - 6*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*\log(b*x + a) + 2*(3*b^2*d^2*e^2*n^2*x^2 + 6*a*b*d^2*e^2*n^2*x + 3*a^2*d^2*e^2*n^2 - 2*(b^2*d^2*e^2*n^2*x^2 + 2*a*b*d^2*e^2*n^2*x + a^2*d^2*e^2*n^2)*\log(b*x + a))*\log(d*x + c))/((a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)*e^2)) - 3/2*A*B^2*\log((b*x + a)^n*e/(d*x + c)^n)^2/(b^3*x^2 + 2*a*b^2*x + a^2*b) + 3/4*(2*d^2*e*n*\log(b*x + a)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) - 2*d^2*e*n*\log(d*x + c)/(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2) + (2*b*d*e*n*x - b*c*e*n + 3*a*d*e*n)/(a^2*b^2*c - a^3*b*d + (b^4*c - a*b^3*d)*x^2 + 2*(a*b^3*c - a^2*b^2*d)*x))*A^2*B/e - 3/2*A^2*B*\log((b*x + a)^n*e/(d*x + c)^n)/(b^3*x^2 + 2*a*b^2*x + a^2*b) - 1/2*A^3/(b^3*x^2 + 2*a*b^2*x + a^2*b)$$

**Fricas [B]** time = 1.49427, size = 4618, normalized size = 11.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^3,x, algorithm="fricas")

[Out] 
$$-1/8*(4*A^3*b^2*c^2 - 8*A^3*a*b*c*d + 4*A^3*a^2*d^2 + 3*(B^3*b^2*c^2 - 16*B^3*a*b*c*d + 15*B^3*a^2*d^2)*n^3 - 4*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*\log(b*x + a)^3 + 4*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*\log(d*x + c)^3 + 4*(B^3*b^2*c^2 - 2*B^3*a*b*c*d + B^3*a^2*d^2)*\log(e)^3 + 6*(A*B^2*b^2*c^2 - 8*A*B^2*a*b*c*d + 7*A*B^2*a^2*d^2)*n^2 + 6*((B^3*b^2*c^2 - 4*B^3*a*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n^2 - (3*B^3*b^2*d^2*n^3 + 2*A*B^2*b^2*d^2*n^2)*x^2 - 2*(2*A*B^2*a*b*d^2*n^2 + (B^3*b^2*c*d + 2*B^3*a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^2*x^2 + 2*B^3*a*b*d^2*n^2*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^2)*\log(e))*\log(b*x + a)^2 + 6*((B^3*b^2*c^2 - 4*B^3*a*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n^2 - (3*B^3*b^2*d^2*n^3 + 2*A*B^2*b^2*d^2*n^2)*x^2 - 2*(2*A*B^2*a*b*d^2*n^2 + (B^3*b^2*c*d + 2*B^3*a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*\log(b*x + a) - 2*(B^3*b^2*d^2*n^2*x^2 + 2*B^3*a*b*d^2*n^2*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^2)*\log(e))*\log(d*x + c)^2 + 6*(2*A*B^2*b^2*c^2 - 4*A*B^2*a*b*c*d + 2*A*B^2*a^2*d^2 - 2*(B^3*b^2*c*d - B^3*a*b*d^2)*n*x + (B^3*b^2*c^2 - 4*B^3*a*b*c*d + 3*B^3*a^2*d^2)*n)*\log(e)^2 + 6*(A^2*B*b^2*c^2 - 4*A^2*B*a*b*c*d + 3*A^2*B*a^2*d^2)*n - 6*(7*(B^3*b^2*c*d - B^3*a*b*d^2)*n^3 + 6*(A*B^2*b^2*c*d - A*B^2*a*b*d^2)*n^2 + 2*(A^2*B*b^2*c*d - A^2*B*a*b*d^2)*n)*x + 6*((B^3*b^2*c^2 - 8*B^3*a*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 4*A*B^2*a*b*c*d)*n^2 - (7*B^3*b^2*d^2*n^3 + 6*A*B^2*b^2*d^2*n^2 + 2*A^2*B*b^2*d^2*n)*x^2 - 2*(B^3*b^2*d^2*n*x^2 + 2*B^3*a*b*d^2*n*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n)*\log(e)^2 + 2*(A^2*B*b^2*c^2 - 2*A^2*B*a*b*c*d)*n - 2*(2*A^2*B*a*b*d^2*n + (3*B^3*b^2*c*d + 4*B^3*a*b*d^2)*n^3 + 2*(A*B^2*b^2*c*d + 2*A*B^2*a*b*d^2)*n^2)*x + 2*((B^3*b^2*c^2 - 4*B^3*a*b*c*d)*n^2 - (3*B^3*b^2*d^2*n^2 + 2*A*B^2*b^2*d^2*n)*x^2 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n - 2*(2*A*B^2*a*b*d^2*n + (B^3*b^2*c*d + 2*B^3*a*b*d^2)*n^2)*x)*\log(e))*\log(b*x + a) - 6*((B^3*b^2*c^2 - 8*B^3*a*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 4*A*B^2*a*b*c*d)*n^2 - (7*B^3*b^2*d^2*n^3 + 6*A*B^2*b^2*d^2*n^2 + 2*A^2*B*b^2*d^2*n)*x^2 - 2*(B^3*b^2*d^2*n^3*x^2 + 2*B^3*a*b*d^2*n^3*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^3)*\log(b*x + a)^2 - 2*(B^3*b^2*d^2*n*x^2 + 2*B^3*a*b*d^2*n*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n)*\log(e)^2 + 2*(A^2*B*b^2*c^2 - 2*A^2*B*a*b*c*d)*n - 2*(2*A^2*B*a*b*d^2*n + (3*B^3*b^2*c*d + 4*B^3*a*b*d^2)*n^3 + 2*(A*B^2*b^2*c*d + 2*A*B^2*a*b*d^2)*n^2)*x + 2*((B^3*b^2*c^2 - 4*B^3*a*b*c*d)*n^3 + 2*(A*B^2*b^2*c^2 - 2*A*B^2*a*b*c*d)*n^2 - (3*B^3*b^2*d^2*n^3 + 2*A*B^2*b^2*d^2*n^2)*x^2 - 2*(2*A*B^2*a*b*d^2*n^2 + (B^3*b^2*c*d + 2*B^3*a*b*d^2)*n^3)*x - 2*(B^3*b^2*d^2*n^2*x^2 + 2*B^3*a*b*d^2*n^2*x - (B^3*b^2*c^2 - 2*B^3*a*b*c*d)*n^2)*\log(e))*\log(d*x + c) + 6*(2*A^2*B*b^2*c^2 - 4*A^2*B*a*b*c*d + 2*A^2*B*a^2*d^2 + (B^3*b^2*c^2 - 8*B^3*a*b*c*d + 7*B^3*a^2*d^2)*n^2 + 2*(A*B^2*b^2*c^2 - 4*A*B^2*a*b*c*d + 3*A*B^2*a^2*d^2)*n - 2*(3*(B^3*b^2*c*d - B^3*a*b*d^2)*n^2 + 2*(A*B^2*b^2*c*d - A*B^2*a*b*d^2)*n)*x)*\log(e))/(a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2 + (b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*x)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*3/(b\*x+a)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)^3}{(bx+a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^3,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^3/(b\*x + a)^3, x)

$$3.170 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^4} dx$$

**Optimal.** Leaf size=611

$$\frac{2b^2B^2n^2(c+dx)^3(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{9(a+bx)^3(bc-ad)^3} - \frac{b^2Bn(c+dx)^3(B \log(e(a+bx)^n(c+dx)^{-n})+A)^2}{3(a+bx)^3(bc-ad)^3} - \frac{b^2(c+dx)^3}{9(a+bx)^3(bc-ad)^3}$$

```
[Out] (-6*B^3*d^2*n^3*(c+d*x))/((b*c-a*d)^3*(a+b*x)) + (3*b*B^3*d*n^3*(c+d*x)^2)/(4*(b*c-a*d)^3*(a+b*x)^2) - (2*b^2*B^3*n^3*(c+d*x)^3)/(27*(b*c-a*d)^3*(a+b*x)^3) - (6*B^2*d^2*n^2*(c+d*x)*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n]))/((b*c-a*d)^3*(a+b*x)) + (3*b*B^2*d*n^2*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n]))/(2*(b*c-a*d)^3*(a+b*x)^2) - (2*b^2*B^2*n^2*(c+d*x)^3*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n]))/(9*(b*c-a*d)^3*(a+b*x)^3) - (3*B*d^2*n*(c+d*x)*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n]))^2/((b*c-a*d)^3*(a+b*x)) + (3*b*B*d*n*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n]))^2/(2*(b*c-a*d)^3*(a+b*x)^2) - (b^2*B*n*(c+d*x)^3*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n]))^2/(3*(b*c-a*d)^3*(a+b*x)^3) - (d^2*(c+d*x)*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n]))^3/((b*c-a*d)^3*(a+b*x)) + (b*d*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n]))^3/((b*c-a*d)^3*(a+b*x)^2) - (b^2*(c+d*x)^3*(A+B*Log[(e*(a+b*x)^n)/(c+d*x)^n]))^3/(3*(b*c-a*d)^3*(a+b*x)^3)
```

**Rubi [C]** time = 3.43242, antiderivative size = 1876, normalized size of antiderivative = 3.07, number of steps used = 66, number of rules used = 16, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$ , Rules used = {6742, 2492, 44, 2514, 2490, 32, 2488, 2411, 2343, 2333, 2315, 2491, 2509, 37, 2506, 6610}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a + b*x)^4, x]
```

```
[Out] -A^3/(3*b*(a+b*x)^3) - (A^2*B*n)/(3*b*(a+b*x)^3) - (2*A*B^2*n^2)/(9*b*(a+b*x)^3) - (2*B^3*n^3)/(27*b*(a+b*x)^3) + (A^2*B*d*n)/(2*b*(b*c-a*d)*(a+b*x)^2) + (5*A*B^2*d*n^2)/(6*b*(b*c-a*d)*(a+b*x)^2) + (5*B^3*d*n^3)/(18*b*(b*c-a*d)*(a+b*x)^2) - (A^2*B*d^2*n)/(b*(b*c-a*d)^2*(a+b*x)) - (11*A*B^2*d^2*n^2)/(3*b*(b*c-a*d)^2*(a+b*x)) - (47*B^3*d^2*n^3)/(9*b*(b*c-a*d)^2*(a+b*x)) + (b*B^3*d*n^3*(c+d*x)^2)/(4*(b*c-a*d)^3*(a+b*x)^2) - (A^2*B*d^3*n*Log[a+b*x])/(b*(b*c-a*d)^3) - (5*A*B^2*d^3*n^2*Log[a+b*x])/(3*b*(b*c-a*d)^3) - (5*B^3*d^3*n^3*Log[a+b*x])/(9*b*(b*c-a*d)^3) + (A^2*B*d^3*n*Log[c+d*x])/(b*(b*c-a*d)^3) + (5*A*B^2*d^3*n^2*Log[c+d*x])/(3*b*(b*c-a*d)^3) + (5*B^3*d^3*n^3*Log[c+d*x])/(9*b*(b*c-a*d)^3) - (A^2*B*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(b*(a+b*x)^3) - (2*A*B^2*n*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(3*b*(a+b*x)^3) - (2*B^3*n^2*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(9*b*(a+b*x)^3) + (A*B^2*d*n*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(b*(b*c-a*d)*(a+b*x)^2) + (B^3*d*n^2*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(3*b*(b*c-a*d)*(a+b*x)^2) - (2*A*B^2*d^2*n*(c+d*x)*Log[(e*(a+b*x)^n)/(c+d*x)^n])/((b*c-a*d)^3*(a+b*x)) - (14*B^3*d^2*n^2*(c+d*x)*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(3*(b*c-a*d)^3*(a+b*x)) + (b*B^3*d*n^2*(c+d*x)^2*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(2*(b*c-a*d)^3*(a+b*x)^2) + (2*A*B^2*d^3*n*Log[-((b*c-a*d)/(d*(a+b*x)))]*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(b*(b*c-a*d)^3) + (2*B^3*d^3*n^2*Log[-((b*c-a*d)/(d*(a+b*x)))]*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(3*b*(b*c-a*d)^3) - (2*A*B^2*d^3*n*Log[(b*c-a*d)/(b*(c+d*x))]*Log[(e*(a+b*x)^n)/(c+d*x)^n])/(b*(b*c-a*d)^3) - (2*B^3*d^3*n^2*Log[(b*c-a*d)/(b*(c+d*x))])
```

```

)]*Log[(e*(a + b*x)^n)/(c + d*x)^n]/(3*b*(b*c - a*d)^3) - (A*B^2*Log[(e*(a
+ b*x)^n)/(c + d*x)^n]^2)/(b*(a + b*x)^3) - (B^3*n*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]^2)/(3*b*(a + b*x)^3) - (2*B^3*d^2*n*(c + d*x)*Log[(e*(a + b*x)^n)
/(c + d*x)^n]^2)/((b*c - a*d)^3*(a + b*x)) + (b*B^3*d*n*(c + d*x)^2*Log[(e*
(a + b*x)^n)/(c + d*x)^n]^2)/(2*(b*c - a*d)^3*(a + b*x)^2) + (B^3*d^3*n*Log
[-((b*c - a*d)/(d*(a + b*x))])*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(b*(b*c
- a*d)^3) - (B^3*d^3*n*Log[(b*c - a*d)/(b*(c + d*x))])*Log[(e*(a + b*x)^n)/(
c + d*x)^n]^2)/(b*(b*c - a*d)^3) - (B^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3)
/(3*b*(a + b*x)^3) - (2*A*B^2*d^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x)
)])/ (b*(b*c - a*d)^3) - (2*B^3*d^3*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x)
)])/ (3*b*(b*c - a*d)^3) - (2*A*B^2*d^3*n^2*PolyLog[2, 1 + (b*c - a*d)/(d*(
a + b*x)])/ (b*(b*c - a*d)^3) - (2*B^3*d^3*n^3*PolyLog[2, 1 + (b*c - a*d)/(
d*(a + b*x)])/ (3*b*(b*c - a*d)^3) - (2*B^3*d^3*n^2*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x)])/ (b*(b*c - a*d)^3) - (2
*B^3*d^3*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 - (b*c - a*d)/(b
*(c + d*x)])/ (b*(b*c - a*d)^3) - (2*B^3*d^3*n^3*PolyLog[3, 1 + (b*c - a*d)
/(d*(a + b*x)])/ (b*(b*c - a*d)^3) + (2*B^3*d^3*n^3*PolyLog[3, 1 - (b*c - a
*d)/(b*(c + d*x)])/ (b*(b*c - a*d)^3)

```

#### Rule 6742

```

Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

```

#### Rule 2492

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :=> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
0] && NeQ[m, -1]

```

#### Rule 44

```

Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :=> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])

```

#### Rule 2514

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFX_), x_Symbol] :=> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]

```

#### Rule 2490

```

Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] :=> Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q]^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0]
]

```

#### Rule 32



Int[((a\_.) + (b\_.)\*(x\_.))^(m\_), x\_Symbol] := Simp[(a + b\*x)^(m + 1)/(b\*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

#### Rule 2488

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.))\*((c\_.) + (d\_.)\*(x\_.))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_.)), x\_Symbol] := -Simp[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d))/h, Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

#### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_.))^(q\_.)\*((h\_.) + (i\_.)\*(x\_.))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

#### Rule 2343

Int[((a\_.) + Log[(c\_.)\*(x\_.)^(n\_.)]\*(b\_.))/((x\_.)\*((d\_.) + (e\_.)\*(x\_.)^(r\_.))), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*Log[c\*x^n])/(x\*(d + e\*x^(r/n))), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]

#### Rule 2333

Int[((a\_.) + Log[(c\_.)\*(x\_.)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_.) + (e\_.)/(x\_.))^(q\_.)\*(x\_.)^(m\_.), x\_Symbol] := Int[(e + d\*x)^q\*(a + b\*Log[c\*x^n])^p, x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]

#### Rule 2315

Int[Log[(c\_.)\*(x\_.)]/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rule 2491

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.))\*((c\_.) + (d\_.)\*(x\_.))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_.))^3, x\_Symbol] := Dist[d/(d\*g - c\*h), Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s/(g + h\*x)^2, x], x] - Dist[h/(d\*g - c\*h), Int[((c + d\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(g + h\*x)^3, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && NeQ[d\*g - c\*h, 0] && IGtQ[s, 0]

#### Rule 2509

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.))\*((c\_.) + (d\_.)\*(x\_.))^(q\_.))^(r\_.)]^(s\_.)\*((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] := Simp[(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s/((m + 1)\*(b\*c - a\*d)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/((m + 1)\*(b\*c - a\*d)), Int[(a + b\*x)^m\*(c + d\*x)^n\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1] && IGtQ[s, 0]

#### Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

### Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_
)^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] := With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^4} dx &= \int \left( \frac{A^3}{(a + bx)^4} + \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^4} + \frac{3AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^4} \right) dx \\
&= -\frac{A^3}{3b(a + bx)^3} + (3A^2B) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^4} dx + (3AB^2) \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^4} dx \\
&= -\frac{A^3}{3b(a + bx)^3} - \frac{A^2B \log(e(a + bx)^n(c + dx)^{-n})}{b(a + bx)^3} - \frac{AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{b(a + bx)^3} \\
&= -\frac{A^3}{3b(a + bx)^3} - \frac{A^2B \log(e(a + bx)^n(c + dx)^{-n})}{b(a + bx)^3} - \frac{AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{b(a + bx)^3} \\
&= -\frac{A^3}{3b(a + bx)^3} - \frac{A^2Bn}{3b(a + bx)^3} + \frac{A^2Bdn}{2b(bc - ad)(a + bx)^2} - \frac{A^2Bd^2n}{b(bc - ad)^2(a + bx)} \\
&= -\frac{A^3}{3b(a + bx)^3} - \frac{A^2Bn}{3b(a + bx)^3} + \frac{A^2Bdn}{2b(bc - ad)(a + bx)^2} - \frac{A^2Bd^2n}{b(bc - ad)^2(a + bx)} \\
&= -\frac{A^3}{3b(a + bx)^3} - \frac{A^2Bn}{3b(a + bx)^3} + \frac{A^2Bdn}{2b(bc - ad)(a + bx)^2} - \frac{A^2Bd^2n}{b(bc - ad)^2(a + bx)} \\
&= -\frac{A^3}{3b(a + bx)^3} - \frac{A^2Bn}{3b(a + bx)^3} + \frac{A^2Bdn}{2b(bc - ad)(a + bx)^2} - \frac{A^2Bd^2n}{b(bc - ad)^2(a + bx)} \\
&= -\frac{A^3}{3b(a + bx)^3} - \frac{A^2Bn}{3b(a + bx)^3} - \frac{2AB^2n^2}{9b(a + bx)^3} + \frac{A^2Bdn}{2b(bc - ad)(a + bx)^2} + \frac{A^2Bd^2n}{6b(bc - ad)^2(a + bx)} \\
&= -\frac{A^3}{3b(a + bx)^3} - \frac{A^2Bn}{3b(a + bx)^3} - \frac{2AB^2n^2}{9b(a + bx)^3} + \frac{A^2Bdn}{2b(bc - ad)(a + bx)^2} + \frac{A^2Bd^2n}{6b(bc - ad)^2(a + bx)} \\
&= -\frac{A^3}{3b(a + bx)^3} - \frac{A^2Bn}{3b(a + bx)^3} - \frac{2AB^2n^2}{9b(a + bx)^3} + \frac{A^2Bdn}{2b(bc - ad)(a + bx)^2} + \frac{A^2Bd^2n}{6b(bc - ad)^2(a + bx)} \\
&= -\frac{A^3}{3b(a + bx)^3} - \frac{A^2Bn}{3b(a + bx)^3} - \frac{2AB^2n^2}{9b(a + bx)^3} - \frac{2B^3n^3}{27b(a + bx)^3} + \frac{A^2Bdn}{2b(bc - ad)(a + bx)^2} + \frac{A^2Bd^2n}{6b(bc - ad)^2(a + bx)} \\
&= -\frac{A^3}{3b(a + bx)^3} - \frac{A^2Bn}{3b(a + bx)^3} - \frac{2AB^2n^2}{9b(a + bx)^3} - \frac{2B^3n^3}{27b(a + bx)^3} + \frac{A^2Bdn}{2b(bc - ad)(a + bx)^2} + \frac{A^2Bd^2n}{6b(bc - ad)^2(a + bx)} \\
&= -\frac{A^3}{3b(a + bx)^3} - \frac{A^2Bn}{3b(a + bx)^3} - \frac{2AB^2n^2}{9b(a + bx)^3} - \frac{2B^3n^3}{27b(a + bx)^3} + \frac{A^2Bdn}{2b(bc - ad)(a + bx)^2} + \frac{A^2Bd^2n}{6b(bc - ad)^2(a + bx)}
\end{aligned}$$

**Mathematica [A]** time = 1.47614, size = 1003, normalized size = 1.64

$$\frac{-36B^3d^3n^3 \log^3(a + bx)(a + bx)^3 + 36B^3d^3n^3 \log^3(c + dx)(a + bx)^3 + 18B^2d^3n^2 \log^2(c + dx)(6A + 11Bn + 6B \log(e(a + bx)^n(c + dx)^{-n}))}{(a + bx)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]^3)/(a + b\*x)^4,x]

[Out] (-36\*B^3\*d^3\*n^3\*(a + b\*x)^3\*Log[a + b\*x]^3 + 36\*B^3\*d^3\*n^3\*(a + b\*x)^3\*Log[c + d\*x]^3 + 18\*B^2\*d^3\*n^2\*(a + b\*x)^3\*Log[c + d\*x]^2\*(6\*A + 11\*B\*n + 6\*B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]) + 18\*B^2\*d^3\*n^2\*(a + b\*x)^3\*Log[a + b\*x]^2\*(6\*A + 11\*B\*n + 6\*B\*n\*Log[c + d\*x] + 6\*B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)])

$$\begin{aligned} & \text{^n])} + 6*B*d^3*n*(a + b*x)^3*\text{Log}[c + d*x]*(18*A^2 + 66*A*B*n + 85*B^2*n^2 + \\ & 6*B*(6*A + 11*B*n)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*\text{Log}[(e*(a + b \\ & *x)^n)/(c + d*x)^n]^2) - (b*c - a*d)*(36*A^3*b^2*c^2 - 72*a*A^3*b*c*d + 36* \\ & a^2*A^3*d^2 + 36*A^2*b^2*B*c^2*n - 126*a*A^2*b*B*c*d*n + 198*a^2*A^2*B*d^2* \\ & n + 24*A*b^2*B^2*c^2*n^2 - 138*a*A*b*B^2*c*d*n^2 + 510*a^2*A*B^2*d^2*n^2 + \\ & 8*b^2*B^3*c^2*n^3 - 73*a*b*B^3*c*d*n^3 + 575*a^2*B^3*d^2*n^3 - 54*A^2*b^2*B \\ & *c*d*n*x + 270*a*A^2*b*B*d^2*n*x - 90*A*b^2*B^2*c*d*n^2*x + 882*a*A*b*B^2*d \\ & ^2*n^2*x - 57*b^2*B^3*c*d*n^3*x + 1077*a*b*B^3*d^2*n^3*x + 108*A^2*b^2*B*d^ \\ & 2*n*x^2 + 396*A*b^2*B^2*d^2*n^2*x^2 + 510*b^2*B^3*d^2*n^3*x^2 + 6*B*(18*A^2 \\ & *(b*c - a*d)^2 + 6*A*B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^2 - \\ & 3*c*d*x + 6*d^2*x^2)) + B^2*n^2*(85*a^2*d^2 + a*b*d*(-23*c + 147*d*x) + b^ \\ & 2*(4*c^2 - 15*c*d*x + 66*d^2*x^2))) * \text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B \\ & ^2*(6*A*(b*c - a*d)^2 + B*n*(11*a^2*d^2 + a*b*d*(-7*c + 15*d*x) + b^2*(2*c^ \\ & 2 - 3*c*d*x + 6*d^2*x^2))) * \text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 36*B^3*(b*c \\ & - a*d)^2 * \text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^3 - 6*B*d^3*n*(a + b*x)^3 * \text{Log}[a \\ & + b*x]*(18*A^2 + 66*A*B*n + 85*B^2*n^2 + 18*B^2*n^2 * \text{Log}[c + d*x]^2 + 6*B*( \\ & 6*A + 11*B*n)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + 18*B^2*\text{Log}[(e*(a + b*x)^n \\ & / (c + d*x)^n]^2 + 6*B*n*\text{Log}[c + d*x]*(6*A + 11*B*n + 6*B*\text{Log}[(e*(a + b*x)^n \\ & )/(c + d*x)^n])))/(108*b*(b*c - a*d)^3*(a + b*x)^3) \end{aligned}$$

**Maple [C]** time = 27.216, size = 175812, normalized size = 287.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^4,x)

[Out] result too large to display

**Maxima [B]** time = 2.60012, size = 4901, normalized size = 8.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^4,x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/3*B^3*\log((b*x + a)^n*e/(d*x + c)^n)^3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^ \\ & 2*x + a^3*b) - 1/108*(18*(6*d^3*e*n*\log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + \\ & 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3*e*n*\log(d*x + c)/(b^4*c^3 - 3*a*b^3*c \\ & ^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - \\ & 7*a*b*c*d*e*n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x)/(a^3*b^ \\ & 3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x \\ & ^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a \\ & ^3*b^3*c*d + a^4*b^2*d^2)*x)) * \log((b*x + a)^n*e/(d*x + c)^n)^2/e + (6*(4*b^ \\ & 3*c^3*e^2*n^2 - 27*a*b^2*c^2*d*e^2*n^2 + 108*a^2*b*c*d^2*e^2*n^2 - 85*a^3*d \\ & ^3*e^2*n^2 + 66*(b^3*c*d^2*e^2*n^2 - a*b^2*d^3*e^2*n^2)*x^2 - 18*(b^3*d^3*e \\ & ^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2* \\ & n^2)*\log(b*x + a)^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3 \\ & *a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)*\log(d*x + c)^2 - 3*(5*b^3*c^2*d*e^2 \\ & *n^2 - 54*a*b^2*c*d^2*e^2*n^2 + 49*a^2*b*d^3*e^2*n^2)*x + 66*(b^3*d^3*e^2*n \\ & ^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2) \end{aligned}$$

$$\begin{aligned}
& * \log(b*x + a) - 6*(11*b^3*d^3*e^2*n^2*x^3 + 33*a*b^2*d^3*e^2*n^2*x^2 + 33*a^2*b*d^3*e^2*n^2*x + 11*a^3*d^3*e^2*n^2 - 6*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2)) * \log(b*x + a) * \log(d*x + c) * \log((b*x + a)^n * e / (d*x + c)^n) / ((a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x) * e) + (8*b^3*c^3*e^3*n^3 - 81*a*b^2*c^2*d*e^3*n^3 + 648*a^2*b*c*d^2*e^3*n^3 - 575*a^3*d^3*e^3*n^3 + 36*(b^3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*e^3*n^3)) * \log(b*x + a)^3 - 36*(b^3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*e^3*n^3) * \log(d*x + c)^3 + 510*(b^3*c*d^2*e^3*n^3 - a*b^2*d^3*e^3*n^3)*x^2 - 198*(b^3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*e^3*n^3) * \log(b*x + a)^2 - 18*(11*b^3*d^3*e^3*n^3*x^3 + 33*a*b^2*d^3*e^3*n^3*x^2 + 33*a^2*b*d^3*e^3*n^3*x + 11*a^3*d^3*e^3*n^3 - 6*(b^3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*e^3*n^3) * \log(b*x + a) * \log(d*x + c)^2 - 3*(19*b^3*c^2*d*e^3*n^3 - 378*a*b^2*c*d^2*e^3*n^3 + 359*a^2*b*d^3*e^3*n^3)*x + 510*(b^3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*e^3*n^3) * \log(b*x + a) - 6*(85*b^3*d^3*e^3*n^3*x^3 + 255*a*b^2*d^3*e^3*n^3*x^2 + 255*a^2*b*d^3*e^3*n^3*x + 85*a^3*d^3*e^3*n^3 + 18*(b^3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*e^3*n^3) * \log(b*x + a)^2 - 66*(b^3*d^3*e^3*n^3*x^3 + 3*a*b^2*d^3*e^3*n^3*x^2 + 3*a^2*b*d^3*e^3*n^3*x + a^3*d^3*e^3*n^3) * \log(b*x + a) * \log(d*x + c)) / ((a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x) * e^2) / e) * B^3 - 1/18*A*B^2*(6*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x) / (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*x)) * \log((b*x + a)^n * e / (d*x + c)^n) / e + (4*b^3*c^3*e^2*n^2 - 27*a*b^2*c^2*d*e^2*n^2 + 108*a^2*b*c*d^2*e^2*n^2 - 85*a^3*d^3*e^2*n^2 + 66*(b^3*c*d^2*e^2*n^2 - a*b^2*d^3*e^2*n^2)*x^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2) * \log(b*x + a)^2 - 18*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2) * \log(d*x + c)^2 - 3*(5*b^3*c^2*d*e^2*n^2 - 54*a*b^2*c*d^2*e^2*n^2 + 49*a^2*b*d^3*e^2*n^2)*x + 66*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2) * \log(b*x + a) - 6*(11*b^3*d^3*e^2*n^2*x^3 + 33*a*b^2*d^3*e^2*n^2*x^2 + 33*a^2*b*d^3*e^2*n^2*x + 11*a^3*d^3*e^2*n^2 - 6*(b^3*d^3*e^2*n^2*x^3 + 3*a*b^2*d^3*e^2*n^2*x^2 + 3*a^2*b*d^3*e^2*n^2*x + a^3*d^3*e^2*n^2) * \log(b*x + a) * \log(d*x + c)) / ((a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3 + (b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*x) * e^2) - A*B^2*log((b*x + a)^n * e / (d*x + c)^n)^2 / (b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b) - 1/6*(6*d^3*e*n*log(b*x + a)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) - 6*d^3*e*n*log(d*x + c)/(b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3) + (6*b^2*d^2*e*n*x^2 + 2*b^2*c^2*e*n - 7*a*b*c*d*e*n + 11*a^2*d^2*e*n - 3*(b^2*c*d*e*n - 5*a*b*d^2*e*n)*x) / (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2 + (b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*x)) * A^2*B/e - A^2*B*log((b*x + a)^n * e / (d*x + c)^n) / (b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b) - 1/3*A^3/(b^4*x^3 + 3*a*b^3*x^2 + 3*a^2*b^2*x + a^3*b)
\end{aligned}$$

**Fricas [B]** time = 1.86161, size = 8181, normalized size = 13.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^4,x, algorithm="fricas")

[Out] 
$$-1/108*(36*A^3*b^3*c^3 - 108*A^3*a*b^2*c^2*d + 108*A^3*a^2*b*c*d^2 - 36*A^3*a^3*d^3 + (8*B^3*b^3*c^3 - 81*B^3*a*b^2*c^2*d + 648*B^3*a^2*b*c*d^2 - 575*B^3*a^3*d^3)*n^3 + 36*(B^3*b^3*d^3*n^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*B^3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^3)*\log(b*x + a)^3 - 36*(B^3*b^3*d^3*n^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*B^3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^3)*\log(d*x + c)^3 + 36*(B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2 - B^3*a^3*d^3)*\log(e)^3 + 6*(4*A*B^2*b^3*c^3 - 27*A*B^2*a*b^2*c^2*d + 108*A*B^2*a^2*b*c*d^2 - 85*A*B^2*a^3*d^3)*n^2 + 6*(85*(B^3*b^3*c*d^2 - B^3*a*b^2*d^3)*n^3 + 66*(A*B^2*b^3*c*d^2 - A*B^2*a*b^2*d^3)*n^2 + 18*(A^2*B*b^3*c*d^2 - A^2*B*a*b^2*d^3)*n)*x^2 + 18*((2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d + 18*B^3*a^2*b*c*d^2)*n^3 + (11*B^3*b^3*d^3*n^3 + 6*A*B^2*b^3*d^3*n^2)*x^3 + 6*(A*B^2*b^3*c^3 - 3*A*B^2*a*b^2*c^2*d + 3*A*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B^2*a*b^2*d^3*n^2 + (2*B^3*b^3*c*d^2 + 9*B^3*a*b^2*d^3)*n^3)*x^2 + 3*(6*A*B^2*a^2*b*d^3*n^2 - (B^3*b^3*c^2*d - 6*B^3*a*b^2*c*d^2 - 6*B^3*a^2*b*d^3)*n^3)*x + 6*(B^3*b^3*d^3*n^2*x^3 + 3*B^3*a*b^2*d^3*n^2*x^2 + 3*B^3*a^2*b*d^3*n^2*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^2)*\log(e))*\log(b*x + a)^2 + 18*((2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d + 18*B^3*a^2*b*c*d^2)*n^3 + (11*B^3*b^3*d^3*n^3 + 6*A*B^2*b^3*d^3*n^2)*x^3 + 6*(A*B^2*b^3*c^3 - 3*A*B^2*a*b^2*c^2*d + 3*A*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B^2*a*b^2*d^3*n^2 + (2*B^3*b^3*c*d^2 + 9*B^3*a*b^2*d^3)*n^3)*x^2 + 3*(6*A*B^2*a^2*b*d^3*n^2 - (B^3*b^3*c^2*d - 6*B^3*a*b^2*c*d^2 - 6*B^3*a^2*b*d^3)*n^3)*x + 6*(B^3*b^3*d^3*n^3*x^3 + 3*B^3*a*b^2*d^3*n^3*x^2 + 3*B^3*a^2*b*d^3*n^3*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^3)*\log(b*x + a) + 6*(B^3*b^3*d^3*n^2*x^3 + 3*B^3*a*b^2*d^3*n^2*x^2 + 3*B^3*a^2*b*d^3*n^2*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n^2)*\log(e))*\log(d*x + c)^2 + 18*(6*A*B^2*b^3*c^3 - 18*A*B^2*a*b^2*c^2*d + 18*A*B^2*a^2*b*c*d^2 - 6*A*B^2*a^3*d^3 + 6*(B^3*b^3*c*d^2 - B^3*a*b^2*d^3)*n)*x^2 - 3*(B^3*b^3*c^2*d - 6*B^3*a*b^2*c*d^2 + 5*B^3*a^2*b*d^3)*n*x + (2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d + 18*B^3*a^2*b*c*d^2 - 11*B^3*a^3*d^3)*n)*\log(e)^2 + 18*(2*A^2*B*b^3*c^3 - 9*A^2*B*a*b^2*c^2*d + 18*A^2*B*a^2*b*c*d^2 - 11*A^2*B*a^3*d^3)*n - 3*((19*B^3*b^3*c^2*d - 378*B^3*a*b^2*c*d^2 + 359*B^3*a^2*b*d^3)*n^3 + 6*(5*A*B^2*b^3*c^2*d - 54*A*B^2*a*b^2*c*d^2 + 49*A*B^2*a^2*b*d^3)*n^2 + 18*(A^2*B*b^3*c^2*d - 6*A^2*B*a*b^2*c*d^2 + 5*A^2*B*a^2*b*d^3)*n)*x + 6*((4*B^3*b^3*c^3 - 27*B^3*a*b^2*c^2*d + 108*B^3*a^2*b*c*d^2)*n^3 + (85*B^3*b^3*d^3*n^3 + 66*A*B^2*b^3*d^3*n^2 + 18*A^2*B*b^3*d^3*n)*x^3 + 6*(2*A*B^2*b^3*c^3 - 9*A*B^2*a*b^2*c^2*d + 18*A*B^2*a^2*b*c*d^2)*n^2 + 3*(18*A^2*B*a*b^2*d^3*n + (22*B^3*b^3*c*d^2 + 63*B^3*a*b^2*d^3)*n^3 + 6*(2*A*B^2*b^3*c*d^2 + 9*A*B^2*a*b^2*d^3)*n^2)*x^2 + 18*(B^3*b^3*d^3*n*x^3 + 3*B^3*a*b^2*d^3*n*x^2 + 3*B^3*a^2*b*d^3*n*x + (B^3*b^3*c^3 - 3*B^3*a*b^2*c^2*d + 3*B^3*a^2*b*c*d^2)*n)*\log(e)^2 + 18*(A^2*B*b^3*c^3 - 3*A^2*B*a*b^2*c^2*d + 3*A^2*B*a^2*b*c*d^2)*n + 3*(18*A^2*B*a^2*b*d^3*n - (5*B^3*b^3*c^2*d - 54*B^3*a*b^2*c*d^2 - 36*B^3*a^2*b*d^3)*n^3 - 6*(A*B^2*b^3*c^2*d - 6*A*B^2*a*b^2*c*d^2 - 6*A*B^2*a^2*b*d^3)*n^2)*x + 6*((11*B^3*b^3*d^3*n^2 + 6*A*B^2*b^3*d^3*n)*x^3 + (2*B^3*b^3*c^3 - 9*B^3*a*b^2*c^2*d + 18*B^3*a^2*b*c*d^2)*n^2 + 3*(6*A*B^2*a*b^2*d^3*n + (2*B^3*b^3*c*d^2 + 9*B^3*a*b^2*d^3)*n^2)*x^2 + 6*(A*B^2*b^3*c^3 - 3*A*B^2*a*b^2*c^2*d + 3*A*B^2*a^2*b*c*d^2)*n + 3*(6*A*B^2*a^2*b*d^3*n - (B^3*b^3*c^2*d - 6*B^3*a*b^2*c*d^2 - 6*B^3*a^2*b*d^3)*n^2)*x)*\log(e))*\log(b*x + a) - 6*((4*B^3*b^3*c^3 - 27*B^3*a*b^2*c^2*d + 108*B^3*a^2*b*c*d^2)*n^3 + (85*B^3*b^3*d^3*n^3 +$$

$$\begin{aligned}
& 66A^2B^2b^3d^3n^2 + 18A^2B^2b^3d^3n)x^3 + 6(2A^2B^2b^3c^3 - 9A^2B^2a^2b^2c^2d + 18A^2B^2a^2b^2c^2d^2)n^2 + 3(18A^2B^2a^2b^2d^3n + (22B^3b^3c^2d^2 + 63B^3a^2b^2d^3)n^3 + 6(2A^2B^2b^3c^2d^2 + 9A^2B^2a^2b^2d^3)n^2)x^2 + 18(B^3b^3d^3n^3x^3 + 3B^3a^2b^2d^3n^3x^2 + 3B^3a^2b^2d^3n^3x + (B^3b^3c^3 - 3B^3a^2b^2c^2d + 3B^3a^2b^2c^2d^2)n^3)\log(bx + a)^2 + 18(B^3b^3d^3n^3x^3 + 3B^3a^2b^2d^3n^3x^2 + 3B^3a^2b^2d^3n^3x + (B^3b^3c^3 - 3B^3a^2b^2c^2d + 3B^3a^2b^2c^2d^2)n^3)\log(e)^2 + 18(A^2B^2b^3c^3 - 3A^2B^2a^2b^2c^2d + 3A^2B^2a^2b^2c^2d^2)n + 3(18A^2B^2a^2b^2d^3n - (5B^3b^3c^2d - 54B^3a^2b^2c^2d^2 - 36B^3a^2b^2d^3)n^3 - 6(A^2B^2b^3c^2d - 6A^2B^2a^2b^2c^2d^2 - 6A^2B^2a^2b^2d^3)n^2)x + 6((2B^3b^3c^3 - 9B^3a^2b^2c^2d + 18B^3a^2b^2c^2d^2)n^3 + (11B^3b^3d^3n^3 + 6A^2B^2b^3d^3n^2)x^3 + 6(A^2B^2b^3c^3 - 3A^2B^2a^2b^2c^2d + 3A^2B^2a^2b^2c^2d^2)n^2 + 3(6A^2B^2a^2b^2d^3n^2 + (2B^3b^3c^2d^2 + 9B^3a^2b^2d^3)n^3)x^2 + 3(6A^2B^2a^2b^2d^3n^2 - (B^3b^3c^2d - 6B^3a^2b^2c^2d^2 - 6B^3a^2b^2d^3)n^3)x + 6(B^3b^3d^3n^2x^3 + 3B^3a^2b^2d^3n^2x^2 + 3B^3a^2b^2d^3n^2x + (B^3b^3c^3 - 3B^3a^2b^2c^2d + 3B^3a^2b^2c^2d^2)n^2)\log(e))\log(bx + a) + 6((11B^3b^3d^3n^2 + 6A^2B^2b^3d^3n)x^3 + (2B^3b^3c^3 - 9B^3a^2b^2c^2d + 18B^3a^2b^2c^2d^2)n^2 + 3(6A^2B^2a^2b^2d^3n + (2B^3b^3c^2d^2 + 9B^3a^2b^2d^3)n^2)x^2 + 6(A^2B^2b^3c^3 - 3A^2B^2a^2b^2c^2d + 3A^2B^2a^2b^2c^2d^2)n + 3(6A^2B^2a^2b^2d^3n - (B^3b^3c^2d - 6B^3a^2b^2c^2d^2 - 6B^3a^2b^2d^3)n^2)x)\log(e))\log(dx + c) + 6(18A^2B^2b^3c^3 - 54A^2B^2a^2b^2c^2d + 54A^2B^2a^2b^2c^2d^2 - 18A^2B^2a^3d^3 + (4B^3b^3c^3 - 27B^3a^2b^2c^2d + 108B^3a^2b^2c^2d^2 - 85B^3a^3d^3)n^2 + 6(11B^3b^3c^2d^2 - B^3a^2b^2d^3)n^2 + 6(A^2B^2b^3c^2d^2 - A^2B^2a^2b^2d^3)n)x^2 + 6(2A^2B^2b^3c^3 - 9A^2B^2a^2b^2c^2d + 18A^2B^2a^2b^2c^2d^2 - 11A^2B^2a^3d^3)n - 3((5B^3b^3c^2d - 54B^3a^2b^2c^2d^2 + 49B^3a^2b^2d^3)n^2 + 6(A^2B^2b^3c^2d - 6A^2B^2a^2b^2c^2d^2 + 5A^2B^2a^2b^2d^3)n)x)\log(e))/(a^3b^4c^3 - 3a^4b^3c^2d + 3a^5b^2c^2d^2 - a^6b^2d^3 + (b^7c^3 - 3a^2b^6c^2d + 3a^2b^5c^2d^2 - a^3b^4d^3)x^3 + 3(a^2b^6c^3 - 3a^2b^5c^2d + 3a^3b^4c^2d^2 - a^4b^3d^3)x^2 + 3(a^2b^5c^3 - 3a^3b^4c^2d + 3a^4b^3c^2d^2 - a^5b^2d^3)x)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*3/(b\*x+a)\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^3}{(bx+a)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^4,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^3/(b\*x + a)^4, x)

$$3.171 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)^5} dx$$

**Optimal.** Leaf size=830

$$\frac{b^3(A+B \log(e(a+bx)^n(c+dx)^{-n}))^3(c+dx)^4}{4(bc-ad)^4(a+bx)^4} - \frac{3b^3Bn(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2(c+dx)^4}{16(bc-ad)^4(a+bx)^4} - \frac{3b^3B^2n^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))}{16(bc-ad)^4(a+bx)^4}$$

[Out]  $(6*B^3*d^3*n^3*(c+d*x))/((b*c-a*d)^4*(a+b*x)) - (9*b*B^3*d^2*n^3*(c+d*x)^2)/(8*(b*c-a*d)^4*(a+b*x)^2) + (2*b^2*B^3*d*n^3*(c+d*x)^3)/(9*(b*c-a*d)^4*(a+b*x)^3) - (3*b^3*B^3*n^3*(c+d*x)^4)/(128*(b*c-a*d)^4*(a+b*x)^4) + (6*B^2*d^3*n^2*(c+d*x)*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n)))/((b*c-a*d)^4*(a+b*x)) - (9*b*B^2*d^2*n^2*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n)))/(4*(b*c-a*d)^4*(a+b*x)^2) + (2*b^2*B^2*d*n^2*(c+d*x)^3*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n)))/(3*(b*c-a*d)^4*(a+b*x)^3) - (3*b^3*B^2*n^2*(c+d*x)^4*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n)))/(32*(b*c-a*d)^4*(a+b*x)^4) + (3*B*d^3*n*(c+d*x)*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n])^2)/((b*c-a*d)^4*(a+b*x)) - (9*b*B*d^2*n*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n])^2)/(4*(b*c-a*d)^4*(a+b*x)^2) + (b^2*B*d*n*(c+d*x)^3*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n])^2)/((b*c-a*d)^4*(a+b*x)^3) - (3*b^3*B*n*(c+d*x)^4*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n])^2)/(16*(b*c-a*d)^4*(a+b*x)^4) + (d^3*(c+d*x)*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n])^3)/((b*c-a*d)^4*(a+b*x)) - (3*b*d^2*(c+d*x)^2*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n])^3)/(2*(b*c-a*d)^4*(a+b*x)^2) + (b^2*d*(c+d*x)^3*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n])^3)/((b*c-a*d)^4*(a+b*x)^3) - (b^3*(c+d*x)^4*(A+B*Log[(e*(a+b*x)^n)/(c+d*x]^n])^3)/(4*(b*c-a*d)^4*(a+b*x)^4)$

**Rubi [C]** time = 4.67497, antiderivative size = 2173, normalized size of antiderivative = 2.62, number of steps used = 93, number of rules used = 16, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$ , Rules used = {6742, 2492, 44, 2514, 2490, 32, 2488, 2411, 2343, 2333, 2315, 2491, 2509, 37, 2506, 6610}

result too large to display

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n])^3/(a + b\*x)^5, x]

[Out]  $-A^3/(4*b*(a+b*x)^4) - (3*A^2*B*n)/(16*b*(a+b*x)^4) - (3*A*B^2*n^2)/(32*b*(a+b*x)^4) - (3*B^3*n^3)/(128*b*(a+b*x)^4) + (A^2*B*d*n)/(4*b*(b*c-a*d)*(a+b*x)^3) + (7*A*B^2*d*n^2)/(24*b*(b*c-a*d)*(a+b*x)^3) + (37*B^3*d*n^3)/(288*b*(b*c-a*d)*(a+b*x)^3) - (3*A^2*B*d^2*n)/(8*b*(b*c-a*d)^2*(a+b*x)^2) - (13*A*B^2*d^2*n^2)/(16*b*(b*c-a*d)^2*(a+b*x)^2) - (7*9*B^3*d^2*n^3)/(192*b*(b*c-a*d)^2*(a+b*x)^2) + (3*A^2*B*d^3*n)/(4*b*(b*c-a*d)^3*(a+b*x)) + (25*A*B^2*d^3*n^2)/(8*b*(b*c-a*d)^3*(a+b*x)) + (451*B^3*d^3*n^3)/(96*b*(b*c-a*d)^3*(a+b*x)) - (3*b*B^3*d^2*n^3*(c+d*x)^2)/(16*(b*c-a*d)^4*(a+b*x)^2) + (3*A^2*B*d^4*n*Log[a+b*x])/(4*b*(b*c-a*d)^4) + (13*A*B^2*d^4*n^2*Log[a+b*x])/(8*b*(b*c-a*d)^4) + (79*B^3*d^4*n^3*Log[a+b*x])/(96*b*(b*c-a*d)^4) - (3*A^2*B*d^4*n*Log[c+d*x])/(4*b*(b*c-a*d)^4) - (13*A*B^2*d^4*n^2*Log[c+d*x])/(8*b*(b*c-a*d)^4) - (79*B^3*d^4*n^3*Log[c+d*x])/(96*b*(b*c-a*d)^4) - (3*A^2*B*Log[(e*(a+b*x)^n)/(c+d*x]^n])/(4*b*(a+b*x)^4) - (3*A*B^2*n*Log[(e*(a+b*x)^n)/(c+d*x]^n])/(8*b*(a+b*x)^4) - (3*B^3*n^2*Log[(e*(a+b*x)^n)/(c+d*x]^n])/(32*b*(a+b*x)^4) + (A*B^2*d*n*Log[(e*(a+b*x)^n)/(c+d*x]^n])/(2*b*(b*c-a*d)*(a+b*x)^3) + (7*B^3*d*n^2*Log[(e*(a+b*x)^n)/(c+d*x]^n])/(2*4*b*(b*c-a*d)*(a+b*x)^3) - (3*A*B^2*d^2*n*Log[(e*(a+b*x)^n)/(c+d*x]^n])/(2*4*b*(b*c-a*d)*(a+b*x)^3)$



$$\begin{aligned} & \frac{1}{(4*b*(b*c - a*d)^2*(a + b*x)^2) - (7*B^3*d^2*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])} \\ & - \frac{1}{(16*b*(b*c - a*d)^2*(a + b*x)^2) + (3*A*B^2*d^3*n*(c + d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])} \\ & - \frac{1}{(2*(b*c - a*d)^4*(a + b*x))} + \frac{1}{(31*B^3*d^3*n^2*(c + d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])} \\ & - \frac{1}{(8*(b*c - a*d)^4*(a + b*x))} - \frac{1}{(3*b*B^3*d^2*n^2*(c + d*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])} \\ & - \frac{1}{(8*(b*c - a*d)^4*(a + b*x)^2)} - \frac{1}{(3*A*B^2*d^4*n*Log[-((b*c - a*d)/(d*(a + b*x))])} \\ & *Log[(e*(a + b*x)^n)/(c + d*x)^n])} \\ & - \frac{1}{(2*b*(b*c - a*d)^4)} - \frac{1}{(7*B^3*d^4*n^2*Log[-((b*c - a*d)/(d*(a + b*x))])} \\ & *Log[(e*(a + b*x)^n)/(c + d*x)^n])} \\ & - \frac{1}{(8*b*(b*c - a*d)^4)} + \frac{1}{(3*A*B^2*d^4*n*Log[(b*c - a*d)/(b*(c + d*x))]} \\ & *Log[(e*(a + b*x)^n)/(c + d*x)^n])} \\ & - \frac{1}{(2*b*(b*c - a*d)^4)} + \frac{1}{(7*B^3*d^4*n^2*Log[(b*c - a*d)/(b*(c + d*x))]} \\ & *Log[(e*(a + b*x)^n)/(c + d*x)^n])} \\ & - \frac{1}{(8*b*(b*c - a*d)^4)} - \frac{1}{(3*A*B^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)} \\ & - \frac{1}{(4*b*(a + b*x)^4)} - \frac{1}{(3*B^3*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)} \\ & - \frac{1}{(16*b*(a + b*x)^4)} + \frac{1}{(B^3*d*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)} \\ & - \frac{1}{(4*b*(b*c - a*d)*(a + b*x)^3)} + \frac{1}{(3*B^3*d^3*n*(c + d*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)} \\ & - \frac{1}{(2*(b*c - a*d)^4*(a + b*x))} - \frac{1}{(3*b*B^3*d^2*n*(c + d*x)^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)} \\ & - \frac{1}{(8*(b*c - a*d)^4*(a + b*x)^2)} - \frac{1}{(3*B^3*d^4*n*Log[-((b*c - a*d)/(d*(a + b*x))]} \\ & *Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)} \\ & - \frac{1}{(4*b*(b*c - a*d)^4)} + \frac{1}{(3*B^3*d^4*n*Log[(b*c - a*d)/(b*(c + d*x))]} \\ & *Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)} \\ & - \frac{1}{(4*b*(b*c - a*d)^4)} - \frac{1}{(B^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3)} \\ & - \frac{1}{(4*b*(a + b*x)^4)} + \frac{1}{(3*A*B^2*d^4*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])} \\ & - \frac{1}{(2*b*(b*c - a*d)^4)} + \frac{1}{(7*B^3*d^4*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])} \\ & - \frac{1}{(8*b*(b*c - a*d)^4)} + \frac{1}{(3*A*B^2*d^4*n^2*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])} \\ & - \frac{1}{(2*b*(b*c - a*d)^4)} + \frac{1}{(7*B^3*d^4*n^3*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])} \\ & - \frac{1}{(8*b*(b*c - a*d)^4)} + \frac{1}{(3*B^3*d^4*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])} \\ & - \frac{1}{(2*b*(b*c - a*d)^4)} + \frac{1}{(3*B^3*d^4*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])} \\ & - \frac{1}{(2*b*(b*c - a*d)^4)} + \frac{1}{(3*B^3*d^4*n^3*PolyLog[3, 1 + (b*c - a*d)/(d*(a + b*x))])} \\ & - \frac{1}{(2*b*(b*c - a*d)^4)} - \frac{1}{(3*B^3*d^4*n^3*PolyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])} \\ & - \frac{1}{(2*b*(b*c - a*d)^4)} \end{aligned}$$

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))^(r_.)]^(s_.)*((g_.) + (h_.)*(x_.))^(m_.), x_Symbol] := Simp[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(a + b*x)*(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0] && NeQ[m, -1]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

#### Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))^(r_.)]^(s_.)*(RFX_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]
```

Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))2, x_Symbol] := Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q]^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] := Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] := Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] := Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2491

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))3, x_Symbol] := Dist[d/(d*g - c*h), Int[Log
[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s/(g + h*x)^2, x], x] - Dist[h/(d*g - c
*h), Int[((c + d*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(g + h*x)^3, x]
, x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0]
&& EqQ[p + q, 0] && EqQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 0]
```

Rule 2509

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1)*Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s)/((m + 1)*(b*c - a*d)), x] - Dist[(p*r*s*(b*c - a*d))/((m + 1)
*(b*c - a*d)), Int[(a + b*x)^m*(c + d*x)^n*Log[e*(f*(a + b*x)^p*(c + d*x)^q
]^r]^(s - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r, s}, x] && NeQ[
b*c - a*d, 0] && EqQ[p + q, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1] && IGtQ[s, 0]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))
^(q_.)]^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^3}{(a + bx)^5} dx &= \int \left( \frac{A^3}{(a + bx)^5} + \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^5} + \frac{3AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^5} \right) dx \\
&= -\frac{A^3}{4b(a + bx)^4} + (3A^2B) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^5} dx + (3AB^2) \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{(a + bx)^5} dx \\
&= -\frac{A^3}{4b(a + bx)^4} - \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{4b(a + bx)^4} - \frac{3AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{4b(a + bx)^4} \\
&= -\frac{A^3}{4b(a + bx)^4} - \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{4b(a + bx)^4} - \frac{3AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{4b(a + bx)^4} \\
&= -\frac{A^3}{4b(a + bx)^4} - \frac{3A^2Bn}{16b(a + bx)^4} + \frac{A^2Bdn}{4b(bc - ad)(a + bx)^3} - \frac{3A^2Bd^2n}{8b(bc - ad)^2(a + bx)^2} \\
&= -\frac{A^3}{4b(a + bx)^4} - \frac{3A^2Bn}{16b(a + bx)^4} + \frac{A^2Bdn}{4b(bc - ad)(a + bx)^3} - \frac{3A^2Bd^2n}{8b(bc - ad)^2(a + bx)^2} \\
&= -\frac{A^3}{4b(a + bx)^4} - \frac{3A^2Bn}{16b(a + bx)^4} + \frac{A^2Bdn}{4b(bc - ad)(a + bx)^3} - \frac{3A^2Bd^2n}{8b(bc - ad)^2(a + bx)^2} \\
&= -\frac{A^3}{4b(a + bx)^4} - \frac{3A^2Bn}{16b(a + bx)^4} - \frac{3AB^2n^2}{32b(a + bx)^4} + \frac{A^2Bdn}{4b(bc - ad)(a + bx)^3} + \frac{A^2Bd^2n}{24b(bc - ad)^2(a + bx)^2} \\
&= -\frac{A^3}{4b(a + bx)^4} - \frac{3A^2Bn}{16b(a + bx)^4} - \frac{3AB^2n^2}{32b(a + bx)^4} + \frac{A^2Bdn}{4b(bc - ad)(a + bx)^3} + \frac{A^2Bd^2n}{24b(bc - ad)^2(a + bx)^2} \\
&= -\frac{A^3}{4b(a + bx)^4} - \frac{3A^2Bn}{16b(a + bx)^4} - \frac{3AB^2n^2}{32b(a + bx)^4} + \frac{A^2Bdn}{4b(bc - ad)(a + bx)^3} + \frac{A^2Bd^2n}{24b(bc - ad)^2(a + bx)^2} \\
&= -\frac{A^3}{4b(a + bx)^4} - \frac{3A^2Bn}{16b(a + bx)^4} - \frac{3AB^2n^2}{32b(a + bx)^4} - \frac{3B^3n^3}{128b(a + bx)^4} + \frac{A^2Bdn}{4b(bc - ad)(a + bx)^3} \\
&= -\frac{A^3}{4b(a + bx)^4} - \frac{3A^2Bn}{16b(a + bx)^4} - \frac{3AB^2n^2}{32b(a + bx)^4} - \frac{3B^3n^3}{128b(a + bx)^4} + \frac{A^2Bdn}{4b(bc - ad)(a + bx)^3} \\
&= -\frac{A^3}{4b(a + bx)^4} - \frac{3A^2Bn}{16b(a + bx)^4} - \frac{3AB^2n^2}{32b(a + bx)^4} - \frac{3B^3n^3}{128b(a + bx)^4} + \frac{A^2Bdn}{4b(bc - ad)(a + bx)^3}
\end{aligned}$$

**Mathematica [A]** time = 2.251, size = 1370, normalized size = 1.65

$$-\frac{-288B^3d^4n^3 \log^3(a + bx)(a + bx)^4 + 288B^3d^4n^3 \log^3(c + dx)(a + bx)^4 + 72B^2d^4n^2 \log^2(c + dx)(12A + 25Bn + 12B \log[c + dx])}{4b^4(a + bx)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^3/(a + b\*x)^5,x]

[Out] -(-288\*B^3\*d^4\*n^3\*(a + b\*x)^4\*Log[a + b\*x]^3 + 288\*B^3\*d^4\*n^3\*(a + b\*x)^4\*Log[c + d\*x]^3 + 72\*B^2\*d^4\*n^2\*(a + b\*x)^4\*Log[c + d\*x]^2\*(12\*A + 25\*B\*n + 12\*B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]) + 72\*B^2\*d^4\*n^2\*(a + b\*x)^4\*Log[a + b\*x]^2\*(12\*A + 25\*B\*n + 12\*B\*n\*Log[c + d\*x] + 12\*B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]))/(4\*b^4\*(a + b\*x)^4)

$$\begin{aligned}
& c + d*x)^n]) + 12*B*d^4*n*(a + b*x)^4*\text{Log}[c + d*x]*(72*A^2 + 300*A*B*n + 415*B^2*n^2 + 12*B*(12*A + 25*B*n)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + 72*B^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2 + (b*c - a*d)*(288*A^3*b^3*c^3 - 864*a*A^3*b^2*c^2*d + 864*a^2*A^3*b*c*d^2 - 288*a^3*A^3*d^3 + 216*A^2*b^3*B*c^3*n - 936*a*A^2*b^2*B*c^2*d*n + 1656*a^2*A^2*b*B*c*d^2*n - 1800*a^3*A^2*B*d^3*n + 108*A*b^3*B^2*c^3*n^2 - 660*a*A*b^2*B^2*c^2*d*n^2 + 1932*a^2*A*b*B^2*c*d^2*n^2 - 4980*a^3*A*B^2*d^3*n^2 + 27*b^3*B^3*c^3*n^3 - 229*a*b^2*B^3*c^2*d*n^3 + 1067*a^2*b*B^3*c*d^2*n^3 - 5845*a^3*B^3*d^3*n^3 - 288*A^2*b^3*B*c^2*d*n*x + 1440*a*A^2*b^2*B*c*d^2*n*x - 3744*a^2*A^2*b*B*d^3*n*x - 336*A*b^3*B^2*c^2*d*n^2*x + 2544*a*A*b^2*B^2*c*d^2*n^2*x - 13008*a^2*A*b*B^2*d^3*n^2*x - 148*b^3*B^3*c^2*d*n^3*x + 1676*a*b^2*B^3*c*d^2*n^3*x - 16468*a^2*b*B^3*d^3*n^3*x + 432*A^2*b^3*B*c*d^2*n*x^2 - 3024*a*A^2*b^2*B*d^3*n*x^2 + 936*A*b^3*B^2*c*d^2*n^2*x^2 - 11736*a*A*b^2*B^2*d^3*n^2*x^2 + 690*b^3*B^3*c*d^2*n^3*x^2 - 15630*a*b^2*B^3*d^3*n^3*x^2 - 864*A^2*b^3*B*d^3*n*x^3 - 3600*A*b^3*B^2*d^3*n^2*x^3 - 4980*b^3*B^3*d^3*n^3*x^3 + 12*B*(72*A^2*(b*c - a*d)^3 + B^2*n^2*(-415*a^3*d^3 + a^2*b*d^2*(161*c - 1084*d*x) + a*b^2*d*(-55*c^2 + 212*c*d*x - 978*d^2*x^2) + b^3*(9*c^3 - 28*c^2*d*x + 78*c*d^2*x^2 - 300*d^3*x^3)) + 12*A*B*n*(-25*a^3*d^3 + a^2*b*d^2*(23*c - 52*d*x) + a*b^2*d*(-13*c^2 + 20*c*d*x - 42*d^2*x^2) + b^3*(3*c^3 - 4*c^2*d*x + 6*c*d^2*x^2 - 12*d^3*x^3)))*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + 72*B^2*(12*A*(b*c - a*d)^3 + B*n*(-25*a^3*d^3 + a^2*b*d^2*(23*c - 52*d*x) + a*b^2*d*(-13*c^2 + 20*c*d*x - 42*d^2*x^2) + b^3*(3*c^3 - 4*c^2*d*x + 6*c*d^2*x^2 - 12*d^3*x^3)))*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 288*B^3*(b*c - a*d)^3*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^3) - 12*B*d^4*n*(a + b*x)^4*\text{Log}[a + b*x]*(72*A^2 + 300*A*B*n + 415*B^2*n^2 + 72*B^2*n^2*\text{Log}[c + d*x]^2 + 12*B*(12*A + 25*B*n)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + 72*B^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2 + 12*B*n*\text{Log}[c + d*x]*(12*A + 25*B*n + 12*B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]))/(1152*b*(b*c - a*d)^4*(a + b*x)^4)
\end{aligned}$$

**Maple [C]** time = 33.085, size = 236754, normalized size = 285.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^5,x)

[Out] result too large to display

**Maxima [B]** time = 3.30885, size = 7128, normalized size = 8.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^5,x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& -1/4*B^3*\text{log}((b*x + a)^n*e/(d*x + c)^n)^3/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/1152*(72*(12*d^4*e*n*\text{log}(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) - 12*d^4*e*n*\text{log}(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a*b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7*a*b^2*d^3*
\end{aligned}$$

$$\begin{aligned}
& e^n) * x^2 + 4 * (b^3 * c^2 * d * e^n - 5 * a * b^2 * c * d^2 * e^n + 13 * a^2 * b * d^3 * e^n) * x) / (a^4 \\
& * b^4 * c^3 - 3 * a^5 * b^3 * c^2 * d + 3 * a^6 * b^2 * c * d^2 - a^7 * b * d^3 + (b^8 * c^3 - 3 * a * b \\
& ^7 * c^2 * d + 3 * a^2 * b^6 * c * d^2 - a^3 * b^5 * d^3) * x^4 + 4 * (a * b^7 * c^3 - 3 * a^2 * b^6 * c^2 * d \\
& + 3 * a^3 * b^5 * c * d^2 - a^4 * b^4 * d^3) * x^3 + 6 * (a^2 * b^6 * c^3 - 3 * a^3 * b^5 * c^2 * d \\
& + 3 * a^4 * b^4 * c * d^2 - a^5 * b^3 * d^3) * x^2 + 4 * (a^3 * b^5 * c^3 - 3 * a^4 * b^4 * c^2 * d + \\
& 3 * a^5 * b^3 * c * d^2 - a^6 * b^2 * d^3) * x) * \log((b * x + a)^n * e / (d * x + c)^n)^2 / e - (12 \\
& * (9 * b^4 * c^4 * e^2 * n^2 - 64 * a * b^3 * c^3 * d * e^2 * n^2 + 216 * a^2 * b^2 * c^2 * d^2 * e^2 * n^2 \\
& - 576 * a^3 * b * c * d^3 * e^2 * n^2 + 415 * a^4 * d^4 * e^2 * n^2 - 300 * (b^4 * c * d^3 * e^2 * n^2 - \\
& a * b^3 * d^4 * e^2 * n^2) * x^3 + 6 * (13 * b^4 * c^2 * d^2 * e^2 * n^2 - 176 * a * b^3 * c * d^3 * e^2 * n^2 \\
& + 163 * a^2 * b^2 * d^4 * e^2 * n^2) * x^2 + 72 * (b^4 * d^4 * e^2 * n^2 * x^4 + 4 * a * b^3 * d^4 * e^2 \\
& * n^2 * x^3 + 6 * a^2 * b^2 * d^4 * e^2 * n^2 * x^2 + 4 * a^3 * b * d^4 * e^2 * n^2 * x + a^4 * d^4 * e^2 \\
& * n^2) * \log(b * x + a)^2 + 72 * (b^4 * d^4 * e^2 * n^2 * x^4 + 4 * a * b^3 * d^4 * e^2 * n^2 * x^3 + \\
& 6 * a^2 * b^2 * d^4 * e^2 * n^2 * x^2 + 4 * a^3 * b * d^4 * e^2 * n^2 * x + a^4 * d^4 * e^2 * n^2) * \log(d * \\
& x + c)^2 - 4 * (7 * b^4 * c^3 * d * e^2 * n^2 - 60 * a * b^3 * c^2 * d^2 * e^2 * n^2 + 324 * a^2 * b^2 * \\
& c * d^3 * e^2 * n^2 - 271 * a^3 * b * d^4 * e^2 * n^2) * x - 300 * (b^4 * d^4 * e^2 * n^2 * x^4 + 4 * a * b \\
& ^3 * d^4 * e^2 * n^2 * x^3 + 6 * a^2 * b^2 * d^4 * e^2 * n^2 * x^2 + 4 * a^3 * b * d^4 * e^2 * n^2 * x + a^4 \\
& * d^4 * e^2 * n^2) * \log(b * x + a) + 12 * (25 * b^4 * d^4 * e^2 * n^2 * x^4 + 100 * a * b^3 * d^4 * e^2 \\
& * n^2 * x^3 + 150 * a^2 * b^2 * d^4 * e^2 * n^2 * x^2 + 100 * a^3 * b * d^4 * e^2 * n^2 * x + 25 * a^4 * \\
& d^4 * e^2 * n^2 - 12 * (b^4 * d^4 * e^2 * n^2 * x^4 + 4 * a * b^3 * d^4 * e^2 * n^2 * x^3 + 6 * a^2 * b^2 \\
& * d^4 * e^2 * n^2 * x^2 + 4 * a^3 * b * d^4 * e^2 * n^2 * x + a^4 * d^4 * e^2 * n^2) * \log(b * x + a) * \log(d * \\
& x + c)) * \log((b * x + a)^n * e / (d * x + c)^n) / ((a^4 * b^5 * c^4 - 4 * a^5 * b^4 * c^3 * d \\
& + 6 * a^6 * b^3 * c^2 * d^2 - 4 * a^7 * b^2 * c * d^3 + a^8 * b * d^4 + (b^9 * c^4 - 4 * a * b^8 * c^3 * \\
& * d + 6 * a^2 * b^7 * c^2 * d^2 - 4 * a^3 * b^6 * c * d^3 + a^4 * b^5 * d^4) * x^4 + 4 * (a * b^8 * c^4 \\
& - 4 * a^2 * b^7 * c^3 * d + 6 * a^3 * b^6 * c^2 * d^2 - 4 * a^4 * b^5 * c * d^3 + a^5 * b^4 * d^4) * x^3 \\
& + 6 * (a^2 * b^7 * c^4 - 4 * a^3 * b^6 * c^3 * d + 6 * a^4 * b^5 * c^2 * d^2 - 4 * a^5 * b^4 * c * d^3 + \\
& a^6 * b^3 * d^4) * x^2 + 4 * (a^3 * b^6 * c^4 - 4 * a^4 * b^5 * c^3 * d + 6 * a^5 * b^4 * c^2 * d^2 - 4 * \\
& a^6 * b^3 * c * d^3 + a^7 * b^2 * d^4) * x) * e) + (27 * b^4 * c^4 * e^3 * n^3 - 256 * a * b^3 * c^3 * d \\
& * e^3 * n^3 + 1296 * a^2 * b^2 * c^2 * d^2 * e^3 * n^3 - 6912 * a^3 * b * c * d^3 * e^3 * n^3 + 5845 * a^4 * \\
& d^4 * e^3 * n^3 - 4980 * (b^4 * c * d^3 * e^3 * n^3 - a * b^3 * d^4 * e^3 * n^3) * x^3 - 288 * (b^4 * \\
& d^4 * e^3 * n^3 * x^4 + 4 * a * b^3 * d^4 * e^3 * n^3 * x^3 + 6 * a^2 * b^2 * d^4 * e^3 * n^3 * x^2 + 4 * \\
& a^3 * b * d^4 * e^3 * n^3 * x + a^4 * d^4 * e^3 * n^3) * \log(b * x + a)^3 + 288 * (b^4 * d^4 * e^3 * n^3 \\
& * x^4 + 4 * a * b^3 * d^4 * e^3 * n^3 * x^3 + 6 * a^2 * b^2 * d^4 * e^3 * n^3 * x^2 + 4 * a^3 * b * d^4 * \\
& e^3 * n^3 * x + a^4 * d^4 * e^3 * n^3) * \log(d * x + c)^3 + 30 * (23 * b^4 * c^2 * d^2 * e^3 * n^3 - \\
& 544 * a * b^3 * c * d^3 * e^3 * n^3 + 521 * a^2 * b^2 * d^4 * e^3 * n^3) * x^2 + 1800 * (b^4 * d^4 * e^3 * \\
& n^3 * x^4 + 4 * a * b^3 * d^4 * e^3 * n^3 * x^3 + 6 * a^2 * b^2 * d^4 * e^3 * n^3 * x^2 + 4 * a^3 * b * d^4 * \\
& e^3 * n^3 * x + a^4 * d^4 * e^3 * n^3) * \log(b * x + a)^2 + 72 * (25 * b^4 * d^4 * e^3 * n^3 * x^4 + \\
& 100 * a * b^3 * d^4 * e^3 * n^3 * x^3 + 150 * a^2 * b^2 * d^4 * e^3 * n^3 * x^2 + 100 * a^3 * b * d^4 * e^3 \\
& * n^3 * x + 25 * a^4 * d^4 * e^3 * n^3 - 12 * (b^4 * d^4 * e^3 * n^3 * x^4 + 4 * a * b^3 * d^4 * e^3 * n^3 \\
& * x^3 + 6 * a^2 * b^2 * d^4 * e^3 * n^3 * x^2 + 4 * a^3 * b * d^4 * e^3 * n^3 * x + a^4 * d^4 * e^3 * n^3) \\
& * \log(b * x + a) * \log(d * x + c)^2 - 4 * (37 * b^4 * c^3 * d * e^3 * n^3 - 456 * a * b^3 * c^2 * d^2 \\
& * e^3 * n^3 + 4536 * a^2 * b^2 * c * d^3 * e^3 * n^3 - 4117 * a^3 * b * d^4 * e^3 * n^3) * x - 4980 * ( \\
& b^4 * d^4 * e^3 * n^3 * x^4 + 4 * a * b^3 * d^4 * e^3 * n^3 * x^3 + 6 * a^2 * b^2 * d^4 * e^3 * n^3 * x^2 + \\
& 4 * a^3 * b * d^4 * e^3 * n^3 * x + a^4 * d^4 * e^3 * n^3) * \log(b * x + a) + 12 * (415 * b^4 * d^4 * e^3 \\
& * n^3 * x^4 + 1660 * a * b^3 * d^4 * e^3 * n^3 * x^3 + 2490 * a^2 * b^2 * d^4 * e^3 * n^3 * x^2 + 166 \\
& 0 * a^3 * b * d^4 * e^3 * n^3 * x + 415 * a^4 * d^4 * e^3 * n^3 + 72 * (b^4 * d^4 * e^3 * n^3 * x^4 + 4 * a \\
& * b^3 * d^4 * e^3 * n^3 * x^3 + 6 * a^2 * b^2 * d^4 * e^3 * n^3 * x^2 + 4 * a^3 * b * d^4 * e^3 * n^3 * x + \\
& a^4 * d^4 * e^3 * n^3) * \log(b * x + a)^2 - 300 * (b^4 * d^4 * e^3 * n^3 * x^4 + 4 * a * b^3 * d^4 * e^3 \\
& * n^3 * x^3 + 6 * a^2 * b^2 * d^4 * e^3 * n^3 * x^2 + 4 * a^3 * b * d^4 * e^3 * n^3 * x + a^4 * d^4 * e^3 \\
& * n^3) * \log(b * x + a) * \log(d * x + c)) / ((a^4 * b^5 * c^4 - 4 * a^5 * b^4 * c^3 * d + 6 * a^6 * b^3 * \\
& c^2 * d^2 - 4 * a^7 * b^2 * c * d^3 + a^8 * b * d^4 + (b^9 * c^4 - 4 * a * b^8 * c^3 * d + 6 * a^2 * \\
& b^7 * c^2 * d^2 - 4 * a^3 * b^6 * c * d^3 + a^4 * b^5 * d^4) * x^4 + 4 * (a * b^8 * c^4 - 4 * a^2 * b^7 * \\
& c^3 * d + 6 * a^3 * b^6 * c^2 * d^2 - 4 * a^4 * b^5 * c * d^3 + a^5 * b^4 * d^4) * x^3 + 6 * (a^2 * b^7 * \\
& c^4 - 4 * a^3 * b^6 * c^3 * d + 6 * a^4 * b^5 * c^2 * d^2 - 4 * a^5 * b^4 * c * d^3 + a^6 * b^3 * d^4) \\
& * x^2 + 4 * (a^3 * b^6 * c^4 - 4 * a^4 * b^5 * c^3 * d + 6 * a^5 * b^4 * c^2 * d^2 - 4 * a^6 * b^3 * c \\
& * d^3 + a^7 * b^2 * d^4) * x) * e^2) / e) * B^3 + 1 / 96 * A * B^2 * (12 * (12 * d^4 * e * n * \log(b * x + \\
& a) / (b^5 * c^4 - 4 * a * b^4 * c^3 * d + 6 * a^2 * b^3 * c^2 * d^2 - 4 * a^3 * b^2 * c * d^3 + a^4 * b * d^4) \\
& - 12 * d^4 * e * n * \log(d * x + c) / (b^5 * c^4 - 4 * a * b^4 * c^3 * d + 6 * a^2 * b^3 * c^2 * d^2 \\
& - 4 * a^3 * b^2 * c * d^3 + a^4 * b * d^4) + (12 * b^3 * d^3 * e * n * x^3 - 3 * b^3 * c^3 * e * n + 13 * a \\
& * b^2 * c^2 * d * e * n - 23 * a^2 * b * c * d^2 * e * n + 25 * a^3 * d^3 * e * n - 6 * (b^3 * c * d^2 * e * n - 7
\end{aligned}$$

$$\begin{aligned} & *a*b^2*d^3*e^n)*x^2 + 4*(b^3*c^2*d*e^n - 5*a*b^2*c*d^2*e^n + 13*a^2*b*d^3*e \\ & *n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c \\ & c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3 \\ & *a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^ \\ & 3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b \\ & ^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*log((b*x + a)^n*(d*x + c)^n \\ & )/e - (9*b^4*c^4*e^2*n^2 - 64*a*b^3*c^3*d*e^2*n^2 + 216*a^2*b^2*c^2*d^2*e^2 \\ & *n^2 - 576*a^3*b*c*d^3*e^2*n^2 + 415*a^4*d^4*e^2*n^2 - 300*(b^4*c*d^3*e^2*n \\ & ^2 - a*b^3*d^4*e^2*n^2)*x^3 + 6*(13*b^4*c^2*d^2*e^2*n^2 - 176*a*b^3*c*d^3*e \\ & ^2*n^2 + 163*a^2*b^2*d^4*e^2*n^2)*x^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d \\ & ^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^ \\ & 4*e^2*n^2)*log(b*x + a)^2 + 72*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x \\ & ^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*l \\ & og(d*x + c)^2 - 4*(7*b^4*c^3*d*e^2*n^2 - 60*a*b^3*c^2*d^2*e^2*n^2 + 324*a^2 \\ & *b^2*c*d^3*e^2*n^2 - 271*a^3*b*d^4*e^2*n^2)*x - 300*(b^4*d^4*e^2*n^2*x^4 + \\ & 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x \\ & + a^4*d^4*e^2*n^2)*log(b*x + a) + 12*(25*b^4*d^4*e^2*n^2*x^4 + 100*a*b^3*d \\ & ^4*e^2*n^2*x^3 + 150*a^2*b^2*d^4*e^2*n^2*x^2 + 100*a^3*b*d^4*e^2*n^2*x + 25 \\ & *a^4*d^4*e^2*n^2 - 12*(b^4*d^4*e^2*n^2*x^4 + 4*a*b^3*d^4*e^2*n^2*x^3 + 6*a^ \\ & 2*b^2*d^4*e^2*n^2*x^2 + 4*a^3*b*d^4*e^2*n^2*x + a^4*d^4*e^2*n^2)*log(b*x + \\ & a))*log(d*x + c))/((a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a \\ & ^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4 \\ & *a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3* \\ & b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b \\ & ^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*x^2 + 4*(a^3* \\ & b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d \\ & ^4)*x)*e^2)) - 3/4*A*B^2*log((b*x + a)^n*(d*x + c)^n)^2/(b^5*x^4 + 4*a*b^ \\ & 4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^4*b) + 1/16*(12*d^4*e*n*log(b*x + a \\ & )/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^ \\ & 4) - 12*d^4*e*n*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - \\ & 4*a^3*b^2*c*d^3 + a^4*b*d^4) + (12*b^3*d^3*e*n*x^3 - 3*b^3*c^3*e*n + 13*a* \\ & b^2*c^2*d*e*n - 23*a^2*b*c*d^2*e*n + 25*a^3*d^3*e*n - 6*(b^3*c*d^2*e*n - 7* \\ & a*b^2*d^3*e*n)*x^2 + 4*(b^3*c^2*d*e*n - 5*a*b^2*c*d^2*e*n + 13*a^2*b*d^3*e* \\ & n)*x)/(a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3 + (b^8*c \\ & ^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*x^4 + 4*(a*b^7*c^3 - 3* \\ & a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*x^3 + 6*(a^2*b^6*c^3 - 3*a^3 \\ & *b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^ \\ & 4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*x))*A^2*B/e - 3/4*A^2*B*log((b*x + \\ & a)^n*(d*x + c)^n)/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + \\ & a^4*b) - 1/4*A^3/(b^5*x^4 + 4*a*b^4*x^3 + 6*a^2*b^3*x^2 + 4*a^3*b^2*x + a^ \\ & 4*b) \end{aligned}$$

**Fricas [B]** time = 2.45033, size = 12382, normalized size = 14.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^5,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/1152*(288*A^3*b^4*c^4 - 1152*A^3*a*b^3*c^3*d + 1728*A^3*a^2*b^2*c^2*d^2 \\ & - 1152*A^3*a^3*b*c*d^3 + 288*A^3*a^4*d^4 + (27*B^3*b^4*c^4 - 256*B^3*a*b^3* \\ & c^3*d + 1296*B^3*a^2*b^2*c^2*d^2 - 6912*B^3*a^3*b*c*d^3 + 5845*B^3*a^4*d^4) \\ & *n^3 - 12*(415*(B^3*b^4*c*d^3 - B^3*a*b^3*d^4)*n^3 + 300*(A*B^2*b^4*c*d^3 - \\ & A*B^2*a*b^3*d^4)*n^2 + 72*(A^2*B*b^4*c*d^3 - A^2*B*a*b^3*d^4)*n)*x^3 - 288 \\ & *(B^3*b^4*d^4*n^3*x^4 + 4*B^3*a*b^3*d^4*n^3*x^3 + 6*B^3*a^2*b^2*d^4*n^3*x^2 \end{aligned}$$

$$\begin{aligned}
& + 4*B^3*a^3*b*d^4*n^3*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n^3*\log(b*x + a)^3 + 288*(B^3*b^4*d^4*n^3*x^4 + 4*B^3*a*b^3*d^4*n^3*x^3 + 6*B^3*a^2*b^2*d^4*n^3*x^2 + 4*B^3*a^3*b*d^4*n^3*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n^3)*\log(d*x + c)^3 + 288*(B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3 + B^3*a^4*d^4)*\log(e)^3 + 12*(9*A*B^2*b^4*c^4 - 64*A*B^2*a*b^3*c^3*d + 216*A*B^2*a^2*b^2*c^2*d^2 - 576*A*B^2*a^3*b*c*d^3 + 415*A*B^2*a^4*d^4)*n^2 + 6*(5*(23*B^3*b^4*c^2*d^2 - 544*B^3*a*b^3*c*d^3 + 521*B^3*a^2*b^2*d^4)*n^3 + 12*(13*A*B^2*b^4*c^2*d^2 - 176*A*B^2*a*b^3*c*d^3 + 163*A*B^2*a^2*b^2*d^4)*n^2 + 72*(A^2*B*b^4*c^2*d^2 - 8*A^2*B*a*b^3*c*d^3 + 7*A^2*B*a^2*b^2*d^4)*n)*x^2 - 72*((25*B^3*b^4*d^4*n^3 + 12*A*B^2*b^4*d^4*n^2)*x^4 - (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3*d + 36*B^3*a^2*b^2*c^2*d^2 - 48*B^3*a^3*b*c*d^3)*n^3 + 4*(12*A*B^2*a*b^3*d^4*n^2 + (3*B^3*b^4*c*d^3 + 22*B^3*a*b^3*d^4)*n^3)*x^3 - 12*(A*B^2*b^4*c^4 - 4*A*B^2*a*b^3*c^3*d + 6*A*B^2*a^2*b^2*c^2*d^2 - 4*A*B^2*a^3*b*c*d^3)*n^2 + 6*(12*A*B^2*a^2*b^2*d^4*n^2 - (B^3*b^4*c^2*d^2 - 8*B^3*a*b^3*c*d^3 - 18*B^3*a^2*b^2*d^4)*n^3)*x^2 + 4*(12*A*B^2*a^3*b*d^4*n^2 + (B^3*b^4*c^3*d - 6*B^3*a*b^3*c^2*d^2 + 18*B^3*a^2*b^2*c*d^3 + 12*B^3*a^3*b*d^4)*n^3)*x + 12*(B^3*b^4*d^4*n^2*x^4 + 4*B^3*a*b^3*d^4*n^2*x^3 + 6*B^3*a^2*b^2*d^4*n^2*x^2 + 4*B^3*a^3*b*d^4*n^2*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n^2)*\log(e))*\log(b*x + a)^2 - 72*((25*B^3*b^4*d^4*n^3 + 12*A*B^2*b^4*d^4*n^2)*x^4 - (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3*d + 36*B^3*a^2*b^2*c^2*d^2 - 48*B^3*a^3*b*c*d^3)*n^3 + 4*(12*A*B^2*a*b^3*d^4*n^2 + (3*B^3*b^4*c*d^3 + 22*B^3*a*b^3*d^4)*n^3)*x^3 - 12*(A*B^2*b^4*c^4 - 4*A*B^2*a*b^3*c^3*d + 6*A*B^2*a^2*b^2*c^2*d^2 - 4*A*B^2*a^3*b*c*d^3)*n^2 + 6*(12*A*B^2*a^2*b^2*d^4*n^2 - (B^3*b^4*c^2*d^2 - 8*B^3*a*b^3*c*d^3 - 18*B^3*a^2*b^2*d^4)*n^3)*x^2 + 4*(12*A*B^2*a^3*b*d^4*n^2 + (B^3*b^4*c^3*d - 6*B^3*a*b^3*c^2*d^2 + 18*B^3*a^2*b^2*c*d^3 + 12*B^3*a^3*b*d^4)*n^3)*x + 12*(B^3*b^4*d^4*n^3*x^4 + 4*B^3*a*b^3*d^4*n^3*x^3 + 6*B^3*a^2*b^2*d^4*n^3*x^2 + 4*B^3*a^3*b*d^4*n^3*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n^3)*\log(b*x + a) + 12*(B^3*b^4*d^4*n^2*x^4 + 4*B^3*a*b^3*d^4*n^2*x^3 + 6*B^3*a^2*b^2*d^4*n^2*x^2 + 4*B^3*a^3*b*d^4*n^2*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n^2)*\log(e))*\log(d*x + c)^2 + 72*(12*A*B^2*b^4*c^4 - 48*A*B^2*a*b^3*c^3*d + 72*A*B^2*a^2*b^2*c^2*d^2 - 48*A*B^2*a^3*b*c*d^3 + 12*A*B^2*a^4*d^4 - 12*(B^3*b^4*c*d^3 - B^3*a*b^3*d^4)*n*x^3 + 6*(B^3*b^4*c^2*d^2 - 8*B^3*a*b^3*c*d^3 + 7*B^3*a^2*b^2*d^4)*n*x^2 - 4*(B^3*b^4*c^3*d - 6*B^3*a*b^3*c^2*d^2 + 18*B^3*a^2*b^2*c*d^3 - 13*B^3*a^3*b*d^4)*n*x + (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3*d + 36*B^3*a^2*b^2*c^2*d^2 - 48*B^3*a^3*b*c*d^3 + 25*B^3*a^4*d^4)*n)*\log(e)^2 + 72*(3*A^2*B*b^4*c^4 - 16*A^2*B*a*b^3*c^3*d + 36*A^2*B*a^2*b^2*c^2*d^2 - 48*A^2*B*a^3*b*c*d^3 + 25*A^2*B*a^4*d^4)*n - 4*((37*B^3*b^4*c^3*d - 456*B^3*a*b^3*c^2*d^2 + 4536*B^3*a^2*b^2*c*d^3 - 4117*B^3*a^3*b*d^4)*n^3 + 12*(7*A*B^2*b^4*c^3*d - 60*A*B^2*a*b^3*c^2*d^2 + 324*A*B^2*a^2*b^2*c*d^3 - 271*A*B^2*a^3*b*d^4)*n^2 + 72*(A^2*B*b^4*c^3*d - 6*A^2*B*a*b^3*c^2*d^2 + 18*A^2*B*a^2*b^2*c*d^3 - 13*A^2*B*a^3*b*d^4)*n)*x - 12*((415*B^3*b^4*d^4*n^3 + 300*A*B^2*b^4*d^4*n^2 + 72*A^2*B*b^4*d^4*n)*x^4 - (9*B^3*b^4*c^4 - 64*B^3*a*b^3*c^3*d + 216*B^3*a^2*b^2*c^2*d^2 - 576*B^3*a^3*b*c*d^3)*n^3 + 4*(72*A^2*B*a*b^3*d^4*n + 5*(15*B^3*b^4*c*d^3 + 68*B^3*a*b^3*d^4)*n^3 + 12*(3*A*B^2*b^4*c*d^3 + 22*A*B^2*a*b^3*d^4)*n^2)*x^3 - 12*(3*A*B^2*b^4*c^4 - 16*A*B^2*a*b^3*c^3*d + 36*A*B^2*a^2*b^2*c^2*d^2 - 48*A*B^2*a^3*b*c*d^3)*n^2 + 6*(72*A^2*B*a^2*b^2*d^4*n - (13*B^3*b^4*c^2*d^2 - 176*B^3*a*b^3*c*d^3 - 252*B^3*a^2*b^2*d^4)*n^3 - 12*(A*B^2*b^4*c^2*d^2 - 8*A*B^2*a*b^3*c*d^3 - 18*A*B^2*a^2*b^2*d^4)*n^2)*x^2 + 72*(B^3*b^4*d^4*n*x^4 + 4*B^3*a*b^3*d^4*n*x^3 + 6*B^3*a^2*b^2*d^4*n*x^2 + 4*B^3*a^3*b*d^4*n*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n)*\log(e)^2 - 72*(A^2*B*b^4*c^4 - 4*A^2*B*a*b^3*c^3*d + 6*A^2*B*a^2*b^2*c^2*d^2 - 4*A^2*B*a^3*b*c*d^3)*n + 4*(72*A^2*B*a^3*b*d^4*n + (7*B^3*b^4*c^3*d - 60*B^3*a*b^3*c^2*d^2 + 324*B^3*a^2*b^2*c*d^3 + 144*B^3*a^3*b*d^4)*n^3 + 12*(A*B^2*b^4*c^3*d - 6*A*B^2*a*b^3*c^2*d^2 + 18*A*B^2*a^2*b^2*c*d^3 + 12*A*B^2*a^3*b*d^4)*n^2)*x + 12*((25*B^3*b^4*d^4*n^2 + 12*A
\end{aligned}$$



$$\begin{aligned}
& *B^2*b^4*d^4*n)*x^4 + 4*(12*A*B^2*a*b^3*d^4*n + (3*B^3*b^4*c*d^3 + 22*B^3*a \\
& *b^3*d^4)*n^2)*x^3 - (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3*d + 36*B^3*a^2*b^2*c \\
& ^2*d^2 - 48*B^3*a^3*b*c*d^3)*n^2 + 6*(12*A*B^2*a^2*b^2*d^4*n - (B^3*b^4*c^2 \\
& *d^2 - 8*B^3*a*b^3*c*d^3 - 18*B^3*a^2*b^2*d^4)*n^2)*x^2 - 12*(A*B^2*b^4*c^4 \\
& - 4*A*B^2*a*b^3*c^3*d + 6*A*B^2*a^2*b^2*c^2*d^2 - 4*A*B^2*a^3*b*c*d^3)*n + \\
& 4*(12*A*B^2*a^3*b*d^4*n + (B^3*b^4*c^3*d - 6*B^3*a*b^3*c^2*d^2 + 18*B^3*a^2 \\
& *b^2*c*d^3 + 12*B^3*a^3*b*d^4)*n^2)*x)*\log(e))*\log(b*x + a) + 12*((415*B^3 \\
& *b^4*d^4*n^3 + 300*A*B^2*b^4*d^4*n^2 + 72*A^2*B*b^4*d^4*n)*x^4 - (9*B^3*b^4 \\
& *c^4 - 64*B^3*a*b^3*c^3*d + 216*B^3*a^2*b^2*c^2*d^2 - 576*B^3*a^3*b*c*d^3)* \\
& n^3 + 4*(72*A^2*B*a*b^3*d^4*n + 5*(15*B^3*b^4*c*d^3 + 68*B^3*a*b^3*d^4)*n^3 \\
& + 12*(3*A*B^2*b^4*c*d^3 + 22*A*B^2*a*b^3*d^4)*n^2)*x^3 - 12*(3*A*B^2*b^4*c \\
& ^4 - 16*A*B^2*a*b^3*c^3*d + 36*A*B^2*a^2*b^2*c^2*d^2 - 48*A*B^2*a^3*b*c*d^3 \\
& )*n^2 + 6*(72*A^2*B*a^2*b^2*d^4*n - (13*B^3*b^4*c^2*d^2 - 176*B^3*a*b^3*c*d \\
& ^3 - 252*B^3*a^2*b^2*d^4)*n^3 - 12*(A*B^2*b^4*c^2*d^2 - 8*A*B^2*a*b^3*c*d^3 \\
& - 18*A*B^2*a^2*b^2*d^4)*n^2)*x^2 + 72*(B^3*b^4*d^4*n^3*x^4 + 4*B^3*a*b^3*d \\
& ^4*n^3*x^3 + 6*B^3*a^2*b^2*d^4*n^3*x^2 + 4*B^3*a^3*b*d^4*n^3*x - (B^3*b^4*c \\
& ^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n^3)*\log \\
& (b*x + a)^2 + 72*(B^3*b^4*d^4*n*x^4 + 4*B^3*a*b^3*d^4*n*x^3 + 6*B^3*a^2*b^2 \\
& *d^4*n*x^2 + 4*B^3*a^3*b*d^4*n*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3 \\
& *a^2*b^2*c^2*d^2 - 4*B^3*a^3*b*c*d^3)*n)*\log(e)^2 - 72*(A^2*B*b^4*c^4 - 4* \\
& A^2*B*a*b^3*c^3*d + 6*A^2*B*a^2*b^2*c^2*d^2 - 4*A^2*B*a^3*b*c*d^3)*n + 4*(7 \\
& 2*A^2*B*a^3*b*d^4*n + (7*B^3*b^4*c^3*d - 60*B^3*a*b^3*c^2*d^2 + 324*B^3*a^2 \\
& *b^2*c*d^3 + 144*B^3*a^3*b*d^4)*n^3 + 12*(A*B^2*b^4*c^3*d - 6*A*B^2*a*b^3*c \\
& ^2*d^2 + 18*A*B^2*a^2*b^2*c*d^3 + 12*A*B^2*a^3*b*d^4)*n^2)*x + 12*((25*B^3* \\
& b^4*d^4*n^3 + 12*A*B^2*b^4*d^4*n^2)*x^4 - (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3 \\
& *d + 36*B^3*a^2*b^2*c^2*d^2 - 48*B^3*a^3*b*c*d^3)*n^3 + 4*(12*A*B^2*a*b^3*d \\
& ^4*n^2 + (3*B^3*b^4*c*d^3 + 22*B^3*a*b^3*d^4)*n^3)*x^3 - 12*(A*B^2*b^4*c^4 \\
& - 4*A*B^2*a*b^3*c^3*d + 6*A*B^2*a^2*b^2*c^2*d^2 - 4*A*B^2*a^3*b*c*d^3)*n^2 \\
& + 6*(12*A*B^2*a^2*b^2*d^4*n^2 - (B^3*b^4*c^2*d^2 - 8*B^3*a*b^3*c*d^3 - 18*B \\
& ^3*a^2*b^2*d^4)*n^3)*x^2 + 4*(12*A*B^2*a^3*b*d^4*n^2 + (B^3*b^4*c^3*d - 6*B \\
& ^3*a*b^3*c^2*d^2 + 18*B^3*a^2*b^2*c*d^3 + 12*B^3*a^3*b*d^4)*n^3)*x + 12*(B^3 \\
& *b^4*d^4*n^2*x^4 + 4*B^3*a*b^3*d^4*n^2*x^3 + 6*B^3*a^2*b^2*d^4*n^2*x^2 + 4 \\
& *B^3*a^3*b*d^4*n^2*x - (B^3*b^4*c^4 - 4*B^3*a*b^3*c^3*d + 6*B^3*a^2*b^2*c^2 \\
& *d^2 - 4*B^3*a^3*b*c*d^3)*n^2)*\log(e))*\log(b*x + a) + 12*((25*B^3*b^4*d^4*n \\
& ^2 + 12*A*B^2*b^4*d^4*n)*x^4 + 4*(12*A*B^2*a*b^3*d^4*n + (3*B^3*b^4*c*d^3 + \\
& 22*B^3*a*b^3*d^4)*n^2)*x^3 - (3*B^3*b^4*c^4 - 16*B^3*a*b^3*c^3*d + 36*B^3* \\
& a^2*b^2*c^2*d^2 - 48*B^3*a^3*b*c*d^3)*n^2 + 6*(12*A*B^2*a^2*b^2*d^4*n - (B^3 \\
& *b^4*c^2*d^2 - 8*B^3*a*b^3*c*d^3 - 18*B^3*a^2*b^2*d^4)*n^2)*x^2 - 12*(A*B^2 \\
& *b^4*c^4 - 4*A*B^2*a*b^3*c^3*d + 6*A*B^2*a^2*b^2*c^2*d^2 - 4*A*B^2*a^3*b*c \\
& *d^3)*n + 4*(12*A*B^2*a^3*b*d^4*n + (B^3*b^4*c^3*d - 6*B^3*a*b^3*c^2*d^2 + \\
& 18*B^3*a^2*b^2*c*d^3 + 12*B^3*a^3*b*d^4)*n^2)*x)*\log(e))*\log(d*x + c) + 12* \\
& (72*A^2*B*b^4*c^4 - 288*A^2*B*a*b^3*c^3*d + 432*A^2*B*a^2*b^2*c^2*d^2 - 288 \\
& *A^2*B*a^3*b*c*d^3 + 72*A^2*B*a^4*d^4 - 12*(25*(B^3*b^4*c*d^3 - B^3*a*b^3*d^4) \\
& *n^2 + 12*(A*B^2*b^4*c*d^3 - A*B^2*a*b^3*d^4)*n)*x^3 + (9*B^3*b^4*c^4 - \\
& 64*B^3*a*b^3*c^3*d + 216*B^3*a^2*b^2*c^2*d^2 - 576*B^3*a^3*b*c*d^3 + 415*B^3 \\
& *a^4*d^4)*n^2 + 6*((13*B^3*b^4*c^2*d^2 - 176*B^3*a*b^3*c*d^3 + 163*B^3*a^2 \\
& *b^2*d^4)*n^2 + 12*(A*B^2*b^4*c^2*d^2 - 8*A*B^2*a*b^3*c*d^3 + 7*A*B^2*a^2*b \\
& ^2*d^4)*n)*x^2 + 12*(3*A*B^2*b^4*c^4 - 16*A*B^2*a*b^3*c^3*d + 36*A*B^2*a^2* \\
& b^2*c^2*d^2 - 48*A*B^2*a^3*b*c*d^3 + 25*A*B^2*a^4*d^4)*n - 4*((7*B^3*b^4*c^ \\
& 3*d - 60*B^3*a*b^3*c^2*d^2 + 324*B^3*a^2*b^2*c*d^3 - 271*B^3*a^3*b*d^4)*n^2 \\
& + 12*(A*B^2*b^4*c^3*d - 6*A*B^2*a*b^3*c^2*d^2 + 18*A*B^2*a^2*b^2*c*d^3 - 1 \\
& 3*A*B^2*a^3*b*d^4)*n)*x)*\log(e))/(a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3 \\
& *c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4 + (b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b \\
& ^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7* \\
& c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*x^3 + 6*(a^2*b^7 \\
& *c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4) \\
& *x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d \\
& ^3 + a^7*b^2*d^4)*x)
\end{aligned}$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*3/(b\*x+a)\*\*5,x)

[Out] Timed out

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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log\left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)^3}{(bx+a)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(b\*x+a)^5,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^3/(b\*x + a)^5, x)

$$3.172 \quad \int \frac{1}{(ag+bgx)^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))} dx$$

**Optimal.** Leaf size=96

$$\frac{e^{\frac{A}{Bn}}(c+dx)(e(a+bx)^n(c+dx)^{-n})^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

[Out] (E^(A/(B\*n)))\*(c + d\*x)\*((e\*(a + b\*x)^n)/(c + d\*x)^n)^(-1)\*ExpIntegralEi[-((A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(B\*n))]/(B\*(b\*c - a\*d)\*g^2\*n\*(a + b\*x))

**Rubi [F]** time = 0.0998305, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))} dx = \int \frac{1}{(ag+bgx)^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))} dx$$

**Mathematica [F]** time = 0.0803575, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])), x]

[Out] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])), x]

**Maple [F]** time = 3.688, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx+ag)^2} \left( A + B \ln \left( \frac{e(bx+a)^n}{(dx+c)^n} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n))), x)

[Out]  $\int \frac{1}{(b*gx+a*g)^2/(A+B*\ln(e*(b*x+a)^n/((d*x+c)^n))}, x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="maxima")`

[Out] `integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)`

**Fricas [A]** time = 1.02981, size = 149, normalized size = 1.55

$$\frac{e^{\left(\frac{B \log(e) + A}{Bn}\right)} \log\_integral \left( \frac{(dx+c)e^{\left(-\frac{B \log(e) + A}{Bn}\right)}}{bx+a} \right)}{(Bbc - Bad)g^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas")`

[Out] `e^((B*log(e) + A)/(B*n))*log_integral((d*x + c)*e^(- (B*log(e) + A)/(B*n))/(b*x + a))/((B*b*c - B*a*d)*g^2*n)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")`

[Out] `integrate(1/((b*g*x + a*g)^2*(B*log((b*x + a)^n*e/(d*x + c)^n) + A)), x)`

### 3.173 $\int (ag + bgx)^4 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right) dx$

**Optimal.** Leaf size=180

$$\frac{g^4(a+bx)^5 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{5b} - \frac{Bg^4x(bc-ad)^4}{5d^4} + \frac{Bg^4(a+bx)^2(bc-ad)^3}{10bd^3} - \frac{Bg^4(a+bx)^3(bc-ad)^2}{15bd^2} + \frac{Bg^4(bc-ad)}{5}$$

[Out]  $-(B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(10*b*d^3) - (B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) + (B*(b*c - a*d)*g^4*(a + b*x)^4)/(20*b*d) + (B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5) + (g^4*(a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(5*b)$

**Rubi [A]** time = 0.124199, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 43}

$$\frac{g^4(a+bx)^5 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{5b} - \frac{Bg^4x(bc-ad)^4}{5d^4} + \frac{Bg^4(a+bx)^2(bc-ad)^3}{10bd^3} - \frac{Bg^4(a+bx)^3(bc-ad)^2}{15bd^2} + \frac{Bg^4(bc-ad)}{5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]$

[Out]  $-(B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(10*b*d^3) - (B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) + (B*(b*c - a*d)*g^4*(a + b*x)^4)/(20*b*d) + (B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5) + (g^4*(a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(5*b)$

#### Rule 2525

$\text{Int}[(a + \text{Log}[(c + \text{RFX})^p] * (b + (d + e*x)^m))]^n, x\_Symbol] := \text{Simp}[(d + e*x)^{m+1} * (a + b * \text{Log}[c * \text{RFX}^p])^n / (e^{m+1}), x] - \text{Dist}[(b * n * p) / (e^{m+1}), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1} * (a + b * \text{Log}[c * \text{RFX}^p])^{n-1} * D[\text{RFX}, x]) / \text{RFX}, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

$\text{Int}[(a + (b + (c + d*x)^n))]^m, x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b + (c + d\*x)^n)] /;

#### Rule 43

$\text{Int}[(a + (b + (c + d*x)^n))]^m, x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m * (c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int (ag + bgx)^4 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right) dx &= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{5b} - \frac{B \int \frac{(-bc+ad)g^5(a+bx)^4}{c+dx} dx}{5bg} \\
&= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{5b} + \frac{(B(bc-ad)g^4) \int \frac{(a+bx)^4}{c+dx} dx}{5b} \\
&= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{5b} + \frac{(B(bc-ad)g^4) \int \left( -\frac{b(bc-ad)^3}{d^4} + \frac{b(bc-ad)^2(a+bx)}{d^3} \right) dx}{5b} \\
&= -\frac{B(bc-ad)^4 g^4 x}{5d^4} + \frac{B(bc-ad)^3 g^4 (a+bx)^2}{10bd^3} - \frac{B(bc-ad)^2 g^4 (a+bx)^3}{15bd^2} + \frac{B(bc-ad) g^4 (a+bx)^4}{20bd}
\end{aligned}$$

**Mathematica [A]** time = 0.100868, size = 142, normalized size = 0.79

$$\frac{g^4 \left( (a+bx)^5 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right) - \frac{B(ad-bc)(6d^2(a+bx)^2(bc-ad)^2 + 4d^3(a+bx)^3(ad-bc) - 12bdx(bc-ad)^3 + 12(bc-ad)^4 \log(c+dx) + 3d^4(a+bx)^4)}{12d^5} \right)}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^4\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]),x]

[Out] (g^4\*(-(B\*(-(b\*c) + a\*d)\*(-12\*b\*d\*(b\*c - a\*d)^3\*x + 6\*d^2\*(b\*c - a\*d)^2\*(a + b\*x)^2 + 4\*d^3\*(-(b\*c) + a\*d)\*(a + b\*x)^3 + 3\*d^4\*(a + b\*x)^4 + 12\*(b\*c - a\*d)^4\*Log[c + d\*x]))/(12\*d^5) + (a + b\*x)^5\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])))/(5\*b)

**Maple [B]** time = 0.201, size = 2930, normalized size = 16.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^4\*(A+B\*ln(e\*(d\*x+c)/(b\*x+a))),x)

[Out] 1/10\*b^4\*e^2\*B\*g^4/d^3/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*c^5+1/15/b\*e^3\*B\*g^4\*d^3/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*a^5-1/15\*b^4\*e^3\*B\*g^4/d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*c^5-2/3\*b^2\*e^3\*B\*g^4/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a^3\*c^2+1/5\*b^4\*e^5\*B\*g^4\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^5\*c^5+2\*b\*B\*g^4/d^2\*ln(b\*(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))-d\*e)\*a^3\*c^2-2\*b^2\*B\*g^4/d^3\*ln(b\*(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))-d\*e)\*a^2\*c^3+b^3\*B\*g^4/d^4\*ln(b\*(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))-d\*e)\*c^4\*a+1/4\*e^4\*B\*g^4\*d^3/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*a^4\*c-1/3\*e^3\*B\*g^4\*d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*a^4\*c+e^5\*A\*g^4/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^5\*a^4\*d^4\*c+1/2\*e^2\*B\*g^4/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a^4\*c\*d+1/5/b\*e\*B\*g^4/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)\*a^5\*d-1/4\*b^3\*e^4\*B\*g^4/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*a\*c^4+2/3\*b\*e^3\*B\*g^4\*d/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*a^3\*c^2-1/2\*b^3\*e^2\*B\*g^4/d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a\*c^4+1/5\*b^4\*e^5\*A\*g^4/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^5\*c^5-1/5\*b^4\*B\*g^4/d^5\*ln(b\*(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))-d\*e)\*c^5-B\*g^4/d\*ln(b\*(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))-d\*e)\*a^4\*c-1/5/b\*e^5\*A\*g^4/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^5\*a^5\*d^5-1/10/b\*e^2\*B\*g^4\*d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a^5-1/20/b\*e^4\*B\*g^4\*d^4/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*a^5+1/20\*b^4\*e^4\*B\*g^4/d/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*c^5-1/5\*b^4\*e\*

$$\begin{aligned}
& B^4 g^4 / d^4 / (-e / (b x + a) * a * d + b * e / (b x + a) * c) * c^5 - e * B^4 g^4 / (-e / (b x + a) * a * d + b * e / (b \\
& * x + a) * c) * a^4 * c + 1 / 5 / b * B^4 g^4 * \ln(b * (d * e / b - e * (a * d - b * c) / b / (b x + a)) - d * e) * a^5 + 2 * b^2 * e^5 * A * g^4 / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^2 * c^3 * d^2 - b^3 * e^5 * A * g^4 / (-e / \\
& (b x + a) * a * d + b * e / (b x + a) * c)^5 * a * c^4 * d + e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) * d^4 / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^4 * c - 1 / 5 / b * e^5 * B^4 g^4 * \ln(d * e / b - e * ( \\
& a * d - b * c) / b / (b x + a)) * d^5 / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^5 - 1 / 2 * b * e^4 * B^4 g^4 * d^2 / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^4 * a^3 * c^2 + 1 / 2 * b^2 * e^4 * B^4 g^4 * d / (-e / (b x \\
& + a) * a * d + b * e / (b x + a) * c)^4 * a^2 * c^3 + b^2 * e^2 * B^4 g^4 / d / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^2 * a^2 * c^3 - 2 * b^2 * e * B^4 g^4 / d^2 / (-e / (b x + a) * a * d + b * e / (b x + a) * c) * a^2 * c^3 + b^3 \\
& * e * B^4 g^4 / d^3 / (-e / (b x + a) * a * d + b * e / (b x + a) * c) * a * c^4 - 2 * b * e^5 * A * g^4 / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^3 * c^2 * d^3 + 2 * b * e * B^4 g^4 / d / (-e / (b x + a) * a * d + b * e / (b x + a) \\
& * c) * a^3 * c^2 + 1 / 3 * b^3 * e^3 * B^4 g^4 / d / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^3 * c^4 * a - 2 * b * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) * d^3 / (-e / (b x + a) * a * d + b * e / (b x + a) * c) \\
& )^5 * a^3 * c^2 + 2 * b^2 * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) * d^2 / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^2 * c^3 - b^3 * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) \\
& ) * d / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a * c^4 + 1 / 5 / b * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) * d^5 / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^10 / (b x + a)^5 + 1 / 5 * b^9 * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) / d^5 / (-e / (b x + a) * a * d + b * e / (b x + a) * c) \\
& )^5 * c^10 / (b x + a)^5 - 252 / 5 * b^4 * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^5 * c^5 / (b x + a)^5 - 2 * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) * d^4 / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^9 * c / (b x + a)^5 + 9 * b * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) * d^3 / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^8 * c^2 / (b x + a)^5 - 24 * b^2 * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) * d^2 / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^7 * c^3 / (b x + a)^5 + 42 * b^3 * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) * d / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^6 * c^4 / (b x + a)^5 + 42 * b^5 * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) / d / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^4 * c^6 / (b x + a)^5 - 24 * b^6 * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) / d^2 / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^3 * c^7 / (b x + a)^5 + 9 * b^7 * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) / d^3 / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a^2 * c^8 / (b x + a)^5 - 2 * b^8 * e^5 * B^4 g^4 * \ln(d * e / b - e * (a * d - b * c) / b / (b x + a)) / d^4 / (-e / (b x + a) * a * d + b * e / (b x + a) * c)^5 * a * c^9 / (b x + a)^5
\end{aligned}$$

**Maxima [B]** time = 1.29752, size = 836, normalized size = 4.64

$$\frac{1}{5} A b^4 g^4 x^5 + A a b^3 g^4 x^4 + 2 A a^2 b^2 g^4 x^3 + 2 A a^3 b g^4 x^2 + \left( x \log \left( \frac{d e x}{b x + a} + \frac{c e}{b x + a} \right) - \frac{a \log(b x + a)}{b} + \frac{c \log(d x + c)}{d} \right) B a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(d\*x+c)/(b\*x+a))),x, algorithm="maxima")

[Out] 1/5\*A\*b^4\*g^4\*x^5 + A\*a\*b^3\*g^4\*x^4 + 2\*A\*a^2\*b^2\*g^4\*x^3 + 2\*A\*a^3\*b\*g^4\*x^2 + (x\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) - a\*log(b\*x + a)/b + c\*log(d\*x + c)/d)\*B\*a^4\*g^4 + 2\*(x^2\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) + a^2\*log(b\*x + a)/b^2 - c^2\*log(d\*x + c)/d^2 + (b\*c - a\*d)\*x/(b\*d))\*B\*a^3\*b\*g^4 + (2\*x^3\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) - 2\*a^3\*log(b\*x + a)/b^3 + 2\*c^3\*log(d\*x + c)/d^3 + ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2))\*B\*a^2\*b^2\*g^4 + 1/6\*(6\*x^4\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) + 6\*a^4\*log(b\*x + a)/b^4 - 6\*c^4\*log(d\*x + c)/d^4 + (2\*(b^3\*c\*d^2 - a\*b^2\*d^3)\*x^3 - 3\*(b^3\*c^2\*d - a^2\*b\*d^3)\*x^2 + 6\*(b^3\*c^3 - a^3\*d^3)\*x)/(b^3\*d^3))\*B\*a\*b^3\*g^4 + 1/60\*(12\*x^5\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) - 12\*a^5\*log(b\*x + a)/b^5 + 12\*c^5\*log(d\*x + c)/d^5 + (3\*(b^4\*c\*d^3 - a\*b^3\*d^4)\*x^4 - 4\*(b^4\*c^2\*d^2 - a^2\*b^2\*d^4)\*x^3 + 6\*(b^4\*c^3\*d - a^3\*b\*d^4)\*x^2 - 12\*(b^4\*c^4 - a^4\*d^4)\*x)/(b^4\*d^4))\*B\*b^4\*g^4 + A\*a^4\*g^4\*x

**Fricas [B]** time = 1.35211, size = 910, normalized size = 5.06

$$12 Ab^5 d^5 g^4 x^5 - 12 Ba^5 d^5 g^4 \log(bx + a) + 3 (Bb^5 cd^4 + (20A - B)ab^4 d^5) g^4 x^4 - 4 (Bb^5 c^2 d^3 - 5 Bab^4 cd^4 - 2(15A - 2B)a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(d\*x+c)/(b\*x+a))),x, algorithm="fricas")

[Out] 1/60\*(12\*A\*b^5\*d^5\*g^4\*x^5 - 12\*B\*a^5\*d^5\*g^4\*log(b\*x + a) + 3\*(B\*b^5\*c\*d^4 + (20\*A - B)\*a\*b^4\*d^5)\*g^4\*x^4 - 4\*(B\*b^5\*c^2\*d^3 - 5\*B\*a\*b^4\*c\*d^4 - 2\*(15\*A - 2\*B)\*a^2\*b^3\*d^5)\*g^4\*x^3 + 6\*(B\*b^5\*c^3\*d^2 - 5\*B\*a\*b^4\*c^2\*d^3 + 10\*B\*a^2\*b^3\*c\*d^4 + 2\*(10\*A - 3\*B)\*a^3\*b^2\*d^5)\*g^4\*x^2 - 12\*(B\*b^5\*c^4\*d - 5\*B\*a\*b^4\*c^3\*d^2 + 10\*B\*a^2\*b^3\*c^2\*d^3 - 10\*B\*a^3\*b^2\*c\*d^4 - (5\*A - 4\*B)\*a^4\*b\*d^5)\*g^4\*x + 12\*(B\*b^5\*c^5 - 5\*B\*a\*b^4\*c^4\*d + 10\*B\*a^2\*b^3\*c^3\*d^2 - 10\*B\*a^3\*b^2\*c^2\*d^3 + 5\*B\*a^4\*b\*c\*d^4)\*g^4\*log(d\*x + c) + 12\*(B\*b^5\*d^5\*g^4\*x^5 + 5\*B\*a\*b^4\*d^5\*g^4\*x^4 + 10\*B\*a^2\*b^3\*d^5\*g^4\*x^3 + 10\*B\*a^3\*b^2\*d^5\*g^4\*x^2 + 5\*B\*a^4\*b\*d^5\*g^4\*x)\*log((d\*e\*x + c\*e)/(b\*x + a))/(b\*d^5)

**Sympy [B]** time = 8.53226, size = 993, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*4\*(A+B\*ln(e\*(d\*x+c)/(b\*x+a))),x)

[Out] A\*b\*\*4\*g\*\*4\*x\*\*5/5 - B\*a\*\*5\*g\*\*4\*log(x + (B\*a\*\*6\*d\*\*5\*g\*\*4/b + 5\*B\*a\*\*5\*c\*d\*\*4\*g\*\*4 - 10\*B\*a\*\*4\*b\*c\*\*2\*d\*\*3\*g\*\*4 + 10\*B\*a\*\*3\*b\*\*2\*c\*\*3\*d\*\*2\*g\*\*4 - 5\*B\*a\*\*2\*b\*\*3\*c\*\*4\*d\*g\*\*4 + B\*a\*b\*\*4\*c\*\*5\*g\*\*4)/(B\*a\*\*5\*d\*\*5\*g\*\*4 + 5\*B\*a\*\*4\*b\*c\*d\*\*4\*g\*\*4 - 10\*B\*a\*\*3\*b\*\*2\*c\*\*2\*d\*\*3\*g\*\*4 + 10\*B\*a\*\*2\*b\*\*3\*c\*\*3\*d\*\*2\*g\*\*4 - 5\*B\*a\*b\*\*4\*c\*\*4\*d\*g\*\*4 + B\*b\*\*5\*c\*\*5\*g\*\*4))/(5\*b) + B\*c\*g\*\*4\*(5\*a\*\*4\*d\*\*4 - 10\*a\*\*3\*b\*c\*d\*\*3 + 10\*a\*\*2\*b\*\*2\*c\*\*2\*d\*\*2 - 5\*a\*b\*\*3\*c\*\*3\*d + b\*\*4\*c\*\*4)\*log(x + (6\*B\*a\*\*5\*c\*d\*\*4\*g\*\*4 - 10\*B\*a\*\*4\*b\*c\*\*2\*d\*\*3\*g\*\*4 + 10\*B\*a\*\*3\*b\*\*2\*c\*\*3\*d\*\*2\*g\*\*4 - 5\*B\*a\*\*2\*b\*\*3\*c\*\*4\*d\*g\*\*4 + B\*a\*b\*\*4\*c\*\*5\*g\*\*4 - B\*a\*c\*g\*\*4\*(5\*a\*\*4\*d\*\*4 - 10\*a\*\*3\*b\*c\*d\*\*3 + 10\*a\*\*2\*b\*\*2\*c\*\*2\*d\*\*2 - 5\*a\*b\*\*3\*c\*\*3\*d + b\*\*4\*c\*\*4) + B\*b\*c\*\*2\*g\*\*4\*(5\*a\*\*4\*d\*\*4 - 10\*a\*\*3\*b\*c\*d\*\*3 + 10\*a\*\*2\*b\*\*2\*c\*\*2\*d\*\*2 - 5\*a\*b\*\*3\*c\*\*3\*d + b\*\*4\*c\*\*4)/d)/(B\*a\*\*5\*d\*\*5\*g\*\*4 + 5\*B\*a\*\*4\*b\*c\*d\*\*4\*g\*\*4 - 10\*B\*a\*\*3\*b\*\*2\*c\*\*2\*d\*\*3\*g\*\*4 + 10\*B\*a\*\*2\*b\*\*3\*c\*\*3\*d\*\*2\*g\*\*4 - 5\*B\*a\*b\*\*4\*c\*\*4\*d\*g\*\*4 + B\*b\*\*5\*c\*\*5\*g\*\*4))/(5\*d\*\*5) + (B\*a\*\*4\*g\*\*4\*x + 2\*B\*a\*\*3\*b\*g\*\*4\*x\*\*2 + 2\*B\*a\*\*2\*b\*\*2\*g\*\*4\*x\*\*3 + B\*a\*b\*\*3\*g\*\*4\*x\*\*4 + B\*b\*\*4\*g\*\*4\*x\*\*5/5)\*log(e\*(c + d\*x)/(a + b\*x)) + x\*\*4\*(20\*A\*a\*b\*\*3\*d\*g\*\*4 - B\*a\*b\*\*3\*d\*g\*\*4 + B\*b\*\*4\*c\*g\*\*4)/(20\*d) + x\*\*3\*(30\*A\*a\*\*2\*b\*\*2\*d\*\*2\*g\*\*4 - 4\*B\*a\*\*2\*b\*\*2\*d\*\*2\*g\*\*4 + 5\*B\*a\*b\*\*3\*c\*d\*g\*\*4 - B\*b\*\*4\*c\*\*2\*g\*\*4)/(15\*d\*\*2) + x\*\*2\*(20\*A\*a\*\*3\*b\*d\*\*3\*g\*\*4 - 6\*B\*a\*\*3\*b\*d\*\*3\*g\*\*4 + 10\*B\*a\*\*2\*b\*\*2\*c\*d\*\*2\*g\*\*4 - 5\*B\*a\*b\*\*3\*c\*\*2\*d\*g\*\*4 + B\*b\*\*4\*c\*\*3\*g\*\*4)/(10\*d\*\*3) + x\*(5\*A\*a\*\*4\*d\*\*4\*g\*\*4 - 4\*B\*a\*\*4\*d\*\*4\*g\*\*4 + 10\*B\*a\*\*3\*b\*c\*d\*\*3\*g\*\*4 - 10\*B\*a\*\*2\*b\*\*2\*c\*\*2\*d\*\*2\*g\*\*4 + 5\*B\*a\*b\*\*3\*c\*\*3\*d\*g\*\*4 - B\*b\*\*4\*c\*\*4\*g\*\*4)/(5\*d\*\*4)

**Giac [B]** time = 89.5295, size = 635, normalized size = 3.53

$$-\frac{Ba^5 g^4 \log(bx + a)}{5b} + \frac{1}{5} (Ab^4 g^4 + Bb^4 g^4) x^5 + \frac{(Bb^4 c g^4 + 20 Aab^3 d g^4 + 19 Bab^3 d g^4) x^4}{20d} - \frac{(Bb^4 c^2 g^4 - 5 Bab^3 c d g^4 - 30 Aa$$

15d



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(d\*x+c)/(b\*x+a))),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/5*B*a^5*g^4*\log(b*x + a)/b + 1/5*(A*b^4*g^4 + B*b^4*g^4)*x^5 + 1/20*(B*b^4*c*g^4 + 20*A*a*b^3*d*g^4 + 19*B*a*b^3*d*g^4)*x^4/d - 1/15*(B*b^4*c^2*g^4 - 5*B*a*b^3*c*d*g^4 - 30*A*a^2*b^2*d^2*g^4 - 26*B*a^2*b^2*d^2*g^4)*x^3/d^2 \\ & + 1/5*(B*b^4*g^4*x^5 + 5*B*a*b^3*g^4*x^4 + 10*B*a^2*b^2*g^4*x^3 + 10*B*a^3*b*g^4*x^2 + 5*B*a^4*g^4*x)*\log((d*x + c)/(b*x + a)) + 1/10*(B*b^4*c^3*g^4 - 5*B*a*b^3*c^2*d*g^4 + 10*B*a^2*b^2*c*d^2*g^4 + 20*A*a^3*b*d^3*g^4 + 14*B*a^3*b*d^3*g^4)*x^2/d^3 - 1/5*(B*b^4*c^4*g^4 - 5*B*a*b^3*c^3*d*g^4 + 10*B*a^2*b^2*c^2*d^2*g^4 - 10*B*a^3*b*c*d^3*g^4 - 5*A*a^4*d^4*g^4 - B*a^4*d^4*g^4)*x/d^4 + 1/5*(B*b^4*c^5*g^4 - 5*B*a*b^3*c^4*d*g^4 + 10*B*a^2*b^2*c^3*d^2*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 5*B*a^4*c*d^4*g^4)*\log(d*x + c)/d^5 \end{aligned}$$

$$3.174 \quad \int (ag + bgx)^3 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right) dx$$

**Optimal.** Leaf size=149

$$\frac{g^3(a+bx)^4 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{4b} + \frac{Bg^3x(bc-ad)^3}{4d^3} - \frac{Bg^3(a+bx)^2(bc-ad)^2}{8bd^2} - \frac{Bg^3(bc-ad)^4 \log(c+dx)}{4bd^4} + \frac{Bg^3(a+bx)^4}{12b}$$

[Out] (B\*(b\*c - a\*d)^3\*g^3\*x)/(4\*d^3) - (B\*(b\*c - a\*d)^2\*g^3\*(a + b\*x)^2)/(8\*b\*d^2) + (B\*(b\*c - a\*d)\*g^3\*(a + b\*x)^3)/(12\*b\*d) - (B\*(b\*c - a\*d)^4\*g^3\*Log[c + d\*x])/(4\*b\*d^4) + (g^3\*(a + b\*x)^4\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(4\*b)

**Rubi [A]** time = 0.101091, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 43}

$$\frac{g^3(a+bx)^4 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{4b} + \frac{Bg^3x(bc-ad)^3}{4d^3} - \frac{Bg^3(a+bx)^2(bc-ad)^2}{8bd^2} - \frac{Bg^3(bc-ad)^4 \log(c+dx)}{4bd^4} + \frac{Bg^3(a+bx)^4}{12b}$$

Antiderivative was successfully verified.

[In] Int[(a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

[Out] (B\*(b\*c - a\*d)^3\*g^3\*x)/(4\*d^3) - (B\*(b\*c - a\*d)^2\*g^3\*(a + b\*x)^2)/(8\*b\*d^2) + (B\*(b\*c - a\*d)\*g^3\*(a + b\*x)^3)/(12\*b\*d) - (B\*(b\*c - a\*d)^4\*g^3\*Log[c + d\*x])/(4\*b\*d^4) + (g^3\*(a + b\*x)^4\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(4\*b)

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned}
\int (ag + bgx)^3 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right) dx &= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{4b} - \frac{B \int \frac{(-bc+ad)g^4(a+bx)^3}{c+dx} dx}{4bg} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{4b} + \frac{(B(bc-ad)g^3) \int \frac{(a+bx)^3}{c+dx} dx}{4b} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{4b} + \frac{(B(bc-ad)g^3) \int \left( \frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)(c+dx)}{d^2} \right) dx}{4b} \\
&= \frac{B(bc-ad)^3 g^3 x}{4d^3} - \frac{B(bc-ad)^2 g^3 (a+bx)^2}{8bd^2} + \frac{B(bc-ad)g^3 (a+bx)^3}{12bd} - \frac{B(bc-ad)(c+dx)(a+bx)^3}{4bd^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0800694, size = 120, normalized size = 0.81

$$\frac{g^3 \left( (a+bx)^4 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right) + \frac{B(bc-ad)(3d^2(a+bx)^2(ad-bc)+6bdx(bc-ad)^2-6(bc-ad)^3 \log(c+dx)+2d^3(a+bx)^3)}{6d^4} \right)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]),x]

[Out] (g^3\*((B\*(b\*c - a\*d)\*(6\*b\*d\*(b\*c - a\*d)^2\*x + 3\*d^2\*(-(b\*c) + a\*d)\*(a + b\*x)^2 + 2\*d^3\*(a + b\*x)^3 - 6\*(b\*c - a\*d)^3\*Log[c + d\*x]))/(6\*d^4) + (a + b\*x)^4\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(4\*b)

**Maple [B]** time = 0.158, size = 2191, normalized size = 14.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^3\*(A+B\*ln(e\*(d\*x+c)/(b\*x+a))),x)

[Out] 2\*e^4\*B\*g^3\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^3/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*a^7/(b\*x+a)^4\*c+3/2\*b\*e^4\*B\*g^3\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*a^2\*c^2-b^2\*e^4\*B\*g^3\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*a\*c^3-1/4\*b^7\*e^4\*B\*g^3\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/d^4/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*c^8/(b\*x+a)^4-35/2\*b^3\*e^4\*B\*g^3\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*a^4\*c^4/(b\*x+a)^4-1/4/b\*e^4\*B\*g^3\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^4/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*a^8/(b\*x+a)^4+14\*b^4\*e^4\*B\*g^3\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/d/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*a^3\*c^5/(b\*x+a)^4-7\*b^5\*e^4\*B\*g^3\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*a^2\*c^6/(b\*x+a)^4+14\*b^2\*e^4\*B\*g^3\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*a^5\*c^3/(b\*x+a)^4\*d-7\*b\*e^4\*B\*g^3\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*a^6\*c^2/(b\*x+a)^4+2\*b^6\*e^4\*B\*g^3\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/d^3/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*a\*c^7/(b\*x+a)^4+1/4\*b^3\*e^4\*A\*g^3/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4\*c^4+1/4\*b^3\*B\*g^3/d^4\*ln(b\*(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))-d\*e)\*c^4-B\*g^3/d\*ln(b\*(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))-d\*e)\*a^3\*c-e\*B\*g^3/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)\*a^3\*c-1/3\*b^2\*e^3\*B\*g^3/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*a\*c^3-3/4\*b\*e^2\*B\*g^3/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a^2\*c^2+1/4\*b^3\*e^4\*B\*g^3\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^4

$$\begin{aligned}
& *c^4 - b^2 * B * g^3 / d^3 * \ln(b * (d * e / b - e * (a * d - b * c)) / b / (b * x + a)) - d * e * c^3 * a + 3 / 2 * b * B * g^3 \\
& / d^2 * \ln(b * (d * e / b - e * (a * d - b * c)) / b / (b * x + a)) - d * e * a^2 * c^2 + 1 / 4 * b * e^4 * A * g^3 / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^4 * a^4 * d^4 + 1 / 4 * b^3 * e * B * g^3 / d^3 / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c) * c^4 + 1 / 4 * b * e * B * g^3 / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c) * a^4 * d + 1 / 4 * b * e^4 * B * g^3 * \ln(d * e / b - e * (a * d - b * c)) / b / (b * x + a)) / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^4 * a^4 * d^4 + 3 / 2 * b * e * B * g^3 / d / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c) * a^2 * c^2 - b^2 * e^4 * A * g^3 / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^4 * c^3 * a * d + 1 / 2 * b * e^3 * B * g^3 / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^3 * a^2 * c^2 * d + 1 / 2 * b^2 * e^2 * B * g^3 / d / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^2 * c^3 * a - e^4 * B * g^3 * \ln(d * e / b - e * (a * d - b * c)) / b / (b * x + a)) * d^3 / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^4 * a^3 * c + 3 / 2 * b * e^4 * A * g^3 / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^4 * a^2 * c^2 * d^2 + 1 / 12 * b * e^3 * B * g^3 * d^3 / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^3 * a^4 - 1 / 8 * b * e^2 * B * g^3 * d^2 / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^2 * a^4 - 1 / 8 * b^3 * e^2 * B * g^3 / d^2 / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^2 * c^4 + 1 / 4 * b * B * g^3 * \ln(b * (d * e / b - e * (a * d - b * c)) / b / (b * x + a)) - d * e * a^4 - b^2 * e * B * g^3 / d^2 / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c) * c^3 * a + 1 / 12 * b^3 * e^3 * B * g^3 / d / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^3 * c^4 - 1 / 3 * e^3 * B * g^3 * d^2 / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^3 * a^3 * c + 1 / 2 * e^2 * B * g^3 * d / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^2 * a^3 * c - e^4 * A * g^3 / (-e / (b * x + a) * a * d + b * e / (b * x + a) * c)^4 * a^3 * d^3 * c
\end{aligned}$$

**Maxima [B]** time = 1.1676, size = 589, normalized size = 3.95

$$\frac{1}{4} A b^3 g^3 x^4 + A a b^2 g^3 x^3 + \frac{3}{2} A a^2 b g^3 x^2 + \left( x \log\left(\frac{d e x}{b x + a} + \frac{c e}{b x + a}\right) - \frac{a \log(b x + a)}{b} + \frac{c \log(d x + c)}{d} \right) B a^3 g^3 + \frac{3}{2} \left( x^2 \log\left(\frac{d e x}{b x + a} + \frac{c e}{b x + a}\right) - \frac{a \log(b x + a)}{b} + \frac{c \log(d x + c)}{d} \right) B a^2 g^3 x + \frac{3}{2} \left( x \log\left(\frac{d e x}{b x + a} + \frac{c e}{b x + a}\right) - \frac{a \log(b x + a)}{b} + \frac{c \log(d x + c)}{d} \right) B a g^3 x^2 + \frac{3}{2} \left( x^2 \log\left(\frac{d e x}{b x + a} + \frac{c e}{b x + a}\right) - \frac{a \log(b x + a)}{b} + \frac{c \log(d x + c)}{d} \right) B g^3 x^3 + \frac{3}{2} \left( x \log\left(\frac{d e x}{b x + a} + \frac{c e}{b x + a}\right) - \frac{a \log(b x + a)}{b} + \frac{c \log(d x + c)}{d} \right) B g^3 x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*(d\*x+c)/(b\*x+a))),x, algorithm="maxima")

[Out] 1/4\*A\*b^3\*g^3\*x^4 + A\*a\*b^2\*g^3\*x^3 + 3/2\*A\*a^2\*b\*g^3\*x^2 + (x\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) - a\*log(b\*x + a)/b + c\*log(d\*x + c)/d)\*B\*a^3\*g^3 + 3/2\*(x^2\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) + a^2\*log(b\*x + a)/b^2 - c^2\*log(d\*x + c)/d^2 + (b\*c - a\*d)\*x/(b\*d))\*B\*a^2\*b\*g^3 + 1/2\*(2\*x^3\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) - 2\*a^3\*log(b\*x + a)/b^3 + 2\*c^3\*log(d\*x + c)/d^3 + ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2))\*B\*a\*b^2\*g^3 + 1/24\*(6\*x^4\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) + 6\*a^4\*log(b\*x + a)/b^4 - 6\*c^4\*log(d\*x + c)/d^4 + (2\*(b^3\*c\*d^2 - a\*b^2\*d^3)\*x^3 - 3\*(b^3\*c^2\*d - a^2\*b\*d^3)\*x^2 + 6\*(b^3\*c^3 - a^3\*d^3)\*x)/(b^3\*d^3))\*B\*b^3\*g^3 + A\*a^3\*g^3\*x

**Fricas [B]** time = 1.15715, size = 664, normalized size = 4.46

$$6 A b^4 d^4 g^3 x^4 - 6 B a^4 d^4 g^3 \log(b x + a) + 2 (B b^4 c d^3 + (12 A - B) a b^3 d^4) g^3 x^3 - 3 (B b^4 c^2 d^2 - 4 B a b^3 c d^3 - 3 (4 A - B) a^2 b^2 d^4) g^3 x^2 + 6 (B b^4 c^3 d - 4 B a^3 b^3 c^2 d^2 + 6 B a^2 b^2 c^3 d + (4 A - 3 B) a^3 b^2 d^4) g^3 x - 6 (B b^4 c^4 - 4 B a^3 b^3 c^3 d + 6 B a^2 b^2 c^2 d^2 - 4 B a^3 b^3 c^2 d^3) g^3 \log(d x + c) + 6 (B b^4 d^4 g^3 x^4 + 4 B a^3 b^3 d^4 g^3 x^3 + 6 B a^2 b^2 d^4 g^3 x^2 + 4 B a^3 b^3 d^4 g^3 x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*(d\*x+c)/(b\*x+a))),x, algorithm="fricas")

[Out] 1/24\*(6\*A\*b^4\*d^4\*g^3\*x^4 - 6\*B\*a^4\*d^4\*g^3\*log(b\*x + a) + 2\*(B\*b^4\*c\*d^3 + (12\*A - B)\*a\*b^3\*d^4)\*g^3\*x^3 - 3\*(B\*b^4\*c^2\*d^2 - 4\*B\*a\*b^3\*c\*d^3 - 3\*(4\*A - B)\*a^2\*b^2\*d^4)\*g^3\*x^2 + 6\*(B\*b^4\*c^3\*d - 4\*B\*a\*b^3\*c^2\*d^2 + 6\*B\*a^2\*b^2\*c^3\*d + (4\*A - 3\*B)\*a^3\*b^2\*d^4)\*g^3\*x - 6\*(B\*b^4\*c^4 - 4\*B\*a\*b^3\*c^3\*d + 6\*B\*a^2\*b^2\*c^2\*d^2 - 4\*B\*a^3\*b^3\*c^2\*d^3)\*g^3\*log(d\*x + c) + 6\*(B\*b^4\*d^4\*g^3\*x^4 + 4\*B\*a^3\*b^3\*d^4\*g^3\*x^3 + 6\*B\*a^2\*b^2\*d^4\*g^3\*x^2 + 4\*B\*a^3\*b^3\*d^4\*g^3\*x)

$$x) \cdot \log((d \cdot e^x + c \cdot e)/(b \cdot x + a)) / (b \cdot d^4)$$

**Sympy [B]** time = 5.8668, size = 719, normalized size = 4.83

$$\frac{Ab^3g^3x^4}{4} - \frac{Ba^4g^3 \log\left(x + \frac{Ba^5d^4g^3 + 4Ba^4cd^3g^3 - 6Ba^3bc^2d^2g^3 + 4Ba^2b^2c^3dg^3 - Bab^3c^4g^3}{Ba^4d^4g^3 + 4Ba^3bcd^3g^3 - 6Ba^2b^2c^2d^2g^3 + 4Bab^3c^3dg^3 - Bb^4c^4g^3}\right)}{4b} + \frac{Bcg^3(2ad - bc)(2a^2d^2 - 2abcd + b^2c^2)}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*3\*(A+B\*ln(e\*(d\*x+c)/(b\*x+a))), x)

[Out]  $A \cdot b^3 \cdot g^3 \cdot x^4 / 4 - B \cdot a^4 \cdot g^3 \cdot \log(x + (B \cdot a^5 \cdot d^4 \cdot g^3 / b + 4 \cdot B \cdot a^4 \cdot c \cdot d^3 \cdot g^3 - 6 \cdot B \cdot a^3 \cdot b \cdot c^2 \cdot d^2 \cdot g^3 + 4 \cdot B \cdot a^2 \cdot b^2 \cdot c^3 \cdot d \cdot g^3 - B \cdot a \cdot b^3 \cdot c^4 \cdot g^3) / (B \cdot a^4 \cdot d^4 \cdot g^3 + 4 \cdot B \cdot a^3 \cdot b \cdot c \cdot d^3 \cdot g^3 - 6 \cdot B \cdot a^2 \cdot b^2 \cdot c^2 \cdot d^2 \cdot g^3 + 4 \cdot B \cdot a \cdot b^3 \cdot c^3 \cdot d \cdot g^3 - B \cdot b^4 \cdot c^4 \cdot g^3)) / (4 \cdot b) + B \cdot c \cdot g^3 \cdot (2 \cdot a \cdot d - b \cdot c) \cdot (2 \cdot a^2 \cdot d^2 - 2 \cdot a \cdot b \cdot c \cdot d + b^2 \cdot c^2) \cdot \log(x + (5 \cdot B \cdot a^4 \cdot c \cdot d^3 \cdot g^3 - 6 \cdot B \cdot a^3 \cdot b \cdot c^2 \cdot d^2 \cdot g^3 + 4 \cdot B \cdot a^2 \cdot b^2 \cdot c^3 \cdot d \cdot g^3 - B \cdot a \cdot b^3 \cdot c^4 \cdot g^3 - B \cdot a \cdot c \cdot g^3 \cdot (2 \cdot a \cdot d - b \cdot c) \cdot (2 \cdot a^2 \cdot d^2 - 2 \cdot a \cdot b \cdot c \cdot d + b^2 \cdot c^2) + B \cdot b \cdot c^2 \cdot g^3 \cdot (2 \cdot a \cdot d - b \cdot c) \cdot (2 \cdot a^2 \cdot d^2 - 2 \cdot a \cdot b \cdot c \cdot d + b^2 \cdot c^2) / d) / (B \cdot a^4 \cdot d^4 \cdot g^3 + 4 \cdot B \cdot a^3 \cdot b \cdot c \cdot d^3 \cdot g^3 - 6 \cdot B \cdot a^2 \cdot b^2 \cdot c^2 \cdot d^2 \cdot g^3 + 4 \cdot B \cdot a \cdot b^3 \cdot c^3 \cdot d \cdot g^3 - B \cdot b^4 \cdot c^4 \cdot g^3)) / (4 \cdot d^4) + (B \cdot a^3 \cdot g^3 \cdot x + 3 \cdot B \cdot a^2 \cdot b \cdot g^3 \cdot x^2 / 2 + B \cdot a \cdot b^2 \cdot g^3 \cdot x^3 + B \cdot b^3 \cdot g^3 \cdot x^4 / 4) \cdot \log(e \cdot (c + d \cdot x) / (a + b \cdot x)) + x^3 \cdot (12 \cdot A \cdot a \cdot b^2 \cdot d \cdot g^3 - B \cdot a \cdot b^2 \cdot d \cdot g^3 + B \cdot b^3 \cdot c \cdot g^3) / (12 \cdot d) + x^2 \cdot (12 \cdot A \cdot a^2 \cdot b \cdot d^2 \cdot g^3 - 3 \cdot B \cdot a^2 \cdot b \cdot d^2 \cdot g^3 + 4 \cdot B \cdot a \cdot b^2 \cdot c \cdot d \cdot g^3 - B \cdot b^3 \cdot c^2 \cdot g^3) / (8 \cdot d^2) + x \cdot (4 \cdot A \cdot a^3 \cdot d^3 \cdot g^3 - 3 \cdot B \cdot a^3 \cdot d^3 \cdot g^3 + 6 \cdot B \cdot a^2 \cdot b \cdot c \cdot d^2 \cdot g^3 - 4 \cdot B \cdot a \cdot b^2 \cdot c^2 \cdot d \cdot g^3 + B \cdot b^3 \cdot c^3 \cdot g^3) / (4 \cdot d^3)$

**Giac [B]** time = 20.9284, size = 460, normalized size = 3.09

$$-\frac{Ba^4g^3 \log(bx + a)}{4b} + \frac{1}{4} (Ab^3g^3 + Bb^3g^3)x^4 + \frac{(Bb^3cg^3 + 12Aab^2dg^3 + 11Bab^2dg^3)x^3}{12d} + \frac{1}{4} (Bb^3g^3x^4 + 4Bab^2g^3x^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*(d\*x+c)/(b\*x+a))), x, algorithm="giac")

[Out]  $-1/4 \cdot B \cdot a^4 \cdot g^3 \cdot \log(b \cdot x + a) / b + 1/4 \cdot (A \cdot b^3 \cdot g^3 + B \cdot b^3 \cdot g^3) \cdot x^4 + 1/12 \cdot (B \cdot b^3 \cdot c \cdot g^3 + 12 \cdot A \cdot a \cdot b^2 \cdot d \cdot g^3 + 11 \cdot B \cdot a \cdot b^2 \cdot d \cdot g^3) \cdot x^3 / d + 1/4 \cdot (B \cdot b^3 \cdot g^3 \cdot x^4 + 4 \cdot B \cdot a \cdot b^2 \cdot g^3 \cdot x^3 + 6 \cdot B \cdot a^2 \cdot b \cdot g^3 \cdot x^2 + 4 \cdot B \cdot a^3 \cdot g^3 \cdot x) \cdot \log((d \cdot x + c) / (b \cdot x + a)) - 1/8 \cdot (B \cdot b^3 \cdot c^2 \cdot g^3 - 4 \cdot B \cdot a \cdot b^2 \cdot c \cdot d \cdot g^3 - 12 \cdot A \cdot a^2 \cdot b \cdot d^2 \cdot g^3 - 9 \cdot B \cdot a^2 \cdot b \cdot d^2 \cdot g^3) \cdot x^2 / d^2 + 1/4 \cdot (B \cdot b^3 \cdot c^3 \cdot g^3 - 4 \cdot B \cdot a \cdot b^2 \cdot c^2 \cdot d \cdot g^3 + 6 \cdot B \cdot a^2 \cdot b \cdot c \cdot d^2 \cdot g^3 + 4 \cdot A \cdot a^3 \cdot d^3 \cdot g^3 + B \cdot a^3 \cdot d^3 \cdot g^3) \cdot x / d^3 - 1/4 \cdot (B \cdot b^3 \cdot c^4 \cdot g^3 - 4 \cdot B \cdot a \cdot b^2 \cdot c^3 \cdot d \cdot g^3 + 6 \cdot B \cdot a^2 \cdot b \cdot c^2 \cdot d^2 \cdot g^3 - 4 \cdot B \cdot a^3 \cdot c \cdot d^3 \cdot g^3) \cdot \log(-d \cdot x - c) / d^4$

$$3.175 \quad \int (ag + bgx)^2 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right) dx$$

**Optimal.** Leaf size=118

$$\frac{g^2(a+bx)^3 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{3b} - \frac{Bg^2x(bc-ad)^2}{3d^2} + \frac{Bg^2(bc-ad)^3 \log(c+dx)}{3bd^3} + \frac{Bg^2(a+bx)^2(bc-ad)}{6bd}$$

[Out]  $-(B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (B*(b*c - a*d)*g^2*(a + b*x)^2)/(6*b*d) + (B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(3*b)$

**Rubi [A]** time = 0.0806846, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 43}

$$\frac{g^2(a+bx)^3 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{3b} - \frac{Bg^2x(bc-ad)^2}{3d^2} + \frac{Bg^2(bc-ad)^3 \log(c+dx)}{3bd^3} + \frac{Bg^2(a+bx)^2(bc-ad)}{6bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]), x]$

[Out]  $-(B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (B*(b*c - a*d)*g^2*(a + b*x)^2)/(6*b*d) + (B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(3*b)$

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned}\int (ag + bgx)^2 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right) dx &= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{3b} - \frac{B \int \frac{(-bc+ad)g^3(a+bx)^2}{c+dx} dx}{3bg} \\ &= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{3b} + \frac{(B(bc-ad)g^2) \int \frac{(a+bx)^2}{c+dx} dx}{3b} \\ &= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{3b} + \frac{(B(bc-ad)g^2) \int \left( -\frac{b(bc-ad)}{d^2} + \frac{b(a+bx)}{d} \right) dx}{3b} \\ &= -\frac{B(bc-ad)^2g^2x}{3d^2} + \frac{B(bc-ad)g^2(a+bx)^2}{6bd} + \frac{B(bc-ad)^3g^2 \log(c+dx)}{3bd^3} + \dots\end{aligned}$$

**Mathematica [A]** time = 0.0506713, size = 99, normalized size = 0.84

$$\frac{g^2 \left( \frac{B(bc-ad)(d(a^2d+4abd x+b^2x(dx-2c))+2(bc-ad)^2 \log(c+dx))}{2d^3} + (a+bx)^3 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right) \right)}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

[Out] (g^2\*((B\*(b\*c - a\*d)\*(d\*(a^2\*d + 4\*a\*b\*d\*x + b^2\*x\*(-2\*c + d\*x)) + 2\*(b\*c - a\*d)^2\*Log[c + d\*x]))/(2\*d^3) + (a + b\*x)^3\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/3\*b)

**Maple [B]** time = 0.159, size = 1537, normalized size = 13.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2\*(A+B\*ln(e\*(d\*x+c)/(b\*x+a))), x)

[Out] -1/3/b\*e^3\*B\*g^2\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^3/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*a^3+1/3/b\*e^3\*B\*g^2\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^3/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*a^6/(b\*x+a)^3+1/3\*b^5\*e^3\*B\*g^2\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/d^3/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*c^6/(b\*x+a)^3-b\*e^3\*B\*g^2\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*c^2\*a\*d-20/3\*b^2\*e^3\*B\*g^2\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*a^3/(b\*x+a)^3\*c^3-B\*g^2/d\*ln(b\*(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))-d\*e)\*a^2\*c+1/3\*b^2\*e^3\*A\*g^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*c^3-1/3\*b^2\*B\*g^2/d^3\*ln(b\*(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))-d\*e)\*c^3-e\*B\*g^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)\*a^2\*c-2\*b^4\*e^3\*B\*g^2\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*a\*c^5/(b\*x+a)^3+5\*b\*e^3\*B\*g^2\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*a^4/(b\*x+a)^3\*c^2+1/2\*e^2\*B\*g^2\*d/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a^2\*c+5\*b^3\*e^3\*B\*g^2\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/d/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*a^2\*c^4/(b\*x+a)^3+e^3\*B\*g^2\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*a^2\*c+b\*e\*B\*g^2/d/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)\*c^2\*a-b\*e^3\*A\*g^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*a\*c^2\*d-1/3\*b^2\*e\*B\*g^2/d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)\*c^3+1/3/b\*e\*B\*g^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)\*a^3\*d-1/3/b\*e^3\*A\*g^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^3\*a^3\*d^3-1/6/b\*e^2\*B\*g^2\*d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a^3-1/2\*b\*e^2\*B\*g^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2

$$c^2 a^2 c^2 + 1/6 b^2 e^2 B g^2 / d / (-e / (b x + a) a^2 d + b e / (b x + a) c)^2 c^3 + b B g^2 / d^2 \ln(b (d e / b - e (a d - b c)) / b (b x + a)) - d e a^2 c^2 + 1/3 b^2 e^3 B g^2 \ln(d e / b - e (a d - b c)) / b (b x + a) / (-e / (b x + a) a^2 d + b e / (b x + a) c)^3 c^3 + e^3 A g^2 / (-e / (b x + a) a^2 d + b e / (b x + a) c)^3 a^2 c d^2 + 1/3 b B g^2 \ln(b (d e / b - e (a d - b c)) / b (b x + a)) - d e a^3 - 2 e^3 B g^2 \ln(d e / b - e (a d - b c)) / b (b x + a) d^2 / (-e / (b x + a) a^2 d + b e / (b x + a) c)^3 a^5 / (b x + a)^3 c$$

**Maxima [B]** time = 1.20332, size = 375, normalized size = 3.18

$$\frac{1}{3} A b^2 g^2 x^3 + A a b g^2 x^2 + \left( x \log\left(\frac{d e x}{b x + a} + \frac{c e}{b x + a}\right) - \frac{a \log(b x + a)}{b} + \frac{c \log(d x + c)}{d} \right) B a^2 g^2 + \left( x^2 \log\left(\frac{d e x}{b x + a} + \frac{c e}{b x + a}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*a*b*g^2 + 1/6*(2*x^3*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

**Fricas [B]** time = 1.06778, size = 467, normalized size = 3.96

$$\frac{2 A b^3 d^3 g^2 x^3 - 2 B a^3 d^3 g^2 \log(b x + a) + (B b^3 c d^2 + (6 A - B) a b^2 d^3) g^2 x^2 - 2 (B b^3 c^2 d - 3 B a b^2 c d^2 - (3 A - 2 B) a^2 b d^3) g^2 x - 2 A a^3 d^3}{6 b d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 - 2*B*a^3*d^3*g^2*log(b*x + a) + (B*b^3*c*d^2 + (6*A - B)*a*b^2*d^3)*g^2*x^2 - 2*(B*b^3*c^2*d - 3*B*a*b^2*c*d^2 - (3*A - 2*B)*a^2*b*d^3)*g^2*x + 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*log(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*g^2*x)*log((d*e*x + c*e)/(b*x + a)))/(b*d^3)
```

**Sympy [B]** time = 4.00286, size = 503, normalized size = 4.26

$$\frac{A b^2 g^2 x^3}{3} - \frac{B a^3 g^2 \log\left(x + \frac{B a^4 d^3 g^2 + 3 B a^3 c d^2 g^2 - 3 B a^2 b c^2 d g^2 + B a b^2 c^3 g^2}{B a^3 d^3 g^2 + 3 B a^2 b c d^2 g^2 - 3 B a b^2 c^2 d g^2 + B b^3 c^3 g^2}\right)}{3 b} + \frac{B c g^2 (3 a^2 d^2 - 3 a b c d + b^2 c^2) \log\left(x + \frac{4 B a^3 c d^2 g^2 - 3 B a^2 b c^2 d g^2}{B a^3 d^3 g^2 + 3 B a^2 b c d^2 g^2 - 3 B a b^2 c^2 d g^2 + B b^3 c^3 g^2}\right)}{3 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)/(b*x+a))),x)
```

```
[Out] A*b**2*g**2*x**3/3 - B*a**3*g**2*log(x + (B*a**4*d**3*g**2/b + 3*B*a**3*c*d**2*g**2 - 3*B*a**2*b*c**2*d*g**2 + B*a*b**2*c**3*g**2)/(B*a**3*d**3*g**2 + 3*B*a**2*b*c*d**2*g**2 - 3*B*a*b**2*c**2*d*g**2 + B*b**3*c**3*g**2))
```



$$\begin{aligned} & (3Ba^2bcd^2g^2 - 3Bab^2c^2dg^2 + Bb^3c^3g^2)/(3b) \\ & + Bc^2g^2(3a^2d^2 - 3abc^2d + b^2c^2)\log(x + (4Ba^3cd^2g^2 - 3Ba^2b^2cd^2g^2 + B^2ab^2c^3g^2 - B^2ac^3g^2(3a^2d^2 - 3abc^2d + b^2c^2) + B^2b^2c^2g^2(3a^2d^2 - 3abc^2d + b^2c^2)/d)/(Ba^3d^3g^2 + 3Ba^2bcd^2g^2 - 3Bab^2c^2dg^2 + Bb^3c^3g^2))/(3d^3) + (Ba^2g^2x + Babg^2x^2 + Bb^2g^2x^3/3)\log(e(c + dx)/(a + bx)) + x^2(6Aabd^2g^2 - B^2abd^2g^2 + B^2b^2c^2g^2)/(6d) + x(3Aa^2d^2g^2 - 2Ba^2d^2g^2 + 3B^2abd^2g^2 - B^2b^2c^2g^2)/(3d^2) \end{aligned}$$

**Giac [B]** time = 4.9494, size = 305, normalized size = 2.58

$$-\frac{Ba^3g^2\log(bx+a)}{3b} + \frac{1}{3}(Ab^2g^2 + Bb^2g^2)x^3 + \frac{(Bb^2cg^2 + 6Aabd^2g^2 + 5Babd^2g^2)x^2}{6d} + \frac{1}{3}(Bb^2g^2x^3 + 3Babg^2x^2 + 3Ba^2g^2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(d\*x+c)/(b\*x+a))),x, algorithm="giac")

[Out]  $-1/3Ba^3g^2\log(bx+a)/b + 1/3(Ab^2g^2 + Bb^2g^2)x^3 + 1/6(Bb^2g^2x^3 + 3Babg^2x^2 + 3Ba^2g^2x)\log((dx+c)/(bx+a)) - 1/3(Bb^2c^2g^2 - 3B^2abd^2g^2 - 3Aa^2d^2g^2 - B^2a^2d^2g^2)x/d^2 + 1/3(Bb^2c^3g^2 - 3B^2abd^2g^2 + 3B^2a^2cd^2g^2)\log(dx+c)/d^3$

$$3.176 \quad \int (ag + bgx) \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right) dx$$

**Optimal.** Leaf size=81

$$\frac{g(a+bx)^2 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{2b} - \frac{Bg(bc-ad)^2 \log(c+dx)}{2bd^2} + \frac{Bgx(bc-ad)}{2d}$$

[Out] (B\*(b\*c - a\*d)\*g\*x)/(2\*d) - (B\*(b\*c - a\*d)^2\*g\*Log[c + d\*x])/(2\*b\*d^2) + (g\*(a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(2\*b)

**Rubi [A]** time = 0.0547031, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$ , Rules used = {2525, 12, 43}

$$\frac{g(a+bx)^2 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{2b} - \frac{Bg(bc-ad)^2 \log(c+dx)}{2bd^2} + \frac{Bgx(bc-ad)}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

[Out] (B\*(b\*c - a\*d)\*g\*x)/(2\*d) - (B\*(b\*c - a\*d)^2\*g\*Log[c + d\*x])/(2\*b\*d^2) + (g\*(a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(2\*b)

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
\int (ag + bgx) \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right) dx &= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{2b} - \frac{B \int \frac{(bc-ad)g^2(-a-bx)}{c+dx} dx}{2bg} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{2b} - \frac{(B(bc-ad)g) \int \frac{-a-bx}{c+dx} dx}{2b} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{2b} - \frac{(B(bc-ad)g) \int \left( -\frac{b}{d} + \frac{bc-ad}{d(c+dx)} \right) dx}{2b} \\
&= \frac{B(bc-ad)gx}{2d} - \frac{B(bc-ad)^2 g \log(c+dx)}{2bd^2} + \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{2b}
\end{aligned}$$

**Mathematica [A]** time = 0.0369662, size = 69, normalized size = 0.85

$$\frac{g \left( (a+bx)^2 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right) + \frac{B(bc-ad)((ad-bc) \log(c+dx)+bdx)}{d^2} \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

[Out] (g\*((B\*(b\*c - a\*d)\*(b\*d\*x + -(b\*c) + a\*d)\*Log[c + d\*x]))/d^2 + (a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(2\*b)

**Maple [B]** time = 0.161, size = 951, normalized size = 11.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)\*(A+B\*ln(e\*(d\*x+c)/(b\*x+a))), x)

[Out] 1/2/b\*e^2\*A\*g/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a^2\*d^2-e^2\*A\*g/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a\*d\*c+1/2\*b\*e^2\*A\*g/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*c^2+1/2/b\*B\*g\*ln(b\*(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))-d\*e)\*a^2-B\*g/d\*ln(b\*(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))-d\*e)\*a\*c+1/2\*b\*B\*g/d^2\*ln(b\*(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))-d\*e)\*c^2+1/2/b\*e\*B\*g/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)\*a^2\*d-e\*B\*g/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)\*a\*c+1/2\*b\*e\*B\*g/d/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)\*c^2+1/2/b\*e^2\*B\*g\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a^2-e^2\*B\*g\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a\*c-1/2/b\*e^2\*B\*g\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a^4/(b\*x+a)^2+2\*e^2\*B\*g\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*c^3/(b\*x+a)^2\*c-3\*b\*e^2\*B\*g\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a^2/(b\*x+a)^2\*c^2+2\*b^2\*e^2\*B\*g\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/d/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*a/(b\*x+a)^2\*c^3+1/2\*b\*e^2\*B\*g\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*c^2-1/2\*b^3\*e^2\*B\*g\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/d^2/(-e/(b\*x+a)\*a\*d+b\*e/(b\*x+a)\*c)^2\*c^4/(b\*x+a)^2

**Maxima [A]** time = 1.16557, size = 193, normalized size = 2.38

$$\frac{1}{2} Abgx^2 + \left( x \log \left( \frac{dex}{bx+a} + \frac{ce}{bx+a} \right) - \frac{a \log(bx+a)}{b} + \frac{c \log(dx+c)}{d} \right) Bag + \frac{1}{2} \left( x^2 \log \left( \frac{dex}{bx+a} + \frac{ce}{bx+a} \right) + \frac{a^2 \log(b)}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(d\*x+c)/(b\*x+a))),x, algorithm="maxima")

[Out]  $\frac{1}{2}A*b*g*x^2 + (x*\log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*\log(b*x + a)/b + c*\log(d*x + c)/d)*B*a*g + \frac{1}{2}*(x^2*\log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*\log(b*x + a)/b^2 - c^2*\log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*B*b*g + A*a*g*x$

**Fricas [A]** time = 1.0672, size = 278, normalized size = 3.43

$$\frac{Ab^2d^2gx^2 - Ba^2d^2g \log(bx + a) + (Bb^2cd + (2A - B)abd^2)gx - (Bb^2c^2 - 2Babcd)g \log(dx + c) + (Bb^2d^2gx^2 + 2Babd^2g)}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(d\*x+c)/(b\*x+a))),x, algorithm="fricas")

[Out]  $\frac{1}{2}*(A*b^2*d^2*g*x^2 - B*a^2*d^2*g*\log(b*x + a) + (B*b^2*c*d + (2*A - B)*a*b*d^2)*g*x - (B*b^2*c^2 - 2*B*a*b*c*d)*g*\log(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*a*b*d^2*g*x)*\log((d*e*x + c*e)/(b*x + a)))/(b*d^2)$

**Sympy [B]** time = 2.54912, size = 257, normalized size = 3.17

$$\frac{Abgx^2}{2} - \frac{Ba^2g \log\left(x + \frac{Ba^3d^2g + 2Ba^2cdg - Babc^2g}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{2b} + \frac{Bcg(2ad - bc) \log\left(x + \frac{3Ba^2cdg - Babc^2g - Bacg(2ad - bc) + \frac{Bbc^2g(2ad - bc)}{d}}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{2d^2} + (Bagx +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*ln(e\*(d\*x+c)/(b\*x+a))),x)

[Out]  $A*b*g*x**2/2 - B*a**2*g*\log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*b) + B*c*g*(2*a*d - b*c)*\log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/(2*d**2) + (B*a*g*x + B*b*g*x**2/2)*\log(e*(c + d*x)/(a + b*x)) + x*(2*A*a*d*g - B*a*d*g + B*b*c*g)/(2*d)$

**Giac [A]** time = 2.27699, size = 153, normalized size = 1.89

$$-\frac{Ba^2g \log(bx + a)}{2b} + \frac{1}{2}(Abg + Bbg)x^2 + \frac{1}{2}(Bbgx^2 + 2Bagx) \log\left(\frac{dx + c}{bx + a}\right) + \frac{(Bbcg + 2Aadg + Badg)x}{2d} - \frac{(Bbc^2g - 2B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(d\*x+c)/(b\*x+a))),x, algorithm="giac")

[Out]  $-1/2*B*a^2*g*\log(b*x + a)/b + 1/2*(A*b*g + B*b*g)*x^2 + 1/2*(B*b*g*x^2 + 2*B*a*g*x)*\log((d*x + c)/(b*x + a)) + 1/2*(B*b*c*g + 2*A*a*d*g + B*a*d*g)*x/d - 1/2*(B*b*c^2*g - 2*B*a*c*d*g)*\log(-d*x - c)/d^2$

$$3.177 \quad \int \frac{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)}{ag+bgx} dx$$

**Optimal.** Leaf size=81

$$-\frac{BPolyLog\left(2, \frac{bc-ad}{d(a+bx)} + 1\right)}{bg} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{bg}$$

[Out] -((Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])))/(b\*g) - (B\*PolyLog[2, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/(b\*g)

**Rubi [A]** time = 0.211494, antiderivative size = 122, normalized size of antiderivative = 1.51, number of steps used = 10, number of rules used = 8, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$ , Rules used = {2524, 12, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{BPolyLog\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg} + \frac{\log(ag + bgx)\left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{bg} - \frac{B \log(ag + bgx) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{bg} + \frac{B \log^2(g(a + bx))}{2bg}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])/(a\*g + b\*g\*x), x]

[Out] (B\*Log[g\*(a + b\*x)]^2)/(2\*b\*g) - (B\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*Log[a\*g + b\*g\*x])/(b\*g) + ((A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])\*Log[a\*g + b\*g\*x])/(b\*g) - (B\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g)

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*(RFX\_), x\_Symbol] :> With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFX, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_.))^(q\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{ag + bgx} dx &= \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag + bgx)}{bg} - \frac{B \int \frac{(a+bx)\left(\frac{de}{a+bx} - \frac{be(c+dx)}{(a+bx)^2}\right) \log(ag+bgx)}{e(c+dx)} dx}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag + bgx)}{bg} - \frac{B \int \frac{(a+bx)\left(\frac{de}{a+bx} - \frac{be(c+dx)}{(a+bx)^2}\right) \log(ag+bgx)}{c+dx} dx}{beg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag + bgx)}{bg} - \frac{B \int \left(-\frac{be \log(ag+bgx)}{a+bx} + \frac{de \log(ag+bgx)}{c+dx}\right) dx}{beg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag + bgx)}{bg} + \frac{B \int \frac{\log(ag+bgx)}{a+bx} dx}{g} - \frac{(Bd) \int \frac{\log(ag+bgx)}{c+dx} dx}{bg} \\
&= -\frac{B \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} + \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag + bgx)}{bg} + B \int \frac{\log\left(\frac{bg(c+dx)}{bcg-adg}\right)}{ag + bgx} dx \\
&= -\frac{B \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} + \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag + bgx)}{bg} + \frac{B \operatorname{Subst}\left(\int \frac{\log(x)}{x} dx, \frac{bg(c+dx)}{bcg-adg}\right)}{bg} \\
&= \frac{B \log^2(g(a + bx))}{2bg} - \frac{B \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} + \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag + bgx)}{bg} - \frac{B \operatorname{Subst}\left(\int \frac{\log(x)}{x} dx, \frac{bg(c+dx)}{bcg-adg}\right)}{bg}
\end{aligned}$$

**Mathematica [A]** time = 0.0427005, size = 95, normalized size = 1.17

$$\frac{\log(g(a + bx)) \left( 2 \left( B \log\left(\frac{e(c+dx)}{a+bx}\right) - B \log\left(\frac{b(c+dx)}{bc-ad}\right) + A \right) + B \log(g(a + bx)) \right) - 2B \operatorname{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right)}{2bg}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x), x]
```

[Out]  $(\text{Log}[g*(a + b*x)]*(B*\text{Log}[g*(a + b*x)] + 2*(A - B*\text{Log}[(b*(c + d*x))/(b*c - a*d)] + B*\text{Log}[(e*(c + d*x))/(a + b*x)])) - 2*B*\text{PolyLog}[2, (d*(a + b*x))/(-b*c + a*d)]/(2*b*g)$

**Maple [B]** time = 0.057, size = 419, normalized size = 5.2

$$-\frac{Aad}{bg(ad-bc)} \ln\left(b\left(\frac{de}{b} - \frac{e(ad-bc)}{b(bx+a)}\right) - de\right) + \frac{Ac}{g(ad-bc)} \ln\left(b\left(\frac{de}{b} - \frac{e(ad-bc)}{b(bx+a)}\right) - de\right) - \frac{Bad}{bg(ad-bc)} \text{dilog}\left(-\frac{1}{de}\left(b\left(\frac{de}{b} - \frac{e(ad-bc)}{b(bx+a)}\right) - de\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g), x)`

[Out]  $-1/b/g/(a*d-b*c)*A*\ln(b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)*a*d+1/g/(a*d-b*c)*A*\ln(b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)*c-1/b/g/(a*d-b*c)*B*\text{dilog}(-b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)/d/e)*a*d+1/g/(a*d-b*c)*B*\text{dilog}(-b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)/d/e)*c-1/b/g/(a*d-b*c)*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*\ln(-b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)/d/e)*a*d+1/g/(a*d-b*c)*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*\ln(-b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)/d/e)*c$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$B\left(\frac{\log(bx+a)\log(dx+c)}{bg} - \int -\frac{bdx \log(e) + bc \log(e) - (2bdx + bc + ad) \log(bx+a)}{b^2d^2gx^2 + abcg + (b^2cg + abd^2g)x} dx\right) + \frac{A \log(bgx + ag)}{bg}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g), x, algorithm="maxima")`

[Out]  $B*(\log(b*x + a)*\log(d*x + c)/(b*g) - \text{integrate}(-(b*d*x*\log(e) + b*c*\log(e) - (2*b*d*x + b*c + a*d)*\log(b*x + a))/(b^2*d^2*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)) + A*\log(b*g*x + a*g)/(b*g)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B \log\left(\frac{dex+ce}{bx+a}\right) + A}{bgx + ag}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g), x, algorithm="fricas")`

[Out] `integral((B*log((d*e*x + c*e)/(b*x + a)) + A)/(b*g*x + a*g), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g), x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log\left(\frac{(dx+c)e}{bx+a}\right) + A}{bgx + ag} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g), x, algorithm="giac")

[Out] integrate((B\*log((d\*x + c)\*e/(b\*x + a)) + A)/(b\*g\*x + a\*g), x)



$$3.178 \quad \int \frac{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^2} dx$$

Optimal. Leaf size=64

$$-\frac{A-B}{bg^2(a+bx)} - \frac{B(c+dx) \log\left(\frac{e(c+dx)}{a+bx}\right)}{g^2(a+bx)(bc-ad)}$$

[Out]  $-\frac{(A - B)}{(b * g^2 * (a + b * x))} - \frac{(B * (c + d * x) * \text{Log}[(e * (c + d * x)) / (a + b * x)])}{(b * c - a * d) * g^2 * (a + b * x)}$

**Rubi [A]** time = 0.0767233, antiderivative size = 101, normalized size of antiderivative = 1.58, number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(c+dx)}{a+bx}\right) + A}{bg^2(a+bx)} + \frac{Bd \log(a+bx)}{bg^2(bc-ad)} - \frac{Bd \log(c+dx)}{bg^2(bc-ad)} + \frac{B}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])/(a\*g + b\*g\*x)^2, x]

[Out]  $\frac{B}{(b * g^2 * (a + b * x))} + \frac{(B * d * \text{Log}[a + b * x])}{(b * (b * c - a * d) * g^2)} - \frac{(B * d * \text{Log}[c + d * x])}{(b * (b * c - a * d) * g^2)} - \frac{(A + B * \text{Log}[(e * (c + d * x)) / (a + b * x)])}{(b * g^2 * (a + b * x))}$

#### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag + bgx)^2} dx &= -\frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{bg^2(a + bx)} + \frac{B \int \frac{-bc+ad}{g(a+bx)^2(c+dx)} dx}{bg} \\
&= -\frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{bg^2(a + bx)} - \frac{(B(bc - ad)) \int \frac{1}{(a+bx)^2(c+dx)} dx}{bg^2} \\
&= -\frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{bg^2(a + bx)} - \frac{(B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)} + \frac{d^2}{(bc-ad)^2(c+dx)}\right) dx}{bg^2} \\
&= \frac{B}{bg^2(a + bx)} + \frac{Bd \log(a + bx)}{b(bc - ad)g^2} - \frac{Bd \log(c + dx)}{b(bc - ad)g^2} - \frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{bg^2(a + bx)}
\end{aligned}$$

**Mathematica [A]** time = 0.0593092, size = 86, normalized size = 1.34

$$\frac{-(bc - ad) \left( B \log\left(\frac{e(c+dx)}{a+bx}\right) + A - B \right) - Bd(a + bx) \log(c + dx) + Bd(a + bx) \log(a + bx)}{bg^2(a + bx)(bc - ad)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])/(a\*g + b\*g\*x)^2, x]

[Out] (B\*d\*(a + b\*x)\*Log[a + b\*x] - B\*d\*(a + b\*x)\*Log[c + d\*x] - (b\*c - a\*d)\*(A - B + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(b\*(b\*c - a\*d)\*g^2\*(a + b\*x))

**Maple [B]** time = 0.051, size = 520, normalized size = 8.1

$$\frac{Ad^2a}{b(ad - bc)^2 g^2} - \frac{Adc}{(ad - bc)^2 g^2} - \frac{Aa^2d^2}{b(ad - bc)^2 g^2 (bx + a)} + 2 \frac{Aadc}{(ad - bc)^2 g^2 (bx + a)} - \frac{bAc^2}{(ad - bc)^2 g^2 (bx + a)} + \frac{Bd^2a}{b(ad - bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g)^2, x)

[Out] 1/b/(a\*d-b\*c)^2/g^2\*A\*d^2\*a-1/(a\*d-b\*c)^2/g^2\*A\*d\*c-1/b/(a\*d-b\*c)^2/g^2\*A/(b\*x+a)\*a^2\*d^2+2/(a\*d-b\*c)^2/g^2\*A/(b\*x+a)\*a\*d\*c-b/(a\*d-b\*c)^2/g^2\*A/(b\*x+a)\*c^2+1/b/(a\*d-b\*c)^2/g^2\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^2\*a-1/(a\*d-b\*c)^2/g^2\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d\*c-1/b/(a\*d-b\*c)^2/g^2\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)\*a^2\*d^2+2/(a\*d-b\*c)^2/g^2\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)\*a\*d\*c-b/(a\*d-b\*c)^2/g^2\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)\*c^2+1/b/(a\*d-b\*c)^2/g^2\*B/(b\*x+a)\*a^2\*d^2-2/(a\*d-b\*c)^2/g^2\*B/(b\*x+a)\*a\*d\*c+b/(a\*d-b\*c)^2/g^2\*B/(b\*x+a)\*c^2-1/b/(a\*d-b\*c)^2/g^2\*B\*d^2\*a+1/(a\*d-b\*c)^2/g^2\*B\*d\*c

**Maxima [B]** time = 1.11624, size = 181, normalized size = 2.83

$$-B \left( \frac{\log\left(\frac{dex}{bx+a} + \frac{ce}{bx+a}\right)}{b^2g^2x + abg^2} - \frac{1}{b^2g^2x + abg^2} - \frac{d \log(bx + a)}{(b^2c - abd)g^2} + \frac{d \log(dx + c)}{(b^2c - abd)g^2} \right) - \frac{A}{b^2g^2x + abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g)^2,x, algorithm="maxima")

[Out]  $-B*(\log(d*e*x/(b*x + a) + c*e/(b*x + a)))/(b^2*g^2*x + a*b*g^2) - 1/(b^2*g^2*x + a*b*g^2) - d*\log(b*x + a)/((b^2*c - a*b*d)*g^2) + d*\log(d*x + c)/((b^2*c - a*b*d)*g^2) - A/(b^2*g^2*x + a*b*g^2)$

**Fricas [A]** time = 1.01728, size = 177, normalized size = 2.77

$$\frac{(A - B)bc - (A - B)ad + (Bbdx + Bbc)\log\left(\frac{dex+ce}{bx+a}\right)}{(b^3c - ab^2d)g^2x + (ab^2c - a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g)^2,x, algorithm="fricas")

[Out]  $-((A - B)*b*c - (A - B)*a*d + (B*b*d*x + B*b*c)*\log((d*e*x + c*e)/(b*x + a)))/((b^3*c - a*b^2*d)*g^2*x + (a*b^2*c - a^2*b*d)*g^2)$

**Sympy [B]** time = 1.8312, size = 231, normalized size = 3.61

$$-\frac{B \log\left(\frac{e(c+dx)}{a+bx}\right)}{abg^2 + b^2g^2x} + \frac{Bd \log\left(x + \frac{\frac{Ba^2d^3}{ad-bc} + \frac{2Babcd^2}{ad-bc} + Ba d^2 - \frac{Bb^2c^2d}{ad-bc} + Bbcd}{2Bbd^2}\right)}{bg^2(ad - bc)} - \frac{Bd \log\left(x + \frac{\frac{Ba^2d^3}{ad-bc} - \frac{2Babcd^2}{ad-bc} + Ba d^2 + \frac{Bb^2c^2d}{ad-bc} + Bbcd}{2Bbd^2}\right)}{bg^2(ad - bc)} - \frac{A - B}{abg^2 + b^2g^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g)\*\*2,x)

[Out]  $-B*\log(e*(c + d*x)/(a + b*x))/(a*b*g**2 + b**2*g**2*x) + B*d*\log(x + (-B*a*a**2*d**3/(a*d - b*c) + 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 - B*b**2*c**2*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) - B*d*\log(x + (B*a**2*d**3/(a*d - b*c) - 2*B*a*b*c*d**2/(a*d - b*c) + B*a*d**2 + B*b**2*c**2*d/(a*d - b*c) + B*b*c*d)/(2*B*b*d**2))/(b*g**2*(a*d - b*c)) - (A - B)/(a*b*g**2 + b**2*g**2*x)$

**Giac [A]** time = 1.32796, size = 151, normalized size = 2.36

$$\frac{Bd \log(bx + a)}{b^2cg^2 - abdg^2} - \frac{Bd \log(dx + c)}{b^2cg^2 - abdg^2} - \frac{B \log\left(\frac{dx+c}{bx+a}\right)}{b^2g^2x + abg^2} - \frac{A}{b^2g^2x + abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g)^2,x, algorithm="giac")

[Out]  $B*d*\log(b*x + a)/(b^2*c*g^2 - a*b*d*g^2) - B*d*\log(d*x + c)/(b^2*c*g^2 - a*b*d*g^2) - B*\log((d*x + c)/(b*x + a))/(b^2*g^2*x + a*b*g^2) - A/(b^2*g^2*x + a*b*g^2)$

$$3.179 \quad \int \frac{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^3} dx$$

**Optimal.** Leaf size=144

$$-\frac{B \log\left(\frac{e(c+dx)}{a+bx}\right) + A}{2bg^3(a+bx)^2} - \frac{Bd^2 \log(a+bx)}{2bg^3(bc-ad)^2} + \frac{Bd^2 \log(c+dx)}{2bg^3(bc-ad)^2} - \frac{Bd}{2bg^3(a+bx)(bc-ad)} + \frac{B}{4bg^3(a+bx)^2}$$

[Out] B/(4\*b\*g^3\*(a + b\*x)^2) - (B\*d)/(2\*b\*(b\*c - a\*d)\*g^3\*(a + b\*x)) - (B\*d^2\*Log[a + b\*x])/(2\*b\*(b\*c - a\*d)^2\*g^3) + (B\*d^2\*Log[c + d\*x])/(2\*b\*(b\*c - a\*d)^2\*g^3) - (A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])/(2\*b\*g^3\*(a + b\*x)^2)

**Rubi [A]** time = 0.104102, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(c+dx)}{a+bx}\right) + A}{2bg^3(a+bx)^2} - \frac{Bd^2 \log(a+bx)}{2bg^3(bc-ad)^2} + \frac{Bd^2 \log(c+dx)}{2bg^3(bc-ad)^2} - \frac{Bd}{2bg^3(a+bx)(bc-ad)} + \frac{B}{4bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])/(a\*g + b\*g\*x)^3,x]

[Out] B/(4\*b\*g^3\*(a + b\*x)^2) - (B\*d)/(2\*b\*(b\*c - a\*d)\*g^3\*(a + b\*x)) - (B\*d^2\*Log[a + b\*x])/(2\*b\*(b\*c - a\*d)^2\*g^3) + (B\*d^2\*Log[c + d\*x])/(2\*b\*(b\*c - a\*d)^2\*g^3) - (A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])/(2\*b\*g^3\*(a + b\*x)^2)

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 44

```
Int[((a_) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)}{(ag + bgx)^3} dx &= -\frac{A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)}{2bg^3(a + bx)^2} + \frac{B \int \frac{-bc+ad}{g^2(a+bx)^3(c+dx)} dx}{2bg} \\
&= -\frac{A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)}{2bg^3(a + bx)^2} - \frac{(B(bc - ad)) \int \frac{1}{(a+bx)^3(c+dx)} dx}{2bg^3} \\
&= -\frac{A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)}{2bg^3(a + bx)^2} - \frac{(B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^3} - \frac{bd}{(bc-ad)^2(a+bx)^2} + \frac{bd^2}{(bc-ad)^3(a+bx)} - \frac{bd^3}{(bc-ad)^4}\right) dx}{2bg^3} \\
&= \frac{B}{4bg^3(a + bx)^2} - \frac{Bd}{2b(bc - ad)g^3(a + bx)} - \frac{Bd^2 \log(a + bx)}{2b(bc - ad)^2g^3} + \frac{Bd^2 \log(c + dx)}{2b(bc - ad)^2g^3} - \frac{A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)}{2bg^3(a + bx)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0983657, size = 128, normalized size = 0.89

$$\frac{(bc - ad) \left( -2aAd + 2B(bc - ad) \log\left(\frac{e^{(c+dx)}}{a+bx}\right) + 3aBd + 2Abc - bBc + 2bBdx \right) - 2Bd^2(a + bx)^2 \log(c + dx) + 2Bd^2(a + bx)^2 \log\left(\frac{e^{(c+dx)}}{a+bx}\right)}{4bg^3(a + bx)^2(bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]))/(a\*g + b\*g\*x)^3,x]

[Out] -(2\*B\*d^2\*(a + b\*x)^2\*Log[a + b\*x] - 2\*B\*d^2\*(a + b\*x)^2\*Log[c + d\*x] + (b\*c - a\*d)\*(2\*A\*b\*c - b\*B\*c - 2\*a\*A\*d + 3\*a\*B\*d + 2\*b\*B\*d\*x + 2\*B\*(b\*c - a\*d)\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(4\*b\*(b\*c - a\*d)^2\*g^3\*(a + b\*x)^2)

**Maple [B]** time = 0.053, size = 753, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g)^3,x)

[Out] -1/2/(a\*d-b\*c)^3/g^3\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^2\*c+1/2/b/(a\*d-b\*c)^3/g^3\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^3\*a+1/2\*b/(a\*d-b\*c)^3/g^3\*B\*d/(b\*x+a)\*c^2+3/2/(a\*d-b\*c)^3/g^3\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)^2\*a^2\*d^2\*c-3/2\*b/(a\*d-b\*c)^3/g^3\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)^2\*a\*d\*c^2+1/4/b/(a\*d-b\*c)^3/g^3\*B/(b\*x+a)^2\*a^3\*d^3+1/2/b/(a\*d-b\*c)^3/g^3\*B\*d^3/(b\*x+a)\*a^2+1/2\*b^2/(a\*d-b\*c)^3/g^3\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)^2\*c^3-1/2/b/(a\*d-b\*c)^3/g^3\*A/(b\*x+a)^2\*a^3\*d^3-3/4/b/(a\*d-b\*c)^3/g^3\*B\*d^3\*a-1/4\*b^2/(a\*d-b\*c)^3/g^3\*B/(b\*x+a)^2\*c^3+1/2\*b^2/(a\*d-b\*c)^3/g^3\*A/(b\*x+a)^2\*c^3+1/2/b/(a\*d-b\*c)^3/g^3\*A\*d^3\*a-1/2/(a\*d-b\*c)^3/g^3\*A\*d^2\*c+3/4\*b/(a\*d-b\*c)^3/g^3\*B/(b\*x+a)^2\*a\*d\*c^2-3/2\*b/(a\*d-b\*c)^3/g^3\*A/(b\*x+a)^2\*a\*d\*c^2-1/(a\*d-b\*c)^3/g^3\*B\*d^2/(b\*x+a)\*a\*c+3/2/(a\*d-b\*c)^3/g^3\*A/(b\*x+a)^2\*a^2\*d^2\*c-3/4/(a\*d-b\*c)^3/g^3\*B/(b\*x+a)^2\*a^2\*d^2\*c-1/2/b/(a\*d-b\*c)^3/g^3\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)^2\*a^3\*d^3+3/4/(a\*d-b\*c)^3/g^3\*B\*d^2\*c

**Maxima [A]** time = 1.24118, size = 344, normalized size = 2.39

$$-\frac{1}{4} B \left( \frac{2 b d x - b c + 3 a d}{(b^4 c - a b^3 d) g^3 x^2 + 2 (a b^3 c - a^2 b^2 d) g^3 x + (a^2 b^2 c - a^3 b d) g^3} + \frac{2 \log\left(\frac{d x}{b x + a} + \frac{c e}{b x + a}\right)}{b^3 g^3 x^2 + 2 a b^2 g^3 x + a^2 b g^3} + \frac{2 d^2 \log(b x + a)}{(b^3 c^2 - 2 a b^2 c d + a^2 d^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g)^3,x, algorithm="maxima")

[Out] 
$$-1/4*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*\log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*\log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*\log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)$$

**Fricas [A]** time = 1.02353, size = 455, normalized size = 3.16

$$\frac{(2A - B)b^2c^2 - 4(A - B)abcd + (2A - 3B)a^2d^2 + 2(Bb^2cd - Babd^2)x - 2(Bb^2d^2x^2 + 2Babd^2x - Bb^2c^2 + 2Babcd)\log\left(\frac{e(c+dx)}{a+bx}\right)}{4\left((b^5c^2 - 2ab^4cd + a^2b^3d^2)g^3x^2 + 2(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2)g^3x + (a^2b^3c^2 - 2a^3b^2cd + a^4bd^2)g^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g)^3,x, algorithm="fricas")

[Out] 
$$-1/4*((2*A - B)*b^2*c^2 - 4*(A - B)*a*b*c*d + (2*A - 3*B)*a^2*d^2 + 2*(B*b^2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x - B*b^2*c^2 + 2*B*a*b*c*d)*\log((d*e*x + c*e)/(b*x + a)))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)$$

**Sympy [B]** time = 3.23947, size = 422, normalized size = 2.93

$$\frac{B \log\left(\frac{e(c+dx)}{a+bx}\right)}{2a^2bg^3 + 4ab^2g^3x + 2b^3g^3x^2} + \frac{Bd^2 \log\left(x + \frac{-\frac{Ba^3d^5}{(ad-bc)^2} + \frac{3Ba^2bcd^4}{(ad-bc)^2} - \frac{3Bab^2c^2d^3}{(ad-bc)^2} + Bad^3 + \frac{Bb^3c^3d^2}{(ad-bc)^2} + Bbcd^2}{2Bbd^3}\right)}{2bg^3(ad-bc)^2} - \frac{Bd^2 \log\left(x + \frac{\frac{Ba^3d^5}{(ad-bc)^2} - \frac{3Ba^2bcd^4}{(ad-bc)^2} + \dots}{2Bbd^3}\right)}{2bg^3(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g)\*\*3,x)

[Out] 
$$-B*\log(e*(c + d*x)/(a + b*x))/(2*a**2*b*g**3 + 4*a*b**2*g**3*x + 2*b**3*g**3*x**2) + B*d**2*\log(x + (-B*a**3*d**5/(a*d - b*c)**2 + 3*B*a**2*b*c*d**4/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 + B*b**3*c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3)))/(2*b*g**3*(a*d - b*c)**2) - B*d**2*\log(x + (B*a**3*d**5/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4/(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**3/(a*d - b*c)**2 + B*a*d**3 - B*b**3*c**3*d**2/(a*d - b*c)**2 + B*b*c*d**2)/(2*B*b*d**3)))/(2*b*g**3*(a*d - b*c)**2) + (-2*A*a*d + 2*A*b*c + 3*B*a*d - B*b*c + 2*B*b*d*x)/(4*a**3*b*d*g**3 - 4*a**2*b**2*c*g**3 + x**2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 - 8*a*b**3*c*g**3))$$

**Giac [A]** time = 1.3898, size = 327, normalized size = 2.27

$$\frac{Bd^2 \log(bx + a)}{2(b^3c^2g^3 - 2ab^2cdg^3 + a^2bd^2g^3)} + \frac{Bd^2 \log(dx + c)}{2(b^3c^2g^3 - 2ab^2cdg^3 + a^2bd^2g^3)} - \frac{B \log\left(\frac{dx+c}{bx+a}\right)}{2(b^3g^3x^2 + 2ab^2g^3x + a^2bg^3)} - \frac{Bd^2 \log(dx + c)}{4(b^4cg^3x^2 - \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] -1/2*B*d^2*log(b*x + a)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) + 1
/2*B*d^2*log(d*x + c)/(b^3*c^2*g^3 - 2*a*b^2*c*d*g^3 + a^2*b*d^2*g^3) - 1/2
*B*log((d*x + c)/(b*x + a))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/4
*(2*B*b*d*x + 2*A*b*c + B*b*c - 2*A*a*d + B*a*d)/(b^4*c*g^3*x^2 - a*b^3*d*g
^3*x^2 + 2*a*b^3*c*g^3*x - 2*a^2*b^2*d*g^3*x + a^2*b^2*c*g^3 - a^3*b*d*g^3)
```

$$3.180 \quad \int \frac{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^4} dx$$

**Optimal.** Leaf size=175

$$-\frac{B \log\left(\frac{e(c+dx)}{a+bx}\right) + A}{3bg^4(a+bx)^3} + \frac{Bd^2}{3bg^4(a+bx)(bc-ad)^2} + \frac{Bd^3 \log(a+bx)}{3bg^4(bc-ad)^3} - \frac{Bd^3 \log(c+dx)}{3bg^4(bc-ad)^3} - \frac{Bd}{6bg^4(a+bx)^2(bc-ad)} + \frac{B}{9bg^4(a+bx)^3}$$

[Out]  $B/(9*b*g^4*(a + b*x)^3) - (B*d)/(6*b*(b*c - a*d)*g^4*(a + b*x)^2) + (B*d^2)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) + (B*d^3*Log[a + b*x])/(3*b*(b*c - a*d)^3*g^4) - (B*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4) - (A + B*Log[(e*(c + d*x))/(a + b*x]))/(3*b*g^4*(a + b*x)^3)$

**Rubi [A]** time = 0.131136, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(c+dx)}{a+bx}\right) + A}{3bg^4(a+bx)^3} + \frac{Bd^2}{3bg^4(a+bx)(bc-ad)^2} + \frac{Bd^3 \log(a+bx)}{3bg^4(bc-ad)^3} - \frac{Bd^3 \log(c+dx)}{3bg^4(bc-ad)^3} - \frac{Bd}{6bg^4(a+bx)^2(bc-ad)} + \frac{B}{9bg^4(a+bx)^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Log}[(e*(c + d*x))/(a + b*x)])/(a*g + b*g*x)^4, x]$

[Out]  $B/(9*b*g^4*(a + b*x)^3) - (B*d)/(6*b*(b*c - a*d)*g^4*(a + b*x)^2) + (B*d^2)/(3*b*(b*c - a*d)^2*g^4*(a + b*x)) + (B*d^3*Log[a + b*x])/(3*b*(b*c - a*d)^3*g^4) - (B*d^3*Log[c + d*x])/(3*b*(b*c - a*d)^3*g^4) - (A + B*Log[(e*(c + d*x))/(a + b*x)])/(3*b*g^4*(a + b*x)^3)$

### Rule 2525

$\text{Int}[(a + \text{Log}[c*(RfX)^p])*(b*x)^n*((d + e*x)^m), x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*(RfX)^p])^n/(e^{m+1}), x] - \text{Dist}[(b*n*p)/(e^{m+1}), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*(RfX)^p])^{n-1}*D[RfX, x])/RfX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RfX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

$\text{Int}[(a)*(u), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b)\*(v)] /; FreeQ[b, x]

### Rule 44

$\text{Int}[(a + (b*x)^m)*((c + d*x)^n), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rubi steps



$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag + bgx)^4} dx &= -\frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a + bx)^3} + \frac{B \int \frac{-bc+ad}{g^3(a+bx)^4(c+dx)} dx}{3bg} \\
&= -\frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a + bx)^3} - \frac{(B(bc - ad)) \int \frac{1}{(a+bx)^4(c+dx)} dx}{3bg^4} \\
&= -\frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{3bg^4(a + bx)^3} - \frac{(B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^4} - \frac{bd}{(bc-ad)^2(a+bx)^3} + \frac{bd^2}{(bc-ad)^3(a+bx)^2} - \frac{bd^3}{(bc-ad)^4(a+bx)}\right) dx}{3bg^4} \\
&= \frac{B}{9bg^4(a + bx)^3} - \frac{Bd}{6b(bc - ad)g^4(a + bx)^2} + \frac{Bd^2}{3b(bc - ad)^2g^4(a + bx)} + \frac{Bd^3 \log(a + bx)}{3b(bc - ad)^3g^4} - \frac{Bd^4}{3b(bc - ad)^4g^4}
\end{aligned}$$

**Mathematica [A]** time = 0.157511, size = 141, normalized size = 0.81

$$\frac{B((bc-ad)(11a^2d^2+abd(15dx-7c)+b^2(2c^2-3cdx+6d^2x^2))-6d^3(a+bx)^3 \log(c+dx)+6d^3(a+bx)^3 \log(a+bx))}{(bc-ad)^3} - 6 \left( B \log\left(\frac{e(c+dx)}{a+bx}\right) + A \right)$$


---


$$18bg^4(a + bx)^3$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])/(a\*g + b\*g\*x)^4, x]

[Out] ((B\*((b\*c - a\*d)\*(11\*a^2\*d^2 + a\*b\*d\*(-7\*c + 15\*d\*x) + b^2\*(2\*c^2 - 3\*c\*d\*x + 6\*d^2\*x^2)) + 6\*d^3\*(a + b\*x)^3\*Log[a + b\*x] - 6\*d^3\*(a + b\*x)^3\*Log[c + d\*x]))/(b\*c - a\*d)^3 - 6\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(18\*b\*g^4\*(a + b\*x)^3)

**Maple [B]** time = 0.056, size = 1012, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g)^4, x)

[Out] -1/3/b/(a\*d-b\*c)^4/g^4\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)^3\*a^4\*d^4-2\*b/(a\*d-b\*c)^4/g^4\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)^3\*a^2\*d^2\*c^2-1/3/(a\*d-b\*c)^4/g^4\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^3\*c+1/9\*b^3/(a\*d-b\*c)^4/g^4\*B/(b\*x+a)^3\*c^4-1/3\*b^3/(a\*d-b\*c)^4/g^4\*A/(b\*x+a)^3\*c^4-1/2/(a\*d-b\*c)^4/g^4\*B\*d^3/(b\*x+a)^2\*a^2\*c+4/3\*b^2/(a\*d-b\*c)^4/g^4\*A/(b\*x+a)^3\*a\*d\*c^3-4/9\*b^2/(a\*d-b\*c)^4/g^4\*B/(b\*x+a)^3\*c^3\*a\*d-4/9/(a\*d-b\*c)^4/g^4\*B/(b\*x+a)^3\*a^3\*d^3\*c-2/3/(a\*d-b\*c)^4/g^4\*B\*d^3/(b\*x+a)\*c\*a+4/3\*b^2/(a\*d-b\*c)^4/g^4\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)^3\*a\*d\*c^3+4/3/(a\*d-b\*c)^4/g^4\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)^3\*a^3\*d^3\*c+1/2\*b/(a\*d-b\*c)^4/g^4\*B\*d^2/(b\*x+a)^2\*c^2\*a-2\*b/(a\*d-b\*c)^4/g^4\*A/(b\*x+a)^3\*a^2\*d^2\*c^2+2/3\*b/(a\*d-b\*c)^4/g^4\*B/(b\*x+a)^3\*a^2\*d^2\*c^2-11/18/b/(a\*d-b\*c)^4/g^4\*B\*d^4\*a+1/3/b/(a\*d-b\*c)^4/g^4\*A\*d^4\*a+4/3/(a\*d-b\*c)^4/g^4\*A/(b\*x+a)^3\*a^3\*d^3\*c-1/3/(a\*d-b\*c)^4/g^4\*A\*d^3\*c+1/6/b/(a\*d-b\*c)^4/g^4\*B\*d^4/(b\*x+a)^2\*a^3-1/6\*b^2/(a\*d-b\*c)^4/g^4\*B\*d/(b\*x+a)^2\*c^3+1/3/b/(a\*d-b\*c)^4/g^4\*B\*d^4/(b\*x+a)\*a^2+1/3/b/(a\*d-b\*c)^4/g^4\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^4\*a-1/3\*b^3/(a\*d-b\*c)^4/g^4\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)^3\*c^4-1/3/b/(a\*d-b\*c)^4/g^4\*A/(b\*x+a)^3\*a^4\*d^4+11/18/(a\*d-b\*c)^4/g^4\*B\*d^3\*c+1/9/b/(a\*d-b\*c)^4/g^4\*B/(b\*x+a)^3\*a^4\*d^4+1/3\*b/(a\*d-b\*c)^4/g^4\*B\*d^2/(b\*x+a)\*c^2

**Maxima [B]** time = 1.26679, size = 578, normalized size = 3.3

$$\frac{1}{18} B \left( \frac{6b^2d^2x^2 + 2b^2c^2 - 7abcd + 11a^2d^2 - 3(b^2cd - 5abd^2)x}{(b^6c^2 - 2ab^5cd + a^2b^4d^2)g^4x^3 + 3(ab^5c^2 - 2a^2b^4cd + a^3b^3d^2)g^4x^2 + 3(a^2b^4c^2 - 2a^3b^3cd + a^4b^2d^2)g^4x + (a^3b^3c^2 - 2a^4b^2cd + a^5b^1d^2)g^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="maxima")
```

```
[Out] 1/18*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) - 6*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

**Fricas [B]** time = 1.02002, size = 826, normalized size = 4.72

$$\frac{2(3A - B)b^3c^3 - 9(2A - B)ab^2c^2d + 18(A - B)a^2bcd^2 - (6A - 11B)a^3d^3 - 6(Bb^3cd^2 - Bab^2d^3)x^2 + 3(Bb^3c^2d - 6Ba^2b^2cd^2)g^4x^3 + 3(ab^6c^3 - 3a^2b^5c^2d + 3a^3b^4cd^2 - a^4b^3d^3)g^4x^2 + 3(a^5b^2c^3 - 3a^4b^3cd^2 + 3a^3b^4d^3)g^4x + (a^6b^1c^3 - 3a^5b^2cd^2 + 3a^4b^3d^3)g^4}{18((b^7c^3 - 3a*b^6*c^2*d + 3a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3a^2*b^5*c^2*d + 3a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3a^3*b^4*c^2*d + 3a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3a^4*b^3*c^2*d + 3a^5*b^2*c*d^2 - a^6*b^1*d^3)*g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="fricas")
```

```
[Out] -1/18*(2*(3*A - B)*b^3*c^3 - 9*(2*A - B)*a*b^2*c^2*d + 18*(A - B)*a^2*b*c*d^2 - (6*A - 11*B)*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log((d*e*x + c*e)/(b*x + a))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b^1*d^3)*g^4)
```

**Sympy [B]** time = 5.22966, size = 656, normalized size = 3.75

$$\frac{B \log\left(\frac{e(c+dx)}{a+bx}\right)}{3a^3bg^4 + 9a^2b^2g^4x + 9ab^3g^4x^2 + 3b^4g^4x^3} + \frac{Bd^3 \log\left(x + \frac{-\frac{Ba^4d^7}{(ad-bc)^3} + \frac{4Ba^3bcd^6}{(ad-bc)^3} - \frac{6Ba^2b^2c^2d^5}{(ad-bc)^3} + \frac{4Bab^3c^3d^4}{(ad-bc)^3} + Bad^4 - \frac{Bb^4c^4d^3}{(ad-bc)^3} + Bbcd^3}{2Bbd^4}\right)}{3bg^4(ad-bc)^3} - \frac{Bd^3 \log\left(\frac{e(c+dx)}{a+bx}\right)}{3a^3bg^4 + 9a^2b^2g^4x + 9ab^3g^4x^2 + 3b^4g^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(c + d*x)/(a + b*x))/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) + B*d**3*log(x + (-B*a**4*d**7/(a*d - b*c)*
```

```

*3 + 4*B*a**3*b*c*d**6/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5/(a*d - b*c)
**3 + 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + B*a*d**4 - B*b**4*c**4*d**3/(a*
d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) - B*d**3*
log(x + (B*a**4*d**7/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6/(a*d - b*c)**3 + 6*
B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4/(a*d - b*c)**3
+ B*a*d**4 + B*b**4*c**4*d**3)/(a*d - b*c)**3 + B*b*c*d**3)/(2*B*b*d**4))/(3
*b*g**4*(a*d - b*c)**3) + (-6*A*a**2*d**2 + 12*A*a*b*c*d - 6*A*b**2*c**2 +
11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**2*c**2 + 6*B*b**2*d**2*x**2 + x*(15*B
*a*b*d**2 - 3*B*b**2*c*d))/(18*a**5*b*d**2*g**4 - 36*a**4*b**2*c*d*g**4 + 1
8*a**3*b**3*c**2*g**4 + x**3*(18*a**2*b**4*d**2*g**4 - 36*a*b**5*c*d*g**4 +
18*b**6*c**2*g**4) + x**2*(54*a**3*b**3*d**2*g**4 - 108*a**2*b**4*c*d*g**4
+ 54*a*b**5*c**2*g**4) + x*(54*a**4*b**2*d**2*g**4 - 108*a**3*b**3*c*d*g**
4 + 54*a**2*b**4*c**2*g**4))

```

---

**Giac [B]** time = 1.39764, size = 609, normalized size = 3.48

$$\frac{Bd^3 \log(bx + a)}{3(b^4c^3g^4 - 3ab^3c^2dg^4 + 3a^2b^2cd^2g^4 - a^3bd^3g^4)} - \frac{Bd^3 \log(dx + c)}{3(b^4c^3g^4 - 3ab^3c^2dg^4 + 3a^2b^2cd^2g^4 - a^3bd^3g^4)} - \frac{1}{3(b^4g^4x^3 + 3a^2b^2c^2g^4x^2 + 3a^3b^3c^2g^4x + a^4b^4c^2g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^4,x, algorithm="giac")
```

```

[Out] 1/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*g
^4 - a^3*b*d^3*g^4) - 1/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g
^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((d*x + c)/(b*x + a))/
(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 1/18*(6*B*b
^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*B*a*b*d^2*x - 6*A*b^2*c^2 - 4*B*b^2*c^2 + 1
2*A*a*b*c*d + 5*B*a*b*c*d - 6*A*a^2*d^2 + 5*B*a^2*d^2)/(b^6*c^2*g^4*x^3 - 2
*a*b^5*c*d*g^4*x^3 + a^2*b^4*d^2*g^4*x^3 + 3*a*b^5*c^2*g^4*x^2 - 6*a^2*b^4*
c*d*g^4*x^2 + 3*a^3*b^3*d^2*g^4*x^2 + 3*a^2*b^4*c^2*g^4*x - 6*a^3*b^3*c*d*g
^4*x + 3*a^4*b^2*d^2*g^4*x + a^3*b^3*c^2*g^4 - 2*a^4*b^2*c*d*g^4 + a^5*b*d^
2*g^4)

```

$$3.181 \quad \int \frac{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag+bgx)^5} dx$$

**Optimal.** Leaf size=206

$$-\frac{B \log\left(\frac{e(c+dx)}{a+bx}\right) + A}{4bg^5(a+bx)^4} - \frac{Bd^3}{4bg^5(a+bx)(bc-ad)^3} + \frac{Bd^2}{8bg^5(a+bx)^2(bc-ad)^2} - \frac{Bd^4 \log(a+bx)}{4bg^5(bc-ad)^4} + \frac{Bd^4 \log(c+dx)}{4bg^5(bc-ad)^4} - \frac{12bg^5(a+bx)^4}{12bg^5(a+bx)^4}$$

[Out] B/(16\*b\*g^5\*(a + b\*x)^4) - (B\*d)/(12\*b\*(b\*c - a\*d)\*g^5\*(a + b\*x)^3) + (B\*d^2)/(8\*b\*(b\*c - a\*d)^2\*g^5\*(a + b\*x)^2) - (B\*d^3)/(4\*b\*(b\*c - a\*d)^3\*g^5\*(a + b\*x)) - (B\*d^4\*Log[a + b\*x])/(4\*b\*(b\*c - a\*d)^4\*g^5) + (B\*d^4\*Log[c + d\*x])/(4\*b\*(b\*c - a\*d)^4\*g^5) - (A + B\*Log[(e\*(c + d\*x))/(a + b\*x])/(4\*b\*g^5\*(a + b\*x)^4)

**Rubi [A]** time = 0.14864, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(c+dx)}{a+bx}\right) + A}{4bg^5(a+bx)^4} - \frac{Bd^3}{4bg^5(a+bx)(bc-ad)^3} + \frac{Bd^2}{8bg^5(a+bx)^2(bc-ad)^2} - \frac{Bd^4 \log(a+bx)}{4bg^5(bc-ad)^4} + \frac{Bd^4 \log(c+dx)}{4bg^5(bc-ad)^4} - \frac{12bg^5(a+bx)^4}{12bg^5(a+bx)^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x])]/(a\*g + b\*g\*x)^5, x]

[Out] B/(16\*b\*g^5\*(a + b\*x)^4) - (B\*d)/(12\*b\*(b\*c - a\*d)\*g^5\*(a + b\*x)^3) + (B\*d^2)/(8\*b\*(b\*c - a\*d)^2\*g^5\*(a + b\*x)^2) - (B\*d^3)/(4\*b\*(b\*c - a\*d)^3\*g^5\*(a + b\*x)) - (B\*d^4\*Log[a + b\*x])/(4\*b\*(b\*c - a\*d)^4\*g^5) + (B\*d^4\*Log[c + d\*x])/(4\*b\*(b\*c - a\*d)^4\*g^5) - (A + B\*Log[(e\*(c + d\*x))/(a + b\*x])/(4\*b\*g^5\*(a + b\*x)^4)

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(ag + bgx)^5} dx &= -\frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a + bx)^4} + \frac{B \int \frac{-bc+ad}{g^4(a+bx)^5(c+dx)} dx}{4bg} \\
&= -\frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a + bx)^4} - \frac{(B(bc - ad)) \int \frac{1}{(a+bx)^5(c+dx)} dx}{4bg^5} \\
&= -\frac{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)}{4bg^5(a + bx)^4} - \frac{(B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^5} - \frac{bd}{(bc-ad)^2(a+bx)^4} + \frac{bd^2}{(bc-ad)^3(a+bx)^3} - \frac{bd^3}{(bc-ad)^4(a+bx)^2}\right) dx}{4bg^5} \\
&= \frac{B}{16bg^5(a + bx)^4} - \frac{Bd}{12b(bc - ad)g^5(a + bx)^3} + \frac{Bd^2}{8b(bc - ad)^2g^5(a + bx)^2} - \frac{Bd^3}{4b(bc - ad)^3g^5(a + bx)}
\end{aligned}$$

**Mathematica [A]** time = 0.202056, size = 166, normalized size = 0.81

$$\frac{B(ad-bc)\left(\frac{12d^3(bc-ad)}{a+bx} - \frac{6d^2(bc-ad)^2}{(a+bx)^2} + \frac{4d(bc-ad)^3}{(a+bx)^3} - \frac{3(bc-ad)^4}{(a+bx)^4} + 12d^4 \log(a+bx) - 12d^4 \log(c+dx)\right) - \frac{B \log\left(\frac{e(c+dx)}{a+bx}\right) + A}{(a+bx)^4}}{4bg^5}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]))/(a\*g + b\*g\*x)^5, x]

[Out] ((B\*(-(b\*c) + a\*d)\*((-3\*(b\*c - a\*d)^4)/(a + b\*x)^4 + (4\*d\*(b\*c - a\*d)^3)/(a + b\*x)^3 - (6\*d^2\*(b\*c - a\*d)^2)/(a + b\*x)^2 + (12\*d^3\*(b\*c - a\*d))/(a + b\*x) + 12\*d^4\*Log[a + b\*x] - 12\*d^4\*Log[c + d\*x]))/(12\*(b\*c - a\*d)^5) - (A + B\*Log[(e\*(c + d\*x))/(a + b\*x]))/(a + b\*x)^4)/(4\*b\*g^5)

**Maple [B]** time = 0.053, size = 1306, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g)^5, x)

[Out] -1/4/(a\*d-b\*c)^5/g^5\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^4\*c+1/4\*b^4/(a\*d-b\*c)^5/g^5\*A/(b\*x+a)^4\*c^5-1/16\*b^4/(a\*d-b\*c)^5/g^5\*B/(b\*x+a)^4\*c^5-5/4\*b^3/(a\*d-b\*c)^5/g^5\*A/(b\*x+a)^4\*a\*d\*c^4-5/2\*b/(a\*d-b\*c)^5/g^5\*A/(b\*x+a)^4\*a^3\*d^3\*c^2+5/8\*b/(a\*d-b\*c)^5/g^5\*B/(b\*x+a)^4\*a^3\*d^3\*c^2+5/4/(a\*d-b\*c)^5/g^5\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)^4\*a^4\*d^4\*c-1/4/b/(a\*d-b\*c)^5/g^5\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)^4\*a^5\*d^5+5/2\*b^2/(a\*d-b\*c)^5/g^5\*A/(b\*x+a)^4\*a^2\*d^2\*c^3-5/8\*b^2/(a\*d-b\*c)^5/g^5\*B/(b\*x+a)^4\*a^2\*d^2\*c^3+5/16\*b^3/(a\*d-b\*c)^5/g^5\*B/(b\*x+a)^4\*a\*d\*c^4+1/2\*b/(a\*d-b\*c)^5/g^5\*B\*d^3/(b\*x+a)^3\*a^2\*c^2-1/3\*b^2/(a\*d-b\*c)^5/g^5\*B\*d^2/(b\*x+a)^3\*a\*c^3+3/8\*b/(a\*d-b\*c)^5/g^5\*B\*d^3/(b\*x+a)^2\*a\*c^2-1/4/(a\*d-b\*c)^5/g^5\*A\*d^4\*c-25/48/b/(a\*d-b\*c)^5/g^5\*B\*d^5\*a+25/48/(a\*d-b\*c)^5/g^5\*B\*d^4\*c+1/4/b/(a\*d-b\*c)^5/g^5\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))\*d^5\*a+1/4\*b^4/(a\*d-b\*c)^5/g^5\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)^4\*c^5+5/2\*b^2/(a\*d-b\*c)^5/g^5\*B\*ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))/(b\*x+a)^4\*a^2\*d^2\*c^3-3/8/(a\*d-b\*c)^5/g^5\*B\*d^4/(b\*x+a)^2\*a^2\*c-1/2/(a\*d-b\*c)^5/g^5\*B\*d^4/(b\*x+a)\*a\*c+1/4/b/(a\*d-b\*c)^5/g^5\*B\*d^5/(b\*x+a)\*a^2+1/4\*b/(a\*d-b\*c)^5/g^5\*B\*d^3/(b\*x+a)\*c^2+1/8/b/(a\*d-b\*c)^5/g^5\*B\*d^5/(b\*x+a)^2\*a^3-1/8\*b^2/(a\*d-b\*c)^5/g^5\*B\*d^2/(b\*x+a)^2\*c^3+1/12/b/(a\*d-b\*c)^5/g^5

$$\begin{aligned} & *B*d^5/(b*x+a)^3*a^4+1/12*b^3/(a*d-b*c)^5/g^5*B*d/(b*x+a)^3*c^4+5/4/(a*d-b*c)^5/g^5*A/(b*x+a)^4*a^4*d^4*c-1/4/b/(a*d-b*c)^5/g^5*A/(b*x+a)^4*a^5*d^5+1/4/b/(a*d-b*c)^5/g^5*A*d^5*a-5/4*b^3/(a*d-b*c)^5/g^5*B*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*a*d*c^4-5/2*b/(a*d-b*c)^5/g^5*B*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*a^3*d^3*c^2+1/16/b/(a*d-b*c)^5/g^5*B/(b*x+a)^4*a^5*d^5-5/16/(a*d-b*c)^5/g^5*B/(b*x+a)^4*a^4*d^4*c-1/3/(a*d-b*c)^5/g^5*B*d^4/(b*x+a)^3*a^3*c \end{aligned}$$

**Maxima [B]** time = 1.1692, size = 873, normalized size = 4.24

$$-\frac{1}{48} B \left( \frac{12 b^3 d^3 x^3 - 3 b^3 c^3 + 13 a b^2 c^2 d - 23 a^2 b c d^2 + 25 a^3 d^3 - 3 a b^7 c^2 d + 3 a^2 b^6 c d^2 - a^3 b^5 d^3}{(b^8 c^3 - 3 a b^7 c^2 d + 3 a^2 b^6 c d^2 - a^3 b^5 d^3) g^5 x^4 + 4 (a b^7 c^3 - 3 a^2 b^6 c^2 d + 3 a^3 b^5 c d^2 - a^4 b^4 d^3) g^5 x^3 + 6 (a^2 b^6 c^3 - 3 a^3 b^5 c^2 d + 3 a^4 b^4 c^2 d^2 - 3 a^5 b^3 c^2 d^3 + 3 a^6 b^2 c^2 d^3 - a^7 b^2 c^2 d^3) g^5 x^2 + 4 (a^2 b^6 c^3 - 3 a^3 b^5 c^2 d + 3 a^4 b^4 c^2 d^2 - a^5 b^3 c^2 d^3) g^5 x + (a^4 b^4 c^3 - 3 a^5 b^3 c^2 d + 3 a^6 b^2 c^2 d^2 - a^7 b^2 c^2 d^3) g^5 + 12 \log(d e x / (b x + a) + c e / (b x + a)) / (b^5 g^5 x^4 + 4 a b^4 g^5 x^3 + 6 a^2 b^3 g^5 x^2 + 4 a^3 b^2 g^5 x + a^4 b g^5) + 12 d^4 \log(b x + a) / (b^5 c^4 - 4 a b^4 c^3 d + 6 a^2 b^3 c^2 d^2 - 4 a^3 b^2 c^2 d^3 + a^4 b d^4) g^5 - 12 d^4 \log(d x + c) / (b^5 c^4 - 4 a b^4 c^3 d + 6 a^2 b^3 c^2 d^2 - 4 a^3 b^2 c^2 d^3 + a^4 b d^4) g^5 \right) - 1/4 * A / (b^5 g^5 x^4 + 4 a b^4 g^5 x^3 + 6 a^2 b^3 g^5 x^2 + 4 a^3 b^2 g^5 x + a^4 b g^5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="maxima")
```

```
[Out] -1/48*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/(b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5 + 12*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c^2*d^3 + a^4*b*d^4)*g^5 - 12*d^4*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c^2*d^3 + a^4*b*d^4)*g^5) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

**Fricas [B]** time = 1.10845, size = 1284, normalized size = 6.23

$$\frac{3(4A - B)b^4c^4 - 16(3A - B)ab^3c^3d + 36(2A - B)a^2b^2c^2d^2 - 48(A - B)a^3bcd^3 + (12A - 25B)a^4d^4 + 12(Bb^4cd^3 - Ba^4b^3cd^3)}{48((b^9c^4 - 4ab^8c^3d + 6a^2b^7c^2d^2 - 4a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4(ab^8c^4 - 4a^2b^7c^3d + 6a^3b^6c^2d^2 - 4a^4b^5cd^3 + 6a^5b^4d^4)g^5x^3 + 4(a^2b^7c^3d + 6a^3b^6c^2d^2 - 4a^4b^5cd^3 + a^5b^4d^4)g^5x^2 + 4(a^3b^6c^3d + 6a^4b^5cd^3 + a^5b^4d^4)g^5x + (a^4b^5cd^3 + 6a^5b^4d^4)g^5 + 12 \log(d e x / (b x + a) + c e / (b x + a)) / (b^5 g^5 x^4 + 4 a b^4 g^5 x^3 + 6 a^2 b^3 g^5 x^2 + 4 a^3 b^2 g^5 x + a^4 b g^5) + 12 d^4 \log(b x + a) / (b^5 c^4 - 4 a b^4 c^3 d + 6 a^2 b^3 c^2 d^2 - 4 a^3 b^2 c^2 d^3 + a^4 b d^4) g^5 - 12 d^4 \log(d x + c) / (b^5 c^4 - 4 a b^4 c^3 d + 6 a^2 b^3 c^2 d^2 - 4 a^3 b^2 c^2 d^3 + a^4 b d^4) g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))/(b*g*x+a*g)^5,x, algorithm="fricas")
```

```
[Out] -1/48*(3*(4*A - B)*b^4*c^4 - 16*(3*A - B)*a*b^3*c^3*d + 36*(2*A - B)*a^2*b^2*c^2*d^2 - 48*(A - B)*a^3*b*c*d^3 + (12*A - 25*B)*a^4*d^4 + 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*x^2 + 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*x - 12*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*b*c*d^3)*log((d*e*x + c*e)/(b*x + a))/(b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*d^4)*g^5 + 12*d^4*log(b*x + a)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c^2*d^3 + a^4*b*d^4)*g^5 - 12*d^4*log(d*x + c)/(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c^2*d^3 + a^4*b*d^4)*g^5) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)
```

$$6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)$$

**Sympy [B]** time = 7.93603, size = 944, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g)\*\*5,x)

[Out]  $-B \cdot \log\left(\frac{e \cdot (c + d \cdot x)}{a + b \cdot x}\right) / (4 \cdot a^{**4} \cdot b \cdot g^{**5} + 16 \cdot a^{**3} \cdot b^{**2} \cdot g^{**5} \cdot x + 24 \cdot a^{**2} \cdot b^{**3} \cdot g^{**5} \cdot x^{**2} + 16 \cdot a \cdot b^{**4} \cdot g^{**5} \cdot x^{**3} + 4 \cdot b^{**5} \cdot g^{**5} \cdot x^{**4}) + B \cdot d^{**4} \cdot \log(x + (-B \cdot a^{**5} \cdot d^{**9} / (a \cdot d - b \cdot c)^{**4} + 5 \cdot B \cdot a^{**4} \cdot b \cdot c \cdot d^{**8} / (a \cdot d - b \cdot c)^{**4} - 10 \cdot B \cdot a^{**3} \cdot b^{**2} \cdot c \cdot d^{**7} / (a \cdot d - b \cdot c)^{**4} + 10 \cdot B \cdot a^{**2} \cdot b^{**3} \cdot c \cdot d^{**6} / (a \cdot d - b \cdot c)^{**4} - 5 \cdot B \cdot a \cdot b^{**4} \cdot c \cdot d^{**5} / (a \cdot d - b \cdot c)^{**4} + B \cdot a \cdot d^{**5} + B \cdot b^{**5} \cdot c \cdot d^{**4} / (a \cdot d - b \cdot c)^{**4} + B \cdot b \cdot c \cdot d^{**4}) / (2 \cdot B \cdot b \cdot d^{**5})) / (4 \cdot b \cdot g^{**5} \cdot (a \cdot d - b \cdot c)^{**4}) - B \cdot d^{**4} \cdot \log(x + (B \cdot a^{**5} \cdot d^{**9} / (a \cdot d - b \cdot c)^{**4} - 5 \cdot B \cdot a^{**4} \cdot b \cdot c \cdot d^{**8} / (a \cdot d - b \cdot c)^{**4} + 10 \cdot B \cdot a^{**3} \cdot b^{**2} \cdot c \cdot d^{**7} / (a \cdot d - b \cdot c)^{**4} - 10 \cdot B \cdot a^{**2} \cdot b^{**3} \cdot c \cdot d^{**6} / (a \cdot d - b \cdot c)^{**4} + 5 \cdot B \cdot a \cdot b^{**4} \cdot c \cdot d^{**5} / (a \cdot d - b \cdot c)^{**4} + B \cdot a \cdot d^{**5} - B \cdot b^{**5} \cdot c \cdot d^{**4} / (a \cdot d - b \cdot c)^{**4} + B \cdot b \cdot c \cdot d^{**4}) / (2 \cdot B \cdot b \cdot d^{**5})) / (4 \cdot b \cdot g^{**5} \cdot (a \cdot d - b \cdot c)^{**4}) + (-12 \cdot A \cdot a^{**3} \cdot d^{**3} + 36 \cdot A \cdot a^{**2} \cdot b \cdot c \cdot d^{**2} - 36 \cdot A \cdot a \cdot b^{**2} \cdot c \cdot d^{**2} + 12 \cdot A \cdot b^{**3} \cdot c \cdot d^{**3} + 25 \cdot B \cdot a^{**3} \cdot d^{**3} - 23 \cdot B \cdot a^{**2} \cdot b \cdot c \cdot d^{**2} + 13 \cdot B \cdot a \cdot b^{**2} \cdot c \cdot d^{**2} - 3 \cdot B \cdot b^{**3} \cdot c \cdot d^{**3} + 12 \cdot B \cdot b^{**3} \cdot d^{**3} \cdot x^{**3} + x^{**2} \cdot (42 \cdot B \cdot a \cdot b^{**2} \cdot d^{**3} - 6 \cdot B \cdot b^{**3} \cdot c \cdot d^{**2}) + x \cdot (52 \cdot B \cdot a^{**2} \cdot b \cdot d^{**3} - 20 \cdot B \cdot a \cdot b^{**2} \cdot c \cdot d^{**2} + 4 \cdot B \cdot b^{**3} \cdot c \cdot d^{**2})) / (48 \cdot a^{**7} \cdot b \cdot d^{**3} \cdot g^{**5} - 144 \cdot a^{**6} \cdot b^{**2} \cdot c \cdot d^{**2} \cdot g^{**5} + 144 \cdot a^{**5} \cdot b^{**3} \cdot c \cdot d^{**2} \cdot g^{**5} - 48 \cdot a^{**4} \cdot b^{**4} \cdot c \cdot d^{**3} \cdot g^{**5} + x^{**4} \cdot (48 \cdot a^{**3} \cdot b^{**5} \cdot d^{**3} \cdot g^{**5} - 144 \cdot a^{**2} \cdot b^{**6} \cdot c \cdot d^{**2} \cdot g^{**5} + 144 \cdot a \cdot b^{**7} \cdot c \cdot d^{**2} \cdot g^{**5} - 48 \cdot b^{**8} \cdot c \cdot d^{**3} \cdot g^{**5}) + x^{**3} \cdot (192 \cdot a^{**4} \cdot b \cdot d^{**3} \cdot g^{**5} - 576 \cdot a^{**3} \cdot b^{**5} \cdot c \cdot d^{**2} \cdot g^{**5} + 576 \cdot a^{**2} \cdot b^{**6} \cdot c \cdot d^{**2} \cdot g^{**5} - 192 \cdot a \cdot b^{**7} \cdot c \cdot d^{**3} \cdot g^{**5}) + x^{**2} \cdot (288 \cdot a^{**5} \cdot b^{**3} \cdot d^{**3} \cdot g^{**5} - 864 \cdot a^{**4} \cdot b^{**4} \cdot c \cdot d^{**2} \cdot g^{**5} + 864 \cdot a^{**3} \cdot b^{**5} \cdot c \cdot d^{**2} \cdot g^{**5} - 288 \cdot a^{**2} \cdot b^{**6} \cdot c \cdot d^{**3} \cdot g^{**5}) + x \cdot (192 \cdot a^{**6} \cdot b \cdot d^{**3} \cdot g^{**5} - 576 \cdot a^{**5} \cdot b^{**3} \cdot c \cdot d^{**2} \cdot g^{**5} + 576 \cdot a^{**4} \cdot b^{**4} \cdot c \cdot d^{**2} \cdot g^{**5} - 192 \cdot a^{**3} \cdot b^{**5} \cdot c \cdot d^{**3} \cdot g^{**5}))$

**Giac [B]** time = 1.35681, size = 967, normalized size = 4.69

$$\frac{Bd^4 \log(bx + a)}{4(b^5c^4g^5 - 4ab^4c^3dg^5 + 6a^2b^3c^2d^2g^5 - 4a^3b^2cd^3g^5 + a^4bd^4g^5)} + \frac{Bd^4 \log(dx + c)}{4(b^5c^4g^5 - 4ab^4c^3dg^5 + 6a^2b^3c^2d^2g^5 - 4a^3b^2cd^3g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))/(b\*g\*x+a\*g)^5,x, algorithm="giac")

[Out]  $-1/4 \cdot B \cdot d^4 \cdot \log(b \cdot x + a) / (b^5 \cdot c^4 \cdot g^5 - 4 \cdot a \cdot b^4 \cdot c^3 \cdot d \cdot g^5 + 6 \cdot a^2 \cdot b^3 \cdot c^2 \cdot d^2 \cdot g^5 - 4 \cdot a^3 \cdot b^2 \cdot c \cdot d^3 \cdot g^5 + a^4 \cdot b \cdot d^4 \cdot g^5) + 1/4 \cdot B \cdot d^4 \cdot \log(d \cdot x + c) / (b^5 \cdot c^4 \cdot g^5 - 4 \cdot a \cdot b^4 \cdot c^3 \cdot d \cdot g^5 + 6 \cdot a^2 \cdot b^3 \cdot c^2 \cdot d^2 \cdot g^5 - 4 \cdot a^3 \cdot b^2 \cdot c \cdot d^3 \cdot g^5 + a^4 \cdot b \cdot d^4 \cdot g^5) - 1/4 \cdot B \cdot \log\left(\frac{d \cdot x + c}{b \cdot x + a}\right) / (b^5 \cdot g^5 \cdot x^4 + 4 \cdot a \cdot b^4 \cdot g^5 \cdot x^3 + 6 \cdot a^2 \cdot b^3 \cdot g^5 \cdot x^2 + 4 \cdot a^3 \cdot b^2 \cdot g^5 \cdot x + a^4 \cdot b \cdot g^5) - 1/48 \cdot (12 \cdot B \cdot b^3 \cdot d^3 \cdot x^3 - 6 \cdot B \cdot b^3 \cdot c \cdot d^2 \cdot x^2 + 42 \cdot B \cdot a \cdot b^2 \cdot d^3 \cdot x^2 + 4 \cdot B \cdot b^3 \cdot c^2 \cdot d \cdot x - 20 \cdot B \cdot a \cdot b^2 \cdot c \cdot d^2 \cdot x + 52 \cdot B \cdot a^2 \cdot b \cdot d^3 \cdot x + 12 \cdot A \cdot b^3 \cdot c^3 + 9 \cdot B \cdot b^3 \cdot c^3 - 36 \cdot A \cdot a \cdot b^2 \cdot c^2 \cdot d - 23 \cdot B \cdot a \cdot b^2 \cdot c^2 \cdot d + 36 \cdot A \cdot a^2 \cdot b \cdot c \cdot d^2 + 13 \cdot B \cdot a^2 \cdot b \cdot c \cdot d^2 - 12 \cdot A \cdot a^3 \cdot d^3 + 13 \cdot B \cdot a^3 \cdot d^3) / (b^8 \cdot c^3 \cdot g^5 \cdot x^4 - 3 \cdot a \cdot b^7 \cdot c^2 \cdot d \cdot g^5 \cdot x^4 + 3 \cdot a^2 \cdot b^6 \cdot c^2 \cdot d \cdot g^5 \cdot x^3 + 12 \cdot a^3 \cdot b^5 \cdot c \cdot d^2 \cdot g^5 \cdot x^3 - 4 \cdot a^4 \cdot b^4 \cdot d^3 \cdot g^5 \cdot x^3 + 6 \cdot a^2 \cdot b^6 \cdot c^3 \cdot g^5 \cdot x^2 - 18 \cdot a^3 \cdot b^5 \cdot c^2 \cdot d \cdot g^5 \cdot x^2 + 18 \cdot a^4 \cdot b^4 \cdot c \cdot d^2 \cdot g^5 \cdot x^2 - 6 \cdot a^5 \cdot b^3 \cdot d^3 \cdot g^5 \cdot x^2 + 4 \cdot a^3 \cdot b^5 \cdot c^3 \cdot g^5 \cdot x - 12 \cdot a^4 \cdot b^4 \cdot c^2 \cdot d \cdot g^5 \cdot x + 12 \cdot a^5 \cdot b^3 \cdot c \cdot d^2 \cdot g^5 \cdot x)$

$$5*x - 4*a^6*b^2*d^3*g^5*x + a^4*b^4*c^3*g^5 - 3*a^5*b^3*c^2*d*g^5 + 3*a^6*b^2*c*d^2*g^5 - a^7*b*d^3*g^5)$$



$$3.182 \quad \int (ag + bgx)^4 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2 dx$$

**Optimal.** Leaf size=503

$$\frac{2B^2g^4(bc-ad)^5 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} - \frac{2Bg^4(bc-ad)^5 \log\left(1 - \frac{d(a+bx)}{b(c+dx)}\right) \left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{5bd^5} - \frac{2Bg^4(c+dx)(bc-ad)}{5bd^5}$$

```
[Out] (13*B^2*(b*c - a*d)^4*g^4*x)/(30*d^4) - (7*B^2*(b*c - a*d)^3*g^4*(a + b*x)^2)/(60*b*d^3) + (B^2*(b*c - a*d)^2*g^4*(a + b*x)^3)/(30*b*d^2) - (5*B^2*(b*c - a*d)^5*g^4*Log[a + b*x])/(6*b*d^5) - (13*B^2*(b*c - a*d)^5*g^4*Log[(c + d*x)/(a + b*x)])/(30*b*d^5) + (B*(b*c - a*d)^3*g^4*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(5*b*d^3) - (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(15*b*d^2) + (B*(b*c - a*d)*g^4*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(10*b*d) - (2*B*(b*c - a*d)^4*g^4*(c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(5*d^5) + (g^4*(a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a + b*x)]^2)/(5*b) - (2*B*(b*c - a*d)^5*g^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(5*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(5*b*d^5)
```

**Rubi [A]** time = 0.818276, antiderivative size = 557, normalized size of antiderivative = 1.11, number of steps used = 28, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{2B^2g^4(bc-ad)^5 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{5bd^5} + \frac{2Bg^4(bc-ad)^5 \log(c+dx) \left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{5bd^5} + \frac{Bg^4(a+bx)^2(bc-ad)^3(B)}{5bd^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (-2*A*B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (13*B^2*(b*c - a*d)^4*g^4*x)/(30*d^4) - (7*B^2*(b*c - a*d)^3*g^4*(a + b*x)^2)/(60*b*d^3) + (B^2*(b*c - a*d)^2*g^4*(a + b*x)^3)/(30*b*d^2) - (5*B^2*(b*c - a*d)^5*g^4*Log[c + d*x])/(6*b*d^5) + (2*B^2*(b*c - a*d)^5*g^4*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(5*b*d^5) - (B^2*(b*c - a*d)^5*g^4*Log[c + d*x]^2)/(5*b*d^5) - (2*B^2*(b*c - a*d)^4*g^4*(a + b*x)*Log[(e*(c + d*x))/(a + b*x)]/(5*b*d^4) + (B*(b*c - a*d)^3*g^4*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(5*b*d^3) - (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(15*b*d^2) + (B*(b*c - a*d)*g^4*(a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(10*b*d) + (2*B*(b*c - a*d)^5*g^4*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(5*b*d^5) + (g^4*(a + b*x)^5*(A + B*Log[(e*(c + d*x))/(a + b*x)]^2)/(5*b) + (2*B^2*(b*c - a*d)^5*g^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]/(5*b*d^5)
```

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^ (n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

Rule 12

$\text{Int}[(a_*)(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 2528

$\text{Int}[(a_*) + \text{Log}[(c_*)(\text{RFx}_*)^{(p_*)}](b_*)^{(n_*)}(\text{RGx}_*), x\_Symbol] \rightarrow \text{With}[\{u = \text{ExpandIntegrand}[(a + b*\text{Log}[c*\text{RFx}^p])^n, \text{RGx}, x]\}, \text{Int}[u, x] /; \text{SumQ}[u]] /; \text{FreeQ}[\{a, b, c, p\}, x] \ \&\& \ \text{RationalFunctionQ}[\text{RFx}, x] \ \&\& \ \text{RationalFunctionQ}[\text{RGx}, x] \ \&\& \ \text{IGtQ}[n, 0]$

Rule 2486

$\text{Int}[\text{Log}[(e_*)((f_*)((a_*) + (b_*)(x_))^{(p_*)}((c_*) + (d_*)(x_))^{(q_*)})^{(r_*)})^{(s_*)}], x\_Symbol] \rightarrow \text{Simp}[(a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/b, x] + \text{Dist}[(q*r*s*(b*c - a*d))/b, \text{Int}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1/(c + d*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, p, q, r, s\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[p + q, 0] \ \&\& \ \text{IGtQ}[s, 0]$

Rule 31

$\text{Int}[(a_*) + (b_*)(x_))^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}[\{a, b\}, x]$

Rule 43

$\text{Int}[(a_*) + (b_*)(x_))^{(m_*)}((c_*) + (d_*)(x_))^{(n_*)}, x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rule 2524

$\text{Int}[(a_*) + \text{Log}[(c_*)(\text{RFx}_*)^{(p_*)}](b_*)^{(n_*)}/((d_*) + (e_*)(x_)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[d + e*x]*(a + b*\text{Log}[c*\text{RFx}^p])^n)/e, x] - \text{Dist}[(b^n*p)/e, \text{Int}[(\text{Log}[d + e*x]*(a + b*\text{Log}[c*\text{RFx}^p])^{(n-1)}*D[\text{RFx}, x])/(\text{RFx}, x)], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{RationalFunctionQ}[\text{RFx}, x] \ \&\& \ \text{IGtQ}[n, 0]$

Rule 2418

$\text{Int}[(a_*) + \text{Log}[(c_*)((d_*) + (e_*)(x_))^{(n_*)}](b_*)^{(p_*)}(\text{RFx}_*), x\_Symbol] \rightarrow \text{With}[\{u = \text{ExpandIntegrand}[(a + b*\text{Log}[c*(d + e*x)^n])^p, \text{RFx}, x]\}, \text{Int}[u, x] /; \text{SumQ}[u]] /; \text{FreeQ}[\{a, b, c, d, e, n\}, x] \ \&\& \ \text{RationalFunctionQ}[\text{RFx}, x] \ \&\& \ \text{IntegerQ}[p]$

Rule 2394

$\text{Int}[(a_*) + \text{Log}[(c_*)((d_*) + (e_*)(x_))^{(n_*)}](b_*)/((f_*) + (g_*)(x_)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)]*(a + b*\text{Log}[c*(d + e*x)^n]))/g, x] - \text{Dist}[(b*e^n)/g, \text{Int}[\text{Log}[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, n\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0]$

Rule 2393

$\text{Int}[(a_*) + \text{Log}[(c_*)((d_*) + (e_*)(x_))](b_*)/((f_*) + (g_*)(x_)), x\_Symbol] \rightarrow \text{Dist}[1/g, \text{Subst}[\text{Int}[(a + b*\text{Log}[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g\}, x] \ \&\& \ \text{NeQ}[e*f - d*g, 0] \ \&\& \ \text{EqQ}[g + c*(e*f - d*g), 0]$

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
 \int (ag + bgx)^4 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2 dx &= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2}{5b} - \frac{(2B) \int \frac{(bc-ad)g^5(a+bx)^4 \left( -A - B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{c+dx}}{5bg} \\
 &= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2}{5b} - \frac{(2B(bc-ad)g^4) \int \frac{(a+bx)^4 \left( -A - B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{c+dx}}{5b} \\
 &= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2}{5b} - \frac{(2B(bc-ad)g^4) \int \left( -\frac{b(bc-ad)^3 \left( -A - B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{d^4} \right)}{5b} \\
 &= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2}{5b} - \frac{(2B(bc-ad)g^4) \int (a+bx)^3 \left( -A - B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{5d} \\
 &= -\frac{2AB(bc-ad)^4 g^4 x}{5d^4} + \frac{B(bc-ad)^3 g^4 (a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{5bd^3} - \frac{2B^2(bc-ad)^4 g^4 (a+bx) \log \left( \frac{e(c+dx)}{a+bx} \right)}{5bd^4} + \frac{B(bc-ad)^3 g^4 (a+bx)^2}{60bd^3} \\
 &= -\frac{2AB(bc-ad)^4 g^4 x}{5d^4} - \frac{2B^2(bc-ad)^5 g^4 \log(c+dx)}{5bd^5} - \frac{2B^2(bc-ad)^4 g^4 (a+bx) \log \left( \frac{e(c+dx)}{a+bx} \right)}{5bd^4} \\
 &= -\frac{2AB(bc-ad)^4 g^4 x}{5d^4} + \frac{13B^2(bc-ad)^4 g^4 x}{30d^4} - \frac{7B^2(bc-ad)^3 g^4 (a+bx)^2}{60bd^3} + \frac{B(bc-ad)^3 g^4 (a+bx)^2}{60bd^3} \\
 &= -\frac{2AB(bc-ad)^4 g^4 x}{5d^4} + \frac{13B^2(bc-ad)^4 g^4 x}{30d^4} - \frac{7B^2(bc-ad)^3 g^4 (a+bx)^2}{60bd^3} + \frac{B(bc-ad)^3 g^4 (a+bx)^2}{60bd^3} \\
 &= -\frac{2AB(bc-ad)^4 g^4 x}{5d^4} + \frac{13B^2(bc-ad)^4 g^4 x}{30d^4} - \frac{7B^2(bc-ad)^3 g^4 (a+bx)^2}{60bd^3} + \frac{B(bc-ad)^3 g^4 (a+bx)^2}{60bd^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.524062, size = 512, normalized size = 1.02

$$g^4 \left( (a + bx)^5 \left( B \log \left( \frac{e^{c+dx}}{a+bx} \right) + A \right)^2 - \frac{B(bc-ad) \left( -12B(bc-ad)^4 \left( 2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) - 12d^2(a+bx)^2(bc-ad)^2}{12d^5} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^4\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2,x]

[Out] (g^4\*((a + b\*x)^5\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2 - (B\*(b\*c - a\*d)\*(24\*A\*b\*d\*(b\*c - a\*d)^3\*x + 24\*B\*(b\*c - a\*d)^4\*Log[a + b\*x] - 4\*B\*(b\*c - a\*d)^2\*(2\*b\*d\*(b\*c - a\*d)\*x - d^2\*(a + b\*x)^2 - 2\*(b\*c - a\*d)^2\*Log[c + d\*x]) - B\*(b\*c - a\*d)\*(6\*b\*d\*(b\*c - a\*d)^2\*x + 3\*d^2\*(-(b\*c) + a\*d)\*(a + b\*x)^2 + 2\*d^3\*(a + b\*x)^3 - 6\*(b\*c - a\*d)^3\*Log[c + d\*x]) - 12\*B\*(b\*c - a\*d)^3\*(b\*d\*x + (-(b\*c) + a\*d)\*Log[c + d\*x]) + 24\*b\*B\*(b\*c - a\*d)^3\*(c + d\*x)\*Log[(e\*(c + d\*x))/(a + b\*x)] - 12\*d^2\*(b\*c - a\*d)^2\*(a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]) + 8\*d^3\*(b\*c - a\*d)\*(a + b\*x)^3\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]) - 6\*d^4\*(a + b\*x)^4\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]) - 24\*(b\*c - a\*d)^4\*Log[c + d\*x]\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]) - 12\*B\*(b\*c - a\*d)^4\*((2\*Log[(d\*(a + b\*x))/(-(b\*c) + a\*d)] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]))/(12\*d^5))/(5\*b)

**Maple [F]** time = 2.313, size = 0, normalized size = 0.

$$\int (bgx + ag)^4 \left( A + B \ln \left( \frac{e(dx + c)}{bx + a} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^4\*(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))^2,x)

[Out] int((b\*g\*x+a\*g)^4\*(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))^2,x)

**Maxima [B]** time = 1.84687, size = 3233, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="maxima")

[Out] 1/5\*A^2\*b^4\*g^4\*x^5 + A^2\*a\*b^3\*g^4\*x^4 + 2\*A^2\*a^2\*b^2\*g^4\*x^3 + 2\*A^2\*a^3\*b\*g^4\*x^2 + 2\*(x\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) - a\*log(b\*x + a)/b + c\*log(d\*x + c)/d)\*A\*B\*a^4\*g^4 + 4\*(x^2\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) + a^2\*log(b\*x + a)/b^2 - c^2\*log(d\*x + c)/d^2 + (b\*c - a\*d)\*x/(b\*d))\*A\*B\*a^3\*b\*g^4 + 2\*(2\*x^3\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) - 2\*a^3\*log(b\*x + a)/b^3 + 2\*c^3\*log(d\*x + c)/d^3 + ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2))\*A\*B\*a^2\*b^2\*g^4 + 1/3\*(6\*x^4\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) + 6\*a^4\*log(b\*x + a)/b^4 - 6\*c^4\*log(d\*x + c)/d^4 + (2\*(b^3\*c\*d^2 - a\*b^2\*d^3)\*x^3 - 3\*(b^3\*c^2\*d - a^2\*b\*d^3)\*x^2 + 6\*(b^3\*c^3 - a^3\*d^3)\*x)/(b^3\*d^3))\*A\*B\*a\*b^3\*g^4 + 1/30\*(12\*x^5\*log(d\*e\*x/(b\*x + a) + c\*e/(b\*x + a)) + 12\*x^4\*log(d\*e\*x/(b\*x + a)) + 12\*x^3\*log(d\*e\*x/(b\*x + a)) + 12\*x^2\*log(d\*e\*x/(b\*x + a)) + 12\*x\*log(d\*e\*x/(b\*x + a)) + 12\*log(d\*e\*x/(b\*x + a)))

$$\begin{aligned}
& x + a) - 12a^5 \log(bx + a)/b^5 + 12c^5 \log(dx + c)/d^5 + (3(b^4c^3d^3 - a^3b^3d^4)x^4 - 4(b^4c^2d^2 - a^2b^2d^4)x^3 + 6(b^4c^3d - a^3b^3d^4)x^2 - 12(b^4c^4 - a^4d^4)x)/(b^4d^4) * A * B * b^4g^4 + A^2a^4g^4 \\
& * x + 1/30 * ((12g^4 \log(e) - 25g^4) * b^4c^5 - (60g^4 \log(e) - 113g^4) * a^3c^4d + 4(30g^4 \log(e) - 49g^4) * a^2b^2c^3d^2 - 12(10g^4 \log(e) - 13g^4) * a^3b^2c^2d^3 + 12(5g^4 \log(e) - 4g^4) * a^4c^3d^4) * B^2 \log(dx + c)/d^5 - 2/5 * (b^5c^5g^4 - 5a^4b^4c^4d^2g^4 + 10a^2b^3c^3d^2g^4 - 10a^3b^2c^2d^3g^4 + 5a^4b^2c^2d^3g^4 - a^5d^5g^4) * (\log(bx + a) * \log(bdx + ad)/(b^2c - ad) + 1) + \operatorname{dilog}(-(bdx + ad)/(b^2c - ad)) * B^2/(bd^5) + 1/60 * (12B^2b^5d^5g^4x^5 \log(e)^2 + 6(b^5c^4d^4g^4 \log(e) + (10g^4 \log(e)^2 - g^4 \log(e)) * a^4d^5) * B^2x^4 - 2((4g^4 \log(e) - g^4) * b^5c^2d^3 - 2(10g^4 \log(e) - g^4) * a^2b^4c^2d^4 - (60g^4 \log(e)^2 - 16g^4 * 4 \log(e) + g^4) * a^2b^3d^5) * B^2x^3 + ((12g^4 \log(e) - 7g^4) * b^5c^3d^2 - 3(20g^4 \log(e) - 9g^4) * a^2b^4c^2d^3 + 3(40g^4 \log(e) - 11g^4) * a^2 * b^3c^2d^4 + (120g^4 \log(e)^2 - 72g^4 \log(e) + 13g^4) * a^3b^2d^5) * B^2x^2 - 2((12g^4 \log(e) - 13g^4) * b^5c^4d - (60g^4 \log(e) - 59g^4) * a^2b^4 * c^3d^2 + 6(20g^4 \log(e) - 17g^4) * a^2b^3c^2d^3 - (120g^4 \log(e) - 79g^4) * a^3b^2c^2d^4 - (30g^4 \log(e)^2 - 48g^4 \log(e) + 23g^4) * a^4b^2d^5) * B^2x + 12(B^2b^5d^5g^4x^5 + 5B^2a^4b^4d^5g^4x^4 + 10B^2a^2b^3d^5g^4x^3 + 10B^2a^3b^2d^5g^4x^2 + 5B^2a^4b^2d^5g^4x + B^2a^5d^5g^4) * \log(bx + a)^2 + 12(B^2b^5d^5g^4x^5 + 5B^2a^4b^4d^5g^4x^4 + 10B^2a^2b^3d^5g^4x^3 + 10B^2a^3b^2d^5g^4x^2 + 5B^2a^4b^2d^5g^4x + (b^5c^5g^4 - 5a^4b^4c^4d^2g^4 + 10a^2b^3c^3d^2g^4 - 10a^3b^2c^2d^3g^4 + 5a^4b^2c^2d^3g^4) * B^2) * \log(dx + c)^2 - 2(12B^2b^5d^5g^4x^5 \log(e) + 3(b^5c^4d^4g^4 + (20g^4 \log(e) - g^4) * a^4d^5) * B^2x^4 - 4(b^5c^2d^3g^4 - 5a^4b^4c^2d^4g^4 - 2(15g^4 \log(e) - 2g^4) * a^2b^3d^5) * B^2x^3 + 6(b^5c^3d^2g^4 - 5a^4b^4c^2d^3g^4 + 10a^2b^3c^2d^4g^4 + 2(10g^4 \log(e) - 3g^4) * a^3b^2d^5) * B^2x^2 - 12(b^5c^4d^2g^4 - 5a^4b^4c^3d^2g^4 + 10a^2b^3c^2d^3g^4 - 10a^3b^2c^2d^4g^4 - (5g^4 \log(e) - 4g^4) * a^4b^2d^5) * B^2x - (12a^4b^4c^4d^2g^4 - 54a^2b^3c^3d^2g^4 + 94a^3b^2c^2d^3g^4 - 77a^4b^2c^2d^4g^4 - (12g^4 \log(e) - 25g^4) * a^5d^5) * B^2) * \log(bx + a) + 2(12B^2b^5d^5g^4x^5 \log(e) + 3(b^5c^4d^4g^4 + (20g^4 \log(e) - g^4) * a^4d^5) * B^2x^4 - 4(b^5c^2d^3g^4 - 5a^4b^4c^2d^4g^4 - 2(15g^4 \log(e) - 2g^4) * a^2b^3d^5) * B^2x^3 + 6(b^5c^3d^2g^4 - 5a^4b^4c^2d^3g^4 + 10a^2b^3c^2d^4g^4 + 2(10g^4 \log(e) - 3g^4) * a^3b^2d^5) * B^2x^2 - 12(b^5c^4d^2g^4 - 5a^4b^4c^3d^2g^4 + 10a^2b^3c^2d^3g^4 - 10a^3b^2c^2d^4g^4 - (5g^4 \log(e) - 4g^4) * a^4b^2d^5) * B^2x - 12(B^2b^5d^5g^4x^5 + 5B^2a^4b^4d^5g^4x^4 + 10B^2a^2b^3d^5g^4x^3 + 10B^2a^3b^2d^5g^4x^2 + 5B^2a^4b^2d^5g^4x + B^2a^5d^5g^4) * \log(bx + a)) * \log(dx + c))/(bd^5)
\end{aligned}$$


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**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( A^2b^4g^4x^4 + 4A^2ab^3g^4x^3 + 6A^2a^2b^2g^4x^2 + 4A^2a^3bg^4x + A^2a^4g^4 + (B^2b^4g^4x^4 + 4B^2ab^3g^4x^3 + 6B^2a^2b^2g^4x^2 + 4B^2a^3bg^4x + B^2a^4g^4) * \log((d * e * x + c * e) / (b * x + a)) \right)^2, x, \operatorname{algorithm} = \text{"fricas"}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas")`

[Out] `integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((d*e*x + c*e)/(b*x + a))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log((d*e*x + c*e)/(b*x + a)), x)`

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*4\*(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))\*\*2,x)

[Out] Timed out

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**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^4 \left( B \log \left( \frac{(dx + c)e}{bx + a} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^4\*(B\*log((d\*x + c)\*e/(b\*x + a)) + A)^2, x)

$$3.183 \quad \int (ag + bgx)^3 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2 dx$$

**Optimal.** Leaf size=420

$$\frac{B^2 g^3 (bc - ad)^4 \text{PolyLog} \left( 2, \frac{d(a+bx)}{b(c+dx)} \right)}{2bd^4} + \frac{Bg^3 (bc - ad)^4 \log \left( 1 - \frac{d(a+bx)}{b(c+dx)} \right) \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{2bd^4} + \frac{Bg^3 (c + dx)(bc - ad)^3}{2bd^4}$$

[Out]  $(-5*B^2*(b*c - a*d)^3*g^3*x)/(12*d^3) + (B^2*(b*c - a*d)^2*g^3*(a + b*x)^2)/(12*b*d^2) + (11*B^2*(b*c - a*d)^4*g^3*\text{Log}[a + b*x])/(12*b*d^4) + (5*B^2*(b*c - a*d)^4*g^3*\text{Log}[(c + d*x)/(a + b*x])/(12*b*d^4) - (B*(b*c - a*d)^2*g^3*(a + b*x)^2*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x]))/(4*b*d^2) + (B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x]))/(6*b*d) + (B*(b*c - a*d)^3*g^3*(c + d*x)*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x]))/(2*d^4) + (g^3*(a + b*x)^4*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x]))^2/(4*b) + (B*(b*c - a*d)^4*g^3*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x]))*\text{Log}[1 - (d*(a + b*x))/(b*(c + d*x))])/(2*b*d^4) - (B^2*(b*c - a*d)^4*g^3*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b*d^4)$

**Rubi [A]** time = 0.648144, antiderivative size = 474, normalized size of antiderivative = 1.13, number of steps used = 24, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{B^2 g^3 (bc - ad)^4 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right)}{2bd^4} - \frac{Bg^3 (bc - ad)^4 \log(c + dx) \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{2bd^4} - \frac{Bg^3 (a + bx)^2 (bc - ad)^2 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{4bd^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^3*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x)])^2, x]$

[Out]  $(A*B*(b*c - a*d)^3*g^3*x)/(2*d^3) - (5*B^2*(b*c - a*d)^3*g^3*x)/(12*d^3) + (B^2*(b*c - a*d)^2*g^3*(a + b*x)^2)/(12*b*d^2) + (11*B^2*(b*c - a*d)^4*g^3*\text{Log}[c + d*x])/(12*b*d^4) - (B^2*(b*c - a*d)^4*g^3*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(2*b*d^4) + (B^2*(b*c - a*d)^4*g^3*\text{Log}[c + d*x]^2)/(4*b*d^4) + (B^2*(b*c - a*d)^3*g^3*(a + b*x)*\text{Log}[(e*(c + d*x))/(a + b*x)])/(2*b*d^3) - (B*(b*c - a*d)^2*g^3*(a + b*x)^2*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x)])))/(4*b*d^2) + (B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x)])))/(6*b*d) - (B*(b*c - a*d)^4*g^3*\text{Log}[c + d*x]*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x)])))/(2*b*d^4) + (g^3*(a + b*x)^4*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x]))^2)/(4*b) - (B^2*(b*c - a*d)^4*g^3*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(2*b*d^4)$

**Rule 2525**

$\text{Int}[(a_.) + \text{Log}[(c_.)*(RFx_)^(p_.)]*(b_.)]^(n_.)*(d_.) + (e_.)*(x_.)]^(m_.), x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^(m + 1)*(a + b*\text{Log}[c*RFx^p])^n/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^(m + 1)*(a + b*\text{Log}[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x] \&\& \text{RationalFunctionQ}[RFx, x] \&\& \text{IGtQ}[n, 0] \&\& (\text{EqQ}[n, 1] || \text{IntegerQ}[m]) \&\& \text{NeQ}[m, -1]$

**Rule 12**

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)
^q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391



Int[Log[(c\_.)\*(d\_) + (e\_.)\*(x\_)^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*(d\_) + (e\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rubi steps

$$\begin{aligned}
 \int (ag + bgx)^3 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2 dx &= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2}{4b} - \frac{B \int \frac{(bc-ad)g^4(a+bx)^3 \left( -A - B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{c+dx}}{2bg} \\
 &= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2}{4b} - \frac{(B(bc-ad)g^3) \int \frac{(a+bx)^3 \left( -A - B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{c+dx}}{2b} \\
 &= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2}{4b} - \frac{(B(bc-ad)g^3) \int \left( \frac{b(bc-ad)^2 \left( -A - B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{d^3} \right)}{2b} \\
 &= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2}{4b} - \frac{(B(bc-ad)g^3) \int (a+bx)^2 \left( -A - B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{2d} \\
 &= \frac{AB(bc-ad)^3 g^3 x}{2d^3} - \frac{B(bc-ad)^2 g^3 (a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{4bd^2} + \frac{B(bc-ad)^2 g^3 (a+bx)^2}{2bd^2} \\
 &= \frac{AB(bc-ad)^3 g^3 x}{2d^3} + \frac{B^2(bc-ad)^3 g^3 (a+bx) \log \left( \frac{e(c+dx)}{a+bx} \right)}{2bd^3} - \frac{B(bc-ad)^2 g^3 (a+bx)^2}{2bd^2} \\
 &= \frac{AB(bc-ad)^3 g^3 x}{2d^3} + \frac{B^2(bc-ad)^4 g^3 \log(c+dx)}{2bd^4} + \frac{B^2(bc-ad)^3 g^3 (a+bx)^2}{2bd^3} \\
 &= \frac{AB(bc-ad)^3 g^3 x}{2d^3} - \frac{5B^2(bc-ad)^3 g^3 x}{12d^3} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{12bd^2} + \frac{11B^2(bc-ad)^3 g^3 (a+bx)^2}{12bd^3} \\
 &= \frac{AB(bc-ad)^3 g^3 x}{2d^3} - \frac{5B^2(bc-ad)^3 g^3 x}{12d^3} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{12bd^2} + \frac{11B^2(bc-ad)^3 g^3 (a+bx)^2}{12bd^3} \\
 &= \frac{AB(bc-ad)^3 g^3 x}{2d^3} - \frac{5B^2(bc-ad)^3 g^3 x}{12d^3} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{12bd^2} + \frac{11B^2(bc-ad)^3 g^3 (a+bx)^2}{12bd^3} \\
 &= \frac{AB(bc-ad)^3 g^3 x}{2d^3} - \frac{5B^2(bc-ad)^3 g^3 x}{12d^3} + \frac{B^2(bc-ad)^2 g^3 (a+bx)^2}{12bd^2} + \frac{11B^2(bc-ad)^3 g^3 (a+bx)^2}{12bd^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.35497, size = 392, normalized size = 0.93

$$g^3 \left( \frac{B(bc-ad) \left( -3B(bc-ad)^3 \left( 2 \operatorname{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) + 2d^3(a+bx)^3 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right) + 3d^2(a+bx)^2(ad-bc) \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{12bd^3} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g^3*((a + b*x)^4*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 + (B*(b*c - a*d)*
6*A*b*d*(b*c - a*d)^2*x + 6*B*(b*c - a*d)^3*Log[a + b*x] - B*(b*c - a*d)*(2
*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) - 3*B*
(b*c - a*d)^2*(b*d*x + (-b*c) + a*d)*Log[c + d*x]) + 6*b*B*(b*c - a*d)^2*(
c + d*x)*Log[(e*(c + d*x))/(a + b*x)] + 3*d^2*(-b*c) + a*d)*(a + b*x)^2*(A
+ B*Log[(e*(c + d*x))/(a + b*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(c + d
*x))/(a + b*x)]) - 6*(b*c - a*d)^3*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a
+ b*x)]) - 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-b*c) + a*d] - Log[c
+ d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4)
)/(4*b)
```

**Maple [F]** time = 1.958, size = 0, normalized size = 0.

$$\int (bgx + ag)^3 \left( A + B \ln \left( \frac{e(dx + c)}{bx + a} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

```
[Out] int((b*g*x+a*g)^3*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

**Maxima [B]** time = 1.68998, size = 2342, normalized size = 5.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] 1/4*A^2*b^3*g^3*x^4 + A^2*a*b^2*g^3*x^3 + 3/2*A^2*a^2*b*g^3*x^2 + 2*(x*log(
d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A*B
*a^3*g^3 + 3*(x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b
^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*B*a^2*b*g^3 + (2*x^3*log
(d*e*x/(b*x + a) + c*e/(b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x
+ c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A
*B*a*b^2*g^3 + 1/12*(6*x^4*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + 6*a^4*log
(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*
g^3 + A^2*a^3*g^3*x - 1/12*((6*g^3*log(e) - 11*g^3)*b^3*c^4 - 2*(12*g^3*log
(e) - 19*g^3)*a*b^2*c^3*d + 9*(4*g^3*log(e) - 5*g^3)*a^2*b*c^2*d^2 - 6*(4*g
^3*log(e) - 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 1/2*(b^4*c^4*g^3 - 4*a
*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d^4*g^3)*(
log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 2*(b^4*c*d^3
*g^3*log(e) + (6*g^3*log(e)^2 - g^3*log(e))*a*b^3*d^4)*B^2*x^3 - ((3*g^3*lo
g(e) - g^3)*b^4*c^2*d^2 - 2*(6*g^3*log(e) - g^3)*a*b^3*c*d^3 - (18*g^3*log(
e)^2 - 9*g^3*log(e) + g^3)*a^2*b^2*d^4)*B^2*x^2 + ((6*g^3*log(e) - 5*g^3)*b
^4*c^3*d - (24*g^3*log(e) - 17*g^3)*a*b^3*c^2*d^2 + (36*g^3*log(e) - 19*g^3
```

$$\begin{aligned} & ) * a^2 * b^2 * c * d^3 + (12 * g^3 * \log(e)^2 - 18 * g^3 * \log(e) + 7 * g^3) * a^3 * b * d^4) * B^2 * \\ & x + 3 * (B^2 * b^4 * d^4 * g^3 * x^4 + 4 * B^2 * a * b^3 * d^4 * g^3 * x^3 + 6 * B^2 * a^2 * b^2 * d^4 * g^3 * \\ & x^2 + 4 * B^2 * a^3 * b * d^4 * g^3 * x + B^2 * a^4 * d^4 * g^3) * \log(b * x + a)^2 + 3 * (B^2 * b^4 * \\ & d^4 * g^3 * x^4 + 4 * B^2 * a * b^3 * d^4 * g^3 * x^3 + 6 * B^2 * a^2 * b^2 * d^4 * g^3 * x^2 + 4 * B^2 * \\ & a^3 * b * d^4 * g^3 * x - (b^4 * c^4 * g^3 - 4 * a * b^3 * c^3 * d * g^3 + 6 * a^2 * b^2 * c^2 * d^2 * g^3 \\ & - 4 * a^3 * b * c * d^3 * g^3) * B^2) * \log(d * x + c)^2 - (6 * B^2 * b^4 * d^4 * g^3 * x^4 * \log(e) + \\ & 2 * (b^4 * c * d^3 * g^3 + (12 * g^3 * \log(e) - g^3) * a * b^3 * d^4) * B^2 * x^3 - 3 * (b^4 * c^2 * d^2 * \\ & g^3 - 4 * a * b^3 * c * d^3 * g^3 - 3 * (4 * g^3 * \log(e) - g^3) * a^2 * b^2 * d^4) * B^2 * x^2 + \\ & 6 * (b^4 * c^3 * d * g^3 - 4 * a * b^3 * c^2 * d^2 * g^3 + 6 * a^2 * b^2 * c * d^3 * g^3 + (4 * g^3 * \log(e) \\ & ) - 3 * g^3) * a^3 * b * d^4) * B^2 * x + (6 * a * b^3 * c^3 * d * g^3 - 21 * a^2 * b^2 * c^2 * d^2 * g^3 + \\ & 26 * a^3 * b * c * d^3 * g^3 + (6 * g^3 * \log(e) - 11 * g^3) * a^4 * d^4) * B^2) * \log(b * x + a) + \\ & (6 * B^2 * b^4 * d^4 * g^3 * x^4 * \log(e) + 2 * (b^4 * c * d^3 * g^3 + (12 * g^3 * \log(e) - g^3) * a * \\ & b^3 * d^4) * B^2 * x^3 - 3 * (b^4 * c^2 * d^2 * g^3 - 4 * a * b^3 * c * d^3 * g^3 - 3 * (4 * g^3 * \log(e) \\ & - g^3) * a^2 * b^2 * d^4) * B^2 * x^2 + 6 * (b^4 * c^3 * d * g^3 - 4 * a * b^3 * c^2 * d^2 * g^3 + 6 * a^2 * \\ & b^2 * c * d^3 * g^3 + (4 * g^3 * \log(e) - 3 * g^3) * a^3 * b * d^4) * B^2 * x - 6 * (B^2 * b^4 * d^4 * \\ & g^3 * x^4 + 4 * B^2 * a * b^3 * d^4 * g^3 * x^3 + 6 * B^2 * a^2 * b^2 * d^4 * g^3 * x^2 + 4 * B^2 * a^3 * \\ & b * d^4 * g^3 * x + B^2 * a^4 * d^4 * g^3) * \log(b * x + a)) * \log(d * x + c)) / (b * d^4) \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(A^2 b^3 g^3 x^3 + 3 A^2 a b^2 g^3 x^2 + 3 A^2 a^2 b g^3 x + A^2 a^3 g^3 + (B^2 b^3 g^3 x^3 + 3 B^2 a b^2 g^3 x^2 + 3 B^2 a^2 b g^3 x + B^2 a^3 g^3) \log\left(\frac{d * x + c}{b * x + a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="fricas")

[Out] integral(A^2\*b^3\*g^3\*x^3 + 3\*A^2\*a\*b^2\*g^3\*x^2 + 3\*A^2\*a^2\*b\*g^3\*x + A^2\*a^3\*g^3 + (B^2\*b^3\*g^3\*x^3 + 3\*B^2\*a\*b^2\*g^3\*x^2 + 3\*B^2\*a^2\*b\*g^3\*x + B^2\*a^3\*g^3)\*log((d\*e\*x + c\*e)/(b\*x + a))^2 + 2\*(A\*B\*b^3\*g^3\*x^3 + 3\*A\*B\*a\*b^2\*g^3\*x^2 + 3\*A\*B\*a^2\*b\*g^3\*x + A\*B\*a^3\*g^3)\*log((d\*e\*x + c\*e)/(b\*x + a)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*3\*(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^3 \left( B \log\left(\frac{dx + c}{bx + a}\right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^3\*(B\*log((d\*x + c)\*e/(b\*x + a)) + A)^2, x)

$$3.184 \quad \int (ag + bgx)^2 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2 dx$$

**Optimal.** Leaf size=335

$$\frac{2B^2g^2(bc-ad)^3 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} - \frac{2Bg^2(bc-ad)^3 \log\left(1 - \frac{d(a+bx)}{b(c+dx)}\right) \left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{3bd^3} - \frac{2Bg^2(c+dx)(bc-ad)^2}{3d^3}$$

[Out] (B^2\*(b\*c - a\*d)^2\*g^2\*x)/(3\*d^2) - (B^2\*(b\*c - a\*d)^3\*g^2\*Log[a + b\*x])/(b\*d^3) - (B^2\*(b\*c - a\*d)^3\*g^2\*Log[(c + d\*x)/(a + b\*x)])/(3\*b\*d^3) + (B\*(b\*c - a\*d)\*g^2\*(a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(3\*b\*d) - (2\*B\*(b\*c - a\*d)^2\*g^2\*(c + d\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(3\*d^3) + (g^2\*(a + b\*x)^3\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]])^2)/(3\*b) - (2\*B\*(b\*c - a\*d)^3\*g^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]])\*Log[1 - (d\*(a + b\*x))/(b\*(c + d\*x)]))/(3\*b\*d^3) + (2\*B^2\*(b\*c - a\*d)^3\*g^2\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x)]))/(3\*b\*d^3)

**Rubi [A]** time = 0.555908, antiderivative size = 389, normalized size of antiderivative = 1.16, number of steps used = 20, number of rules used = 13, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.406$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{2B^2g^2(bc-ad)^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3bd^3} + \frac{2Bg^2(bc-ad)^3 \log(c+dx) \left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{3bd^3} - \frac{2ABg^2x(bc-ad)^2}{3d^2} + \frac{Bg^2(a+bx)^2}{3d^2}$$

Antiderivative was successfully verified.

[In] Int[(a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2,x]

[Out] (-2\*A\*B\*(b\*c - a\*d)^2\*g^2\*x)/(3\*d^2) + (B^2\*(b\*c - a\*d)^2\*g^2\*x)/(3\*d^2) - (B^2\*(b\*c - a\*d)^3\*g^2\*Log[c + d\*x])/(b\*d^3) + (2\*B^2\*(b\*c - a\*d)^3\*g^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))\*Log[c + d\*x])/(3\*b\*d^3) - (B^2\*(b\*c - a\*d)^3\*g^2\*Log[c + d\*x]^2)/(3\*b\*d^3) - (2\*B^2\*(b\*c - a\*d)^2\*g^2\*(a + b\*x)\*Log[(e\*(c + d\*x))/(a + b\*x)]/(3\*b\*d^2) + (B\*(b\*c - a\*d)\*g^2\*(a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(3\*b\*d) + (2\*B\*(b\*c - a\*d)^3\*g^2\*Log[c + d\*x]\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(3\*b\*d^3) + (g^2\*(a + b\*x)^3\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]])^2)/(3\*b) + (2\*B^2\*(b\*c - a\*d)^3\*g^2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]/(3\*b\*d^3)

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
 \int (ag + bgx)^2 \left( A + B \log \left( \frac{e(c + dx)}{a + bx} \right) \right)^2 dx &= \frac{g^2(a + bx)^3 \left( A + B \log \left( \frac{e(c + dx)}{a + bx} \right) \right)^2}{3b} - \frac{(2B) \int \frac{(bc - ad)g^3(a + bx)^2 \left( -A - B \log \left( \frac{e(c + dx)}{a + bx} \right) \right)}{c + dx}}{3bg} \\
 &= \frac{g^2(a + bx)^3 \left( A + B \log \left( \frac{e(c + dx)}{a + bx} \right) \right)^2}{3b} - \frac{(2B(bc - ad)g^2) \int \frac{(a + bx)^2 \left( -A - B \log \left( \frac{e(c + dx)}{a + bx} \right) \right)}{c + dx}}{3b} \\
 &= \frac{g^2(a + bx)^3 \left( A + B \log \left( \frac{e(c + dx)}{a + bx} \right) \right)^2}{3b} - \frac{(2B(bc - ad)g^2) \int \left( -\frac{b(bc - ad) \left( -A - B \log \left( \frac{e(c + dx)}{a + bx} \right) \right)}{d^2} \right)}{3b} \\
 &= \frac{g^2(a + bx)^3 \left( A + B \log \left( \frac{e(c + dx)}{a + bx} \right) \right)^2}{3b} - \frac{(2B(bc - ad)g^2) \int (a + bx) \left( -A - B \log \left( \frac{e(c + dx)}{a + bx} \right) \right)}{3d} \\
 &= -\frac{2AB(bc - ad)^2 g^2 x}{3d^2} + \frac{B(bc - ad)g^2(a + bx)^2 \left( A + B \log \left( \frac{e(c + dx)}{a + bx} \right) \right)}{3bd} + \frac{2B(bc - ad)g^2(a + bx)^2}{3bd} \\
 &= -\frac{2AB(bc - ad)^2 g^2 x}{3d^2} - \frac{2B^2(bc - ad)^2 g^2(a + bx) \log \left( \frac{e(c + dx)}{a + bx} \right)}{3bd^2} + \frac{B(bc - ad)g^2(a + bx)^2}{3bd} \\
 &= -\frac{2AB(bc - ad)^2 g^2 x}{3d^2} - \frac{2B^2(bc - ad)^3 g^2 \log(c + dx)}{3bd^3} - \frac{2B^2(bc - ad)^2 g^2(a + bx)}{3bd^2} \\
 &= -\frac{2AB(bc - ad)^2 g^2 x}{3d^2} + \frac{B^2(bc - ad)^2 g^2 x}{3d^2} - \frac{B^2(bc - ad)^3 g^2 \log(c + dx)}{bd^3} - \frac{2B^2(bc - ad)^2 g^2(a + bx)}{3bd^2} \\
 &= -\frac{2AB(bc - ad)^2 g^2 x}{3d^2} + \frac{B^2(bc - ad)^2 g^2 x}{3d^2} - \frac{B^2(bc - ad)^3 g^2 \log(c + dx)}{bd^3} + \frac{2B^2(bc - ad)^2 g^2(a + bx)}{3bd^2} \\
 &= -\frac{2AB(bc - ad)^2 g^2 x}{3d^2} + \frac{B^2(bc - ad)^2 g^2 x}{3d^2} - \frac{B^2(bc - ad)^3 g^2 \log(c + dx)}{bd^3} + \frac{2B^2(bc - ad)^2 g^2(a + bx)}{3bd^2} \\
 &= -\frac{2AB(bc - ad)^2 g^2 x}{3d^2} + \frac{B^2(bc - ad)^2 g^2 x}{3d^2} - \frac{B^2(bc - ad)^3 g^2 \log(c + dx)}{bd^3} + \frac{2B^2(bc - ad)^2 g^2(a + bx)}{3bd^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.238731, size = 290, normalized size = 0.87

$$\frac{g^2 \left( (a + bx)^3 \left( B \log \left( \frac{e(c + dx)}{a + bx} \right) + A \right)^2 - \frac{B(bc - ad) \left( -B(bc - ad)^2 \left( 2 \text{PolyLog} \left( 2, \frac{b(c + dx)}{bc - ad} \right) + \log(c + dx) \right) \left( 2 \log \left( \frac{d(a + bx)}{ad - bc} \right) - \log(c + dx) \right) \right) - d^2(a + bx)^2 \left( B \log \left( \frac{e(c + dx)}{a + bx} \right) + A \right)^2}{3b}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2,x]
```

```
[Out] (g^2*((a + b*x)^3*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2 - (B*(b*c - a*d)*(
2*A*b*d*(b*c - a*d)*x + 2*B*(b*c - a*d)^2*Log[a + b*x] - B*(b*c - a*d)*(b*d
*x + (-b*c) + a*d)*Log[c + d*x]) + 2*b*B*(b*c - a*d)*(c + d*x)*Log[(e*(c +
d*x))/(a + b*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x))/(a + b*x)]) -
2*(b*c - a*d)^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]) - B*(b*c
- a*d)^2*((2*Log[(d*(a + b*x))/(-b*c) + a*d]] - Log[c + d*x])*Log[c + d*x]
+ 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/d^3)/(3*b)
```

**Maple [F]** time = 1.943, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( \frac{e(dx + c)}{bx + a} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

```
[Out] int((b*g*x+a*g)^2*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)
```

**Maxima [B]** time = 1.76125, size = 1582, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*A^2*b^2*g^2*x^3 + A^2*a*b*g^2*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*x
+ a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A*B*a^2*g^2 + 2*(x^2*log(d*e*
x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2
+ (b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + 1/3*(2*x^3*log(d*e*x/(b*x + a) + c*e/(
b*x + a)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a
*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*g^2 + A^2*a^2*g^2
*x + 1/3*((2*g^2*log(e) - 3*g^2)*b^2*c^3 - (6*g^2*log(e) - 7*g^2)*a*b*c^2*d
+ 2*(3*g^2*log(e) - 2*g^2)*a^2*c*d^2)*B^2*log(d*x + c)/d^3 - 2/3*(b^3*c^3*
g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*x + a)*lo
g((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(
b*d^3) + 1/3*(B^2*b^3*d^3*g^2*x^3*log(e)^2 + (b^3*c*d^2*g^2*log(e) + (3*g^2
*log(e)^2 - g^2*log(e))*a*b^2*d^3)*B^2*x^2 - ((2*g^2*log(e) - g^2)*b^3*c^2*
d - 2*(3*g^2*log(e) - g^2)*a*b^2*c*d^2 - (3*g^2*log(e)^2 - 4*g^2*log(e) + g
^2)*a^2*b*d^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B
^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3
+ 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2
*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 - (2*B^2*b^3*d^3*g^2*x^
3*log(e) + (b^3*c*d^2*g^2 + (6*g^2*log(e) - g^2)*a*b^2*d^3)*B^2*x^2 - 2*(b^
3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 - (3*g^2*log(e) - 2*g^2)*a^2*b*d^3)*B^2*x -
(2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 - (2*g^2*log(e) - 3*g^2)*a^3*d^3)*B
^2)*log(b*x + a) + (2*B^2*b^3*d^3*g^2*x^3*log(e) + (b^3*c*d^2*g^2 + (6*g^2*
log(e) - g^2)*a*b^2*d^3)*B^2*x^2 - 2*(b^3*c^2*d*g^2 - 3*a*b^2*c*d^2*g^2 - (
3*g^2*log(e) - 2*g^2)*a^2*b*d^3)*B^2*x - 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b
^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a))*log
(d*x + c))/(b*d^3)
```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2 b^2 g^2 x^2 + 2 A^2 a b g^2 x + A^2 a^2 g^2 + (B^2 b^2 g^2 x^2 + 2 B^2 a b g^2 x + B^2 a^2 g^2) \log \left( \frac{d e x + c e}{b x + a} \right)^2 + 2 (A B b^2 g^2 x^2 + 2 A B a b g^2 x + A B a^2 g^2) \log \left( \frac{d e x + c e}{b x + a} \right), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="fricas")

[Out] integral(A^2\*b^2\*g^2\*x^2 + 2\*A^2\*a\*b\*g^2\*x + A^2\*a^2\*g^2 + (B^2\*b^2\*g^2\*x^2 + 2\*B^2\*a\*b\*g^2\*x + B^2\*a^2\*g^2)\*log((d\*e\*x + c\*e)/(b\*x + a))^2 + 2\*(A\*B\*b^2\*g^2\*x^2 + 2\*A\*B\*a\*b\*g^2\*x + A\*B\*a^2\*g^2)\*log((d\*e\*x + c\*e)/(b\*x + a)), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*2\*(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (b g x + a g)^2 \left( B \log \left( \frac{(d x + c) e}{b x + a} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^2\*(B\*log((d\*x + c)\*e/(b\*x + a)) + A)^2, x)



$$3.185 \quad \int (ag + bgx) \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2 dx$$

**Optimal.** Leaf size=202

$$\frac{B^2 g(bc - ad)^2 \text{PolyLog} \left( 2, \frac{d(a+bx)}{b(c+dx)} \right)}{bd^2} + \frac{Bg(bc - ad)^2 \log \left( 1 - \frac{d(a+bx)}{b(c+dx)} \right) \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{bd^2} + \frac{Bg(c + dx)(bc - ad) \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{d^2}$$

[Out] (B^2\*(b\*c - a\*d)^2\*g\*Log[a + b\*x])/(b\*d^2) + (B\*(b\*c - a\*d)\*g\*(c + d\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/d^2 + (g\*(a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]^2)/(2\*b) + (B\*(b\*c - a\*d)^2\*g\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]\*Log[1 - (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*d^2) - (B^2\*(b\*c - a\*d)^2\*g\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*d^2)

**Rubi [A]** time = 0.417675, antiderivative size = 284, normalized size of antiderivative = 1.41, number of steps used = 16, number of rules used = 12, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{B^2 g(bc - ad)^2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right)}{bd^2} - \frac{Bg(bc - ad)^2 \log(c + dx) \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{bd^2} + \frac{g(a + bx)^2 \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right)}{2b}$$

Antiderivative was successfully verified.

[In] Int[(a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2,x]

[Out] (A\*B\*(b\*c - a\*d)\*g\*x)/d + (B^2\*(b\*c - a\*d)^2\*g\*Log[c + d\*x])/(b\*d^2) - (B^2\*(b\*c - a\*d)^2\*g\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x])/(b\*d^2) + (B^2\*(b\*c - a\*d)^2\*g\*Log[c + d\*x]^2)/(2\*b\*d^2) + (B^2\*(b\*c - a\*d)\*g\*(a + b\*x)\*Log[(e\*(c + d\*x))/(a + b\*x)]/(b\*d) - (B\*(b\*c - a\*d)^2\*g\*Log[c + d\*x]\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))/(b\*d^2) + (g\*(a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]^2)/(2\*b) - (B^2\*(b\*c - a\*d)^2\*g\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/(b\*d^2)

#### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFX^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFX^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFX_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFX, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFX, x] && IntegerQ[p]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
]^n))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_))^(n_.)]*(b_.)/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rubi steps

$$\begin{aligned}
\int (ag + bgx) \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2 dx &= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2}{2b} - \frac{B \int \frac{(bc-ad)g^2(a+bx) \left( -A - B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{c+dx} dx}{bg} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2}{2b} - \frac{(B(bc-ad)g) \int \frac{(a+bx) \left( -A - B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{c+dx}}{b} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2}{2b} - \frac{(B(bc-ad)g) \int \left( \frac{b \left( -A - B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{d} + \right)}{b} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)^2}{2b} - \frac{(B(bc-ad)g) \int \left( -A - B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{d} \\
&= \frac{AB(bc-ad)gx}{d} - \frac{B(bc-ad)^2 g \log(c+dx) \left( A + B \log \left( \frac{e(c+dx)}{a+bx} \right) \right)}{bd^2} + \frac{g(a+bx)^2}{2b} \\
&= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)g(a+bx) \log \left( \frac{e(c+dx)}{a+bx} \right)}{bd} - \frac{B(bc-ad)^2 g \log(c+dx)}{bd^2} \\
&= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g \log(c+dx)}{bd^2} + \frac{B^2(bc-ad)g(a+bx) \log \left( \frac{e(c+dx)}{a+bx} \right)}{bd} \\
&= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g \log(c+dx)}{bd^2} + \frac{B^2(bc-ad)g(a+bx) \log \left( \frac{e(c+dx)}{a+bx} \right)}{bd} \\
&= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g \log(c+dx)}{bd^2} - \frac{B^2(bc-ad)^2 g \log \left( -\frac{d(a+bx)}{bc-ad} \right)}{bd^2} \\
&= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g \log(c+dx)}{bd^2} - \frac{B^2(bc-ad)^2 g \log \left( -\frac{d(a+bx)}{bc-ad} \right)}{bd^2} \\
&= \frac{AB(bc-ad)gx}{d} + \frac{B^2(bc-ad)^2 g \log(c+dx)}{bd^2} - \frac{B^2(bc-ad)^2 g \log \left( -\frac{d(a+bx)}{bc-ad} \right)}{bd^2}
\end{aligned}$$

**Mathematica [A]** time = 0.185652, size = 203, normalized size = 1.

$$g \left( \frac{B(bc-ad) \left( -B(bc-ad) \left( 2 \operatorname{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) - 2(bc-ad) \log(c+dx) \left( B \log \left( \frac{e(c+dx)}{a+bx} \right) + A \right) + 2bB(c+dx) \log \left( \frac{e(c+dx)}{a+bx} \right) + 2g(a+bx)^2}{d^2} \right)$$


---

2b

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2,x]

[Out] (g\*((a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2 + (B\*(b\*c - a\*d)\*(2\*A\*b\*d\*x + 2\*B\*(b\*c - a\*d)\*Log[a + b\*x] + 2\*b\*B\*(c + d\*x)\*Log[(e\*(c + d\*x))/(a + b\*x]) - 2\*(b\*c - a\*d)\*Log[c + d\*x]\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]) - B\*(b\*c - a\*d)\*((2\*Log[(d\*(a + b\*x))/(-b\*c) + a\*d]) - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]))/d^2))/(2\*b)

**Maple [F]** time = 1.7, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln \left( \frac{e(dx+c)}{bx+a} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)`

[Out] `int((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)`

**Maxima [B]** time = 1.69013, size = 836, normalized size = 4.14

$$\frac{1}{2} A^2 b g x^2 + 2 \left( x \log \left( \frac{d e x}{b x + a} + \frac{c e}{b x + a} \right) - \frac{a \log(b x + a)}{b} + \frac{c \log(d x + c)}{d} \right) A B a g + \left( x^2 \log \left( \frac{d e x}{b x + a} + \frac{c e}{b x + a} \right) + \frac{a^2 \log(b x + a)}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima")`

[Out] `1/2*A^2*b*g*x^2 + 2*(x*log(d*e*x/(b*x + a) + c*e/(b*x + a)) - a*log(b*x + a)/b + c*log(d*x + c)/d)*A*B*a*g + (x^2*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))*A*B*b*g + A^2*a*g*x - ((g*log(e) - g)*b*c^2 - (2*g*log(e) - g)*a*c*d)*B^2*log(d*x + c)/d^2 + (b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*log(e)^2 + 2*(b^2*c*d*g*log(e) + (g*log(e)^2 - g*log(e))*a*b*d^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*log(d*x + c)^2 - 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x + ((g*log(e) - g)*a^2*d^2 + a*b*c*d*g)*B^2)*log(b*x + a) + 2*(B^2*b^2*d^2*g*x^2*log(e) + ((2*g*log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x - (B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log(b*x + a))*log(d*x + c))/(b*d^2)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2 b g x + A^2 a g + (B^2 b g x + B^2 a g) \log \left( \frac{d e x + c e}{b x + a} \right)^2 + 2 (A B b g x + A B a g) \log \left( \frac{d e x + c e}{b x + a} \right), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)*(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas")`

[Out] `integral(A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((d*e*x + c*e)/(b*x + a))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((d*e*x + c*e)/(b*x + a)), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)*(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag) \left( B \log \left( \frac{(dx + c)e}{bx + a} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)\*(B\*log((d\*x + c)\*e/(b\*x + a)) + A)^2, x)

$$3.186 \quad \int \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag+bgx} dx$$

**Optimal.** Leaf size=128

$$\frac{2BPolyLog\left(2, \frac{b(c+dx)}{d(a+bx)}\right)\left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{bg} + \frac{2B^2PolyLog\left(3, \frac{b(c+dx)}{d(a+bx)}\right)}{bg} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)^2}{bg}$$

[Out] -((Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2)/(b\*g)) - (2\*B\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])\*PolyLog[2, (b\*(c + d\*x))/(d\*(a + b\*x))])/(b\*g) + (2\*B^2\*PolyLog[3, (b\*(c + d\*x))/(d\*(a + b\*x))])/(b\*g)

**Rubi [B]** time = 4.42328, antiderivative size = 719, normalized size of antiderivative = 5.62, number of steps used = 47, number of rules used = 24, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$ , Rules used = {2524, 12, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6691, 6741, 6742, 2499, 2302, 30, 2396, 2433, 2374, 6589, 2500, 2440, 2434, 2375, 2317}

$$\frac{2ABPolyLog\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg} + \frac{2B^2PolyLog\left(2, -\frac{d(a+bx)}{bc-ad}\right)\left(-\log\left(\frac{e(c+dx)}{a+bx}\right) + \log\left(\frac{1}{a+bx}\right) + \log(c + dx)\right)}{bg} - \frac{2B^2PolyLog\left(3, -\frac{d(a+bx)}{bc-ad}\right)}{bg}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2/(a\*g + b\*g\*x), x]

[Out] (A\*B\*Log[g\*(a + b\*x)]^2)/(b\*g) + (B^2\*Log[g\*(a + b\*x)]^3)/(3\*b\*g) - (B^2\*Log[(a + b\*x)^(-1)]^2\*Log[c + d\*x])/(b\*g) - (2\*B^2\*Log[(a + b\*x)^(-1)]\*Log[g\*(a + b\*x)]\*Log[c + d\*x])/(b\*g) - (B^2\*Log[g\*(a + b\*x)]^2\*Log[c + d\*x])/(b\*g) + (B^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x]^2)/(b\*g) - (B^2\*Log[g\*(a + b\*x)]\*Log[c + d\*x]^2)/(b\*g) + (B^2\*Log[(a + b\*x)^(-1)]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g) + (B^2\*Log[g\*(a + b\*x)]^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g) - (2\*A\*B\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*Log[a\*g + b\*g\*x])/(b\*g) + (2\*B^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*(Log[(a + b\*x)^(-1)] + Log[c + d\*x] - Log[(e\*(c + d\*x))/(a + b\*x)])\*Log[a\*g + b\*g\*x])/(b\*g) + ((A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2\*Log[a\*g + b\*g\*x])/(b\*g) - (B^2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*Log[a\*g + b\*g\*x]^2)/(b\*g) + (B^2\*Log[(e\*(c + d\*x))/(a + b\*x)]\*Log[a\*g + b\*g\*x]^2)/(b\*g) - (2\*A\*B\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g) - (2\*B^2\*Log[(a + b\*x)^(-1)]\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g) + (2\*B^2\*(Log[(a + b\*x)^(-1)] + Log[c + d\*x] - Log[(e\*(c + d\*x))/(a + b\*x)])\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g) + (2\*B^2\*Log[c + d\*x]\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g) - (2\*B^2\*PolyLog[3, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g) - (2\*B^2\*PolyLog[3, (b\*(c + d\*x))/(b\*c - a\*d)])/(b\*g)

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(Rfx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 6691

Int[(u\_)^(m\_.)\*((a\_.)\*(u\_)^(n\_) + (v\_))^(p\_.)\*(w\_), x\_Symbol] := Int[u^(m + n\*p)\*(a + v/u^n)^p\*w, x] /; FreeQ[{a, m, n}, x] && IntegerQ[p] && !GtQ[n, 0] && !FreeQ[v, x]

#### Rule 6741

Int[u\_, x\_Symbol] := With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2499

```
Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.))*((c_.) + (d_.)*(x_.))^(q_.)
)^(r_.)]*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_.))^(n_.)]*(t_.))^(m_.))/((j_.
) + (k_.)*(x_.)), x_Symbol] := Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q]^r)]/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]
```

Rule 2302

```
Int[((a_.) + Log[(c_.)*(x_.)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] := Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2396

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_)/((f_.) + (g_.
)*(x_.)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]
```

Rule 2433

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_.))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_.))^(r_.), x_Sym
bol] := Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]
```

Rule 2374

```
Int[(Log[(d_.)*((e_) + (f_.)*(x_.)^(m_.))]*((a_.) + Log[(c_.)*(x_.)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_.))^(p_.)]/((d_.) + (e_.)*(x_.)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 2500



```

Int[(Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_)^(p_.))*((c_.) + (d_.)*(x_)^(q_.))
)^(r_.)]*((s_.) + Log[(i_.)*(g_.) + (h_.)*(x_)^(n_.)]*(t_.))/((j_.) + (k
_.)*(x_)), x_Symbol] := Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ
[b*c - a*d, 0]

```

#### Rule 2440

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_)^(m_.)]*(g_.))*((k_) + (l_.)*(x_)^(r_.)), x_Symbol] :=
Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f +
g*Log[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; FreeQ[{a,
b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]

```

#### Rule 2434

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + Log[(h_.)
*((i_.) + (j_.)*(x_)^(m_.)]*(g_.)))/(x_), x_Symbol] := Simp[Log[x]*(a + b
*Log[c*(d + e*x)^n])*(f + g*Log[h*(i + j*x)^m]), x] + (-Dist[e*g*m, Int[(Lo
g[x]*(a + b*Log[c*(d + e*x)^n]))/(d + e*x), x], x] - Dist[b*j*n, Int[(Log[x
]*(f + g*Log[h*(i + j*x)^m)))/(i + j*x), x], x]) /; FreeQ[{a, b, c, d, e, f
, g, h, i, j, m, n}, x] && EqQ[e*i - d*j, 0]

```

#### Rule 2375

```

Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))^(r_.)]*((a_.) + Log[(c_.)*(x_)^(n
_.)]*(b_.))^(p_.))/(x_), x_Symbol] := Simp[(Log[d*(e + f*x^m)^r]*(a + b*Log[
c*x^n])^(p + 1))/(b*n*(p + 1)), x] - Dist[(f*m*r)/(b*n*(p + 1)), Int[(x^(m
- 1)*(a + b*Log[c*x^n])^(p + 1))/(e + f*x^m), x], x] /; FreeQ[{a, b, c, d,
e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d*e, 1]

```

#### Rule 2317

```

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] := Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b
, c, d, e, n}, x] && IGtQ[p, 0]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{ag + bgx} dx &= \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{(a+bx)\left(\frac{de}{a+bx} - \frac{be(c+dx)}{(a+bx)^2}\right)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag+bgx)}{e(c+dx)} dx}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{(a+bx)\left(\frac{de}{a+bx} - \frac{be(c+dx)}{(a+bx)^2}\right)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag+bgx)}{c+dx} dx}{beg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{\left(de - \frac{be(c+dx)}{a+bx}\right)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag+bgx)}{c+dx} dx}{beg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{(bc-ad)e\left(-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag+bgx)}{(a+bx)(c+dx)} dx}{beg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B(bc-ad)) \int \frac{\left(-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag+bgx)}{(a+bx)(c+dx)} dx}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B(bc-ad)) \int \left(\frac{b\left(-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag+bgx)}{(bc-ad)(a+bx)} + \frac{d(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)) \log(ag+bgx)}{bc-ad}\right) dx}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{\left(-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)\right) \log(ag+bgx)}{a+bx} dx}{g} - \frac{(2Bd) \int \frac{A \log(ag+bgx)}{a+bx} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \left(\frac{A \log(ag+bgx)}{-a-bx} + \frac{B \log\left(\frac{e(c+dx)}{a+bx}\right) \log(ag+bgx)}{-a-bx}\right) dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2AB) \int \frac{\log(ag+bgx)}{-a-bx} dx}{g} - \frac{(2B^2) \int \frac{\log\left(\frac{e(c+dx)}{a+bx}\right) \log(ag+bgx)}{-a-bx} dx}{g} \\
&= -\frac{2AB \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} + \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2 \log(ag + bgx)}{bg} + \frac{B^2 \log\left(\frac{e(c+dx)}{a+bx}\right) \log(ag+bgx)}{bg} \\
&= -\frac{2AB \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} + \frac{2B^2 \log\left(\frac{b(c+dx)}{bc-ad}\right) \left(\log\left(\frac{1}{a+bx}\right) + \log(c + dx) - \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{bg} \\
&= \frac{AB \log^2(g(a + bx))}{bg} - \frac{2B^2 \log\left(\frac{1}{a+bx}\right) \log(g(a + bx)) \log(c + dx)}{bg} - \frac{B^2 \log(g(a + bx)) \log^2(c + dx)}{bg} \\
&= \frac{AB \log^2(g(a + bx))}{bg} - \frac{2B^2 \log\left(\frac{1}{a+bx}\right) \log(g(a + bx)) \log(c + dx)}{bg} + \frac{B^2 \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log^2(c + dx)}{bg} \\
&= \frac{AB \log^2(g(a + bx))}{bg} + \frac{B^2 \log^3(g(a + bx))}{3bg} - \frac{B^2 \log^2\left(\frac{1}{a+bx}\right) \log(c + dx)}{bg} - \frac{2B^2 \log\left(\frac{1}{a+bx}\right) \log(g(a + bx)) \log(c + dx)}{bg} \\
&= \frac{AB \log^2(g(a + bx))}{bg} + \frac{B^2 \log^3(g(a + bx))}{3bg} - \frac{B^2 \log^2\left(\frac{1}{a+bx}\right) \log(c + dx)}{bg} - \frac{2B^2 \log\left(\frac{1}{a+bx}\right) \log(g(a + bx)) \log(c + dx)}{bg} \\
&= \frac{AB \log^2(g(a + bx))}{bg} + \frac{B^2 \log^3(g(a + bx))}{3bg} - \frac{B^2 \log^2\left(\frac{1}{a+bx}\right) \log(c + dx)}{bg} - \frac{2B^2 \log\left(\frac{1}{a+bx}\right) \log(g(a + bx)) \log(c + dx)}{bg} \\
&= \frac{AB \log^2(g(a + bx))}{bg} + \frac{B^2 \log^3(g(a + bx))}{3bg} - \frac{B^2 \log^2\left(\frac{1}{a+bx}\right) \log(c + dx)}{bg} - \frac{2B^2 \log\left(\frac{1}{a+bx}\right) \log(g(a + bx)) \log(c + dx)}{bg}
\end{aligned}$$

**Mathematica [A]** time = 0.276452, size = 251, normalized size = 1.96

$$2AB\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right) - 2B^2\text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) \log\left(\frac{e(c+dx)}{a+bx}\right) + 2B^2\text{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right) + A^2 \log(a+bx) + 2AB$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2/(a\*g + b\*g\*x), x]

[Out] 
$$\begin{aligned} & -(A*B*\text{Log}[a/b + x]^2) + A^2*\text{Log}[a + b*x] + 2*A*B*\text{Log}[a/b + x]*\text{Log}[a + b*x] \\ & - 2*A*B*\text{Log}[c/d + x]*\text{Log}[a + b*x] + 2*A*B*\text{Log}[c/d + x]*\text{Log}[(d*(a + b*x))/( \\ & -(b*c) + a*d)] + 2*A*B*\text{Log}[a + b*x]*\text{Log}[(e*(c + d*x))/(a + b*x)] - B^2*\text{Log}[ \\ & -(b*c) + a*d]/(d*(a + b*x))*\text{Log}[(e*(c + d*x))/(a + b*x)]^2 + 2*A*B*\text{PolyLo} \\ & \text{g}[2, (b*(c + d*x))/(b*c - a*d)] - 2*B^2*\text{Log}[(e*(c + d*x))/(a + b*x)]*\text{PolyLo} \\ & \text{g}[2, (b*(c + d*x))/(d*(a + b*x))] + 2*B^2*\text{PolyLog}[3, (b*(c + d*x))/(d*(a + \\ & b*x))]/(b*g) \end{aligned}$$

**Maple [B]** time = 0.067, size = 906, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g), x)

[Out] 
$$\begin{aligned} & -1/b/g/(a*d-b*c)*A^2*\ln(b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)*a*d+1/g/(a*d-b \\ & *c)*A^2*\ln(b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)*c-1/b/g/(a*d-b*c)*B^2*\ln(d* \\ & e/b-e*(a*d-b*c)/b/(b*x+a))^2*\ln(1-b/d/e*(d*e/b-e*(a*d-b*c)/b/(b*x+a)))*a*d+ \\ & 1/g/(a*d-b*c)*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*\ln(1-b/d/e*(d*e/b-e*(a* \\ & d-b*c)/b/(b*x+a)))*c-2/b/g/(a*d-b*c)*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*po \\ & lylog(2,b/d/e*(d*e/b-e*(a*d-b*c)/b/(b*x+a)))*a*d+2/g/(a*d-b*c)*B^2*\ln(d*e/b \\ & -e*(a*d-b*c)/b/(b*x+a))*polylog(2,b/d/e*(d*e/b-e*(a*d-b*c)/b/(b*x+a)))*c+2/ \\ & b/g/(a*d-b*c)*B^2*polylog(3,b/d/e*(d*e/b-e*(a*d-b*c)/b/(b*x+a)))*a*d-2/g/(a \\ & *d-b*c)*B^2*polylog(3,b/d/e*(d*e/b-e*(a*d-b*c)/b/(b*x+a)))*c-2/b/g/(a*d-b*c) \\ & )*A*B*dilog(-(b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)/d/e)*a*d+2/g/(a*d-b*c)*A \\ & *B*dilog(-(b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)/d/e)*c-2/b/g/(a*d-b*c)*A*B* \\ & \ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*\ln(-(b*(d*e/b-e*(a*d-b*c)/b/(b*x+a))-d*e)/ \\ & /e)*a*d+2/g/(a*d-b*c)*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*\ln(-(b*(d*e/b-e*( \\ & a*d-b*c)/b/(b*x+a))-d*e)/d/e)*c \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{B^2 \log(bx + a) \log(dx + c)^2}{bg} + \frac{A^2 \log(bgx + ag)}{bg} - \int \frac{B^2 bc \log(e)^2 + 2 ABbc \log(e) + (B^2 bdx + B^2 bc) \log(bx + a)^2}{bg}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g), x, algorithm="maxima")

[Out] 
$$\begin{aligned} & B^2*\log(b*x + a)*\log(d*x + c)^2/(b*g) + A^2*\log(b*g*x + a*g)/(b*g) - \text{integr} \\ & \text{ate}(-(B^2*b*c*\log(e)^2 + 2*A*B*b*c*\log(e) + (B^2*b*d*x + B^2*b*c)*\log(b*x + \\ & a)^2 + (B^2*b*d*\log(e)^2 + 2*A*B*b*d*\log(e))*x - 2*(B^2*b*c*\log(e) + A*B*b \\ & *c + (B^2*b*d*\log(e) + A*B*b*d)*x)*\log(b*x + a) + 2*(B^2*b*c*\log(e) + A*B*b \end{aligned}$$

$*c + (B^2*b*d*log(e) + A*B*b*d)*x - (2*B^2*b*d*x + (b*c + a*d)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log\left(\frac{dex+ce}{bx+a}\right)^2 + 2AB \log\left(\frac{dex+ce}{bx+a}\right) + A^2}{bgx + ag}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g),x, algorithm="fricas")

[Out] integral((B^2\*log((d\*e\*x + c\*e)/(b\*x + a))^2 + 2\*A\*B\*log((d\*e\*x + c\*e)/(b\*x + a)) + A^2)/(b\*g\*x + a\*g), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))\*\*2/(b\*g\*x+a\*g),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(dx+c)e}{bx+a}\right) + A\right)^2}{bgx + ag} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g),x, algorithm="giac")

[Out] integrate((B\*log((d\*x + c)\*e/(b\*x + a)) + A)^2/(b\*g\*x + a\*g), x)

$$3.187 \quad \int \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^2} dx$$

**Optimal.** Leaf size=153

$$\frac{(c+dx)\left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)^2}{g^2(a+bx)(bc-ad)} + \frac{2AB(c+dx)}{g^2(a+bx)(bc-ad)} + \frac{2B^2(c+dx) \log\left(\frac{e(c+dx)}{a+bx}\right)}{g^2(a+bx)(bc-ad)} - \frac{2B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

[Out]  $(2AB(c+dx))/((b*c - a*d)*g^2*(a + b*x)) - (2B^2*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) + (2B^2*(c + d*x)*Log[(e*(c + d*x))/(a + b*x)])/((b*c - a*d)*g^2*(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(c + d*x))/(a + b*x)])^2)/((b*c - a*d)*g^2*(a + b*x))$

**Rubi [C]** time = 0.762186, antiderivative size = 470, normalized size of antiderivative = 3.07, number of steps used = 26, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2B^2 d \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg^2(bc-ad)} - \frac{2B^2 d \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bg^2(bc-ad)} + \frac{2Bd \log(a+bx)\left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{bg^2(bc-ad)} - \frac{2Bd \log(c+dx)}{bg^2(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2/(a\*g + b\*g\*x)^2, x]

[Out]  $(-2*B^2)/(b*g^2*(a + b*x)) - (2*B^2*d*Log[a + b*x])/(b*(b*c - a*d)*g^2) + (B^2*d*Log[a + b*x]^2)/(b*(b*c - a*d)*g^2) + (2*B^2*d*Log[c + d*x])/(b*(b*c - a*d)*g^2) - (2*B^2*d*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*(b*c - a*d)*g^2) + (B^2*d*Log[c + d*x]^2)/(b*(b*c - a*d)*g^2) - (2*B^2*d*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)*g^2) + (2*B*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(b*g^2*(a + b*x)) + (2*B*d*Log[a + b*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)*g^2) - (2*B*d*Log[c + d*x]*(A + B*Log[(e*(c + d*x))/(a + b*x)]))/(b*(b*c - a*d)*g^2) - (A + B*Log[(e*(c + d*x))/(a + b*x)])^2/(b*g^2*(a + b*x)) - (2*B^2*d*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)*g^2) - (2*B^2*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d])/(b*(b*c - a*d)*g^2)$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u

]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 44

Int[((a\_) + (b\_)\*(x\_)^(m\_))\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 2524

Int[((a\_) + Log[(c\_)\*(RFx\_)^(p\_)]\*(b\_))^(n\_)/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]\*(b\_))^(p\_)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]\*(b\_))^(p\_)\*((f\_) + (g\_)\*(x\_)^(q\_)), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]\*(b\_))/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))]\*(b\_))/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)^2}{(ag + bgx)^2} dx &= -\frac{\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)^2}{bg^2(a + bx)} + \frac{(2B) \int \frac{(bc-ad)\left(-A-B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)}{g(a+bx)^2(c+dx)} dx}{bg} \\
&= -\frac{\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)^2}{bg^2(a + bx)} + \frac{(2B(bc - ad)) \int \frac{-A-B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)}{(a+bx)^2(c+dx)} dx}{bg^2} \\
&= -\frac{\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)^2}{bg^2(a + bx)} + \frac{(2B(bc - ad)) \int \left(\frac{b(-A-B \log\left(\frac{e^{(c+dx)}}{a+bx}\right))}{(bc-ad)(a+bx)^2} - \frac{bd(-A-B \log\left(\frac{e^{(c+dx)}}{a+bx}\right))}{(bc-ad)^2(a+bx)} + \dots\right) dx}{bg^2} \\
&= -\frac{\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)^2}{bg^2(a + bx)} + \frac{(2B) \int \frac{-A-B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)}{(a+bx)^2} dx}{g^2} - \frac{(2Bd) \int \frac{-A-B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)}{a+bx} dx}{(bc - ad)g^2} + \dots \\
&= \frac{2B\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)}{bg^2(a + bx)} + \frac{2Bd \log(a + bx)\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)}{b(bc - ad)g^2} - \frac{2Bd \log(c + dx)\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)}{b(bc - ad)g^2} + \dots \\
&= \frac{2B\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)}{bg^2(a + bx)} + \frac{2Bd \log(a + bx)\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)}{b(bc - ad)g^2} - \frac{2Bd \log(c + dx)\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)}{b(bc - ad)g^2} + \dots \\
&= \frac{2B\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)}{bg^2(a + bx)} + \frac{2Bd \log(a + bx)\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)}{b(bc - ad)g^2} - \frac{2Bd \log(c + dx)\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)}{b(bc - ad)g^2} + \dots \\
&= -\frac{2B^2}{bg^2(a + bx)} - \frac{2B^2d \log(a + bx)}{b(bc - ad)g^2} + \frac{2B^2d \log(c + dx)}{b(bc - ad)g^2} + \frac{2B\left(A + B \log\left(\frac{e^{(c+dx)}}{a+bx}\right)\right)}{bg^2(a + bx)} + \dots \\
&= -\frac{2B^2}{bg^2(a + bx)} - \frac{2B^2d \log(a + bx)}{b(bc - ad)g^2} + \frac{2B^2d \log(c + dx)}{b(bc - ad)g^2} - \frac{2B^2d \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c + dx)}{b(bc - ad)g^2} + \dots \\
&= -\frac{2B^2}{bg^2(a + bx)} - \frac{2B^2d \log(a + bx)}{b(bc - ad)g^2} + \frac{B^2d \log^2(a + bx)}{b(bc - ad)g^2} + \frac{2B^2d \log(c + dx)}{b(bc - ad)g^2} - \frac{2B^2d \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c + dx)}{b(bc - ad)g^2} + \dots \\
&= -\frac{2B^2}{bg^2(a + bx)} - \frac{2B^2d \log(a + bx)}{b(bc - ad)g^2} + \frac{B^2d \log^2(a + bx)}{b(bc - ad)g^2} + \frac{2B^2d \log(c + dx)}{b(bc - ad)g^2} - \frac{2B^2d \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c + dx)}{b(bc - ad)g^2} + \dots
\end{aligned}$$

**Mathematica [C]** time = 0.494578, size = 314, normalized size = 2.05

$$\frac{B\left(-Bd(a+bx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left[2,\frac{d(a+bx)}{ad-bc}\right]\right)+Bd(a+bx)\left(2\text{PolyLog}\left[2,\frac{b(c+dx)}{bc-ad}\right]+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)\right)}{b^2g^2(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2/(a\*g + b\*g\*x)^2,x]

[Out] -(((A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2 + (B\*(2\*B\*(b\*c - a\*d + d\*(a + b\*x))\*Log[a + b\*x] - d\*(a + b\*x)\*Log[c + d\*x]) - 2\*(b\*c - a\*d)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x])) - 2\*d\*(a + b\*x)\*Log[a + b\*x]\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x])) + 2\*d\*(a + b\*x)\*Log[c + d\*x]\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x])) - B\*d\*(a + b\*x)\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-(b\*c) + a\*d)]) + B\*d\*(a + b\*x)\*((2\*Log[(d\*(a + b\*x))/(-(b\*c) + a\*d)] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/(b\*c - a\*d)/(b\*g^2\*(a + b\*x))

**Maple [B]** time = 0.05, size = 1251, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x)`

[Out] 
$$\frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A^2 d^2 a^{-1} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A^2 d*c+2 \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A*B*d*c-2 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A*B*d^2 a+2 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A*B/(b*x+a) a^2 d^2-1 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a) a^2 d^2+2 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a) a^2 d^2-2 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A*B*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a) c^2+2 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A*B*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a)) d^2 a+2 \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a) a*d*c-4 \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A*B/(b*x+a) a*d*c-4 \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a) a*d*c-1 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A^2/(b*x+a) a^2 d^2-b \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A^2/(b*x+a) c^2-2 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2/(b*x+a) c^2-1 \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2 d*c+2 \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a)) d*c+4 \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2/(b*x+a) a*d*c+2 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A*B/(b*x+a) c^2-2 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2/(b*x+a) a^2 d^2+2 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a) c^2-2 \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A*B*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a)) d*c+1 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2 d^2 a-b \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a) c^2-2 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a)) d^2 a+2 \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A^2/(b*x+a) a*d*c+2 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2 d^2 a-2 \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} B^2 d*c+4 \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A*B*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a) a*d*c-2 \frac{1}{b} \frac{1}{(a*d-b*c)^2} \frac{1}{g^2} A*B*ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a) a^2 d^2$$

**Maxima [B]** time = 1.29471, size = 562, normalized size = 3.67

$$\left( 2 \left( \frac{1}{b^2 g^2 x + a b g^2} + \frac{d \log(bx + a)}{(b^2 c - a b d) g^2} - \frac{d \log(dx + c)}{(b^2 c - a b d) g^2} \right) \log \left( \frac{d e x}{b x + a} + \frac{c e}{b x + a} \right) + \frac{(b d x + a d) \log(bx + a)^2 + (b d x + a d) \log(dx + c)}{(b^2 c - a b d) g^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^2,x, algorithm="maxima")`

[Out] 
$$\frac{2 \left( \frac{1}{b^2 g^2 x + a b g^2} + d \log(bx + a) / ((b^2 c - a b d) g^2) - d \log(dx + c) / ((b^2 c - a b d) g^2) \right) \log \left( \frac{d e x}{b x + a} + \frac{c e}{b x + a} \right) + ((b d x + a d) \log(bx + a)^2 + (b d x + a d) \log(dx + c)^2 - 2 b c + 2 a d - 2 (b d x + a d) \log(bx + a) + 2 (b d x + a d - (b d x + a d) \log(bx + a)) \log(dx + c)) / (a b^2 c g^2 - a^2 b d g^2 + (b^3 c g^2 - a b^2 d g^2) x) * B^2 - 2 A B (\log(d e x / (b x + a) + c e / (b x + a)) / (b^2 g^2 x + a b g^2) - 1 / (b^2 g^2 x + a b g^2) - d \log(bx + a) / ((b^2 c - a b d) g^2) + d \log(dx + c) / ((b^2 c - a b d) g^2)) - B^2 \log(d e x / (b x + a) + c e / (b x + a))^2 / (b^2 g^2 x + a b g^2) - A^2 / (b^2 g^2 x + a b g^2)}$$



**Fricas [A]** time = 1.01157, size = 319, normalized size = 2.08

$$\frac{(A^2 - 2AB + 2B^2)bc - (A^2 - 2AB + 2B^2)ad + (B^2bdx + B^2bc) \log\left(\frac{dex+ce}{bx+a}\right)^2 + 2((AB - B^2)bdx + (AB - B^2)bc) \log\left(\frac{dex+ce}{bx+a}\right)}{(b^3c - ab^2d)g^2x + (ab^2c - a^2bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g)^2,x, algorithm="fricas")

[Out] -((A^2 - 2\*A\*B + 2\*B^2)\*b\*c - (A^2 - 2\*A\*B + 2\*B^2)\*a\*d + (B^2\*b\*d\*x + B^2\*b\*c)\*log((d\*e\*x + c\*e)/(b\*x + a))^2 + 2\*((A\*B - B^2)\*b\*d\*x + (A\*B - B^2)\*b\*c)\*log((d\*e\*x + c\*e)/(b\*x + a)))/((b^3\*c - a\*b^2\*d)\*g^2\*x + (a\*b^2\*c - a^2\*b\*d)\*g^2)

**Sympy [B]** time = 3.7579, size = 430, normalized size = 2.81

$$\frac{2Bd(A - B) \log\left(x + \frac{2ABad^2 + 2ABbcd - 2B^2ad^2 - 2B^2bcd - \frac{2Ba^2d^3(A-B)}{ad-bc} + \frac{4Babcd^2(A-B)}{ad-bc} - \frac{2Bt^2c^2d(A-B)}{ad-bc}}{4ABbd^2 - 4B^2bd^2}\right)}{bg^2(ad - bc)} - \frac{2Bd(A - B) \log\left(x + \frac{2ABad^2 + 2ABbcd - 2B^2ad^2 - 2B^2bcd - \frac{2Ba^2d^3(A-B)}{ad-bc} + \frac{4Babcd^2(A-B)}{ad-bc} - \frac{2Bt^2c^2d(A-B)}{ad-bc}}{4ABbd^2 - 4B^2bd^2}\right)}{bg^2(ad - bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))\*\*2/(b\*g\*x+a\*g)\*\*2,x)

[Out] 2\*B\*d\*(A - B)\*log(x + (2\*A\*B\*a\*d\*\*2 + 2\*A\*B\*b\*c\*d - 2\*B\*\*2\*a\*d\*\*2 - 2\*B\*\*2\*b\*c\*d - 2\*B\*a\*\*2\*d\*\*3\*(A - B)/(a\*d - b\*c) + 4\*B\*a\*b\*c\*d\*\*2\*(A - B)/(a\*d - b\*c) - 2\*B\*b\*\*2\*c\*\*2\*d\*(A - B)/(a\*d - b\*c))/(4\*A\*B\*b\*d\*\*2 - 4\*B\*\*2\*b\*d\*\*2))/(b\*g\*\*2\*(a\*d - b\*c)) - 2\*B\*d\*(A - B)\*log(x + (2\*A\*B\*a\*d\*\*2 + 2\*A\*B\*b\*c\*d - 2\*B\*\*2\*a\*d\*\*2 - 2\*B\*\*2\*b\*c\*d + 2\*B\*a\*\*2\*d\*\*3\*(A - B)/(a\*d - b\*c) - 4\*B\*a\*b\*c\*d\*\*2\*(A - B)/(a\*d - b\*c) + 2\*B\*b\*\*2\*c\*\*2\*d\*(A - B)/(a\*d - b\*c))/(4\*A\*B\*b\*d\*\*2 - 4\*B\*\*2\*b\*d\*\*2))/(b\*g\*\*2\*(a\*d - b\*c)) + (-2\*A\*B + 2\*B\*\*2)\*log(e\*(c + d\*x)/(a + b\*x))/(a\*b\*g\*\*2 + b\*\*2\*g\*\*2\*x) + (B\*\*2\*c + B\*\*2\*d\*x)\*log(e\*(c + d\*x)/(a + b\*x))\*\*2/(a\*\*2\*d\*g\*\*2 - a\*b\*c\*g\*\*2 + a\*b\*d\*g\*\*2\*x - b\*\*2\*c\*g\*\*2\*x) - (A\*\*2 - 2\*A\*B + 2\*B\*\*2)/(a\*b\*g\*\*2 + b\*\*2\*g\*\*2\*x)

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{dx+ce}{bx+a}\right) + A\right)^2}{(bgx + ag)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g)^2,x, algorithm="giac")

[Out] integrate((B\*log((d\*x + c)\*e/(b\*x + a)) + A)^2/(b\*g\*x + a\*g)^2, x)

$$3.188 \quad \int \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^3} dx$$

**Optimal.** Leaf size=296

$$\frac{bB(c+dx)^2 \left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{2g^3(a+bx)^2(bc-ad)^2} - \frac{b(c+dx)^2 \left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)^2}{2g^3(a+bx)^2(bc-ad)^2} + \frac{d(c+dx) \left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)^2}{g^3(a+bx)(bc-ad)^2} - \frac{2ABd(c+dx)}{g^3(a+bx)(bc-ad)^2}$$

[Out]  $(-2ABd(c+dx))/((b^2c - a^2d)^2g^3(a+bx)) + (2B^2d(c+dx))/((b^2c - a^2d)^2g^3(a+bx)) - (b^2B^2d(c+dx)^2)/(4(b^2c - a^2d)^2g^3(a+bx)^2) - (2B^2d(c+dx) \log((e(c+dx))/(a+bx)))/((b^2c - a^2d)^2g^3(a+bx)) + (b^2B^2d(c+dx)^2(A + B \log((e(c+dx))/(a+bx))))/(2(b^2c - a^2d)^2g^3(a+bx)^2) + (d(c+dx)(A + B \log((e(c+dx))/(a+bx))))^2/((b^2c - a^2d)^2g^3(a+bx)) - (b^2(c+dx)^2(A + B \log((e(c+dx))/(a+bx))))^2/(2(b^2c - a^2d)^2g^3(a+bx)^2)$

**Rubi [C]** time = 0.910413, antiderivative size = 578, normalized size of antiderivative = 1.95, number of steps used = 30, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2d^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg^3(bc-ad)^2} + \frac{B^2d^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bg^3(bc-ad)^2} - \frac{Bd^2 \log(a+bx) \left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{bg^3(bc-ad)^2} + \frac{Bd^2 \log(c+dx) \left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{bg^3(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]))^2/(a\*g + b\*g\*x)^3, x]

[Out]  $-B^2/(4b^2g^3(a+bx)^2) + (3B^2d)/(2b^2(b^2c - a^2d)g^3(a+bx)) + (3B^2d^2 \log[a+bx])/(2b^2(b^2c - a^2d)^2g^3) - (B^2d^2 \log[a+bx]^2)/(2b^2(b^2c - a^2d)^2g^3) - (3B^2d^2 \log[c+d*x])/(2b^2(b^2c - a^2d)^2g^3) + (B^2d^2 \log[-((d(a+bx))/(b^2c - a^2d))] \log[c+d*x])/(b^2(b^2c - a^2d)^2g^3) - (B^2d^2 \log[c+d*x]^2)/(2b^2(b^2c - a^2d)^2g^3) + (B^2d^2 \log[a+bx] \log[(b^2(c+d*x))/(b^2c - a^2d)])/(b^2(b^2c - a^2d)^2g^3) + (B^2(A + B \log((e(c+dx))/(a+bx))))/(2b^2g^3(a+bx)^2) - (B^2d^2(A + B \log((e(c+dx))/(a+bx))))/(b^2(b^2c - a^2d)g^3(a+bx)) - (B^2d^2 \log[a+bx] \log[(e(c+dx))/(a+bx)])/(b^2(b^2c - a^2d)^2g^3) + (B^2d^2 \log[c+d*x] \log[(e(c+dx))/(a+bx)])/(b^2(b^2c - a^2d)^2g^3) - (A + B \log((e(c+dx))/(a+bx)))^2/(2b^2g^3(a+bx)^2) + (B^2d^2 \text{PolyLog}[2, -((d(a+bx))/(b^2c - a^2d))])/(b^2(b^2c - a^2d)^2g^3) + (B^2d^2 \text{PolyLog}[2, (b^2(c+d*x))/(b^2c - a^2d)])/(b^2(b^2c - a^2d)^2g^3)$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.)/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

## Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag + bgx)^3} dx &= -\frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2bg^3(a + bx)^2} + \frac{B \int \frac{(bc-ad)\left(-A - B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{g^2(a+bx)^3(c+dx)} dx}{bg} \\
&= -\frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2bg^3(a + bx)^2} + \frac{(B(bc - ad)) \int \frac{-A - B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^3(c+dx)} dx}{bg^3} \\
&= -\frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2bg^3(a + bx)^2} + \frac{(B(bc - ad)) \int \left(\frac{b\left(-A - B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)(a+bx)^3} - \frac{bd\left(-A - B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^2(a+bx)^2} + \frac{bd^2\left(-A - B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3}\right) dx}{bg^3} \\
&= -\frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{2bg^3(a + bx)^2} + \frac{B \int \frac{-A - B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^3} dx}{g^3} + \frac{(Bd^2) \int \frac{-A - B \log\left(\frac{e(c+dx)}{a+bx}\right)}{a+bx} dx}{(bc - ad)^2g^3} - \frac{(Bd^3) \int \frac{-A - B \log\left(\frac{e(c+dx)}{a+bx}\right)}{c+dx} dx}{b(bc - ad)g^3} \\
&= \frac{B\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2bg^3(a + bx)^2} - \frac{Bd\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc - ad)g^3(a + bx)} - \frac{Bd^2 \log(a + bx)\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc - ad)^2g^3} \\
&= \frac{B\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2bg^3(a + bx)^2} - \frac{Bd\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc - ad)g^3(a + bx)} - \frac{Bd^2 \log(a + bx)\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc - ad)^2g^3} \\
&= \frac{B\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2bg^3(a + bx)^2} - \frac{Bd\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc - ad)g^3(a + bx)} - \frac{Bd^2 \log(a + bx)\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{b(bc - ad)^2g^3} \\
&= -\frac{B^2}{4bg^3(a + bx)^2} + \frac{3B^2d}{2b(bc - ad)g^3(a + bx)} + \frac{3B^2d^2 \log(a + bx)}{2b(bc - ad)^2g^3} - \frac{3B^2d^2 \log(c + dx)}{2b(bc - ad)^2g^3} + \frac{B\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2bg^3(a + bx)^2} \\
&= -\frac{B^2}{4bg^3(a + bx)^2} + \frac{3B^2d}{2b(bc - ad)g^3(a + bx)} + \frac{3B^2d^2 \log(a + bx)}{2b(bc - ad)^2g^3} - \frac{3B^2d^2 \log(c + dx)}{2b(bc - ad)^2g^3} + \frac{B\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{2bg^3(a + bx)^2} \\
&= -\frac{B^2}{4bg^3(a + bx)^2} + \frac{3B^2d}{2b(bc - ad)g^3(a + bx)} + \frac{3B^2d^2 \log(a + bx)}{2b(bc - ad)^2g^3} - \frac{B^2d^2 \log^2(a + bx)}{2b(bc - ad)^2g^3} - \frac{3B^2d^2 \log(c + dx)}{2b(bc - ad)^2g^3} \\
&= -\frac{B^2}{4bg^3(a + bx)^2} + \frac{3B^2d}{2b(bc - ad)g^3(a + bx)} + \frac{3B^2d^2 \log(a + bx)}{2b(bc - ad)^2g^3} - \frac{B^2d^2 \log^2(a + bx)}{2b(bc - ad)^2g^3} - \frac{3B^2d^2 \log(c + dx)}{2b(bc - ad)^2g^3}
\end{aligned}$$

**Mathematica [C]** time = 0.476015, size = 444, normalized size = 1.5

$$\frac{B\left(-2Bd^2(a+bx)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+2Bd^2(a+bx)^2\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)-4Bd^2(a+bx)^2\log(a+bx)\log(c+dx)\right)}{4b^2g^3(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2/(a\*g + b\*g\*x)^3,x]

[Out] (-2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2 + (B\*(4\*B\*d\*(a + b\*x)\*(b\*c - a\*d + d\*(a + b\*x)\*Log[a + b\*x] - d\*(a + b\*x)\*Log[c + d\*x]) - B\*((b\*c - a\*d)^2 + 2\*d\*(-(b\*c) + a\*d)\*(a + b\*x) - 2\*d^2\*(a + b\*x)^2\*Log[a + b\*x] + 2\*d^2\*(a + b\*x)^2\*Log[c + d\*x]) + 2\*(b\*c - a\*d)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x])) + 4\*d\*(-(b\*c) + a\*d)\*(a + b\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]) - 4\*d^2\*(a + b\*x)^2\*Log[a + b\*x]\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]) + 4\*d^2\*(a + b\*x)^2\*Log[c + d\*x]\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]) - 2\*B\*d^2\*(a +

$$b*x)^2*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-b*c) + a*d]) + 2*B*d^2*(a + b*x)^2*((2*\text{Log}[(d*(a + b*x))/(-b*c) + a*d]) - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2/(4*b*g^3*(a + b*x)^2)$$

**Maple [B]** time = 0.053, size = 1934, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g)^3,x)

[Out] 
$$\frac{b}{(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d/(b*x+a)*c^{2-3/2}/b/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^3*a+3/2/(a*d-b*c)^3/g^3*A*B*d^2*c+b/(a*d-b*c)^3/g^3*A*B*d/(b*x+a)*c^{2+1}/b/(a*d-b*c)^3/g^3*A*B*d^3/(b*x+a)*a^{2-3/4}/b/(a*d-b*c)^3/g^3*B^2/(b*x+a)^2*c^2*a*d+1/2/b/(a*d-b*c)^3/g^3*A*B/(b*x+a)^2*a^3*d^3-3/2*b/(a*d-b*c)^3/g^3*A^2/(b*x+a)^2*a*d*c^{2+1}/b/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^3/(b*x+a)*a^{2-3/2}/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^2*a^2*d^2*c+1/b/(a*d-b*c)^3/g^3*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^3*a-1/2/b/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a)^2*a^3*d^3-3/2/(a*d-b*c)^3/g^3*A*B/(b*x+a)^2*a^2*d^2*c-2/(a*d-b*c)^3/g^3*A*B*d^2/(b*x+a)*a*c+b^2/(a*d-b*c)^3/g^3*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^2*c^3+1/2/b/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^2*a^3*d^3-3/2/b/(a*d-b*c)^3/g^3*A*B*d^3*a+3/2/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a)^2*a^2*d^2*c-2/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^2/(b*x+a)*a*c+1/2/b/(a*d-b*c)^3/g^3*A^2*d^3*a-1/2/(a*d-b*c)^3/g^3*A^2*d^2*c-7/4/(a*d-b*c)^3/g^3*B^2*d^2*c-1/2/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*d^2*c+3/4/(a*d-b*c)^3/g^3*B^2/(b*x+a)^2*a^2*d^2*c+3/(a*d-b*c)^3/g^3*B^2*d^2/(b*x+a)*c*a+1/2*b^2/(a*d-b*c)^3/g^3*A^2/(b*x+a)^2*c^3+1/4*b^2/(a*d-b*c)^3/g^3*B^2/(b*x+a)^2*c^3+3/2/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^2*c-1/4/b/(a*d-b*c)^3/g^3*B^2/(b*x+a)^2*a^3*d^3-3/2/b/(a*d-b*c)^3/g^3*B^2*d^3/(b*x+a)*a^2-1/(a*d-b*c)^3/g^3*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^2*c+7/4/b/(a*d-b*c)^3/g^3*B^2*d^3*a-1/2*b^2/(a*d-b*c)^3/g^3*A*B/(b*x+a)^2*c^3-1/2/b/(a*d-b*c)^3/g^3*A^2/(b*x+a)^2*a^3*d^3-1/2*b^2/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^2*c^3+1/2/b/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*d^3*a+1/2*b^2/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a)^2*c^3-3/2*b/(a*d-b*c)^3/g^3*B^2*d/(b*x+a)*c^2+3/2/(a*d-b*c)^3/g^3*A^2/(b*x+a)^2*a^2*d^2*c-3*b/(a*d-b*c)^3/g^3*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^2*c^2*a*d-1/b/(a*d-b*c)^3/g^3*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^2*a^3*d^3-3/2*b/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a)^2*c^2*a*d+3/2*b/(a*d-b*c)^3/g^3*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^2*c^2*a*d+3/2*b/(a*d-b*c)^3/g^3*A*B/(b*x+a)^2*c^2*a*d+3/(a*d-b*c)^3/g^3*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^2*a^2*d^2*c$$

**Maxima [B]** time = 1.54714, size = 1143, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g)^3,x, algorithm="maxima")

```
[Out] -1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3))*log(d*e*x/(b*x + a) + c*e/(b*x + a)) + (b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2))*log(b*x + a))*log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 - 1/2*A*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*log(d*e*x/(b*x + a) + c*e/(b*x + a))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*B^2*log(d*e*x/(b*x + a) + c*e/(b*x + a))^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

---

**Fricas [A]** time = 1.0752, size = 767, normalized size = 2.59

$$\frac{(2A^2 - 2AB + B^2)b^2c^2 - 4(A^2 - 2AB + 2B^2)abcd + (2A^2 - 6AB + 7B^2)a^2d^2 - 2(B^2b^2d^2x^2 + 2B^2abd^2x - B^2b^2c^2 + 4(b^5c^2 - 2ab^4cd +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*A^2 - 2*A*B + B^2)*b^2*c^2 - 4*(A^2 - 2*A*B + 2*B^2)*a*b*c*d + (2*A^2 - 6*A*B + 7*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b^2*c^2 + 2*B^2*a*b*c*d)*log((d*e*x + c*e)/(b*x + a))^2 + 2*((2*A*B - 3*B^2)*b^2*c*d - (2*A*B - 3*B^2)*a*b*d^2)*x - 2*((2*A*B - 3*B^2)*b^2*d^2*x^2 - (2*A*B - B^2)*b^2*c^2 + 4*(A*B - B^2)*a*b*c*d - 2*(B^2*b^2*c*d - 2*(A*B - B^2)*a*b*d^2)*x)*log((d*e*x + c*e)/(b*x + a))/((b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

---

**Sympy [B]** time = 6.71012, size = 892, normalized size = 3.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)/(b*x+a)))**2/(b*g*x+a*g)**3,x)
```

```
[Out] B*d**2*(2*A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 3*B**2*a*d**3 - 3*B**2*b*c*d**2 - B*a**3*d**5*(2*A - 3*B)/(a*d - b*c)**2 + 3*B*a**2*b*c*d**4*(2*A - 3*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**3*(2*A - 3*B)/(a*d - b*c)**2 + B*b**3*c**3*d**2*(2*A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 6*B**2*b*d**3))/(2*b*g**3*(a*d - b*c)**2) - B*d**2*(2*A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 3*B**2*a*d**3 - 3*B**2*b*c*d**2 + B*a**3*d**5*(2*A - 3*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**4*(2*A - 3*B)/(a*d - b*c)**2 + 3*B*a
```

```

b**2*c**2*d**3*(2*A - 3*B)/(a*d - b*c)**2 - B*b**3*c**3*d**2*(2*A - 3*B)/(a
*d - b*c)**2)/(4*A*B*b*d**3 - 6*B**2*b*d**3))/(2*b*g**3*(a*d - b*c)**2) + (
2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(c +
d*x)/(a + b*x))**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*g
**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*g
**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**3
*x**2) + (-2*A*B*a*d + 2*A*B*b*c + 3*B**2*a*d - B**2*b*c + 2*B**2*b*d*x)*lo
g(e*(c + d*x)/(a + b*x))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + 4*a**2*b**
2*d*g**3*x - 4*a*b**3*c*g**3*x + 2*a*b**3*d*g**3*x**2 - 2*b**4*c*g**3*x**2)
+ (-2*A**2*a*d + 2*A**2*b*c + 6*A*B*a*d - 2*A*B*b*c - 7*B**2*a*d + B**2*b*
c + x*(4*A*B*b*d - 6*B**2*b*d))/(4*a**3*b*d*g**3 - 4*a**2*b**2*c*g**3 + x**
2*(4*a*b**3*d*g**3 - 4*b**4*c*g**3) + x*(8*a**2*b**2*d*g**3 - 8*a*b**3*c*g*
*3))

```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(dx+c)e}{bx+a}\right) + A\right)^2}{(bgx + ag)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^3,x, algorithm="giac")
```

```
[Out] integrate((B*log((d*x + c)*e/(b*x + a)) + A)^2/(b*g*x + a*g)^3, x)
```

$$3.189 \quad \int \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^4} dx$$

**Optimal.** Leaf size=399

$$\frac{2b^2B(c+dx)^3 \left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{9g^4(a+bx)^3(bc-ad)^3} - \frac{2Bd^3 \log\left(\frac{c+dx}{a+bx}\right) \left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{3bg^4(bc-ad)^3} + \frac{2Bd^2(c+dx) \left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{g^4(a+bx)(bc-ad)^3} - \frac{bBd}{3bg^4}$$

[Out]  $(-2*B^2*d^2*(c+d*x))/((b*c-a*d)^3*g^4*(a+b*x)) + (b*B^2*d*(c+d*x)^2)/(2*(b*c-a*d)^3*g^4*(a+b*x)^2) - (2*b^2*B^2*(c+d*x)^3)/(27*(b*c-a*d)^3*g^4*(a+b*x)^3) + (B^2*d^3*Log[(c+d*x)/(a+b*x)]^2)/(3*b*(b*c-a*d)^3*g^4) + (2*B*d^2*(c+d*x)*(A+B*Log[(e*(c+d*x))/(a+b*x]]))/((b*c-a*d)^3*g^4*(a+b*x)) - (b*B*d*(c+d*x)^2*(A+B*Log[(e*(c+d*x))/(a+b*x]]))/((b*c-a*d)^3*g^4*(a+b*x)^2) + (2*b^2*B*(c+d*x)^3*(A+B*Log[(e*(c+d*x))/(a+b*x]]))/(9*(b*c-a*d)^3*g^4*(a+b*x)^3) - (2*B*d^3*Log[(c+d*x)/(a+b*x)]*(A+B*Log[(e*(c+d*x))/(a+b*x]]))/(3*b*(b*c-a*d)^3*g^4) - (A+B*Log[(e*(c+d*x))/(a+b*x]]^2)/(3*b*g^4*(a+b*x)^3)$

**Rubi [C]** time = 1.07406, antiderivative size = 680, normalized size of antiderivative = 1.7, number of steps used = 34, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{2B^2d^3 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{3bg^4(bc-ad)^3} - \frac{2B^2d^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3bg^4(bc-ad)^3} + \frac{2Bd^3 \log(a+bx) \left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}{3bg^4(bc-ad)^3} - \frac{2Bd^3 \log(c+dx)}{3bg^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2/(a\*g + b\*g\*x)^4, x]

[Out]  $(-2*B^2)/(27*b*g^4*(a+b*x)^3) + (5*B^2*d)/(18*b*(b*c-a*d)*g^4*(a+b*x)^2) - (11*B^2*d^2)/(9*b*(b*c-a*d)^2*g^4*(a+b*x)) - (11*B^2*d^3*Log[a+b*x])/(9*b*(b*c-a*d)^3*g^4) + (B^2*d^3*Log[a+b*x]^2)/(3*b*(b*c-a*d)^3*g^4) + (11*B^2*d^3*Log[c+d*x])/(9*b*(b*c-a*d)^3*g^4) - (2*B^2*d^3*Log[-((d*(a+b*x))/(b*c-a*d))]*Log[c+d*x])/(3*b*(b*c-a*d)^3*g^4) + (B^2*d^3*Log[c+d*x]^2)/(3*b*(b*c-a*d)^3*g^4) - (2*B^2*d^3*Log[a+b*x]*Log[(b*(c+d*x))/(b*c-a*d)])/(3*b*(b*c-a*d)^3*g^4) + (2*B*(A+B*Log[(e*(c+d*x))/(a+b*x]]))/(9*b*g^4*(a+b*x)^3) - (B*d*(A+B*Log[(e*(c+d*x))/(a+b*x]]))/(3*b*(b*c-a*d)*g^4*(a+b*x)^2) + (2*B*d^2*(A+B*Log[(e*(c+d*x))/(a+b*x]]))/(3*b*(b*c-a*d)^2*g^4*(a+b*x)) + (2*B*d^3*Log[a+b*x]*(A+B*Log[(e*(c+d*x))/(a+b*x]]))/(3*b*(b*c-a*d)^3*g^4) - (2*B*d^3*Log[c+d*x]*(A+B*Log[(e*(c+d*x))/(a+b*x]]))/(3*b*(b*c-a*d)^3*g^4) - (A+B*Log[(e*(c+d*x))/(a+b*x]]^2)/(3*b*g^4*(a+b*x)^3) - (2*B^2*d^3*PolyLog[2, -((d*(a+b*x))/(b*c-a*d))])/(3*b*(b*c-a*d)^3*g^4) - (2*B^2*d^3*PolyLog[2, (b*(c+d*x))/(b*c-a*d)])/(3*b*(b*c-a*d)^3*g^4)$

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]



Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 2528

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

Rule 2524

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]

Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(Rfx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

**Rule 2391**

Int[Log[(c\_.)\*(d\_) + (e\_.)\*(x\_)^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
 \int \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag + bgx)^4} dx &= -\frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a + bx)^3} + \frac{(2B) \int \frac{(bc-ad)\left(-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{g^3(a+bx)^4(c+dx)} dx}{3bg} \\
 &= -\frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a + bx)^3} + \frac{(2B(bc - ad)) \int \frac{-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^4(c+dx)} dx}{3bg^4} \\
 &= -\frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a + bx)^3} + \frac{(2B(bc - ad)) \int \left(\frac{b\left(-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd\left(-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^2(a+bx)^3} + \frac{bd^2\left(-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^3(a+bx)^2}\right) dx}{3bg^4} \\
 &= -\frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{3bg^4(a + bx)^3} + \frac{(2B) \int \frac{-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^4} dx}{3g^4} - \frac{(2Bd^3) \int \frac{-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)}{a+bx} dx}{3(bc - ad)^3g^4} + \frac{(2Bbd^3) \int \frac{-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^2} dx}{3(bc - ad)^2g^4} \\
 &= \frac{2B\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9bg^4(a + bx)^3} - \frac{Bd\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc - ad)g^4(a + bx)^2} + \frac{2Bd^2\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc - ad)^2g^4(a + bx)} + \frac{2Bbd^3\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc - ad)^3g^4} \\
 &= \frac{2B\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9bg^4(a + bx)^3} - \frac{Bd\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc - ad)g^4(a + bx)^2} + \frac{2Bd^2\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc - ad)^2g^4(a + bx)} + \frac{2Bbd^3\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc - ad)^3g^4} \\
 &= \frac{2B\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{9bg^4(a + bx)^3} - \frac{Bd\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc - ad)g^4(a + bx)^2} + \frac{2Bd^2\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc - ad)^2g^4(a + bx)} + \frac{2Bbd^3\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{3b(bc - ad)^3g^4} \\
 &= -\frac{2B^2}{27bg^4(a + bx)^3} + \frac{5B^2d}{18b(bc - ad)g^4(a + bx)^2} - \frac{11B^2d^2}{9b(bc - ad)^2g^4(a + bx)} - \frac{11B^2d^3 \log(a + bx)}{9b(bc - ad)^3g^4} \\
 &= -\frac{2B^2}{27bg^4(a + bx)^3} + \frac{5B^2d}{18b(bc - ad)g^4(a + bx)^2} - \frac{11B^2d^2}{9b(bc - ad)^2g^4(a + bx)} - \frac{11B^2d^3 \log(a + bx)}{9b(bc - ad)^3g^4} \\
 &= -\frac{2B^2}{27bg^4(a + bx)^3} + \frac{5B^2d}{18b(bc - ad)g^4(a + bx)^2} - \frac{11B^2d^2}{9b(bc - ad)^2g^4(a + bx)} - \frac{11B^2d^3 \log(a + bx)}{9b(bc - ad)^3g^4} \\
 &= -\frac{2B^2}{27bg^4(a + bx)^3} + \frac{5B^2d}{18b(bc - ad)g^4(a + bx)^2} - \frac{11B^2d^2}{9b(bc - ad)^2g^4(a + bx)} - \frac{11B^2d^3 \log(a + bx)}{9b(bc - ad)^3g^4}
 \end{aligned}$$

**Mathematica [C]** time = 0.790161, size = 585, normalized size = 1.47

$$\frac{B\left(-18Bd^3(a+bx)^3\left(\log(a+bx)\left(\log(a+bx)-2 \log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2 \text{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right)\right)+18Bd^3(a+bx)^3\left(2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2 \log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)\right)}{27b^2g^4(a+bx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2/(a\*g + b\*g\*x)^4, x]

[Out] -(18\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2 + (B\*(36\*B\*d^2\*(a + b\*x)^2\*(b\*c - a\*d + d\*(a + b\*x)\*Log[a + b\*x] - d\*(a + b\*x)\*Log[c + d\*x]) - 9\*B\*d\*(a +

$$\begin{aligned}
& b*x)*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*\text{Log}[ \\
& a + b*x] + 2*d^2*(a + b*x)^2*\text{Log}[c + d*x]) + 2*B*(2*(b*c - a*d)^3 - 3*d*(b* \\
& c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*\text{Lo} \\
& \text{g}[a + b*x] - 6*d^3*(a + b*x)^3*\text{Log}[c + d*x]) - 12*(b*c - a*d)^3*(A + B*\text{Log}[ \\
& (e*(c + d*x))/(a + b*x)]) + 18*d*(b*c - a*d)^2*(a + b*x)*(A + B*\text{Log}[(e*(c + \\
& d*x))/(a + b*x)]) + 36*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*\text{Log}[(e*(c + d \\
& *x))/(a + b*x)]) - 36*d^3*(a + b*x)^3*\text{Log}[a + b*x]*(A + B*\text{Log}[(e*(c + d*x)) \\
& / (a + b*x)]) + 36*d^3*(a + b*x)^3*\text{Log}[c + d*x]*(A + B*\text{Log}[(e*(c + d*x))/(a \\
& + b*x)]) - 18*B*d^3*(a + b*x)^3*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + \\
& d*x))/(b*c - a*d)])) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)] + 18*B*d \\
& ^3*(a + b*x)^3*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \text{Log}[c + d*x])* \text{Log}[c \\
& + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)]))/ (b*c - a*d)^3/(54*b*g^ \\
& 4*(a + b*x)^3)
\end{aligned}$$

**Maple [B]** time = 0.055, size = 2758, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((A+B*\ln(e*(d*x+c)/(b*x+a)))^2/(b*g*x+a*g)^4,x)$

[Out] 
$$\begin{aligned}
& -1/3/(a*d-b*c)^4/g^4*A^2*d^3*c+8/3/(a*d-b*c)^4/g^4*A*B*\ln(d*e/b-e*(a*d-b*c) \\
& /b/(b*x+a))/(b*x+a)^3*a^3*d^3*c-2*b/(a*d-b*c)^4/g^4*B^2*\ln(d*e/b-e*(a*d-b*c) \\
& )/b/(b*x+a)^2/(b*x+a)^3*a^2*d^2*c^2-2/3/b/(a*d-b*c)^4/g^4*A*B*\ln(d*e/b-e*( \\
& a*d-b*c)/b/(b*x+a))/(b*x+a)^3*a^4*d^4+4/3*b^2/(a*d-b*c)^4/g^4*B^2*\ln(d*e/b- \\
& e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a)^3*c^3*a*d+b/(a*d-b*c)^4/g^4*B^2*\ln(d*e/b-e \\
& *(a*d-b*c)/b/(b*x+a))*d^2/(b*x+a)^2*a*c^2+4/3*b/(a*d-b*c)^4/g^4*B^2*\ln(d*e/ \\
& b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^3*a^2*d^2*c^2-8/9*b^2/(a*d-b*c)^4/g^4*B^2* \\
& \ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^3*a*d*c^3+8/3*b^2/(a*d-b*c)^4/g^4*A \\
& *B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^3*c^3*a*d-4*b/(a*d-b*c)^4/g^4*A* \\
& B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^3*a^2*d^2*c^2-8/9*b^2/(a*d-b*c)^4 \\
& /g^4*A*B/(b*x+a)^3*a*d*c^3+4/3*b/(a*d-b*c)^4/g^4*A*B/(b*x+a)^3*a^2*d^2*c^2+ \\
& b/(a*d-b*c)^4/g^4*A*B*d^2/(b*x+a)^2*a*c^2-4/3/(a*d-b*c)^4/g^4*B^2*\ln(d*e/b- \\
& e*(a*d-b*c)/b/(b*x+a))*d^3/(b*x+a)*a*c+1/3/b/(a*d-b*c)^4/g^4*B^2*\ln(d*e/b-e \\
& *(a*d-b*c)/b/(b*x+a))*d^4/(b*x+a)^2*a^3-1/3*b^2/(a*d-b*c)^4/g^4*B^2*\ln(d*e/ \\
& b-e*(a*d-b*c)/b/(b*x+a))*d/(b*x+a)^2*c^3-1/3/b/(a*d-b*c)^4/g^4*B^2*\ln(d*e/b \\
& -e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a)^3*a^4*d^4+2/9/b/(a*d-b*c)^4/g^4*B^2*\ln(d* \\
& e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^3*a^4*d^4+85/54/b/(a*d-b*c)^4/g^4*B^2*d^ \\
& 4*a+8/27/(a*d-b*c)^4/g^4*B^2/(b*x+a)^3*a^3*d^3*c+1/3/b/(a*d-b*c)^4/g^4*B^2* \\
& \ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*d^4*a-11/9/b/(a*d-b*c)^4/g^4*B^2*\ln(d*e/b \\
& -e*(a*d-b*c)/b/(b*x+a))*d^4*a+2/9*b^3/(a*d-b*c)^4/g^4*B^2*\ln(d*e/b-e*(a*d-b \\
& *c)/b/(b*x+a))/(b*x+a)^3*c^4-1/3*b^3/(a*d-b*c)^4/g^4*B^2*\ln(d*e/b-e*(a*d-b* \\
& c)/b/(b*x+a))^2/(b*x+a)^3*c^4-1/3*b^3/(a*d-b*c)^4/g^4*A^2/(b*x+a)^3*c^4-2/2 \\
& 7*b^3/(a*d-b*c)^4/g^4*B^2/(b*x+a)^3*c^4-1/3/(a*d-b*c)^4/g^4*B^2*\ln(d*e/b-e* \\
& (a*d-b*c)/b/(b*x+a))^2*d^3*c-2/3/(a*d-b*c)^4/g^4*A*B*\ln(d*e/b-e*(a*d-b*c)/b \\
& / (b*x+a))*d^3*c+5/6/(a*d-b*c)^4/g^4*B^2*d^3/(b*x+a)^2*a^2*c+4/3/(a*d-b*c)^4 \\
& /g^4*A^2/(b*x+a)^3*a^3*d^3*c+22/9/(a*d-b*c)^4/g^4*B^2*d^3/(b*x+a)*a*c+2/9/b \\
& / (a*d-b*c)^4/g^4*A*B/(b*x+a)^3*a^4*d^4-1/3*b^2/(a*d-b*c)^4/g^4*A*B*d/(b*x+a \\
& )^2*c^3+1/3/b/(a*d-b*c)^4/g^4*A*B*d^4/(b*x+a)^2*a^3+4/3*b^2/(a*d-b*c)^4/g^4 \\
& *A^2/(b*x+a)^3*a*d*c^3-5/6*b/(a*d-b*c)^4/g^4*B^2*d^2/(b*x+a)^2*c^2*a+2/3*b/ \\
& (a*d-b*c)^4/g^4*A*B*d^2/(b*x+a)*c^2+2/3/b/(a*d-b*c)^4/g^4*A*B*d^4/(b*x+a)*a \\
& ^2+11/9/(a*d-b*c)^4/g^4*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^3*c-11/9/b/(a \\
& *d-b*c)^4/g^4*A*B*d^4*a+11/9/(a*d-b*c)^4/g^4*A*B*d^3*c-2/27/b/(a*d-b*c)^4/g \\
& ^4*B^2/(b*x+a)^3*a^4*d^4-11/9/b/(a*d-b*c)^4/g^4*B^2*d^4/(b*x+a)*a^2-11/9*b/ \\
& (a*d-b*c)^4/g^4*B^2*d^2/(b*x+a)*c^2-1/3/b/(a*d-b*c)^4/g^4*A^2/(b*x+a)^3*a^4 \\
& *d^4+8/27*b^2/(a*d-b*c)^4/g^4*B^2/(b*x+a)^3*c^3*a*d-4/9*b/(a*d-b*c)^4/g^4*B
\end{aligned}$$

$$\begin{aligned} & \frac{1}{2} \frac{1}{(bx+a)^3} \frac{1}{a^2 d^2 c^2 - 2b} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{A^2} \frac{1}{(bx+a)^3} \frac{1}{a^2 d^2 c^2 - 8/9} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{AB} \frac{1}{(bx+a)^3} \frac{1}{a^3 d^3 c^2 - 4/3} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{AB} \frac{1}{d^3} \frac{1}{(bx+a)^3} \frac{1}{a^2 c^2 + 2/3} \frac{1}{b} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{B^2} \frac{1}{\ln} \frac{1}{(d^2 e/b - e(ad-bc)/b)} \frac{1}{(bx+a)} \frac{1}{d^4} \frac{1}{(bx+a)^2} \frac{1}{a^2 + 2/3} \frac{1}{b} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{B^2} \frac{1}{\ln} \frac{1}{(d^2 e/b - e(ad-bc)/b)} \frac{1}{(bx+a)} \frac{1}{d^2} \frac{1}{(bx+a)^2} \frac{1}{a^2 - 1} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{B^2} \frac{1}{\ln} \frac{1}{(d^2 e/b - e(ad-bc)/b)} \frac{1}{(bx+a)} \frac{1}{d^3} \frac{1}{(bx+a)^2} \frac{1}{a^2 c^2 - 8/9} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{B^2} \frac{1}{\ln} \frac{1}{(d^2 e/b - e(ad-bc)/b)} \frac{1}{(bx+a)} \frac{1}{(bx+a)^3} \frac{1}{a^3 d^3 c^2 + 4/3} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{B^2} \frac{1}{\ln} \frac{1}{(d^2 e/b - e(ad-bc)/b)} \frac{1}{(bx+a)} \frac{1}{(bx+a)^2} \frac{1}{(bx+a)^3} \frac{1}{a^3 d^3 c^2 + 9/b^3} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{AB} \frac{1}{(bx+a)^3} \frac{1}{c^4 - 5/18} \frac{1}{b} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{B^2} \frac{1}{d^4} \frac{1}{(bx+a)^2} \frac{1}{a^3 + 5/18} \frac{1}{b^2} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{B^2} \frac{1}{d} \frac{1}{(bx+a)^2} \frac{1}{c^3 - 85/54} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{B^2} \frac{1}{d^3} \frac{1}{c^2 + 1/3} \frac{1}{b} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{A^2} \frac{1}{d^4} \frac{1}{a^2} \frac{1}{a + 2/3} \frac{1}{b} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{AB} \frac{1}{\ln} \frac{1}{(d^2 e/b - e(ad-bc)/b)} \frac{1}{(bx+a)} \frac{1}{d^4} \frac{1}{a^2} \frac{1}{3} \frac{1}{b^3} \frac{1}{(ad-bc)^4} \frac{1}{g^4} \frac{1}{AB} \frac{1}{\ln} \frac{1}{(d^2 e/b - e(ad-bc)/b)} \frac{1}{(bx+a)} \frac{1}{(bx+a)^3} \frac{1}{c^4} \end{aligned}$$

**Maxima [B]** time = 1.78074, size = 1917, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g)^4,x, algorithm="maxima")

[Out]  $\frac{1}{54} \cdot (6 \cdot ((6b^2d^2x^2 + 2b^2c^2 - 7a^2b^2cd + 11a^2d^2 - 3(b^2cd - 5abd^2))x) / ((b^6c^2 - 2a^2b^5cd + a^2b^4d^2)g^4x^3 + 3(a^2b^5c^2 - 2a^2b^4cd + a^3b^3d^2)g^4x^2 + 3(a^2b^4c^2 - 2a^3b^3cd + a^4b^2d^2)g^4x + (a^3b^3c^2 - 2a^4b^2cd + a^5bd^2)g^4) + 6d^3 \log(bx+a) / ((b^4c^3 - 3a^2b^3c^2d + 3a^2b^2cd^2 - a^3bd^3)g^4) - 6d^3 \log(dx+c) / ((b^4c^3 - 3a^2b^3c^2d + 3a^2b^2cd^2 - a^3bd^3)g^4)) \cdot \log(dx+c) / (bx+a) + ce/(bx+a) - (4b^3c^3 - 27a^2b^2c^2d + 108a^2b^2cd^2 - 85a^3d^3 + 66(b^3cd^2 - abd^3))x^2 - 18(b^3d^3x^3 + 3a^2b^2d^3x^2 + 3a^2bd^3x + a^3d^3) \log(bx+a)^2 - 18(b^3d^3x^3 + 3a^2b^2d^3x^2 + 3a^2bd^3x + a^3d^3) \log(dx+c)^2 - 3(5b^3c^2d - 54a^2b^2cd^2 + 49a^2bd^3)x + 66(b^3d^3x^3 + 3a^2b^2d^3x^2 + 3a^2bd^3x + a^3d^3) \log(bx+a) - 6(11b^3d^3x^3 + 33a^2b^2d^3x^2 + 33a^2bd^3x + 11a^3d^3 - 6(b^3d^3x^3 + 3a^2b^2d^3x^2 + 3a^2bd^3x + a^3d^3) \log(bx+a)) \log(dx+c) / (a^3b^4c^3g^4 - 3a^4b^3c^2dg^4 + 3a^5b^2cd^2g^4 - a^6bd^3g^4 + (b^7c^3g^4 - 3a^2b^6c^2dg^4 + 3a^2b^5cd^2g^4 - a^3b^4d^3g^4)x^3 + 3(a^6c^3g^4 - 3a^2b^5c^2dg^4 + 3a^3b^4cd^2g^4 - a^4b^3d^3g^4)x^2 + 3(a^2b^5c^3g^4 - 3a^3b^4cd^2dg^4 + 3a^4b^3cd^2g^4 - a^5b^2d^3g^4)x) \cdot B^2 + 1/9 \cdot AB \cdot ((6b^2d^2x^2 + 2b^2c^2 - 7a^2b^2cd + 11a^2d^2 - 3(b^2cd - 5abd^2))x) / ((b^6c^2 - 2a^2b^5cd + a^2b^4d^2)g^4x^3 + 3(a^2b^5c^2 - 2a^2b^4cd + a^3b^3d^2)g^4x^2 + 3(a^2b^4c^2 - 2a^3b^3cd + a^4b^2d^2)g^4x + (a^3b^3c^2 - 2a^4b^2cd + a^5bd^2)g^4) - 6 \log(dx+c) / (bx+a) + ce/(bx+a) / (b^4g^4x^3 + 3a^2b^3g^4x^2 + 3a^2b^2g^4x + a^3bg^4) + 6d^3 \log(bx+a) / ((b^4c^3 - 3a^2b^3c^2d + 3a^2b^2cd^2 - a^3bd^3)g^4) - 6d^3 \log(dx+c) / ((b^4c^3 - 3a^2b^3c^2d + 3a^2b^2cd^2 - a^3bd^3)g^4)) - 1/3 \cdot B^2 \cdot \log(dx+c) / (bx+a) + ce/(bx+a) ^2 / (b^4g^4x^3 + 3a^2b^3g^4x^2 + 3a^2b^2g^4x + a^3bg^4) - 1/3 \cdot A^2 / (b^4g^4x^3 + 3a^2b^3g^4x^2 + 3a^2b^2g^4x + a^3bg^4)$

**Fricas [A]** time = 1.30216, size = 1388, normalized size = 3.48

$$2(9A^2 - 6AB + 2B^2)b^3c^3 - 27(2A^2 - 2AB + B^2)ab^2c^2d + 54(A^2 - 2AB + 2B^2)a^2bcd^2 - (18A^2 - 66AB + 85B^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g)^4,x, algorithm="fricas")

[Out] 
$$-1/54*(2*(9*A^2 - 6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 - 2*A*B + B^2)*a*b^2*c^2*d + 54*(A^2 - 2*A*B + 2*B^2)*a^2*b*c*d^2 - (18*A^2 - 66*A*B + 85*B^2)*a^3*d^3 - 6*((6*A*B - 11*B^2)*b^3*c*d^2 - (6*A*B - 11*B^2)*a*b^2*d^3)*x^2 + 18*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2)*\log((d*e*x + c*e)/(b*x + a))^2 + 3*((6*A*B - 5*B^2)*b^3*c^2*d - 18*(2*A*B - 3*B^2)*a*b^2*c*d^2 + (30*A*B - 49*B^2)*a^2*b*d^3)*x + 6*((6*A*B - 11*B^2)*b^3*d^3*x^3 + 2*(3*A*B - B^2)*b^3*c^3 - 9*(2*A*B - B^2)*a*b^2*c^2*d + 18*(A*B - B^2)*a^2*b*c*d^2 - 3*(2*B^2*b^3*c*d^2 - 3*(2*A*B - 3*B^2)*a*b^2*d^3)*x^2 + 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 6*(A*B - B^2)*a^2*b*d^3)*x*\log((d*e*x + c*e)/(b*x + a)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)$$

**Sympy [B]** time = 34.789, size = 1544, normalized size = 3.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))\*\*2/(b\*g\*x+a\*g)\*\*4,x)

[Out] 
$$B*d**3*(6*A - 11*B)*\log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 - 11*B**2*a*d**4 - 11*B**2*b*c*d**3 - B*a**4*d**7*(6*A - 11*B)/(a*d - b*c)**3 + 4*B*a**3*b*c*d**6*(6*A - 11*B)/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**5*(6*A - 11*B)/(a*d - b*c)**3 + 4*B*a*b**3*c**3*d**4*(6*A - 11*B)/(a*d - b*c)**3 - B*b**4*c**4*d**3*(6*A - 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 - 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) - B*d**3*(6*A - 11*B)*\log(x + (6*A*B*a*d**4 + 6*A*B*b*c*d**3 - 11*B**2*a*d**4 - 11*B**2*b*c*d**3 + B*a**4*d**7*(6*A - 11*B)/(a*d - b*c)**3 - 4*B*a**3*b*c*d**6*(6*A - 11*B)/(a*d - b*c)**3 + 6*B*a**2*b**2*c**2*d**5*(6*A - 11*B)/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**4*(6*A - 11*B)/(a*d - b*c)**3 + B*b**4*c**4*d**3*(6*A - 11*B)/(a*d - b*c)**3)/(12*A*B*b*d**4 - 22*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*\log(e*(c + d*x)/(a + b*x))**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 + 11*B**2*a**2*d**2 - 7*B**2*a*b*c*d + 15*B**2*a*b*d**2*x + 2*B**2*b**2*c**2 - 3*B**2*b**2*c*d*x + 6*B**2*b**2*d**2*x**2)*\log(e*(c + d*x)/(a + b*x))/(9*a**5*b*d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 27*a**4*b**2*d**2$$

$2g^4x + 9a^3b^3c^2g^4 - 54a^3b^3cdg^4x + 27a^3b^3d^2g^4x^2 + 27a^2b^4c^2g^4x - 54a^2b^4cdg^4x^2 + 9a^2b^4d^2g^4x^3 + 27ab^5c^2g^4x^2 - 18ab^5cdg^4x^3 + 9b^6c^2g^4x^3) + (-18A^2a^2d^2 + 36A^2abc d - 18A^2b^2c^2 + 66ABa^2d^2 - 42ABab^2cd + 12ABb^2c^2 - 85B^2a^2d^2 + 23B^2abc d - 4B^2b^2c^2 + x^2(36ABb^2d^2 - 66B^2b^2d^2) + x(90ABab^2d^2 - 18ABb^2cd - 147B^2abd^2 + 15B^2b^2cd)) / (54a^5b^2d^2g^4 - 108a^4b^2cdg^4 + 54a^3b^3c^2g^4 + x^3(54a^2b^4d^2g^4 - 108ab^5cdg^4 + 54b^6c^2g^4) + x^2(162a^3b^3d^2g^4 - 324a^2b^4cdg^4 + 162ab^5c^2g^4) + x(162a^4b^2d^2g^4 - 324a^3b^3cdg^4 + 162a^2b^4c^2g^4))$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(dx+c)e}{bx+a}\right) + A\right)^2}{(bgx + ag)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g)^4,x, algorithm="giac")

[Out] integrate((B\*log((d\*x + c)\*e/(b\*x + a)) + A)^2/(b\*g\*x + a\*g)^4, x)

$$3.190 \quad \int \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag+bgx)^5} dx$$

**Optimal.** Leaf size=498

$$\frac{2b^2Bd(c+dx)^3\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4} + \frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{8g^5(a+bx)^4(bc-ad)^4} + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2bg^5(bc-ad)^4}$$

[Out]  $(2*B^2*d^3*(c+d*x))/((b*c-a*d)^4*g^5*(a+b*x)) - (3*b*B^2*d^2*(c+d*x)^2)/(4*(b*c-a*d)^4*g^5*(a+b*x)^2) + (2*b^2*B^2*d*(c+d*x)^3)/(9*(b*c-a*d)^4*g^5*(a+b*x)^3) - (b^3*B^2*(c+d*x)^4)/(32*(b*c-a*d)^4*g^5*(a+b*x)^4) - (B^2*d^4*Log[(c+d*x)/(a+b*x)]^2)/(4*b*(b*c-a*d)^4*g^5) - (2*B*d^3*(c+d*x)*(A+B*Log[(e*(c+d*x))/(a+b*x])))/((b*c-a*d)^4*g^5*(a+b*x)) + (3*b*B*d^2*(c+d*x)^2*(A+B*Log[(e*(c+d*x))/(a+b*x])))/(2*(b*c-a*d)^4*g^5*(a+b*x)^2) - (2*b^2*B*d*(c+d*x)^3*(A+B*Log[(e*(c+d*x))/(a+b*x])))/(3*(b*c-a*d)^4*g^5*(a+b*x)^3) + (b^3*B*(c+d*x)^4*(A+B*Log[(e*(c+d*x))/(a+b*x])))/(8*(b*c-a*d)^4*g^5*(a+b*x)^4) + (B*d^4*Log[(c+d*x)/(a+b*x)]*(A+B*Log[(e*(c+d*x))/(a+b*x])))/(2*b*(b*c-a*d)^4*g^5) - (A+B*Log[(e*(c+d*x))/(a+b*x)])^2/(4*b*g^5*(a+b*x)^4)$

**Rubi [C]** time = 1.26372, antiderivative size = 763, normalized size of antiderivative = 1.53, number of steps used = 38, number of rules used = 11, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2d^4\text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{2bg^5(bc-ad)^4} + \frac{B^2d^4\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{2bg^5(bc-ad)^4} - \frac{Bd^4\log(a+bx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2bg^5(bc-ad)^4} + \frac{Bd^4\log(c+dx)\left(B\log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}{2bg^5(bc-ad)^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2/(a\*g + b\*g\*x)^5, x]

[Out]  $-B^2/(32*b*g^5*(a+b*x)^4) + (7*B^2*d)/(72*b*(b*c-a*d)*g^5*(a+b*x)^3) - (13*B^2*d^2)/(48*b*(b*c-a*d)^2*g^5*(a+b*x)^2) + (25*B^2*d^3)/(24*b*(b*c-a*d)^3*g^5*(a+b*x)) + (25*B^2*d^4*Log[a+b*x])/(24*b*(b*c-a*d)^4*g^5) - (B^2*d^4*Log[a+b*x]^2)/(4*b*(b*c-a*d)^4*g^5) - (25*B^2*d^4*Log[c+d*x])/(24*b*(b*c-a*d)^4*g^5) + (B^2*d^4*Log[-((d*(a+b*x))/(b*c-a*d))]*Log[c+d*x])/(2*b*(b*c-a*d)^4*g^5) - (B^2*d^4*Log[c+d*x]^2)/(4*b*(b*c-a*d)^4*g^5) + (B^2*d^4*Log[a+b*x]*Log[(b*(c+d*x))/(b*c-a*d)])/(2*b*(b*c-a*d)^4*g^5) + (B*(A+B*Log[(e*(c+d*x))/(a+b*x])))/(8*b*g^5*(a+b*x)^4) - (B*d*(A+B*Log[(e*(c+d*x))/(a+b*x])))/(6*b*(b*c-a*d)*g^5*(a+b*x)^3) + (B*d^2*(A+B*Log[(e*(c+d*x))/(a+b*x])))/(4*b*(b*c-a*d)^2*g^5*(a+b*x)^2) - (B*d^3*(A+B*Log[(e*(c+d*x))/(a+b*x])))/(2*b*(b*c-a*d)^3*g^5*(a+b*x)) - (B*d^4*Log[a+b*x]*(A+B*Log[(e*(c+d*x))/(a+b*x])))/(2*b*(b*c-a*d)^4*g^5) + (B*d^4*Log[c+d*x]*(A+B*Log[(e*(c+d*x))/(a+b*x])))/(2*b*(b*c-a*d)^4*g^5) - (A+B*Log[(e*(c+d*x))/(a+b*x)])^2/(4*b*g^5*(a+b*x)^4) + (B^2*d^4*PolyLog[2, -((d*(a+b*x))/(b*c-a*d))])/(2*b*(b*c-a*d)^4*g^5) + (B^2*d^4*PolyLog[2, (b*(c+d*x))/(b*c-a*d)])/(2*b*(b*c-a*d)^4*g^5)$

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^n\*(d\_.) + (e\_.)\*(x\_)]^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1))

```
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

### Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393



```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))]*(b_.))/((f_.) + (g_.)*(x_.)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{(ag + bgx)^5} dx &= -\frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B \int \frac{(bc-ad)\left(-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{g^4(a+bx)^5(c+dx)} dx}{2bg} \\
&= -\frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad)) \int \frac{-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^5(c+dx)} dx}{2bg^5} \\
&= -\frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{(B(bc-ad)) \int \left(\frac{b\left(-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)(a+bx)^5} - \frac{bd\left(-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{(bc-ad)^2(a+bx)^4}\right) dx}{2bg^5} \\
&= -\frac{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2}{4bg^5(a+bx)^4} + \frac{B \int \frac{-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)}{(a+bx)^5} dx}{2g^5} + \frac{(Bd^4) \int \frac{-A-B \log\left(\frac{e(c+dx)}{a+bx}\right)}{a+bx} dx}{2(bc-ad)^4g^5} - \frac{(Bd^5) \int \frac{1}{a+bx} dx}{2g^5} \\
&= \frac{B\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8bg^5(a+bx)^4} - \frac{Bd\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b(bc-ad)^2g^5(a+bx)^2} - \frac{Bd^3}{2b} \\
&= \frac{B\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8bg^5(a+bx)^4} - \frac{Bd\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b(bc-ad)^2g^5(a+bx)^2} - \frac{Bd^3}{2b} \\
&= \frac{B\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{8bg^5(a+bx)^4} - \frac{Bd\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{6b(bc-ad)g^5(a+bx)^3} + \frac{Bd^2\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{4b(bc-ad)^2g^5(a+bx)^2} - \frac{Bd^3}{2b} \\
&= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{25B^2d^3}{24b(bc-ad)g^5} \\
&= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{25B^2d^3}{24b(bc-ad)g^5} \\
&= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{25B^2d^3}{24b(bc-ad)g^5} \\
&= -\frac{B^2}{32bg^5(a+bx)^4} + \frac{7B^2d}{72b(bc-ad)g^5(a+bx)^3} - \frac{13B^2d^2}{48b(bc-ad)^2g^5(a+bx)^2} + \frac{25B^2d^3}{24b(bc-ad)g^5}
\end{aligned}$$

**Mathematica [C]** time = 1.06083, size = 748, normalized size = 1.5

$$B\left(-72Bd^4(a+bx)^4\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+72Bd^4(a+bx)^4\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2/(a\*g + b\*g\*x)^5,x]

[Out]  $(-72*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x)])^2 + (B*(144*B*d^3*(a + b*x)^3*(b*c - a*d + d*(a + b*x)*\text{Log}[a + b*x] - d*(a + b*x)*\text{Log}[c + d*x]) - 36*B*d^2*(a + b*x)^2*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*\text{Log}[a + b*x] + 2*d^2*(a + b*x)^2*\text{Log}[c + d*x]) + 8*B*d*(a + b*x)*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*\text{Log}[a + b*x] - 6*d^3*(a + b*x)^3*\text{Log}[c + d*x]) - 3*B*(3*(b*c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*\text{Log}[a + b*x] + 12*d^4*(a + b*x)^4*\text{Log}[c + d*x]) + 36*(b*c - a*d)^4*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x)]) + 48*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x)]) + 72*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x)]) + 144*d^3*(-(b*c) + a*d)*(a + b*x)^3*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x)]) - 144*d^4*(a + b*x)^4*\text{Log}[a + b*x]*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x)]) + 144*d^4*(a + b*x)^4*\text{Log}[c + d*x]*(A + B*\text{Log}[(e*(c + d*x))/(a + b*x)]) - 72*B*d^4*(a + b*x)^4*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 72*B*d^4*(a + b*x)^4*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^4/(288*b*g^5*(a + b*x)^4)$

**Maple [B]** time = 0.054, size = 3717, normalized size = 7.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g)^5,x)

[Out]  $-5/4*b^2/(a*d-b*c)^5/g^5*A*B/(b*x+a)^4*a^2*d^2*c^3+5/8*b^3/(a*d-b*c)^5/g^5*A*B/(b*x+a)^4*a*d*c^4+5/4*b/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*a^3*d^3*c^2-5/4*b^2/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*a^2*d^2*c^3+5/8*b^3/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*a*d*c^4+b/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^3/(b*x+a)^3*a^2*c^2-1/2/b/(a*d-b*c)^5/g^5*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*a^5*d^5-2/3*b^2/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^2/(b*x+a)^3*c^3*a-5/2*b/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a)^4*a^3*d^3*c^2+5/2*b^2/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a)^4*a^2*d^2*c^3+3/4*b/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^3/(b*x+a)^2*c^2*a-5/4*b^3/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a)^4*c^4*a*d+5/2/(a*d-b*c)^5/g^5*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*a^4*d^4*c-5*b/(a*d-b*c)^5/g^5*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*a^3*d^3*c^2+5*b^2/(a*d-b*c)^5/g^5*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*a^2*d^2*c^3-5/2*b^3/(a*d-b*c)^5/g^5*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*a*d*c^4+5/2*b^2/(a*d-b*c)^5/g^5*A^2/(b*x+a)^4*a^2*d^2*c^3-5/2*b/(a*d-b*c)^5/g^5*A^2/(b*x+a)^4*a^3*d^3*c^2+1/6*b^3/(a*d-b*c)^5/g^5*A*B*d/(b*x+a)^3*c^4+1/4/b/(a*d-b*c)^5/g^5*A*B*d^5/(b*x+a)^2*a^3-1/4*b^2/(a*d-b*c)^5/g^5*A*B*d^2/(b*x+a)^2*c^3+1/6/b/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^5/(b*x+a)^3*a^4-7/12*b/(a*d-b*c)^5/g^5*B^2*d^3/(b*x+a)^3*a^2*c^2-13/16*b/(a*d-b*c)^5/g^5*B^2*d^3/(b*x+a)^2*a*c^2+1/2*b^4/(a*d-b*c)^5/g^5*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*c^5+1/6/b/(a*d-b*c)^5/g^5*A*B*d^5/(b*x+a)^3*a^4-5/16*b/(a*d-b*c)^5/g^5*B^2/(b*x+a)^4*a^3*d^3*c^2+5/16*b^2/(a*d-b*c)^5/g^5*B^2/(b*x+a)^4*a^2*d^2*c^3-5/32*b^3/(a*d-b*c)^5/g^5*B^2/(b*x+a)^4*a*d*c^4+1/8/b/(a*d-b*c)^5/g^5*A*B/(b*x+a)^4*a^5*d^5-2/3/(a*d-b*c)^5/g^5*A*B*d^4/(b*x+a)^3*a^3*c-5/8/(a*d-b*c)^5/g^5*A*B/(b*x+a)^4*a^4*d^4*c-3/4/(a*d-b*c)^5/g^5$

$$5A*B*d^4/(b*x+a)^2*a^2*c-1/(a*d-b*c)^5/g^5*A*B*d^4/(b*x+a)*a*c+1/8/b/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*a^5*d^5+1/2/b/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^5/(b*x+a)*a^2+1/2*b/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^3/(b*x+a)*c^2-2/3/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^4/(b*x+a)^3*a^3*c-5/8/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*a^4*d^4*c-1/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^4/(b*x+a)*a*c+5/4/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a)^4*a^4*d^4*c-3/4/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^4/(b*x+a)^2*a^2*c-1/4/b/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a)^4*a^5*d^5+1/6*b^3/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d/(b*x+a)^3*c^4+1/4/b/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^5/(b*x+a)^2*a^3+7/18*b^2/(a*d-b*c)^5/g^5*B^2*d^2/(b*x+a)^3*a*c^3-5/4*b^3/(a*d-b*c)^5/g^5*A^2/(b*x+a)^4*a*d*c^4+1/2/b/(a*d-b*c)^5/g^5*A*B*d^5/(b*x+a)*a^2+1/2*b/(a*d-b*c)^5/g^5*A*B*d^3/(b*x+a)*c^2-1/4*b^2/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^2/(b*x+a)^2*c^3+1/2/b/(a*d-b*c)^5/g^5*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^5*a+3/4*b/(a*d-b*c)^5/g^5*A*B*d^3/(b*x+a)^2*a*c^2+5/4*b/(a*d-b*c)^5/g^5*A*B/(b*x+a)^4*a^3*d^3*c^2+b/(a*d-b*c)^5/g^5*A*B*d^3/(b*x+a)^3*a^2*c^2-2/3*b^2/(a*d-b*c)^5/g^5*A*B*d^2/(b*x+a)^3*a*c^3+1/4*b^4/(a*d-b*c)^5/g^5*A^2/(b*x+a)^4*c^5+1/32*b^4/(a*d-b*c)^5/g^5*B^2/(b*x+a)^4*c^5+25/24/(a*d-b*c)^5/g^5*A*B*d^4*c-25/24/b/(a*d-b*c)^5/g^5*A*B*d^5*a-1/8*b^4/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))/(b*x+a)^4*c^5+25/12/(a*d-b*c)^5/g^5*B^2*d^4/(b*x+a)*a*c+13/16/(a*d-b*c)^5/g^5*B^2*d^4/(b*x+a)^2*a^2*c+7/18/(a*d-b*c)^5/g^5*B^2*d^4/(b*x+a)^3*a^3*c-1/4/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*d^4*c-1/2/(a*d-b*c)^5/g^5*A*B*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^4*c-1/8*b^4/(a*d-b*c)^5/g^5*A*B/(b*x+a)^4*c^5-1/32/b/(a*d-b*c)^5/g^5*B^2/(b*x+a)^4*a^5*d^5-25/24/b/(a*d-b*c)^5/g^5*B^2*d^5/(b*x+a)*a^2-25/24*b/(a*d-b*c)^5/g^5*B^2*d^3/(b*x+a)*c^2+25/24/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^4*c-1/4/(a*d-b*c)^5/g^5*A^2*d^4*c+415/288/b/(a*d-b*c)^5/g^5*B^2*d^5*a-415/288/(a*d-b*c)^5/g^5*B^2*d^4*c+5/32/(a*d-b*c)^5/g^5*B^2/(b*x+a)^4*a^4*d^4*c+5/4/(a*d-b*c)^5/g^5*A^2/(b*x+a)^4*a^4*d^4*c+13/48*b^2/(a*d-b*c)^5/g^5*B^2*d^2/(b*x+a)^2*c^3-1/4/b/(a*d-b*c)^5/g^5*A^2/(b*x+a)^4*a^5*d^5-7/72*b^3/(a*d-b*c)^5/g^5*B^2*d/(b*x+a)^3*c^4-13/48/b/(a*d-b*c)^5/g^5*B^2*d^5/(b*x+a)^2*a^3-7/72/b/(a*d-b*c)^5/g^5*B^2*d^5/(b*x+a)^3*a^4+1/4/b/(a*d-b*c)^5/g^5*A^2*d^5*a-25/24/b/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))*d^5*a+1/4/b/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2*d^5*a+1/4*b^4/(a*d-b*c)^5/g^5*B^2*\ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))^2/(b*x+a)^4*c^5$$

**Maxima [B]** time = 2.29015, size = 2865, normalized size = 5.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g)^5,x, algorithm="maxima")

[Out] 
$$-1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3))*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)$$

$$\begin{aligned}
& 4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))\log(d*e*x/ \\
& (b*x + a) + c*e/(b*x + a)) + (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2* \\
& d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*( \\
& 13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + \\
& 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*\log(b*x + a \\
& )^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x \\
& + a^4*d^4)*\log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2 \\
& *c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2 \\
& *d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*\log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100 \\
& *a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b \\
& ^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4) \\
& *\log(b*x + a))*\log(d*x + c))/(a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6 \\
& *b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a \\
& *b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4* \\
& g^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 - \\
& 4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6 \\
& *c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5 \\
& )*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - \\
& 4*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x)*B^2 - 1/24*A*B*((12*b^3*d^3*x^3 \\
& - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - \\
& 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c \\
& ^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 \\
& - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 \\
& - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 \\
& - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - \\
& 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 12*\log(d*e*x/(b*x + \\
& a) + c*e/(b*x + a))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4* \\
& a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*\log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d \\
& + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*\log(d*x + \\
& c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b* \\
& d^4)*g^5)) - 1/4*B^2*\log(d*e*x/(b*x + a) + c*e/(b*x + a))^2/(b^5*g^5*x^4 + \\
& 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^ \\
& 2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^ \\
& 4*b*g^5)
\end{aligned}$$

**Fricas [B]** time = 1.38743, size = 2152, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g)^5,x, algorithm="fricas")

[Out]  $-1/288*(9*(8*A^2 - 4*A*B + B^2)*b^4*c^4 - 32*(9*A^2 - 6*A*B + 2*B^2)*a*b^3*c^3*d + 216*(2*A^2 - 2*A*B + B^2)*a^2*b^2*c^2*d^2 - 288*(A^2 - 2*A*B + 2*B^2)*a^3*b*c*d^3 + (72*A^2 - 300*A*B + 415*B^2)*a^4*d^4 + 12*((12*A*B - 25*B^2)*b^4*c*d^3 - (12*A*B - 25*B^2)*a*b^3*d^4)*x^3 - 6*((12*A*B - 13*B^2)*b^4*c^2*d^2 - 16*(6*A*B - 11*B^2)*a*b^3*c*d^3 + (84*A*B - 163*B^2)*a^2*b^2*d^4)*x^2 - 72*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*B^2*a^3*b*c*d^3)*\log((d*e*x + c*e)/(b*x + a))^2 + 4*((12*A*B - 7*B^2)*b^4*c^3*d - 12*(6*A*B - 5*B^2)*a*b^3*c^2*d^2 + 108*(2*A*B - 3*B^2)*a^2*b^2*c*d^3 - (156*A*B - 271*B^2)*a^3*b*d^4)*x - 12*((12*A*B - 25*B^2)*b^4*d^4*x^4 - 3*(4*A*B - B^2)*b^4*c^4 + 16*(3*A*B - B^2)*a*b^3*c^3*d - 36*(2*A*B - B^2)*a^2*b^2*c^2*d^2 + 48*(A*B - B^2)*a^3*b*c*d^3 - 4*(3*B^2*b^4*c*d^3 - 2*(6*A*B - 11*B^2)*a*b^3*d^4)*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 6*$

$$(2AB - 3B^2)a^2b^2d^4x^2 - 4(B^2b^4c^3d - 6B^2ab^3c^2d^2 + 18B^2a^2b^2cd^3 - 12(AB - B^2)a^3bd^4)x \log\left(\frac{dex + ce}{bx + a}\right) / ((b^9c^4 - 4ab^8c^3d + 6a^2b^7c^2d^2 - 4a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4(ab^8c^4 - 4a^2b^7c^3d + 6a^3b^6c^2d^2 - 4a^4b^5cd^3 + a^5b^4d^4)g^5x^3 + 6(a^2b^7c^4 - 4a^3b^6c^3d + 6a^4b^5c^2d^2 - 4a^5b^4cd^3 + a^6b^3d^4)g^5x^2 + 4(a^3b^6c^4 - 4a^4b^5cd^3 + 6a^5b^4c^2d^2 - 4a^6b^3cd^3 + a^7b^2d^4)g^5x + (a^4b^5c^4 - 4a^5b^4c^3d + 6a^6b^3c^2d^2 - 4a^7b^2cd^3 + a^8bd^4)g^5)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)/(b\*x+a)))\*\*2/(b\*g\*x+a\*g)\*\*5,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(dx+c)e}{bx+a}\right) + A\right)^2}{(bgx + ag)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2/(b\*g\*x+a\*g)^5,x, algorithm="giac")

[Out] integrate((B\*log((d\*x + c)\*e/(b\*x + a)) + A)^2/(b\*g\*x + a\*g)^5, x)

$$3.191 \quad \int \frac{(ag+bgx)^2}{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{(ag + bgx)^2}{B \log\left(\frac{e(c+dx)}{a+bx}\right) + A}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

**Rubi [A]** time = 0.1956, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(ag + bgx)^2}{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

[Out] a^2\*g^2\*Defer[Int][(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]^(-1), x] + 2\*a\*b\*g^2\*Defer[Int][x/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x] + b^2\*g^2\*Defer[Int][x^2/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

Rubi steps

$$\begin{aligned} \int \frac{(ag + bgx)^2}{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx &= \int \left( \frac{a^2g^2}{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)} + \frac{2abg^2x}{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)} + \frac{b^2g^2x^2}{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)} \right) dx \\ &= (a^2g^2) \int \frac{1}{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx + (2abg^2) \int \frac{x}{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx + (b^2g^2) \int \frac{x^2}{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.586605, size = 0, normalized size = 0.

$$\int \frac{(ag + bgx)^2}{A + B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

[Out] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

**Maple [A]** time = 1.224, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln\left(\frac{e(dx + c)}{bx + a}\right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

[Out] `int((b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{B \log\left(\frac{(dx+c)e}{bx+a}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")`

[Out] `integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{b^2g^2x^2 + 2abg^2x + a^2g^2}{B \log\left(\frac{dex+ce}{bx+a}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")`

[Out] `integral((b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2)/(B*log((d*e*x + c*e)/(b*x + a)) + A), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{B \log\left(\frac{(dx+c)e}{bx+a}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)*e/(b*x + a)) + A), x)
```



$$3.192 \quad \int \frac{ag+bgx}{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx$$

**Optimal.** Leaf size=32

$$\text{Unintegrable}\left(\frac{ag+bgx}{B \log\left(\frac{e(c+dx)}{a+bx}\right)+A}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

**Rubi [A]** time = 0.10057, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{ag+bgx}{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

[Out] a\*g\*Defer[Int][(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]<sup>-1</sup>), x] + b\*g\*Defer[Int][x/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

Rubi steps

$$\begin{aligned} \int \frac{ag+bgx}{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx &= \int \left( \frac{ag}{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)} + \frac{bgx}{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)} \right) dx \\ &= (ag) \int \frac{1}{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx + (bg) \int \frac{x}{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.236994, size = 0, normalized size = 0.

$$\int \frac{ag+bgx}{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

[Out] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]), x]

**Maple [A]** time = 1.047, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln\left(\frac{e(dx + c)}{bx + a}\right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

[Out] `int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bgx + ag}{B \log\left(\frac{(dx+c)e}{bx+a}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")`

[Out] `integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bgx + ag}{B \log\left(\frac{dex+ce}{bx+a}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")`

[Out] `integral((b*g*x + a*g)/(B*log((d*e*x + c*e)/(b*x + a)) + A), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bgx + ag}{B \log\left(\frac{(dx+c)e}{bx+a}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")`

[Out] `integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A), x)`

$$3.193 \quad \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{1}{(ag+bgx)\left(B \log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)}, x\right)$$

[Out] Unintegrable[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])), x]

**Rubi [A]** time = 0.0730861, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)} dx = \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)} dx$$

**Mathematica [A]** time = 0.235204, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])), x]

[Out] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])), x]

**Maple [A]** time = 1.347, size = 0, normalized size = 0.

$$\int \frac{1}{bgx+ag} \left(A+B \ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(d\*x+c)/(b\*x+a))), x)

[Out] `int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)\left(B \log\left(\frac{(dx+c)e}{bx+a}\right) + A\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")`

[Out] `integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{Abgx + Aag + (Bbgx + Bag)\log\left(\frac{dex+ce}{bx+a}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")`

[Out] `integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((d*e*x + c*e)/(b*x + a))), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)\left(B \log\left(\frac{(dx+c)e}{bx+a}\right) + A\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")`

[Out] `integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)), x)`

$$3.194 \quad \int \frac{1}{(ag+bgx)^2 \left( A+B \log\left(\frac{e(c+dx)}{a+bx}\right) \right)} dx$$

**Optimal.** Leaf size=53

$$\frac{e^{-\frac{A}{B}} \operatorname{Ei}\left(\frac{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B e g^2 (bc - ad)}$$

[Out]  $-(\operatorname{ExpIntegralEi}[(A + B \operatorname{Log}[(e*(c + d*x))/(a + b*x)])/B]/(B*(b*c - a*d)*e*E^{(A/B)*g^2}))$

**Rubi [F]** time = 0.0858857, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag + bgx)^2 \left( A + B \log\left(\frac{e(c+dx)}{a+bx}\right) \right)} dx$$

Verification is Not applicable to the result.

[In]  $\operatorname{Int}[1/((a*g + b*g*x)^2*(A + B*\operatorname{Log}[(e*(c + d*x))/(a + b*x)])),x]$

[Out]  $\operatorname{Defer}[\operatorname{Int}[1/((a*g + b*g*x)^2*(A + B*\operatorname{Log}[(e*(c + d*x))/(a + b*x)])), x]$

Rubi steps

$$\int \frac{1}{(ag + bgx)^2 \left( A + B \log\left(\frac{e(c+dx)}{a+bx}\right) \right)} dx = \int \frac{1}{(ag + bgx)^2 \left( A + B \log\left(\frac{e(c+dx)}{a+bx}\right) \right)} dx$$

**Mathematica [A]** time = 0.0639659, size = 50, normalized size = 0.94

$$\frac{e^{-\frac{A}{B}} \operatorname{Ei}\left(\frac{A}{B} + \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B e g^2 (ad - bc)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[1/((a*g + b*g*x)^2*(A + B*\operatorname{Log}[(e*(c + d*x))/(a + b*x)])),x]$

[Out]  $\operatorname{ExpIntegralEi}[A/B + \operatorname{Log}[(e*(c + d*x))/(a + b*x)]]/(B*(-(b*c) + a*d)*e*E^{(A/B)*g^2})$

**Maple [A]** time = 0.286, size = 69, normalized size = 1.3

$$-\frac{1}{e(ad - bc)g^2B} e^{-\frac{A}{B}} \operatorname{Ei}\left(1, -\ln\left(\frac{de}{b} - \frac{e(ad - bc)}{b(bx + a)}\right) - \frac{A}{B}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*g*x+a*g)^2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

[Out] `-1/e/(a*d-b*c)/g^2/B*exp(-A/B)*Ei(1,-ln(d*e/b-e*(a*d-b*c)/b/(b*x+a))-A/B)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(dx+c)e}{bx+a} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")`

[Out] `integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)), x)`

**Fricas [A]** time = 0.979911, size = 109, normalized size = 2.06

$$\frac{e^{\left(-\frac{A}{B}\right)} \log\_integral\left(\frac{(dex+ce)e^{\frac{A}{B}}}{bx+a}\right)}{(Bbc - Bad)eg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")`

[Out] `-e^(-A/B)*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a))/((B*b*c - B*a*d)*e*g^2)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(dx+c)e}{bx+a} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")`

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

$$3.195 \quad \int \frac{1}{(ag+bgx)^3 \left( A+B \log\left(\frac{e(c+dx)}{a+bx}\right) \right)} dx$$

**Optimal.** Leaf size=109

$$\frac{de^{-\frac{A}{B}} \operatorname{Ei}\left(\frac{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{Be^3(bc-ad)^2} - \frac{be^{-\frac{2A}{B}} \operatorname{Ei}\left(\frac{2\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{Be^2g^3(bc-ad)^2}$$

[Out] (d\*ExpIntegralEi[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]])/B])/(B\*(b\*c - a\*d)^2\*e\*E^(A/B)\*g^3) - (b\*ExpIntegralEi[(2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]])/B])/(B\*(b\*c - a\*d)^2\*e^2\*E^((2\*A)/B)\*g^3)

**Rubi [F]** time = 0.0719976, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag + bgx)^3 \left( A + B \log\left(\frac{e(c+dx)}{a+bx}\right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]])),x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]])), x]

Rubi steps

$$\int \frac{1}{(ag + bgx)^3 \left( A + B \log\left(\frac{e(c+dx)}{a+bx}\right) \right)} dx = \int \frac{1}{(ag + bgx)^3 \left( A + B \log\left(\frac{e(c+dx)}{a+bx}\right) \right)} dx$$

**Mathematica [A]** time = 0.157746, size = 89, normalized size = 0.82

$$\frac{e^{-\frac{2A}{B}} \left( de^{A/B} \operatorname{Ei}\left(\frac{A}{B} + \log\left(\frac{e(c+dx)}{a+bx}\right)\right) - b \operatorname{Ei}\left(\frac{2\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right) \right)}{Be^2g^3(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]])),x]

[Out] (d\*e\*E^(A/B)\*ExpIntegralEi[A/B + Log[(e\*(c + d\*x))/(a + b\*x]]) - b\*ExpIntegralEi[(2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]])/B])/(B\*(b\*c - a\*d)^2\*e^2\*E^((2\*A)/B)\*g^3)

**Maple [F]** time = 1.425, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( A + B \ln\left(\frac{e(dx + c)}{bx + a}\right) \right)^{-1}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

[Out] `int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log \left( \frac{(dx+c)e}{bx+a} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="maxima")`

[Out] `integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)), x)`

**Fricas [A]** time = 0.976792, size = 286, normalized size = 2.62

$$\frac{\left( dee^{\frac{A}{B}} \log\_integral \left( \frac{(dex+ce)e^{\frac{A}{B}}}{bx+a} \right) - b \log\_integral \left( \frac{(d^2e^2x^2+2cde^2x+c^2e^2)e^{\frac{2A}{B}}}{b^2x^2+2abx+a^2} \right) \right) e^{-\frac{2A}{B}}}{(Bb^2c^2 - 2Babcd + Ba^2d^2)e^2g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="fricas")`

[Out] `(d*e*e^(A/B)*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a)) - b*log_integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)*e^(2*A/B)/(b^2*x^2 + 2*a*b*x + a^2)))*e^(-2*A/B)/((B*b^2*c^2 - 2*B*a*b*c*d + B*a^2*d^2)*e^2*g^3)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)/(b*x+a))),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log \left( \frac{(dx+c)e}{bx+a} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a))),x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)), x)
```

$$3.196 \quad \int \frac{(ag+bgx)^2}{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left[\frac{(ag+bgx)^2}{\left(B \log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}, x\right]$$

[Out] Unintegrable[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2, x]

**Rubi [A]** time = 0.206335, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2,x]

[Out] a^2\*g^2\*Defer[Int][(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^(-2), x] + 2\*a\*b\*g^2\*Defer[Int][x/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2, x] + b^2\*g^2\*Defer[Int][x^2/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{(ag+bgx)^2}{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx &= \int \left[ \frac{a^2g^2}{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} + \frac{2abg^2x}{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} + \frac{b^2g^2x^2}{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} \right] dx \\ &= (a^2g^2) \int \frac{1}{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx + (2abg^2) \int \frac{x}{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx + (b^2g^2) \int \frac{x^2}{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 1.3554, size = 0, normalized size = 0.

$$\int \frac{(ag+bgx)^2}{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2,x]

[Out] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2, x]

**Maple [A]** time = 1.11, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( \frac{e(dx + c)}{bx + a} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))^2,x)

[Out] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{b^3 dg^2 x^4 + a^3 cg^2 + (b^3 cg^2 + 3 ab^2 dg^2) x^3 + 3 (ab^2 cg^2 + a^2 bdg^2) x^2 + (3 a^2 bcg^2 + a^3 dg^2) x}{(bc - ad) B^2 \log(bx + a) - (bc - ad) B^2 \log(dx + c) - (bc - ad) AB - (bc \log(e) - ad \log(e)) B^2} + \int \frac{4 b^3 dg^2 x^3 + 3}{(bc - ad) B^2 \log(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="maxima")

[Out]  $-(b^3 d g^2 x^4 + a^3 c g^2 + (b^3 c g^2 + 3 a b^2 d g^2) x^3 + 3 (a b^2 c g^2 + a^2 b d g^2) x^2 + (3 a^2 b c g^2 + a^3 d g^2) x) / ((b c - a d) B^2 \log(b x + a) - (b c - a d) B^2 \log(d x + c) - (b c - a d) A B - (b c \log(e) - a d \log(e)) B^2) + \text{integrate}((4 b^3 d g^2 x^3 + 3 a^2 b c g^2 + a^3 d g^2 + 3 (b^3 c g^2 + 3 a b^2 d g^2) x^2 + 6 (a b^2 c g^2 + a^2 b d g^2) x) / ((b c - a d) B^2 \log(b x + a) - (b c - a d) B^2 \log(d x + c) - (b c - a d) A B - (b c \log(e) - a d \log(e)) B^2), x)$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b^2 g^2 x^2 + 2 a b g^2 x + a^2 g^2}{B^2 \log \left( \frac{d e x + c e}{b x + a} \right)^2 + 2 A B \log \left( \frac{d e x + c e}{b x + a} \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="fricas")

[Out] integral((b^2\*g^2\*x^2 + 2\*a\*b\*g^2\*x + a^2\*g^2)/(B^2\*log((d\*e\*x + c\*e)/(b\*x + a))^2 + 2\*A\*B\*log((d\*e\*x + c\*e)/(b\*x + a)) + A^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*2/(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))\*\*2,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{\left(B \log\left(\frac{(dx+c)e}{bx+a}\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^2/(B\*log((d\*x + c)\*e/(b\*x + a)) + A)^2, x)

$$3.197 \quad \int \frac{ag+bgx}{\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$$

**Optimal.** Leaf size=32

$$\text{Unintegrable}\left(\frac{ag + bgx}{\left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)^2}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2, x]

**Rubi [A]** time = 0.106045, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{ag + bgx}{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2, x]

[Out] a\*g\*Defer[Int][(A + B\*Log[(e\*(c + d\*x))/(a + b\*x))]^(-2), x] + b\*g\*Defer[Int][x/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x))]^2, x]

Rubi steps

$$\begin{aligned} \int \frac{ag + bgx}{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx &= \int \left( \frac{ag}{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} + \frac{bgx}{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} \right) dx \\ &= (ag) \int \frac{1}{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx + (bg) \int \frac{x}{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.944556, size = 0, normalized size = 0.

$$\int \frac{ag + bgx}{\left(A + B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2, x]

[Out] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])^2, x]

**Maple [A]** time = 1.129, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln\left(\frac{e(dx + c)}{bx + a}\right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)`

[Out] `int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{b^2 d g x^3 + a^2 c g + (b^2 c g + 2 a b d g) x^2 + (2 a b c g + a^2 d g) x}{(b c - a d) B^2 \log(b x + a) - (b c - a d) B^2 \log(d x + c) - (b c - a d) A B - (b c \log(e) - a d \log(e)) B^2} + \int \frac{1}{(b c - a d) B^2 \log(b x + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="maxima")`

[Out] `-(b^2*d*g*x^3 + a^2*c*g + (b^2*c*g + 2*a*b*d*g)*x^2 + (2*a*b*c*g + a^2*d*g)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b^2*d*g*x^2 + 2*a*b*c*g + a^2*d*g + 2*(b^2*c*g + 2*a*b*d*g)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) - (b*c - a*d)*A*B - (b*c*log(e) - a*d*log(e))*B^2), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b g x + a g}{B^2 \log\left(\frac{d e x + c e}{b x + a}\right)^2 + 2 A B \log\left(\frac{d e x + c e}{b x + a}\right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="fricas")`

[Out] `integral((b*g*x + a*g)/(B^2*log((d*e*x + c*e)/(b*x + a))^2 + 2*A*B*log((d*e*x + c*e)/(b*x + a)) + A^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)/(b*x+a)))**2,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{b g x + a g}{\left(B \log\left(\frac{d x + c e}{b x + a}\right) + A\right)^2} d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)*e/(b*x + a)) + A)^2, x)
```



$$3.198 \quad \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{1}{(ag+bgx)\left(B \log\left(\frac{e(c+dx)}{a+bx}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]))^2), x]

**Rubi [A]** time = 0.0786841, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]))^2), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]))^2), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx = \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$$

**Mathematica [A]** time = 0.514056, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]))^2), x]

[Out] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x]))^2), x]

**Maple [A]** time = 1.079, size = 0, normalized size = 0.

$$\int \frac{1}{bgx+ag}\left(A+B \ln\left(\frac{e(dx+c)}{bx+a}\right)\right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))^2,x)

[Out] int(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$d \int \frac{1}{(bcg - adg)B^2 \log(bx + a) - (bcg - adg)B^2 \log(dx + c) - (bcg - adg)AB - (bcg \log(e) - adg \log(e))B^2} dx - \frac{1}{(bcg - adg)B^2 \log(bx + a) - (bcg - adg)B^2 \log(dx + c) - (bcg - adg)AB - (bcg \log(e) - adg \log(e))B^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)/(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="maxima")

[Out] d\*integrate(1/((b\*c\*g - a\*d\*g)\*B^2\*log(b\*x + a) - (b\*c\*g - a\*d\*g)\*B^2\*log(d\*x + c) - (b\*c\*g - a\*d\*g)\*A\*B - (b\*c\*g\*log(e) - a\*d\*g\*log(e))\*B^2), x) - (d\*x + c)/((b\*c\*g - a\*d\*g)\*B^2\*log(b\*x + a) - (b\*c\*g - a\*d\*g)\*B^2\*log(d\*x + c)) - (b\*c\*g - a\*d\*g)\*A\*B - (b\*c\*g\*log(e) - a\*d\*g\*log(e))\*B^2

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2bgx + A^2ag + (B^2bgx + B^2ag) \log\left(\frac{dex+ce}{bx+a}\right)^2 + 2(ABbgx + ABag) \log\left(\frac{dex+ce}{bx+a}\right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)/(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="fricas")

[Out] integral(1/(A^2\*b\*g\*x + A^2\*a\*g + (B^2\*b\*g\*x + B^2\*a\*g)\*log((d\*e\*x + c\*e)/(b\*x + a))^2 + 2\*(A\*B\*b\*g\*x + A\*B\*a\*g)\*log((d\*e\*x + c\*e)/(b\*x + a))), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag) \left( B \log\left(\frac{(dx+c)e}{bx+a}\right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)*e/(b*x + a)) + A)^2), x)
```

$$3.199 \quad \int \frac{1}{(ag+bgx)^2 \left( A+B \log\left(\frac{e(c+dx)}{a+bx}\right) \right)^2} dx$$

**Optimal.** Leaf size=104

$$\frac{c+dx}{Bg^2(a+bx)(bc-ad) \left( B \log\left(\frac{e(c+dx)}{a+bx}\right) + A \right)} - \frac{e^{-\frac{A}{B}} \text{Ei}\left(\frac{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2 eg^2(bc-ad)}$$

[Out]  $-(\text{ExpIntegralEi}[(A + B \cdot \text{Log}[(e \cdot (c + d \cdot x)) / (a + b \cdot x)]) / B] / (B^2 \cdot (b \cdot c - a \cdot d) \cdot e \cdot E^{(A/B) \cdot g^2}) + (c + d \cdot x) / (B \cdot (b \cdot c - a \cdot d) \cdot g^2 \cdot (a + b \cdot x) \cdot (A + B \cdot \text{Log}[(e \cdot (c + d \cdot x)) / (a + b \cdot x)]))$

**Rubi [F]** time = 0.0886496, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)^2 \left( A+B \log\left(\frac{e(c+dx)}{a+bx}\right) \right)^2} dx$$

Verification is Not applicable to the result.

[In]  $\text{Int}[1/((a \cdot g + b \cdot g \cdot x)^2 \cdot (A + B \cdot \text{Log}[(e \cdot (c + d \cdot x)) / (a + b \cdot x)]))^2, x]$

[Out]  $\text{Defer}[\text{Int}[1/((a \cdot g + b \cdot g \cdot x)^2 \cdot (A + B \cdot \text{Log}[(e \cdot (c + d \cdot x)) / (a + b \cdot x)]))^2, x]$

Rubi steps

$$\int \frac{1}{(ag+bgx)^2 \left( A+B \log\left(\frac{e(c+dx)}{a+bx}\right) \right)^2} dx = \int \frac{1}{(ag+bgx)^2 \left( A+B \log\left(\frac{e(c+dx)}{a+bx}\right) \right)^2} dx$$

**Mathematica [A]** time = 0.124988, size = 88, normalized size = 0.85

$$\frac{e^{-\frac{A}{B}} \text{Ei}\left(\frac{A}{B} + \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{e} - \frac{B(c+dx)}{(a+bx) \left( B \log\left(\frac{e(c+dx)}{a+bx}\right) + A \right)} \Bigg/ B^2 g^2 (ad - bc)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/((a \cdot g + b \cdot g \cdot x)^2 \cdot (A + B \cdot \text{Log}[(e \cdot (c + d \cdot x)) / (a + b \cdot x)]))^2, x]$

[Out]  $(\text{ExpIntegralEi}[A/B + \text{Log}[(e \cdot (c + d \cdot x)) / (a + b \cdot x)]] / (e \cdot E^{(A/B)}) - (B \cdot (c + d \cdot x)) / ((a + b \cdot x) \cdot (A + B \cdot \text{Log}[(e \cdot (c + d \cdot x)) / (a + b \cdot x)]))) / (B^2 \cdot (- (b \cdot c) + a \cdot d) \cdot g^2)$

**Maple [B]** time = 0.16, size = 258, normalized size = 2.5

$$-\frac{d}{(ad-bc)g^2B^2b} \left( \ln \left( \frac{de}{b} - \frac{e(ad-bc)}{b(bx+a)} \right) + \frac{A}{B} \right)^{-1} + \frac{ad}{(ad-bc)g^2B^2b(bx+a)} \left( \ln \left( \frac{de}{b} - \frac{e(ad-bc)}{b(bx+a)} \right) + \frac{A}{B} \right)^{-1} - \frac{1}{(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))^2,x)

[Out] -1/(a\*d-b\*c)/g^2/B^2/(ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))+A/B)\*d/b+1/(a\*d-b\*c)/g^2/B^2/(ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))+A/B)/b/(b\*x+a)\*a\*d-1/(a\*d-b\*c)/g^2/B^2/(ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))+A/B)/(b\*x+a)\*c-1/e/(a\*d-b\*c)/g^2/B^2\*exp(-A/B)\*Ei(1,-ln(d\*e/b-e\*(a\*d-b\*c)/b/(b\*x+a))-A/B)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(abcg^2 - a^2dg^2)AB + (abcg^2 \log(e) - a^2dg^2 \log(e))B^2 + ((b^2cg^2 - abdg^2)AB + (b^2cg^2 \log(e) - abdg^2 \log(e))B^2)x + \dots}{dx + c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="maxima")

[Out] (d\*x + c)/((a\*b\*c\*g^2 - a^2\*d\*g^2)\*A\*B + (a\*b\*c\*g^2\*log(e) - a^2\*d\*g^2\*log(e))\*B^2 + ((b^2\*c\*g^2 - a\*b\*d\*g^2)\*A\*B + (b^2\*c\*g^2\*log(e) - a\*b\*d\*g^2\*log(e))\*B^2)\*x - ((b^2\*c\*g^2 - a\*b\*d\*g^2)\*B^2\*x + (a\*b\*c\*g^2 - a^2\*d\*g^2)\*B^2)\*log(b\*x + a) + ((b^2\*c\*g^2 - a\*b\*d\*g^2)\*B^2\*x + (a\*b\*c\*g^2 - a^2\*d\*g^2)\*B^2)\*log(d\*x + c) + integrate(1/(B^2\*a^2\*g^2\*log(e) + A\*B\*a^2\*g^2 + (B^2\*b^2\*g^2\*log(e) + A\*B\*b^2\*g^2)\*x^2 + 2\*(B^2\*a\*b\*g^2\*log(e) + A\*B\*a\*b\*g^2)\*x - (B^2\*b^2\*g^2\*x^2 + 2\*B^2\*a\*b\*g^2\*x + B^2\*a^2\*g^2)\*log(b\*x + a) + (B^2\*b^2\*g^2\*x^2 + 2\*B^2\*a\*b\*g^2\*x + B^2\*a^2\*g^2)\*log(d\*x + c)), x)

**Fricas [B]** time = 0.970356, size = 439, normalized size = 4.22

$$\frac{(Bdex + Bce)e^{\frac{A}{B}} - \left( Abx + Aa + (Bbx + Ba) \log \left( \frac{dex+ce}{bx+a} \right) \right) \log\_integral \left( \frac{(dex+ce)e^{\frac{A}{B}}}{bx+a} \right)}{\left( (B^3b^2c - B^3abd)eg^2x + (B^3abc - B^3a^2d)eg^2 \right) e^{\frac{A}{B}} \log \left( \frac{dex+ce}{bx+a} \right) + \left( (AB^2b^2c - AB^2abd)eg^2x + (AB^2abc - AB^2a^2d)eg^2 \right) e^{\frac{A}{B}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="fricas")

[Out] ((B\*d\*e\*x + B\*c\*e)\*e^(A/B) - (A\*b\*x + A\*a + (B\*b\*x + B\*a)\*log((d\*e\*x + c\*e)/(b\*x + a)))\*log\_integral((d\*e\*x + c\*e)\*e^(A/B)/(b\*x + a)))/(((B^3\*b^2\*c - B^3\*a\*b\*d)\*e\*g^2\*x + (B^3\*a\*b\*c - B^3\*a^2\*d)\*e\*g^2)\*e^(A/B)\*log((d\*e\*x + c\*e)/(b\*x + a)) + ((A\*B^2\*b^2\*c - A\*B^2\*a\*b\*d)\*e\*g^2\*x + (A\*B^2\*a\*b\*c - A\*B^2\*a^2\*d)\*e\*g^2)\*e^(A/B))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)\*\*2/(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))\*\*2,x)

[Out] Timed out

**Giac [B]** time = 1.41631, size = 344, normalized size = 3.31

$$-\frac{\operatorname{Ei}\left(\frac{A}{B} + \log\left(\frac{dx+c}{bx+a}\right) + 1\right) e^{\left(-\frac{A}{B}-1\right)}}{B^2bcg^2 - B^2adg^2} + \frac{B^2b^2cg^2x \log\left(\frac{dx+c}{bx+a}\right) - B^2abdg^2x \log\left(\frac{dx+c}{bx+a}\right) + ABb^2cg^2x + B^2b^2cg^2x - ABabdg^2x - B^2adg^2x}{B^2b^2cg^2x \log\left(\frac{dx+c}{bx+a}\right) - B^2abdg^2x \log\left(\frac{dx+c}{bx+a}\right) + ABb^2cg^2x + B^2b^2cg^2x - ABabdg^2x - B^2adg^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="giac")

[Out] -Ei(A/B + log((d\*x + c)/(b\*x + a)) + 1)\*e^(-A/B - 1)/(B^2\*b\*c\*g^2 - B^2\*a\*d\*g^2) + (d\*x + c)/(B^2\*b^2\*c\*g^2\*x\*log((d\*x + c)/(b\*x + a)) - B^2\*a\*b\*d\*g^2\*x\*log((d\*x + c)/(b\*x + a)) + A\*B\*b^2\*c\*g^2\*x + B^2\*b^2\*c\*g^2\*x - A\*B\*a\*b\*d\*g^2\*x - B^2\*a\*b\*d\*g^2\*x + B^2\*a\*b\*c\*g^2\*log((d\*x + c)/(b\*x + a)) - B^2\*a^2\*d\*g^2\*log((d\*x + c)/(b\*x + a)) + A\*B\*a\*b\*c\*g^2 + B^2\*a\*b\*c\*g^2 - A\*B\*a^2\*d\*g^2 - B^2\*a^2\*d\*g^2)

$$3.200 \quad \int \frac{1}{(ag+bgx)^3 \left( A+B \log\left( \frac{e(c+dx)}{a+bx} \right) \right)^2} dx$$

**Optimal.** Leaf size=159

$$-\frac{2be^{-\frac{2A}{B}} \operatorname{Ei}\left(\frac{2\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{B^2 e^2 g^3 (bc-ad)^2} + \frac{de^{-\frac{A}{B}} \operatorname{Ei}\left(\frac{A+B \log\left(\frac{e(c+dx)}{a+bx}\right)}{B}\right)}{B^2 e g^3 (bc-ad)^2} + \frac{c+dx}{Bg^3(a+bx)^2(bc-ad)\left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}$$

[Out] (d\*ExpIntegralEi[(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])/B])/(B^2\*(b\*c - a\*d)^2\*e\*E^(A/B)\*g^3) - (2\*b\*ExpIntegralEi[(2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])/B])/(B^2\*(b\*c - a\*d)^2\*e^2\*E^((2\*A)/B)\*g^3) + (c + d\*x)/(B\*(b\*c - a\*d)\*g^3\*(a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]))

**Rubi [F]** time = 0.0797503, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag + bgx)^3 \left( A + B \log\left( \frac{e(c+dx)}{a+bx} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]^2), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]^2), x]

Rubi steps

$$\int \frac{1}{(ag + bgx)^3 \left( A + B \log\left( \frac{e(c+dx)}{a+bx} \right) \right)^2} dx = \int \frac{1}{(ag + bgx)^3 \left( A + B \log\left( \frac{e(c+dx)}{a+bx} \right) \right)^2} dx$$

**Mathematica [A]** time = 0.431304, size = 135, normalized size = 0.85

$$\frac{-\frac{2be^{-\frac{2A}{B}} \operatorname{Ei}\left(\frac{2\left(A+B \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{B}\right)}{e^2} + \frac{de^{-\frac{A}{B}} \operatorname{Ei}\left(\frac{A}{B} + \log\left(\frac{e(c+dx)}{a+bx}\right)\right)}{e} + \frac{B(c+dx)(bc-ad)}{(a+bx)^2\left(B \log\left(\frac{e(c+dx)}{a+bx}\right) + A\right)}}{B^2 g^3 (bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)]^2), x]

[Out] ((d\*ExpIntegralEi[A/B + Log[(e\*(c + d\*x))/(a + b\*x)]])/(e\*E^(A/B)) - (2\*b\*ExpIntegralEi[(2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])/B])/(e^2\*E^((2\*A)/B)) + (B\*(b\*c - a\*d)\*(c + d\*x))/((a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x))/(a + b\*x)])))/(B^2\*(b\*c - a\*d)^2\*g^3)

**Maple [F]** time = 1.582, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3} \left( A + B \ln \left( \frac{e(dx + c)}{bx + a} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^3/(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))^2,x)

[Out] int(1/(b\*g\*x+a\*g)^3/(A+B\*ln(e\*(d\*x+c)/(b\*x+a)))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="maxima")

[Out] (d\*x + c)/((a^2\*b\*c\*g^3 - a^3\*d\*g^3)\*A\*B + (a^2\*b\*c\*g^3\*log(e) - a^3\*d\*g^3\*log(e))\*B^2 + ((b^3\*c\*g^3 - a\*b^2\*d\*g^3)\*A\*B + (b^3\*c\*g^3\*log(e) - a\*b^2\*d\*g^3\*log(e))\*B^2)\*x^2 + 2\*((a\*b^2\*c\*g^3 - a^2\*b\*d\*g^3)\*A\*B + (a\*b^2\*c\*g^3\*log(e) - a^2\*b\*d\*g^3\*log(e))\*B^2)\*x - ((b^3\*c\*g^3 - a\*b^2\*d\*g^3)\*B^2\*x^2 + 2\*(a\*b^2\*c\*g^3 - a^2\*b\*d\*g^3)\*B^2\*x + (a^2\*b\*c\*g^3 - a^3\*d\*g^3)\*B^2)\*log(b\*x + a) + ((b^3\*c\*g^3 - a\*b^2\*d\*g^3)\*B^2\*x^2 + 2\*(a\*b^2\*c\*g^3 - a^2\*b\*d\*g^3)\*B^2\*x + (a^2\*b\*c\*g^3 - a^3\*d\*g^3)\*B^2)\*log(d\*x + c) - integrate(-(b\*d\*x + 2\*b\*c - a\*d)/(((b^4\*c\*g^3 - a\*b^3\*d\*g^3)\*A\*B + (b^4\*c\*g^3\*log(e) - a\*b^3\*d\*g^3\*log(e))\*B^2)\*x^3 + (a^3\*b\*c\*g^3 - a^4\*d\*g^3)\*A\*B + (a^3\*b\*c\*g^3\*log(e) - a^4\*d\*g^3\*log(e))\*B^2 + 3\*((a\*b^3\*c\*g^3 - a^2\*b^2\*d\*g^3)\*A\*B + (a\*b^3\*c\*g^3\*log(e) - a^2\*b^2\*d\*g^3\*log(e))\*B^2)\*x^2 + 3\*((a^2\*b^2\*c\*g^3 - a^3\*b\*d\*g^3)\*A\*B + (a^2\*b^2\*c\*g^3\*log(e) - a^3\*b\*d\*g^3\*log(e))\*B^2)\*x - ((b^4\*c\*g^3 - a\*b^3\*d\*g^3)\*B^2\*x^3 + 3\*(a\*b^3\*c\*g^3 - a^2\*b^2\*d\*g^3)\*B^2\*x^2 + 3\*(a^2\*b^2\*c\*g^3 - a^3\*b\*d\*g^3)\*B^2\*x + (a^3\*b\*c\*g^3 - a^4\*d\*g^3)\*B^2)\*log(b\*x + a) + ((b^4\*c\*g^3 - a\*b^3\*d\*g^3)\*B^2\*x^3 + 3\*(a\*b^3\*c\*g^3 - a^2\*b^2\*d\*g^3)\*B^2\*x^2 + 3\*(a^2\*b^2\*c\*g^3 - a^3\*b\*d\*g^3)\*B^2\*x + (a^3\*b\*c\*g^3 - a^4\*d\*g^3)\*B^2)\*log(d\*x + c)), x)

**Fricas [B]** time = 1.06759, size = 1212, normalized size = 7.62

$$\left( (Bbcd - Bad^2)e^2x + (Bbc^2 - Bacd)e^2 \right) e^{\left( \frac{2A}{B} \right)} - 2 \left( Ab^3x^2 + 2Aab^2x + Aa^2b + (Bb^3x^2 + 2Bab^2x + Ba^2b) \log \right)$$

$$\left( (B^3b^4c^2 - 2B^3ab^3cd + B^3a^2b^2d^2) e^2g^3x^2 + 2(B^3ab^3c^2 - 2B^3a^2b^2cd + B^3a^3bd^2) e^2g^3x + (B^3a^2b^2c^2 - 2B^3a^3bcd + B^3a^4d^2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*(d\*x+c)/(b\*x+a)))^2,x, algorithm="fricas")

[Out] (((B\*b\*c\*d - B\*a\*d^2)\*e^2\*x + (B\*b\*c^2 - B\*a\*c\*d)\*e^2)\*e^(2\*A/B) - 2\*(A\*b^3\*x^2 + 2\*A\*a\*b^2\*x + A\*a^2\*b + (B\*b^3\*x^2 + 2\*B\*a\*b^2\*x + B\*a^2\*b)\*log((d\*e



```
*x + c*e)/(b*x + a))*log_integral((d^2*e^2*x^2 + 2*c*d*e^2*x + c^2*e^2)*e^(2*A/B)/(b^2*x^2 + 2*a*b*x + a^2)) + ((B*b^2*d*e*x^2 + 2*B*a*b*d*e*x + B*a^2*d*e)*e^(A/B)*log((d*e*x + c*e)/(b*x + a)) + (A*b^2*d*e*x^2 + 2*A*a*b*d*e*x + A*a^2*d*e)*e^(A/B))*log_integral((d*e*x + c*e)*e^(A/B)/(b*x + a)))/((B^3*b^4*c^2 - 2*B^3*a*b^3*c*d + B^3*a^2*b^2*d^2)*e^2*g^3*x^2 + 2*(B^3*a*b^3*c^2 - 2*B^3*a^2*b^2*c*d + B^3*a^3*b*d^2)*e^2*g^3*x + (B^3*a^2*b^2*c^2 - 2*B^3*a^3*b*c*d + B^3*a^4*d^2)*e^2*g^3)*e^(2*A/B)*log((d*e*x + c*e)/(b*x + a)) + ((A*B^2*b^4*c^2 - 2*A*B^2*a*b^3*c*d + A*B^2*a^2*b^2*d^2)*e^2*g^3*x^2 + 2*(A*B^2*a*b^3*c^2 - 2*A*B^2*a^2*b^2*c*d + A*B^2*a^3*b*d^2)*e^2*g^3*x + (A*B^2*a^2*b^2*c^2 - 2*A*B^2*a^3*b*c*d + A*B^2*a^4*d^2)*e^2*g^3)*e^(2*A/B))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)/(b*x+a)))*2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log\left(\frac{(dx+c)e}{bx+a}\right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)/(b*x+a)))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)*e/(b*x + a)) + A)^2), x)
```

### 3.201 $\int (ag + bgx)^4 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right) dx$

**Optimal.** Leaf size=182

$$\frac{g^4(a+bx)^5 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)}{5b} - \frac{2Bg^4x(bc-ad)^4}{5d^4} + \frac{Bg^4(a+bx)^2(bc-ad)^3}{5bd^3} - \frac{2Bg^4(a+bx)^3(bc-ad)^2}{15bd^2} + \frac{2Bg^4(bc-a}{5$$

[Out]  $(-2*B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(5*b*d^3) - (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) + (B*(b*c - a*d)*g^4*(a + b*x)^4)/(10*b*d) + (2*B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5) + (g^4*(a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(5*b)$

**Rubi [A]** time = 0.117856, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 43}

$$\frac{g^4(a+bx)^5 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)}{5b} - \frac{2Bg^4x(bc-ad)^4}{5d^4} + \frac{Bg^4(a+bx)^2(bc-ad)^3}{5bd^3} - \frac{2Bg^4(a+bx)^3(bc-ad)^2}{15bd^2} + \frac{2Bg^4(bc-a}{5$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*g + b*g*x)^4*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]), x]$

[Out]  $(-2*B*(b*c - a*d)^4*g^4*x)/(5*d^4) + (B*(b*c - a*d)^3*g^4*(a + b*x)^2)/(5*b*d^3) - (2*B*(b*c - a*d)^2*g^4*(a + b*x)^3)/(15*b*d^2) + (B*(b*c - a*d)*g^4*(a + b*x)^4)/(10*b*d) + (2*B*(b*c - a*d)^5*g^4*Log[c + d*x])/(5*b*d^5) + (g^4*(a + b*x)^5*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(5*b)$

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol]
:> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
\int (ag + bgx)^4 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right) dx &= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{5b} - \frac{B \int \frac{2(-bc+ad)g^5(a+bx)^4}{c+dx} dx}{5bg} \\
&= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{5b} + \frac{(2B(bc-ad)g^4) \int \frac{(a+bx)^4}{c+dx} dx}{5b} \\
&= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{5b} + \frac{(2B(bc-ad)g^4) \int \left( -\frac{b(bc-ad)^3}{d^4} + \frac{b(bc-ad)^2}{d^3} \right) dx}{5b} \\
&= -\frac{2B(bc-ad)^4 g^4 x}{5d^4} + \frac{B(bc-ad)^3 g^4 (a+bx)^2}{5bd^3} - \frac{2B(bc-ad)^2 g^4 (a+bx)^3}{15bd^2}
\end{aligned}$$

**Mathematica [A]** time = 0.100684, size = 144, normalized size = 0.79

$$\frac{g^4 \left( (a+bx)^5 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right) - \frac{B(ad-bc)(6d^2(a+bx)^2(bc-ad)^2 + 4d^3(a+bx)^3(ad-bc) - 12bdx(bc-ad)^3 + 12(bc-ad)^4 \log(c+dx) + 3d^4(a+bx)^4)}{6d^5} \right)}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^4\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]), x]

[Out] (g^4\*(-(B\*(-(b\*c) + a\*d))\*(-12\*b\*d\*(b\*c - a\*d)^3\*x + 6\*d^2\*(b\*c - a\*d)^2\*(a + b\*x)^2 + 4\*d^3\*(-(b\*c) + a\*d)\*(a + b\*x)^3 + 3\*d^4\*(a + b\*x)^4 + 12\*(b\*c - a\*d)^4\*Log[c + d\*x]))/(6\*d^5) + (a + b\*x)^5\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]))/(5\*b)

**Maple [B]** time = 0.415, size = 1030, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^4\*(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2)), x)

[Out] 2/5/b\*g^4\*B\*a^5\*ln(1/(b\*x+a))-2/5/b\*g^4\*B\*a^5\*ln(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)+1/5\*b^4\*B\*ln(e\*(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x^5\*g^4+1/10\*b^4\*g^4\*B\*c/d\*x^4+1/5\*b^4\*g^4\*B\*c^3/d^3\*x^2-2/15\*b^4\*g^4\*B\*c^2/d^2\*x^3+2\*g^4\*B\*a^4/d\*ln(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)\*c-2\*g^4\*B\*a^4/d\*ln(1/(b\*x+a))\*c+2\*b\*B\*ln(e\*(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x^2\*a^3\*g^4-2/5\*b^4\*g^4\*B\*c^4/d^4\*x+2/5\*b^4\*g^4\*B\*c^5/d^5\*ln(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)-2/5\*b^4\*g^4\*B\*c^5/d^5\*ln(1/(b\*x+a))+2\*b^2\*B\*ln(e\*(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x^3\*a^2\*g^4+b^3\*B\*ln(e\*(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x^4\*a\*g^4+9/5\*b^2\*g^4\*B\*a^2/d^3\*c^3-2/5\*b^3\*g^4\*B\*a/d^4\*c^4-47/15\*b\*g^4\*B\*a^3/d^2\*c^2+77/30\*g^4\*B\*a^4/d\*c-5/6/b\*g^4\*B\*a^5+1/5\*b^4\*A\*x^5\*g^4+A\*x\*a^4\*g^4-8/5\*B\*x\*a^4\*g^4+1/5/b\*A\*a^5\*g^4+1/5/b\*B\*ln(e\*(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*a^5\*g^4+b^3\*A\*x^4\*a\*g^4+2\*b^2\*A\*x^3\*a^2\*g^4+2\*b\*A\*x^2\*a^3\*g^4-1/10\*b^3\*B\*x^4\*a\*g^4-8/15\*b^2\*B\*x^3\*a^2\*g^4-6/5\*b\*B\*x^2\*a^3\*g^4+B\*ln(e\*(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x\*a^4\*g^4+4\*b\*g^4\*B\*c/d\*x\*a^3+4\*b\*g^4\*B\*a^3/d^2\*ln(1/(b\*x+a))\*c^2-4\*b\*g^4\*B\*a^3/d^2\*ln(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)\*c^2+2\*b^3\*g^4\*B\*a/d^4\*ln(1/(b\*x+a))\*c^4-2\*b^3\*g^4\*B\*a/d^4\*ln(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)\*c^4+4\*b^2\*g^4\*B\*a^2/d^3\*ln(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)\*c^3-b^3\*g^4\*B\*c^2/d^2\*x^2\*a-4\*b^2\*g^4\*B\*a^2/d^3\*ln(1/(b\*x+a))\*c^3+2\*b^2\*g^4\*B\*c/d\*x^2\*a^2-4\*b^2\*g^4\*B\*c^2/d^

$$2*x*a^2+2*b^3*g^4*B*c^3/d^3*a*x+2/3*b^3*g^4*B*c/d*x^3*a$$

**Maxima [B]** time = 1.34761, size = 1191, normalized size = 6.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2)),x, algorithm="maxima")

[Out]  $\frac{1}{5}A*b^4*g^4*x^5 + A*a*b^3*g^4*x^4 + 2*A*a^2*b^2*g^4*x^3 + 2*A*a^3*b*g^4*x^2 + (x*\log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2)) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*\log(b*x + a)/b + 2*c*\log(d*x + c)/d)*B*a^4*g^4 + 2*(x^2*\log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2)) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*\log(b*x + a)/b^2 - 2*c^2*\log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*B*a^3*b*g^4 + 2*(x^3*\log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2)) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*\log(b*x + a)/b^3 + 2*c^3*\log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b^2*g^4 + 1/3*(3*x^4*\log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2)) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 6*a^4*\log(b*x + a)/b^4 - 6*c^4*\log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a*b^3*g^4 + 1/30*(6*x^5*\log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2)) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 12*a^5*\log(b*x + a)/b^5 + 12*c^5*\log(d*x + c)/d^5 + (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*b^4*g^4 + A*a^4*g^4*x$

**Fricas [B]** time = 1.27888, size = 952, normalized size = 5.23

$$6Ab^5d^5g^4x^5 - 12Ba^5d^5g^4 \log(bx + a) + 3(Bb^5cd^4 + (10A - B)ab^4d^5)g^4x^4 - 4(Bb^5c^2d^3 - 5Bab^4cd^4 - (15A - 4B)a^2b^3c^2d^5)g^4x^3 + 6(Bb^5c^3d^2 - 5B*a*b^4*c^2*d^3 + 10*B*a^2*b^3*c*d^4 + 2*(5A - 3B)*a^3*b^2*d^5)g^4x^2 - 6*(2*B*b^5*c^4*d - 10*B*a*b^4*c^3*d^2 + 20*B*a^2*b^3*c^2*d^3 - 20*B*a^3*b^2*c*d^4 - (5A - 8B)*a^4*b*d^5)g^4x + 12*(B*b^5*c^5 - 5B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5B*a^4*b*c*d^4)g^4*\log(d*x + c) + 6*(B*b^5*d^5*g^4*x^5 + 5B*a*b^4*d^5*g^4*x^4 + 10B*a^2*b^3*d^5*g^4*x^3 + 10B*a^3*b^2*d^5*g^4*x^2 + 5B*a^4*b*d^5*g^4*x)*\log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))/(b*d^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2)),x, algorithm="fricas")

[Out]  $\frac{1}{30}(6A*b^5*d^5*g^4*x^5 - 12B*a^5*d^5*g^4*\log(b*x + a) + 3*(B*b^5*c*d^4 + (10A - B)*a*b^4*d^5)*g^4*x^4 - 4*(B*b^5*c^2*d^3 - 5B*a*b^4*c^2*d^4 - (15A - 4B)*a^2*b^3*d^5)*g^4*x^3 + 6*(B*b^5*c^3*d^2 - 5B*a*b^4*c^2*d^3 + 10B*a^2*b^3*c*d^4 + 2*(5A - 3B)*a^3*b^2*d^5)*g^4*x^2 - 6*(2*B*b^5*c^4*d - 10*B*a*b^4*c^3*d^2 + 20*B*a^2*b^3*c^2*d^3 - 20*B*a^3*b^2*c*d^4 - (5A - 8B)*a^4*b*d^5)*g^4*x + 12*(B*b^5*c^5 - 5B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5B*a^4*b*c*d^4)*g^4*\log(d*x + c) + 6*(B*b^5*d^5*g^4*x^5 + 5B*a*b^4*d^5*g^4*x^4 + 10B*a^2*b^3*d^5*g^4*x^3 + 10B*a^3*b^2*d^5*g^4*x^2 + 5B*a^4*b*d^5*g^4*x)*\log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/(b*d^5)$

**Sympy [B]** time = 8.81681, size = 1018, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*4\*(A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2)),x)

[Out]  $A*b**4*g**4*x**5/5 - 2*B*a**5*g**4*\log(x + (2*B*a**6*d**5*g**4/b + 10*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b**4*c**5*g**4)/(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*b) + 2*B*c*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)*\log(x + (12*B*a**5*c*d**4*g**4 - 20*B*a**4*b*c**2*d**3*g**4 + 20*B*a**3*b**2*c**3*d**2*g**4 - 10*B*a**2*b**3*c**4*d*g**4 + 2*B*a*b**4*c**5*g**4 - 2*B*a*c**g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4) + 2*B*b*c**2*g**4*(5*a**4*d**4 - 10*a**3*b*c*d**3 + 10*a**2*b**2*c**2*d**2 - 5*a*b**3*c**3*d + b**4*c**4)/d)/(2*B*a**5*d**5*g**4 + 10*B*a**4*b*c*d**4*g**4 - 20*B*a**3*b**2*c**2*d**3*g**4 + 20*B*a**2*b**3*c**3*d**2*g**4 - 10*B*a*b**4*c**4*d*g**4 + 2*B*b**5*c**5*g**4))/(5*d**5) + (B*a**4*g**4*x + 2*B*a**3*b*g**4*x**2 + 2*B*a**2*b**2*g**4*x**3 + B*a*b**3*g**4*x**4 + B*b**4*g**4*x**5/5)*\log(e*(c + d*x)**2/(a + b*x)**2) + x**4*(10*A*a*b**3*d*g**4 - B*a*b**3*d*g**4 + B*b**4*c*g**4)/(10*d) + x**3*(30*A*a**2*b**2*d**2*g**4 - 8*B*a**2*b**2*d**2*g**4 + 10*B*a*b**3*c*d*g**4 - 2*B*b**4*c**2*g**4)/(15*d**2) + x**2*(10*A*a**3*b*d**3*g**4 - 6*B*a**3*b*d**3*g**4 + 10*B*a**2*b**2*c*d**2*g**4 - 5*B*a*b**3*c**2*d*g**4 + B*b**4*c**3*g**4)/(5*d**3) + x*(5*A*a**4*d**4*g**4 - 8*B*a**4*d**4*g**4 + 20*B*a**3*b*c*d**3*g**4 - 20*B*a**2*b**2*c**2*d**2*g**4 + 10*B*a*b**3*c**3*d*g**4 - 2*B*b**4*c**4*g**4)/(5*d**4)$

**Giac [B]** time = 162.637, size = 666, normalized size = 3.66

$$-\frac{2Ba^5g^4 \log(bx+a)}{5b} + \frac{1}{5}(Ab^4g^4 + Bb^4g^4)x^5 + \frac{(Bb^4cg^4 + 10Aab^3dg^4 + 9Bab^3dg^4)x^4}{10d} - \frac{2(Bb^4c^2g^4 - 5Bab^3cdg^4 - \dots)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2)),x, algorithm="giac")

[Out]  $-2/5*B*a^5*g^4*\log(b*x + a)/b + 1/5*(A*b^4*g^4 + B*b^4*g^4)*x^5 + 1/10*(B*b^4*c*g^4 + 10*A*a*b^3*d*g^4 + 9*B*a*b^3*d*g^4)*x^4/d - 2/15*(B*b^4*c^2*g^4 - 5*B*a*b^3*c*d*g^4 - 15*A*a^2*b^2*d^2*g^4 - 11*B*a^2*b^2*d^2*g^4)*x^3/d^2 + 1/5*(B*b^4*g^4*x^5 + 5*B*a*b^3*g^4*x^4 + 10*B*a^2*b^2*g^4*x^3 + 10*B*a^3*b*g^4*x^2 + 5*B*a^4*g^4*x)*\log((d^2*x^2 + 2*c*d*x + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) + 1/5*(B*b^4*c^3*g^4 - 5*B*a*b^3*c^2*d*g^4 + 10*B*a^2*b^2*c*d^2*g^4 + 10*A*a^3*b*d^3*g^4 + 4*B*a^3*b*d^3*g^4)*x^2/d^3 - 1/5*(2*B*b^4*c^4*g^4 - 10*B*a*b^3*c^3*d*g^4 + 20*B*a^2*b^2*c^2*d^2*g^4 - 20*B*a^3*b*c*d^3*g^4 - 5*A*a^4*d^4*g^4 + 3*B*a^4*d^4*g^4)*x/d^4 + 2/5*(B*b^4*c^5*g^4 - 5*B*a*b^3*c^4*d*g^4 + 10*B*a^2*b^2*c^3*d^2*g^4 - 10*B*a^3*b*c^2*d^3*g^4 + 5*B*a^4*c*d^4*g^4)*\log(d*x + c)/d^5$

$$3.202 \quad \int (ag + bgx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right) dx$$

**Optimal.** Leaf size=151

$$\frac{g^3(a+bx)^4 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)}{4b} + \frac{Bg^3x(bc-ad)^3}{2d^3} - \frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd^2} - \frac{Bg^3(bc-ad)^4 \log(c+dx)}{2bd^4} + \frac{Bg^3(a+bx)^4}{6b}$$

[Out] (B\*(b\*c - a\*d)^3\*g^3\*x)/(2\*d^3) - (B\*(b\*c - a\*d)^2\*g^3\*(a + b\*x)^2)/(4\*b\*d^2) + (B\*(b\*c - a\*d)\*g^3\*(a + b\*x)^3)/(6\*b\*d) - (B\*(b\*c - a\*d)^4\*g^3\*Log[c + d\*x])/(2\*b\*d^4) + (g^3\*(a + b\*x)^4\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]))/(4\*b)

**Rubi [A]** time = 0.0982844, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 43}

$$\frac{g^3(a+bx)^4 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)}{4b} + \frac{Bg^3x(bc-ad)^3}{2d^3} - \frac{Bg^3(a+bx)^2(bc-ad)^2}{4bd^2} - \frac{Bg^3(bc-ad)^4 \log(c+dx)}{2bd^4} + \frac{Bg^3(a+bx)^4}{6b}$$

Antiderivative was successfully verified.

[In] Int[(a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]),x]

[Out] (B\*(b\*c - a\*d)^3\*g^3\*x)/(2\*d^3) - (B\*(b\*c - a\*d)^2\*g^3\*(a + b\*x)^2)/(4\*b\*d^2) + (B\*(b\*c - a\*d)\*g^3\*(a + b\*x)^3)/(6\*b\*d) - (B\*(b\*c - a\*d)^4\*g^3\*Log[c + d\*x])/(2\*b\*d^4) + (g^3\*(a + b\*x)^4\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]))/(4\*b)

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 43

```
Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned}
\int (ag + bgx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right) dx &= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{4b} - \frac{B \int \frac{2(-bc+ad)g^4(a+bx)^3}{c+dx} dx}{4bg} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{4b} + \frac{(B(bc-ad)g^3) \int \frac{(a+bx)^3}{c+dx} dx}{2b} \\
&= \frac{g^3(a+bx)^4 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{4b} + \frac{(B(bc-ad)g^3) \int \left( \frac{b(bc-ad)^2}{d^3} - \frac{b(bc-ad)}{d} \right) dx}{2b} \\
&= \frac{B(bc-ad)^3 g^3 x}{2d^3} - \frac{B(bc-ad)^2 g^3 (a+bx)^2}{4bd^2} + \frac{B(bc-ad)g^3(a+bx)^3}{6bd} - \frac{B(bc-ad)g^3(a+bx)^4}{4bd}
\end{aligned}$$

**Mathematica [A]** time = 0.07413, size = 122, normalized size = 0.81

$$\frac{g^3 \left( (a+bx)^4 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right) + \frac{B(bc-ad)(3d^2(a+bx)^2(ad-bc)+6bdx(bc-ad)^2-6(bc-ad)^3 \log(c+dx)+2d^3(a+bx)^3)}{3d^4} \right)}{4b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]),x]

[Out] (g^3\*((B\*(b\*c - a\*d)\*(6\*b\*d\*(b\*c - a\*d)^2\*x + 3\*d^2\*(-(b\*c) + a\*d)\*(a + b\*x)^2 + 2\*d^3\*(a + b\*x)^3 - 6\*(b\*c - a\*d)^3\*Log[c + d\*x]))/(3\*d^4) + (a + b\*x)^4\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]))/(4\*b)

**Maple [B]** time = 0.243, size = 788, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^3\*(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2)),x)

[Out] 1/4\*A\*g^3\*x^4\*b^3+A\*g^3\*x\*a^3-3/2\*B\*x\*a^3\*g^3+1/2\*b^3\*g^3\*B\*c^4/d^4\*ln(1/(b\*x+a))+b^2\*B\*ln(e\*(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x^3\*a\*g^3-2\*g^3\*B\*a^3/d\*ln(1/(b\*x+a))\*c+2\*g^3\*B\*a^3/d\*ln(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)\*c-1/2\*b^3\*g^3\*B\*c^4/d^4\*ln(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)+3/2\*b\*B\*ln(e\*(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x^2\*a^2\*g^3+1/4/b\*A\*a^4\*g^3-11/12/b\*B\*a^4\*g^3+1/2/d^3\*B\*g^3\*b^2\*a\*c^3+A\*g^3\*x^3\*a\*b^2-3/4\*B\*g^3\*b\*a^2\*x^2+13/6/d\*B\*a^3\*c\*g^3+3/2\*A\*g^3\*x^2\*a^2\*b-3\*b\*g^3\*B\*a^2/d^2\*ln(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)\*c^2+2\*b^2\*g^3\*B\*a/d^3\*ln(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)\*c^3+3\*b\*g^3\*B\*a^2/d^2\*ln(1/(b\*x+a))\*c^2-7/4/d^2\*B\*g^3\*b\*a^2\*c^2-1/6\*B\*g^3\*b^2\*a\*x^3+B\*ln(e\*(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x\*a^3\*g^3+1/4/b\*B\*ln(e\*(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*a^4\*g^3-1/2/b\*g^3\*B\*a^4\*ln(1/(b\*x+a))+1/4\*b^3\*B\*ln(e\*(1/(b\*x+a))\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x^4\*g^3+1/6/d\*B\*g^3\*b^3\*c\*x^3-1/4/d^2\*B\*g^3\*b^3\*c^2\*x^2+1/2/d^3\*B\*g^3\*b^3\*c^3\*x-2\*b^2\*g^3\*B\*a/d^3\*ln(1/(b\*x+a))\*c^3+1/d\*B\*g^3\*b^2\*a\*x^2\*c-2/d^2\*B\*g^3\*b^2\*a\*x\*c^2+3/d\*B\*g^3\*a^2\*b\*c\*x

**Maxima [B]** time = 1.37302, size = 871, normalized size = 5.77

$$\frac{1}{4} Ab^3 g^3 x^4 + Aab^2 g^3 x^3 + \frac{3}{2} Aa^2 b g^3 x^2 + \left( x \log \left( \frac{d^2 e x^2}{b^2 x^2 + 2 abx + a^2} + \frac{2 cdex}{b^2 x^2 + 2 abx + a^2} + \frac{c^2 e}{b^2 x^2 + 2 abx + a^2} \right) - \frac{2 a \log(b)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxima")
```

```
[Out] 1/4*A*b^3*g^3*x^4 + A*a*b^2*g^3*x^3 + 3/2*A*a^2*b*g^3*x^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*B*a^3*g^3 + 3/2*(x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*B*a^2*b*g^3 + (x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2*g^3 + 1/12*(3*x^4*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*g^3 + A*a^3*g^3*x
```

**Fricas [B]** time = 1.18112, size = 709, normalized size = 4.7

$$3 Ab^4 d^4 g^3 x^4 - 6 Ba^4 d^4 g^3 \log(bx + a) + 2 (Bb^4 cd^3 + (6A - B)ab^3 d^4) g^3 x^3 - 3 (Bb^4 c^2 d^2 - 4 Bab^3 cd^3 - 3(2A - B)a^2 b^2 d^4) g^3 x^2 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*A*b^4*d^4*g^3*x^4 - 6*B*a^4*d^4*g^3*log(b*x + a) + 2*(B*b^4*c*d^3 + (6*A - B)*a*b^3*d^4)*g^3*x^3 - 3*(B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - 3*(2*A - B)*a^2*b^2*d^4)*g^3*x^2 + 6*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 6*B*a^2*b^2*c*d^3 + (2*A - 3*B)*a^3*b*d^4)*g^3*x - 6*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3)*g^3*log(d*x + c) + 3*(B*b^4*d^4*g^3*x^4 + 4*B*a*b^3*d^4*g^3*x^3 + 6*B*a^2*b^2*d^4*g^3*x^2 + 4*B*a^3*b*d^4*g^3*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/(b*d^4)
```

**Sympy [B]** time = 6.02591, size = 722, normalized size = 4.78

$$\frac{Ab^3 g^3 x^4}{4} - \frac{Ba^4 g^3 \log \left( x + \frac{Ba^5 d^4 g^3 + 4Ba^4 cd^3 g^3 - 6Ba^3 bc^2 d^2 g^3 + 4Ba^2 b^2 c^3 dg^3 - Bab^3 c^4 g^3}{Ba^4 d^4 g^3 + 4Ba^3 bcd^3 g^3 - 6Ba^2 b^2 c^2 d^2 g^3 + 4Bab^3 c^3 dg^3 - Bb^4 c^4 g^3} \right)}{2b} + \frac{Bcg^3 (2ad - bc) (2a^2 d^2 - 2abcd + b^2 c^2) \log \left( \dots \right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```



```
[Out] A*b**3*g**3*x**4/4 - B*a**4*g**3*log(x + (B*a**5*d**4*g**3/b + 4*B*a**4*c*d
**3*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3
*c**4*g**3)/(B*a**4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2
*d**2*g**3 + 4*B*a*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*b) + B*c*g**3*(
2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)*log(x + (5*B*a**4*c*d**3
*g**3 - 6*B*a**3*b*c**2*d**2*g**3 + 4*B*a**2*b**2*c**3*d*g**3 - B*a*b**3*c*
**4*g**3 - B*a*c*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2) +
B*b*c**2*g**3*(2*a*d - b*c)*(2*a**2*d**2 - 2*a*b*c*d + b**2*c**2)/d)/(B*a**
4*d**4*g**3 + 4*B*a**3*b*c*d**3*g**3 - 6*B*a**2*b**2*c**2*d**2*g**3 + 4*B*a
*b**3*c**3*d*g**3 - B*b**4*c**4*g**3))/(2*d**4) + (B*a**3*g**3*x + 3*B*a**2
*b*g**3*x**2/2 + B*a*b**2*g**3*x**3 + B*b**3*g**3*x**4/4)*log(e*(c + d*x)**
2/(a + b*x)**2) + x**3*(6*A*a*b**2*d*g**3 - B*a*b**2*d*g**3 + B*b**3*c*g**3
)/(6*d) + x**2*(6*A*a**2*b*d**2*g**3 - 3*B*a**2*b*d**2*g**3 + 4*B*a*b**2*c*
d*g**3 - B*b**3*c**2*g**3)/(4*d**2) + x*(2*A*a**3*d**3*g**3 - 3*B*a**3*d**3
*g**3 + 6*B*a**2*b*c*d**2*g**3 - 4*B*a*b**2*c**2*d*g**3 + B*b**3*c**3*g**3)
/(2*d**3)
```

---

**Giac [B]** time = 26.1149, size = 491, normalized size = 3.25

$$-\frac{Ba^4g^3 \log(bx + a)}{2b} + \frac{1}{4}(Ab^3g^3 + Bb^3g^3)x^4 + \frac{(Bb^3cg^3 + 6Aab^2dg^3 + 5Bab^2dg^3)x^3}{6d} + \frac{1}{4}(Bb^3g^3x^4 + 4Bab^2g^3x^3 + 6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac
")
```

```
[Out] -1/2*B*a^4*g^3*log(b*x + a)/b + 1/4*(A*b^3*g^3 + B*b^3*g^3)*x^4 + 1/6*(B*b^
3*c*g^3 + 6*A*a*b^2*d*g^3 + 5*B*a*b^2*d*g^3)*x^3/d + 1/4*(B*b^3*g^3*x^4 + 4
*B*a*b^2*g^3*x^3 + 6*B*a^2*b*g^3*x^2 + 4*B*a^3*g^3*x)*log((d^2*x^2 + 2*c*d*
x + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - 1/4*(B*b^3*c^2*g^3 - 4*B*a*b^2*c*d*g^
3 - 6*A*a^2*b*d^2*g^3 - 3*B*a^2*b*d^2*g^3)*x^2/d^2 + 1/2*(B*b^3*c^3*g^3 - 4
*B*a*b^2*c^2*d*g^3 + 6*B*a^2*b*c*d^2*g^3 + 2*A*a^3*d^3*g^3 - B*a^3*d^3*g^3)
*x/d^3 - 1/2*(B*b^3*c^4*g^3 - 4*B*a*b^2*c^3*d*g^3 + 6*B*a^2*b*c^2*d^2*g^3 -
4*B*a^3*c*d^3*g^3)*log(-d*x - c)/d^4
```

$$3.203 \quad \int (ag + bgx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right) dx$$

**Optimal.** Leaf size=120

$$\frac{g^2(a+bx)^3 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)}{3b} - \frac{2Bg^2x(bc-ad)^2}{3d^2} + \frac{2Bg^2(bc-ad)^3 \log(c+dx)}{3bd^3} + \frac{Bg^2(a+bx)^2(bc-ad)}{3bd}$$

[Out]  $(-2*B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (B*(b*c - a*d)*g^2*(a + b*x)^2)/(3*b*d) + (2*B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b)$

**Rubi [A]** time = 0.0783567, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 43}

$$\frac{g^2(a+bx)^3 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)}{3b} - \frac{2Bg^2x(bc-ad)^2}{3d^2} + \frac{2Bg^2(bc-ad)^3 \log(c+dx)}{3bd^3} + \frac{Bg^2(a+bx)^2(bc-ad)}{3bd}$$

Antiderivative was successfully verified.

[In] Int[(a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]),x]

[Out]  $(-2*B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (B*(b*c - a*d)*g^2*(a + b*x)^2)/(3*b*d) + (2*B*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b*d^3) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b)$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RfX\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RfX^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RfX^p])^(n - 1)\*D[RfX, x])/RfX, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RfX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

### Rubi steps

$$\begin{aligned}
\int (ag + bgx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right) dx &= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{3b} - \frac{B \int \frac{2(-bc+ad)g^3(a+bx)^2}{c+dx} dx}{3bg} \\
&= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{3b} + \frac{(2B(bc-ad)g^2) \int \frac{(a+bx)^2}{c+dx} dx}{3b} \\
&= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{3b} + \frac{(2B(bc-ad)g^2) \int \left( -\frac{b(bc-ad)}{d^2} + \frac{b(a+bx)}{d} \right) dx}{3b} \\
&= -\frac{2B(bc-ad)^2 g^2 x}{3d^2} + \frac{B(bc-ad)g^2(a+bx)^2}{3bd} + \frac{2B(bc-ad)^3 g^2 \log(c+dx)}{3bd^3}
\end{aligned}$$

**Mathematica [A]** time = 0.050639, size = 98, normalized size = 0.82

$$\frac{g^2 \left( \frac{B(bc-ad)(d(a^2d+4abdx+b^2x(dx-2c))+2(bc-ad)^2 \log(c+dx)}{d^3} + (a+bx)^3 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right) \right)}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]), x]

[Out] (g^2\*((B\*(b\*c - a\*d)\*(d\*(a^2\*d + 4\*a\*b\*d\*x + b^2\*x\*(-2\*c + d\*x)) + 2\*(b\*c - a\*d)^2\*Log[c + d\*x]))/d^3 + (a + b\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]))/(3\*b)

**Maple [B]** time = 0.241, size = 569, normalized size = 4.7

$$\frac{5g^2Ba^2c}{3d} + 2\frac{g^2Ba^2c}{d} \ln\left(\frac{ad}{bx+a} - \frac{bc}{bx+a} - d\right) + \frac{b^2g^2Bcx^2}{3d} - \frac{2b^2g^2Bc^2x}{3d^2} - \frac{2bg^2Bac^2}{3d^2} - \frac{2g^2Ba^3}{3b} \ln\left(\frac{ad}{bx+a} - \frac{bc}{bx+a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2\*(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2)), x)

[Out] 5/3\*g^2\*B\*a^2/d\*c+2\*g^2\*B\*a^2/d\*ln(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)\*c+1/3\*b^2\*g^2\*B\*c/d\*x^2-2/3\*b^2\*g^2\*B\*c^2/d^2\*x-2/3\*b\*g^2\*B\*a/d^2\*c^2-2/3/b\*g^2\*B\*a^3\*ln(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)+1/3\*b^2\*A\*x^3\*g^2+1/3\*b^2\*B\*ln(e\*(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x^3\*g^2+b\*B\*ln(e\*(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x^2\*a\*g^2+B\*ln(e\*(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x\*a^2\*g^2+1/3/b\*B\*ln(e\*(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*a^3\*g^2+2/3/b\*g^2\*B\*a^3\*ln(1/(b\*x+a))+2/3\*b^2\*g^2\*B\*c^3/d^3\*ln(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)-2/3\*b^2\*g^2\*B\*c^3/d^3\*ln(1/(b\*x+a))-2\*b\*g^2\*B\*a/d^2\*ln(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)\*c^2+2\*b\*g^2\*B\*a/d^2\*ln(1/(b\*x+a))\*c^2-2\*g^2\*B\*a^2/d\*ln(1/(b\*x+a))\*c+2\*b\*g^2\*B\*a/d\*c\*x-4/3\*B\*x\*a^2\*g^2-1/3\*b\*B\*x^2\*a\*g^2+b\*A\*x^2\*a\*g^2+A\*x\*a^2\*g^2+1/3/b\*A\*a^3\*g^2-1/b\*g^2\*B\*a^3

**Maxima [B]** time = 1.33054, size = 589, normalized size = 4.91

$$\frac{1}{3}Ab^2g^2x^3 + Aabg^2x^2 + \left( x \log \left( \frac{d^2ex^2}{b^2x^2 + 2abx + a^2} + \frac{2cdex}{b^2x^2 + 2abx + a^2} + \frac{c^2e}{b^2x^2 + 2abx + a^2} \right) - \frac{2a \log(bx+a)}{b} + \frac{2c}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxima")
```

```
[Out] 1/3*A*b^2*g^2*x^3 + A*a*b*g^2*x^2 + (x*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*log(b*x + a)/b + 2*c*log(d*x + c)/d)*B*a^2*g^2 + (x^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*log(b*x + a)/b^2 - 2*c^2*log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*B*a*b*g^2 + 1/3*(x^3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a^3*log(b*x + a)/b^3 + 2*c^3*log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*g^2 + A*a^2*g^2*x
```

**Fricas [B]** time = 1.11278, size = 508, normalized size = 4.23

$$\frac{Ab^3d^3g^2x^3 - 2Ba^3d^3g^2 \log(bx + a) + (Bb^3cd^2 + (3A - B)ab^2d^3)g^2x^2 - (2Bb^3c^2d - 6Bab^2cd^2 - (3A - 4B)a^2bd^3)g^2x + 3bd^3}{3bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2*(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fricas")
```

```
[Out] 1/3*(A*b^3*d^3*g^2*x^3 - 2*B*a^3*d^3*g^2*log(b*x + a) + (B*b^3*c*d^2 + (3*A - B)*a*b^2*d^3)*g^2*x^2 - (2*B*b^3*c^2*d - 6*B*a*b^2*c*d^2 - (3*A - 4*B)*a^2*b*d^3)*g^2*x + 2*(B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*g^2*log(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*a*b^2*d^3*g^2*x^2 + 3*B*a^2*b*d^3*g^2*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/(b*d^3)
```

**Sympy [B]** time = 4.15602, size = 527, normalized size = 4.39

$$\frac{Ab^2g^2x^3}{3} - \frac{2Ba^3g^2 \log\left(x + \frac{2Ba^4d^3g^2 + 6Ba^3cd^2g^2 - 6Ba^2bc^2dg^2 + 2Bab^2c^3g^2}{2Ba^3d^3g^2 + 6Ba^2bcd^2g^2 - 6Bab^2c^2dg^2 + 2Bb^3c^3g^2}\right)}{3b} + \frac{2Bcg^2(3a^2d^2 - 3abcd + b^2c^2) \log\left(x + \frac{8Ba^3cd^2g^2 - 6Ba^2b^2c^2g^2}{8Ba^3cd^2g^2 - 6Ba^2b^2c^2g^2}\right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)
```

```
[Out] A*b**2*g**2*x**3/3 - 2*B*a**3*g**2*log(x + (2*B*a**4*d**3*g**2/b + 6*B*a**3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2)/(2*B*a**3*d**3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g**2))/(3*b) + 2*B*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)*log(x + (8*B*a**3*c*d**2*g**2 - 6*B*a**2*b*c**2*d*g**2 + 2*B*a*b**2*c**3*g**2 - 2*B*a*c*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2) + 2*B*b*c**2*g**2*(3*a**2*d**2 - 3*a*b*c*d + b**2*c**2)/d)/(2*B*a**3*d**3*g**2 + 6*B*a**2*b*c*d**2*g**2 - 6*B*a*b**2*c**2*d*g**2 + 2*B*b**3*c**3*g**2))/(3*d**3) + (B*a**2*g**2*x + B*a*b*g**2*x**2 + B*b**2*g**2*x**3/3)*log(e*(c + d*x)**2/(a + b*x)**2) + x**2*(3*A*a*b*d*g**2 - B*a*b*d*g**2 + B*b**2*c*g**2)/(3*d) + x*(3*A*a**2*d**2*g**2 - 4*B*a**2*d**2*g**2 + 6*B*a*b*c*d*g**2 - 2*B*b**2*c**2*g**2)/(3*d**2)
```

---

**Giac [B]** time = 4.72178, size = 335, normalized size = 2.79

$$-\frac{2Ba^3g^2 \log(bx+a)}{3b} + \frac{1}{3}(Ab^2g^2 + Bb^2g^2)x^3 + \frac{(Bb^2cg^2 + 3Aabdg^2 + 2Babdg^2)x^2}{3d} + \frac{1}{3}(Bb^2g^2x^3 + 3Babg^2x^2 + 3B$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2)),x, algorithm="giac")

[Out] 
$$-2/3*B*a^3*g^2*\log(b*x + a)/b + 1/3*(A*b^2*g^2 + B*b^2*g^2)*x^3 + 1/3*(B*b^2*c*g^2 + 3*A*a*b*d*g^2 + 2*B*a*b*d*g^2)*x^2/d + 1/3*(B*b^2*g^2*x^3 + 3*B*a*b*g^2*x^2 + 3*B*a^2*g^2*x)*\log((d^2*x^2 + 2*c*d*x + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) - 1/3*(2*B*b^2*c^2*g^2 - 6*B*a*b*c*d*g^2 - 3*A*a^2*d^2*g^2 + B*a^2*d^2*g^2)*x/d^2 + 2/3*(B*b^2*c^3*g^2 - 3*B*a*b*c^2*d*g^2 + 3*B*a^2*c*d^2*g^2)*\log(d*x + c)/d^3$$

$$3.204 \quad \int (ag + bgx) \left( A + B \log \left( \frac{e^{(c+dx)^2}}{(a+bx)^2} \right) \right) dx$$

**Optimal.** Leaf size=78

$$\frac{g(a+bx)^2 \left( B \log \left( \frac{e^{(c+dx)^2}}{(a+bx)^2} \right) + A \right)}{2b} - \frac{Bg(bc-ad)^2 \log(c+dx)}{bd^2} + \frac{Bgx(bc-ad)}{d}$$

[Out] (B\*(b\*c - a\*d)\*g\*x)/d - (B\*(b\*c - a\*d)^2\*g\*Log[c + d\*x])/(b\*d^2) + (g\*(a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]))/(2\*b)

**Rubi [A]** time = 0.0523818, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$ , Rules used = {2525, 12, 43}

$$\frac{g(a+bx)^2 \left( B \log \left( \frac{e^{(c+dx)^2}}{(a+bx)^2} \right) + A \right)}{2b} - \frac{Bg(bc-ad)^2 \log(c+dx)}{bd^2} + \frac{Bgx(bc-ad)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]),x]

[Out] (B\*(b\*c - a\*d)\*g\*x)/d - (B\*(b\*c - a\*d)^2\*g\*Log[c + d\*x])/(b\*d^2) + (g\*(a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]))/(2\*b)

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rubi steps

$$\begin{aligned}
\int (ag + bgx) \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right) dx &= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{2b} - \frac{B \int \frac{2(bc-ad)g^2(-a-bx)}{c+dx} dx}{2bg} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{2b} - \frac{(B(bc-ad)g) \int \frac{-a-bx}{c+dx} dx}{b} \\
&= \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{2b} - \frac{(B(bc-ad)g) \int \left( -\frac{b}{d} + \frac{bc-ad}{d(c+dx)} \right) dx}{b} \\
&= \frac{B(bc-ad)gx}{d} - \frac{B(bc-ad)^2 g \log(c+dx)}{bd^2} + \frac{g(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{2b}
\end{aligned}$$

**Mathematica [A]** time = 0.037048, size = 72, normalized size = 0.92

$$\frac{g \left( (a+bx)^2 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right) - \frac{2B(ad-bc)((ad-bc) \log(c+dx)+bdx)}{d^2} \right)}{2b}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]),x]

[Out] (g\*((-2\*B\*(-(b\*c) + a\*d)\*(b\*d\*x + (-b\*c) + a\*d)\*Log[c + d\*x]))/d^2 + (a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]))/(2\*b)

**Maple [B]** time = 0.242, size = 340, normalized size = 4.4

$$\frac{bAx^2g}{2} + Axag + \frac{Aa^2g}{2b} + \frac{bBx^2g}{2} \ln \left( \frac{e}{b^2} \left( \frac{ad}{bx+a} - \frac{bc}{bx+a} - d \right)^2 \right) + B \ln \left( \frac{e}{b^2} \left( \frac{ad}{bx+a} - \frac{bc}{bx+a} - d \right)^2 \right) xag + \frac{gBa^2}{2b} \ln \left( \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)\*(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2)),x)

[Out] 1/2\*b\*A\*x^2\*g+A\*x\*a\*g+1/2/b\*A\*a^2\*g+1/2\*b\*B\*ln(e\*(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x^2\*g+B\*ln(e\*(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*x\*a\*g+1/2/b\*B\*ln(e\*(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)\*a^2\*g+1/b\*g\*B\*ln(1/(b\*x+a))\*a^2-2\*g\*B/d\*ln(1/(b\*x+a))\*a\*c+b\*g\*B/d^2\*ln(1/(b\*x+a))\*c^2-B\*x\*a\*g-1/b\*g\*B\*a^2+b\*g\*B/d\*c\*x+g\*B/d\*a\*c-1/b\*g\*B\*ln(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)\*a^2+2\*g\*B/d\*ln(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)\*a\*c-b\*g\*B/d^2\*ln(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)\*c^2

**Maxima [B]** time = 1.24947, size = 338, normalized size = 4.33

$$\frac{1}{2} Abgx^2 + \left( x \log \left( \frac{d^2ex^2}{b^2x^2 + 2abx + a^2} + \frac{2cdex}{b^2x^2 + 2abx + a^2} + \frac{c^2e}{b^2x^2 + 2abx + a^2} \right) - \frac{2a \log(bx+a)}{b} + \frac{2c \log(dx+c)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2)),x, algorithm="maxima")

[Out]  $1/2*A*b*g*x^2 + (x*\log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))) - 2*a*\log(b*x + a)/b + 2*c*\log(d*x + c)/d*B*a*g + 1/2*(x^2*\log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))) + 2*a^2*\log(b*x + a)/b^2 - 2*c^2*\log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*B*b*g + A*a*g*x$

**Fricas [A]** time = 1.10473, size = 329, normalized size = 4.22

$$\frac{Ab^2d^2gx^2 - 2Ba^2d^2g \log(bx + a) + 2(Bb^2cd + (A - B)abd^2)gx - 2(Bb^2c^2 - 2Babcd)g \log(dx + c) + (Bb^2d^2gx^2 + 2Babgdx + Aa^2d^2g)}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2)),x, algorithm="fricas")

[Out]  $1/2*(A*b^2*d^2*g*x^2 - 2*B*a^2*d^2*g*\log(b*x + a) + 2*(B*b^2*c*d + (A - B)*a*b*d^2)*g*x - 2*(B*b^2*c^2 - 2*B*a*b*c*d)*g*\log(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*a*b*d^2*g*x)*\log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/(b*d^2)$

**Sympy [B]** time = 2.54484, size = 253, normalized size = 3.24

$$\frac{Abgx^2}{2} - \frac{Ba^2g \log\left(x + \frac{Ba^3d^2g + 2Ba^2cdg - Babc^2g}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{b} + \frac{Bcg(2ad - bc) \log\left(x + \frac{3Ba^2cdg - Babc^2g - Bacg(2ad - bc) + \frac{Bbc^2g(2ad - bc)}{d}}{Ba^2d^2g + 2Babcdg - Bb^2c^2g}\right)}{d^2} + (Bagx + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2)),x)

[Out]  $A*b*g*x**2/2 - B*a**2*g*\log(x + (B*a**3*d**2*g/b + 2*B*a**2*c*d*g - B*a*b*c**2*g)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/b + B*c*g*(2*a*d - b*c)*\log(x + (3*B*a**2*c*d*g - B*a*b*c**2*g - B*a*c*g*(2*a*d - b*c) + B*b*c**2*g*(2*a*d - b*c)/d)/(B*a**2*d**2*g + 2*B*a*b*c*d*g - B*b**2*c**2*g))/d**2 + (B*a*g*x + B*b*g*x**2/2)*\log(e*(c + d*x)**2/(a + b*x)**2) + x*(A*a*d*g - B*a*d*g + B*b*c*g)/d$

**Giac [A]** time = 1.74365, size = 173, normalized size = 2.22

$$-\frac{Ba^2g \log(bx + a)}{b} + \frac{1}{2}(Abg + Bbg)x^2 + \frac{1}{2}(Bbgx^2 + 2Bagx) \log\left(\frac{d^2x^2 + 2cdx + c^2}{b^2x^2 + 2abx + a^2}\right) + \frac{(Bbcg + Aadg)x}{d} - \frac{(Bbc^2g - 2Babcdg)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2)),x, algorithm="giac")

[Out]  $-B*a^2*g*\log(b*x + a)/b + 1/2*(A*b*g + B*b*g)*x^2 + 1/2*(B*b*g*x^2 + 2*B*a*g*x)*\log((d^2*x^2 + 2*c*d*x + c^2)/(b^2*x^2 + 2*a*b*x + a^2)) + (B*b*c*g + A*a*d*g)*x/d - (B*b*c^2*g - 2*B*a*c*d*g)*\log(-d*x - c)/d^2$



$$3.205 \quad \int \frac{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag+bgx} dx$$

**Optimal.** Leaf size=83

$$-\frac{2B \text{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right)}{bg} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right) \left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)}{bg}$$

[Out] -((Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(b\*g)) - (2\*B\*PolyLog[2, 1 + (b\*c - a\*d)/(d\*(a + b\*x))])/(b\*g)

**Rubi [A]** time = 0.291895, antiderivative size = 121, normalized size of antiderivative = 1.46, number of steps used = 10, number of rules used = 8, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$ , Rules used = {2524, 12, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{2B \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg} + \frac{\log(ag + bgx) \left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)}{bg} - \frac{2B \log(ag + bgx) \log\left(\frac{b(c+dx)}{bc-ad}\right)}{bg} + \frac{B \log^2(g(a + bx))}{bg}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(a\*g + b\*g\*x), x]

[Out] (B\*Log[g\*(a + b\*x)]^2)/(b\*g) - (2\*B\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*Log[a\*g + b\*g\*x])/(b\*g) + ((A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])\*Log[a\*g + b\*g\*x])/(b\*g) - (2\*B\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/(b\*g)

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_.))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
 \int \frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{ag + bgx} dx &= \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \log(ag + bgx)}{bg} - \frac{B \int \frac{(a+bx)^2 \left(\frac{2de(c+dx)}{(a+bx)^2} - \frac{2be(c+dx)^2}{(a+bx)^3}\right) \log(ag+bgx)}{e(c+dx)^2} dx}{bg} \\
 &= \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \log(ag + bgx)}{bg} - \frac{B \int \frac{(a+bx)^2 \left(\frac{2de(c+dx)}{(a+bx)^2} - \frac{2be(c+dx)^2}{(a+bx)^3}\right) \log(ag+bgx)}{(c+dx)^2} dx}{beg} \\
 &= \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \log(ag + bgx)}{bg} - \frac{B \int \left(-\frac{2be \log(ag+bgx)}{a+bx} + \frac{2de \log(ag+bgx)}{c+dx}\right) dx}{beg} \\
 &= \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \log(ag + bgx)}{bg} + \frac{(2B) \int \frac{\log(ag+bgx)}{a+bx} dx}{g} - \frac{(2Bd) \int \frac{\log(ag+bgx)}{c+dx} dx}{bg} \\
 &= -\frac{2B \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} + \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \log(ag + bgx)}{bg} + (2B) \int \frac{\log\left(\frac{bg(c+dx)}{bcg-ad}\right)}{ag + bgx} dx \\
 &= -\frac{2B \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} + \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \log(ag + bgx)}{bg} + \frac{(2B) \text{Subst}\left(\int \frac{\log\left(\frac{bg(c+dx)}{bcg-ad}\right)}{x} dx\right)}{b} \\
 &= \frac{B \log^2(g(a + bx))}{bg} - \frac{2B \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} + \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \log(ag + bgx)}{bg}
 \end{aligned}$$

**Mathematica [A]** time = 0.0366503, size = 87, normalized size = 1.05

$$\frac{\log(a + bx) \left( B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) - 2B \log\left(\frac{b(c+dx)}{bc-ad}\right) + B \log(a + bx) + A \right) - 2B \text{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right)}{bg}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(a\*g + b\*g\*x), x]

[Out]  $(\text{Log}[a + b*x]*(A + B*\text{Log}[a + b*x] - 2*B*\text{Log}[(b*(c + d*x))/(b*c - a*d)] + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]) - 2*B*\text{PolyLog}[2, (d*(a + b*x))/(-b*c + a*d)])/(b*g)$

**Maple [B]** time = 0.335, size = 265, normalized size = 3.2

$$-\frac{A \ln((bx+a)^{-1})}{bg} - \frac{B \ln((bx+a)^{-1})}{bg} \ln\left(\frac{e}{b^2} \left(\frac{ad}{bx+a} - \frac{bc}{bx+a} - d\right)^2\right) + 2 \frac{Bad}{bg(ad-bc)} \text{dilog}\left(-\frac{1}{d} \left(\frac{ad-bc}{bx+a} - d\right)\right) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g), x)`

[Out]  $-1/b/g*A*\ln(1/(b*x+a))-1/b/g*B*\ln(1/(b*x+a))*\ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)+2/b/g*B*\text{dilog}(-1/(b*x+a)*(a*d-b*c)-d)/d)/(a*d-b*c)*a*d-2/g*B*d\text{ilog}(-1/(b*x+a)*(a*d-b*c)-d)/d)/(a*d-b*c)*c+2/b/g*B*\ln(1/(b*x+a))*\ln(-1/(b*x+a)*(a*d-b*c)-d)/d)/(a*d-b*c)*a*d-2/g*B*\ln(1/(b*x+a))*\ln(-1/(b*x+a)*(a*d-b*c)-d)/d)/(a*d-b*c)*c$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$B \left( \frac{2 \log(bx+a) \log(dx+c)}{bg} - \int -\frac{bdx \log(e) + bc \log(e) - 2(2bdx + bc + ad) \log(bx+a)}{b^2dgx^2 + abcg + (b^2cg + abdg)x} dx \right) + \frac{A \log(bgx+ag)}{bg}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g), x, algorithm="maxima")`

[Out]  $B*(2*\log(b*x + a)*\log(d*x + c)/(b*g) - \text{integrate}(-(b*d*x*\log(e) + b*c*\log(e) - 2*(2*b*d*x + b*c + a*d)*\log(b*x + a))/(b^2*d*g*x^2 + a*b*c*g + (b^2*c*g + a*b*d*g)*x), x)) + A*\log(b*g*x + a*g)/(b*g)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B \log\left(\frac{d^2ex^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right) + A}{bgx + ag}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g), x, algorithm="fricas")`

[Out]  $\text{integral}((B*\log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + A)/(b*g*x + a*g), x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2))/(b\*g\*x+a\*g),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A}{bgx + ag} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))/(b\*g\*x+a\*g),x, algorithm="giac")

[Out] integrate((B\*log((d\*x + c)^2\*e/(b\*x + a)^2) + A)/(b\*g\*x + a\*g), x)

$$3.206 \quad \int \frac{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^2} dx$$

**Optimal.** Leaf size=102

$$-\frac{A(c+dx)}{g^2(a+bx)(bc-ad)} - \frac{B(c+dx) \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{g^2(a+bx)(bc-ad)} + \frac{2B(c+dx)}{g^2(a+bx)(bc-ad)}$$

[Out]  $-\frac{A(c+dx)}{g^2(a+bx)(bc-ad)} - \frac{B(c+dx) \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{g^2(a+bx)(bc-ad)} + \frac{2B(c+dx)}{g^2(a+bx)(bc-ad)}$

**Rubi [A]** time = 0.07582, antiderivative size = 105, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A}{bg^2(a+bx)} + \frac{2Bd \log(a+bx)}{bg^2(bc-ad)} - \frac{2Bd \log(c+dx)}{bg^2(bc-ad)} + \frac{2B}{bg^2(a+bx)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(a\*g + b\*g\*x)^2,x]

[Out]  $\frac{2B}{bg^2(a+bx)} + \frac{2Bd \log(a+bx)}{bg^2(bc-ad)} - \frac{2Bd \log(c+dx)}{bg^2(bc-ad)} + \frac{2B}{bg^2(a+bx)}$

#### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 44

Int[(a\_ + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e^{(c+dx)^2}}{(a+bx)^2}\right)}{(ag + bgx)^2} dx &= -\frac{A + B \log\left(\frac{e^{(c+dx)^2}}{(a+bx)^2}\right)}{bg^2(a + bx)} + \frac{B \int \frac{2(-bc+ad)}{g(a+bx)^2(c+dx)} dx}{bg} \\
&= -\frac{A + B \log\left(\frac{e^{(c+dx)^2}}{(a+bx)^2}\right)}{bg^2(a + bx)} - \frac{(2B(bc - ad)) \int \frac{1}{(a+bx)^2(c+dx)} dx}{bg^2} \\
&= -\frac{A + B \log\left(\frac{e^{(c+dx)^2}}{(a+bx)^2}\right)}{bg^2(a + bx)} - \frac{(2B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^2} - \frac{bd}{(bc-ad)^2(a+bx)} + \frac{d^2}{(bc-ad)^2(c+dx)}\right) dx}{bg^2} \\
&= \frac{2B}{bg^2(a + bx)} + \frac{2Bd \log(a + bx)}{b(bc - ad)g^2} - \frac{2Bd \log(c + dx)}{b(bc - ad)g^2} - \frac{A + B \log\left(\frac{e^{(c+dx)^2}}{(a+bx)^2}\right)}{bg^2(a + bx)}
\end{aligned}$$

**Mathematica [A]** time = 0.0515359, size = 89, normalized size = 0.87

$$\frac{-(bc - ad) \left( B \log\left(\frac{e^{(c+dx)^2}}{(a+bx)^2}\right) + A - 2B \right) - 2Bd(a + bx) \log(c + dx) + 2Bd(a + bx) \log(a + bx)}{bg^2(a + bx)(bc - ad)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(a\*g + b\*g\*x)^2,x]

[Out] (2\*B\*d\*(a + b\*x)\*Log[a + b\*x] - 2\*B\*d\*(a + b\*x)\*Log[c + d\*x] - (b\*c - a\*d)\*(A - 2\*B + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]))/(b\*(b\*c - a\*d)\*g^2\*(a + b\*x))

**Maple [B]** time = 0.062, size = 212, normalized size = 2.1

$$-\frac{A}{bg^2(bx + a)} - \frac{B}{bg^2(bx + a)} \ln\left(\frac{e}{b^2} \left(\frac{ad}{bx + a} - \frac{bc}{bx + a} - d\right)^2\right) + 2 \frac{Bad}{bg^2(ad - bc)(bx + a)} - 2 \frac{Bc}{g^2(ad - bc)(bx + a)} + 2 \frac{A}{bg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))/(b\*g\*x+a\*g)^2,x)

[Out] -1/b/g^2\*A/(b\*x+a)-1/b/g^2\*B/(b\*x+a)\*ln(e\*(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)+2/b/g^2\*B/(a\*d-b\*c)/(b\*x+a)\*a\*d-2/g^2\*B/(a\*d-b\*c)/(b\*x+a)\*c+2/b/g^2\*B\*d^2/(a\*d-b\*c)^2\*ln(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)\*a-2/g^2\*B\*d/(a\*d-b\*c)^2\*ln(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)\*c

**Maxima [A]** time = 1.23322, size = 252, normalized size = 2.47

$$-B \left( \frac{\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2} + \frac{2cdex}{b^2x^2+2abx+a^2} + \frac{c^2e}{b^2x^2+2abx+a^2}\right)}{b^2g^2x + abg^2} - \frac{2}{b^2g^2x + abg^2} - \frac{2d \log(bx + a)}{(b^2c - abd)g^2} + \frac{2d \log(dx + c)}{(b^2c - abd)g^2} \right) - \frac{A}{b^2g^2x + abg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))/(b\*g\*x+a\*g)^2,x, algorithm="maxima")

[Out]  $-B(\log(d^2 e x^2 / (b^2 x^2 + 2 a b x + a^2) + 2 c d e x / (b^2 x^2 + 2 a b x + a^2) + c^2 e / (b^2 x^2 + 2 a b x + a^2))) / (b^2 g^2 x + a b g^2) - 2 / (b^2 g^2 x + a b g^2) - 2 d \log(b x + a) / ((b^2 c - a b d) g^2) + 2 d \log(d x + c) / ((b^2 c - a b d) g^2) - A / (b^2 g^2 x + a b g^2)$

**Fricas [A]** time = 1.0378, size = 228, normalized size = 2.24

$$\frac{(A - 2B)bc - (A - 2B)ad + (Bbdx + Bbc) \log\left(\frac{d^2 ex^2 + 2cdex + c^2 e}{b^2 x^2 + 2abx + a^2}\right)}{(b^3 c - ab^2 d)g^2 x + (ab^2 c - a^2 bd)g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="fricas")`

[Out]  $-((A - 2B) * b * c - (A - 2B) * a * d + (B * b * d * x + B * b * c) * \log((d^2 * e * x^2 + 2 * c * d * e * x + c^2 * e) / (b^2 * x^2 + 2 * a * b * x + a^2))) / ((b^3 * c - a * b^2 * d) * g^2 * x + (a * b^2 * c - a^2 * b * d) * g^2)$

**Sympy [B]** time = 1.93865, size = 253, normalized size = 2.48

$$\frac{B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{abg^2 + b^2g^2x} + \frac{2Bd \log\left(x + \frac{-\frac{2Ba^2d^3}{ad-bc} + \frac{4Babcd^2}{ad-bc} + 2Bad^2 - \frac{2Bb^2c^2d}{ad-bc} + 2Bbcd}{4Bbd^2}\right)}{bg^2(ad-bc)} - \frac{2Bd \log\left(x + \frac{\frac{2Ba^2d^3}{ad-bc} - \frac{4Babcd^2}{ad-bc} + 2Bad^2 + \frac{2Bb^2c^2d}{ad-bc} + 2Bbcd}{4Bbd^2}\right)}{bg^2(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**2,x)`

[Out]  $-B \log(e * (c + d * x) ** 2 / (a + b * x) ** 2) / (a * b * g ** 2 + b ** 2 * g ** 2 * x) + 2 * B * d * \log(x + (-2 * B * a ** 2 * d ** 3 / (a * d - b * c) + 4 * B * a * b * c * d ** 2 / (a * d - b * c) + 2 * B * a * d ** 2 - 2 * B * b ** 2 * c ** 2 * d / (a * d - b * c) + 2 * B * b * c * d) / (4 * B * b * d ** 2)) / (b * g ** 2 * (a * d - b * c)) - 2 * B * d * \log(x + (2 * B * a ** 2 * d ** 3 / (a * d - b * c) - 4 * B * a * b * c * d ** 2 / (a * d - b * c) + 2 * B * a * d ** 2 + 2 * B * b ** 2 * c ** 2 * d / (a * d - b * c) + 2 * B * b * c * d) / (4 * B * b * d ** 2)) / (b * g ** 2 * (a * d - b * c)) - (A - 2 * B) / (a * b * g ** 2 + b ** 2 * g ** 2 * x)$

**Giac [A]** time = 1.38116, size = 254, normalized size = 2.49

$$-\left(2(b^2 c g^2 - a b d g^2) \left( \frac{d \log\left(\left|\frac{bcg}{bgx+ag} - \frac{adg}{bgx+ag} + d\right|\right)}{b^4 c^2 g^4 - 2 a b^3 c d g^4 + a^2 b^2 d^2 g^4} - \frac{1}{(b^2 c g^2 - a b d g^2)(bgx + ag)bg} \right) + \frac{\log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right)}{(bgx + ag)bg} \right) B - \frac{A}{(bgx + ag)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^2,x, algorithm="giac")`

[Out]  $-(2 * (b^2 * c * g^2 - a * b * d * g^2) * (d * \log(\text{abs}(b * c * g / (b * g * x + a * g) - a * d * g / (b * g * x + a * g) + d)) / (b^4 * c^2 * g^4 - 2 * a * b^3 * c * d * g^4 + a^2 * b^2 * d^2 * g^4) - 1 / ((b^2 * c * g^2 - a * b * d * g^2) * (b * g * x + a * g) * b * g)) + \log((d * x + c)^2 * e / (b * x + a)^2) / ((b * g * x + a * g) * b * g)) * B - A / ((b * g * x + a * g) * b * g)$

$$3.207 \quad \int \frac{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^3} dx$$

**Optimal.** Leaf size=139

$$-\frac{B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A}{2bg^3(a+bx)^2} - \frac{Bd^2 \log(a+bx)}{bg^3(bc-ad)^2} + \frac{Bd^2 \log(c+dx)}{bg^3(bc-ad)^2} - \frac{Bd}{bg^3(a+bx)(bc-ad)} + \frac{B}{2bg^3(a+bx)^2}$$

[Out] B/(2\*b\*g^3\*(a + b\*x)^2) - (B\*d)/(b\*(b\*c - a\*d)\*g^3\*(a + b\*x)) - (B\*d^2\*Log[a + b\*x])/(b\*(b\*c - a\*d)^2\*g^3) + (B\*d^2\*Log[c + d\*x])/(b\*(b\*c - a\*d)^2\*g^3) - (A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(2\*b\*g^3\*(a + b\*x)^2)

**Rubi [A]** time = 0.100561, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A}{2bg^3(a+bx)^2} - \frac{Bd^2 \log(a+bx)}{bg^3(bc-ad)^2} + \frac{Bd^2 \log(c+dx)}{bg^3(bc-ad)^2} - \frac{Bd}{bg^3(a+bx)(bc-ad)} + \frac{B}{2bg^3(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(a\*g + b\*g\*x)^3,x]

[Out] B/(2\*b\*g^3\*(a + b\*x)^2) - (B\*d)/(b\*(b\*c - a\*d)\*g^3\*(a + b\*x)) - (B\*d^2\*Log[a + b\*x])/(b\*(b\*c - a\*d)^2\*g^3) + (B\*d^2\*Log[c + d\*x])/(b\*(b\*c - a\*d)^2\*g^3) - (A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(2\*b\*g^3\*(a + b\*x)^2)

#### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps



$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag + bgx)^3} dx &= -\frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a + bx)^2} + \frac{B \int \frac{2(-bc+ad)}{g^2(a+bx)^3(c+dx)} dx}{2bg} \\
&= -\frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a + bx)^2} - \frac{(B(bc - ad)) \int \frac{1}{(a+bx)^3(c+dx)} dx}{bg^3} \\
&= -\frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a + bx)^2} - \frac{(B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^3} - \frac{bd}{(bc-ad)^2(a+bx)^2} + \frac{bd^2}{(bc-ad)^3(a+bx)} - \frac{bd^3}{(bc-ad)^4}\right) dx}{bg^3} \\
&= \frac{B}{2bg^3(a + bx)^2} - \frac{Bd}{b(bc - ad)g^3(a + bx)} - \frac{Bd^2 \log(a + bx)}{b(bc - ad)^2g^3} + \frac{Bd^2 \log(c + dx)}{b(bc - ad)^2g^3} - \frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2bg^3(a + bx)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.0962121, size = 128, normalized size = 0.92

$$\frac{(bc - ad) \left( -aAd + B(bc - ad) \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + 3aBd + Abc - bBc + 2Bdx \right) - 2Bd^2(a + bx)^2 \log(c + dx) + 2Bd^2(a + bx)^2 \log(a + bx)}{2bg^3(a + bx)^2(bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(a\*g + b\*g\*x)^3, x]

[Out] -(2\*B\*d^2\*(a + b\*x)^2\*Log[a + b\*x] - 2\*B\*d^2\*(a + b\*x)^2\*Log[c + d\*x] + (b\*c - a\*d)\*(A\*b\*c - b\*B\*c - a\*A\*d + 3\*a\*B\*d + 2\*b\*B\*d\*x + B\*(b\*c - a\*d)\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]))/(2\*b\*(b\*c - a\*d)^2\*g^3\*(a + b\*x)^2)

**Maple [B]** time = 0.06, size = 300, normalized size = 2.2

$$-\frac{A}{2b(bx + a)^2g^3} - \frac{B}{2b(bx + a)^2g^3} \ln\left(\frac{e}{b^2} \left(\frac{ad}{bx + a} - \frac{bc}{bx + a} - d\right)^2\right) + \frac{Ba^2d^2}{2bg^3(ad - bc)^2(bx + a)^2} - \frac{Badc}{g^3(ad - bc)^2(bx + a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))/(b\*g\*x+a\*g)^3, x)

[Out] -1/2/b/(b\*x+a)^2/g^3\*A-1/2/b/g^3\*B/(b\*x+a)^2\*ln(e\*(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)+1/2/b/g^3\*B/(a\*d-b\*c)^2/(b\*x+a)^2\*a^2\*d^2-1/g^3\*B/(a\*d-b\*c)^2/(b\*x+a)^2\*a\*d\*c+1/2\*b/g^3\*B/(a\*d-b\*c)^2\*c^2/(b\*x+a)^2+1/b/g^3\*B/(a\*d-b\*c)^2\*d^2/(b\*x+a)\*a-1/g^3\*B/(a\*d-b\*c)^2\*d/(b\*x+a)\*c+1/b/g^3\*B\*d^3/(a\*d-b\*c)^3\*ln(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)\*a-1/g^3\*B\*d^2/(a\*d-b\*c)^3\*ln(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)\*c

**Maxima [B]** time = 1.21986, size = 413, normalized size = 2.97

$$-\frac{1}{2}B \left( \frac{2bdx - bc + 3ad}{(b^4c - ab^3d)g^3x^2 + 2(ab^3c - a^2b^2d)g^3x + (a^2b^2c - a^3bd)g^3} + \frac{\log\left(\frac{d^2ex^2}{b^2x^2+2abx+a^2} + \frac{2cdx}{b^2x^2+2abx+a^2} + \frac{c^2e}{b^2x^2+2abx+a^2}\right)}{b^3g^3x^2 + 2ab^2g^3x + a^2bg^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))/(b\*g\*x+a\*g)^3,x, algorithm="maxima")

[Out] -1/2\*B\*((2\*b\*d\*x - b\*c + 3\*a\*d)/((b^4\*c - a\*b^3\*d)\*g^3\*x^2 + 2\*(a\*b^3\*c - a^2\*b^2\*d)\*g^3\*x + (a^2\*b^2\*c - a^3\*b\*d)\*g^3) + log(d^2\*e\*x^2/(b^2\*x^2 + 2\*a\*b\*x + a^2) + 2\*c\*d\*e\*x/(b^2\*x^2 + 2\*a\*b\*x + a^2) + c^2\*e/(b^2\*x^2 + 2\*a\*b\*x + a^2))/(b^3\*g^3\*x^2 + 2\*a\*b^2\*g^3\*x + a^2\*b\*g^3) + 2\*d^2\*log(b\*x + a)/((b^3\*c^2 - 2\*a\*b^2\*c\*d + a^2\*b\*d^2)\*g^3) - 2\*d^2\*log(d\*x + c)/((b^3\*c^2 - 2\*a\*b^2\*c\*d + a^2\*b\*d^2)\*g^3)) - 1/2\*A/(b^3\*g^3\*x^2 + 2\*a\*b^2\*g^3\*x + a^2\*b\*g^3)

**Fricas [A]** time = 1.00497, size = 495, normalized size = 3.56

$$\frac{(A - B)b^2c^2 - 2(A - 2B)abcd + (A - 3B)a^2d^2 + 2(Bb^2cd - Babd^2)x - (Bb^2d^2x^2 + 2Babd^2x - Bb^2c^2 + 2Babcd) \log\left(\frac{d^2e^2x^2 + 2cde^2x + c^2e}{b^2x^2 + 2abx + a^2}\right)}{2\left((b^5c^2 - 2ab^4cd + a^2b^3d^2)g^3x^2 + 2(ab^4c^2 - 2a^2b^3cd + a^3b^2d^2)g^3x + (a^2b^3c^2 - 2a^3b^2cd + a^4bd^2)g^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))/(b\*g\*x+a\*g)^3,x, algorithm="fricas")

[Out] -1/2\*((A - B)\*b^2\*c^2 - 2\*(A - 2\*B)\*a\*b\*c\*d + (A - 3\*B)\*a^2\*d^2 + 2\*(B\*b^2\*c\*d - B\*a\*b\*d^2)\*x - (B\*b^2\*d^2\*x^2 + 2\*B\*a\*b\*d^2\*x - B\*b^2\*c^2 + 2\*B\*a\*b\*c\*d)\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2)))/((b^5\*c^2 - 2\*a\*b^4\*c\*d + a^2\*b^3\*d^2)\*g^3\*x^2 + 2\*(a\*b^4\*c^2 - 2\*a^2\*b^3\*c\*d + a^3\*b^2\*d^2)\*g^3\*x + (a^2\*b^3\*c^2 - 2\*a^3\*b^2\*c\*d + a^4\*b\*d^2)\*g^3)

**Sympy [B]** time = 3.26856, size = 418, normalized size = 3.01

$$\frac{B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2a^2bg^3 + 4ab^2g^3x + 2b^3g^3x^2} + \frac{Bd^2 \log\left(x + \frac{-\frac{Ba^3d^5}{(ad-bc)^2} + \frac{3Ba^2bcd^4}{(ad-bc)^2} - \frac{3Bab^2c^2d^3}{(ad-bc)^2} + Bad^3 + \frac{Bb^3c^3d^2}{(ad-bc)^2} + Bbcd^2}{2Bbd^3}\right)}{bg^3(ad-bc)^2} - \frac{Bd^2 \log\left(x + \frac{Ba^3d^5}{(ad-bc)^2} - \frac{3Ba^2bcd^4}{(ad-bc)^2} + \dots\right)}{bg^3(ad-bc)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2))/(b\*g\*x+a\*g)\*\*3,x)

[Out] -B\*log(e\*(c + d\*x)\*\*2/(a + b\*x)\*\*2)/(2\*a\*\*2\*b\*g\*\*3 + 4\*a\*b\*\*2\*g\*\*3\*x + 2\*b\*\*3\*g\*\*3\*x\*\*2) + B\*d\*\*2\*log(x + (-B\*a\*\*3\*d\*\*5/(a\*d - b\*c)\*\*2 + 3\*B\*a\*\*2\*b\*c\*d\*\*4/(a\*d - b\*c)\*\*2 - 3\*B\*a\*b\*\*2\*c\*\*2\*d\*\*3/(a\*d - b\*c)\*\*2 + B\*a\*d\*\*3 + B\*b\*\*3\*c\*\*3\*d\*\*2/(a\*d - b\*c)\*\*2 + B\*b\*c\*d\*\*2)/(2\*B\*b\*d\*\*3))/(b\*g\*\*3\*(a\*d - b\*c)\*\*2) - B\*d\*\*2\*log(x + (B\*a\*\*3\*d\*\*5/(a\*d - b\*c)\*\*2 - 3\*B\*a\*\*2\*b\*c\*d\*\*4/(a\*d - b\*c)\*\*2 + 3\*B\*a\*b\*\*2\*c\*\*2\*d\*\*3/(a\*d - b\*c)\*\*2 + B\*a\*d\*\*3 - B\*b\*\*3\*c\*\*3\*d\*\*2/(a\*d - b\*c)\*\*2 + B\*b\*c\*d\*\*2)/(2\*B\*b\*d\*\*3))/(b\*g\*\*3\*(a\*d - b\*c)\*\*2) + (-A\*a\*d + A\*b\*c + 3\*B\*a\*d - B\*b\*c + 2\*B\*b\*d\*x)/(2\*a\*\*3\*b\*d\*g\*\*3 - 2\*a\*\*2\*b\*\*2\*c\*g\*\*3 + x\*\*2\*(2\*a\*b\*\*3\*d\*g\*\*3 - 2\*b\*\*4\*c\*g\*\*3) + x\*(4\*a\*\*2\*b\*\*2\*d\*g\*\*3 - 4\*a\*b\*\*3\*c\*g\*\*3))

**Giac [A]** time = 1.34159, size = 350, normalized size = 2.52

$$-\frac{Bd^2 \log(bx + a)}{b^3c^2g^3 - 2ab^2cdg^3 + a^2bd^2g^3} + \frac{Bd^2 \log(dx + c)}{b^3c^2g^3 - 2ab^2cdg^3 + a^2bd^2g^3} - \frac{B \log\left(\frac{d^2x^2 + 2cdx + c^2}{b^2x^2 + 2abx + a^2}\right)}{2(b^3g^3x^2 + 2ab^2g^3x + a^2bg^3)} - \frac{1}{2(b^4cg^3x^2 - ab^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))/(b\*g\*x+a\*g)^3,x, algorithm="giac")

[Out] -B\*d^2\*log(b\*x + a)/(b^3\*c^2\*g^3 - 2\*a\*b^2\*c\*d\*g^3 + a^2\*b\*d^2\*g^3) + B\*d^2\*log(d\*x + c)/(b^3\*c^2\*g^3 - 2\*a\*b^2\*c\*d\*g^3 + a^2\*b\*d^2\*g^3) - 1/2\*B\*log((d^2\*x^2 + 2\*c\*d\*x + c^2)/(b^2\*x^2 + 2\*a\*b\*x + a^2))/(b^3\*g^3\*x^2 + 2\*a\*b^2\*g^3\*x + a^2\*b\*g^3) - 1/2\*(2\*B\*b\*d\*x + A\*b\*c - A\*a\*d + 2\*B\*a\*d)/(b^4\*c\*g^3\*x^2 - a\*b^3\*d\*g^3\*x^2 + 2\*a\*b^3\*c\*g^3\*x - 2\*a^2\*b^2\*d\*g^3\*x + a^2\*b^2\*c\*g^3 - a^3\*b\*d\*g^3)

$$3.208 \quad \int \frac{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^4} dx$$

**Optimal.** Leaf size=177

$$-\frac{B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A}{3bg^4(a+bx)^3} + \frac{2Bd^2}{3bg^4(a+bx)(bc-ad)^2} + \frac{2Bd^3 \log(a+bx)}{3bg^4(bc-ad)^3} - \frac{2Bd^3 \log(c+dx)}{3bg^4(bc-ad)^3} - \frac{Bd}{3bg^4(a+bx)^2(bc-ad)} + \frac{1}{9bg^4}$$

[Out] (2\*B)/(9\*b\*g^4\*(a + b\*x)^3) - (B\*d)/(3\*b\*(b\*c - a\*d)\*g^4\*(a + b\*x)^2) + (2\*B\*d^2)/(3\*b\*(b\*c - a\*d)^2\*g^4\*(a + b\*x)) + (2\*B\*d^3\*Log[a + b\*x])/(3\*b\*(b\*c - a\*d)^3\*g^4) - (2\*B\*d^3\*Log[c + d\*x])/(3\*b\*(b\*c - a\*d)^3\*g^4) - (A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(3\*b\*g^4\*(a + b\*x)^3)

**Rubi [A]** time = 0.120882, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A}{3bg^4(a+bx)^3} + \frac{2Bd^2}{3bg^4(a+bx)(bc-ad)^2} + \frac{2Bd^3 \log(a+bx)}{3bg^4(bc-ad)^3} - \frac{2Bd^3 \log(c+dx)}{3bg^4(bc-ad)^3} - \frac{Bd}{3bg^4(a+bx)^2(bc-ad)} + \frac{1}{9bg^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(a\*g + b\*g\*x)^4, x]

[Out] (2\*B)/(9\*b\*g^4\*(a + b\*x)^3) - (B\*d)/(3\*b\*(b\*c - a\*d)\*g^4\*(a + b\*x)^2) + (2\*B\*d^2)/(3\*b\*(b\*c - a\*d)^2\*g^4\*(a + b\*x)) + (2\*B\*d^3\*Log[a + b\*x])/(3\*b\*(b\*c - a\*d)^3\*g^4) - (2\*B\*d^3\*Log[c + d\*x])/(3\*b\*(b\*c - a\*d)^3\*g^4) - (A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(3\*b\*g^4\*(a + b\*x)^3)

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 44

```
Int[((a_) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag + bgx)^4} dx &= -\frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a + bx)^3} + \frac{B \int \frac{2(-bc+ad)}{g^3(a+bx)^4(c+dx)} dx}{3bg} \\
&= -\frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a + bx)^3} - \frac{(2B(bc - ad)) \int \frac{1}{(a+bx)^4(c+dx)} dx}{3bg^4} \\
&= -\frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3bg^4(a + bx)^3} - \frac{(2B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^4} - \frac{bd}{(bc-ad)^2(a+bx)^3} + \frac{bd^2}{(bc-ad)^3(a+bx)^2} - \frac{bd^3}{(bc-ad)^4(a+bx)}\right) dx}{3bg^4} \\
&= \frac{2B}{9bg^4(a + bx)^3} - \frac{Bd}{3b(bc - ad)g^4(a + bx)^2} + \frac{2Bd^2}{3b(bc - ad)^2g^4(a + bx)} + \frac{2Bd^3 \log(a + bx)}{3b(bc - ad)^3g^4} - \frac{2Bd^4}{3b(bc - ad)^4g^4}
\end{aligned}$$

**Mathematica [A]** time = 0.120337, size = 140, normalized size = 0.79

$$\frac{B(6d^2(a+bx)^2(bc-ad) - 6d^3(a+bx)^3 \log(c+dx) - 3d(a+bx)(bc-ad)^2 + 2(bc-ad)^3 + 6d^3(a+bx)^3 \log(a+bx))}{(bc-ad)^3} - 3 \left( B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A \right)$$


---


$$9bg^4(a + bx)^3$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(a\*g + b\*g\*x)^4, x]

[Out] ((B\*(2\*(b\*c - a\*d)^3 - 3\*d\*(b\*c - a\*d)^2\*(a + b\*x) + 6\*d^2\*(b\*c - a\*d)\*(a + b\*x)^2 + 6\*d^3\*(a + b\*x)^3\*Log[a + b\*x] - 6\*d^3\*(a + b\*x)^3\*Log[c + d\*x]))/(b\*c - a\*d)^3 - 3\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(9\*b\*g^4\*(a + b\*x)^3)

**Maple [B]** time = 0.059, size = 427, normalized size = 2.4

$$-\frac{A}{3b(bx+a)^3g^4} - \frac{B}{3b(bx+a)^3g^4} \ln\left(\frac{e}{b^2} \left(\frac{ad}{bx+a} - \frac{bc}{bx+a} - d\right)^2\right) + \frac{2Ba^3d^3}{9bg^4(ad-bc)^3(bx+a)^3} - \frac{2Ba^2d^2c}{3g^4(ad-bc)^3(bx+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))/(b\*g\*x+a\*g)^4, x)

[Out] -1/3/b/(b\*x+a)^3/g^4\*A - 1/3/b/g^4\*B/(b\*x+a)^3\*ln(e\*(1/(b\*x+a)\*a\*d - b\*c/(b\*x+a) - d)^2/b^2) + 2/9/b/g^4\*B\*a^3\*d^3/(a\*d - b\*c)^3/(b\*x+a)^3 - 2/3/g^4\*B\*a^2\*d^2/(a\*d - b\*c)^3/(b\*x+a)^3\*c + 2/3\*b/g^4\*B\*a\*d/(a\*d - b\*c)^3/(b\*x+a)^3\*c^2 + 1/3/b/g^4\*B\*a^2\*d^3/(a\*d - b\*c)^3/(b\*x+a)^2 - 2/3/g^4\*B\*a\*d^2/(a\*d - b\*c)^3/(b\*x+a)^2\*c + 2/3/b/g^4\*B\*a\*d^3/(a\*d - b\*c)^3/(b\*x+a) + 2/3/b/g^4\*B\*a\*d^4/(a\*d - b\*c)^4\*ln(1/(b\*x+a)\*a\*d - b\*c/(b\*x+a) - d) - 2/9\*b^2/g^4\*B\*c^3/(a\*d - b\*c)^3/(b\*x+a)^3 + 1/3\*b/g^4\*B\*c^2/(a\*d - b\*c)^3/(b\*x+a)^2\*d - 2/3/g^4\*B\*c/(a\*d - b\*c)^3/(b\*x+a)\*d^2 - 2/3/g^4\*B\*c\*d^3/(a\*d - b\*c)^4\*ln(1/(b\*x+a)\*a\*d - b\*c/(b\*x+a) - d)

**Maxima [B]** time = 1.31059, size = 648, normalized size = 3.66

$$\frac{1}{9} B \left( \frac{6b^2d^2x^2 + 2b^2c^2 - 7abcd + 11a^2d^2 - 3(b^2cd - 5abd^2)x}{(b^6c^2 - 2ab^5cd + a^2b^4d^2)g^4x^3 + 3(ab^5c^2 - 2a^2b^4cd + a^3b^3d^2)g^4x^2 + 3(a^2b^4c^2 - 2a^3b^3cd + a^4b^2d^2)g^4x + (a^3b^3c^2 - 2a^4b^2cd + a^5b^1d^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="maxima")
```

```
[Out] 1/9*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) - 3*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 1/3*A/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

**Fricas [B]** time = 1.07831, size = 869, normalized size = 4.91

$$\frac{(3A - 2B)b^3c^3 - 9(A - B)ab^2c^2d + 9(A - 2B)a^2bcd^2 - (3A - 11B)a^3d^3 - 6(Bb^3cd^2 - Bab^2d^3)x^2 + 3(Bb^3c^2d - 6Bab^2cd^2 - 3A^2b^3c^2d + 3A^3b^2cd^2 - a^3b^3d^3)g^4x^2 + 3(9(b^7c^3 - 3ab^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3)g^4x^3 + 3(ab^6c^3 - 3a^2b^5c^2d + 3a^3b^4cd^2 - a^4b^3d^3)g^4x^2 + 3(a^2b^5c^2d - 2a^3b^4cd^2 + 3a^4b^3d^3)g^4x + (a^3b^3c^2d - 2a^4b^2cd^2 + a^5b^2d^3)g^4)}{9((b^7c^3 - 3ab^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3)g^4x^3 + 3(ab^6c^3 - 3a^2b^5c^2d + 3a^3b^4cd^2 - a^4b^3d^3)g^4x^2 + 3(a^2b^5c^2d - 2a^3b^4cd^2 + 3a^4b^3d^3)g^4x + (a^3b^3c^2d - 2a^4b^2cd^2 + a^5b^2d^3)g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="fricas")
```

```
[Out] -1/9*((3*A - 2*B)*b^3*c^3 - 9*(A - B)*a*b^2*c^2*d + 9*(A - 2*B)*a^2*b*c*d^2 - (3*A - 11*B)*a^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*x + 3*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x + B*b^3*c^3 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^7*c^3 - 3*a*b^6*c^2*d + 3*a^2*b^5*c*d^2 - a^3*b^4*d^3)*g^4*x^3 + 3*(a*b^6*c^3 - 3*a^2*b^5*c^2*d + 3*a^3*b^4*c*d^2 - a^4*b^3*d^3)*g^4*x^2 + 3*(a^2*b^5*c^3 - 3*a^3*b^4*c^2*d + 3*a^4*b^3*c*d^2 - a^5*b^2*d^3)*g^4*x + (a^3*b^4*c^3 - 3*a^4*b^3*c^2*d + 3*a^5*b^2*c*d^2 - a^6*b*d^3)*g^4)
```

**Sympy [B]** time = 5.36037, size = 677, normalized size = 3.82

$$\frac{B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{3a^3bg^4 + 9a^2b^2g^4x + 9ab^3g^4x^2 + 3b^4g^4x^3} + \frac{2Bd^3 \log\left(x + \frac{-\frac{2Ba^4d^7}{(ad-bc)^3} + \frac{8Ba^3bcd^6}{(ad-bc)^3} - \frac{12Ba^2b^2c^2d^5}{(ad-bc)^3} + \frac{8Bab^3c^3d^4}{(ad-bc)^3} + 2Bad^4 - \frac{2Bb^4c^4d^3}{(ad-bc)^3} + 2Bbcd^3}{4Bbd^4}\right)}{3bg^4(ad-bc)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))/(b*g*x+a*g)**4,x)
```

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(3*a**3*b*g**4 + 9*a**2*b**2*g**4*x + 9*a*b**3*g**4*x**2 + 3*b**4*g**4*x**3) + 2*B*d**3*log(x + (-2*B*a**4*d**7/(a*d - b*c)**3 + 8*B*a**3*b*c*d**6/(a*d - b*c)**3 - 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 + 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 - 2*B*b**4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) - 2*B*d**3*log(x + (2*B*a**4*d**7/(a*d - b*c)**3 - 8*B*a**3*b*c*d**6/(a*d - b*c)**3 - 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 + 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 - 2*B*b**4*c**4*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3)
```

$$\frac{6/(a*d - b*c)**3 + 12*B*a**2*b**2*c**2*d**5/(a*d - b*c)**3 - 8*B*a*b**3*c**3*d**4/(a*d - b*c)**3 + 2*B*a*d**4 + 2*B*b**4*c**4*d**3/(a*d - b*c)**3 + 2*B*b*c*d**3)/(4*B*b*d**4))/(3*b*g**4*(a*d - b*c)**3) + (-3*A*a**2*d**2 + 6*A*a*b*c*d - 3*A*b**2*c**2 + 11*B*a**2*d**2 - 7*B*a*b*c*d + 2*B*b**2*c**2 + 6*B*b**2*d**2*x**2 + x*(15*B*a*b*d**2 - 3*B*b**2*c*d))/(9*a**5*b*d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 9*a**3*b**3*c**2*g**4 + x**3*(9*a**2*b**4*d**2*g**4 - 18*a*b**5*c*d*g**4 + 9*b**6*c**2*g**4) + x**2*(27*a**3*b**3*d**2*g**4 - 54*a**2*b**4*c*d*g**4 + 27*a*b**5*c**2*g**4) + x*(27*a**4*b**2*d**2*g**4 - 54*a**3*b**3*c*d*g**4 + 27*a**2*b**4*c**2*g**4))$$

**Giac [B]** time = 1.38949, size = 639, normalized size = 3.61

$$\frac{2Bd^3 \log(bx + a)}{3(b^4c^3g^4 - 3ab^3c^2dg^4 + 3a^2b^2cd^2g^4 - a^3bd^3g^4)} - \frac{2Bd^3 \log(dx + c)}{3(b^4c^3g^4 - 3ab^3c^2dg^4 + 3a^2b^2cd^2g^4 - a^3bd^3g^4)} - \frac{B \log(b^4g^4x^3 + 3a^2g^4x^2 + 3a^2g^4x + a^3g^4)}{3(b^4g^4x^3 + 3a^2g^4x^2 + 3a^2g^4x + a^3g^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^4,x, algorithm="giac")
```

```
[Out] 2/3*B*d^3*log(b*x + a)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 2/3*B*d^3*log(d*x + c)/(b^4*c^3*g^4 - 3*a*b^3*c^2*d*g^4 + 3*a^2*b^2*c*d^2*g^4 - a^3*b*d^3*g^4) - 1/3*B*log((d^2*x^2 + 2*c*d*x + c^2)/(b^2*x^2 + 2*a*b*x + a^2))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4) + 1/9*(6*B*b^2*d^2*x^2 - 3*B*b^2*c*d*x + 15*B*a*b*d^2*x - 3*A*b^2*c^2 - B*b^2*c^2 + 6*A*a*b*c*d - B*a*b*c*d - 3*A*a^2*d^2 + 8*B*a^2*d^2)/(b^6*c^2*g^4*x^3 - 2*a*b^5*c*d*g^4*x^3 + a^2*b^4*d^2*g^4*x^3 + 3*a*b^5*c^2*g^4*x^2 - 6*a^2*b^4*c*d*g^4*x^2 + 3*a^3*b^3*d^2*g^4*x^2 + 3*a^2*b^4*c^2*g^4*x - 6*a^3*b^3*c*d*g^4*x + 3*a^4*b^2*d^2*g^4*x + a^3*b^3*c^2*g^4 - 2*a^4*b^2*c*d*g^4 + a^5*b*d^2*g^4)
```

$$3.209 \quad \int \frac{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag+bgx)^5} dx$$

**Optimal.** Leaf size=208

$$-\frac{B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A}{4bg^5(a+bx)^4} - \frac{Bd^3}{2bg^5(a+bx)(bc-ad)^3} + \frac{Bd^2}{4bg^5(a+bx)^2(bc-ad)^2} - \frac{Bd^4 \log(a+bx)}{2bg^5(bc-ad)^4} + \frac{Bd^4 \log(c+dx)}{2bg^5(bc-ad)^4} - \frac{6bg^5(a+bx)^4}{6bg^5(a+bx)^4}$$

[Out] B/(8\*b\*g^5\*(a + b\*x)^4) - (B\*d)/(6\*b\*(b\*c - a\*d)\*g^5\*(a + b\*x)^3) + (B\*d^2)/(4\*b\*(b\*c - a\*d)^2\*g^5\*(a + b\*x)^2) - (B\*d^3)/(2\*b\*(b\*c - a\*d)^3\*g^5\*(a + b\*x)) - (B\*d^4\*Log[a + b\*x])/(2\*b\*(b\*c - a\*d)^4\*g^5) + (B\*d^4\*Log[c + d\*x])/(2\*b\*(b\*c - a\*d)^4\*g^5) - (A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(4\*b\*g^5\*(a + b\*x)^4)

**Rubi [A]** time = 0.142715, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {2525, 12, 44}

$$-\frac{B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A}{4bg^5(a+bx)^4} - \frac{Bd^3}{2bg^5(a+bx)(bc-ad)^3} + \frac{Bd^2}{4bg^5(a+bx)^2(bc-ad)^2} - \frac{Bd^4 \log(a+bx)}{2bg^5(bc-ad)^4} + \frac{Bd^4 \log(c+dx)}{2bg^5(bc-ad)^4} - \frac{6bg^5(a+bx)^4}{6bg^5(a+bx)^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(a\*g + b\*g\*x)^5,x]

[Out] B/(8\*b\*g^5\*(a + b\*x)^4) - (B\*d)/(6\*b\*(b\*c - a\*d)\*g^5\*(a + b\*x)^3) + (B\*d^2)/(4\*b\*(b\*c - a\*d)^2\*g^5\*(a + b\*x)^2) - (B\*d^3)/(2\*b\*(b\*c - a\*d)^3\*g^5\*(a + b\*x)) - (B\*d^4\*Log[a + b\*x])/(2\*b\*(b\*c - a\*d)^4\*g^5) + (B\*d^4\*Log[c + d\*x])/(2\*b\*(b\*c - a\*d)^4\*g^5) - (A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(4\*b\*g^5\*(a + b\*x)^4)

#### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rubi steps



$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(ag + bgx)^5} dx &= -\frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a + bx)^4} + \frac{B \int \frac{2(-bc+ad)}{g^4(a+bx)^5(c+dx)} dx}{4bg} \\
&= -\frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a + bx)^4} - \frac{(B(bc - ad)) \int \frac{1}{(a+bx)^5(c+dx)} dx}{2bg^5} \\
&= -\frac{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{4bg^5(a + bx)^4} - \frac{(B(bc - ad)) \int \left(\frac{b}{(bc-ad)(a+bx)^5} - \frac{bd}{(bc-ad)^2(a+bx)^4} + \frac{bd^2}{(bc-ad)^3(a+bx)^3} - \frac{bd^3}{(bc-ad)^4(a+bx)^2}\right) dx}{2bg^5} \\
&= \frac{B}{8bg^5(a + bx)^4} - \frac{Bd}{6b(bc - ad)g^5(a + bx)^3} + \frac{Bd^2}{4b(bc - ad)^2g^5(a + bx)^2} - \frac{Bd^3}{2b(bc - ad)^3g^5(a + bx)}
\end{aligned}$$

**Mathematica [A]** time = 0.189059, size = 162, normalized size = 0.78

$$\frac{B(6d^2(a+bx)^2(bc-ad)^2+12d^3(a+bx)^3(ad-bc)+12d^4(a+bx)^4 \log(c+dx)+4d(a+bx)(ad-bc)^3+3(bc-ad)^4-12d^4(a+bx)^4 \log(a+bx))}{(bc-ad)^4} - 6 \left( B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + \frac{24bg^5(a + bx)^4}{24bg^5(a + bx)^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(a\*g + b\*g\*x)^5, x]

[Out] ((B\*(3\*(b\*c - a\*d)^4 + 4\*d\*(-(b\*c) + a\*d)^3\*(a + b\*x) + 6\*d^2\*(b\*c - a\*d)^2\*(a + b\*x)^2 + 12\*d^3\*(-(b\*c) + a\*d)\*(a + b\*x)^3 - 12\*d^4\*(a + b\*x)^4\*Log[a + b\*x] + 12\*d^4\*(a + b\*x)^4\*Log[c + d\*x]))/(b\*c - a\*d)^4 - 6\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]))/(24\*b\*g^5\*(a + b\*x)^4)

**Maple [B]** time = 0.064, size = 587, normalized size = 2.8

$$-\frac{A}{4b(bx+a)^4g^5} - \frac{B}{4b(bx+a)^4g^5} \ln\left(\frac{e}{b^2} \left(\frac{ad}{bx+a} - \frac{bc}{bx+a} - d\right)^2\right) + \frac{Ba^4d^4}{8bg^5(ad-bc)^4(bx+a)^4} - \frac{Ba^3d^3c}{2g^5(ad-bc)^4(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))/(b\*g\*x+a\*g)^5, x)

[Out] -1/4/b/(b\*x+a)^4/g^5\*A-1/4/b/g^5\*B/(b\*x+a)^4\*ln(e\*(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)^2/b^2)+1/8/b/g^5\*B\*a^4\*d^4/(a\*d-b\*c)^4/(b\*x+a)^4-1/2/g^5\*B\*a^3\*d^3/(a\*d-b\*c)^4/(b\*x+a)^4\*c+3/4\*b/g^5\*B\*a^2\*d^2/(a\*d-b\*c)^4/(b\*x+a)^4\*c^2-1/2\*b^2/g^5\*B\*a\*d/(a\*d-b\*c)^4/(b\*x+a)^4\*c^3+1/6/b/g^5\*B\*a^3\*d^4/(a\*d-b\*c)^4/(b\*x+a)^3-1/2/g^5\*B\*a^2\*d^3/(a\*d-b\*c)^4/(b\*x+a)^3\*c+1/2\*b/g^5\*B\*a\*d^2/(a\*d-b\*c)^4/(b\*x+a)^3\*c^2+1/4/b/g^5\*B\*a^2\*d^4/(a\*d-b\*c)^4/(b\*x+a)^2-1/2/g^5\*B\*a\*d^3/(a\*d-b\*c)^4/(b\*x+a)^2\*c+1/2/b/g^5\*B\*a\*d^4/(a\*d-b\*c)^4/(b\*x+a)+1/2/b/g^5\*B\*a\*d^5/(a\*d-b\*c)^5\*ln(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)+1/8\*b^3/g^5\*B\*c^4/(a\*d-b\*c)^4/(b\*x+a)^4-1/6\*b^2/g^5\*B\*c^3/(a\*d-b\*c)^4/(b\*x+a)^3\*d+1/4\*b/g^5\*B\*c^2/(a\*d-b\*c)^4/(b\*x+a)^2\*d^2-1/2/g^5\*B\*c/(a\*d-b\*c)^4/(b\*x+a)\*d^3-1/2/g^5\*B\*c\*d^4/(a\*d-b\*c)^5\*ln(1/(b\*x+a)\*a\*d-b\*c/(b\*x+a)-d)

**Maxima [B]** time = 1.3591, size = 944, normalized size = 4.54

$$-\frac{1}{24} B \left( \frac{12b^3d^3x^3 - 3b^3c^3 + 13ab^2c^2d - 23a^2bcd^2 + 2(b^8c^3 - 3ab^7c^2d + 3a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4(ab^7c^3 - 3a^2b^6c^2d + 3a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6(a^2b^6c^3 - 3a^3b^5cd^2 + 3a^4b^4d^3)g^5x^2 + 6(a^2b^6c^3 - 3a^3b^5cd^2 + 3a^4b^4d^3)g^5x + 6(a^2b^6c^3 - 3a^3b^5cd^2 + 3a^4b^4d^3)g^5}{(b^8c^3 - 3ab^7c^2d + 3a^2b^6cd^2 - a^3b^5d^3)g^5x^4 + 4(ab^7c^3 - 3a^2b^6c^2d + 3a^3b^5cd^2 - a^4b^4d^3)g^5x^3 + 6(a^2b^6c^3 - 3a^3b^5cd^2 + 3a^4b^4d^3)g^5x^2 + 6(a^2b^6c^3 - 3a^3b^5cd^2 + 3a^4b^4d^3)g^5x + 6(a^2b^6c^3 - 3a^3b^5cd^2 + 3a^4b^4d^3)g^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))/(b\*g\*x+a\*g)^5,x, algorithm="maxima")

[Out] 
$$-1/24*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 6*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 1/4*A/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)$$

**Fricas [B]** time = 1.09048, size = 1328, normalized size = 6.38

$$\frac{3(2A - B)b^4c^4 - 8(3A - 2B)ab^3c^3d + 36(A - B)a^2b^2c^2d^2 - 24(A - 2B)a^3bcd^3 + (6A - 25B)a^4d^4 + 12(Bb^4cd^3 - Ba^4cd^3)}{24((b^9c^4 - 4ab^8c^3d + 6a^2b^7c^2d^2 - 4a^3b^6cd^3 + a^4b^5d^4)g^5x^4 + 4(ab^8c^4 - 4a^2b^7c^3d + 6a^3b^6c^2d^2 - 4a^4b^5cd^3 + a^5b^4d^4)g^5x^3 + 6(a^2b^7c^4 - 4a^3b^6c^3d + 6a^4b^5c^2d^2 - 4a^5b^4c*d^3 + a^6b^3d^4)g^5x^2 + 4(a^3b^6c^4 - 4a^4b^5c^3d + 6a^5b^4c^2d^2 - 4a^6b^3c*d^3 + a^7b^2d^4)g^5x + (a^4b^5c^4 - 4a^5b^4c^3d + 6a^6b^3c^2d^2 - 4a^7b^2c*d^3 + a^8b*d^4)g^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))/(b\*g\*x+a\*g)^5,x, algorithm="fricas")

[Out] 
$$-1/24*(3*(2*A - B)*b^4*c^4 - 8*(3*A - 2*B)*a*b^3*c^3*d + 36*(A - B)*a^2*b^2*c^2*d^2 - 24*(A - 2*B)*a^3*b*c*d^3 + (6*A - 25*B)*a^4*d^4 + 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*x^2 + 4*(B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*x - 6*(B*b^4*d^4*x^4 + 4*B*a*b^3*d^4*x^3 + 6*B*a^2*b^2*d^4*x^2 + 4*B*a^3*b*d^4*x - B*b^4*c^4 + 4*B*a*b^3*c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*b*c*d^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^9*c^4 - 4*a*b^8*c^3*d + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5)$$

**Sympy [B]** time = 8.01469, size = 947, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2))/(b\*g\*x+a\*g)\*\*5,x)

```
[Out] -B*log(e*(c + d*x)**2/(a + b*x)**2)/(4*a**4*b*g**5 + 16*a**3*b**2*g**5*x +
24*a**2*b**3*g**5*x**2 + 16*a*b**4*g**5*x**3 + 4*b**5*g**5*x**4) + B*d**4*log(x + (-B*a**5*d**9/(a*d - b*c)**4 + 5*B*a**4*b*c*d**8/(a*d - b*c)**4 - 10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 + 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4 - 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 + B*b**5*c**5*d**4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) - B*d**4*log(x + (B*a**5*d**9/(a*d - b*c)**4 - 5*B*a**4*b*c*d**8/(a*d - b*c)**4 + 10*B*a**3*b**2*c**2*d**7/(a*d - b*c)**4 - 10*B*a**2*b**3*c**3*d**6/(a*d - b*c)**4 + 5*B*a*b**4*c**4*d**5/(a*d - b*c)**4 + B*a*d**5 - B*b**5*c**5*d**4/(a*d - b*c)**4 + B*b*c*d**4)/(2*B*b*d**5))/(2*b*g**5*(a*d - b*c)**4) + (-6*A*a**3*d**3 + 18*A*a**2*b*c*d**2 - 18*A*a*b**2*c**2*d + 6*A*b**3*c**3 + 25*B*a**3*d**3 - 23*B*a**2*b*c*d**2 + 13*B*a*b**2*c**2*d - 3*B*b**3*c**3 + 12*B*b**3*d**3*x**3 + x**2*(42*B*a*b**2*d**3 - 6*B*b**3*c*d**2) + x*(52*B*a**2*b*d**3 - 20*B*a*b**2*c*d**2 + 4*B*b**3*c**2*d))/(24*a**7*b*d**3*g**5 - 72*a**6*b**2*c*d**2*g**5 + 72*a**5*b**3*c**2*d*g**5 - 24*a**4*b**4*c**3*g**5 + x**4*(24*a**3*b**5*d**3*g**5 - 72*a**2*b**6*c*d**2*g**5 + 72*a*b**7*c**2*d*g**5 - 24*b**8*c**3*g**5) + x**3*(96*a**4*b**4*d**3*g**5 - 288*a**3*b**5*c*d**2*g**5 + 288*a**2*b**6*c**2*d*g**5 - 96*a*b**7*c**3*g**5) + x**2*(144*a**5*b**3*d**3*g**5 - 432*a**4*b**4*c*d**2*g**5 + 432*a**3*b**5*c**2*d*g**5 - 144*a**2*b**6*c**3*g**5) + x*(96*a**6*b**2*d**3*g**5 - 288*a**5*b**3*c*d**2*g**5 + 288*a**4*b**4*c**2*d*g**5 - 96*a**3*b**5*c**3*g**5))
```

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**Giac [B]** time = 1.39681, size = 562, normalized size = 2.7

$$\frac{Bd^4 \log\left(-\frac{bcg}{bgx+ag} + \frac{adg}{bgx+ag} - d\right)}{2(b^5c^4g^5 - 4ab^4c^3dg^5 + 6a^2b^3c^2d^2g^5 - 4a^3b^2cd^3g^5 + a^4bd^4g^5)} - \frac{Bd^3}{2(b^3c^3g^3 - 3ab^2c^2dg^3 + 3a^2bcd^2g^3 - a^3d^3g^3)}(bgx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))/(b*g*x+a*g)^5,x, algorithm="giac")
```

```
[Out] 1/2*B*d^4*log(-b*c*g/(b*g*x + a*g) + a*d*g/(b*g*x + a*g) - d)/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 + a^4*b*d^4*g^5) - 1/2*B*d^3/((b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*(b*g*x + a*g)*b*g) + 1/4*B*d^2/((b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(b*g*x + a*g)^2*b*g^2) - 1/4*B*log((b^2*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + a*g)^2 + 2*b*c*d*g/(b*g*x + a*g) - 2*a*d^2*g/(b*g*x + a*g) + d^2)/b^2)/((b*g*x + a*g)^4*b*g) - 1/6*B*d/((b*g*x + a*g)^3*(b*c - a*d)*b*g^2) - 1/8*(2*A*b^3*g^3 + B*b^3*g^3)/((b*g*x + a*g)^4*b^4*g^4)
```

$$3.210 \quad \int (ag + bgx)^4 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=515

$$\frac{8B^2g^4(bc-ad)^5 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{5bd^5} - \frac{4Bg^4(bc-ad)^5 \log\left(1 - \frac{d(a+bx)}{b(c+dx)}\right) \left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)}{5bd^5} - \frac{4Bg^4(c+dx)(bc-ad)^4}{5d^5}$$

[Out]  $(26B^2(b^2c - a^2d)^4g^4x)/(15d^4) - (7B^2(b^2c - a^2d)^3g^4(a + bx)^2)/(15b^2d^3) + (2B^2(b^2c - a^2d)^2g^4(a + bx)^3)/(15b^2d^2) - (10B^2(b^2c - a^2d)^5g^4 \text{Log}[a + bx])/(3b^2d^5) - (26B^2(b^2c - a^2d)^5g^4 \text{Log}[(c + dx)/(a + bx)])/(15b^2d^5) + (2B(b^2c - a^2d)^3g^4(a + bx)^2(A + B \text{Log}[(e(c + dx)^2)/(a + bx)^2]))/(5b^2d^3) - (4B(b^2c - a^2d)^2g^4(a + bx)^3(A + B \text{Log}[(e(c + dx)^2)/(a + bx)^2]))/(15b^2d^2) + (B(b^2c - a^2d)g^4(a + bx)^4(A + B \text{Log}[(e(c + dx)^2)/(a + bx)^2]))/(5bd) - (4B(b^2c - a^2d)^4g^4(c + dx)(A + B \text{Log}[(e(c + dx)^2)/(a + bx)^2]))/(5d^5) + (g^4(a + bx)^5(A + B \text{Log}[(e(c + dx)^2)/(a + bx)^2])^2)/(5b) - (4B(b^2c - a^2d)^5g^4(A + B \text{Log}[(e(c + dx)^2)/(a + bx)^2]) \text{Log}[1 - (d(a + bx))/(b(c + dx))])/(5b^2d^5) + (8B^2(b^2c - a^2d)^5g^4 \text{PolyLog}[2, (d(a + bx))/(b(c + dx))])/(5b^2d^5)$

**Rubi [A]** time = 0.864275, antiderivative size = 569, normalized size of antiderivative = 1.1, number of steps used = 28, number of rules used = 13, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{8B^2g^4(bc-ad)^5 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{5bd^5} + \frac{4Bg^4(bc-ad)^5 \log(c+dx) \left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)}{5bd^5} + \frac{2Bg^4(a+bx)^2(bc-ad)^3 \left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)}{5bd^3}$$

Antiderivative was successfully verified.

[In] Int[(a\*g + b\*g\*x)^4\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2,x]

[Out]  $(-4AB(b^2c - a^2d)^4g^4x)/(5d^4) + (26B^2(b^2c - a^2d)^4g^4x)/(15d^4) - (7B^2(b^2c - a^2d)^3g^4(a + bx)^2)/(15b^2d^3) + (2B^2(b^2c - a^2d)^2g^4(a + bx)^3)/(15b^2d^2) - (10B^2(b^2c - a^2d)^5g^4 \text{Log}[c + dx])/(3b^2d^5) + (8B^2(b^2c - a^2d)^5g^4 \text{Log}[-((d(a + bx))/(b^2c - a^2d))] \text{Log}[c + dx])/(5b^2d^5) - (4B^2(b^2c - a^2d)^5g^4 \text{Log}[c + dx]^2)/(5b^2d^5) - (4B^2(b^2c - a^2d)^4g^4(a + bx) \text{Log}[(e(c + dx)^2)/(a + bx)^2])/(5b^2d^4) + (2B(b^2c - a^2d)^3g^4(a + bx)^2(A + B \text{Log}[(e(c + dx)^2)/(a + bx)^2]))/(5b^2d^3) - (4B(b^2c - a^2d)^2g^4(a + bx)^3(A + B \text{Log}[(e(c + dx)^2)/(a + bx)^2]))/(15b^2d^2) + (B(b^2c - a^2d)g^4(a + bx)^4(A + B \text{Log}[(e(c + dx)^2)/(a + bx)^2]))/(5bd) + (4B(b^2c - a^2d)^5g^4 \text{Log}[c + dx](A + B \text{Log}[(e(c + dx)^2)/(a + bx)^2]))/(5b^2d^5) + (g^4(a + bx)^5(A + B \text{Log}[(e(c + dx)^2)/(a + bx)^2])^2)/(5b) + (8B^2(b^2c - a^2d)^5g^4 \text{PolyLog}[2, (b(c + dx))/(b^2c - a^2d)])/(5b^2d^5)$

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFX^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^n\_\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^n\_/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]

Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^p\_\*(RFX\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFX, x] && IntegerQ[p]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*(f\_. + (g\_.)\*(x\_)^(q\_.))/(x\_), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
 \int (ag + bgx)^4 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2 dx &= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2}{5b} - \frac{(2B) \int \frac{2(bc-ad)g^5(a+bx)^4 \left( -A - B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{c+dx}}{5bg} \\
 &= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2}{5b} - \frac{(4B(bc-ad)g^4) \int \frac{(a+bx)^4 \left( -A - B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{c+dx}}{5b} \\
 &= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2}{5b} - \frac{(4B(bc-ad)g^4) \int \left( -\frac{b(bc-ad)^3 \left( -A - B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{d^4} \right)}{5b} \\
 &= \frac{g^4(a+bx)^5 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2}{5b} - \frac{(4B(bc-ad)g^4) \int (a+bx)^3 \left( -A - B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{5d} \\
 &= -\frac{4AB(bc-ad)^4 g^4 x}{5d^4} + \frac{2B(bc-ad)^3 g^4 (a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{5bd^3} - \frac{4B^2(bc-ad)^4 g^4 (a+bx) \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right)}{5bd^4} + \frac{2B(bc-ad)^4 g^4 (a+bx)^2}{5bd^4} \\
 &= -\frac{4AB(bc-ad)^4 g^4 x}{5d^4} + \frac{26B^2(bc-ad)^4 g^4 x}{15d^4} - \frac{7B^2(bc-ad)^3 g^4 (a+bx)^2}{15bd^3} + \frac{2B^2(bc-ad)^4 g^4 (a+bx)^2}{15bd^3} \\
 &= -\frac{4AB(bc-ad)^4 g^4 x}{5d^4} + \frac{26B^2(bc-ad)^4 g^4 x}{15d^4} - \frac{7B^2(bc-ad)^3 g^4 (a+bx)^2}{15bd^3} + \frac{2B^2(bc-ad)^4 g^4 (a+bx)^2}{15bd^3} \\
 &= -\frac{4AB(bc-ad)^4 g^4 x}{5d^4} + \frac{26B^2(bc-ad)^4 g^4 x}{15d^4} - \frac{7B^2(bc-ad)^3 g^4 (a+bx)^2}{15bd^3} + \frac{2B^2(bc-ad)^4 g^4 (a+bx)^2}{15bd^3}
 \end{aligned}$$

**Mathematica [A]** time = 0.481292, size = 524, normalized size = 1.02

$$g^4 \left( (a + bx)^5 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)^2 - \frac{B(bc-ad) \left( -12B(bc-ad)^4 \left( 2 \operatorname{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) - 6d^2(a+bx)^2(bc-d)}{\right.$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^4\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2,x]

[Out] (g^4\*((a + b\*x)^5\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2 - (B\*(b\*c - a\*d)\*(12\*A\*b\*d\*(b\*c - a\*d)^3\*x + 24\*B\*(b\*c - a\*d)^4\*Log[c + d\*x] - 4\*B\*(b\*c - a\*d)^2\*(2\*b\*d\*(b\*c - a\*d)\*x - d^2\*(a + b\*x)^2 - 2\*(b\*c - a\*d)^2\*Log[c + d\*x]) - B\*(b\*c - a\*d)\*(6\*b\*d\*(b\*c - a\*d)^2\*x + 3\*d^2\*(-(b\*c) + a\*d)\*(a + b\*x)^2 + 2\*d^3\*(a + b\*x)^3 - 6\*(b\*c - a\*d)^3\*Log[c + d\*x]) - 12\*B\*(b\*c - a\*d)^3\*(b\*d\*x + (-b\*c) + a\*d)\*Log[c + d\*x]) + 12\*B\*d\*(b\*c - a\*d)^3\*(a + b\*x)\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2] - 6\*d^2\*(b\*c - a\*d)^2\*(a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]) + 4\*d^3\*(b\*c - a\*d)\*(a + b\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]) - 3\*d^4\*(a + b\*x)^4\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]) - 12\*(b\*c - a\*d)^4\*Log[c + d\*x]\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]) - 12\*B\*(b\*c - a\*d)^4\*((2\*Log[(d\*(a + b\*x))/(-b\*c) + a\*d]) - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/(3\*d^5))/(5\*b)

**Maple [F]** time = 1.913, size = 0, normalized size = 0.

$$\int (bgx + ag)^4 \left( A + B \ln \left( \frac{e(dx + c)^2}{(bx + a)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^4\*(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

[Out] int((b\*g\*x+a\*g)^4\*(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

**Maxima [B]** time = 2.02881, size = 3591, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="maxima")

[Out] 1/5\*A^2\*b^4\*g^4\*x^5 + A^2\*a\*b^3\*g^4\*x^4 + 2\*A^2\*a^2\*b^2\*g^4\*x^3 + 2\*A^2\*a^3\*b\*g^4\*x^2 + 2\*(x\*log(d^2\*e\*x^2/(b^2\*x^2 + 2\*a\*b\*x + a^2) + 2\*c\*d\*e\*x/(b^2\*x^2 + 2\*a\*b\*x + a^2) + c^2\*e/(b^2\*x^2 + 2\*a\*b\*x + a^2)) - 2\*a\*log(b\*x + a)/b + 2\*c\*log(d\*x + c)/d)\*A\*B\*a^4\*g^4 + 4\*(x^2\*log(d^2\*e\*x^2/(b^2\*x^2 + 2\*a\*b\*x + a^2) + 2\*c\*d\*e\*x/(b^2\*x^2 + 2\*a\*b\*x + a^2) + c^2\*e/(b^2\*x^2 + 2\*a\*b\*x + a^2)) + 2\*a^2\*log(b\*x + a)/b^2 - 2\*c^2\*log(d\*x + c)/d^2 + 2\*(b\*c - a\*d)\*x/(b\*d))\*A\*B\*a^3\*b\*g^4 + 4\*(x^3\*log(d^2\*e\*x^2/(b^2\*x^2 + 2\*a\*b\*x + a^2) + 2\*c\*d\*e\*x/(b^2\*x^2 + 2\*a\*b\*x + a^2) + c^2\*e/(b^2\*x^2 + 2\*a\*b\*x + a^2)) - 2\*a^3\*log(b\*x + a)/b^3 + 2\*c^3\*log(d\*x + c)/d^3 + ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*

$$\begin{aligned}
& (b^2c^2 - a^2d^2)x/(b^2d^2)) * A * B * a^2 * b^2 * g^4 + 2/3 * (3x^4 * \log(d^2 * e * x^2 / (b^2 * x^2 + 2 * a * b * x + a^2)) + 2 * c * d * e * x / (b^2 * x^2 + 2 * a * b * x + a^2) + c^2 * e / (b^2 * x^2 + 2 * a * b * x + a^2)) + 6 * a^4 * \log(b * x + a) / b^4 - 6 * c^4 * \log(d * x + c) / d^4 \\
& + (2 * (b^3 * c * d^2 - a * b^2 * d^3) * x^3 - 3 * (b^3 * c^2 * d - a^2 * b * d^3) * x^2 + 6 * (b^3 * c^3 - a^3 * d^3) * x) / (b^3 * d^3) * A * B * a * b^3 * g^4 + 1/15 * (6 * x^5 * \log(d^2 * e * x^2 / (b^2 * x^2 + 2 * a * b * x + a^2)) + 2 * c * d * e * x / (b^2 * x^2 + 2 * a * b * x + a^2) + c^2 * e / (b^2 * x^2 + 2 * a * b * x + a^2)) - 12 * a^5 * \log(b * x + a) / b^5 + 12 * c^5 * \log(d * x + c) / d^5 + (3 * (b^4 * c * d^3 - a * b^3 * d^4) * x^4 - 4 * (b^4 * c^2 * d^2 - a^2 * b^2 * d^4) * x^3 + 6 * (b^4 * c^3 * d - a^3 * b * d^4) * x^2 - 12 * (b^4 * c^4 - a^4 * d^4) * x) / (b^4 * d^4) * A * B * b^4 * g^4 + \\
& A^2 * a^4 * g^4 * x + 2/15 * ((6 * g^4 * \log(e) - 25 * g^4) * b^4 * c^5 - (30 * g^4 * \log(e) - 113 * g^4) * a * b^3 * c^4 * d + 4 * (15 * g^4 * \log(e) - 49 * g^4) * a^2 * b^2 * c^3 * d^2 - 12 * (5 * g^4 * \log(e) - 13 * g^4) * a^3 * b * c^2 * d^3 + 6 * (5 * g^4 * \log(e) - 8 * g^4) * a^4 * c * d^4) * B^2 * \log(d * x + c) / d^5 - 8/5 * (b^5 * c^5 * g^4 - 5 * a * b^4 * c^4 * d * g^4 + 10 * a^2 * b^3 * c^3 * d^2 * g^4 - 10 * a^3 * b^2 * c^2 * d^3 * g^4 + 5 * a^4 * b * c * d^4 * g^4 - a^5 * d^5 * g^4) * (\log(b * x + a) * \log((b * d * x + a * d) / (b * c - a * d)) + 1) + \operatorname{dilog}(-(b * d * x + a * d) / (b * c - a * d)) * B^2 / (b * d^5) + 1/15 * (3 * B^2 * b^5 * d^5 * g^4 * x^5 * \log(e)^2 + 3 * (b^5 * c * d^4 * g^4 * \log(e) + (5 * g^4 * \log(e)^2 - g^4 * \log(e)) * a * b^4 * d^5) * B^2 * x^4 - 2 * ((2 * g^4 * \log(e) - g^4) * b^5 * c^2 * d^3 - 2 * (5 * g^4 * \log(e) - g^4) * a * b^4 * c * d^4 - (15 * g^4 * \log(e)^2 - 8 * g^4 * \log(e) + g^4) * a^2 * b^3 * d^5) * B^2 * x^3 + ((6 * g^4 * \log(e) - 7 * g^4) * b^5 * c^3 * d^2 - 3 * (10 * g^4 * \log(e) - 9 * g^4) * a * b^4 * c^2 * d^3 + 3 * (20 * g^4 * \log(e) - 11 * g^4) * a^2 * b^3 * c * d^4 + (30 * g^4 * \log(e)^2 - 36 * g^4 * \log(e) + 13 * g^4) * a^3 * b^2 * d^5) * B^2 * x^2 - (2 * (6 * g^4 * \log(e) - 13 * g^4) * b^5 * c^4 * d - 2 * (30 * g^4 * \log(e) - 59 * g^4) * a * b^4 * c^3 * d^2 + 12 * (10 * g^4 * \log(e) - 17 * g^4) * a^2 * b^3 * c^2 * d^3 - 2 * (60 * g^4 * \log(e) - 79 * g^4) * a^3 * b^2 * c * d^4 - (15 * g^4 * \log(e)^2 - 48 * g^4 * \log(e) + 46 * g^4) * a^4 * b * d^5) * B^2 * x + 12 * (B^2 * b^5 * d^5 * g^4 * x^5 + 5 * B^2 * a * b^4 * d^5 * g^4 * x^4 + 10 * B^2 * a^2 * b^3 * d^5 * g^4 * x^3 + 10 * B^2 * a^3 * b^2 * d^5 * g^4 * x^2 + 5 * B^2 * a^4 * b * d^5 * g^4 * x + B^2 * a^5 * d^5 * g^4) * \log(b * x + a)^2 + 12 * (B^2 * b^5 * d^5 * g^4 * x^5 + 5 * B^2 * a * b^4 * d^5 * g^4 * x^4 + 10 * B^2 * a^2 * b^3 * d^5 * g^4 * x^3 + 10 * B^2 * a^3 * b^2 * d^5 * g^4 * x^2 + 5 * B^2 * a^4 * b * d^5 * g^4 * x + (b^5 * c^5 * g^4 - 5 * a * b^4 * c^4 * d * g^4 + 10 * a^2 * b^3 * c^3 * d^2 * g^4 - 10 * a^3 * b^2 * c^2 * d^3 * g^4 + 5 * a^4 * b * c * d^4 * g^4) * B^2) * \log(d * x + c)^2 - 2 * (6 * B^2 * b^5 * d^5 * g^4 * x^5 * \log(e) + 3 * (b^5 * c * d^4 * g^4 + (10 * g^4 * \log(e) - g^4) * a * b^4 * d^5) * B^2 * x^4 - 4 * (b^5 * c^2 * d^3 * g^4 - 5 * a * b^4 * c * d^4 * g^4 - (15 * g^4 * \log(e) - 4 * g^4) * a^2 * b^3 * d^5) * B^2 * x^3 + 6 * (b^5 * c^3 * d^2 * g^4 - 5 * a * b^4 * c^2 * d^3 * g^4 + 10 * a^2 * b^3 * c * d^4 * g^4 + 2 * (5 * g^4 * \log(e) - 3 * g^4) * a^3 * b^2 * d^5) * B^2 * x^2 - 6 * (2 * b^5 * c^4 * d * g^4 - 10 * a * b^4 * c^3 * d^2 * g^4 + 20 * a^2 * b^3 * c^2 * d^3 * g^4 - 20 * a^3 * b^2 * c * d^4 * g^4 - (5 * g^4 * \log(e) - 8 * g^4) * a^4 * b * d^5) * B^2 * x - (12 * a * b^4 * c^4 * d * g^4 - 54 * a^2 * b^3 * c^3 * d^2 * g^4 + 94 * a^3 * b^2 * c^2 * d^3 * g^4 - 77 * a^4 * b * c * d^4 * g^4 - (6 * g^4 * \log(e) - 25 * g^4) * a^5 * d^5) * B^2) * \log(b * x + a) + 2 * (6 * B^2 * b^5 * d^5 * g^4 * x^5 * \log(e) + 3 * (b^5 * c * d^4 * g^4 + (10 * g^4 * \log(e) - g^4) * a * b^4 * d^5) * B^2 * x^4 - 4 * (b^5 * c^2 * d^3 * g^4 - 5 * a * b^4 * c * d^4 * g^4 - (15 * g^4 * \log(e) - 4 * g^4) * a^2 * b^3 * d^5) * B^2 * x^3 + 6 * (b^5 * c^3 * d^2 * g^4 - 5 * a * b^4 * c^2 * d^3 * g^4 + 10 * a^2 * b^3 * c * d^4 * g^4 + 2 * (5 * g^4 * \log(e) - 3 * g^4) * a^3 * b^2 * d^5) * B^2 * x^2 - 6 * (2 * b^5 * c^4 * d * g^4 - 10 * a * b^4 * c^3 * d^2 * g^4 + 20 * a^2 * b^3 * c^2 * d^3 * g^4 - 20 * a^3 * b^2 * c * d^4 * g^4 - (5 * g^4 * \log(e) - 8 * g^4) * a^4 * b * d^5) * B^2 * x - 12 * (B^2 * b^5 * d^5 * g^4 * x^5 + 5 * B^2 * a * b^4 * d^5 * g^4 * x^4 + 10 * B^2 * a^2 * b^3 * d^5 * g^4 * x^3 + 10 * B^2 * a^3 * b^2 * d^5 * g^4 * x^2 + 5 * B^2 * a^4 * b * d^5 * g^4 * x + B^2 * a^5 * d^5 * g^4) * \log(b * x + a)) * \log(d * x + c)) / (b * d^5)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( A^2 b^4 g^4 x^4 + 4 A^2 a b^3 g^4 x^3 + 6 A^2 a^2 b^2 g^4 x^2 + 4 A^2 a^3 b g^4 x + A^2 a^4 g^4 + (B^2 b^4 g^4 x^4 + 4 B^2 a b^3 g^4 x^3 + 6 B^2 a^2 b^2 g^4 x^2 + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^4\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="fricas")



```
[Out] integral(A^2*b^4*g^4*x^4 + 4*A^2*a*b^3*g^4*x^3 + 6*A^2*a^2*b^2*g^4*x^2 + 4*
A^2*a^3*b*g^4*x + A^2*a^4*g^4 + (B^2*b^4*g^4*x^4 + 4*B^2*a*b^3*g^4*x^3 + 6*
B^2*a^2*b^2*g^4*x^2 + 4*B^2*a^3*b*g^4*x + B^2*a^4*g^4)*log((d^2*e*x^2 + 2*c
*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*(A*B*b^4*g^4*x^4 + 4*A*B*a
*b^3*g^4*x^3 + 6*A*B*a^2*b^2*g^4*x^2 + 4*A*B*a^3*b*g^4*x + A*B*a^4*g^4)*log
((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**4*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^4 \left( B \log \left( \frac{(dx + c)^2 e}{(bx + a)^2} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^4*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi
ac")
```

```
[Out] integrate((b*g*x + a*g)^4*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

$$3.211 \quad \int (ag + bgx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=422

$$\frac{2B^2g^3(bc-ad)^4 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd^4} + \frac{Bg^3(bc-ad)^4 \log\left(1 - \frac{d(a+bx)}{b(c+dx)}\right) \left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)}{bd^4} + \frac{Bg^3(c+dx)(bc-ad)^3}{d^4}$$

[Out]  $(-5B^2(b*c - a*d)^3g^3x)/(3d^3) + (B^2(b*c - a*d)^2g^3(a + b*x)^2)/(3*b*d^2) + (11B^2(b*c - a*d)^4g^3*Log[a + b*x])/(3*b*d^4) + (5B^2(b*c - a*d)^4g^3*Log[(c + d*x)/(a + b*x)])/(3*b*d^4) - (B*(b*c - a*d)^2g^3*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2]/(a + b*x)^2]))/(2*b*d^2) + (B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(c + d*x)^2]/(a + b*x)^2]))/(3*b*d) + (B*(b*c - a*d)^3g^3*(c + d*x)*(A + B*Log[(e*(c + d*x)^2]/(a + b*x)^2]))/d^4 + (g^3*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2]/(a + b*x)^2])^2)/(4*b) + (B*(b*c - a*d)^4g^3*(A + B*Log[(e*(c + d*x)^2]/(a + b*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(b*d^4) - (2B^2(b*c - a*d)^4g^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^4)$

**Rubi [A]** time = 0.740716, antiderivative size = 469, normalized size of antiderivative = 1.11, number of steps used = 24, number of rules used = 13, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{2B^2g^3(bc-ad)^4 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bd^4} - \frac{Bg^3(bc-ad)^4 \log(c+dx) \left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)}{bd^4} - \frac{Bg^3(a+bx)^2(bc-ad)^2 \left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)}{2bd^2}$$

Antiderivative was successfully verified.

[In] Int[(a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2])^2,x]

[Out]  $(A*B*(b*c - a*d)^3g^3x)/d^3 - (5B^2(b*c - a*d)^3g^3x)/(3d^3) + (B^2(b*c - a*d)^2g^3(a + b*x)^2)/(3*b*d^2) + (11B^2(b*c - a*d)^4g^3*Log[c + d*x])/(3*b*d^4) - (2B^2(b*c - a*d)^4g^3*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d^4) + (B^2(b*c - a*d)^4g^3*Log[c + d*x]^2)/(b*d^4) + (B^2(b*c - a*d)^3g^3*(a + b*x)*Log[(e*(c + d*x)^2]/(a + b*x)^2])/(b*d^3) - (B*(b*c - a*d)^2g^3*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2]/(a + b*x)^2]))/(2*b*d^2) + (B*(b*c - a*d)*g^3*(a + b*x)^3*(A + B*Log[(e*(c + d*x)^2]/(a + b*x)^2]))/(3*b*d) - (B*(b*c - a*d)^4g^3*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2]/(a + b*x)^2]))/(b*d^4) + (g^3*(a + b*x)^4*(A + B*Log[(e*(c + d*x)^2]/(a + b*x)^2])^2)/(4*b) - (2B^2(b*c - a*d)^4g^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d])/(b*d^4)$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 43

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7\*m + 4\*n + 4, 0]) || LtQ[9\*m + 5\*(n + 1), 0] || GtQ[m + n + 2, 0])

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(RFX\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFX, x] && IntegerQ[p]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_)^(q\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rubi steps

$$\begin{aligned}
 \int (ag + bgx)^3 \left( A + B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)^2 dx &= \frac{g^3(a + bx)^4 \left( A + B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)^2}{4b} - \frac{B \int \frac{2(bc - ad)g^4(a + bx)^3 \left( -A - B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)}{c + dx}}{2bg} \\
 &= \frac{g^3(a + bx)^4 \left( A + B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)^2}{4b} - \frac{(B(bc - ad)g^3) \int \frac{(a + bx)^3 \left( -A - B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)}{c + dx}}{b} \\
 &= \frac{g^3(a + bx)^4 \left( A + B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)^2}{4b} - \frac{(B(bc - ad)g^3) \int \left( \frac{b(bc - ad)^2 \left( -A - B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)}{d^3} \right)}{b} \\
 &= \frac{g^3(a + bx)^4 \left( A + B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)^2}{4b} - \frac{(B(bc - ad)g^3) \int (a + bx)^2 \left( -A - B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)}{d} \\
 &= \frac{AB(bc - ad)^3 g^3 x}{d^3} - \frac{B(bc - ad)^2 g^3 (a + bx)^2 \left( A + B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)}{2bd^2} + \frac{B(bc - ad)^2 g^3 (a + bx)^2}{2bd^2} \\
 &= \frac{AB(bc - ad)^3 g^3 x}{d^3} + \frac{B^2(bc - ad)^3 g^3 (a + bx) \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right)}{bd^3} - \frac{B(bc - ad)^2 g^3 (a + bx)^2}{2bd^2} \\
 &= \frac{AB(bc - ad)^3 g^3 x}{d^3} + \frac{2B^2(bc - ad)^4 g^3 \log(c + dx)}{bd^4} + \frac{B^2(bc - ad)^3 g^3 (a + bx)^2}{bd^3} \\
 &= \frac{AB(bc - ad)^3 g^3 x}{d^3} - \frac{5B^2(bc - ad)^3 g^3 x}{3d^3} + \frac{B^2(bc - ad)^2 g^3 (a + bx)^2}{3bd^2} + \frac{11B^2(bc - ad)^3 g^3 (a + bx)^2}{3bd^2} \\
 &= \frac{AB(bc - ad)^3 g^3 x}{d^3} - \frac{5B^2(bc - ad)^3 g^3 x}{3d^3} + \frac{B^2(bc - ad)^2 g^3 (a + bx)^2}{3bd^2} + \frac{11B^2(bc - ad)^3 g^3 (a + bx)^2}{3bd^2} \\
 &= \frac{AB(bc - ad)^3 g^3 x}{d^3} - \frac{5B^2(bc - ad)^3 g^3 x}{3d^3} + \frac{B^2(bc - ad)^2 g^3 (a + bx)^2}{3bd^2} + \frac{11B^2(bc - ad)^3 g^3 (a + bx)^2}{3bd^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.324396, size = 402, normalized size = 0.95

$$g^3 \left( \frac{2B(bc-ad) \left( -6B(bc-ad)^3 \left( 2 \operatorname{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) + 2d^3(a+bx)^3 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right) + 3d^2(a+bx)^2(ad-bc) \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)}{\dots} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2,x]

[Out] (g^3\*((a + b\*x)^4\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2 + (2\*B\*(b\*c - a\*d)\*(6\*A\*b\*d\*(b\*c - a\*d)^2\*x + 12\*B\*(b\*c - a\*d)^3\*Log[c + d\*x] - 2\*B\*(b\*c - a\*d)\*(2\*b\*d\*(b\*c - a\*d)\*x - d^2\*(a + b\*x)^2 - 2\*(b\*c - a\*d)^2\*Log[c + d\*x]) - 6\*B\*(b\*c - a\*d)^2\*(b\*d\*x + (-b\*c) + a\*d)\*Log[c + d\*x]) + 6\*B\*d\*(b\*c - a\*d)^2\*(a + b\*x)\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2] + 3\*d^2\*(-b\*c) + a\*d)\*(a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]) + 2\*d^3\*(a + b\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]) - 6\*(b\*c - a\*d)^3\*Log[c + d\*x]\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]) - 6\*B\*(b\*c - a\*d)^3\*((2\*Log[(d\*(a + b\*x))/(-b\*c) + a\*d] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/(3\*d^4))/(4\*b)

**Maple [F]** time = 1.569, size = 0, normalized size = 0.

$$\int (bgx + ag)^3 \left( A + B \ln \left( \frac{e(dx + c)^2}{(bx + a)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^3\*(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

[Out] int((b\*g\*x+a\*g)^3\*(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

**Maxima [B]** time = 1.95392, size = 2633, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="maxima")

[Out] 1/4\*A^2\*b^3\*g^3\*x^4 + A^2\*a\*b^2\*g^3\*x^3 + 3/2\*A^2\*a^2\*b\*g^3\*x^2 + 2\*(x\*log(d^2\*e\*x^2/(b^2\*x^2 + 2\*a\*b\*x + a^2) + 2\*c\*d\*e\*x/(b^2\*x^2 + 2\*a\*b\*x + a^2) + c^2\*e/(b^2\*x^2 + 2\*a\*b\*x + a^2)) - 2\*a\*log(b\*x + a)/b + 2\*c\*log(d\*x + c)/d)\*A\*B\*a^3\*g^3 + 3\*(x^2\*log(d^2\*e\*x^2/(b^2\*x^2 + 2\*a\*b\*x + a^2) + 2\*c\*d\*e\*x/(b^2\*x^2 + 2\*a\*b\*x + a^2) + c^2\*e/(b^2\*x^2 + 2\*a\*b\*x + a^2)) + 2\*a^2\*log(b\*x + a)/b^2 - 2\*c^2\*log(d\*x + c)/d^2 + 2\*(b\*c - a\*d)\*x/(b\*d))\*A\*B\*a^2\*b\*g^3 + 2\*(x^3\*log(d^2\*e\*x^2/(b^2\*x^2 + 2\*a\*b\*x + a^2) + 2\*c\*d\*e\*x/(b^2\*x^2 + 2\*a\*b\*x + a^2) + c^2\*e/(b^2\*x^2 + 2\*a\*b\*x + a^2)) - 2\*a^3\*log(b\*x + a)/b^3 + 2\*c^3\*log(d\*x + c)/d^3 + ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2))\*A\*B\*a\*b^2\*g^3 + 1/6\*(3\*x^4\*log(d^2\*e\*x^2/(b^2\*x^2 + 2\*a\*b\*x + a^2) + 2\*c\*d\*e\*x/(b^2\*x^2 + 2\*a\*b\*x + a^2) + c^2\*e/(b^2\*x^2 + 2\*a\*b\*x + a^2)) + 6\*a^4\*log(b\*x + a)/b^4 - 6\*c^4\*log(d\*x + c)/d^4 + (2\*(b^3\*c\*d^2 - a\*b^2

```
*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d
^3))*A*B*b^3*g^3 + A^2*a^3*g^3*x - 1/3*((3*g^3*log(e) - 11*g^3)*b^3*c^4 - 2
*(6*g^3*log(e) - 19*g^3)*a*b^2*c^3*d + 9*(2*g^3*log(e) - 5*g^3)*a^2*b*c^2*d
^2 - 6*(2*g^3*log(e) - 3*g^3)*a^3*c*d^3)*B^2*log(d*x + c)/d^4 + 2*(b^4*c^4*
g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3 + a^4*d
^4*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x +
a*d)/(b*c - a*d)))*B^2/(b*d^4) + 1/12*(3*B^2*b^4*d^4*g^3*x^4*log(e)^2 + 4*(
b^4*c*d^3*g^3*log(e) + (3*g^3*log(e)^2 - g^3*log(e))*a*b^3*d^4)*B^2*x^3 - 2
*((3*g^3*log(e) - 2*g^3)*b^4*c^2*d^2 - 4*(3*g^3*log(e) - g^3)*a*b^3*c*d^3 -
(9*g^3*log(e)^2 - 9*g^3*log(e) + 2*g^3)*a^2*b^2*d^4)*B^2*x^2 + 4*((3*g^3*1
og(e) - 5*g^3)*b^4*c^3*d - (12*g^3*log(e) - 17*g^3)*a*b^3*c^2*d^2 + (18*g^3
*log(e) - 19*g^3)*a^2*b^2*c*d^3 + (3*g^3*log(e)^2 - 9*g^3*log(e) + 7*g^3)*a
^3*b*d^4)*B^2*x + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2
*a^2*b^2*d^4*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a
)^2 + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4
*g^3*x^2 + 4*B^2*a^3*b*d^4*g^3*x - (b^4*c^4*g^3 - 4*a*b^3*c^3*d*g^3 + 6*a^2
*b^2*c^2*d^2*g^3 - 4*a^3*b*c*d^3*g^3)*B^2)*log(d*x + c)^2 - 4*(3*B^2*b^4*d^
4*g^3*x^4*log(e) + 2*(b^4*c*d^3*g^3 + (6*g^3*log(e) - g^3)*a*b^3*d^4)*B^2*x
^3 - 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3 - 3*(2*g^3*log(e) - g^3)*a^2*b^
2*d^4)*B^2*x^2 + 6*(b^4*c^3*d*g^3 - 4*a*b^3*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g
^3 + (2*g^3*log(e) - 3*g^3)*a^3*b*d^4)*B^2*x + (6*a*b^3*c^3*d*g^3 - 21*a^2*
b^2*c^2*d^2*g^3 + 26*a^3*b*c*d^3*g^3 + (3*g^3*log(e) - 11*g^3)*a^4*d^4)*B^2
)*log(b*x + a) + 4*(3*B^2*b^4*d^4*g^3*x^4*log(e) + 2*(b^4*c*d^3*g^3 + (6*g^
3*log(e) - g^3)*a*b^3*d^4)*B^2*x^3 - 3*(b^4*c^2*d^2*g^3 - 4*a*b^3*c*d^3*g^3
- 3*(2*g^3*log(e) - g^3)*a^2*b^2*d^4)*B^2*x^2 + 6*(b^4*c^3*d*g^3 - 4*a*b^3
*c^2*d^2*g^3 + 6*a^2*b^2*c*d^3*g^3 + (2*g^3*log(e) - 3*g^3)*a^3*b*d^4)*B^2*
x - 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*a*b^3*d^4*g^3*x^3 + 6*B^2*a^2*b^2*d^4*g^
3*x^2 + 4*B^2*a^3*b*d^4*g^3*x + B^2*a^4*d^4*g^3)*log(b*x + a))*log(d*x + c
)/(b*d^4)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(A^2 b^3 g^3 x^3 + 3 A^2 a b^2 g^3 x^2 + 3 A^2 a^2 b g^3 x + A^2 a^3 g^3 + (B^2 b^3 g^3 x^3 + 3 B^2 a b^2 g^3 x^2 + 3 B^2 a^2 b g^3 x + B^2 a^3 g^3) \log\left(\frac{d^2 e}{b^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^3*(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="fr
icas")
```

```
[Out] integral(A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*a^
3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2*a^
3*g^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2
*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*
log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**3*(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^3 \left( B \log \left( \frac{(dx + c)^2 e}{(bx + a)^2} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^3\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^3\*(B\*log((d\*x + c)^2\*e/(b\*x + a)^2) + A)^2, x)

**3.212**  $\int (ag + bgx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2 dx$

**Optimal.** Leaf size=343

$$\frac{8B^2g^2(bc - ad)^3 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3bd^3} - \frac{4Bg^2(bc - ad)^3 \log\left(1 - \frac{d(a+bx)}{b(c+dx)}\right) \left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)}{3bd^3} - \frac{4Bg^2(c + dx)(bc - ad)^2}{3d^3}$$

```
[Out] (4*B^2*(b*c - a*d)^2*g^2*x)/(3*d^2) - (4*B^2*(b*c - a*d)^3*g^2*Log[a + b*x])/(b*d^3) - (4*B^2*(b*c - a*d)^3*g^2*Log[(c + d*x)/(a + b*x)])/(3*b*d^3) + (2*B*(b*c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b*d) - (4*B*(b*c - a*d)^2*g^2*(c + d*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*d^3) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(3*b) - (4*B*(b*c - a*d)^3*g^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3) + (8*B^2*(b*c - a*d)^3*g^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b*d^3)
```

**Rubi [A]** time = 0.625311, antiderivative size = 397, normalized size of antiderivative = 1.16, number of steps used = 20, number of rules used = 13, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.382$ , Rules used = {2525, 12, 2528, 2486, 31, 43, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{8B^2g^2(bc - ad)^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3bd^3} + \frac{4Bg^2(bc - ad)^3 \log(c + dx) \left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)}{3bd^3} - \frac{4ABg^2x(bc - ad)^2}{3d^2} + \frac{2Bg^2(a + b*x)^2}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

```
[Out] (-4*A*B*(b*c - a*d)^2*g^2*x)/(3*d^2) + (4*B^2*(b*c - a*d)^2*g^2*x)/(3*d^2) - (4*B^2*(b*c - a*d)^3*g^2*Log[c + d*x])/(b*d^3) + (8*B^2*(b*c - a*d)^3*g^2*Log[-((d*(a + b*x))/(b*c - a*d))] * Log[c + d*x])/(3*b*d^3) - (4*B^2*(b*c - a*d)^3*g^2*Log[c + d*x]^2)/(3*b*d^3) - (4*B^2*(b*c - a*d)^2*g^2*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(3*b*d^2) + (2*B*(b*c - a*d)*g^2*(a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b*d) + (4*B*(b*c - a*d)^3*g^2*Log[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(3*b*d^3) + (g^2*(a + b*x)^3*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(3*b) + (8*B^2*(b*c - a*d)^3*g^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(3*b*d^3)
```

Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2528



```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*RFX^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFX, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

#### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFX^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFX_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFX, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFX, x] && IntegerQ[p]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_.))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rubi steps

$$\begin{aligned}
\int (ag + bgx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2 dx &= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2}{3b} - \frac{(2B) \int \frac{2(bc-ad)g^3(a+bx)^2 \left( -A - B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{c+dx}}{3bg} \\
&= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2}{3b} - \frac{(4B(bc-ad)g^2) \int \frac{(a+bx)^2 \left( -A - B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{c+dx}}{3b} \\
&= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2}{3b} - \frac{(4B(bc-ad)g^2) \int \left( -\frac{b(bc-ad) \left( -A - B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{d^2} \right)}{3b} \\
&= \frac{g^2(a+bx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2}{3b} - \frac{(4B(bc-ad)g^2) \int (a+bx) \left( -A - B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{3d} \\
&= -\frac{4AB(bc-ad)^2 g^2 x}{3d^2} + \frac{2B(bc-ad)g^2(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{3bd} + \frac{4B^2(bc-ad)^2 g^2 (a+bx) \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right)}{3bd^2} \\
&= -\frac{4AB(bc-ad)^2 g^2 x}{3d^2} - \frac{4B^2(bc-ad)^2 g^2 (a+bx) \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right)}{3bd^2} + \frac{2B(bc-ad)g^2(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{3bd} \\
&= -\frac{4AB(bc-ad)^2 g^2 x}{3d^2} - \frac{8B^2(bc-ad)^3 g^2 \log(c+dx)}{3bd^3} - \frac{4B^2(bc-ad)^2 g^2 (a+bx) \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right)}{3bd} \\
&= -\frac{4AB(bc-ad)^2 g^2 x}{3d^2} + \frac{4B^2(bc-ad)^2 g^2 x}{3d^2} - \frac{4B^2(bc-ad)^3 g^2 \log(c+dx)}{bd^3} \\
&= -\frac{4AB(bc-ad)^2 g^2 x}{3d^2} + \frac{4B^2(bc-ad)^2 g^2 x}{3d^2} - \frac{4B^2(bc-ad)^3 g^2 \log(c+dx)}{bd^3} + \frac{2B(bc-ad)g^2(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{3bd} \\
&= -\frac{4AB(bc-ad)^2 g^2 x}{3d^2} + \frac{4B^2(bc-ad)^2 g^2 x}{3d^2} - \frac{4B^2(bc-ad)^3 g^2 \log(c+dx)}{bd^3} + \frac{2B(bc-ad)g^2(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{3bd}
\end{aligned}$$

**Mathematica [A]** time = 0.228467, size = 298, normalized size = 0.87

$$g^2 \left( (a+bx)^3 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)^2 - \frac{2B(bc-ad) \left( -2B(bc-ad)^2 \left( 2 \operatorname{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \right) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) - d^2(a+bx)^2 \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)}{3d^2} + \frac{4B^2(bc-ad)^2 g^2 x}{3d^2} - \frac{4B^2(bc-ad)^3 g^2 \log(c+dx)}{bd^3} + \frac{2B(bc-ad)g^2(a+bx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)}{3bd} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2,x]

[Out]  $(g^2((a + b*x)^3(A + B\text{Log}[\frac{e(c + d*x)^2}{(a + b*x)^2}])^2 - (2*B*(b*c - a*d)*(2*A*b*d*(b*c - a*d)*x + 4*B*(b*c - a*d)^2*\text{Log}[c + d*x] - 2*B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*\text{Log}[c + d*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*\text{Log}[\frac{e(c + d*x)^2}{(a + b*x)^2} - d^2*(a + b*x)^2*(A + B\text{Log}[\frac{e(c + d*x)^2}{(a + b*x)^2}]) - 2*(b*c - a*d)^2*\text{Log}[c + d*x]*(A + B\text{Log}[\frac{e(c + d*x)^2}{(a + b*x)^2}]) - 2*B*(b*c - a*d)^2*((2*\text{Log}[\frac{d(a + b*x)}{-(b*c) + a*d}]) - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, \frac{b(c + d*x)}{b*c - a*d}])))/d^3)/3*b)$

**Maple [F]** time = 1.52, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( \frac{e(dx + c)^2}{(bx + a)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2\*(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

[Out] int((b\*g\*x+a\*g)^2\*(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

**Maxima [B]** time = 2.05687, size = 1800, normalized size = 5.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="maxima")

[Out]  $\frac{1}{3}A^2b^2g^2x^3 + A^2a*b*g^2x^2 + 2*(x*\log(d^2e*x^2/(b^2x^2 + 2*a*b*x + a^2)) + 2*c*d*e*x/(b^2x^2 + 2*a*b*x + a^2) + c^2e/(b^2x^2 + 2*a*b*x + a^2)) - 2*a*\log(b*x + a)/b + 2*c*\log(d*x + c)/d)*A*B*a^2g^2 + 2*(x^2*\log(d^2e*x^2/(b^2x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2x^2 + 2*a*b*x + a^2) + c^2e/(b^2x^2 + 2*a*b*x + a^2)) + 2*a^2*\log(b*x + a)/b^2 - 2*c^2*\log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*A*B*a*b*g^2 + \frac{2}{3}*(x^3*\log(d^2e*x^2/(b^2x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2x^2 + 2*a*b*x + a^2) + c^2e/(b^2x^2 + 2*a*b*x + a^2)) - 2*a^3*\log(b*x + a)/b^3 + 2*c^3*\log(d*x + c)/d^3 + ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2g^2 + A^2a^2g^2x + \frac{4}{3}*((g^2*\log(e) - 3*g^2)*b^2*c^3 - (3*g^2*\log(e) - 7*g^2)*a*b*c^2*d + (3*g^2*\log(e) - 4*g^2)*a^2*c*d^2)*B^2*\log(d*x + c)/d^3 - \frac{8}{3}*(b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2 - a^3*d^3*g^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + \text{dilog}(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^3) + \frac{1}{3}*(B^2*b^3*d^3*g^2*x^3*\log(e)^2 + (2*b^3*c*d^2*g^2*\log(e) + (3*g^2*\log(e)^2 - 2*g^2*\log(e))*a*b^2*d^3)*B^2*x^2 - (4*(g^2*\log(e) - g^2)*b^3*c^2*d - 4*(3*g^2*\log(e) - 2*g^2)*a*b^2*c*d^2 - (3*g^2*\log(e)^2 - 8*g^2*\log(e) + 4*g^2)*a^2*b*d^3)*B^2*x + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + B^2*a^3*d^3*g^2)*log(b*x + a)^2 + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*a*b^2*d^3*g^2*x^2 + 3*B^2*a^2*b*d^3*g^2*x + (b^3*c^3*g^2 - 3*a*b^2*c^2*d*g^2 + 3*a^2*b*c*d^2*g^2)*B^2)*log(d*x + c)^2 - 4*(B^2*b^3*d^3*g^2*x^3*\log(e) + (b^3*c*d^2*g^2 + (3*g^2*\log(e) - g^2)*a*b^2*d^3)*B^2*x^2 - (2*b^3*c^2*d*g^2 - 6*a*b^2*c*d^2*g^2 - (3*g^2*\log(e) - 4*g^2)*a^2*b*d^3)*B^2*x - (2*a*b^2*c^2*d*g^2 - 5*a^2*b*c*d^2*g^2 - (g^2*\log(e) -$

$$3g^2a^3d^3B^2 \log(bx+a) + 4(B^2b^3d^3g^2x^3 \log(e) + (b^3cd^2g^2 + (3g^2 \log(e) - g^2)ab^2d^3)B^2x^2 - (2b^3c^2d^2g^2 - 6ab^2cd^2g^2 - (3g^2 \log(e) - 4g^2)a^2bd^3)B^2x - 2(B^2b^3d^3g^2x^3 + 3B^2ab^2d^3g^2x^2 + 3B^2a^2bd^3g^2x + B^2a^3d^3g^2) \log(bx+a)) \log(dx+c) / (bd^3)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2b^2g^2x^2 + 2A^2abg^2x + A^2a^2g^2 + (B^2b^2g^2x^2 + 2B^2abg^2x + B^2a^2g^2) \log \left( \frac{d^2ex^2 + 2cdex + c^2e}{b^2x^2 + 2abx + a^2} \right)^2 + 2(ABb^2g^2 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="fricas")

[Out] integral(A^2\*b^2\*g^2\*x^2 + 2\*A^2\*a\*b\*g^2\*x + A^2\*a^2\*g^2 + (B^2\*b^2\*g^2\*x^2 + 2\*B^2\*a\*b\*g^2\*x + B^2\*a^2\*g^2)\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2))^2 + 2\*(A\*B\*b^2\*g^2\*x^2 + 2\*A\*B\*a\*b\*g^2\*x + A\*B\*a^2\*g^2)\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*2\*(A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( B \log \left( \frac{(dx + c)^2 e}{(bx + a)^2} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^2\*(B\*log((d\*x + c)^2\*e/(b\*x + a)^2) + A)^2, x)

$$3.213 \quad \int (ag + bgx) \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=211

$$\frac{4B^2g(bc-ad)^2 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd^2} + \frac{2Bg(bc-ad)^2 \log\left(1 - \frac{d(a+bx)}{b(c+dx)}\right) \left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)}{bd^2} + \frac{2Bg(c+dx)(bc-ad)}{bd^2}$$

```
[Out] (4*B^2*(b*c - a*d)^2*g*Log[a + b*x])/(b*d^2) + (2*B*(b*c - a*d)*g*(c + d*x)
*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/d^2 + (g*(a + b*x)^2*(A + B*Log[
(e*(c + d*x)^2)/(a + b*x)^2])^2)/(2*b) + (2*B*(b*c - a*d)^2*g*(A + B*Log[(e
*(c + d*x)^2)/(a + b*x)^2])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/(b*d^2) -
(4*B^2*(b*c - a*d)^2*g*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d^2)
```

**Rubi [A]** time = 0.504235, antiderivative size = 291, normalized size of antiderivative = 1.38, number of steps used = 16, number of rules used = 12, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$ , Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2394, 2393, 2391, 2390, 2301}

$$\frac{4B^2g(bc-ad)^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bd^2} - \frac{2Bg(bc-ad)^2 \log(c+dx) \left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)}{bd^2} + \frac{g(a+bx)^2 \left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2, x]
```

```
[Out] (2*A*B*(b*c - a*d)*g*x)/d + (4*B^2*(b*c - a*d)^2*g*Log[c + d*x])/(b*d^2) -
(4*B^2*(b*c - a*d)^2*g*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b*d
^2) + (2*B^2*(b*c - a*d)^2*g*Log[c + d*x]^2)/(b*d^2) + (2*B^2*(b*c - a*d)*g
*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2])/(b*d) - (2*B*(b*c - a*d)^2*g*L
og[c + d*x]*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*d^2) + (g*(a + b*x
)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(2*b) - (4*B^2*(b*c - a*d)^
2*g*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*d^2)
```

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*RFX^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFX_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFX, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFX, x] && IntegerQ[p]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)
]^n))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_))^(n_.)]*(b_.)/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rubi steps

$$\begin{aligned}
 \int (ag + bgx) \left( A + B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)^2 dx &= \frac{g(a + bx)^2 \left( A + B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)^2}{2b} - \frac{B \int \frac{2(bc - ad)g^2(a + bx) \left( -A - B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)}{c + dx}}{bg} \\
 &= \frac{g(a + bx)^2 \left( A + B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)^2}{2b} - \frac{(2B(bc - ad)g) \int \frac{(a + bx) \left( -A - B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)}{c + dx}}{b} \\
 &= \frac{g(a + bx)^2 \left( A + B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)^2}{2b} - \frac{(2B(bc - ad)g) \int \left( \frac{b \left( -A - B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)}{d} \right)}{b} \\
 &= \frac{g(a + bx)^2 \left( A + B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)^2}{2b} - \frac{(2B(bc - ad)g) \int \left( -A - B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)}{d} \\
 &= \frac{2AB(bc - ad)gx}{d} - \frac{2B(bc - ad)^2g \log(c + dx) \left( A + B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) \right)}{bd^2} + \dots \\
 &= \frac{2AB(bc - ad)gx}{d} + \frac{2B^2(bc - ad)g(a + bx) \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right)}{bd} - \frac{2B(bc - ad)^2g}{bd} \\
 &= \frac{2AB(bc - ad)gx}{d} + \frac{4B^2(bc - ad)^2g \log(c + dx)}{bd^2} + \frac{2B^2(bc - ad)g(a + bx)}{bd} \\
 &= \frac{2AB(bc - ad)gx}{d} + \frac{4B^2(bc - ad)^2g \log(c + dx)}{bd^2} + \frac{2B^2(bc - ad)g(a + bx)}{bd} \\
 &= \frac{2AB(bc - ad)gx}{d} + \frac{4B^2(bc - ad)^2g \log(c + dx)}{bd^2} - \frac{4B^2(bc - ad)^2g \log \left( -\frac{c + dx}{a + bx} \right)}{bd^2} \\
 &= \frac{2AB(bc - ad)gx}{d} + \frac{4B^2(bc - ad)^2g \log(c + dx)}{bd^2} - \frac{4B^2(bc - ad)^2g \log \left( -\frac{c + dx}{a + bx} \right)}{bd^2} \\
 &= \frac{2AB(bc - ad)gx}{d} + \frac{4B^2(bc - ad)^2g \log(c + dx)}{bd^2} - \frac{4B^2(bc - ad)^2g \log \left( -\frac{c + dx}{a + bx} \right)}{bd^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.173962, size = 195, normalized size = 0.92

$$\frac{g \left( \frac{4B(bc - ad) \left( (2aBd - 2bBc) \text{PolyLog} \left( 2, \frac{b(c + dx)}{bc - ad} \right) - (bc - ad) \log(c + dx) \left( B \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) + 2B \log \left( \frac{d(a + bx)}{ad - bc} \right) + A - 2B \right) + Bd(a + bx) \log \left( \frac{e(c + dx)^2}{(a + bx)^2} \right) + B(bc - ad) \log^2(c + dx) \right)}{d^2} \right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*g + b*g*x)*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2,x]
```

```
[Out] (g*((a + b*x)^2*(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]))^2 + (4*B*(b*c - a*d)*(A*b*d*x + B*(b*c - a*d)*Log[c + d*x]^2 + B*d*(a + b*x)*Log[(e*(c + d*x)^2)/(a + b*x)^2] - (b*c - a*d)*Log[c + d*x]*(A - 2*B + 2*B*Log[(d*(a + b*x))/(-b*c + a*d)] + B*Log[(e*(c + d*x)^2)/(a + b*x)^2]) + (-2*b*B*c + 2*a*B*d)*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/d^2)/(2*b)
```

**Maple [F]** time = 1.257, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln \left( \frac{e(dx + c)^2}{(bx + a)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)\*(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

[Out] int((b\*g\*x+a\*g)\*(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

**Maxima [B]** time = 1.67505, size = 986, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="maxima")

[Out]  $\frac{1}{2}A^2b^2g^2x^2 + 2*(x*\log(d^2e*x^2/(b^2*x^2 + 2*a*b*x + a^2)) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) - 2*a*\log(b*x + a)/b + 2*c*\log(d*x + c)/d)*A*B*a*g + (x^2*\log(d^2e*x^2/(b^2*x^2 + 2*a*b*x + a^2)) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + 2*a^2*\log(b*x + a)/b^2 - 2*c^2*\log(d*x + c)/d^2 + 2*(b*c - a*d)*x/(b*d))*A*B*b*g + A^2*a*g*x - 2*((g*\log(e) - 2*g)*b*c^2 - 2*(g*\log(e) - g)*a*c*d)*B^2*\log(d*x + c)/d^2 + 4*(b^2*c^2*g - 2*a*b*c*d*g + a^2*d^2*g)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*\log(e)^2 + 2*(2*b^2*c*d*g*\log(e) + (g*\log(e)^2 - 2*g*\log(e))*a*b*d^2)*B^2*x + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*\log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x - (b^2*c^2*g - 2*a*b*c*d*g)*B^2)*\log(d*x + c)^2 - 4*(B^2*b^2*d^2*g*x^2*\log(e) + 2*((g*\log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x + ((g*\log(e) - 2*g)*a^2*d^2 + 2*a*b*c*d*g)*B^2)*\log(b*x + a) + 4*(B^2*b^2*d^2*g*x^2*\log(e) + 2*((g*\log(e) - g)*a*b*d^2 + b^2*c*d*g)*B^2*x - 2*(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*\log(b*x + a))*\log(d*x + c))/(b*d^2)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2bgx + A^2ag + (B^2bgx + B^2ag) \log \left( \frac{d^2ex^2 + 2cdex + c^2e}{b^2x^2 + 2abx + a^2} \right)^2 + 2(ABbgx + ABag) \log \left( \frac{d^2ex^2 + 2cdex + c^2e}{b^2x^2 + 2abx + a^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="fricas")

[Out] integral(A^2\*b\*g\*x + A^2\*a\*g + (B^2\*b\*g\*x + B^2\*a\*g)\*log((d^2e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2))^2 + 2\*(A\*B\*b\*g\*x + A\*B\*a\*g)\*log((d^2e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2)), x)



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (bgx + ag) \left( B \log \left( \frac{(dx + c)^2 e}{(bx + a)^2} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)\*(B\*log((d\*x + c)^2\*e/(b\*x + a)^2) + A)^2, x)

$$3.214 \quad \int \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag+bgx} dx$$

**Optimal.** Leaf size=132

$$\frac{4BPolyLog\left(2, \frac{b(c+dx)}{d(a+bx)}\right)\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)}{bg} + \frac{8B^2PolyLog\left(3, \frac{b(c+dx)}{d(a+bx)}\right)}{bg} - \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)^2}{bg}$$

[Out]  $-(\text{Log}[-((b*c - a*d)/(d*(a + b*x))])*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2])^2)/(b*g) - (4*B*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/(b*g) + (8*B^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/(b*g)$

**Rubi [B]** time = 4.0796, antiderivative size = 740, normalized size of antiderivative = 5.61, number of steps used = 46, number of rules used = 23, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.676$ , Rules used = {2524, 12, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 6742, 2499, 2302, 30, 2396, 2433, 2374, 6589, 2500, 2440, 2434, 2375, 2317}

$$-\frac{4ABPolyLog\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg} + \frac{4B^2PolyLog\left(2, -\frac{d(a+bx)}{bc-ad}\right)\left(-\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + \log\left(\frac{1}{(a+bx)^2}\right) + \log((c+dx)^2)\right)}{bg} - \frac{8B^2PolyLog\left(3, -\frac{d(a+bx)}{bc-ad}\right)}{bg}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2/(a\*g + b\*g\*x), x]

[Out]  $(2*A*B*\text{Log}[g*(a + b*x)]^2)/(b*g) + (4*B^2*\text{Log}[g*(a + b*x)]^3)/(3*b*g) - (B^2*\text{Log}[(a + b*x)^{-2}]^2*\text{Log}[c + d*x])/(b*g) - (4*B^2*\text{Log}[(a + b*x)^{-2}]*\text{Log}[g*(a + b*x)]*\text{Log}[c + d*x])/(b*g) - (4*B^2*\text{Log}[g*(a + b*x)]^2*\text{Log}[c + d*x])/(b*g) + (B^2*\text{Log}[(a + b*x)^{-2}]^2*\text{Log}[(b*(c + d*x))/(b*c - a*d)])/(b*g) + (4*B^2*\text{Log}[g*(a + b*x)]^2*\text{Log}[(b*(c + d*x))/(b*c - a*d)])/(b*g) + (B^2*\text{Log}[-((d*(a + b*x))/(b*c - a*d))] * \text{Log}[(c + d*x)^2]^2)/(b*g) - (B^2*\text{Log}[g*(a + b*x)] * \text{Log}[(c + d*x)^2]^2)/(b*g) - (4*A*B*\text{Log}[(b*(c + d*x))/(b*c - a*d)] * \text{Log}[a*g + b*g*x])/(b*g) + (4*B^2*\text{Log}[(b*(c + d*x))/(b*c - a*d)] * (\text{Log}[(a + b*x)^{-2}] + \text{Log}[(c + d*x)^2] - \text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]) * \text{Log}[a*g + b*g*x])/(b*g) + ((A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2])^2 * \text{Log}[a*g + b*g*x])/(b*g) - (4*B^2*\text{Log}[(b*(c + d*x))/(b*c - a*d)] * \text{Log}[a*g + b*g*x]^2)/(b*g) + (2*B^2*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2] * \text{Log}[a*g + b*g*x]^2)/(b*g) - (4*A*B*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g) - (4*B^2*\text{Log}[(a + b*x)^{-2}] * PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g) + (4*B^2*(\text{Log}[(a + b*x)^{-2}] + \text{Log}[(c + d*x)^2] - \text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]) * PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b*g) + (4*B^2*\text{Log}[(c + d*x)^2] * PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(b*g) - (8*B^2*PolyLog[3, -((d*(a + b*x))/(b*c - a*d))])/(b*g) - (8*B^2*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])/(b*g)$

### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /;

FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(Rfx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2499

```

Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^((r_.))*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.))^(m_.)))/((j_.)
) + (k_.)*(x_)), x_Symbol] := Simp[((s + t*Log[i*(g + h*x)^n])^(m + 1)*Log[
e*(f*(a + b*x)^p*(c + d*x)^q]^r)/(k*n*t*(m + 1)), x] + (-Dist[(b*p*r)/(k*n
*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(a + b*x), x], x] - Dis
t[(d*q*r)/(k*n*t*(m + 1)), Int[(s + t*Log[i*(g + h*x)^n])^(m + 1)/(c + d*x)
, x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, m, n, p, q, r},
x] && NeQ[b*c - a*d, 0] && EqQ[h*j - g*k, 0] && IGtQ[m, 0]

```

### Rule 2302

```

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] := Dist[1/(
b*n), Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p},
x]

```

### Rule 30

```

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]

```

### Rule 2396

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)/((f_.) + (g_.)
*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d
+ e*x)^n])^p)/g, x] - Dist[(b*e*n*p)/g, Int[(Log[(e*(f + g*x))/(e*f - d*g)]
*(a + b*Log[c*(d + e*x)^n])^(p - 1))/(d + e*x), x], x] /; FreeQ[{a, b, c, d
, e, f, g, n, p}, x] && NeQ[e*f - d*g, 0] && IGtQ[p, 1]

```

### Rule 2433

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + Log
[(h_.)*((i_.) + (j_.)*(x_))^(m_.)]*(g_.))*((k_.) + (l_.)*(x_))^(r_.), x_Sym
bol] := Dist[1/e, Subst[Int[((k*x)/d)^r*(a + b*Log[c*x^n])^p*(f + g*Log[h*(
e*i - d*j)/e + (j*x)/e]^m), x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e,
f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e*k - d*l, 0]

```

### Rule 2374

```

Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] := -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x
^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
&& EqQ[d*e, 1]

```

### Rule 6589

```

Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]

```

### Rule 2500

```

Int[(Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)
)^((r_.))*((s_.) + Log[(i_.)*((g_.) + (h_.)*(x_))^(n_.)]*(t_.)))/((j_.) + (k
_.)*(x_)), x_Symbol] := Dist[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r] - Log[(a
+ b*x)^(p*r)] - Log[(c + d*x)^(q*r)], Int[(s + t*Log[i*(g + h*x)^n])/(j + k
*x), x], x] + (Int[(Log[(a + b*x)^(p*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k
*x), x] + Int[(Log[(c + d*x)^(q*r)]*(s + t*Log[i*(g + h*x)^n]))/(j + k*x),
x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ

```

[b\*c - a\*d, 0]

#### Rule 2440

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + Log[(h\_.) \* ((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.))\*((k\_) + (l\_.)\*(x\_)^(r\_.)), x\_Symbol] :> Dist[1/l, Subst[Int[x^r\*(a + b\*Log[c\*(-((e\*k - d\*l)/l) + (e\*x)/l)^n])\*(f + g\*Log[h\*(-((j\*k - i\*l)/l) + (j\*x)/l)^m]), x], x, k + l\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]

#### Rule 2434

Int[(((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))\*((f\_.) + Log[(h\_.) \* ((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.)))/(x\_), x\_Symbol] :> Simp[Log[x]\*(a + b\*Log[c\*(d + e\*x)^n])\*(f + g\*Log[h\*(i + j\*x)^m]), x] + (-Dist[e\*g\*m, Int[(Log[x]\*(a + b\*Log[c\*(d + e\*x)^n])]/(d + e\*x), x], x] - Dist[b\*j\*n, Int[(Log[x]\*(f + g\*Log[h\*(i + j\*x)^m])]/(i + j\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, m, n}, x] && EqQ[e\*i - d\*j, 0]

#### Rule 2375

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))^(r\_.)]\*(a\_.) + Log[(c\_.)\*(x\_)^(n\_.) \* (b\_.)]^(p\_.))/(x\_), x\_Symbol] :> Simp[(Log[d\*(e + f\*x^m)^r]\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[(f\*m\*r)/(b\*n\*(p + 1)), Int[(x^(m - 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(e + f\*x^m), x], x] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d\*e, 1]

#### Rule 2317

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.)]^(p\_.)/((d\_) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[1 + (e\*x)/d]\*(a + b\*Log[c\*x^n])^p)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[1 + (e\*x)/d]\*(a + b\*Log[c\*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{ag + bgx} dx &= \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{(a+bx)^2 \left(\frac{2de(c+dx)}{(a+bx)^2} - \frac{2be(c+dx)^2}{(a+bx)^3}\right) \left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \log}{e(c+dx)^2}}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{(a+bx)^2 \left(\frac{2de(c+dx)}{(a+bx)^2} - \frac{2be(c+dx)^2}{(a+bx)^3}\right) \left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \log}{(c+dx)^2}}{beg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(2B) \int \frac{2(bc-ad)e\left(-A - B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \log(ag + bgx)}{(a+bx)(c+dx)} dx}{beg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(4B(bc - ad)) \int \frac{\left(-A - B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \log(ag + bgx)}{(a+bx)(c+dx)} dx}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(4B(bc - ad)) \int \frac{b\left(-A - B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \log(ag + bgx)}{(bc-ad)(a+bx)} dx}{bg} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(4B) \int \frac{\left(-A - B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right) \log(ag + bgx)}{a+bx} dx}{g} - \frac{(4Bd) \int}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(4B) \int \left(\frac{A \log(ag + bgx)}{-a - bx} + \frac{B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) \log(ag + bgx)}{-a - bx}\right) dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 \log(ag + bgx)}{bg} - \frac{(4AB) \int \frac{\log(ag + bgx)}{-a - bx} dx}{g} - \frac{(4B^2) \int \frac{\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) \log(ag + bgx)}{-a - bx} dx}{g} \\
&= -\frac{4AB \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} + \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2 \log(ag + bgx)}{bg} + \frac{2B^2 \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) \log(ag + bgx)}{bg} \\
&= -\frac{4AB \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(ag + bgx)}{bg} + \frac{4B^2 \log\left(\frac{b(c+dx)}{bc-ad}\right) \left(\log\left(\frac{1}{(a+bx)^2}\right) + \log((c + dx)^2) - \log(c + dx)\right) \log(ag + bgx)}{bg} \\
&= \frac{2AB \log^2(g(a + bx))}{bg} - \frac{4B^2 \log\left(\frac{1}{(a+bx)^2}\right) \log(g(a + bx)) \log(c + dx)}{bg} - \frac{B^2 \log(g(a + bx)) \log(c + dx)}{bg} \\
&= \frac{2AB \log^2(g(a + bx))}{bg} - \frac{4B^2 \log\left(\frac{1}{(a+bx)^2}\right) \log(g(a + bx)) \log(c + dx)}{bg} + \frac{B^2 \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(g(a + bx)) \log(c + dx)}{bg} \\
&= \frac{2AB \log^2(g(a + bx))}{bg} + \frac{4B^2 \log^3(g(a + bx))}{3bg} - \frac{B^2 \log^2\left(\frac{1}{(a+bx)^2}\right) \log(c + dx)}{bg} - \frac{4B^2 \log\left(\frac{1}{(a+bx)^2}\right) \log(c + dx)}{bg} \\
&= \frac{2AB \log^2(g(a + bx))}{bg} + \frac{4B^2 \log^3(g(a + bx))}{3bg} - \frac{B^2 \log^2\left(\frac{1}{(a+bx)^2}\right) \log(c + dx)}{bg} - \frac{4B^2 \log\left(\frac{1}{(a+bx)^2}\right) \log(c + dx)}{bg} \\
&= \frac{2AB \log^2(g(a + bx))}{bg} + \frac{4B^2 \log^3(g(a + bx))}{3bg} - \frac{B^2 \log^2\left(\frac{1}{(a+bx)^2}\right) \log(c + dx)}{bg} - \frac{4B^2 \log\left(\frac{1}{(a+bx)^2}\right) \log(c + dx)}{bg}
\end{aligned}$$

**Mathematica [A]** time = 0.330015, size = 257, normalized size = 1.95

$$4AB\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right) - 4B^2\text{PolyLog}\left(2, \frac{b(c+dx)}{d(a+bx)}\right) \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + 8B^2\text{PolyLog}\left(3, \frac{b(c+dx)}{d(a+bx)}\right) + A^2 \log(a+bx) + 2AB$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2/(a\*g + b\*g\*x), x]

[Out] (-2\*A\*B\*Log[a/b + x]^2 + A^2\*Log[a + b\*x] + 4\*A\*B\*Log[a/b + x]\*Log[a + b\*x] - 4\*A\*B\*Log[c/d + x]\*Log[a + b\*x] + 4\*A\*B\*Log[c/d + x]\*Log[(d\*(a + b\*x))/(-(b\*c) + a\*d)] + 2\*A\*B\*Log[a + b\*x]\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2] - B^2\*Log[-(b\*c) + a\*d]/(d\*(a + b\*x)))\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]^2 + 4\*A\*B\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)] - 4\*B^2\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]\*PolyLog[2, (b\*(c + d\*x))/(d\*(a + b\*x))] + 8\*B^2\*PolyLog[3, (b\*(c + d\*x))/(d\*(a + b\*x))]/(b\*g)

**Maple [F]** time = 1.391, size = 0, normalized size = 0.

$$\int \frac{1}{bgx + ag} \left( A + B \ln \left( \frac{e(dx + c)^2}{(bx + a)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g), x)

[Out] int((A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{4B^2 \log(bx + a) \log(dx + c)^2}{bg} + \frac{A^2 \log(bgx + ag)}{bg} - \int \frac{B^2bc \log(e)^2 + 2ABbc \log(e) + 4(B^2bdx + B^2bc) \log(bx + a)}{bgx + ag} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g), x, algorithm="maxima")

[Out] 4\*B^2\*log(b\*x + a)\*log(d\*x + c)^2/(b\*g) + A^2\*log(b\*g\*x + a\*g)/(b\*g) - integrate(-(B^2\*b\*c\*log(e)^2 + 2\*A\*B\*b\*c\*log(e) + 4\*(B^2\*b\*d\*x + B^2\*b\*c)\*log(b\*x + a)^2 + (B^2\*b\*d\*log(e)^2 + 2\*A\*B\*b\*d\*log(e))\*x - 4\*(B^2\*b\*c\*log(e) + A\*B\*b\*c + (B^2\*b\*d\*log(e) + A\*B\*b\*d)\*x)\*log(b\*x + a) + 4\*(B^2\*b\*c\*log(e) + A\*B\*b\*c + (B^2\*b\*d\*log(e) + A\*B\*b\*d)\*x - 2\*(2\*B^2\*b\*d\*x + (b\*c + a\*d)\*B^2)\*log(b\*x + a))\*log(d\*x + c))/(b^2\*d\*g\*x^2 + a\*b\*c\*g + (b^2\*c\*g + a\*b\*d\*g)\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B^2 \log\left(\frac{d^2ex^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)^2 + 2AB \log\left(\frac{d^2ex^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right) + A^2}{bgx + ag}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="fricas")
```

```
[Out] integral((B^2*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 2*A*B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + A^2)/(b*g*x + a*g), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g),x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log\left( \frac{(dx+c)^2 e}{(bx+a)^2} \right) + A \right)^2}{bgx + ag} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g),x, algorithm="giac")
```

```
[Out] integrate((B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2/(b*g*x + a*g), x)
```



$$3.215 \quad \int \frac{\left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2}{(ag+bgx)^2} dx$$

**Optimal.** Leaf size=157

$$-\frac{(c+dx) \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)^2}{g^2(a+bx)(bc-ad)} + \frac{4AB(c+dx)}{g^2(a+bx)(bc-ad)} + \frac{4B^2(c+dx) \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right)}{g^2(a+bx)(bc-ad)} - \frac{8B^2(c+dx)}{g^2(a+bx)(bc-ad)}$$

[Out]  $(4AB(c+dx))/((b*c - a*d)*g^2*(a + b*x)) - (8B^2*(c + d*x))/((b*c - a*d)*g^2*(a + b*x)) + (4B^2*(c + d*x)*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2])/((b*c - a*d)*g^2*(a + b*x)) - ((c + d*x)*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]))/((b*c - a*d)*g^2*(a + b*x))$

**Rubi [C]** time = 0.923751, antiderivative size = 480, normalized size of antiderivative = 3.06, number of steps used = 26, number of rules used = 11, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{8B^2 d \text{PolyLog} \left( 2, -\frac{d(a+bx)}{bc-ad} \right)}{bg^2(bc-ad)} - \frac{8B^2 d \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right)}{bg^2(bc-ad)} + \frac{4Bd \log(a+bx) \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)}{bg^2(bc-ad)} - \frac{4Bd \log(c+dx)}{bg^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^2, x]$

[Out]  $(-8*B^2)/(b*g^2*(a + b*x)) - (8*B^2*d*\text{Log}[a + b*x])/(b*(b*c - a*d)*g^2) + (4*B^2*d*\text{Log}[a + b*x]^2)/(b*(b*c - a*d)*g^2) + (8*B^2*d*\text{Log}[c + d*x])/(b*(b*c - a*d)*g^2) - (8*B^2*d*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(b*(b*c - a*d)*g^2) + (4*B^2*d*\text{Log}[c + d*x]^2)/(b*(b*c - a*d)*g^2) - (8*B^2*d*\text{Log}[a + b*x]*\text{Log}[(b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)*g^2) + (4*B*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*g^2*(a + b*x)) + (4*B*d*\text{Log}[a + b*x]*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)*g^2) - (4*B*d*\text{Log}[c + d*x]*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]))/(b*(b*c - a*d)*g^2) - (A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2])^2/(b*g^2*(a + b*x)) - (8*B^2*d*\text{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))])/(b*(b*c - a*d)*g^2) - (8*B^2*d*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(b*(b*c - a*d)*g^2)$

### Rule 2525

$\text{Int}[(a_. + \text{Log}[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(d_. + (e_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^(m + 1)*(a + b*\text{Log}[c*Rfx^p])^n/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^(m + 1)*(a + b*\text{Log}[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x] \&\& \text{RationalFunctionQ}[Rfx, x] \&\& \text{IGtQ}[n, 0] \&\& (\text{EqQ}[n, 1] || \text{IntegerQ}[m]) \&\& \text{NeQ}[m, -1]$

### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& !\text{MatchQ}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$

### Rule 2528

$\text{Int}[(a_. + \text{Log}[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x\_Symbol] \rightarrow \text{With}[\{u = \text{ExpandIntegrand}[(a + b*\text{Log}[c*Rfx^p])^n, RGx, x]\}, \text{Int}[u, x] /; \text{SumQ}[u$

]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 44

Int[((a\_) + (b\_)\*(x\_)^(m\_))\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

#### Rule 2524

Int[((a\_) + Log[(c\_)\*(RFx\_)^(p\_)])\*(b\_)^(n\_)/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))])\*(b\_)^(p\_)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))])\*(b\_)^(p\_)\*((f\_) + (g\_)\*(x\_)^(q\_)), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))])\*(b\_)/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))])\*(b\_)/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_)\*((d\_) + (e\_)\*(x\_)^(n\_))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag + bgx)^2} dx &= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg^2(a + bx)} + \frac{(2B) \int \frac{2(bc-ad)\left(-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{g(a+bx)^2(c+dx)} dx}{bg} \\
&= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg^2(a + bx)} + \frac{(4B(bc - ad)) \int \frac{-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^2(c+dx)} dx}{bg^2} \\
&= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg^2(a + bx)} + \frac{(4B(bc - ad)) \int \left(\frac{b\left(-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)(a+bx)^2} - \frac{bd\left(-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^2(a+bx)}\right) dx}{bg^2} \\
&= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{bg^2(a + bx)} + \frac{(4B) \int \frac{-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^2} dx}{g^2} - \frac{(4Bd) \int \frac{-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a+bx} dx}{(bc - ad)g^2} \\
&= \frac{4B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^2(a + bx)} + \frac{4Bd \log(a + bx)\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc - ad)g^2} - \frac{4Bd \log(c + dx)}{b(bc - ad)g^2} \\
&= \frac{4B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^2(a + bx)} + \frac{4Bd \log(a + bx)\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc - ad)g^2} - \frac{4Bd \log(c + dx)}{b(bc - ad)g^2} \\
&= \frac{4B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^2(a + bx)} + \frac{4Bd \log(a + bx)\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc - ad)g^2} - \frac{4Bd \log(c + dx)}{b(bc - ad)g^2} \\
&= -\frac{8B^2}{bg^2(a + bx)} - \frac{8B^2d \log(a + bx)}{b(bc - ad)g^2} + \frac{8B^2d \log(c + dx)}{b(bc - ad)g^2} + \frac{4B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^2(a + bx)} \\
&= -\frac{8B^2}{bg^2(a + bx)} - \frac{8B^2d \log(a + bx)}{b(bc - ad)g^2} + \frac{8B^2d \log(c + dx)}{b(bc - ad)g^2} - \frac{8B^2d \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c + dx)}{b(bc - ad)g^2} \\
&= -\frac{8B^2}{bg^2(a + bx)} - \frac{8B^2d \log(a + bx)}{b(bc - ad)g^2} + \frac{4B^2d \log^2(a + bx)}{b(bc - ad)g^2} + \frac{8B^2d \log(c + dx)}{b(bc - ad)g^2} - \frac{8B^2d \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c + dx)}{b(bc - ad)g^2} \\
&= -\frac{8B^2}{bg^2(a + bx)} - \frac{8B^2d \log(a + bx)}{b(bc - ad)g^2} + \frac{4B^2d \log^2(a + bx)}{b(bc - ad)g^2} + \frac{8B^2d \log(c + dx)}{b(bc - ad)g^2} - \frac{8B^2d \log\left(-\frac{d(a+bx)}{bc-ad}\right) \log(c + dx)}{b(bc - ad)g^2}
\end{aligned}$$

**Mathematica [C]** time = 0.475139, size = 322, normalized size = 2.05

$$\frac{4B\left(-Bd(a+bx)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+Bd(a+bx)\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)\right)}{g^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2/(a\*g + b\*g\*x)^2,x]

[Out] -(((A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2 + (4\*B\*(2\*B\*(b\*c - a\*d + d\*(a + b\*x)\*Log[a + b\*x] - d\*(a + b\*x)\*Log[c + d\*x]) - (b\*c - a\*d)\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]) - d\*(a + b\*x)\*Log[a + b\*x]\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]) + d\*(a + b\*x)\*Log[c + d\*x]\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]) - B\*d\*(a + b\*x)\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c + a\*d)])) + B\*d\*(a

$$+ b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)/(b*g^2*(a + b*x))$$

**Maple [B]** time = 0.069, size = 452, normalized size = 2.9

$$-\frac{A^2}{bg^2(bx+a)} - \frac{B^2}{bg^2(bx+a)} \left( \ln \left( \frac{e}{b^2} \left( \frac{ad}{bx+a} - \frac{bc}{bx+a} - d \right)^2 \right) \right)^2 - 8 \frac{B^2}{bg^2(bx+a)} + 4 \frac{B^2}{bg^2(bx+a)} \ln \left( \frac{e}{b^2} \left( \frac{ad}{bx+a} - \frac{bc}{bx+a} - d \right)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x)
```

```
[Out] -1/b/g^2*A^2/(b*x+a)-1/b/g^2/(b*x+a)*B^2*ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)^2-8/b/g^2*B^2/(b*x+a)+4/b/g^2*B^2/(b*x+a)*ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)-4/b/g^2*B^2*d/(a*d-b*c)*ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)+1/b/g^2*B^2*d/(a*d-b*c)*ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)^2-2/b/g^2*A*B/(b*x+a)*ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)+4/b/g^2*A*B/(a*d-b*c)/(b*x+a)*a*d-4/g^2*A*B/(a*d-b*c)/(b*x+a)*c+4/b/g^2*A*B*d^2/(a*d-b*c)^2*ln(1/(b*x+a)*a*d-b*c/(b*x+a)-d)*a-4/g^2*A*B*d/(a*d-b*c)^2*ln(1/(b*x+a)*a*d-b*c/(b*x+a)-d)*c
```

**Maxima [B]** time = 1.51879, size = 774, normalized size = 4.93

$$4 \left( \left( \frac{1}{b^2 g^2 x + a b g^2} + \frac{d \log(bx + a)}{(b^2 c - a b d) g^2} - \frac{d \log(dx + c)}{(b^2 c - a b d) g^2} \right) \log \left( \frac{d^2 e x^2}{b^2 x^2 + 2 a b x + a^2} + \frac{2 c d e x}{b^2 x^2 + 2 a b x + a^2} + \frac{c^2 e}{b^2 x^2 + 2 a b x + a^2} \right) \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^2,x, algorithm="maxima")
```

```
[Out] 4*((1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + ((b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x + c))/(a*b^2*c*g^2 - a^2*b*d*g^2 + (b^3*c*g^2 - a*b^2*d*g^2)*x)*B^2 - 2*A*B*(log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/(b^2*g^2*x + a*b*g^2) - 2/(b^2*g^2*x + a*b*g^2) - 2*d*log(b*x + a)/((b^2*c - a*b*d)*g^2) + 2*d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - B^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))^2/(b^2*g^2*x + a*b*g^2) - A^2/(b^2*g^2*x + a*b*g^2)
```

**Fricas [A]** time = 1.04988, size = 416, normalized size = 2.65

$$\frac{(A^2 - 4 AB + 8 B^2)bc - (A^2 - 4 AB + 8 B^2)ad + (B^2 bdx + B^2 bc) \log \left( \frac{d^2 e x^2 + 2 c d e x + c^2 e}{b^2 x^2 + 2 a b x + a^2} \right)^2 + 2 \left( (AB - 2 B^2) bdx + (AB - 2 B^2) a \right)}{(b^3 c - a b^2 d) g^2 x + (a b^2 c - a^2 b d) g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g)^2,x, algorithm="fricas")

[Out] -((A^2 - 4\*A\*B + 8\*B^2)\*b\*c - (A^2 - 4\*A\*B + 8\*B^2)\*a\*d + (B^2\*b\*d\*x + B^2\*b\*c)\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2))^2 + 2\*((A\*B - 2\*B^2)\*b\*d\*x + (A\*B - 2\*B^2)\*b\*c)\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2)))/((b^3\*c - a\*b^2\*d)\*g^2\*x + (a\*b^2\*c - a^2\*b\*d)\*g^2)

**Sympy [B]** time = 3.73447, size = 450, normalized size = 2.87

$$\frac{4Bd(A - 2B) \log\left(x + \frac{4ABad^2 + 4ABbcd - 8B^2ad^2 - 8B^2bcd - \frac{4Ba^2d^3(A-2B)}{ad-bc} + \frac{8Babcd^2(A-2B)}{ad-bc} - \frac{4Bb^2c^2d(A-2B)}{ad-bc}}{8ABbd^2 - 16B^2bd^2}\right)}{bg^2(ad - bc)} - \frac{4Bd(A - 2B) \log\left(x + \frac{4ABad^2 + 4ABbcd - 8B^2ad^2 - 8B^2bcd - \frac{4Ba^2d^3(A-2B)}{ad-bc} + \frac{8Babcd^2(A-2B)}{ad-bc} - \frac{4Bb^2c^2d(A-2B)}{ad-bc}}{8ABbd^2 - 16B^2bd^2}\right)}{bg^2(ad - bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2))\*\*2/(b\*g\*x+a\*g)\*\*2,x)

[Out] 4\*B\*d\*(A - 2\*B)\*log(x + (4\*A\*B\*a\*d\*\*2 + 4\*A\*B\*b\*c\*d - 8\*B\*\*2\*a\*d\*\*2 - 8\*B\*\*2\*b\*c\*d - 4\*B\*a\*\*2\*d\*\*3\*(A - 2\*B)/(a\*d - b\*c) + 8\*B\*a\*b\*c\*d\*\*2\*(A - 2\*B)/(a\*d - b\*c) - 4\*B\*b\*\*2\*c\*\*2\*d\*(A - 2\*B)/(a\*d - b\*c))/(8\*A\*B\*b\*d\*\*2 - 16\*B\*\*2\*b\*d\*\*2))/(b\*g\*\*2\*(a\*d - b\*c)) - 4\*B\*d\*(A - 2\*B)\*log(x + (4\*A\*B\*a\*d\*\*2 + 4\*A\*B\*b\*c\*d - 8\*B\*\*2\*a\*d\*\*2 - 8\*B\*\*2\*b\*c\*d + 4\*B\*a\*\*2\*d\*\*3\*(A - 2\*B)/(a\*d - b\*c) - 8\*B\*a\*b\*c\*d\*\*2\*(A - 2\*B)/(a\*d - b\*c) + 4\*B\*b\*\*2\*c\*\*2\*d\*(A - 2\*B)/(a\*d - b\*c))/(8\*A\*B\*b\*d\*\*2 - 16\*B\*\*2\*b\*d\*\*2))/(b\*g\*\*2\*(a\*d - b\*c)) + (-2\*A\*B + 4\*B\*\*2)\*log(e\*(c + d\*x)\*\*2/(a + b\*x)\*\*2)/(a\*b\*g\*\*2 + b\*\*2\*g\*\*2\*x) + (B\*\*2\*c + B\*\*2\*d\*x)\*log(e\*(c + d\*x)\*\*2/(a + b\*x)\*\*2)\*\*2/(a\*\*2\*d\*g\*\*2 - a\*b\*c\*g\*\*2 + a\*b\*d\*g\*\*2\*x - b\*\*2\*c\*g\*\*2\*x) - (A\*\*2 - 4\*A\*B + 8\*B\*\*2)/(a\*b\*g\*\*2 + b\*\*2\*g\*\*2\*x)

**Giac [B]** time = 1.43896, size = 505, normalized size = 3.22

$$-\left(\frac{B^2d}{b^2cg^2 - abdg^2} + \frac{B^2}{(bgx + ag)bg}\right) \log\left(\frac{\frac{b^2c^2g^2}{(bgx+ag)^2} - \frac{2abcdg^2}{(bgx+ag)^2} + \frac{a^2d^2g^2}{(bgx+ag)^2} + \frac{2bcdg}{bgx+ag} - \frac{2ad^2g}{bgx+ag} + d^2}{b^2}\right)^2 - \frac{4(ABd - B^2d) \log\left(\frac{b^2c^2g^2}{(bgx+ag)^2} - \frac{2abcdg^2}{(bgx+ag)^2} + \frac{a^2d^2g^2}{(bgx+ag)^2} + \frac{2bcdg}{bgx+ag} - \frac{2ad^2g}{bgx+ag} + d^2\right)}{b^2cg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g)^2,x, algorithm="giac")

[Out] -(B^2\*d/(b^2\*c\*g^2 - a\*b\*d\*g^2) + B^2/((b\*g\*x + a\*g)\*b\*g))\*log((b^2\*c^2\*g^2/(b\*g\*x + a\*g)^2 - 2\*a\*b\*c\*d\*g^2/(b\*g\*x + a\*g)^2 + a^2\*d^2\*g^2/(b\*g\*x + a\*g)^2 + 2\*b\*c\*d\*g/(b\*g\*x + a\*g) - 2\*a\*d^2\*g/(b\*g\*x + a\*g) + d^2)/b^2)^2 - 4\*(A\*B\*d - B^2\*d)\*log(b\*c\*g/(b\*g\*x + a\*g) - a\*d\*g/(b\*g\*x + a\*g) + d)/(b^2\*c\*g^2 - a\*b\*d\*g^2) - 2\*(A\*B - B^2)\*log((b^2\*c^2\*g^2/(b\*g\*x + a\*g)^2 - 2\*a\*b\*c\*d\*g^2/(b\*g\*x + a\*g)^2 + a^2\*d^2\*g^2/(b\*g\*x + a\*g)^2 + 2\*b\*c\*d\*g/(b\*g\*x + a\*g) - 2\*a\*d^2\*g/(b\*g\*x + a\*g) + d^2)/b^2)/((b\*g\*x + a\*g)\*b\*g) - (A^2 - 2\*A\*B + 5\*B^2)/((b\*g\*x + a\*g)\*b\*g)

**3.216** 
$$\int \frac{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^3} dx$$

**Optimal.** Leaf size=299

$$\frac{bB(c+dx)^2\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^3(a+bx)^2(bc-ad)^2} - \frac{b(c+dx)^2\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{2g^3(a+bx)^2(bc-ad)^2} + \frac{d(c+dx)\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}{g^3(a+bx)(bc-ad)^2} - \frac{4ABd(c+dx)}{g^3(a+bx)(bc-ad)^2}$$

[Out]  $(-4ABd(c+dx))/((b^2c-ad)^2g^3(a+bx)) + (8B^2d(c+dx))/((b^2c-ad)^2g^3(a+bx)) - (b^2B^2d(c+dx)^2)/((b^2c-ad)^2g^3(a+bx)^2) - (4B^2d(c+dx)*Log[(e*(c+dx)^2)/(a+bx)^2])/((b^2c-ad)^2g^3(a+bx)) + (b^2B*(c+dx)^2*(A+B*Log[(e*(c+dx)^2)/(a+bx)^2]))/((b^2c-ad)^2g^3(a+bx)^2) + (d*(c+dx)*(A+B*Log[(e*(c+dx)^2)/(a+bx)^2]))^2/((b^2c-ad)^2g^3(a+bx)) - (b*(c+dx)^2*(A+B*Log[(e*(c+dx)^2)/(a+bx)^2]))^2/(2*(b^2c-ad)^2g^3(a+bx)^2)$

**Rubi [C]** time = 1.08082, antiderivative size = 578, normalized size of antiderivative = 1.93, number of steps used = 30, number of rules used = 11, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{4B^2d^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg^3(bc-ad)^2} + \frac{4B^2d^2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bg^3(bc-ad)^2} - \frac{2Bd^2 \log(a+bx)\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg^3(bc-ad)^2} + \frac{2Bd^2 \log(c+dx)}{bg^3(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Log[(e*(c + d*x)^2)/(a + b*x)^2])^2/(a*g + b*g*x)^3,x]`

[Out]  $(-B^2/(b^2g^3(a+bx)^2)) + (6B^2d)/(b*(b^2c-ad)*g^3(a+bx)) + (6B^2d^2*Log[a+bx])/((b^2c-ad)^2g^3) - (2B^2d^2*Log[a+bx]^2)/(b*(b^2c-ad)^2g^3) - (6B^2d^2*Log[c+d*x])/((b^2c-ad)^2g^3) + (4B^2d^2*Log[-((d*(a+bx))/(b^2c-ad))]*Log[c+d*x])/((b^2c-ad)^2g^3) - (2B^2d^2*Log[c+d*x]^2)/(b*(b^2c-ad)^2g^3) + (4B^2d^2*Log[a+bx]*Log[(b*(c+d*x))/(b^2c-ad)])/((b^2c-ad)^2g^3) + (B*(A+B*Log[(e*(c+d*x)^2)/(a+bx)^2]))/(b^2g^3(a+bx)^2) - (2B*d*(A+B*Log[(e*(c+d*x)^2)/(a+bx)^2]))/(b*(b^2c-ad)*g^3(a+bx)) - (2B*d^2*Log[a+bx]*(A+B*Log[(e*(c+d*x)^2)/(a+bx)^2]))/(b*(b^2c-ad)^2g^3) + (2B*d^2*Log[c+d*x]*(A+B*Log[(e*(c+d*x)^2)/(a+bx)^2]))/(b*(b^2c-ad)^2g^3) - (A+B*Log[(e*(c+d*x)^2)/(a+bx)^2])^2/(2*b^2g^3(a+bx)^2) + (4B^2d^2*PolyLog[2, -((d*(a+bx))/(b^2c-ad))])/((b^2c-ad)^2g^3) + (4B^2d^2*PolyLog[2, (b*(c+d*x))/(b^2c-ad)])/((b^2c-ad)^2g^3)$

**Rule 2525**

`Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]`

**Rule 12**

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.), x_Symbol] := Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))])*(b_.)/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

## Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag + bgx)^3} dx &= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{B \int \frac{2(bc-ad)\left(-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{g^2(a+bx)^3(c+dx)} dx}{bg} \\
&= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B(bc-ad)) \int \frac{-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^3(c+dx)} dx}{bg^3} \\
&= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B(bc-ad)) \int \left(\frac{b\left(-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)(a+bx)^3} - \frac{bd\left(-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^2(a+bx)^2} + \dots\right) dx}{bg^3} \\
&= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{2bg^3(a+bx)^2} + \frac{(2B) \int \frac{-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^3} dx}{g^3} + \frac{(2Bd^2) \int \frac{-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a+bx} dx}{(bc-ad)^2g^3} - \dots \\
&= \frac{B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^3(a+bx)^2} - \frac{2Bd\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{2Bd^2 \log(a+bx)\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)^2g^3} - \dots \\
&= \frac{B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^3(a+bx)^2} - \frac{2Bd\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{2Bd^2 \log(a+bx)\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)^2g^3} - \dots \\
&= \frac{B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^3(a+bx)^2} - \frac{2Bd\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)g^3(a+bx)} - \frac{2Bd^2 \log(a+bx)\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{b(bc-ad)^2g^3} - \dots \\
&= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2 \log(a+bx)}{b(bc-ad)^2g^3} - \frac{6B^2d^2 \log(c+dx)}{b(bc-ad)^2g^3} + \frac{B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{bg^3(a+bx)^2} \\
&= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2 \log(a+bx)}{b(bc-ad)^2g^3} - \frac{6B^2d^2 \log(c+dx)}{b(bc-ad)^2g^3} + \frac{4B^2d^2 \log^2(a+bx)}{b(bc-ad)^2g^3} \\
&= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2 \log(a+bx)}{b(bc-ad)^2g^3} - \frac{2B^2d^2 \log^2(a+bx)}{b(bc-ad)^2g^3} - \frac{6B^2d^2 \log^2(c+dx)}{b(bc-ad)^2g^3} \\
&= -\frac{B^2}{bg^3(a+bx)^2} + \frac{6B^2d}{b(bc-ad)g^3(a+bx)} + \frac{6B^2d^2 \log(a+bx)}{b(bc-ad)^2g^3} - \frac{2B^2d^2 \log^2(a+bx)}{b(bc-ad)^2g^3} - \frac{6B^2d^2 \log^2(c+dx)}{b(bc-ad)^2g^3}
\end{aligned}$$

**Mathematica [C]** time = 0.484111, size = 452, normalized size = 1.51

$$\frac{\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)^2 - \frac{2B\left(-2Bd^2(a+bx)^2\left(\log(a+bx)\left(\log(a+bx) - 2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right) - 2\text{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right)\right) + 2Bd^2(a+bx)^2\left(2\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right) + \log(c+dx)\right)\right)}{b^2g^3(a+bx)^2}}{b^2g^3(a+bx)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2/(a\*g + b\*g\*x)^3, x]

[Out] -((A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2 - (2\*B\*(4\*B\*d\*(a + b\*x)\*(b\*c - a\*d + d\*(a + b\*x)\*Log[a + b\*x] - d\*(a + b\*x)\*Log[c + d\*x]) - B\*((b\*c - a\*d)^2 + 2\*d\*(-(b\*c) + a\*d)\*(a + b\*x) - 2\*d^2\*(a + b\*x)^2\*Log[a + b\*x] + 2\*d^2\*(a + b\*x)^2\*Log[c + d\*x]) + (b\*c - a\*d)^2\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]))/b^2g^3(a+bx)^2)



$$b*x)^2]) + 2*d*(-(b*c) + a*d)*(a + b*x)*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]) - 2*d^2*(a + b*x)^2*\text{Log}[a + b*x]*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]) + 2*d^2*(a + b*x)^2*\text{Log}[c + d*x]*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]) - 2*B*d^2*(a + b*x)^2*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(- (b*c) + a*d)]) + 2*B*d^2*(a + b*x)^2*((2*\text{Log}[(d*(a + b*x))/(- (b*c) + a*d)] - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2/(2*b*g^3*(a + b*x)^2)$$

**Maple [B]** time = 0.07, size = 664, normalized size = 2.2

$$-\frac{A^2}{2b(bx+a)^2g^3} - \frac{B^2}{b(bx+a)^2g^3} + \frac{B^2}{b(bx+a)^2g^3} \ln\left(\frac{e}{b^2}\left(\frac{ad}{bx+a} - \frac{bc}{bx+a} - d\right)^2\right) - \frac{B^2}{2b(bx+a)^2g^3} \left(\ln\left(\frac{e}{b^2}\left(\frac{ad}{bx+a} - \frac{bc}{bx+a} - d\right)^2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g)^3,x)

[Out] 
$$-1/2/b/(b*x+a)^2/g^3*A^2-1/b/g^3*B^2/(b*x+a)^2+1/b/g^3*B^2/(b*x+a)^2*\ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)-1/2/b/g^3*B^2/(b*x+a)^2*\ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)^2-6/b/g^3*B^2*d/(a*d-b*c)/(b*x+a)-3/b/g^3*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)*\ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)+1/2/b/g^3*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)*\ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)^2+2/b/g^3*B^2*d/(a*d-b*c)/(b*x+a)*\ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)-1/b/g^3*A*B/(b*x+a)^2*\ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)+1/b/g^3*A*B/(a*d-b*c)^2/(b*x+a)^2*a^2*d^2-2/g^3*A*B/(a*d-b*c)^2/(b*x+a)^2*a*d*c+b/g^3*A*B/(a*d-b*c)^2*c^2/(b*x+a)^2+2/b/g^3*A*B/(a*d-b*c)^2*d^2/(b*x+a)*a-2/g^3*A*B/(a*d-b*c)^2*d/(b*x+a)*c+2/b/g^3*A*B*d^3/(a*d-b*c)^3*\ln(1/(b*x+a)*a*d-b*c/(b*x+a)-d)*a-2/g^3*A*B*d^2/(a*d-b*c)^3*\ln(1/(b*x+a)*a*d-b*c/(b*x+a)-d)*c$$

**Maxima [B]** time = 1.55689, size = 1351, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g)^3,x, algorithm="maxima")

[Out] 
$$-(((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*\log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*\log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3))*\log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + (b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*\log(b*x + a))*\log(d*x + c))/(a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^3)*x))*B^2 - A*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + \log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b$$

$$\begin{aligned} & \frac{2*x^2 + 2*a*b*x + a^2}{b^2*x^2 + 2*a*b*x + a^2} + \frac{c^2*e}{(b^2*x^2 + 2*a*b*x + a^2)} / \frac{(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)}{(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3} \\ & + \frac{2*d^2*\log(b*x + a)}{(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3} - \frac{2*d^2*\log(d*x + c)}{(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3} \\ & - \frac{1}{2*B^2*\log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))} \\ & \frac{2}{(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)} - \frac{1}{2*A^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)} \end{aligned}$$

**Fricas [A]** time = 1.09039, size = 846, normalized size = 2.83

$$\frac{(A^2 - 2AB + 2B^2)b^2c^2 - 2(A^2 - 4AB + 8B^2)abcd + (A^2 - 6AB + 14B^2)a^2d^2 - (B^2b^2d^2x^2 + 2B^2abd^2x - B^2b^2c^2 + 2ab^2c^2)}{2((b^5c^2 - 2ab^4c^2 - 2a^2b^4c^2 + a^4b^4d^2)*g^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2/(b*g*x+a*g)^3,x, algorithm="fricas")
```

```
[Out] -1/2*((A^2 - 2*A*B + 2*B^2)*b^2*c^2 - 2*(A^2 - 4*A*B + 8*B^2)*a*b*c*d + (A^2 - 6*A*B + 14*B^2)*a^2*d^2 - (B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2*x - B^2*b^2*c^2 + 2*B^2*a*b*c*d)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 + 4*((A*B - 3*B^2)*b^2*c*d - (A*B - 3*B^2)*a*b*d^2)*x - 2*((A*B - 3*B^2)*b^2*d^2*x^2 - (A*B - B^2)*b^2*c^2 + 2*(A*B - 2*B^2)*a*b*c*d - 2*(B^2*b^2*c*d - (A*B - 2*B^2)*a*b*d^2)*x)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/(b^5*c^2 - 2*a*b^4*c*d + a^2*b^3*d^2)*g^3*x^2 + 2*(a*b^4*c^2 - 2*a^2*b^3*c*d + a^3*b^2*d^2)*g^3*x + (a^2*b^3*c^2 - 2*a^3*b^2*c*d + a^4*b*d^2)*g^3)
```

**Sympy [B]** time = 6.77515, size = 877, normalized size = 2.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2/(b*g*x+a*g)**3,x)
```

```
[Out] 2*B*d**2*(A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 6*B**2*a*d**3 - 6*B**2*b*c*d**2 - 2*B*a**3*d**5*(A - 3*B))/(a*d - b*c)**2 + 6*B*a**2*b*c*d**4*(A - 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A - 3*B)/(a*d - b*c)**2 + 2*B*b**3*c**3*d**2*(A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) - 2*B*d**2*(A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 6*B**2*a*d**3 - 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A - 3*B))/(a*d - b*c)**2 - 6*B*a**2*b*c*d**4*(A - 3*B)/(a*d - b*c)**2 + 6*B*a*b**2*c**2*d**3*(A - 3*B)/(a*d - b*c)**2 - 2*B*b**3*c**3*d**2*(A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + (2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(c + d*x)**2/(a + b*x)**2)**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**3*x**2) + (-A*B*a*d + A*B*b*c + 3*B**2*a*d - B**2*b*c + 2*B**2*b*d*x)*log(e*(c + d*x)**2/(a + b*x)**2)/(a**3*b*d*g**3 - a**2*b**2*c*g**3 + 2*a**2*b**2*d*g**3*x - 2*a*b**3*c*g**3*x + a*b**3*d*g**3*x**2 - b**4*c*g**3*x**2) + (-A**2*a*d + A**2*b*c + 6*A*B*a*d - 2*A*B*b*c - 14*B**2*a*d + 2*B**2*b*c + x*(4*A*B*b*d - 12*B**2*b*d))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + x**2*(2*a*b**2*c*d - 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A - 3*B))/(a*d - b*c)**2 + 6*B*a**2*b*c*d**4*(A - 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A - 3*B)/(a*d - b*c)**2 + 2*B*b**3*c**3*d**2*(A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) - 2*B*d**2*(A - 3*B)*log(x + (2*A*B*a*d**3 + 2*A*B*b*c*d**2 - 6*B**2*a*d**3 - 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A - 3*B))/(a*d - b*c)**2 - 6*B*a**2*b*c*d**4*(A - 3*B)/(a*d - b*c)**2 + 6*B*a*b**2*c**2*d**3*(A - 3*B)/(a*d - b*c)**2 - 2*B*b**3*c**3*d**2*(A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2) + (2*B**2*a*c*d + 2*B**2*a*d**2*x - B**2*b*c**2 + B**2*b*d**2*x**2)*log(e*(c + d*x)**2/(a + b*x)**2)**2/(2*a**4*d**2*g**3 - 4*a**3*b*c*d*g**3 + 4*a**3*b*d**2*g**3*x + 2*a**2*b**2*c**2*g**3 - 8*a**2*b**2*c*d*g**3*x + 2*a**2*b**2*d**2*g**3*x**2 + 4*a*b**3*c**2*g**3*x - 4*a*b**3*c*d*g**3*x**2 + 2*b**4*c**2*g**3*x**2) + (-A*B*a*d + A*B*b*c + 3*B**2*a*d - B**2*b*c + 2*B**2*b*d*x)*log(e*(c + d*x)**2/(a + b*x)**2)/(a**3*b*d*g**3 - a**2*b**2*c*g**3 + 2*a**2*b**2*d*g**3*x - 2*a*b**3*c*g**3*x + a*b**3*d*g**3*x**2 - b**4*c*g**3*x**2) + (-A**2*a*d + A**2*b*c + 6*A*B*a*d - 2*A*B*b*c - 14*B**2*a*d + 2*B**2*b*c + x*(4*A*B*b*d - 12*B**2*b*d))/(2*a**3*b*d*g**3 - 2*a**2*b**2*c*g**3 + x**2*(2*a*b**2*c*d - 6*B**2*b*c*d**2 + 2*B*a**3*d**5*(A - 3*B))/(a*d - b*c)**2 + 6*B*a**2*b*c*d**4*(A - 3*B)/(a*d - b*c)**2 - 6*B*a*b**2*c**2*d**3*(A - 3*B)/(a*d - b*c)**2 + 2*B*b**3*c**3*d**2*(A - 3*B)/(a*d - b*c)**2)/(4*A*B*b*d**3 - 12*B**2*b*d**3))/(b*g**3*(a*d - b*c)**2)
```

$3*d*g**3 - 2*b**4*c*g**3) + x*(4*a**2*b**2*d*g**3 - 4*a*b**3*c*g**3))$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A\right)^2}{(bgx + ag)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g)^3,x, algorithm="giac")

[Out] integrate((B\*log((d\*x + c)^2\*e/(b\*x + a)^2) + A)^2/(b\*g\*x + a\*g)^3, x)

**3.217** 
$$\int \frac{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^4} dx$$

**Optimal.** Leaf size=407

$$\frac{4b^2B(c+dx)^3\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{9g^4(a+bx)^3(bc-ad)^3} - \frac{4Bd^3 \log\left(\frac{c+dx}{a+bx}\right)\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bg^4(bc-ad)^3} + \frac{4Bd^2(c+dx)\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{g^4(a+bx)(bc-ad)^3} - \frac{2Bd}{g^4(a+bx)(bc-ad)^3}$$

[Out]  $(-8B^2d^2(c+dx))/((b*c - a*d)^3g^4(a+bx)) + (2b*B^2d^2(c+dx)^2)/((b*c - a*d)^3g^4(a+bx)^2) - (8b^2B^2d^2(c+dx)^3)/(27*(b*c - a*d)^3g^4(a+bx)^3) + (4B^2d^3*Log[(c+dx)/(a+bx)]^2)/(3*b*(b*c - a*d)^3g^4) + (4*B*d^2*(c+dx)*(A+B*Log[(e*(c+dx)^2)/(a+bx)^2]))/((b*c - a*d)^3g^4(a+bx)) - (2*b*B*d*(c+dx)^2*(A+B*Log[(e*(c+dx)^2)/(a+bx)^2]))/((b*c - a*d)^3g^4(a+bx)^2) + (4*b^2*B*(c+dx)^3*(A+B*Log[(e*(c+dx)^2)/(a+bx)^2]))/(9*(b*c - a*d)^3g^4(a+bx)^3) - (4*B*d^3*Log[(c+dx)/(a+bx)]*(A+B*Log[(e*(c+dx)^2)/(a+bx)^2]))/(3*b*(b*c - a*d)^3g^4) - (A+B*Log[(e*(c+dx)^2)/(a+bx)^2])^2/(3*b*g^4(a+bx)^3)$

**Rubi [C]** time = 1.22808, antiderivative size = 692, normalized size of antiderivative = 1.7, number of steps used = 34, number of rules used = 11, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{8B^2d^3 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{3bg^4(bc-ad)^3} - \frac{8B^2d^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3bg^4(bc-ad)^3} + \frac{4Bd^3 \log(a+bx)\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3bg^4(bc-ad)^3} - \frac{4Bd^3 \log(c+dx)}{3bg^4(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2/(a\*g + b\*g\*x)^4, x]

[Out]  $(-8B^2)/(27*b*g^4(a+bx)^3) + (10*B^2*d)/(9*b*(b*c - a*d)*g^4(a+bx)^2) - (44*B^2*d^2)/(9*b*(b*c - a*d)^2*g^4(a+bx)) - (44*B^2*d^3*Log[a+bx])/(9*b*(b*c - a*d)^3*g^4) + (4*B^2*d^3*Log[a+bx]^2)/(3*b*(b*c - a*d)^3*g^4) + (44*B^2*d^3*Log[c+dx])/(9*b*(b*c - a*d)^3*g^4) - (8*B^2*d^3*Log[-((d*(a+bx))/(b*c - a*d))]*Log[c+dx])/(3*b*(b*c - a*d)^3*g^4) + (4*B^2*d^3*Log[c+dx]^2)/(3*b*(b*c - a*d)^3*g^4) - (8*B^2*d^3*Log[a+bx]*Log[(b*(c+dx))/(b*c - a*d)])/(3*b*(b*c - a*d)^3*g^4) + (4*B*(A+B*Log[(e*(c+dx)^2)/(a+bx)^2]))/(9*b*g^4(a+bx)^3) - (2*B*d*(A+B*Log[(e*(c+dx)^2)/(a+bx)^2]))/(3*b*(b*c - a*d)*g^4(a+bx)^2) + (4*B*d^2*(A+B*Log[(e*(c+dx)^2)/(a+bx)^2]))/(3*b*(b*c - a*d)^2*g^4(a+bx)) + (4*B*d^3*Log[a+bx]*(A+B*Log[(e*(c+dx)^2)/(a+bx)^2]))/(3*b*(b*c - a*d)^3*g^4) - (4*B*d^3*Log[c+dx]*(A+B*Log[(e*(c+dx)^2)/(a+bx)^2]))/(3*b*(b*c - a*d)^3*g^4) - (A+B*Log[(e*(c+dx)^2)/(a+bx)^2])^2/(3*b*g^4(a+bx)^3) - (8*B^2*d^3*PolyLog[2, -((d*(a+bx))/(b*c - a*d))])/(3*b*(b*c - a*d)^3*g^4) - (8*B^2*d^3*PolyLog[2, (b*(c+dx))/(b*c - a*d)])/(3*b*(b*c - a*d)^3*g^4)$

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||

IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

### Rule 44

Int[((a\_) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]

### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(Rfx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c

(e\*f - d\*g), 0]

**Rule 2391**

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :-Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned} \int \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag + bgx)^4} dx &= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a + bx)^3} + \frac{(2B) \int \frac{2(bc-ad)\left(-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{g^3(a+bx)^4(c+dx)} dx}{3bg} \\ &= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a + bx)^3} + \frac{(4B(bc - ad)) \int \frac{-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^4(c+dx)} dx}{3bg^4} \\ &= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a + bx)^3} + \frac{(4B(bc - ad)) \int \left(\frac{b\left(-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)(a+bx)^4} - \frac{bd\left(-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^2(a+bx)^3} + \dots\right) dx}{3bg^4} \\ &= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{3bg^4(a + bx)^3} + \frac{(4B) \int \frac{-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^4} dx}{3g^4} - \frac{(4Bd^3) \int \frac{-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a+bx} dx}{3(bc - ad)^3g^4} + \dots \\ &= \frac{4B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9bg^4(a + bx)^3} - \frac{2Bd\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc - ad)g^4(a + bx)^2} + \frac{4Bd^2\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc - ad)^2g^4(a + bx)} + \dots \\ &= \frac{4B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9bg^4(a + bx)^3} - \frac{2Bd\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc - ad)g^4(a + bx)^2} + \frac{4Bd^2\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc - ad)^2g^4(a + bx)} + \dots \\ &= \frac{4B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{9bg^4(a + bx)^3} - \frac{2Bd\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc - ad)g^4(a + bx)^2} + \frac{4Bd^2\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc - ad)^2g^4(a + bx)} + \dots \\ &= -\frac{8B^2}{27bg^4(a + bx)^3} + \frac{10B^2d}{9b(bc - ad)g^4(a + bx)^2} - \frac{44B^2d^2}{9b(bc - ad)^2g^4(a + bx)} - \frac{44B^2d^3 \log(a + bx)}{9b(bc - ad)^3g^4} \\ &= -\frac{8B^2}{27bg^4(a + bx)^3} + \frac{10B^2d}{9b(bc - ad)g^4(a + bx)^2} - \frac{44B^2d^2}{9b(bc - ad)^2g^4(a + bx)} - \frac{44B^2d^3 \log(a + bx)}{9b(bc - ad)^3g^4} \\ &= -\frac{8B^2}{27bg^4(a + bx)^3} + \frac{10B^2d}{9b(bc - ad)g^4(a + bx)^2} - \frac{44B^2d^2}{9b(bc - ad)^2g^4(a + bx)} - \frac{44B^2d^3 \log(a + bx)}{9b(bc - ad)^3g^4} \\ &= -\frac{8B^2}{27bg^4(a + bx)^3} + \frac{10B^2d}{9b(bc - ad)g^4(a + bx)^2} - \frac{44B^2d^2}{9b(bc - ad)^2g^4(a + bx)} - \frac{44B^2d^3 \log(a + bx)}{9b(bc - ad)^3g^4} \end{aligned}$$

**Mathematica [C]** time = 0.720682, size = 598, normalized size = 1.47

$$2B\left(-18Bd^3(a+bx)^3\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+18Bd^3(a+bx)^3\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2/(a\*g + b\*g\*x)^4,x]

[Out]  $-(9*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (2*B*(36*B*d^2*(a + b*x)^2*(b*c - a*d + d*(a + b*x)*\text{Log}[a + b*x] - d*(a + b*x)*\text{Log}[c + d*x]) - 9*B*d*(a + b*x)*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*\text{Log}[a + b*x] + 2*d^2*(a + b*x)^2*\text{Log}[c + d*x]) + 2*B*(2*(b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 + 6*d^3*(a + b*x)^3*\text{Log}[a + b*x] - 6*d^3*(a + b*x)^3*\text{Log}[c + d*x]) - 6*(b*c - a*d)^3*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]) + 9*d*(b*c - a*d)^2*(a + b*x)*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]) + 18*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]) - 18*d^3*(a + b*x)^3*\text{Log}[a + b*x]*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]) + 18*d^3*(a + b*x)^3*\text{Log}[c + d*x]*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2]) - 18*B*d^3*(a + b*x)^3*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 18*B*d^3*(a + b*x)^3*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^3)/(27*b*g^4*(a + b*x)^3)$

**Maple [B]** time = 0.075, size = 947, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g)^4,x)

[Out]  $-1/3/b/(b*x+a)^3/g^4*A^2-8/27/b/g^4*B^2/(b*x+a)^3+4/9/b/g^4*B^2/(b*x+a)^3*1n(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)-1/3/b/g^4*B^2/(b*x+a)^3*1n(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)^2-10/9/b/g^4*B^2*d/(a*d-b*c)/(b*x+a)^2-44/9/b/g^4*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)-22/9/b/g^4*d^3*B^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*1n(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)+1/3/b/g^4*d^3*B^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)*1n(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)^2+2/3/b/g^4*B^2*d/(a*d-b*c)/(b*x+a)^2*1n(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)+4/3/b/g^4*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)*1n(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)-2/3/b/g^4*A*B/(b*x+a)^3*1n(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)+4/9/b/g^4*A*B*a^3*d^3/(a*d-b*c)^3/(b*x+a)^3-4/3/g^4*A*B*a^2*d^2/(a*d-b*c)^3/(b*x+a)^3*c+4/3*b/g^4*A*B*a*d/(a*d-b*c)^3/(b*x+a)^3*c^2+2/3/b/g^4*A*B*a^2*d^3/(a*d-b*c)^3/(b*x+a)^2-4/3/g^4*A*B*a*d^2/(a*d-b*c)^3/(b*x+a)^2*c+4/3/b/g^4*A*B*a*d^3/(a*d-b*c)^3/(b*x+a)+4/3/b/g^4*A*B*a*d^4/(a*d-b*c)^4*1n(1/(b*x+a)*a*d-b*c/(b*x+a)-d)-4/9*b^2/g^4*A*B*c^3/(a*d-b*c)^3/(b*x+a)^3+2/3*b/g^4*A*B*c^2/(a*d-b*c)^3/(b*x+a)^2*d-4/3/g^4*A*B*c/(a*d-b*c)^3/(b*x+a)*d^2-4/3/g^4*A*B*c*d^3/(a*d-b*c)^4*1n(1/(b*x+a)*a*d-b*c/(b*x+a)-d)$

**Maxima [B]** time = 1.88066, size = 2128, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g)^4,x, algorithm="maxima")

[Out]  $2/27*(3*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2$

$$\begin{aligned}
& 2 - 2a^2b^4cd + a^3b^3d^2)g^4x^2 + 3(a^2b^4c^2 - 2a^3b^3cd + \\
& a^4b^2d^2)g^4x + (a^3b^3c^2 - 2a^4b^2cd + a^5bd^2)g^4) + 6d^3 \\
& 3\log(bx + a)/((b^4c^3 - 3a^2b^3c^2d + 3a^2b^2cd^2 - a^3bd^3)g^4 \\
& ) - 6d^3\log(dx + c)/((b^4c^3 - 3a^2b^3c^2d + 3a^2b^2cd^2 - a^3bd^3)g^4) \\
& ) * \log(d^2ex^2/(b^2x^2 + 2abx + a^2) + 2cdex/(b^2x^2 + 2 \\
& abx + a^2) + c^2e/(b^2x^2 + 2abx + a^2)) - (4b^3c^3 - 27a^2b^2c^2d \\
& + 108a^2b^2cd^2 - 85a^3d^3 + 66(b^3cd^2 - ab^2d^3)x^2 - 18(b \\
& ^3d^3x^3 + 3a^2b^2d^3x^2 + 3a^2bd^3x + a^3d^3) * \log(bx + a)^2 - 18 \\
& * (b^3d^3x^3 + 3a^2b^2d^3x^2 + 3a^2bd^3x + a^3d^3) * \log(dx + c)^2 - \\
& 3(5b^3c^2d - 54a^2b^2cd^2 + 49a^2bd^3)x + 66(b^3d^3x^3 + 3a^2 \\
& b^2d^3x^2 + 3a^2bd^3x + a^3d^3) * \log(bx + a) - 6(11b^3d^3x^3 + 3 \\
& 3a^2b^2d^3x^2 + 33a^2bd^3x + 11a^3d^3 - 6(b^3d^3x^3 + 3a^2b^2d^3 \\
& 3x^2 + 3a^2bd^3x + a^3d^3) * \log(bx + a)) * \log(dx + c) / (a^3b^4c^3g \\
& ^4 - 3a^4b^3c^2d^2g^4 + 3a^5b^2cd^2g^4 - a^6bd^3g^4 + (b^7c^3g \\
& ^4 - 3a^2b^6c^2d^2g^4 + 3a^2b^5cd^2g^4 - a^3b^4d^3g^4) * x^3 + 3(a^ \\
& b^6c^3g^4 - 3a^2b^5c^2d^2g^4 + 3a^3b^4cd^2g^4 - a^4b^3d^3g^4) * \\
& x^2 + 3(a^2b^5c^3g^4 - 3a^3b^4cd^2g^4 + 3a^4b^3cd^2g^4 - a^5b^2 \\
& d^3g^4) * x) * B^2 + 2/9AB * ((6b^2d^2x^2 + 2b^2c^2 - 7ab^2cd + 11 \\
& a^2d^2 - 3(b^2cd - 5abd^2)x) / ((b^6c^2 - 2ab^5cd + a^2b^4d^2) \\
& ) * g^4x^3 + 3(a^2b^5c^2 - 2a^2b^4cd + a^3b^3d^2) * g^4x^2 + 3(a^2b^4 \\
& c^2 - 2a^3b^3cd + a^4b^2d^2) * g^4x + (a^3b^3c^2 - 2a^4b^2cd + \\
& a^5bd^2) * g^4) - 3\log(d^2ex^2/(b^2x^2 + 2abx + a^2) + 2cdex/(b \\
& ^2x^2 + 2abx + a^2) + c^2e/(b^2x^2 + 2abx + a^2)) / (b^4g^4x^3 + 3 \\
& ab^3g^4x^2 + 3a^2b^2g^4x + a^3bg^4) + 6d^3\log(bx + a) / ((b^4c^3 \\
& - 3a^2b^3c^2d + 3a^2b^2cd^2 - a^3bd^3) * g^4) - 6d^3\log(dx + c) / \\
& ((b^4c^3 - 3a^2b^3c^2d + 3a^2b^2cd^2 - a^3bd^3) * g^4) - 1/3B^2\log \\
& (d^2ex^2/(b^2x^2 + 2abx + a^2) + 2cdex/(b^2x^2 + 2abx + a^2) \\
& + c^2e/(b^2x^2 + 2abx + a^2))^2 / (b^4g^4x^3 + 3a^2b^3g^4x^2 + 3a^2 \\
& b^2g^4x + a^3bg^4) - 1/3A^2 / (b^4g^4x^3 + 3a^2b^3g^4x^2 + 3a^2b^ \\
& ^2g^4x + a^3bg^4)
\end{aligned}$$

**Fricas [A]** time = 1.12827, size = 1474, normalized size = 3.62

$$(9A^2 - 12AB + 8B^2)b^3c^3 - 27(A^2 - 2AB + 2B^2)ab^2c^2d + 27(A^2 - 4AB + 8B^2)a^2bcd^2 - (9A^2 - 66AB + 170B^2)a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(dx+c)^2/(bx+a)^2))^2/(b\*gx+a\*g)^4,x, algorithm="fricas")

[Out] -1/27\*((9A^2 - 12AB + 8B^2)\*b^3c^3 - 27\*(A^2 - 2AB + 2B^2)\*a^2b^2cd^2 + 27\*(A^2 - 4AB + 8B^2)\*a^2b^2cd^2 - (9A^2 - 66AB + 170B^2)\*a^3d^3 - 12\*((3AB - 11B^2)\*b^3cd^2 - (3AB - 11B^2)\*ab^2d^3)x^2 + 9\*(B^2b^3d^3x^3 + 3B^2ab^2d^3x^2 + 3B^2a^2bd^3x + B^2b^3c^3 - 3B^2ab^2c^2d + 3B^2a^2bcd^2) \* log((d^2ex^2 + 2cdex + c^2e)/(b^2x^2 + 2abx + a^2))^2 + 6\*((3AB - 5B^2)\*b^3c^2d - 18\*(AB - 3B^2)\*ab^2cd^2 + (15AB - 49B^2)\*a^2bd^3)x + 6\*((3AB - 11B^2)\*b^3d^3x^3 + (3AB - 2B^2)\*b^3c^3 - 9\*(AB - B^2)\*ab^2c^2d + 9\*(AB - 2B^2)\*a^2bcd^2 - 3\*(2B^2b^3cd^2 - 3\*(AB - 3B^2)\*ab^2d^3)x^2 + 3\*(B^2b^3c^2d - 6B^2ab^2cd^2 + 3\*(AB - 2B^2)\*a^2bd^3)x) \* log((d^2ex^2 + 2cdex + c^2e)/(b^2x^2 + 2abx + a^2)) / ((b^7c^3 - 3a^2b^6c^2d + 3a^2b^5cd^2 - a^3b^4d^3) \* g^4x^3 + 3(a^2b^6c^3 - 3a^2b^5c^2d + 3a^3b^4cd^2 - a^4b^3d^3) \* g^4x^2 + 3(a^2b^5c^3 - 3a^3b^4c^2d + 3a^4b^3cd^2 - a^5b^2d^3) \* g^4x + (a^3b^4c^3 - 3a^4b^3cd^2 + 3a^5b^2cd^2 - a^6bd^3) \* g^4)



**Sympy [B]** time = 35.508, size = 1561, normalized size = 3.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2))\*\*2/(b\*g\*x+a\*g)\*\*4,x)

[Out]  $4*B*d**3*(3*A - 11*B)*\log(x + (12*A*B*a*d**4 + 12*A*B*b*c*d**3 - 44*B**2*a*d**4 - 44*B**2*b*c*d**3 - 4*B*a**4*d**7*(3*A - 11*B)/(a*d - b*c)**3 + 16*B*a**3*b*c*d**6*(3*A - 11*B)/(a*d - b*c)**3 - 24*B*a**2*b**2*c**2*d**5*(3*A - 11*B)/(a*d - b*c)**3 + 16*B*a*b**3*c**3*d**4*(3*A - 11*B)/(a*d - b*c)**3 - 4*B*b**4*c**4*d**3*(3*A - 11*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 - 88*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) - 4*B*d**3*(3*A - 11*B)*\log(x + (12*A*B*a*d**4 + 12*A*B*b*c*d**3 - 44*B**2*a*d**4 - 44*B**2*b*c*d**3 + 4*B*a**4*d**7*(3*A - 11*B)/(a*d - b*c)**3 - 16*B*a**3*b*c*d**6*(3*A - 11*B)/(a*d - b*c)**3 + 24*B*a**2*b**2*c**2*d**5*(3*A - 11*B)/(a*d - b*c)**3 - 16*B*a*b**3*c**3*d**4*(3*A - 11*B)/(a*d - b*c)**3 + 4*B*b**4*c**4*d**3*(3*A - 11*B)/(a*d - b*c)**3)/(24*A*B*b*d**4 - 88*B**2*b*d**4))/(9*b*g**4*(a*d - b*c)**3) + (3*B**2*a**2*c*d**2 + 3*B**2*a**2*d**3*x - 3*B**2*a*b*c**2*d + 3*B**2*a*b*d**3*x**2 + B**2*b**2*c**3 + B**2*b**2*d**3*x**3)*\log(e*(c + d*x)**2/(a + b*x)**2)**2/(3*a**6*d**3*g**4 - 9*a**5*b*c*d**2*g**4 + 9*a**5*b*d**3*g**4*x + 9*a**4*b**2*c**2*d*g**4 - 27*a**4*b**2*c*d**2*g**4*x + 9*a**4*b**2*d**3*g**4*x**2 - 3*a**3*b**3*c**3*g**4 + 27*a**3*b**3*c**2*d*g**4*x - 27*a**3*b**3*c*d**2*g**4*x**2 + 3*a**3*b**3*d**3*g**4*x**3 - 9*a**2*b**4*c**3*g**4*x + 27*a**2*b**4*c**2*d*g**4*x**2 - 9*a**2*b**4*c*d**2*g**4*x**3 - 9*a*b**5*c**3*g**4*x**2 + 9*a*b**5*c**2*d*g**4*x**3 - 3*b**6*c**3*g**4*x**3) + (-6*A*B*a**2*d**2 + 12*A*B*a*b*c*d - 6*A*B*b**2*c**2 + 22*B**2*a**2*d**2 - 14*B**2*a*b*c*d + 30*B**2*a*b*d**2*x + 4*B**2*b**2*c**2 - 6*B**2*b**2*c*d*x + 12*B**2*b**2*d**2*x**2)*\log(e*(c + d*x)**2/(a + b*x)**2)/(9*a**5*b*d**2*g**4 - 18*a**4*b**2*c*d*g**4 + 27*a**4*b**2*d**2*g**4*x + 9*a**3*b**3*c**2*g**4 - 54*a**3*b**3*c*d*g**4*x + 27*a**3*b**3*d**2*g**4*x**2 + 27*a**2*b**4*c**2*g**4*x - 54*a**2*b**4*c*d*g**4*x**2 + 9*a**2*b**4*d**2*g**4*x**3 + 27*a*b**5*c**2*g**4*x**2 - 18*a*b**5*c*d*g**4*x**3 + 9*b**6*c**2*g**4*x**3) + (-9*A**2*a**2*d**2 + 18*A**2*a*b*c*d - 9*A**2*b**2*c**2 + 66*A*B*a**2*d**2 - 42*A*B*a*b*c*d + 12*A*B*b**2*c**2 - 170*B**2*a**2*d**2 + 46*B**2*a*b*c*d - 8*B**2*b**2*c**2 + x**2*(36*A*B*b**2*d**2 - 132*B**2*b**2*d**2) + x*(90*A*B*a*b*d**2 - 18*A*B*b**2*c*d - 294*B**2*a*b*d**2 + 30*B**2*b**2*c*d))/(27*a**5*b*d**2*g**4 - 54*a**4*b**2*c*d*g**4 + 27*a**3*b**3*c**2*g**4 + x**3*(27*a**2*b**4*d**2*g**4 - 54*a*b**5*c*d*g**4 + 27*b**6*c**2*g**4) + x**2*(81*a**3*b**3*d**2*g**4 - 162*a**2*b**4*c*d*g**4 + 81*a*b**5*c**2*g**4) + x*(81*a**4*b**2*d**2*g**4 - 162*a**3*b**3*c*d*g**4 + 81*a**2*b**4*c**2*g**4))$

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A\right)^2}{(bgx + ag)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g)^4,x, algorithm="giac")

[Out] integrate((B\*log((d\*x + c)^2\*e/(b\*x + a)^2) + A)^2/(b\*g\*x + a\*g)^4, x)

$$3.218 \quad \int \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag+bgx)^5} dx$$

**Optimal.** Leaf size=501

$$-\frac{4b^2Bd(c+dx)^3\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{3g^5(a+bx)^3(bc-ad)^4} + \frac{b^3B(c+dx)^4\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{4g^5(a+bx)^4(bc-ad)^4} + \frac{Bd^4\log\left(\frac{c+dx}{a+bx}\right)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg^5(bc-ad)^4} - \frac{Bd^4\log(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg^5(bc-ad)^4}$$

[Out]  $(8*B^2*d^3*(c+d*x))/((b*c-a*d)^4*g^5*(a+b*x)) - (3*b*B^2*d^2*(c+d*x)^2)/((b*c-a*d)^4*g^5*(a+b*x)^2) + (8*b^2*B^2*d*(c+d*x)^3)/(9*(b*c-a*d)^4*g^5*(a+b*x)^3) - (b^3*B^2*(c+d*x)^4)/(8*(b*c-a*d)^4*g^5*(a+b*x)^4) - (B^2*d^4*Log[(c+d*x)/(a+b*x)]^2)/(b*(b*c-a*d)^4*g^5) - (4*B*d^3*(c+d*x)*(A+B*Log[(e*(c+d*x)^2]/(a+b*x)^2]))/((b*c-a*d)^4*g^5*(a+b*x)) + (3*b*B*d^2*(c+d*x)^2*(A+B*Log[(e*(c+d*x)^2]/(a+b*x)^2]))/((b*c-a*d)^4*g^5*(a+b*x)^2) - (4*b^2*B*d*(c+d*x)^3*(A+B*Log[(e*(c+d*x)^2]/(a+b*x)^2]))/(3*(b*c-a*d)^4*g^5*(a+b*x)^3) + (b^3*B*(c+d*x)^4*(A+B*Log[(e*(c+d*x)^2]/(a+b*x)^2]))/(4*(b*c-a*d)^4*g^5*(a+b*x)^4) + (B*d^4*Log[(c+d*x)/(a+b*x)]*(A+B*Log[(e*(c+d*x)^2]/(a+b*x)^2]))/(b*(b*c-a*d)^4*g^5) - (A+B*Log[(e*(c+d*x)^2]/(a+b*x)^2])^2/(4*b*g^5*(a+b*x)^4)$

**Rubi [C]** time = 1.42886, antiderivative size = 758, normalized size of antiderivative = 1.51, number of steps used = 38, number of rules used = 11, integrand size = 34,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.324$ , Rules used = {2525, 12, 2528, 44, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2B^2d^4\text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{bg^5(bc-ad)^4} + \frac{2B^2d^4\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{bg^5(bc-ad)^4} - \frac{Bd^4\log(a+bx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg^5(bc-ad)^4} + \frac{Bd^4\log(c+dx)\left(B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}{bg^5(bc-ad)^4}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2])^2/(a\*g + b\*g\*x)^5, x]

[Out]  $-B^2/(8*b*g^5*(a+b*x)^4) + (7*B^2*d)/(18*b*(b*c-a*d)*g^5*(a+b*x)^3) - (13*B^2*d^2)/(12*b*(b*c-a*d)^2*g^5*(a+b*x)^2) + (25*B^2*d^3)/(6*b*(b*c-a*d)^3*g^5*(a+b*x)) + (25*B^2*d^4*Log[a+b*x])/(6*b*(b*c-a*d)^4*g^5) - (B^2*d^4*Log[a+b*x]^2)/(b*(b*c-a*d)^4*g^5) - (25*B^2*d^4*Log[c+d*x])/(6*b*(b*c-a*d)^4*g^5) + (2*B^2*d^4*Log[-((d*(a+b*x))/(b*c-a*d))]*Log[c+d*x])/(b*(b*c-a*d)^4*g^5) - (B^2*d^4*Log[c+d*x]^2)/(b*(b*c-a*d)^4*g^5) + (2*B^2*d^4*Log[a+b*x]*Log[(b*(c+d*x))/(b*c-a*d)])/(b*(b*c-a*d)^4*g^5) + (B*(A+B*Log[(e*(c+d*x)^2]/(a+b*x)^2]))/(4*b*g^5*(a+b*x)^4) - (B*d*(A+B*Log[(e*(c+d*x)^2]/(a+b*x)^2]))/(3*b*(b*c-a*d)*g^5*(a+b*x)^3) + (B*d^2*(A+B*Log[(e*(c+d*x)^2]/(a+b*x)^2]))/(2*b*(b*c-a*d)^2*g^5*(a+b*x)^2) - (B*d^3*(A+B*Log[(e*(c+d*x)^2]/(a+b*x)^2]))/(b*(b*c-a*d)^3*g^5*(a+b*x)) - (B*d^4*Log[a+b*x]*(A+B*Log[(e*(c+d*x)^2]/(a+b*x)^2]))/(b*(b*c-a*d)^4*g^5) + (B*d^4*Log[c+d*x]*(A+B*Log[(e*(c+d*x)^2]/(a+b*x)^2]))/(b*(b*c-a*d)^4*g^5) - (A+B*Log[(e*(c+d*x)^2]/(a+b*x)^2])^2/(4*b*g^5*(a+b*x)^4) + (2*B^2*d^4*PolyLog[2, -((d*(a+b*x))/(b*c-a*d))])/(b*(b*c-a*d)^4*g^5) + (2*B^2*d^4*PolyLog[2, (b*(c+d*x))/(b*c-a*d)])/(b*(b*c-a*d)^4*g^5)$

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1))

, x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 2528

Int[((a\_) + Log[(c\_)\*(RFx\_)^(p\_)]\*(b\_))^(n\_)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

### Rule 44

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x)^m\*(c + d\*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])

### Rule 2524

Int[((a\_) + Log[(c\_)\*(RFx\_)^(p\_)]\*(b\_))^(n\_)/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

### Rule 2418

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)]\*(b\_))^(p\_)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

### Rule 2390

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)]\*(b\_))^(p\_)\*((f\_) + (g\_)\*(x\_))^(q\_), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_) + Log[(c\_)\*(x\_)]^(n\_)]\*(b\_)/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 2394

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)]\*(b\_))/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

**Rule 2391**

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\int \frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{(ag + bgx)^5} dx = -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a + bx)^4} + \frac{B \int \frac{2(bc-ad)\left(-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{g^4(a+bx)^5(c+dx)} dx}{2bg}$$

$$= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a + bx)^4} + \frac{(B(bc - ad)) \int \frac{-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^5(c+dx)} dx}{bg^5}$$

$$= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a + bx)^4} + \frac{(B(bc - ad)) \int \left(\frac{b\left(-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)(a+bx)^5} - \frac{bd\left(-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{(bc-ad)^2(a+bx)^4} + \dots\right) dx}{bg^5}$$

$$= -\frac{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2}{4bg^5(a + bx)^4} + \frac{B \int \frac{-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{(a+bx)^5} dx}{g^5} + \frac{(Bd^4) \int \frac{-A-B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{a+bx} dx}{(bc - ad)^4 g^5} - \frac{(Bd^5)}{g^5}$$

$$= \frac{B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4bg^5(a + bx)^4} - \frac{Bd\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc - ad)g^5(a + bx)^3} + \frac{Bd^2\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b(bc - ad)^2g^5(a + bx)^2} - \frac{Bd^3}{b(bc - ad)^3g^5}$$

$$= \frac{B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4bg^5(a + bx)^4} - \frac{Bd\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc - ad)g^5(a + bx)^3} + \frac{Bd^2\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b(bc - ad)^2g^5(a + bx)^2} - \frac{Bd^3}{b(bc - ad)^3g^5}$$

$$= \frac{B\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{4bg^5(a + bx)^4} - \frac{Bd\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{3b(bc - ad)g^5(a + bx)^3} + \frac{Bd^2\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)}{2b(bc - ad)^2g^5(a + bx)^2} - \frac{Bd^3}{b(bc - ad)^3g^5}$$

$$= -\frac{B^2}{8bg^5(a + bx)^4} + \frac{7B^2d}{18b(bc - ad)g^5(a + bx)^3} - \frac{13B^2d^2}{12b(bc - ad)^2g^5(a + bx)^2} + \frac{25B^2d^3}{6b(bc - ad)^3g^5}$$

$$= -\frac{B^2}{8bg^5(a + bx)^4} + \frac{7B^2d}{18b(bc - ad)g^5(a + bx)^3} - \frac{13B^2d^2}{12b(bc - ad)^2g^5(a + bx)^2} + \frac{25B^2d^3}{6b(bc - ad)^3g^5}$$

$$= -\frac{B^2}{8bg^5(a + bx)^4} + \frac{7B^2d}{18b(bc - ad)g^5(a + bx)^3} - \frac{13B^2d^2}{12b(bc - ad)^2g^5(a + bx)^2} + \frac{25B^2d^3}{6b(bc - ad)^3g^5}$$

$$= -\frac{B^2}{8bg^5(a + bx)^4} + \frac{7B^2d}{18b(bc - ad)g^5(a + bx)^3} - \frac{13B^2d^2}{12b(bc - ad)^2g^5(a + bx)^2} + \frac{25B^2d^3}{6b(bc - ad)^3g^5}$$

**Mathematica [C]** time = 0.987847, size = 762, normalized size = 1.52

$$B\left(-72Bd^4(a+bx)^4\left(\log(a+bx)\left(\log(a+bx)-2 \log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2 \text{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right)\right)+72Bd^4(a+bx)^4\left(2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2 \log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2/(a\*g + b\*g\*x)^5,x]

[Out] 
$$\begin{aligned} & (-18*(A + B*\text{Log}[(e*(c + d*x)^2)/(a + b*x)^2])^2 + (B*(144*B*d^3*(a + b*x)^3 \\ & *(b*c - a*d + d*(a + b*x)*\text{Log}[a + b*x] - d*(a + b*x)*\text{Log}[c + d*x]) - 36*B*d \\ & ^2*(a + b*x)^2*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b \\ & *x)^2*\text{Log}[a + b*x] + 2*d^2*(a + b*x)^2*\text{Log}[c + d*x]) + 8*B*d*(a + b*x)*(2*( \\ & b*c - a*d)^3 - 3*d*(b*c - a*d)^2*(a + b*x) + 6*d^2*(b*c - a*d)*(a + b*x)^2 \\ & + 6*d^3*(a + b*x)^3*\text{Log}[a + b*x] - 6*d^3*(a + b*x)^3*\text{Log}[c + d*x]) - 3*B*(3 \\ & *(b*c - a*d)^4 + 4*d*(-(b*c) + a*d)^3*(a + b*x) + 6*d^2*(b*c - a*d)^2*(a + \\ & b*x)^2 + 12*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 12*d^4*(a + b*x)^4*\text{Log}[a + b*x \\ & ] + 12*d^4*(a + b*x)^4*\text{Log}[c + d*x]) + 18*(b*c - a*d)^4*(A + B*\text{Log}[(e*(c + \\ & d*x)^2)/(a + b*x)^2]) + 24*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*\text{Log}[(e*(c + \\ & d*x)^2)/(a + b*x)^2]) + 36*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*\text{Log}[(e*(c + \\ & d*x)^2)/(a + b*x)^2]) + 72*d^3*(-(b*c) + a*d)*(a + b*x)^3*(A + B*\text{Log}[(e*(c \\ & + d*x)^2)/(a + b*x)^2]) - 72*d^4*(a + b*x)^4*\text{Log}[a + b*x]*(A + B*\text{Log}[(e*(c \\ & + d*x)^2)/(a + b*x)^2]) + 72*d^4*(a + b*x)^4*\text{Log}[c + d*x]*(A + B*\text{Log}[(e*(c \\ & + d*x)^2)/(a + b*x)^2]) - 72*B*d^4*(a + b*x)^4*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] \\ & - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + \\ & a*d)]) + 72*B*d^4*(a + b*x)^4*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \text{Log}[ \\ & c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - \\ & a*d)^4)/(72*b*g^5*(a + b*x)^4) \end{aligned}$$

**Maple [B]** time = 0.078, size = 1285, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g)^5,x)

[Out] 
$$\begin{aligned} & 1/b/g^5*A*B*a*d^4/(a*d-b*c)^4/(b*x+a)+1/2/b/g^5*A*B*a^2*d^4/(a*d-b*c)^4/(b \\ & x+a)^2-1/3*b^2/g^5*A*B*c^3/(a*d-b*c)^4/(b*x+a)^3*d+1/b/g^5*A*B*a*d^5/(a*d-b \\ & *c)^5*\ln(1/(b*x+a)*a*d-b*c/(b*x+a)-d)+1/3/b/g^5*B^2*d/(a*d-b*c)/(b*x+a)^3*1 \\ & n(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)+1/2/b/g^5*B^2*d^2/(a^2*d^2-2*a*b*c \\ & *d+b^2*c^2)/(b*x+a)^2*\ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)+1/b/g^5*d^3 \\ & *B^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)*\ln(e*(1/(b*x+a)* \\ & a*d-b*c/(b*x+a)-d)^2/b^2)-1/g^5*A*B*c*d^4/(a*d-b*c)^5*\ln(1/(b*x+a)*a*d-b*c/ \\ & (b*x+a)-d)+1/4*b^3/g^5*A*B*c^4/(a*d-b*c)^4/(b*x+a)^4-1/g^5*A*B*c/(a*d-b*c)^ \\ & 4/(b*x+a)*d^3-1/g^5*A*B*a^2*d^3/(a*d-b*c)^4/(b*x+a)^3*c+1/4/b/g^5*B^2/(b*x+ \\ & a)^4*\ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)-1/4/b/(b*x+a)^4/g^5*A^2-1/8/ \\ & b/g^5*B^2/(b*x+a)^4-1/g^5*A*B*a*d^3/(a*d-b*c)^4/(b*x+a)^2*c-1/g^5*A*B*a^3*d \\ & ^3/(a*d-b*c)^4/(b*x+a)^4*c+1/2*b/g^5*A*B*c^2/(a*d-b*c)^4/(b*x+a)^2*d^2+1/3/ \\ & b/g^5*A*B*a^3*d^4/(a*d-b*c)^4/(b*x+a)^3+1/4/b/g^5*A*B*a^4*d^4/(a*d-b*c)^4/( \\ & b*x+a)^4-1/4/b/g^5*B^2/(b*x+a)^4*\ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)^ \\ & 2+b/g^5*A*B*a*d^2/(a*d-b*c)^4/(b*x+a)^3*c^2+3/2*b/g^5*A*B*a^2*d^2/(a*d-b*c) \\ & ^4/(b*x+a)^4*c^2-b^2/g^5*A*B*a*d/(a*d-b*c)^4/(b*x+a)^4*c^3-25/6/b/g^5*d^3*B \\ & ^2/(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(b*x+a)-25/12/b/g^5*d^4*B^ \\ & 2/(a^4*d^4-4*a^3*b*c*d^3+6*a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*\ln(e*(1/( \\ & b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)+1/4/b/g^5*d^4*B^2/(a^4*d^4-4*a^3*b*c*d^3+6 \\ & *a^2*b^2*c^2*d^2-4*a*b^3*c^3*d+b^4*c^4)*\ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^ \\ & 2/b^2)^2-1/2/b/g^5*A*B/(b*x+a)^4*\ln(e*(1/(b*x+a)*a*d-b*c/(b*x+a)-d)^2/b^2)- \\ & 13/12/b/g^5*B^2*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(b*x+a)^2-7/18/b/g^5*B^2*d/ \\ & (a*d-b*c)/(b*x+a)^3 \end{aligned}$$

**Maxima [B]** time = 2.398, size = 3075, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g)^5,x, algorithm="maxima")

[Out] 
$$-1/72*(6*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2)) + (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 12*(25*b^4*d^4*x^4 + 100*a*b^3*d^4*x^3 + 150*a^2*b^2*d^4*x^2 + 100*a^3*b*d^4*x + 25*a^4*d^4 - 12*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a))*log(d*x + c))/((a^4*b^5*c^4*g^5 - 4*a^5*b^4*c^3*d*g^5 + 6*a^6*b^3*c^2*d^2*g^5 - 4*a^7*b^2*c*d^3*g^5 + a^8*b*d^4*g^5 + (b^9*c^4*g^5 - 4*a*b^8*c^3*d*g^5 + 6*a^2*b^7*c^2*d^2*g^5 - 4*a^3*b^6*c*d^3*g^5 + a^4*b^5*d^4*g^5)*x^4 + 4*(a*b^8*c^4*g^5 - 4*a^2*b^7*c^3*d*g^5 + 6*a^3*b^6*c^2*d^2*g^5 - 4*a^4*b^5*c*d^3*g^5 + a^5*b^4*d^4*g^5)*x^3 + 6*(a^2*b^7*c^4*g^5 - 4*a^3*b^6*c^3*d*g^5 + 6*a^4*b^5*c^2*d^2*g^5 - 4*a^5*b^4*c*d^3*g^5 + a^6*b^3*d^4*g^5)*x^2 + 4*(a^3*b^6*c^4*g^5 - 4*a^4*b^5*c^3*d*g^5 + 6*a^5*b^4*c^2*d^2*g^5 - 4*a^6*b^3*c*d^3*g^5 + a^7*b^2*d^4*g^5)*x))*B^2 - 1/12*A*B*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 6*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)) - 1/4*B^2*log(d^2*e*x^2/(b^2*x^2 + 2*a*b*x + a^2) + 2*c*d*e*x/(b^2*x^2 + 2*a*b*x + a^2) + c^2*e/(b^2*x^2 + 2*a*b*x + a^2))^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5) - 1/4*A^2/(b^5*g^5*x^4 + 4*a*b^4*g^5*x^3 + 6*a^2*b^3*g^5*x^2 + 4*a^3*b^2*g^5*x + a^4*b*g^5)$$

**Fricas [B]** time = 1.19125, size = 2229, normalized size = 4.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g)^5,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/72*(9*(2*A^2 - 2*A*B + B^2)*b^4*c^4 - 8*(9*A^2 - 12*A*B + 8*B^2)*a*b^3*c^3*d \\ & + 108*(A^2 - 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 72*(A^2 - 4*A*B + 8*B^2)*a^3*b*c*d^3 \\ & + (18*A^2 - 150*A*B + 415*B^2)*a^4*d^4 + 12*((6*A*B - 25*B^2)*b^4*c*d^3 - (6*A*B - 25*B^2)*a*b^3*d^4)*x^3 \\ & - 6*((6*A*B - 13*B^2)*b^4*c^2*d^2 - 16*(3*A*B - 11*B^2)*a*b^3*c*d^3 + (42*A*B - 163*B^2)*a^2*b^2*d^4)*x^2 \\ & - 18*(B^2*b^4*d^4*x^4 + 4*B^2*a*b^3*d^4*x^3 + 6*B^2*a^2*b^2*d^4*x^2 + 4*B^2*a^3*b*d^4*x - B^2*b^4*c^4 \\ & + 4*B^2*a*b^3*c^3*d - 6*B^2*a^2*b^2*c^2*d^2 + 4*B^2*a^3*b*c*d^3)*\log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))^2 \\ & + 4*((6*A*B - 7*B^2)*b^4*c^3*d - 12*(3*A*B - 5*B^2)*a*b^3*c^2*d^2 + 108*(A*B - 3*B^2)*a^2*b^2*c*d^3 \\ & - (78*A*B - 271*B^2)*a^3*b*d^4)*x - 6*((6*A*B - 25*B^2)*b^4*d^4*x^4 - 3*(2*A*B - B^2)*b^4*c^4 \\ & + 8*(3*A*B - 2*B^2)*a*b^3*c^3*d - 36*(A*B - B^2)*a^2*b^2*c^2*d^2 + 24*(A*B - 2*B^2)*a^3*b*c*d^3 - 4 \\ & *(3*B^2*b^4*c*d^3 - 2*(3*A*B - 11*B^2)*a*b^3*d^4)*x^3 + 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 \\ & + 6*(A*B - 3*B^2)*a^2*b^2*d^4)*x^2 - 4*(B^2*b^4*c^3*d - 6*B^2*a*b^3*c^2*d^2 + 18*B^2*a^2*b^2*c*d^3 \\ & - 6*(A*B - 2*B^2)*a^3*b*d^4)*x*\log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)))/((b^9*c^4 - 4*a*b^8*c^3*d \\ & + 6*a^2*b^7*c^2*d^2 - 4*a^3*b^6*c*d^3 + a^4*b^5*d^4)*g^5*x^4 + 4*(a*b^8*c^4 - 4*a^2*b^7*c^3*d \\ & + 6*a^3*b^6*c^2*d^2 - 4*a^4*b^5*c*d^3 + a^5*b^4*d^4)*g^5*x^3 + 6*(a^2*b^7*c^4 - 4*a^3*b^6*c^3*d \\ & + 6*a^4*b^5*c^2*d^2 - 4*a^5*b^4*c*d^3 + a^6*b^3*d^4)*g^5*x^2 + 4*(a^3*b^6*c^4 - 4*a^4*b^5*c^3*d \\ & + 6*a^5*b^4*c^2*d^2 - 4*a^6*b^3*c*d^3 + a^7*b^2*d^4)*g^5*x + (a^4*b^5*c^4 - 4*a^5*b^4*c^3*d \\ & + 6*a^6*b^3*c^2*d^2 - 4*a^7*b^2*c*d^3 + a^8*b*d^4)*g^5) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2))\*\*2/(b\*g\*x+a\*g)\*\*5,x)

[Out] Timed out

**Giac [A]** time = 1.68893, size = 1172, normalized size = 2.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2/(b\*g\*x+a\*g)^5,x, algorithm="giac")

[Out] 
$$\begin{aligned} & 1/4*(B^2*d^4/(b^5*c^4*g^5 - 4*a*b^4*c^3*d*g^5 + 6*a^2*b^3*c^2*d^2*g^5 - 4*a^3*b^2*c*d^3*g^5 \\ & + a^4*b*d^4*g^5) - B^2/((b*g*x + a*g)^4*b*g))*\log((b^2*c^2*g^2/(b*g*x + a*g)^2 - 2*a*b*c*d*g^2/(b*g*x + a*g)^2 + a^2*d^2*g^2/(b*g*x + \end{aligned}$$

$$\begin{aligned}
& a^2g^2 + 2b^2cdg/(b^2gx + ag) - 2ad^2g/(b^2gx + ag) + d^2/b^2 - \\
& 1/12*(12B^2d^3/((b^3c^3g^3 - 3a^2b^2c^2dg^3 + 3a^2b^2cd^2g^3 - a^3d^3g^3)*(b^2gx + ag)*b^2g) - 6B^2d^2/((b^2c^2g - 2a^2b^2cdg + a^2d^2g)*(b^2gx + ag)^2*b^2g^2) + 4B^2d/((b^2gx + ag)^3*(b^2c - a^2d)*b^2g^2) \\
& + 3*(2A^2B^2b^3g^3 + B^2b^3g^3)/((b^2gx + ag)^4*b^4g^4))*\log((b^2c^2g^2/(b^2gx + ag)^2 - 2a^2b^2cdg^2/(b^2gx + ag)^2 + a^2d^2g^2/(b^2gx + ag)^2 + 2b^2cdg/(b^2gx + ag) - 2ad^2g/(b^2gx + ag) + d^2)/b^2) + 1/ \\
& 6*(6A^2B^2d^4 - 19B^2d^4)*\log(-b^2cg/(b^2gx + ag) + adg/(b^2gx + ag) - \\
& d)/(b^5c^4g^5 - 4a^2b^4c^3dg^5 + 6a^2b^3c^2d^2g^5 - 4a^3b^2cd^3g^5 + a^4b^2d^4g^5) - 1/6*(6A^2B^2d^3 - 19B^2d^3)/((b^3c^3g^3 - 3a^2b^2c^2dg^3 + 3a^2b^2cd^2g^3 - a^3d^3g^3)*(b^2gx + ag)*b^2g) + 1/12 \\
& *(6A^2B^2b^2d^2 - 7B^2b^2d^2)/((b^2c^2g - 2a^2b^2cdg + a^2d^2g)*(b^2gx + ag)^2*b^2g^2) - 1/18*(6A^2B^2b^2dg - B^2b^2dg)/((b^2gx + ag)^3*(b^2c - a^2d)*b^3g^3) - 1/8*(2A^2b^3g^3 + 2A^2B^2b^3g^3 + B^2b^3g^3)/((b^2gx + ag)^4*b^4g^4)
\end{aligned}$$



$$3.219 \quad \int \frac{(ag+bgx)^2}{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable}\left(\frac{(ag + bgx)^2}{B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]), x]

**Rubi [A]** time = 0.202201, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(ag + bgx)^2}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]), x]

[Out] a^2\*g^2\*Defer[Int][(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^(-1), x] + 2\*a\*b\*g^2\*Defer[Int][x/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]), x] + b^2\*g^2\*Defer[Int][x^2/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]), x]

Rubi steps

$$\begin{aligned} \int \frac{(ag + bgx)^2}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx &= \int \left( \frac{a^2g^2}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} + \frac{2abg^2x}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} + \frac{b^2g^2x^2}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} \right) dx \\ &= (a^2g^2) \int \frac{1}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx + (2abg^2) \int \frac{x}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx + (b^2g^2) \int \frac{x^2}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.156989, size = 0, normalized size = 0.

$$\int \frac{(ag + bgx)^2}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]), x]

[Out] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]), x]

**Maple [A]** time = 1.127, size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( \frac{e(dx + c)^2}{(bx + a)^2} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2)),x)

[Out] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{B \log \left( \frac{e(dx + c)^2}{(bx + a)^2} \right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2)),x, algorithm="maxima")

[Out] integrate((b\*g\*x + a\*g)^2/(B\*log((d\*x + c)^2\*e/(b\*x + a)^2) + A), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b^2 g^2 x^2 + 2 a b g^2 x + a^2 g^2}{B \log \left( \frac{d^2 e x^2 + 2 c d e x + c^2 e}{b^2 x^2 + 2 a b x + a^2} \right) + A}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2)),x, algorithm="fricas")

[Out] integral((b^2\*g^2\*x^2 + 2\*a\*b\*g^2\*x + a^2\*g^2)/(B\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2)) + A), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)\*\*2/(A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{B \log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2)),x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)^2/(B\*log((d\*x + c)^2\*e/(b\*x + a)^2) + A), x)

$$3.220 \quad \int \frac{ag+bgx}{A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{ag + bgx}{B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]), x]

**Rubi [A]** time = 0.100778, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{ag + bgx}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]), x]

[Out] a\*g\*Defer[Int][(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^(-1), x] + b\*g\*Defer[Int][x/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]), x]

Rubi steps

$$\begin{aligned} \int \frac{ag + bgx}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx &= \int \left( \frac{ag}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} + \frac{bgx}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} \right) dx \\ &= (ag) \int \frac{1}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx + (bg) \int \frac{x}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.115382, size = 0, normalized size = 0.

$$\int \frac{ag + bgx}{A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]), x]

[Out] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2]), x]

**Maple [A]** time = 0.962, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln \left( \frac{e(dx+c)^2}{(bx+a)^2} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)`

[Out] `int((b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bgx + ag}{B \log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxima")`

[Out] `integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{bgx + ag}{B \log\left(\frac{d^2ex^2 + 2cdex + c^2e}{b^2x^2 + 2abx + a^2}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fricas")`

[Out] `integral((b*g*x + a*g)/(B*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2)) + A), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bgx + ag}{B \log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)/(B*log((d*x + c)^2*e/(b*x + a)^2) + A), x)
```

$$3.221 \quad \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable}\left(\frac{1}{(ag+bgx)\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)}, x\right)$$

[Out] Unintegrable[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])), x]

**Rubi [A]** time = 0.0731757, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} dx = \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} dx$$

**Mathematica [A]** time = 0.0703185, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])), x]

[Out] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])), x]

**Maple [A]** time = 1.298, size = 0, normalized size = 0.

$$\int \frac{1}{bgx+ag}\left(A+B \ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)`

[Out] `int(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag) \left( B \log \left( \frac{(dx+c)^2 e}{(bx+a)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxima")`

[Out] `integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{Abgx + Aag + (Bbgx + Bag) \log \left( \frac{d^2ex^2 + 2cdex + c^2e}{b^2x^2 + 2abx + a^2} \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fricas")`

[Out] `integral(1/(A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag) \left( B \log \left( \frac{(dx+c)^2 e}{(bx+a)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

$$3.222 \quad \int \frac{1}{(ag+bgx)^2 \left( A+B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)} dx$$

**Optimal.** Leaf size=91

$$\frac{e^{-\frac{A}{2B}(c+dx)} \operatorname{Ei} \left( \frac{A+B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right)}{2B} \right)}{2Bg^2(a+bx)(bc-ad) \sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

[Out] -((c + d\*x)\*ExpIntegralEi[(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2)]/(2\*B)))/(2\*B\*(b\*c - a\*d)\*E^(A/(2\*B))\*g^2\*(a + b\*x)\*Sqrt[(e\*(c + d\*x)^2]/(a + b\*x)^2])

**Rubi [F]** time = 0.0872545, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag + bgx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2))),x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2))), x]

Rubi steps

$$\int \frac{1}{(ag + bgx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)} dx = \int \frac{1}{(ag + bgx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)} dx$$

**Mathematica [F]** time = 0.076942, size = 0, normalized size = 0.

$$\int \frac{1}{(ag + bgx)^2 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2))),x]

[Out] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2))), x]

**Maple [F]** time = 1.228, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( A + B \ln \left( \frac{e(dx + c)^2}{(bx + a)^2} \right) \right)^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2)),x)

[Out] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2)),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(dx+c)^2 e}{(bx+a)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2)),x, algorithm="maxima")

[Out] integrate(1/((b\*g\*x + a\*g)^2\*(B\*log((d\*x + c)^2\*e/(b\*x + a)^2) + A)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{Ab^2g^2x^2 + 2Aabg^2x + Aa^2g^2 + (Bb^2g^2x^2 + 2Babg^2x + Ba^2g^2) \log \left( \frac{d^2ex^2 + 2cdex + c^2e}{b^2x^2 + 2abx + a^2} \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2)),x, algorithm="fricas")

[Out] integral(1/(A\*b^2\*g^2\*x^2 + 2\*A\*a\*b\*g^2\*x + A\*a^2\*g^2 + (B\*b^2\*g^2\*x^2 + 2\*B\*a\*b\*g^2\*x + B\*a^2\*g^2)\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2))), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)\*\*2/(A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2)),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(dx+c)^2 e}{(bx+a)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="gi  
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

$$3.223 \quad \int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)} dx$$

**Optimal.** Leaf size=151

$$\frac{de^{-\frac{A}{2B}}(c+dx)\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{2B}\right)}{2Bg^3(a+bx)(bc-ad)^2\sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} - \frac{be^{-\frac{A}{B}}\text{Ei}\left(\frac{A+B\log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)}{B}\right)}{2Beg^3(bc-ad)^2}$$

[Out] (d\*(c + d\*x)\*ExpIntegralEi[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/(2\*B)]) / (2\*B\*(b\*c - a\*d)^2\*E^(A/(2\*B))\*g^3\*(a + b\*x)\*Sqrt[(e\*(c + d\*x)^2)/(a + b\*x)^2]) - (b\*ExpIntegralEi[(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])/B])/(2\*B\*(b\*c - a\*d)^2\*e\*E^(A/B)\*g^3)

**Rubi [F]** time = 0.0736238, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag + bgx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])),x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])), x]

Rubi steps

$$\int \frac{1}{(ag + bgx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)} dx = \int \frac{1}{(ag + bgx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)} dx$$

**Mathematica [F]** time = 0.0806264, size = 0, normalized size = 0.

$$\int \frac{1}{(ag + bgx)^3 \left( A + B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])),x]

[Out] Integrate[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])), x]

**Maple [F]** time = 1.371, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( A + B \ln \left( \frac{e(dx + c)^2}{(bx + a)^2} \right) \right)^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)`

[Out] `int(1/(b*g*x+a*g)^3/(A+B*ln(e*(d*x+c)^2/(b*x+a)^2)),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log \left( \frac{(dx+c)^2 e}{(bx+a)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="maxima")`

[Out] `integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{Ab^3g^3x^3 + 3Aab^2g^3x^2 + 3Aa^2bg^3x + Aa^3g^3 + (Bb^3g^3x^3 + 3Bab^2g^3x^2 + 3Ba^2bg^3x + Ba^3g^3) \log \left( \frac{d^2ex^2 + 2cdex + c^2}{b^2x^2 + 2abx + a^2} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="fricas")`

[Out] `integral(1/(A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3 + (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((d^2*e*x^2 + 2*c*d*e*x + c^2*e)/(b^2*x^2 + 2*a*b*x + a^2))), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*g*x+a*g)**3/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2)),x)`

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log \left( \frac{(dx+c)^2 e}{(bx+a)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^3/(A+B*log(e*(d*x+c)^2/(b*x+a)^2)),x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^3*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)), x)
```

$$3.224 \quad \int \frac{(ag+bgx)^2}{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable}\left(\frac{(ag+bgx)^2}{\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2, x]

**Rubi [A]** time = 0.211595, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0$ ., Rules used = {}

$$\int \frac{(ag+bgx)^2}{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2, x]

[Out] a^2\*g^2\*Defer[Int][(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^(-2), x] + 2\*a\*b\*g^2\*Defer[Int][x/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2, x] + b^2\*g^2\*Defer[Int][x^2/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{(ag+bgx)^2}{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx &= \int \left( \frac{a^2g^2}{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} + \frac{2abg^2x}{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} + \frac{b^2g^2x^2}{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} \right) dx \\ &= (a^2g^2) \int \frac{1}{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx + (2abg^2) \int \frac{x}{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx + (b^2g^2) \int \frac{x^2}{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.460496, size = 0, normalized size = 0.

$$\int \frac{(ag+bgx)^2}{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2, x]

[Out] Integrate[(a\*g + b\*g\*x)^2/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2, x]



**Maple [A]** time = 1., size = 0, normalized size = 0.

$$\int (bgx + ag)^2 \left( A + B \ln \left( \frac{e(dx + c)^2}{(bx + a)^2} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

[Out] int((b\*g\*x+a\*g)^2/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{b^3 dg^2 x^4 + a^3 cg^2 + (b^3 cg^2 + 3 ab^2 dg^2)x^3 + 3(ab^2 cg^2 + a^2 bdg^2)x^2 + (3 a^2 bcg^2 + a^3 dg^2)x}{2(2(bc - ad)B^2 \log(bx + a) - 2(bc - ad)B^2 \log(dx + c) - (bc - ad)AB - (bc \log(e) - ad \log(e))B^2)} + \int \frac{1}{2(2(bc - ad)B^2 \log(bx + a) - 2(bc - ad)B^2 \log(dx + c) - (bc - ad)AB - (bc \log(e) - ad \log(e))B^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="maxima")

[Out] -1/2\*(b^3\*d\*g^2\*x^4 + a^3\*c\*g^2 + (b^3\*c\*g^2 + 3\*a\*b^2\*d\*g^2)\*x^3 + 3\*(a\*b^2\*c\*g^2 + a^2\*b\*d\*g^2)\*x^2 + (3\*a^2\*b\*c\*g^2 + a^3\*d\*g^2)\*x)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) - (b\*c - a\*d)\*A\*B - (b\*c\*log(e) - a\*d\*log(e))\*B^2) + integrate(1/2\*(4\*b^3\*d\*g^2\*x^3 + 3\*a^2\*b\*c\*g^2 + a^3\*d\*g^2 + 3\*(b^3\*c\*g^2 + 3\*a\*b^2\*d\*g^2)\*x^2 + 6\*(a\*b^2\*c\*g^2 + a^2\*b\*d\*g^2)\*x)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) - (b\*c - a\*d)\*A\*B - (b\*c\*log(e) - a\*d\*log(e))\*B^2), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{b^2 g^2 x^2 + 2 abg^2 x + a^2 g^2}{B^2 \log \left( \frac{d^2 ex^2 + 2 cdex + c^2 e}{b^2 x^2 + 2 abx + a^2} \right)^2 + 2 AB \log \left( \frac{d^2 ex^2 + 2 cdex + c^2 e}{b^2 x^2 + 2 abx + a^2} \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="fricas")

[Out] integral((b^2\*g^2\*x^2 + 2\*a\*b\*g^2\*x + a^2\*g^2)/(B^2\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2))^2 + 2\*A\*B\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2)) + A^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bgx + ag)^2}{\left(B \log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((b*g*x + a*g)^2/(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2, x)
```

$$3.225 \quad \int \frac{ag+bgx}{\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$$

**Optimal.** Leaf size=34

$$\text{Unintegrable}\left(\frac{ag + bgx}{\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right) + A\right)^2}, x\right)$$

[Out] Unintegrable[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2, x]

**Rubi [A]** time = 0.109584, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{ag + bgx}{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2, x]

[Out] a\*g\*Defer[Int][(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^(-2), x] + b\*g\*Defer[Int][x/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{ag + bgx}{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx &= \int \left( \frac{ag}{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} + \frac{bgx}{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} \right) dx \\ &= (ag) \int \frac{1}{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx + (bg) \int \frac{x}{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.341781, size = 0, normalized size = 0.

$$\int \frac{ag + bgx}{\left(A + B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2, x]

[Out] Integrate[(a\*g + b\*g\*x)/(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2, x]

**Maple [A]** time = 1.026, size = 0, normalized size = 0.

$$\int (bgx + ag) \left( A + B \ln \left( \frac{e(dx + c)^2}{(bx + a)^2} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*g\*x+a\*g)/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

[Out] int((b\*g\*x+a\*g)/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{b^2dgx^3 + a^2cg + (b^2cg + 2abdg)x^2 + (2abcbg + a^2dg)x}{2(2(bc - ad)B^2 \log(bx + a) - 2(bc - ad)B^2 \log(dx + c) - (bc - ad)AB - (bc \log(e) - ad \log(e))B^2)} + \int \frac{1}{2(2(bc - ad)B^2 \log(bx + a) - 2(bc - ad)B^2 \log(dx + c) - (bc - ad)AB - (bc \log(e) - ad \log(e))B^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="maxima")

[Out] -1/2\*(b^2\*d\*g\*x^3 + a^2\*c\*g + (b^2\*c\*g + 2\*a\*b\*d\*g)\*x^2 + (2\*a\*b\*c\*g + a^2\*d\*g)\*x)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) - (b\*c - a\*d)\*A\*B - (b\*c\*log(e) - a\*d\*log(e))\*B^2) + integrate(1/2\*(3\*b^2\*d\*g\*x^2 + 2\*a\*b\*c\*g + a^2\*d\*g + 2\*(b^2\*c\*g + 2\*a\*b\*d\*g)\*x)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) - (b\*c - a\*d)\*A\*B - (b\*c\*log(e) - a\*d\*log(e))\*B^2), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{bgx + ag}{B^2 \log \left( \frac{d^2ex^2 + 2cdex + c^2e}{b^2x^2 + 2abx + a^2} \right)^2 + 2AB \log \left( \frac{d^2ex^2 + 2cdex + c^2e}{b^2x^2 + 2abx + a^2} \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="fricas")

[Out] integral((b\*g\*x + a\*g)/(B^2\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2))^2 + 2\*A\*B\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2)) + A^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2))\*\*2,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{bgx + ag}{\left(B \log\left(\frac{(dx+c)^2 e}{(bx+a)^2}\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*g\*x+a\*g)/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="giac")

[Out] integrate((b\*g\*x + a\*g)/(B\*log((d\*x + c)^2\*e/(b\*x + a)^2) + A)^2, x)

$$3.226 \quad \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$$

**Optimal.** Leaf size=36

$$\text{Unintegrable}\left(\frac{1}{(ag+bgx)\left(B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2), x]

**Rubi [A]** time = 0.0821686, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx = \int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$$

**Mathematica [A]** time = 0.143205, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)\left(A+B \log\left(\frac{e(c+dx)^2}{(a+bx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2), x]

[Out] Integrate[1/((a\*g + b\*g\*x)\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2), x]

**Maple [A]** time = 1.024, size = 0, normalized size = 0.

$$\int \frac{1}{bgx+ag}\left(A+B \ln\left(\frac{e(dx+c)^2}{(bx+a)^2}\right)\right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

[Out] int(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$d \int \frac{1}{2(2(bcg - adg)B^2 \log(bx + a) - 2(bcg - adg)B^2 \log(dx + c) - (bcg - adg)AB - (bcg \log(e) - adg \log(e))B^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="maxima")

[Out] d\*integrate(1/2/(2\*(b\*c\*g - a\*d\*g)\*B^2\*log(b\*x + a) - 2\*(b\*c\*g - a\*d\*g)\*B^2\*log(d\*x + c) - (b\*c\*g - a\*d\*g)\*A\*B - (b\*c\*g\*log(e) - a\*d\*g\*log(e))\*B^2), x) - 1/2\*(d\*x + c)/(2\*(b\*c\*g - a\*d\*g)\*B^2\*log(b\*x + a) - 2\*(b\*c\*g - a\*d\*g)\*B^2\*log(d\*x + c) - (b\*c\*g - a\*d\*g)\*A\*B - (b\*c\*g\*log(e) - a\*d\*g\*log(e))\*B^2)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2bgx + A^2ag + (B^2bgx + B^2ag) \log\left(\frac{d^2ex^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)^2 + 2(ABbgx + ABag) \log\left(\frac{d^2ex^2+2cdex+c^2e}{b^2x^2+2abx+a^2}\right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="fricas")

[Out] integral(1/(A^2\*b\*g\*x + A^2\*a\*g + (B^2\*b\*g\*x + B^2\*a\*g)\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2))^2 + 2\*(A\*B\*b\*g\*x + A\*B\*a\*g)\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2))), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)/(A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag) \left( B \log\left(\frac{(dx+c)^2e}{(bx+a)^2}\right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="gi  
ac")
```

```
[Out] integrate(1/((b*g*x + a*g)*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```



$$3.227 \quad \int \frac{1}{(ag+bgx)^2 \left( A+B \log\left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2} dx$$

**Optimal.** Leaf size=147

$$\frac{c+dx}{2Bg^2(a+bx)(bc-ad) \left( B \log\left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)} - \frac{e^{-\frac{A}{2B}(c+dx)} \text{Ei}\left( \frac{A+B \log\left( \frac{e(c+dx)^2}{(a+bx)^2} \right)}{2B} \right)}{4B^2g^2(a+bx)(bc-ad) \sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}}$$

[Out]  $-\left( (c+dx) \text{ExpIntegralEi}\left[ \frac{A+B \log\left( \frac{e(c+dx)^2}{(a+bx)^2} \right)}{2B} \right] \right) / \left( 4B^2(bc-ad) e^{A/(2B)} g^2(a+bx) \sqrt{\frac{e(c+dx)^2}{(a+bx)^2}} + (c+dx) / (2B(bc-ad) g^2(a+bx) (A+B \log\left( \frac{e(c+dx)^2}{(a+bx)^2} \right)) \right)$

**Rubi [F]** time = 0.0930418, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)^2 \left( A+B \log\left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)^2 \left( A+B \log\left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2} dx = \int \frac{1}{(ag+bgx)^2 \left( A+B \log\left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2} dx$$

**Mathematica [F]** time = 0.182028, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)^2 \left( A+B \log\left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2), x]

[Out] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2)/(a + b\*x)^2])^2), x]

**Maple [F]** time = 1.406, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2} \left( A + B \ln \left( \frac{e(dx + c)^2}{(bx + a)^2} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

[Out] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2 \left( (abcg^2 - a^2dg^2)AB + (abcg^2 \log(e) - a^2dg^2 \log(e))B^2 + \left( (b^2cg^2 - abdg^2)AB + (b^2cg^2 \log(e) - abdg^2 \log(e))B^2 \right) x - 2 \right) dx +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="maxima")

[Out] 1/2\*(d\*x + c)/((a\*b\*c\*g^2 - a^2\*d\*g^2)\*A\*B + (a\*b\*c\*g^2\*log(e) - a^2\*d\*g^2\*log(e))\*B^2 + ((b^2\*c\*g^2 - a\*b\*d\*g^2)\*A\*B + (b^2\*c\*g^2\*log(e) - a\*b\*d\*g^2\*log(e))\*B^2)\*x - 2\*((b^2\*c\*g^2 - a\*b\*d\*g^2)\*B^2\*x + (a\*b\*c\*g^2 - a^2\*d\*g^2)\*B^2)\*log(b\*x + a) + 2\*((b^2\*c\*g^2 - a\*b\*d\*g^2)\*B^2\*x + (a\*b\*c\*g^2 - a^2\*d\*g^2)\*B^2)\*log(d\*x + c) + integrate(1/2/(B^2\*a^2\*g^2\*log(e) + A\*B\*a^2\*g^2 + (B^2\*b^2\*g^2\*log(e) + A\*B\*b^2\*g^2)\*x^2 + 2\*(B^2\*a\*b\*g^2\*log(e) + A\*B\*a\*b\*g^2)\*x - 2\*(B^2\*b^2\*g^2\*x^2 + 2\*B^2\*a\*b\*g^2\*x + B^2\*a^2\*g^2)\*log(b\*x + a) + 2\*(B^2\*b^2\*g^2\*x^2 + 2\*B^2\*a\*b\*g^2\*x + B^2\*a^2\*g^2)\*log(d\*x + c)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2b^2g^2x^2 + 2A^2abg^2x + A^2a^2g^2 + (B^2b^2g^2x^2 + 2B^2abg^2x + B^2a^2g^2) \log \left( \frac{d^2ex^2 + 2cdex + c^2e}{b^2x^2 + 2abx + a^2} \right)^2} + 2(ABb^2g^2x^2 + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^2/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="fricas")

[Out] integral(1/(A^2\*b^2\*g^2\*x^2 + 2\*A^2\*a\*b\*g^2\*x + A^2\*a^2\*g^2 + (B^2\*b^2\*g^2\*x^2 + 2\*B^2\*a\*b\*g^2\*x + B^2\*a^2\*g^2)\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2))^2 + 2\*(A\*B\*b^2\*g^2\*x^2 + 2\*A\*B\*a\*b\*g^2\*x + A\*B\*a^2\*g^2)\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2))), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)**2/(A+B*ln(e*(d*x+c)**2/(b*x+a)**2))**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(dx+c)^2 e}{(bx+a)^2} \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*g*x+a*g)^2/(A+B*log(e*(d*x+c)^2/(b*x+a)^2))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*g*x + a*g)^2*(B*log((d*x + c)^2*e/(b*x + a)^2) + A)^2), x)
```

$$3.228 \quad \int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2} dx$$

**Optimal.** Leaf size=206

$$\frac{de^{-\frac{A}{2B}}(c+dx) \operatorname{Ei} \left( \frac{A+B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right)}{2B} \right) - be^{-\frac{A}{B}} \operatorname{Ei} \left( \frac{A+B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right)}{B} \right)}{4B^2 g^3 (a+bx)(bc-ad)^2 \sqrt{\frac{e(c+dx)^2}{(a+bx)^2}}} - \frac{1}{2B^2 e g^3 (bc-ad)^2} + \frac{c+dx}{2B g^3 (a+bx)^2 (bc-ad) \left( B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) + A \right)}$$

[Out] (d\*(c + d\*x)\*ExpIntegralEi[(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2)]/(2\*B)))/(4\*B^2\*(b\*c - a\*d)^2\*E^(A/(2\*B))\*g^3\*(a + b\*x)\*Sqrt[(e\*(c + d\*x)^2)/(a + b\*x)^2]) - (b\*ExpIntegralEi[(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2)]/B))/(2\*B^2\*(b\*c - a\*d)^2\*e\*E^(A/B)\*g^3) + (c + d\*x)/(2\*B\*(b\*c - a\*d)\*g^3\*(a + b\*x)^2\*(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2]))

**Rubi [F]** time = 0.0850021, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2])^2), x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2])^2), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2} dx = \int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2} dx$$

**Mathematica [F]** time = 0.354644, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)^3 \left( A+B \log \left( \frac{e(c+dx)^2}{(a+bx)^2} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2])^2), x]

[Out] Integrate[1/((a\*g + b\*g\*x)^3\*(A + B\*Log[(e\*(c + d\*x)^2]/(a + b\*x)^2])^2), x]

**Maple [F]** time = 1.622, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3} \left( A + B \ln \left( \frac{e(dx + c)^2}{(bx + a)^2} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^3/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

[Out] int(1/(b\*g\*x+a\*g)^3/(A+B\*ln(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="maxima")

[Out] 1/2\*(d\*x + c)/((a^2\*b\*c\*g^3 - a^3\*d\*g^3)\*A\*B + (a^2\*b\*c\*g^3\*log(e) - a^3\*d\*g^3\*log(e))\*B^2 + ((b^3\*c\*g^3 - a\*b^2\*d\*g^3)\*A\*B + (b^3\*c\*g^3\*log(e) - a\*b^2\*d\*g^3\*log(e))\*B^2)\*x^2 + 2\*((a\*b^2\*c\*g^3 - a^2\*b\*d\*g^3)\*A\*B + (a\*b^2\*c\*g^3\*log(e) - a^2\*b\*d\*g^3\*log(e))\*B^2)\*x - 2\*((b^3\*c\*g^3 - a\*b^2\*d\*g^3)\*B^2\*x^2 + 2\*(a\*b^2\*c\*g^3 - a^2\*b\*d\*g^3)\*B^2\*x + (a^2\*b\*c\*g^3 - a^3\*d\*g^3)\*B^2)\*log(b\*x + a) + 2\*((b^3\*c\*g^3 - a\*b^2\*d\*g^3)\*B^2\*x^2 + 2\*(a\*b^2\*c\*g^3 - a^2\*b\*d\*g^3)\*B^2\*x + (a^2\*b\*c\*g^3 - a^3\*d\*g^3)\*B^2)\*log(d\*x + c) - integrate(-1/2\*(b\*d\*x + 2\*b\*c - a\*d)/((b^4\*c\*g^3 - a\*b^3\*d\*g^3)\*A\*B + (b^4\*c\*g^3\*log(e) - a\*b^3\*d\*g^3\*log(e))\*B^2)\*x^3 + (a^3\*b\*c\*g^3 - a^4\*d\*g^3)\*A\*B + (a^3\*b\*c\*g^3\*log(e) - a^4\*d\*g^3\*log(e))\*B^2 + 3\*((a\*b^3\*c\*g^3 - a^2\*b^2\*d\*g^3)\*A\*B + (a\*b^3\*c\*g^3\*log(e) - a^2\*b^2\*d\*g^3\*log(e))\*B^2)\*x^2 + 3\*((a^2\*b^2\*c\*g^3 - a^3\*b\*d\*g^3)\*A\*B + (a^2\*b^2\*c\*g^3\*log(e) - a^3\*b\*d\*g^3\*log(e))\*B^2)\*x - 2\*((b^4\*c\*g^3 - a\*b^3\*d\*g^3)\*B^2\*x^3 + 3\*(a\*b^3\*c\*g^3 - a^2\*b^2\*d\*g^3)\*B^2\*x^2 + 3\*(a^2\*b^2\*c\*g^3 - a^3\*b\*d\*g^3)\*B^2\*x + (a^3\*b\*c\*g^3 - a^4\*d\*g^3)\*B^2)\*log(b\*x + a) + 2\*((b^4\*c\*g^3 - a\*b^3\*d\*g^3)\*B^2\*x^3 + 3\*(a\*b^3\*c\*g^3 - a^2\*b^2\*d\*g^3)\*B^2\*x^2 + 3\*(a^2\*b^2\*c\*g^3 - a^3\*b\*d\*g^3)\*B^2\*x + (a^3\*b\*c\*g^3 - a^4\*d\*g^3)\*B^2)\*log(d\*x + c)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2 b^3 g^3 x^3 + 3 A^2 a b^2 g^3 x^2 + 3 A^2 a^2 b g^3 x + A^2 a^3 g^3 + (B^2 b^3 g^3 x^3 + 3 B^2 a b^2 g^3 x^2 + 3 B^2 a^2 b g^3 x + B^2 a^3 g^3) \log \left( \frac{e(dx + c)^2}{(bx + a)^2} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="fricas")

[Out] integral(1/(A^2\*b^3\*g^3\*x^3 + 3\*A^2\*a\*b^2\*g^3\*x^2 + 3\*A^2\*a^2\*b\*g^3\*x + A^2\*a^3\*g^3 + (B^2\*b^3\*g^3\*x^3 + 3\*B^2\*a\*b^2\*g^3\*x^2 + 3\*B^2\*a^2\*b\*g^3\*x + B^2\*a^3\*g^3)\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2)))^2)

+ 2\*(A\*B\*b^3\*g^3\*x^3 + 3\*A\*B\*a\*b^2\*g^3\*x^2 + 3\*A\*B\*a^2\*b\*g^3\*x + A\*B\*a^3\*g^3)\*log((d^2\*e\*x^2 + 2\*c\*d\*e\*x + c^2\*e)/(b^2\*x^2 + 2\*a\*b\*x + a^2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)\*\*3/(A+B\*ln(e\*(d\*x+c)\*\*2/(b\*x+a)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^3 \left( B \log \left( \frac{(dx+c)^2 e}{(bx+a)^2} \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*g\*x+a\*g)^3/(A+B\*log(e\*(d\*x+c)^2/(b\*x+a)^2))^2,x, algorithm="giac")

[Out] integrate(1/((b\*g\*x + a\*g)^3\*(B\*log((d\*x + c)^2\*e/(b\*x + a)^2) + A)^2), x)

$$3.229 \quad \int \frac{1}{(ag+bgx)^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))} dx$$

**Optimal.** Leaf size=96

$$\frac{e^{\frac{A}{Bn}}(c+dx)(e(a+bx)^n(c+dx)^{-n})^{\frac{1}{n}} \operatorname{Ei}\left(-\frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{Bn}\right)}{Bg^2n(a+bx)(bc-ad)}$$

[Out] (E^(A/(B\*n)))\*(c + d\*x)\*((e\*(a + b\*x)^n)/(c + d\*x)^n)^(-1)\*ExpIntegralEi[-((A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(B\*n))]/(B\*(b\*c - a\*d)\*g^2\*n\*(a + b\*x))

**Rubi [F]** time = 0.102474, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(ag+bgx)^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))} dx$$

Verification is Not applicable to the result.

[In] Int[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])),x]

[Out] Defer[Int][1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])), x]

Rubi steps

$$\int \frac{1}{(ag+bgx)^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))} dx = \int \frac{1}{(ag+bgx)^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))} dx$$

**Mathematica [F]** time = 0.0590735, size = 0, normalized size = 0.

$$\int \frac{1}{(ag+bgx)^2(A+B \log(e(a+bx)^n(c+dx)^{-n}))} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])),x]

[Out] Integrate[1/((a\*g + b\*g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])), x]

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{1}{(bgx+ag)^2} \left( A + B \ln \left( \frac{e(bx+a)^n}{(dx+c)^n} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*g\*x+a\*g)^2/(A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n))),x)

[Out]  $\text{int}(1/(b*g*x+a*g)^2/(A+B*\ln(e*(b*x+a)^n/((d*x+c)^n))),x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(1/(b*g*x+a*g)^2/(A+B*\log(e*(b*x+a)^n/((d*x+c)^n))),x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(1/((b*g*x + a*g)^2*(B*\log((b*x + a)^n*e/(d*x + c)^n) + A)), x)$

**Fricas [A]** time = 1.04724, size = 149, normalized size = 1.55

$$\frac{e^{\left(\frac{B \log(e)+A}{Bn}\right)} \log\_integral \left( \frac{(dx+c)e^{\left(-\frac{B \log(e)+A}{Bn}\right)}}{bx+a} \right)}{(Bbc - Bad)g^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(1/(b*g*x+a*g)^2/(A+B*\log(e*(b*x+a)^n/((d*x+c)^n))),x, \text{algorithm}="fricas")$

[Out]  $e^{\left(\frac{B*\log(e) + A}{B*n}\right)}*\log\_integral((d*x + c)*e^{\left(-\frac{B*\log(e) + A}{B*n}\right)}/(b*x + a))/((B*b*c - B*a*d)*g^2*n)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(1/(b*g*x+a*g)**2/(A+B*\ln(e*(b*x+a)**n/((d*x+c)**n))),x)$

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bgx + ag)^2 \left( B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(1/(b*g*x+a*g)^2/(A+B*\log(e*(b*x+a)^n/((d*x+c)^n))),x, \text{algorithm}="giac")$

[Out]  $\text{integrate}(1/((b*g*x + a*g)^2*(B*\log((b*x + a)^n*e/(d*x + c)^n) + A)), x)$



$$3.230 \quad \int (f + gx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right) dx$$

**Optimal.** Leaf size=355

$$\frac{Bg^2x^2(bc - ad)(a^2d^2g^2 - abdg(5df - cg) + b^2(c^2g^2 - 5cdfg + 10d^2f^2))}{10b^3d^3} + \frac{Bgx(bc - ad)(-a^2bd^2g^2(5df - cg) + a^3d^3g^2)}{10b^3d^3}$$

```
[Out] (B*(b*c - a*d)*g*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*d*f*g^2 - c^3*g^3))*x)/(5*b^4*d^4) - (B*(b*c - a*d)*g^2*(a^2*d^2*g^2 - a*b*d*g*(5*d*f - c*g) + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(10*b^3*d^3) - (B*(b*c - a*d)*g^3*(5*b*d*f - b*c*g - a*d*g)*x^3)/(15*b^2*d^2) - (B*(b*c - a*d)*g^4*x^4)/(20*b*d) - (B*(b*f - a*g)^5*Log[a + b*x])/(5*b^5*g) + ((f + g*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*g) + (B*(d*f - c*g)^5*Log[c + d*x])/(5*d^5*g)
```

**Rubi [A]** time = 0.556875, antiderivative size = 339, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2525, 12, 72}

$$\frac{Bg^2x^2(bc - ad)(a^2d^2g^2 - abdg(5df - cg) + b^2(c^2g^2 - 5cdfg + 10d^2f^2))}{10b^3d^3} + \frac{Bgx(-10a^2b^2d^4f^2g + 5a^3bd^4fg^2 - a^4d^4g^2)}{10b^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

```
[Out] (B*g*(10*a*b^3*d^4*f^3 - 10*a^2*b^2*d^4*f^2*g + 5*a^3*b*d^4*f*g^2 - a^4*d^4*g^3 - b^4*c*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*d*f*g^2 - c^3*g^3))*x)/(5*b^4*d^4) - (B*(b*c - a*d)*g^2*(a^2*d^2*g^2 - a*b*d*g*(5*d*f - c*g) + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(10*b^3*d^3) - (B*(b*c - a*d)*g^3*(5*b*d*f - b*c*g - a*d*g)*x^3)/(15*b^2*d^2) - (B*(b*c - a*d)*g^4*x^4)/(20*b*d) - (B*(b*f - a*g)^5*Log[a + b*x])/(5*b^5*g) + ((f + g*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*g) + (B*(d*f - c*g)^5*Log[c + d*x])/(5*d^5*g)
```

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[(((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[(((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int (f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right) dx = \frac{(f + gx)^5 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{5g} - \frac{B \int \frac{(bc - ad)(f + gx)^5}{(a + bx)(c + dx)} dx}{5g}$$

$$= \frac{(f + gx)^5 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{5g} - \frac{(B(bc - ad)) \int \frac{(f + gx)^5}{(a + bx)(c + dx)} dx}{5g}$$

$$= \frac{(f + gx)^5 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{5g} - \frac{(B(bc - ad)) \int \left( \frac{g^2(-a^3d^3g^3 + a^2bd^2g^2(5df - cg) - ab^2dg^2)}{(a + bx)(c + dx)} \right) dx}{5g}$$

$$= \frac{B(bc - ad)g \left( a^3d^3g^3 - a^2bd^2g^2(5df - cg) + ab^2dg \left( 10d^2f^2 - 5cdfg + c^2g^2 \right) \right) - 1}{5b^4d^4}$$

**Mathematica [A]** time = 0.586314, size = 279, normalized size = 0.79

$$\frac{Bg^2x(ad-bc)(6a^2bd^2g^2(-2cg+10df+dgx)-12a^3d^3g^3-2ab^2dg(6c^2g^2-3cdg(10f+gx)+d^2(60f^2+15fgx+2g^2x^2)))+b^3(6c^2dg^2(10f+gx)-12c^3g^3-2cd^2g(60f^2+15fgx+2g^2x^2))}{12b^4d^4}$$

5g

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

```
[Out] ((B*(-(b*c) + a*d)*g^2*x*(-12*a^3*d^3*g^3 + 6*a^2*b*d^2*g^2*(10*d*f - 2*c*g + d*g*x) - 2*a*b^2*d*g*(6*c^2*g^2 - 3*c*d*g*(10*f + g*x) + d^2*(60*f^2 + 15*f*g*x + 2*g^2*x^2)) + b^3*(-12*c^3*g^3 + 6*c^2*d*g^2*(10*f + g*x) - 2*c*d^2*g*(60*f^2 + 15*f*g*x + 2*g^2*x^2) + d^3*(120*f^3 + 60*f^2*g*x + 20*f*g^2*x^2 + 3*g^3*x^3)))/(12*b^4*d^4) - (B*(b*f - a*g)^5*Log[a + b*x])/b^5 + (f + g*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x])) + (B*(d*f - c*g)^5*Log[c + d*x])/d^5)/(5*g)
```

**Maple [B]** time = 0.231, size = 14719, normalized size = 41.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^4*(A+B*ln(e*(b*x+a)/(d*x+c))), x)
```

```
[Out] result too large to display
```

**Maxima [A]** time = 1.22073, size = 801, normalized size = 2.26

$$\frac{1}{5} Ag^4x^5 + Afg^3x^4 + 2Afg^2x^3 + 2Afgx^2 + \left( x \log \left( \frac{bex}{dx + c} + \frac{ae}{dx + c} \right) + \frac{a \log(bx + a)}{b} - \frac{c \log(dx + c)}{d} \right) Bf^4 + 2 \left( x^2 \log \left( \frac{e(bx + a)}{d(x + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))), x, algorithm="maxima")
```

```
[Out] 1/5*A*g^4*x^5 + A*f*g^3*x^4 + 2*A*f^2*g^2*x^3 + 2*A*f^3*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*f^4 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*f^3*g + (2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f^2*g^2 + 1/6*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*f*g^3 + 1/60*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4))*B*g^4 + A*f^4*x
```

---

**Fricas [A]** time = 3.51736, size = 1283, normalized size = 3.61

$$12 Ab^5 d^5 g^4 x^5 + 3 (20 Ab^5 d^5 f g^3 - (Bb^5 cd^4 - Bab^4 d^5) g^4) x^4 + 4 (30 Ab^5 d^5 f^2 g^2 - 5 (Bb^5 cd^4 - Bab^4 d^5) f g^3 + (Bb^5 c^2 d^3 - Bab^4 d^5) f^2 g) x^3 + 6 (20 Ab^5 d^5 f^3 g - 10 (Bb^5 cd^4 - Bab^4 d^5) f^2 g^2 + 5 (Bb^5 c^2 d^3 - Bab^4 d^5) f g^3 - (Bb^5 c^3 d^2 - Bab^3 b^2 d^5) g^4) x^2 + 12 (5 Ab^5 d^5 f^4 - 10 (Bb^5 cd^4 - Bab^4 d^5) f^3 g + 10 (Bb^5 c^2 d^3 - Bab^4 d^5) f^2 g^2 - 5 (Bb^5 c^3 d^2 - Bab^3 b^2 d^5) f g^3 + (Bb^5 c^4 d - Bab^4 b d^5) g^4) x + 12 (5 Bab^4 d^5 f^4 - 10 Bab^4 d^5 f^3 g + 10 Bab^3 b^2 d^5 f^2 g^2 - 5 Bab^4 b d^5 f g^3 + Bab^5 c^5 g^4) \log(d*x + c) + 12 (Bb^5 d^5 g^4 x^5 + 5 Bab^5 d^5 f g^3 x^4 + 10 Bab^5 d^5 f^2 g^2 x^3 + 10 Bab^5 d^5 f^3 g x^2 + 5 Bab^5 d^5 f^4 x) \log((b*e*x + a*e)/(d*x + c)) / (b^5 d^5)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^4*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*g^4*x^5 + 3*(20*A*b^5*d^5*f*g^3 - (B*b^5*c*d^4 - B*a*b^4*d^5)*g^4)*x^4 + 4*(30*A*b^5*d^5*f^2*g^2 - 5*(B*b^5*c*d^4 - B*a*b^4*d^5)*f*g^3 + (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^4)*x^3 + 6*(20*A*b^5*d^5*f^3*g - 10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^2*g^2 + 5*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f*g^3 - (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g^4)*x^2 + 12*(5*A*b^5*d^5*f^4 - 10*(B*b^5*c*d^4 - B*a*b^4*d^5)*f^3*g + 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*f^2*g^2 - 5*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*f*g^3 + (B*b^5*c^4*d - B*a^4*b*d^5)*g^4)*x + 12*(5*B*a*b^4*d^5*f^4 - 10*B*a^2*b^3*d^5*f^3*g + 10*B*a^3*b^2*d^5*f^2*g^2 - 5*B*a^4*b*d^5*f*g^3 + B*a^5*d^5*g^4)*log(b*x + a) - 12*(5*B*b^5*c*d^4*f^4 - 10*B*b^5*c^2*d^3*f^3*g + 10*B*b^5*c^3*d^2*f^2*g^2 - 5*B*b^5*c^4*d*f*g^3 + B*b^5*c^5*g^4)*log(d*x + c) + 12*(B*b^5*d^5*g^4*x^5 + 5*B*b^5*d^5*f*g^3*x^4 + 10*B*b^5*d^5*f^2*g^2*x^3 + 10*B*b^5*d^5*f^3*g*x^2 + 5*B*b^5*d^5*f^4*x)*log((b*e*x + a*e)/(d*x + c))/(b^5*d^5)
```

---

**Sympy [B]** time = 36.98, size = 1528, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**4*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g**4*x**5/5 + B*a*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4)*log(x + (B*a**5*c*d**4*g**4 - 5*B*a**4*b*c*d**4*f*g**3 + 10*B*a**3*b**2*c*d**4*f**2*g**2 - 10*B*a**2*b**3*c*d**4*f**3*g + B*a**2*d**5*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4))/b + B*a*b**4*c**5*g**4 - 5*B*a*b**4*c**4*d*f*g**3 + 10*B*a*b**4*c**3*d**2*f**2*g**2 - 10*B*a*b**4*c**2*d**3*f**3*g + 10*B*a*b**4*c*d**4*f**4 - B*a*c*d**4*(a**4*g**4 - 5*a**3*b*f*g**3 + 10*a**2*b**2*f**2*g**2 - 10*a*b**3*f**3*g + 5*b**4*f**4))/(B*a**5*d**5*g**4 - 5*B*a**4*b*d**5*f*g**3 + 10*B*a**3*b**2*d**5*f**2*g**2 - 10*B*a**2*b**3*d**5*f**3*g
```

$$\begin{aligned}
&g + 5Bab^4d^5f^4 + Bb^5c^5g^4 - 5Bb^5c^4dfg^3 + 10B \\
&b^5c^3d^2f^2g^2 - 10Bb^5c^2d^3f^3g + 5Bb^5c^4d^4f^4 \\
&)) / (5b^5) - Bc(c^4g^4 - 5c^3dfg^3 + 10c^2d^2f^2g^2 - \\
&10cd^3f^3g + 5d^4f^4) \log(x + (B^5cd^4g^4 - 5B^4bc \\
&d^4fg^3 + 10B^3b^2cd^4f^2g^2 - 10B^2b^3cd^4f^3 \\
&g + B^4c^5g^4 - 5B^4b^4c^4dfg^3 + 10B^4b^4c^3d^2f^2 \\
&g^2 - 10B^4b^4c^2d^3f^3g + 10B^4b^4cd^4f^4 - B^4b^4c \\
&^4c(c^4g^4 - 5c^3dfg^3 + 10c^2d^2f^2g^2 - 10cd^3f^3 \\
&g + 5d^4f^4) + Bb^5c^2(c^4g^4 - 5c^3dfg^3 + 10c^2d^2 \\
&f^2g^2 - 10cd^3f^3g + 5d^4f^4) / d) / (B^5d^5g^4 - 5B^4 \\
&4bd^5fg^3 + 10B^3b^2d^5f^2g^2 - 10B^2b^3d^5f^3g \\
&g + 5B^4ab^4d^5f^4 + Bb^5c^5g^4 - 5Bb^5c^4dfg^3 + 10B \\
&b^5c^3d^2f^2g^2 - 10Bb^5c^2d^3f^3g + 5Bb^5c^4d^4f^4) \\
&)) / (5d^5) + (Bf^4x + 2Bf^3gx^2 + 2Bf^2g^2x^3 + Bfg^3 \\
&x^4 + Bg^4x^5/5) \log(e^a + bx) / (c + dx) + x^4(20Abdfg^3 + \\
&B^4d^4g^4 - B^4bc^4g^4) / (20bd) - x^3(-30A^2b^2d^2f^2g^2 + B^4a \\
&^2d^2g^4 - 5B^4abd^2fg^3 - B^4b^2c^2g^4 + 5B^4b^2cdfg^3 \\
&)) / (15b^2d^2) + x^2(20A^2b^3d^3f^3g + B^4a^3d^3g^4 - 5B^4a^2 \\
&b^3d^3fg^3 + 10B^4ab^2d^3f^2g^2 - B^4b^3c^3g^4 + 5B^4b^3c^2 \\
&d^2fg^3 - 10B^4b^3cd^2f^2g^2) / (10b^3d^3) - x(-5A^4b^4d \\
&^4f^4 + B^4a^4d^4g^4 - 5B^4a^3bd^4fg^3 + 10B^4a^2b^2d^4f^2 \\
&g^2 - 10B^4ab^3d^4f^3g - B^4b^4c^4g^4 + 5B^4b^4c^3dfg^3 \\
&g^3 - 10B^4b^4c^2d^2f^2g^2 + 10B^4b^4cd^3f^3g) / (5b^4d^4)
\end{aligned}$$


---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^4\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="giac")

[Out] Timed out

### 3.231 $\int (f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right) dx$

**Optimal.** Leaf size=227

$$\frac{Bgx(bc - ad) \left( a^2 d^2 g^2 - abdg(4df - cg) + b^2 (c^2 g^2 - 4cdfg + 6d^2 f^2) \right)}{4b^3 d^3} + \frac{(f + gx)^4 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right)}{4g} - \frac{Bg^2 x^2 (bc - ad)}{4g}$$

```
[Out] -(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*x)/(4*b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3)/(12*b*d) - (B*(b*f - a*g)^4*Log[a + b*x])/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*g) + (B*(d*f - c*g)^4*Log[c + d*x])/(4*d^4*g)
```

**Rubi [A]** time = 0.341136, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2525, 12, 72}

$$\frac{Bgx(bc - ad) \left( a^2 d^2 g^2 - abdg(4df - cg) + b^2 (c^2 g^2 - 4cdfg + 6d^2 f^2) \right)}{4b^3 d^3} + \frac{(f + gx)^4 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right)}{4g} - \frac{Bg^2 x^2 (bc - ad)}{4g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x]]),x]
```

```
[Out] -(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*x)/(4*b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3)/(12*b*d) - (B*(b*f - a*g)^4*Log[a + b*x])/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x]]))/(4*g) + (B*(d*f - c*g)^4*Log[c + d*x])/(4*d^4*g)
```

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned}
\int (f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right) dx &= \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{4g} - \frac{B \int \frac{(bc-ad)(f+gx)^4}{(a+bx)(c+dx)} dx}{4g} \\
&= \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{4g} - \frac{(B(bc - ad)) \int \frac{(f+gx)^4}{(a+bx)(c+dx)} dx}{4g} \\
&= \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{4g} - \frac{(B(bc - ad)) \int \left( \frac{g^2(a^2d^2g^2 - abdg(4df - cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2))}{b^3d^3} \right) dx}{4g} \\
&= -\frac{B(bc - ad)g(a^2d^2g^2 - abdg(4df - cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2))x}{4b^3d^3} - \frac{B(b^3d^3g^4x^3)}{4g}
\end{aligned}$$

**Mathematica [A]** time = 0.261989, size = 215, normalized size = 0.95

$$\frac{(f + gx)^4 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right) - \frac{B(6bdg^2x(bc-ad)(a^2d^2g^2 + abdg(cg - 4df) + b^2(c^2g^2 - 4cdfg + 6d^2f^2)) + 3b^2d^2g^3x^2(bc-ad)(-adg - bcg + 4bdf) + 2b^3d^3g^4x^3)}{6b^4d^4}}{4g}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

[Out] ((f + g\*x)^4\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]) - (B\*(6\*b\*d\*(b\*c - a\*d)\*g^2\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(-4\*d\*f + c\*g) + b^2\*(6\*d^2\*f^2 - 4\*c\*d\*f\*g + c^2\*g^2))\*x + 3\*b^2\*d^2\*(b\*c - a\*d)\*g^3\*(4\*b\*d\*f - b\*c\*g - a\*d\*g)\*x^2 + 2\*b^3\*d^3\*(b\*c - a\*d)\*g^4\*x^3 + 6\*d^4\*(b\*f - a\*g)^4\*Log[a + b\*x] - 6\*b^4\*(d\*f - c\*g)^4\*Log[c + d\*x]))/(6\*b^4\*d^4))/(4\*g)

**Maple [B]** time = 0.2, size = 8605, normalized size = 37.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^3\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c))), x)

[Out] result too large to display

**Maxima [A]** time = 1.20154, size = 560, normalized size = 2.47

$$\frac{1}{4} Ag^3x^4 + Afg^2x^3 + \frac{3}{2} Af^2gx^2 + \left( x \log \left( \frac{bex}{dx+c} + \frac{ae}{dx+c} \right) + \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) Bf^3 + \frac{3}{2} \left( x^2 \log \left( \frac{bex}{dx+c} + \frac{ae}{dx+c} \right) + \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) Bf^3 + \frac{3}{2} \left( x^2 \log \left( \frac{bex}{dx+c} + \frac{ae}{dx+c} \right) + \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) Bf^3 + \frac{3}{2} \left( x^2 \log \left( \frac{bex}{dx+c} + \frac{ae}{dx+c} \right) + \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) Bf^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))), x, algorithm="maxima")

[Out] 1/4\*A\*g^3\*x^4 + A\*f\*g^2\*x^3 + 3/2\*A\*f^2\*g\*x^2 + (x\*log(b\*e\*x/(d\*x + c)) + a\*e/(d\*x + c)) + a\*log(b\*x + a)/b - c\*log(d\*x + c)/d)\*B\*f^3 + 3/2\*(x^2\*log(b\*e\*x/(d\*x + c)) + a\*e/(d\*x + c)) - a^2\*log(b\*x + a)/b^2 + c^2\*log(d\*x + c)/d^2

$$2 - (b*c - a*d)*x/(b*d))*B*f^2*g + 1/2*(2*x^3*\log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*\log(b*x + a)/b^3 - 2*c^3*\log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*f*g^2 + 1/24*(6*x^4*\log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*\log(b*x + a)/b^4 + 6*c^4*\log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*g^3 + A*f^3*x$$

**Fricas [B]** time = 1.83661, size = 895, normalized size = 3.94

$$6 Ab^4 d^4 g^3 x^4 + 2 (12 Ab^4 d^4 f g^2 - (Bb^4 cd^3 - Bab^3 d^4) g^3) x^3 + 3 (12 Ab^4 d^4 f^2 g - 4 (Bb^4 cd^3 - Bab^3 d^4) f g^2 + (Bb^4 c^2 d^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*g^3*x^4 + 2*(12*A*b^4*d^4*f*g^2 - (B*b^4*c*d^3 - B*a*b^3*d^4)*g^3)*x^3 + 3*(12*A*b^4*d^4*f^2*g - 4*(B*b^4*c*d^3 - B*a*b^3*d^4)*f*g^2 + (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g^3)*x^2 + 6*(4*A*b^4*d^4*f^3 - 6*(B*b^4*c*d^3 - B*a*b^3*d^4)*f^2*g + 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*f*g^2 - (B*b^4*c^3*d - B*a^3*b*d^4)*g^3)*x + 6*(4*B*a*b^3*d^4*f^3 - 6*B*a^2*b^2*d^4*f^2*g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*log(b*x + a) - 6*(4*B*b^4*c*d^3*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b^4*c^4*g^3)*log(d*x + c) + 6*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4*d^4*f^2*g*x^2 + 4*B*b^4*d^4*f^3*x)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d^4)
```

**Sympy [B]** time = 20.5577, size = 1049, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g**3*x**4/4 - B*a*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)*log(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B*a**2*d**4*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)/b + B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g - 8*B*a*b**3*c*d**3*f**3 - B*a*c*d**3*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)))/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(4*b**4) + B*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)*log(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g - 8*B*a*b**3*c*d**3*f**3 - B*a*b**3*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2) + B*b**4*c**2*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)/d)/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(4*d**4) + (B*f**3*x + 3*B*f**2*g*x**2/2 + B*f*g**2*x**3 + B*g**3*x**4/4)*log(e*(a + b*x)/(c + d*x)) + x**3*(1/24*(6*A*b*d*f*g**2 + B*a*d*g**3 - B*b*c*g**3)/(12*b*d) - x**2*(-12*A*b**2*d**2*f**2*g + B*a**2*d**2*g**3 - 4*B*a*b*d**2*f*g**2 - B*b**2*c**2*g**3 + 4*B*b**2*c*d*f*g**2)/(8*b**2*d**2) + x*(4*A*b**3*d**3*f**3 + B*a**3*d**3*g**3 - 4
```

$$\frac{*B*a**2*b*d**3*f*g**2 + 6*B*a*b**2*d**3*f**2*g - B*b**3*c**3*g**3 + 4*B*b**3*c**2*d*f*g**2 - 6*B*b**3*c*d**2*f**2*g}{(4*b**3*d**3)}$$

---

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="giac")

[Out] Timed out



$$3.232 \quad \int (f + gx)^2 \left( A + B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) \right) dx$$

**Optimal.** Leaf size=150

$$\frac{(f + gx)^3 \left( B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) + A \right)}{3g} - \frac{Bgx(bc - ad)(-adg - bcg + 3bdf)}{3b^2d^2} - \frac{B(bf - ag)^3 \log(a + bx)}{3b^3g} - \frac{Bg^2x^2(bc - ad)}{6bd} + \frac{B}{6bd}$$

```
[Out] -(B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*x)/(3*b^2*d^2) - (B*(b*c - a*d)*g^2*x^2)/(6*b*d) - (B*(b*f - a*g)^3*Log[a + b*x])/(3*b^3*g) + ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*g) + (B*(d*f - c*g)^3*Log[c + d*x])/(3*d^3*g)
```

**Rubi [A]** time = 0.169498, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2525, 12, 72}

$$\frac{(f + gx)^3 \left( B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) + A \right)}{3g} - \frac{Bgx(bc - ad)(-adg - bcg + 3bdf)}{3b^2d^2} - \frac{B(bf - ag)^3 \log(a + bx)}{3b^3g} - \frac{Bg^2x^2(bc - ad)}{6bd} + \frac{B}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

```
[Out] -(B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*x)/(3*b^2*d^2) - (B*(b*c - a*d)*g^2*x^2)/(6*b*d) - (B*(b*f - a*g)^3*Log[a + b*x])/(3*b^3*g) + ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*g) + (B*(d*f - c*g)^3*Log[c + d*x])/(3*d^3*g)
```

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 72

```
Int[((e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

### Rubi steps

$$\begin{aligned} \int (f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right) dx &= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{3g} - \frac{B \int \frac{(bc - ad)(f + gx)^3}{(a + bx)(c + dx)} dx}{3g} \\ &= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{3g} - \frac{(B(bc - ad)) \int \frac{(f + gx)^3}{(a + bx)(c + dx)} dx}{3g} \\ &= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{3g} - \frac{(B(bc - ad)) \int \left( \frac{g^2(3bdf - bcg - adg)}{b^2d^2} + \frac{g^3x}{bd} + \frac{(b}{b^2(bc - ad)} \right) dx}{3g} \\ &= -\frac{B(bc - ad)g(3bdf - bcg - adg)x}{3b^2d^2} - \frac{B(bc - ad)g^2x^2}{6bd} - \frac{B(bf - ag)^3 \log(a + bx)}{3b^3g} \end{aligned}$$

**Mathematica [A]** time = 0.131928, size = 142, normalized size = 0.95

$$\frac{(f + gx)^3 \left( B \log \left( \frac{e(a + bx)}{c + dx} \right) + A \right) - \frac{B(b^2d^2g^3x^2(bc - ad) + 2bdg^2x(bc - ad)(-adg - bcg + 3bdf) + 2d^3(bf - ag)^3 \log(a + bx) - 2b^3(df - cg)^3 \log(c + dx))}{2b^3d^3}}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]), x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (B*(2*b*d*(b*c - a*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*(b*f - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(2*b^3*d^3)/(3*g)
```

**Maple [B]** time = 0.184, size = 4406, normalized size = 29.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(A+B*ln(e*(b*x+a)/(d*x+c))), x)
```

```
[Out] -1/6*e^2/d^3*B*g^2*b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^3-1/3*e^3/d^3*A*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*b^3*c^3-e^2/d*A*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2*c+e^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2*f-e/d*A/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*f^2*b*c+e/d*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^2*g^2/(d*x+c)*a^2-e^2/d^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^4*f/(d*x+c)^2-2*e^3*d^2*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^5/(d*x+c)^3*c-e^2*d^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^4*f/(d*x+c)^2-2*e^2/d*A*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*b*c*f*a+e/d^3*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^4*g^2/(d*x+c)*b+e*d*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*f^2/(d*x+c)*a^2+1/3*e^3*A*g^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^3+e^2*d*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^4*c/(d*x+c)^2+5*e^3*d*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^4*c^2/(d*x+c)^3+5*e^3/d*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^3*a^2*c^4/(d*x+c)^3*b-2*e^2/d*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a*f*b*c-2*e^3/d^2*B*g^2*ln(b*e/d+(a
```

$$\begin{aligned}
& *d-b*c)*e/d/(d*x+c))*b^2/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^3*a*c^5/(d*x+c)^3-2* \\
& e/d^2*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)))/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)*c^3*f \\
& *g/(d*x+c)*b+4*e/d*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)))/(d*e/(d*x+c))*a-e/(d*x+ \\
& c)*b*c)*c^2*f*g/(d*x+c))*a-4*e^2/d^2*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)))/( \\
& d*e/(d*x+c))*a-e/(d*x+c)*b*c)^2*a*c^4/(d*x+c)^2*b-2*e*B*ln(b*e/d+(a*d-b*c)*e \\
& /d/(d*x+c))/b/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)*c*f*g/(d*x+c))*a^-2-1/2*e^2/d*B*g \\
& ^2/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^2*a^2*c+e*B*g/b/(d*e/(d*x+c))*a-e/(d*x+c)*b \\
& *c)*a^2*f+e/d^2*B*g^2/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)*c^2*a-2/3*e/d^3*B*g^2/( \\
& d*e/(d*x+c))*a-e/(d*x+c)*b*c)*c^3*b+e/d^2*A/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)*c^ \\
& 2*g^2*a-e/d^3*A/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)*c^3*g^2*b-e^2/d^3*A*g^2/(d*e/ \\
& (d*x+c))*a-e/(d*x+c)*b*c)^2*b^2*c^3+e/d*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)))/(d \\
& *e/(d*x+c))*a-e/(d*x+c)*b*c)*f^2/(d*x+c))*c^2*b+e^2/d^3*B*g^2*ln(b*e/d+(a*d-b \\
& *c)*e/d/(d*x+c))*b^2/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^2*c^5/(d*x+c)^2+2*e/d^2* \\
& B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)))/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)*c^2*f*g*b+e \\
& ^2/d^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b^2/(d*e/(d*x+c))*a-e/(d*x+c)*b*c \\
& )^2*c^2*f-2*e/d*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)))/(d*e/(d*x+c))*a-e/(d*x+c)* \\
& b*c)*c*f*g*a+1/3*e^3*d^3*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b^3/(d*e/(d* \\
& x+c))*a-e/(d*x+c)*b*c)^3*a^6/(d*x+c)^3+6*e^2/d*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/ \\
& (d*x+c))/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^2*a^2*c^3/(d*x+c)^2-4*e^2*B*g^2*ln(b \\
& *e/d+(a*d-b*c)*e/d/(d*x+c))/b/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^2*a^3*c^2/(d*x+ \\
& c)^2-6*e^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c))*a-e/(d*x+c)*b*c \\
& )^2*c^2*f/(d*x+c)^2*a^2+1/3*e^3/d^3*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b \\
& ^3/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^3*c^6/(d*x+c)^3-e^3/d*B*g^2*ln(b*e/d+(a*d- \\
& b*c)*e/d/(d*x+c))/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^3*a^2*c*b+2*e^2/d^2*B*g^2*ln \\
& (b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^2*c^2*b*a+e^3/ \\
& d^2*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^3*a \\
& *c^2*b^2-2*e/d^2*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c))*a-e/(d*x+c) \\
& *b*c)*c^3*g^2/(d*x+c))*a+e*A/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)*f^2*a-B/b*ln(d*(b \\
& *e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*f^2*a-1/3*B*g^2/b^3*ln(d*(b*e/d+(a*d-b*c)* \\
& e/d/(d*x+c))-b*e)*a^3+1/3/d^3*B*g^2*ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e) \\
& *c^3+1/d*B*ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*f^2*c+e*B*ln(b*e/d+(a*d- \\
& b*c)*e/d/(d*x+c))/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)*f^2*a+1/3*e^3*B*g^2*ln(b*e/ \\
& d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^3*a^3+B*g/b^2*ln(d*( \\
& b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*a^2*f-1/3*e*B*g^2/b^2/(d*e/(d*x+c))*a-e/(d \\
& *x+c)*b*c)*a^3+e^2*A*g/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^2*a^2*f+1/6*e^2*B*g^2/ \\
& b/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^2*a^3-1/d^2*B*ln(d*(b*e/d+(a*d-b*c)*e/d/(d* \\
& x+c))-b*e)*c^2*f*g-e^2/d^3*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+ \\
& c))*a-e/(d*x+c)*b*c)^2*c^3*b^2+e/d^2*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/ \\
& (d*x+c))*a-e/(d*x+c)*b*c)*c^2*g^2*a-e/d*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d \\
& *e/(d*x+c))*a-e/(d*x+c)*b*c)*f^2*b*c-e^2/d*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x \\
& +c))/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^2*a^2*c-1/3*e^3/d^3*B*g^2*ln(b*e/d+(a*d- \\
& b*c)*e/d/(d*x+c))/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^3*c^3*b^3+e^3/d^2*A*g^2/(d* \\
& e/(d*x+c))*a-e/(d*x+c)*b*c)^3*a*b^2*c^2-2*e/d*A/(d*e/(d*x+c))*a-e/(d*x+c)*b*c \\
& )*c*f*g*a-2*e*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c))*a-e/(d*x+c)*b* \\
& c)*f^2/(d*x+c))*a*c-20/3*e^3*B*g^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x \\
& +c))*a-e/(d*x+c)*b*c)^3*a^3*c^3/(d*x+c)^3-e/d^3*B*ln(b*e/d+(a*d-b*c)*e/d/(d* \\
& x+c))/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)*c^3*g^2*b+2*e^2/d^2*A*g^2/(d*e/(d*x+c))* \\
& a-e/(d*x+c)*b*c)^2*a*c^2*b+e^2/d^2*A*g/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^2*b^2* \\
& c^2*f+1/2*e^2/d^2*B*g^2/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^2*a*c^2*b-e^3/d*A*g^2 \\
& /d*e/(d*x+c))*a-e/(d*x+c)*b*c)^3*a^2*b*c+e/d^2*B*g/(d*e/(d*x+c))*a-e/(d*x+c) \\
& *b*c)*c^2*f*b+2*e/d^2*A/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)*c^2*f*g*b-2*e/d*B*g/( \\
& d*e/(d*x+c))*a-e/(d*x+c)*b*c)*c*f*a+4*e^2/d*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+ \\
& c))*b/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^2*c^3*f/(d*x+c)^2*a+4*e^2*d*B*g*ln(b*e/ \\
& d+(a*d-b*c)*e/d/(d*x+c))/b/(d*e/(d*x+c))*a-e/(d*x+c)*b*c)^2*a^3*f/(d*x+c)^2* \\
& c
\end{aligned}$$

**Maxima [A]** time = 1.1324, size = 354, normalized size = 2.36

$$\frac{1}{3} Ag^2x^3 + Afgx^2 + \left( x \log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) Bf^2 + \left( x^2 \log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2 \log(bx+a)}{b} + \frac{c^2 \log(dx+c)}{d} \right) Bf^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

```
[Out] 1/3*A*g^2*x^3 + A*f*g*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*f^2 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*f*g + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*g^2 + A*f^2*x
```

**Fricas [A]** time = 1.31593, size = 581, normalized size = 3.87

$$2 Ab^3d^3g^2x^3 + (6 Ab^3d^3fg - (Bb^3cd^2 - Bab^2d^3)g^2)x^2 + 2(3 Ab^3d^3f^2 - 3(Bb^3cd^2 - Bab^2d^3)fg + (Bb^3c^2d - Ba^2bd^3)g^2)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

```
[Out] 1/6*(2*A*b^3*d^3*g^2*x^3 + (6*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2)*x^2 + 2*(3*A*b^3*d^3*f^2 - 3*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g + (B*b^3*c^2*d - B*a^2*b*d^3)*g^2)*x + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*a^3*d^3*g^2)*log(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b^3*c^3*g^2)*log(d*x + c) + 2*(B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x^2 + 3*B*b^3*d^3*f^2*x)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d^3)
```

**Sympy [B]** time = 10.6912, size = 688, normalized size = 4.59

$$\frac{Ag^2x^3}{3} + \frac{Ba(a^2g^2 - 3abfg + 3b^2f^2) \log\left(x + \frac{Ba^3cd^2g^2 - 3Ba^2bcd^2fg + \frac{Ba^2d^3(a^2g^2 - 3abfg + 3b^2f^2)}{b} + Bab^2c^3g^2 - 3Bab^2c^2dfg + 6Bab^2cd^2f^2 - Bacd^2(a^2g^2 - 3abfg + 3b^2f^2)}{Ba^3d^3g^2 - 3Ba^2bd^3fg + 3Bab^2d^3f^2 + Bb^3c^3g^2 - 3Bb^3c^2dfg + 3Bb^3cd^2f^2}\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

```
[Out] A*g**2*x**3/3 + B*a*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2)*log(x + (B*a**3*c*d**2*g**2 - 3*B*a**2*b*c*d**2*f*g + B*a**2*d**3*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2)/b + B*a*b**2*c**3*g**2 - 3*B*a*b**2*c**2*d*f*g + 6*B*a*b**2*c*d**2*f**2 - B*a*c*d**2*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2))/(B*a**3*d**3*g**2 - 3*B*a**2*b*d**3*f*g + 3*B*a*b**2*d**3*f**2 + B*b**3*c**3*g**2 - 3*B*b**3*c**2*d*f*g + 3*B*b**3*c*d**2*f**2))/(3*b**3) - B*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2)*log(x + (B*a**3*c*d**2*g**2 - 3*B*a**2*b*c*d**2*f*g + B*a*b**2*c**3*g**2 - 3*B*a*b**2*c**2*d*f*g + 6*B*a*b**2*c*d**2*f**2 - B*a*b**2*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2) + B*b**3*c**2*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2)/d)/(B*a**3*d**3*g**2 - 3*B*a**2*b*d**3*f*g + 3*B*a*b**2*d**3*f**2))
```

```

**3*f**2 + B*b**3*c**3*g**2 - 3*B*b**3*c**2*d*f*g + 3*B*b**3*c*d**2*f**2))/
(3*d**3) + (B*f**2*x + B*f*g*x**2 + B*g**2*x**3/3)*log(e*(a + b*x)/(c + d*x
)) + x**2*(6*A*b*d*f*g + B*a*d*g**2 - B*b*c*g**2)/(6*b*d) - x*(-3*A*b**2*d*
*2*f**2 + B*a**2*d**2*g**2 - 3*B*a*b*d**2*f*g - B*b**2*c**2*g**2 + 3*B*b**2
*c*d*f*g)/(3*b**2*d**2)

```

---

**Giac [A]** time = 15.9632, size = 346, normalized size = 2.31

$$\frac{1}{3}(Ag^2 + Bg^2)x^3 + \frac{1}{3}(Bg^2x^3 + 3Bfgx^2 + 3Bf^2x)\log\left(\frac{bx+a}{dx+c}\right) + \frac{(6Abdfg + 6Bbdfg - Bbcg^2 + Badg^2)x^2}{6bd} + \frac{(3Bab^2d^2f^2 - 3Bab^2d^2fg + B^2a^2d^2g^2)\log(bx+a)}{b^3d^2} - \frac{1}{3}(3B^2cd^2f^2 - 3B^2cd^2fg + B^2c^3g^2)\log(-dx-c)/d^3 + \frac{1}{3}(3A^2b^2d^2f^2 + 3A^2b^2d^2fg - 3A^2b^2c^2d^2fg + 3A^2ab^2d^2f^2 + B^2b^2c^2g^2 - B^2a^2d^2g^2) \frac{x}{b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

```
[Out] 1/3*(A*g^2 + B*g^2)*x^3 + 1/3*(B*g^2*x^3 + 3*B*f*g*x^2 + 3*B*f^2*x)*log((b*
x + a)/(d*x + c)) + 1/6*(6*A*b*d*f*g + 6*B*b*d*f*g - B*b*c*g^2 + B*a*d*g^2)
*x^2/(b*d) + 1/3*(3*B*a*b^2*f^2 - 3*B*a^2*b*f*g + B*a^3*g^2)*log(b*x + a)/b
^3 - 1/3*(3*B*c*d^2*f^2 - 3*B*c^2*d*f*g + B*c^3*g^2)*log(-d*x - c)/d^3 + 1/
3*(3*A*b^2*d^2*f^2 + 3*A*b^2*d^2fg - 3*A*b^2*c^2d^2fg + 3*A^2ab^2d^2f^2 +
B*b^2*c^2g^2 - B*a^2*d^2g^2)*x/(b^2*d^2)
```

$$3.233 \quad \int (f + gx) \left( A + B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) \right) dx$$

**Optimal.** Leaf size=109

$$\frac{(f + gx)^2 \left( B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) + A \right)}{2g} - \frac{B(bf - ag)^2 \log(a + bx)}{2b^2g} - \frac{Bgx(bc - ad)}{2bd} + \frac{B(df - cg)^2 \log(c + dx)}{2d^2g}$$

[Out]  $-(B*(b*c - a*d)*g*x)/(2*b*d) - (B*(b*f - a*g)^2*\text{Log}[a + b*x])/(2*b^2*g) + ((f + g*x)^2*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(2*g) + (B*(d*f - c*g)^2*\text{Log}[c + d*x])/(2*d^2*g)$

**Rubi [A]** time = 0.0975082, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$ , Rules used = {2525, 12, 72}

$$\frac{(f + gx)^2 \left( B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) + A \right)}{2g} - \frac{B(bf - ag)^2 \log(a + bx)}{2b^2g} - \frac{Bgx(bc - ad)}{2bd} + \frac{B(df - cg)^2 \log(c + dx)}{2d^2g}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]), x]$

[Out]  $-(B*(b*c - a*d)*g*x)/(2*b*d) - (B*(b*f - a*g)^2*\text{Log}[a + b*x])/(2*b^2*g) + ((f + g*x)^2*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)]))/(2*g) + (B*(d*f - c*g)^2*\text{Log}[c + d*x])/(2*d^2*g)$

### Rule 2525

$\text{Int}[(a_.) + \text{Log}[(c_.)*(\text{RFx}_.)^{(p_.)}*(b_.)^{(n_.)}*((d_.) + (e_.)*(x_.)^{(m_.)})], x\_Symbol] \rightarrow \text{Simp}[(d + e*x)^{(m + 1)}*(a + b*\text{Log}[c*\text{RFx}^p])^n/(e*(m + 1)), x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{(m + 1)}*(a + b*\text{Log}[c*\text{RFx}^p])^{(n - 1)}*D[\text{RFx}, x])/(\text{RFx}, x), x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

$\text{Int}[(a_)*(u_), x\_Symbol] \rightarrow \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 72

$\text{Int}[(e_.) + (f_.)*(x_.)^{(p_.)}/(((a_.) + (b_.)*(x_.)*((c_.) + (d_.)*(x_.)])), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

### Rubi steps

$$\begin{aligned}
\int (f + gx) \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right) dx &= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{2g} - \frac{B \int \frac{(bc - ad)(f + gx)^2}{(a + bx)(c + dx)} dx}{2g} \\
&= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{2g} - \frac{(B(bc - ad)) \int \frac{(f + gx)^2}{(a + bx)(c + dx)} dx}{2g} \\
&= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{2g} - \frac{(B(bc - ad)) \int \left( \frac{g^2}{bd} + \frac{(bf - ag)^2}{b(bc - ad)(a + bx)} + \frac{df}{d(-bc + a)} \right) dx}{2g} \\
&= -\frac{B(bc - ad)gx}{2bd} - \frac{B(bf - ag)^2 \log(a + bx)}{2b^2g} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{2g}
\end{aligned}$$

**Mathematica [A]** time = 0.107345, size = 114, normalized size = 1.05

$$\frac{b \left( d \left( Bg^2x(ad - bc) + Abd(f + gx)^2 \right) + bBd^2(f + gx)^2 \log \left( \frac{e(a + bx)}{c + dx} \right) + bB(df - cg)^2 \log(c + dx) \right) - Bd^2(bf - ag)^2 \log(a + bx)}{2b^2d^2g}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]]), x]

[Out]  $(-(B*d^2*(b*f - a*g)^2*Log[a + b*x]) + b*(d*(B*(-(b*c) + a*d)*g^2*x + A*b*d*(f + g*x)^2) + b*B*d^2*(f + g*x)^2*Log[(e*(a + b*x))/(c + d*x)] + b*B*(d*f - c*g)^2*Log[c + d*x]))/(2*b^2*d^2*g)$

**Maple [B]** time = 0.174, size = 1809, normalized size = 16.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c))), x)

[Out]  $e*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*f*a^{1/2}*e^{2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^{2+1/2}*e*B*g/b/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^{2-e/d*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c*g*a^{-3}*e^{2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2/(d*x+c)^2*c^{2+1/2}*e^2/d^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*c^2+e/d^2*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^2*g*b-e/d*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*f*b*c^{-2}*e*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*f/(d*x+c)*a^{c+1/2}*e^2/d^2*A*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*b^2*c^2-e/d*A/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c*g*a-e/d*B*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*a^{c+1/2}*e/d^2*B*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^2*b+e/d^2*A/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^2*g*b+e*A/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*f*a^{1/2}*B*g/b^2*ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*a^{-2}-B/b*ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*f*a^{-1/2}/d^2*B*ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*c^2*g+1/d*B*ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*f*c+2*e/d*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^2*g/(d*x+c)*a^{-1/2}*e^2*d^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^4/(d*x+c)^{2-1/2}*e^2/d^2*B*g*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b^2/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2$

$+c)*b*c)^2*c^4/(d*x+c)^2+e/d*B*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*f/(d*x+c)*c^2*b-e*B*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c*g/(d*x+c)*a^2+e*d*B*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*f/(d*x+c)*a^2-e/d^2*B*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*c^3*g/(d*x+c)*b-e^2/d*A*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a*b*c-e^2/d*B*g*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a*b*c+2*e^2*d*B*g*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^3/(d*x+c)^2*c+2*e^2/d*B*g*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a/(d*x+c)^2*c^3*b-e/d*A/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)*f*b*c+1/2*e^2*A*g/(d*e/(d*x+c)*a-e/(d*x+c)*b*c)^2*a^2$

**Maxima [A]** time = 1.19729, size = 189, normalized size = 1.73

$$\frac{1}{2} Agx^2 + \left( x \log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a \log(bx+a)}{b} - \frac{c \log(dx+c)}{d} \right) Bf + \frac{1}{2} \left( x^2 \log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2 \log(bx+a)}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="maxima")

[Out]  $\frac{1}{2}A*g*x^2 + (x*\log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*\log(b*x + a)/b - c*\log(d*x + c)/d)*B*f + \frac{1}{2}*(x^2*\log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*\log(b*x + a)/b^2 + c^2*\log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*g + A*f*x$

**Fricas [A]** time = 1.35456, size = 321, normalized size = 2.94

$$\frac{Ab^2d^2gx^2 + (2Ab^2d^2f - (Bb^2cd - Babd^2)g)x + (2Babd^2f - Ba^2d^2g)\log(bx+a) - (2Bb^2cdf - Bb^2c^2g)\log(dx+c) + (A^2g^2 - 2Agf + f^2)}{2b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="fricas")

[Out]  $\frac{1}{2}*(A*b^2*d^2*g*x^2 + (2*A*b^2*d^2*f - (B*b^2*c*d - B*a*b*d^2)*g)*x + (2*B*a*b*d^2*f - B*a^2*d^2*g)*\log(b*x + a) - (2*B*b^2*c*d*f - B*b^2*c^2*g)*\log(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*\log((b*e*x + a*e)/(d*x + c)))/(b^2*d^2)$

**Sympy [B]** time = 4.78579, size = 325, normalized size = 2.98

$$\frac{Agx^2}{2} - \frac{Ba(ag - 2bf) \log\left(x + \frac{Ba^2cdg + \frac{Ba^2d^2(ag-2bf)}{b} + Babc^2g - 4Babcdf - Bacd(ag-2bf)}{Ba^2d^2g - 2Babd^2f + Bb^2c^2g - 2Bb^2cdf}\right)}{2b^2} + \frac{Bc(cg - 2df) \log\left(x + \frac{Ba^2cdg + Babc^2g - 4Babcdf - Bacd(ag-2bf)}{Ba^2d^2g - 2Babd^2f + Bb^2c^2g - 2Bb^2cdf}\right)}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c))),x)

[Out]  $A*g*x**2/2 - B*a*(a*g - 2*b*f)*\log(x + (B*a**2*c*d*g + B*a**2*d**2*(a*g - 2*b*f)/b + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*c*d*(a*g - 2*b*f))/(B*a**2*d**2)$



$$2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f)/(2*b**2) + B*c*(c*g - 2*d*f)*\log(x + (B*a**2*c*d*g + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*b*c*(c*g - 2*d*f) + B*b**2*c**2*(c*g - 2*d*f)/d)/(B*a**2*d**2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/(2*d**2) + (B*f*x + B*g*x**2/2)*\log(e*(a + b*x)/(c + d*x)) + x*(2*A*b*d*f + B*a*d*g - B*b*c*g)/(2*b*d)$$

**Giac [A]** time = 2.52862, size = 171, normalized size = 1.57

$$\frac{1}{2}(Ag + Bg)x^2 + \frac{1}{2}(Bgx^2 + 2Bfx)\log\left(\frac{bx + a}{dx + c}\right) + \frac{(2Abdf + 2Bbdf - Bbcg + Badg)x}{2bd} + \frac{(2Babf - Ba^2g)\log(bx + a)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*log(e\*(b\*x+a)/(d\*x+c))),x, algorithm="giac")

[Out] 1/2\*(A\*g + B\*g)\*x^2 + 1/2\*(B\*g\*x^2 + 2\*B\*f\*x)\*log((b\*x + a)/(d\*x + c)) + 1/2\*(2\*A\*b\*d\*f + 2\*B\*b\*d\*f - B\*b\*c\*g + B\*a\*d\*g)\*x/(b\*d) + 1/2\*(2\*B\*a\*b\*f - B\*a^2\*g)\*log(b\*x + a)/b^2 - 1/2\*(2\*B\*c\*d\*f - B\*c^2\*g)\*log(-d\*x - c)/d^2

$$3.234 \quad \int \left( A + B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) \right) dx$$

**Optimal.** Leaf size=52

$$\frac{B(a+bx) \log \left( \frac{e^{(a+bx)}}{c+dx} \right)}{b} - \frac{B(bc-ad) \log(c+dx)}{bd} + Ax$$

[Out] A\*x + (B\*(a + b\*x)\*Log[(e\*(a + b\*x))/(c + d\*x)]/b - (B\*(b\*c - a\*d)\*Log[c + d\*x])/(b\*d)

**Rubi [A]** time = 0.027389, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {2486, 31}

$$\frac{B(a+bx) \log \left( \frac{e^{(a+bx)}}{c+dx} \right)}{b} - \frac{B(bc-ad) \log(c+dx)}{bd} + Ax$$

Antiderivative was successfully verified.

[In] Int[A + B\*Log[(e\*(a + b\*x))/(c + d\*x)], x]

[Out] A\*x + (B\*(a + b\*x)\*Log[(e\*(a + b\*x))/(c + d\*x)]/b - (B\*(b\*c - a\*d)\*Log[c + d\*x])/(b\*d)

#### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_.))^(p\_.))\*((c\_.) + (d\_.)\*(x\_.))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] :> Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_.))^(-1), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned} \int \left( A + B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) \right) dx &= Ax + B \int \log \left( \frac{e^{(a+bx)}}{c+dx} \right) dx \\ &= Ax + \frac{B(a+bx) \log \left( \frac{e^{(a+bx)}}{c+dx} \right)}{b} - \frac{(B(bc-ad)) \int \frac{1}{c+dx} dx}{b} \\ &= Ax + \frac{B(a+bx) \log \left( \frac{e^{(a+bx)}}{c+dx} \right)}{b} - \frac{B(bc-ad) \log(c+dx)}{bd} \end{aligned}$$

**Mathematica [A]** time = 0.007627, size = 52, normalized size = 1.

$$\frac{B(a+bx) \log \left( \frac{e^{(a+bx)}}{c+dx} \right)}{b} - \frac{B(bc-ad) \log(c+dx)}{bd} + Ax$$

Antiderivative was successfully verified.

[In] Integrate[A + B\*Log[(e\*(a + b\*x))/(c + d\*x)],x]

[Out] A\*x + (B\*(a + b\*x)\*Log[(e\*(a + b\*x))/(c + d\*x)]/b - (B\*(b\*c - a\*d)\*Log[c + d\*x])/(b\*d)

**Maple [B]** time = 0.149, size = 418, normalized size = 8.

$$Ax - \frac{Ba}{b} \ln \left( d \left( \frac{be}{d} + \frac{e(ad-bc)}{(dx+c)d} \right) - be \right) + \frac{Bc}{d} \ln \left( d \left( \frac{be}{d} + \frac{e(ad-bc)}{(dx+c)d} \right) - be \right) + eBa \ln \left( \frac{be}{d} + \frac{e(ad-bc)}{(dx+c)d} \right) \left( \frac{ade}{dx+c} - \frac{be}{dx+c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(A+B\*ln(e\*(b\*x+a)/(d\*x+c)),x)

[Out] A\*x-B/b\*ln(d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))-b\*e)\*a+B/d\*ln(d\*(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))-b\*e)\*c+e\*B\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)\*a-e\*B\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)/d\*b\*c+e\*B\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/b/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)/(d\*x+c)\*a^2\*d-2\*e\*B\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)/(d\*x+c)\*a\*c+e\*B\*ln(b\*e/d+(a\*d-b\*c)\*e/d/(d\*x+c))/(d\*e/(d\*x+c)\*a-e/(d\*x+c)\*b\*c)/d/(d\*x+c)\*c^2\*b

**Maxima [A]** time = 1.16848, size = 73, normalized size = 1.4

$$\left( x \log \left( \frac{(bx+a)e}{dx+c} \right) + \frac{ae \log(bx+a)}{b} - \frac{ce \log(dx+c)}{d} \right) B + Ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B\*log(e\*(b\*x+a)/(d\*x+c)),x, algorithm="maxima")

[Out] (x\*log((b\*x + a)\*e/(d\*x + c)) + (a\*e\*log(b\*x + a)/b - c\*e\*log(d\*x + c)/d)/e)\*B + A\*x

**Fricas [A]** time = 1.30415, size = 132, normalized size = 2.54

$$\frac{Bbdx \log \left( \frac{bex+ae}{dx+c} \right) + Abdx + Bad \log (bx + a) - Bbc \log (dx + c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B\*log(e\*(b\*x+a)/(d\*x+c)),x, algorithm="fricas")

[Out] (B\*b\*d\*x\*log((b\*e\*x + a\*e)/(d\*x + c)) + A\*b\*d\*x + B\*a\*d\*log(b\*x + a) - B\*b\*c\*log(d\*x + c))/(b\*d)

**Sympy [A]** time = 1.16599, size = 83, normalized size = 1.6

$$Ax + \frac{Ba \log\left(x + \frac{\frac{Ba^2d}{b} + Bac}{Bad + Bbc}\right)}{b} - \frac{Bc \log\left(x + \frac{Bac + \frac{Bbc^2}{d}}{Bad + Bbc}\right)}{d} + Bx \log\left(\frac{e(a + bx)}{c + dx}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B\*ln(e\*(b\*x+a)/(d\*x+c)),x)

[Out] A\*x + B\*a\*log(x + (B\*a\*\*2\*d/b + B\*a\*c)/(B\*a\*d + B\*b\*c))/b - B\*c\*log(x + (B\*a\*c + B\*b\*c\*\*2/d)/(B\*a\*d + B\*b\*c))/d + B\*x\*log(e\*(a + b\*x)/(c + d\*x))

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B\*log(e\*(b\*x+a)/(d\*x+c)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.235 \quad \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx} dx$$

**Optimal.** Leaf size=140

$$\frac{BPolyLog\left(2, \frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{BPolyLog\left(2, \frac{d(f+gx)}{df-cg}\right)}{g} + \frac{\log(f+gx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{g} - \frac{B \log(f+gx) \log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g}$$

[Out] -((B\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*Log[f + g\*x])/g) + ((A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]\*Log[f + g\*x])/g + (B\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*Log[f + g\*x])/g - (B\*PolyLog[2, (b\*(f + g\*x))/(b\*f - a\*g)])/g + (B\*PolyLog[2, (d\*(f + g\*x))/(d\*f - c\*g)])/g

**Rubi [A]** time = 0.246955, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {2524, 12, 2418, 2394, 2393, 2391}

$$\frac{BPolyLog\left(2, \frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{BPolyLog\left(2, \frac{d(f+gx)}{df-cg}\right)}{g} + \frac{\log(f+gx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{g} - \frac{B \log(f+gx) \log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])/(f + g\*x), x]

[Out] -((B\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*Log[f + g\*x])/g) + ((A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]\*Log[f + g\*x])/g + (B\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*Log[f + g\*x])/g - (B\*PolyLog[2, (b\*(f + g\*x))/(b\*f - a\*g)])/g + (B\*PolyLog[2, (d\*(f + g\*x))/(d\*f - c\*g)])/g

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*(RFx\_), x\_Symbol] :> With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_.)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{f + gx} dx &= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f + gx)}{g} - \frac{B \int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2} + \frac{be}{c+dx}\right) \log(f+gx)}{e(a+bx)} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f + gx)}{g} - \frac{B \int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2} + \frac{be}{c+dx}\right) \log(f+gx)}{a+bx} dx}{eg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f + gx)}{g} - \frac{B \int \left(\frac{be \log(f+gx)}{a+bx} - \frac{de \log(f+gx)}{c+dx}\right) dx}{eg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f + gx)}{g} - \frac{(bB) \int \frac{\log(f+gx)}{a+bx} dx}{g} + \frac{(Bd) \int \frac{\log(f+gx)}{c+dx} dx}{g} \\
&= -\frac{B \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f + gx)}{g} + \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f + gx)}{g} + \frac{B \log\left(-\frac{g(c+dx)}{df-cg}\right) \log(f + gx)}{g} \\
&= -\frac{B \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f + gx)}{g} + \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f + gx)}{g} + \frac{B \log\left(-\frac{g(c+dx)}{df-cg}\right) \log(f + gx)}{g} \\
&= -\frac{B \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f + gx)}{g} + \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f + gx)}{g} + \frac{B \log\left(-\frac{g(c+dx)}{df-cg}\right) \log(f + gx)}{g}
\end{aligned}$$

**Mathematica [A]** time = 0.0571215, size = 115, normalized size = 0.82

$$\frac{-B \operatorname{PolyLog}\left(2, \frac{b(f+gx)}{bf-ag}\right) + B \operatorname{PolyLog}\left(2, \frac{d(f+gx)}{df-cg}\right) + \log(f + gx) \left( B \log\left(\frac{e(a+bx)}{c+dx}\right) - B \log\left(\frac{g(a+bx)}{ag-bf}\right) + A + B \log\left(\frac{g(c+dx)}{cg-df}\right) \right)}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x), x]
```

```
[Out] ((A - B*Log[(g*(a + b*x))/(-(b*f) + a*g)] + B*Log[(e*(a + b*x))/(c + d*x)]
+ B*Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] - B*PolyLog[2, (b*(f +
g*x))/(b*f - a*g)] + B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)]/g
```

**Maple [B]** time = 0.545, size = 1400, normalized size = 10.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f),x)`

[Out] 
$$\begin{aligned} & dA/g/(a*d-b*c)*\ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*c*g-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)*a-A/g/(a*d-b*c)*\ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*c*g-d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)*b*c-dA/g/(a*d-b*c)*\ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*a+A/g/(a*d-b*c)*\ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)*b*c+d*B/(a*d-b*c)*\operatorname{dilog}(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*c*a-B/(a*d-b*c)*\operatorname{dilog}(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*c^2*b-d^2*B/g/(a*d-b*c)*\operatorname{dilog}(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*f*a+d*B/g/(a*d-b*c)*\operatorname{dilog}(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*f*b*c+d*B/(a*d-b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*\ln(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*c*a-B/(a*d-b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*\ln(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*c^2*b-d^2*B/g/(a*d-b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*\ln(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*f*a+d*B/g/(a*d-b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*\ln(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*f*b*c-d*B/g/(a*d-b*c)*\operatorname{dilog}(-(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)/b/e)*a+B/g/(a*d-b*c)*\operatorname{dilog}(-(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)/b/e)*b*c-d*B/g/(a*d-b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*\ln(-(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)/b/e)*a+B/g/(a*d-b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*\ln(-(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)/b/e)*b*c \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-B \int -\frac{\log(bx+a) - \log(dx+c) + \log(e)}{gx+f} dx + \frac{A \log(gx+f)}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="maxima")`

[Out] `-B*integrate(-(log(b*x + a) - log(d*x + c) + log(e))/(g*x + f), x) + A*log(g*x + f)/g`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{B \log\left(\frac{bex+ae}{dx+c}\right) + A}{gx+f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(g*x+f),x, algorithm="fricas")`

[Out] `integral((B*log((b*e*x + a*e)/(d*x + c)) + A)/(g*x + f), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A}{gx+f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f),x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)\*e/(d\*x + c)) + A)/(g\*x + f), x)



$$3.236 \quad \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^2} dx$$

Optimal. Leaf size=87

$$\frac{(a+bx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{(f+gx)(bf-ag)} + \frac{B(bc-ad) \log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

[Out] ((a + b\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]]))/((b\*f - a\*g)\*(f + g\*x)) + (B\*(b\*c - a\*d)\*Log[(f + g\*x)/(c + d\*x]])/((b\*f - a\*g)\*(d\*f - c\*g))

**Rubi [A]** time = 0.105829, antiderivative size = 113, normalized size of antiderivative = 1.3, number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2525, 12, 72}

$$-\frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{g(f+gx)} + \frac{B(bc-ad) \log(f+gx)}{(bf-ag)(df-cg)} + \frac{bB \log(a+bx)}{g(bf-ag)} - \frac{Bd \log(c+dx)}{g(df-cg)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])/(f + g\*x)^2, x]

[Out] (b\*B\*Log[a + b\*x])/(g\*(b\*f - a\*g)) - (A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])/(g\*(f + g\*x)) - (B\*d\*Log[c + d\*x])/(g\*(d\*f - c\*g)) + (B\*(b\*c - a\*d)\*Log[f + g\*x])/((b\*f - a\*g)\*(d\*f - c\*g))

#### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 72

Int[((e\_.) + (f\_.)\*(x\_.))^(p\_.)/(((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))), x\_Symbol] := Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

#### Rubi steps

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(f + gx)^2} dx = -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{g(f + gx)} + \frac{B \int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)} dx}{g}$$

$$= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{g(f + gx)} + \frac{(B(bc - ad)) \int \frac{1}{(a+bx)(c+dx)(f+gx)} dx}{g}$$

$$= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{g(f + gx)} + \frac{(B(bc - ad)) \int \left(\frac{b^2}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2}{(bc-ad)(-df+cg)(c+dx)} + \frac{g^2}{(bf-ag)(df-cg)}\right) dx}{g}$$

$$= \frac{bB \log(a + bx)}{g(bf - ag)} - \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{g(f + gx)} - \frac{Bd \log(c + dx)}{g(df - cg)} + \frac{B(bc - ad) \log(f + gx)}{(bf - ag)(df - cg)}$$

**Mathematica [A]** time = 0.151138, size = 105, normalized size = 1.21

$$\frac{B(b \log(a+bx)(df-cg)+\log(c+dx)(adg-bdf)+g(bc-ad) \log(f+gx))}{(bf-ag)(df-cg)} - \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right)+A}{f+gx}}{g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)) + (B*(b*(d*f - c*g)*Log[a + b*x] + (-b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*Log[f + g*x]))/(b*f - a*g)*(d*f - c*g))/g
```

**Maple [B]** time = 0.149, size = 926, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))/(g*x+f)^2,x)
```

```
[Out] e*d*A/(b*e/d*c*g+e/(d*x+c)*a*c*g-e/(d*x+c)*a*d*f-e/d/(d*x+c)*b*c^2*g+e/(d*x+c)*b*c*f-a*e*g)/(c*g-d*f)*a-e*A/(b*e/d*c*g+e/(d*x+c)*a*c*g-e/(d*x+c)*a*d*f-e/d/(d*x+c)*b*c^2*g+e/(d*x+c)*b*c*f-a*e*g)/(c*g-d*f)*b*c-d*B/(a*g-b*f)*ln((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(c*g-d*f)*a+B/(a*g-b*f)*ln((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(c*g-d*f)*b*c+e*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(a*g-b*f)/(b*e/d*c*g+e/(d*x+c)*a*c*g-e/(d*x+c)*a*d*f-e/d/(d*x+c)*b*c^2*g+e/(d*x+c)*b*c*f-a*e*g)*b*a-e/d*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(a*g-b*f)/(b*e/d*c*g+e/(d*x+c)*a*c*g-e/(d*x+c)*a*d*f-e/d/(d*x+c)*b*c^2*g+e/(d*x+c)*b*c*f-a*e*g)*b^2*c+e*d*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(a*g-b*f)/(b*e/d*c*g+e/(d*x+c)*a*c*g-e/(d*x+c)*a*d*f-e/d/(d*x+c)*b*c^2*g+e/(d*x+c)*b*c*f-a*e*g)/(d*x+c)*a^2-2*e*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(a*g-b*f)/(b*e/d*c*g+e/(d*x+c)*a*c*g-e/(d*x+c)*a*d*f-e/d/(d*x+c)*b*c^2*g+e/(d*x+c)*b*c*f-a*e*g)/(d*x+c)*a*b*c+e/d*B*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))/(a*g-b*f)/(b*e/d*c*g+e/(d*x+c)*a*c*g-e/(d*x+c)*a*d*f-e/d/(d*x+c)*b*c^2*g+e/(d*x+c)*b*c*f-a*e*g)/(d*x+c)*b^2*c^2
```

**Maxima [A]** time = 1.12098, size = 186, normalized size = 2.14

$$B \left( \frac{b \log(bx + a)}{bfg - ag^2} - \frac{d \log(dx + c)}{dfg - cg^2} + \frac{(bc - ad) \log(gx + f)}{bdf^2 + acg^2 - (bc + ad)fg} - \frac{\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)}{g^2x + fg} \right) - \frac{A}{g^2x + fg}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^2,x, algorithm="maxima")

[Out] B\*(b\*log(b\*x + a)/(b\*f\*g - a\*g^2) - d\*log(d\*x + c)/(d\*f\*g - c\*g^2) + (b\*c - a\*d)\*log(g\*x + f)/(b\*d\*f^2 + a\*c\*g^2 - (b\*c + a\*d)\*f\*g) - log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c))/(g^2\*x + f\*g)) - A/(g^2\*x + f\*g)

**Fricas [B]** time = 21.9531, size = 559, normalized size = 6.43

$$\frac{Abdf^2 + Aacg^2 - (Abc + Aad)fg - (Bbdf^2 - Bbcfg + (Bbdfg - Bbcg^2)x) \log(bx + a) + (Bbdf^2 - Badfg + (Bbdfg - Bbcg^2)x) \log(dx + c) - ((Bb*c - Bb*a*d)*g^2*x + (Bb*c - Bb*a*d)*f*g) \log(g*x + f) + (Bb*d*f^2 + Bb*a*c*g^2 - (Bb*c + Bb*a*d)*f*g) \log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)}{bdf^3g + acfg^3 - (bc + ad)f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^2,x, algorithm="fricas")

[Out] -(A\*b\*d\*f^2 + A\*a\*c\*g^2 - (A\*b\*c + A\*a\*d)\*f\*g - (B\*b\*d\*f^2 - B\*b\*c\*f\*g + (B\*b\*d\*f\*g - B\*b\*c\*g^2)\*x)\*log(b\*x + a) + (B\*b\*d\*f^2 - B\*a\*d\*f\*g + (B\*b\*d\*f\*g - B\*a\*d\*g^2)\*x)\*log(d\*x + c) - ((B\*b\*c - B\*a\*d)\*g^2\*x + (B\*b\*c - B\*a\*d)\*f\*g)\*log(g\*x + f) + (B\*b\*d\*f^2 + B\*a\*c\*g^2 - (B\*b\*c + B\*a\*d)\*f\*g)\*log((b\*e\*x + a\*e)/(d\*x + c))/(b\*d\*f^3\*g + a\*c\*f\*g^3 - (b\*c + a\*d)\*f^2\*g^2 + (b\*d\*f^2\*g^2 + a\*c\*g^4 - (b\*c + a\*d)\*f\*g^3)\*x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)\*\*2,x)

[Out] Timed out

**Giac [B]** time = 1.31311, size = 378, normalized size = 4.34

$$\frac{(Bbc - Bad) \log(gx + f)}{bdf^2 - bcfg - adfg + acg^2} - \frac{B \log\left(\frac{bx+a}{dx+c}\right)}{g^2x + fg} - \frac{(Bbc - Bad) \log(|bdx^2 + bcx + adx + ac|)}{2(bdf^2 - bcfg - adfg + acg^2)} - \frac{A + B}{g^2x + fg} + \frac{(2Bb^2cdf - 2Bb^2cd)}{2(g^2x + fg)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^2,x, algorithm="giac")

```
[Out] (B*b*c - B*a*d)*log(g*x + f)/(b*d*f^2 - b*c*f*g - a*d*f*g + a*c*g^2) - B*log((b*x + a)/(d*x + c))/(g^2*x + f*g) - 1/2*(B*b*c - B*a*d)*log(abs(b*d*x^2 + b*c*x + a*d*x + a*c))/(b*d*f^2 - b*c*f*g - a*d*f*g + a*c*g^2) - (A + B)/(g^2*x + f*g) + 1/2*(2*B*b^2*c*d*f - 2*B*a*b*d^2*f - B*b^2*c^2*g + B*a^2*d^2*g)*log(abs((2*b*d*x + b*c + a*d - abs(-b*c + a*d))/(2*b*d*x + b*c + a*d + abs(-b*c + a*d))))/((b*d*f^2*g - b*c*f*g^2 - a*d*f*g^2 + a*c*g^3)*abs(-b*c + a*d))
```

$$3.237 \quad \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3} dx$$

**Optimal.** Leaf size=183

$$-\frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{2g(f+gx)^2} + \frac{b^2 B \log(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)}{2(f+gx)(bf-ag)(df-cg)} + \frac{B(bc-ad) \log(f+gx)(-adg-bcg+2bdf)}{2(bf-ag)^2(df-cg)^2}$$

[Out]  $-(B*(b*c - a*d))/(2*(b*f - a*g)*(d*f - c*g)*(f + g*x)) + (b^2*B*\text{Log}[a + b*x])/ (2*g*(b*f - a*g)^2) - (A + B*\text{Log}[(e*(a + b*x))/(c + d*x]))/(2*g*(f + g*x)^2) - (B*d^2*\text{Log}[c + d*x])/(2*g*(d*f - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*\text{Log}[f + g*x])/(2*(b*f - a*g)^2*(d*f - c*g)^2)$

**Rubi [A]** time = 0.185454, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2525, 12, 72}

$$-\frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{2g(f+gx)^2} + \frac{b^2 B \log(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)}{2(f+gx)(bf-ag)(df-cg)} + \frac{B(bc-ad) \log(f+gx)(-adg-bcg+2bdf)}{2(bf-ag)^2(df-cg)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(A + B*\text{Log}[(e*(a + b*x))/(c + d*x]))/(f + g*x)^3, x]$

[Out]  $-(B*(b*c - a*d))/(2*(b*f - a*g)*(d*f - c*g)*(f + g*x)) + (b^2*B*\text{Log}[a + b*x])/ (2*g*(b*f - a*g)^2) - (A + B*\text{Log}[(e*(a + b*x))/(c + d*x]))/(2*g*(f + g*x)^2) - (B*d^2*\text{Log}[c + d*x])/(2*g*(d*f - c*g)^2) + (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*\text{Log}[f + g*x])/(2*(b*f - a*g)^2*(d*f - c*g)^2)$

#### Rule 2525

$\text{Int}[(a + \text{Log}[(c + \text{RFX})^p])*(b + (d + e*x)^m)^n, x\_Symbol] := \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^n/(e*(m+1)), x] - \text{Dist}[(b*n*p)/(e*(m+1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^{n-1}*D[\text{RFX}, x])/RFX, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, p\}, x \&\& \text{RationalFunctionQ}[\text{RFX}, x] \&\& \text{IGtQ}[n, 0] \&\& (\text{EqQ}[n, 1] || \text{IntegerQ}[m]) \&\& \text{NeQ}[m, -1]$

#### Rule 12

$\text{Int}[(a + u), x\_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& !\text{MatchQ}[u, (b + v) /; \text{FreeQ}[b, x]]$

#### Rule 72

$\text{Int}[(e + f*x)^p/((a + b*x)*(c + d*x)), x\_Symbol] := \text{Int}[\text{ExpandIntegrand}[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{IntegerQ}[p]$

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3} dx &= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2} + \frac{B \int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)^2} dx}{2g} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2} + \frac{(B(bc-ad)) \int \frac{1}{(a+bx)(c+dx)(f+gx)^2} dx}{2g} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2} + \frac{(B(bc-ad)) \int \left( \frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d^3}{(bc-ad)(-df+cg)^2(c+dx)} + \frac{g^2}{(bf-ag)(df-cg)^2} \right) dx}{2g} \\
&= -\frac{B(bc-ad)}{2(bf-ag)(df-cg)(f+gx)} + \frac{b^2 B \log(a+bx)}{2g(bf-ag)^2} - \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{2g(f+gx)^2} - \frac{Bd^2 \log(c+dx)}{2g(df-cg)^2} + \dots
\end{aligned}$$

**Mathematica [A]** time = 0.645492, size = 169, normalized size = 0.92

$$\frac{B(bc-ad) \left( \frac{b^2 \log(a+bx)}{(bc-ad)(bf-ag)^2} + \frac{\frac{d^2 \log(c+dx)}{ad-bc} + \frac{g(cg-df)}{(f+gx)(bf-ag)} - \frac{g \log(f+gx)(adg+bcg-2bdf)}{(bf-ag)^2} \right) - \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{(f+gx)^2}}{2g}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))/(f + g\*x)^3,x]

[Out] (-((A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))/(f + g\*x)^2) + B\*(b\*c - a\*d)\*((b^2\*Log[a + b\*x])/((b\*c - a\*d)\*(b\*f - a\*g)^2) + ((g\*(-(d\*f) + c\*g))/((b\*f - a\*g)\*(f + g\*x)) + (d^2\*Log[c + d\*x])/(-(b\*c) + a\*d) - (g\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g)\*Log[f + g\*x])/((b\*f - a\*g)^2)/(d\*f - c\*g)^2))/(2\*g)

**Maple [B]** time = 0.175, size = 5274, normalized size = 28.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^3,x)

[Out] result too large to display

**Maxima [B]** time = 1.24118, size = 474, normalized size = 2.59

$$\frac{1}{2} \left( \frac{b^2 \log(bx+a)}{b^2 f^2 g - 2 abf g^2 + a^2 g^3} - \frac{d^2 \log(dx+c)}{d^2 f^2 g - 2 cdf g^2 + c^2 g^3} + \frac{(2(b^2 cd - abd^2)f - (b^2 c^2 - a^2 d^2)g) \log(gx+f)}{b^2 d^2 f^4 + a^2 c^2 g^4 - 2(b^2 cd + abd^2)f^3 g + (b^2 c^2 + 4abcd + a^2 d^2)f^2 g^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^3,x, algorithm="maxima")

[Out] 1/2\*(b^2\*log(b\*x + a)/(b^2\*f^2\*g - 2\*a\*b\*f\*g^2 + a^2\*g^3) - d^2\*log(d\*x + c)/(d^2\*f^2\*g - 2\*c\*d\*f\*g^2 + c^2\*g^3) + (2\*(b^2\*c\*d - a\*b\*d^2)\*f - (b^2\*c^2

$$- a^2 d^2 g \log(gx + f) / (b^2 d^2 f^4 + a^2 c^2 g^4 - 2(b^2 c d + a b d^2) f^3 g + (b^2 c^2 + 4 a b c d + a^2 d^2) f^2 g^2 - 2(a b c^2 + a^2 c d) f g^3) - (b c - a d) / (b d f^3 + a c f g^2 - (b c + a d) f^2 g + (b d f^2 g + a c g^3 - (b c + a d) f g^2) x) - \log(b e x / (d x + c) + a e / (d x + c)) / (g^3 x^2 + 2 f g^2 x + f^2 g) * B - 1/2 A / (g^3 x^2 + 2 f g^2 x + f^2 g)$$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^3,x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)\*\*3,x)

[Out] Timed out

**Giac [B]** time = 1.74853, size = 1110, normalized size = 6.07

$$\frac{(2 B b^2 c d f - 2 B a b d^2 f - B b^2 c^2 g + B a^2 d^2 g) \log(gx + f)}{2 (b^2 d^2 f^4 - 2 b^2 c d f^3 g - 2 a b d^2 f^3 g + b^2 c^2 f^2 g^2 + 4 a b c d f^2 g^2 + a^2 d^2 f^2 g^2 - 2 a b c^2 f g^3 - 2 a^2 c d f g^3 + a^2 c^2 g^4)} - \frac{B}{2 (g^3 x^2 + 2 f g^2 x + f^2 g)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^3,x, algorithm="giac")

[Out]  $1/2 * (2 * B * b^2 * c * d * f - 2 * B * a * b * d^2 * f - B * b^2 * c^2 * g + B * a^2 * d^2 * g) * \log(gx + f) / (b^2 * d^2 * f^4 - 2 * b^2 * c * d * f^3 * g - 2 * a * b * d^2 * f^3 * g + b^2 * c^2 * f^2 * g^2 + 4 * a * b * c * d * f^2 * g^2 + a^2 * d^2 * f^2 * g^2 - 2 * a * b * c^2 * f * g^3 - 2 * a^2 * c * d * f * g^3 + a^2 * c^2 * g^4) - 1/2 * B * \log((b * x + a) / (d * x + c)) / (g^3 * x^2 + 2 * f * g^2 * x + f^2 * g) - 1/4 * (2 * B * b^2 * c * d * f - 2 * B * a * b * d^2 * f - B * b^2 * c^2 * g + B * a^2 * d^2 * g) * \log(\text{abs}(b * d * x^2 + b * c * x + a * d * x + a * c)) / (b^2 * d^2 * f^4 - 2 * b^2 * c * d * f^3 * g - 2 * a * b * d^2 * f^3 * g + b^2 * c^2 * f^2 * g^2 + 4 * a * b * c * d * f^2 * g^2 + a^2 * d^2 * f^2 * g^2 - 2 * a * b * c^2 * f * g^3 - 2 * a^2 * c * d * f * g^3 + a^2 * c^2 * g^4) - 1/2 * (B * b * c * g^2 * x - B * a * d * g^2 * x + A * b * d * f^2 + B * b * d * f^2 - A * b * c * f * g - A * a * d * f * g - 2 * B * a * d * f * g + A * a * c * g^2 + B * a * c * g^2) / (b * d * f^2 * g^3 * x^2 - b * c * f * g^4 * x^2 - a * d * f * g^4 * x^2 + a * c * g^5 * x^2 + 2 * b * d * f^3 * g^2 * x - 2 * b * c * f^2 * g^3 * x - 2 * a * d * f^2 * g^3 * x + 2 * a * c * f * g^4 * x + b * d * f^4 * g - b * c * f^3 * g^2 - a * d * f^3 * g^2 + a * c * f^2 * g^3) + 1/4 * (2 * B * b^3 * c * d^2 * f^2 - 2 * B * a * b^2 * d^3 * f^2 - 2 * B * b^3 * c^2 * d * f * g + 2 * B * a^2 * b * d^3 * f * g + B * b^3 * c^3 * g^2 - B * a * b^2 * c^2 * d * g^2 + B * a^2 * b * c * d^2 * g^2 - B * a^3 * d^3 * g^2) * \log(\text{abs}((2 * b * d * x + b * c + a * d - \text{abs}(-b * c + a * d)) / (2 * b * d * x + b * c + a * d + \text{abs}(-b * c + a * d)))) / ((b^2 * d^2 * f^4 * g - 2 * b^2 * c * d * f^3 * g^2 - 2 * a * b * d^2 * f^3 * g^2 + b^2 * c^2 * f^2 * g^3 + 4 * a * b * c * d * f^2 * g^2 + a^2 * d^2 * f^2 * g^2 - 2 * a * b * c^2 * f * g^3 - 2 * a^2 * c * d * f * g^3 + a^2 * c^2 * g^4) * (g^3 * x^2 + 2 * f * g^2 * x + f^2 * g))$

$$f^2g^3 + a^2d^2f^2g^3 - 2abc^2fg^4 - 2a^2cdfg^4 + a^2c^2g^5) \cdot \text{abs}(-bc + ad)$$



$$3.238 \quad \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4} dx$$

**Optimal.** Leaf size=275

$$\frac{B(bc-ad) \log(f+gx) (a^2 d^2 g^2 - abdg(3df-cg) + b^2 (c^2 g^2 - 3cdfg + 3d^2 f^2))}{3(bf-ag)^3 (df-cg)^3} - \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{3g(f+gx)^3} + \frac{b^3 B \log(a+bx)}{3g(bf-ag)}$$

```
[Out] -(B*(b*c - a*d))/(6*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*B*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*g*(f + g*x)^3) - (B*d^3*Log[c + d*x])/(3*g*(d*f - c*g)^3) + (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

**Rubi [A]** time = 0.396054, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2525, 12, 72}

$$\frac{B(bc-ad) \log(f+gx) (a^2 d^2 g^2 - abdg(3df-cg) + b^2 (c^2 g^2 - 3cdfg + 3d^2 f^2))}{3(bf-ag)^3 (df-cg)^3} - \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}{3g(f+gx)^3} + \frac{b^3 B \log(a+bx)}{3g(bf-ag)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^4, x]
```

```
[Out] -(B*(b*c - a*d))/(6*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*B*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*g*(f + g*x)^3) - (B*d^3*Log[c + d*x])/(3*g*(d*f - c*g)^3) + (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^4} dx &= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3} + \frac{B \int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)^3} dx}{3g} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3} + \frac{(B(bc-ad)) \int \frac{1}{(a+bx)(c+dx)(f+gx)^3} dx}{3g} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{3g(f+gx)^3} + \frac{(B(bc-ad)) \int \left( \frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)} + \frac{d^4}{(bc-ad)(-df+cg)^3(c+dx)} + \frac{g^2}{(bf-ag)(df-cg)^3} \right) dx}{3g} \\
&= -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^2} - \frac{B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^3 B \log(a+bx)}{3g(bf-ag)^3} - \frac{A+B}{3g}
\end{aligned}$$

**Mathematica [A]** time = 0.926759, size = 260, normalized size = 0.95

$$\frac{B(bc-ad) \left( \frac{g \log(f+gx)(a^2 d^2 g^2 + abdg(cg-3df) + b^2(c^2 g^2 - 3cdfg + 3d^2 f^2))}{(bf-ag)^3(df-cg)^3} + \frac{b^3 \log(a+bx)}{(bc-ad)(bf-ag)^3} + \frac{d^3 \log(c+dx)}{(bc-ad)(cg-df)^3} + \frac{g(adg+bcg-2bdf)}{(f+gx)(bf-ag)^2(df-cg)^2} - \frac{A+B}{2(f+gx)} \right)}{3g}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))/(f + g\*x)^4, x]

[Out] 
$$\begin{aligned}
&-\left(\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^3} + \frac{B(bc-ad)(-g/(2(bf-ag)(df-cg)(f+gx)^2) + (g*(-2bdf+bcg+adg))/((bf-ag)^2(df-cg)^2(f+gx)) + (b^3 \log(a+bx))/((bc-ad)(bf-ag)^3) + (d^3 \log(c+dx))/((bc-ad)(cg-df)^3) + (g(adg+bcg-2bdf))/((f+gx)(bf-ag)^2(df-cg)^2) - (A+B)/(2(f+gx)))}{3g}\right)
\end{aligned}$$

**Maple [B]** time = 0.203, size = 18285, normalized size = 66.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^4, x)

[Out] result too large to display

**Maxima [B]** time = 1.5379, size = 1145, normalized size = 4.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^4, x, algorithm="maxima")

[Out] 
$$\begin{aligned}
&\frac{1}{6} \left( \frac{2b^3 \log(bx+a)}{b^3 f^3 g - 3a b^2 f^2 g^2 + 3a^2 b f g^3 - a^3 g^4} - \frac{2d^3 \log(dx+c)}{d^3 f^3 g - 3c d^2 f^2 g^2 + 3c^2 d f g^3 - c^3 g^4} + 2(3(b^3 c d^2 - a b^2 d^3) f^2 - 3(b^3 c^2 d - a^2 b d^3) f g + \dots \right)
\end{aligned}$$

$$\begin{aligned} & (b^3c^3 - a^3d^3)g^2 \log(gx + f) / (b^3d^3f^6 + a^3c^3g^6 - 3(b^3c^3d^2 + a^3b^2d^3)f^5g + 3(b^3c^2d + 3a^2b^2c^2d^2 + a^2b^2d^3)f^4g^2 \\ & - (b^3c^3 + 9a^2b^2c^2d + 9a^2b^2c^2d^2 + a^3d^3)f^3g^3 + 3(a^2b^2c^3 + 3a^2b^2c^2d + a^3c^2d^2)f^2g^4 - 3(a^2b^2c^3 + a^3c^2d)f^2g^5 \\ & - (5(b^2c^2d - a^2bd^2)f^2 - 3(b^2c^2 - a^2d^2)f^2g + (abc^2 - a^2cd)g^2 + 2(2(b^2cd - abd^2)f^2g - (b^2c^2 - a^2d^2)g^2)x) / (b^2d^2f^6 + a^2c^2f^2g^4 - 2(b^2cd + abd^2)f^5g + (b^2c^2 + 4abc^2 + a^2d^2)f^4g^2 - 2(abc^2 + a^2cd)f^3g^3 + (b^2d^2f^4g^2 + a^2c^2g^6 - 2(b^2cd + abd^2)f^3g^3 + (b^2c^2 + 4abc^2 + a^2d^2)f^2g^4 - 2(abc^2 + a^2cd)f^2g^5)x^2 + 2(b^2d^2f^5g + a^2c^2f^2g^5 - 2(b^2cd + abd^2)f^4g^2 + (b^2c^2 + 4abc^2 + a^2d^2)f^3g^3 - 2(abc^2 + a^2cd)f^2g^4)x) - 2 \log(bex/(dx + c) + ae/(dx + c)) / (g^4x^3 + 3f^2g^2x + f^3g) * B - 1/3A / (g^4x^3 + 3f^2g^2x + f^3g) \end{aligned}$$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^4,x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)\*\*4,x)

[Out] Timed out

**Giac [B]** time = 4.30114, size = 2765, normalized size = 10.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^4,x, algorithm="giac")

[Out] 
$$\begin{aligned} & 1/3*(3B*b^3c^2d^2f^2 - 3B*a^2b^2d^3f^2 - 3B*b^3c^2d^2f^2g + 3B*a^2b^2d^3f^2g + B*b^3c^3g^2 - B*a^3d^3g^2) \log(gx + f) / (b^3d^3f^6 - 3b^3c^3d^2f^5g - 3a^2b^2d^3f^5g + 3b^3c^2d^2f^4g^2 + 9a^2b^2c^2d^2f^4g^2 + 3a^2b^2d^3f^4g^2 - b^3c^3f^3g^3 - 9a^2b^2c^2d^2f^3g^3 - 9a^2b^2c^2d^2f^3g^3 - a^3d^3f^3g^3 + 3a^2b^2c^3f^2g^4 + 9a^2b^2c^2d^2f^2g^4 + 3a^3c^2d^2f^2g^4 - 3a^2b^2c^3f^2g^5 - 3a^3c^2d^2f^2g^5 + a^3c^3g^6) - 1/3B \log((b*x + a)/(d*x + c)) / (g^4x^3 + 3f^2g^2x + f^3g) - 1/6*(3B*b^3c^2d^2f^2 - 3B*a^2b^2d^3f^2 - 3B*b^3c^2d^2f^2g + 3B*a^2b^2d^3f^2g + B*b^3c^3g^2 - B*a^3d^3g^2) \log(\text{abs}(b*d*x^2 + b*c \end{aligned}$$

$$\begin{aligned}
& *x + a*d*x + a*c)) / (b^3*d^3*f^6 - 3*b^3*c*d^2*f^5*g - 3*a*b^2*d^3*f^5*g + 3 \\
& *b^3*c^2*d*f^4*g^2 + 9*a*b^2*c*d^2*f^4*g^2 + 3*a^2*b*d^3*f^4*g^2 - b^3*c^3* \\
& f^3*g^3 - 9*a*b^2*c^2*d*f^3*g^3 - 9*a^2*b*c*d^2*f^3*g^3 - a^3*d^3*f^3*g^3 + \\
& 3*a*b^2*c^3*f^2*g^4 + 9*a^2*b*c^2*d*f^2*g^4 + 3*a^3*c*d^2*f^2*g^4 - 3*a^2* \\
& b*c^3*f*g^5 - 3*a^3*c^2*d*f*g^5 + a^3*c^3*g^6) - 1/6*(4*B*b^2*c*d*f*g^3*x^2 \\
& - 4*B*a*b*d^2*f*g^3*x^2 - 2*B*b^2*c^2*g^4*x^2 + 2*B*a^2*d^2*g^4*x^2 + 9*B* \\
& b^2*c*d*f^2*g^2*x - 9*B*a*b*d^2*f^2*g^2*x - 5*B*b^2*c^2*f*g^3*x + 5*B*a^2*d \\
& ^2*f*g^3*x + B*a*b*c^2*g^4*x - B*a^2*c*d*g^4*x + 2*A*b^2*d^2*f^4 + 2*B*b^2* \\
& d^2*f^4 - 4*A*b^2*c*d*f^3*g + B*b^2*c*d*f^3*g - 4*A*a*b*d^2*f^3*g - 9*B*a*b \\
& *d^2*f^3*g + 2*A*b^2*c^2*f^2*g^2 - B*b^2*c^2*f^2*g^2 + 8*A*a*b*c*d*f^2*g^2 \\
& + 8*B*a*b*c*d*f^2*g^2 + 2*A*a^2*d^2*f^2*g^2 + 5*B*a^2*d^2*f^2*g^2 - 4*A*a*b \\
& *c^2*f*g^3 - 3*B*a*b*c^2*f*g^3 - 4*A*a^2*c*d*f*g^3 - 5*B*a^2*c*d*f*g^3 + 2* \\
& A*a^2*c^2*g^4 + 2*B*a^2*c^2*g^4) / (b^2*d^2*f^4*g^4*x^3 - 2*b^2*c*d*f^3*g^5*x \\
& ^3 - 2*a*b*d^2*f^3*g^5*x^3 + b^2*c^2*f^2*g^6*x^3 + 4*a*b*c*d*f^2*g^6*x^3 + \\
& a^2*d^2*f^2*g^6*x^3 - 2*a*b*c^2*f*g^7*x^3 - 2*a^2*c*d*f*g^7*x^3 + a^2*c^2*g \\
& ^8*x^3 + 3*b^2*d^2*f^5*g^3*x^2 - 6*b^2*c*d*f^4*g^4*x^2 - 6*a*b*d^2*f^4*g^4* \\
& x^2 + 3*b^2*c^2*f^3*g^5*x^2 + 12*a*b*c*d*f^3*g^5*x^2 + 3*a^2*d^2*f^3*g^5*x^ \\
& 2 - 6*a*b*c^2*f^2*g^6*x^2 - 6*a^2*c*d*f^2*g^6*x^2 + 3*a^2*c^2*f*g^7*x^2 + 3 \\
& *b^2*d^2*f^6*g^2*x - 6*b^2*c*d*f^5*g^3*x - 6*a*b*d^2*f^5*g^3*x + 3*b^2*c^2* \\
& f^4*g^4*x + 12*a*b*c*d*f^4*g^4*x + 3*a^2*d^2*f^4*g^4*x - 6*a*b*c^2*f^3*g^5* \\
& x - 6*a^2*c*d*f^3*g^5*x + 3*a^2*c^2*f^2*g^6*x + b^2*d^2*f^7*g - 2*b^2*c*d*f \\
& ^6*g^2 - 2*a*b*d^2*f^6*g^2 + b^2*c^2*f^5*g^3 + 4*a*b*c*d*f^5*g^3 + a^2*d^2* \\
& f^5*g^3 - 2*a*b*c^2*f^4*g^4 - 2*a^2*c*d*f^4*g^4 + a^2*c^2*f^3*g^5) + 1/6*(2 \\
& *B*b^4*c*d^3*f^3 - 2*B*a*b^3*d^4*f^3 - 3*B*b^4*c^2*d^2*f^2*g + 3*B*a^2*b^2* \\
& d^4*f^2*g + 3*B*b^4*c^3*d*f*g^2 - 3*B*a*b^3*c^2*d^2*f*g^2 + 3*B*a^2*b^2*c*d \\
& ^3*f*g^2 - 3*B*a^3*b*d^4*f*g^2 - B*b^4*c^4*g^3 + B*a*b^3*c^3*d*g^3 - B*a^3* \\
& b*c*d^3*g^3 + B*a^4*d^4*g^3)*log(abs((2*b*d*x + b*c + a*d - abs(-b*c + a*d) \\
& )) / (2*b*d*x + b*c + a*d + abs(-b*c + a*d))) / ((b^3*d^3*f^6*g - 3*b^3*c*d^2*f \\
& ^5*g^2 - 3*a*b^2*d^3*f^5*g^2 + 3*b^3*c^2*d*f^4*g^3 + 9*a*b^2*c*d^2*f^4*g^3 \\
& + 3*a^2*b*d^3*f^4*g^3 - b^3*c^3*f^3*g^4 - 9*a*b^2*c^2*d*f^3*g^4 - 9*a^2*b*c \\
& *d^2*f^3*g^4 - a^3*d^3*f^3*g^4 + 3*a*b^2*c^3*f^2*g^5 + 9*a^2*b*c^2*d*f^2*g^ \\
& 5 + 3*a^3*c*d^2*f^2*g^5 - 3*a^2*b*c^3*f*g^6 - 3*a^3*c^2*d*f*g^6 + a^3*c^3*g \\
& ^7)*abs(-b*c + a*d))
\end{aligned}$$

$$3.239 \quad \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(f+gx)^5} dx$$

**Optimal.** Leaf size=379

$$\frac{B(bc-ad)(a^2d^2g^2 - abd g(3df - cg) + b^2(c^2g^2 - 3cdfg + 3d^2f^2))}{4(f+gx)(bf-ag)^3(df-cg)^3} - \frac{B(bc-ad) \log(f+gx)(-adg - bcg + 2bdf)}{4(bf-ag)^3}$$

```
[Out] -(B*(b*c - a*d))/(12*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)*
(2*b*d*f - b*c*g - a*d*g))/(8*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (B
*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*
f*g + c^2*g^2)))/(4*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*Log[a +
b*x])/(4*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(4*g*(f +
g*x)^4) - (B*d^4*Log[c + d*x])/(4*g*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d
*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f
*g + c^2*g^2))*Log[f + g*x])/(4*(b*f - a*g)^4*(d*f - c*g)^4)
```

**Rubi [A]** time = 0.617712, antiderivative size = 379, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2525, 12, 72}

$$\frac{B(bc-ad)(a^2d^2g^2 - abd g(3df - cg) + b^2(c^2g^2 - 3cdfg + 3d^2f^2))}{4(f+gx)(bf-ag)^3(df-cg)^3} - \frac{B(bc-ad) \log(f+gx)(-adg - bcg + 2bdf)}{4(bf-ag)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(f + g*x)^5, x]
```

```
[Out] -(B*(b*c - a*d))/(12*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)*
(2*b*d*f - b*c*g - a*d*g))/(8*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (B
*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*
f*g + c^2*g^2)))/(4*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*Log[a +
b*x])/(4*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(4*g*(f +
g*x)^4) - (B*d^4*Log[c + d*x])/(4*g*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d
*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f
*g + c^2*g^2))*Log[f + g*x])/(4*(b*f - a*g)^4*(d*f - c*g)^4)
```

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 72

```
Int[((e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

;/ FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rubi steps

$$\int \frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(f + gx)^5} dx = -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f + gx)^4} + \frac{B \int \frac{bc-ad}{(a+bx)(c+dx)(f+gx)^4} dx}{4g}$$

$$= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f + gx)^4} + \frac{(B(bc - ad)) \int \frac{1}{(a+bx)(c+dx)(f+gx)^4} dx}{4g}$$

$$= -\frac{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)}{4g(f + gx)^4} + \frac{(B(bc - ad)) \int \left( \frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5}{(bc-ad)(-df+cg)^4(c+dx)} + \frac{g^2}{(bf-ag)(df-cg)} \right) dx}{4g}$$

$$= -\frac{B(bc - ad)}{12(bf - ag)(df - cg)(f + gx)^3} - \frac{B(bc - ad)(2bdf - bcg - adg)}{8(bf - ag)^2(df - cg)^2(f + gx)^2} - \frac{B(bc - ad)(a^2d^2g^2 - abcf)}{4(bf - ag)(df - cg)}$$

**Mathematica [A]** time = 1.15871, size = 355, normalized size = 0.94

$$\frac{B(bc - ad) \left( -\frac{g(a^2d^2g^2 + abdgcg - 3df) + b^2(c^2g^2 - 3cdfg + 3d^2f^2)}{(f+gx)(bf-ag)^3(df-cg)^3} - \frac{g \log(f+gx)(adg+bcg-2bdf)(a^2d^2g^2 - 2abd^2fg + b^2(c^2g^2 - 2cdfg + 2d^2f^2))}{(bf-ag)^4(df-cg)^4} + \frac{b^4 \log(a+bx)}{(bc-ad)(bf-ag)} \right)}{4g}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))/(f + g\*x)^5, x]

[Out] (-((A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))/(f + g\*x)^4) + B\*(b\*c - a\*d)\*(-g/(3\*(b\*f - a\*g)\*(d\*f - c\*g)\*(f + g\*x)^3) + (g\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g))/(2\*(b\*f - a\*g)^2\*(d\*f - c\*g)^2\*(f + g\*x)^2) - (g\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(-3\*d\*f + c\*g) + b^2\*(3\*d^2\*f^2 - 3\*c\*d\*f\*g + c^2\*g^2)))/((b\*f - a\*g)^3\*(d\*f - c\*g)^3\*(f + g\*x)) + (b^4\*Log[a + b\*x])/((b\*c - a\*d)\*(b\*f - a\*g)^4) - (d^4\*Log[c + d\*x])/((b\*c - a\*d)\*(d\*f - c\*g)^4) - (g\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g)\*(-2\*a\*b\*d^2\*f\*g + a^2\*d^2\*g^2 + b^2\*(2\*d^2\*f^2 - 2\*c\*d\*f\*g + c^2\*g^2))\*Log[f + g\*x])/((b\*f - a\*g)^4\*(d\*f - c\*g)^4))/(4\*g)

**Maple [B]** time = 0.259, size = 44893, normalized size = 118.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^5, x)

[Out] result too large to display

**Maxima [B]** time = 1.89596, size = 2372, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^5,x, algorithm="maxima")

[Out]  $\frac{1}{24} \cdot (6b^4 \log(bx+a)/(b^4 f^4 g - 4ab^3 f^3 g^2 + 6a^2 b^2 f^2 g^3 - 4a^3 b f g^4 + a^4 g^5) - 6d^4 \log(dx+c)/(d^4 f^4 g - 4cd^3 f^3 g^2 + 6c^2 d^2 f^2 g^3 - 4c^3 d f g^4 + c^4 g^5) + 6(4(b^4 c d^3 - ab^3 d^4) f^3 - 6(b^4 c^2 d^2 - a^2 b^2 d^4) f^2 g + 4(b^4 c^3 d - a^3 b d^4) f g^2 - (b^4 c^4 - a^4 d^4) g^3) \log(gx+f) / (b^4 d^4 f^8 + a^4 c^4 g^8 - 4(b^4 c d^3 + ab^3 d^4) f^7 g + 2(3b^4 c^2 d^2 + 8a^2 b^3 c d^3 + 3a^2 b^2 d^4) f^6 g^2 - 4(b^4 c^3 d + 6a^2 b^3 c^2 d^2 + 6a^2 b^2 c d^3 + a^3 b d^4) f^5 g^3 + (b^4 c^4 + 16a^2 b^3 c^3 d + 36a^2 b^2 c^2 d^2 + 16a^3 b c d^3 + a^4 d^4) f^4 g^4 - 4(ab^3 c^4 + 6a^2 b^2 c^3 d + 6a^3 b c^2 d^2 + a^4 c d^3) f^3 g^5 + 2(3a^2 b^2 c^4 + 8a^3 b c^3 d + 3a^4 c^2 d^2) f^2 g^6 - 4(a^3 b c^4 + a^4 c^3 d) f g^7) - (26(b^3 c d^2 - ab^2 d^3) f^4 - 31(b^3 c^2 d - a^2 b d^3) f^3 g + (11b^3 c^3 + 15a^2 b^2 c^2 d - 15a^2 b c d^2 - 11a^3 d^3) f^2 g^2 - 7(ab^2 c^3 - a^3 c d^2) f g^3 + 2(a^2 b c^3 - a^3 c^2 d) g^4 + 6(3(b^3 c d^2 - ab^2 d^3) f^2 g^2 - 3(b^3 c^2 d - a^2 b d^3) f g^3 + (b^3 c^3 - a^3 d^3) g^4) x^2 + 3(14(b^3 c d^2 - ab^2 d^3) f^3 g - 15(b^3 c^2 d - a^2 b d^3) f^2 g^2 + (5b^3 c^3 + 3a^2 b^2 c^2 d - 3a^2 b c d^2 - 5a^3 d^3) f g^3 - (ab^2 c^3 - a^3 c d^2) g^4) x) / (b^3 d^3 f^9 + a^3 c^3 f^3 g^6 - 3(b^3 c d^2 + ab^2 d^3) f^8 g + 3(b^3 c^2 d + 3a^2 b^2 c d^2 + a^2 b d^3) f^7 g^2 - (b^3 c^3 + 9a^2 b^2 c^2 d + 9a^2 b b c d^2 + a^3 d^3) f^6 g^3 + 3(ab^2 c^3 + 3a^2 b c^2 d + a^3 c d^2) f^5 g^4 - 3(a^2 b c^3 + a^3 c^2 d) f^4 g^5 + (b^3 d^3 f^6 g^3 + a^3 c^3 g^9 - 3(b^3 c d^2 + ab^2 d^3) f^5 g^4 + 3(b^3 c^2 d + 3a^2 b^2 c d^2 + a^2 b d^3) f^4 g^5 - (b^3 c^3 + 9a^2 b^2 c^2 d + 9a^2 b b c d^2 + a^3 d^3) f^3 g^6 + 3(ab^2 c^3 + 3a^2 b c^2 d + a^3 c d^2) f^2 g^7 - 3(a^2 b c^3 + a^3 c^2 d) f g^8) x^3 + 3(b^3 d^3 f^7 g^2 + a^3 c^3 f g^8 - 3(b^3 c d^2 + ab^2 d^3) f^6 g^3 + 3(b^3 c^2 d + 3a^2 b^2 c d^2 + a^2 b d^3) f^5 g^4 - (b^3 c^3 + 9a^2 b^2 c^2 d + 9a^2 b b c d^2 + a^3 d^3) f^4 g^5 + 3(ab^2 c^3 + 3a^2 b c^2 d + a^3 c d^2) f^3 g^6 - 3(a^2 b c^3 + a^3 c^2 d) f^2 g^7) x^2 + 3(b^3 d^3 f^8 g + a^3 c^3 f^2 g^7 - 3(b^3 c d^2 + ab^2 d^3) f^7 g^2 + 3(b^3 c^2 d + 3a^2 b^2 c d^2 + a^2 b d^3) f^6 g^3 - (b^3 c^3 + 9a^2 b^2 c^2 d + 9a^2 b b c d^2 + a^3 d^3) f^5 g^4 + 3(ab^2 c^3 + 3a^2 b c^2 d + a^3 c d^2) f^4 g^5 - 3(a^2 b c^3 + a^3 c^2 d) f^3 g^6) x) - 6 \log(bex/(dx+c) + ae/(dx+c)) / (g^5 x^4 + 4f g^4 x^3 + 6f^2 g^3 x^2 + 4f^3 g^2 x + f^4 g) * B - 1/4 A / (g^5 x^4 + 4f g^4 x^3 + 6f^2 g^3 x^2 + 4f^3 g^2 x + f^4 g)$

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^5,x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)\*\*5,x)

[Out] Timed out

**Giac [B]** time = 11.7247, size = 5516, normalized size = 14.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))/(g\*x+f)^5,x, algorithm="giac")

[Out] 
$$\frac{1}{4} \cdot (4Bb^4c^3d^3f^3 - 4B^2ab^3d^4f^3 - 6B^2b^4c^2d^2f^2g + 6B^2a^2b^2d^4f^2g + 4B^2b^4c^3d^2f^2g^2 - 4B^2a^3b^3d^4f^2g^2 - B^2b^4c^4g^3 + B^2a^4d^4g^3) \cdot \log(gx + f) / (b^4d^4f^8 - 4b^4c^3d^3f^7g - 4a^3b^3d^4f^7g + 6b^4c^2d^2f^6g^2 + 16a^2b^2d^4f^6g^2 + 6a^2b^2d^4f^6g^2 - 4b^4c^3d^2f^5g^3 - 24a^2b^2c^2d^2f^5g^3 - 24a^2b^2c^2d^3f^5g^3 - 4a^3b^3d^4f^5g^3 + b^4c^4f^4g^4 + 16a^2b^3c^3d^2f^4g^4 + 36a^2b^2c^2d^2f^4g^4 + 16a^3b^3c^3d^2f^4g^4 + a^4d^4f^4g^4 - 4a^2b^3c^4f^3g^5 - 24a^2b^2c^3d^2f^3g^5 - 24a^3b^3c^2d^2f^3g^5 - 4a^4c^3d^3f^3g^5 + 6a^2b^2c^4f^2g^6 + 16a^3b^3c^3d^2f^2g^6 + 6a^4c^2d^2f^2g^6 - 4a^3b^3c^4f^2g^7 - 4a^4c^3d^2f^2g^7 + a^4c^4g^8) - \frac{1}{4} B \log((b^4d^4f^8 - 4b^4c^3d^3f^7g - 4a^3b^3d^4f^7g + 6b^4c^2d^2f^6g^2 + 16a^2b^2d^4f^6g^2 - 4b^4c^3d^2f^5g^3 - 24a^2b^2c^2d^2f^5g^3 - 24a^2b^2c^2d^3f^5g^3 - 4a^3b^3d^4f^5g^3 + b^4c^4f^4g^4 + 16a^2b^3c^3d^2f^4g^4 + 36a^2b^2c^2d^2f^4g^4 + 16a^3b^3c^3d^2f^4g^4 + a^4d^4f^4g^4 - 4a^2b^3c^4f^3g^5 - 24a^2b^2c^3d^2f^3g^5 - 24a^3b^3c^2d^2f^3g^5 - 4a^4c^3d^3f^3g^5 + 6a^2b^2c^4f^2g^6 + 16a^3b^3c^3d^2f^2g^6 + 6a^4c^2d^2f^2g^6 - 4a^3b^3c^4f^2g^7 - 4a^4c^3d^2f^2g^7 + a^4c^4g^8) - \frac{1}{8} (4B^2b^4c^3d^3f^3 - 4B^2a^2b^3d^4f^3 - 6B^2b^4c^2d^2f^2g + 6B^2a^2b^2d^4f^2g + 4B^2b^4c^3d^2f^2g^2 - 4B^2a^3b^3d^4f^2g^2 - B^2b^4c^4g^3 + B^2a^4d^4g^3) \cdot \log(\text{abs}(b^2d^2x^2 + b^2cx + a^2dx + a^2c)) / (b^4d^4f^8 - 4b^4c^3d^3f^7g - 4a^3b^3d^4f^7g + 6b^4c^2d^2f^6g^2 + 16a^2b^2d^4f^6g^2 - 4b^4c^3d^2f^5g^3 - 24a^2b^2c^2d^2f^5g^3 - 24a^2b^2c^2d^3f^5g^3 - 4a^3b^3d^4f^5g^3 + b^4c^4f^4g^4 + 16a^2b^3c^3d^2f^4g^4 + 36a^2b^2c^2d^2f^4g^4 + 16a^3b^3c^3d^2f^4g^4 + a^4d^4f^4g^4 - 4a^2b^3c^4f^3g^5 - 24a^2b^2c^3d^2f^3g^5 - 24a^3b^3c^2d^2f^3g^5 - 4a^4c^3d^3f^3g^5 + 6a^2b^2c^4f^2g^6 + 16a^3b^3c^3d^2f^2g^6 + 6a^4c^2d^2f^2g^6 - 4a^3b^3c^4f^2g^7 - 4a^4c^3d^2f^2g^7 + a^4c^4g^8) - \frac{1}{24} (18B^2b^3c^3d^2f^2g^4x^3 - 18B^2a^2b^2d^3f^2g^4x^3 - 18B^2b^3c^2d^2f^2g^5x^3 + 18B^2a^2b^2d^3f^2g^5x^3 + 6B^2b^3c^3g^6x^3 - 6B^2a^3d^3g^6x^3 + 60B^2b^3c^3d^2f^3g^3x^2 - 60B^2a^2b^2d^3f^3g^3x^2 - 63B^2b^3c^2d^2f^2g^4x^2 + 63B^2a^2b^2d^3f^2g^4x^2 + 21B^2b^3c^3f^2g^5x^2 + 9B^2a^2b^2c^2d^2f^2g^5x^2 - 9B^2a^2b^2c^2d^2f^2g^5x^2 - 21B^2a^3d^3f^2g^5x^2 - 3B^2a^2b^2c^3g^6x^2 + 3B^2a^3c^3d^2g^6x^2 + 68B^2b^3c^3d^2f^4g^2x - 68B^2a^2b^2d^3f^4g^2x - 76B^2b^3c^2d^2f^3g^3x + 76B^2a^2b^2d^3f^3g^3x + 26B^2b^3c^3f^2g^4x + 24B^2a^2b^2c^2d^2f^2g^4x - 24B^2a^2b^2c^2d^2f^2g^4x - 26B^2a^3d^3f^2g^4x - 10B^2a^2b^2c^3f^2g^5x + 10B^2a^3c^3d^2f^2g^5x + 2B^2a^2b^2c^3g^6x - 2B^2a^3c^2d^2g^6x + 6A^2b^3d^3f^6 + 6B^2b^3d^3f^6 - 18A^2b^3c^3d^2f^5g + 8B^2b^3c^3d^2f^5g - 18A^2a^2b^2d^3f^5g - 44B^2a^2b^2d^3f^5g + 18A^2b^3c^2d^2f^4g^2 - 13B^2b^3c^2d^2f^4g^2 + 54A^2a^2b^2c^2d^2f^4g^2 + 54B^2a^2b^2c^2d^2f^4g^2 + 18A^2a^2b^2d^3f^4g^2 + 49B^2a^2b^2d^3f^4g^2 - 6A^2b^3c^3f^3g^3 + 5B^2b^3c^3f^3g^3 - 54A^2a^2b^2c^2d^2f^3g^3 - 39B^2a^2b^2c^2d^2f^3g^3 - 54A^2a^2b^2c^2d^2f^3g^3 - 69B^2a^2b^2c^2d^2f^3g^3 - 6A^2a^3d^3f^3g^3 - 17B^2a^3d^3f^3g^3 + 18A^2a^2b^2c^3f^2g^4 + 11B^2a^2b^2c^3f^2g^4 + 54A^2a^2b^2c^2d^2f^2g^4 + 54B^2a^2b^2c^2d^2f^2g^4 + 18A^2a^3c^3d^2f^2g^4 + 25B^2a^3c^3d^2f^2g^4 - 18A^2a^2b^2c^3f^2g^5 - 16B^2a^2b^2c^3f^2g^5 - 18A^2a^3c^2d^2f^2g^5 - 20B^2a^3c^2d^2f^2g^5 + 6A^2a^3c^3g^6 + 6B^2a^3c^3g^6) / (b^3d^3f^6g^5x^4 - 3b^3c^3d^2f^5g^6x^4 - 3a^2b^2d^3f^5g^6x^4 + 3b^3c^2d^2f^4g^7x^4 + 9a^2b^2c^2d^2f^4g^7x^4 + 3a^2b^2d^3f^4g^7x^4 - b^3c^3f^3g^8x^4 - 9a^2b^2c^2d^2f^3g^8x^4 - 9a^2b^2c^2d^2f^3g^8x^4 - a^3d^3f^3g^8x^4 + 3a^2b^2c^3f^2g^9x^4 + 9a^2b^2c^2d^2f^2g^9x^4 + 3a^3c^3d^2f^2g^9x^4 - 3a^2b^2c^3f^2g^10x^4 - 3a^3c^3d^2f^2g^10x^4 + a^3c^3g^11x^4 + 4$$



$$\begin{aligned}
& b^3 d^3 f^7 g^4 x^3 - 12 b^3 c d^2 f^6 g^5 x^3 - 12 a b^2 d^3 f^6 g^5 x^3 + \\
& 12 b^3 c^2 d f^5 g^6 x^3 + 36 a b^2 c d^2 f^5 g^6 x^3 + 12 a^2 b d^3 f^5 g^6 x^3 - 4 b^3 c^3 f^4 g^7 x^3 - 36 a b^2 c^2 d f^4 g^7 x^3 - 36 a^2 b c d^2 f^4 g^7 x^3 - 4 a^3 d^3 f^4 g^7 x^3 + 12 a b^2 c^3 f^3 g^8 x^3 + 36 a^2 b c^2 d f^3 g^8 x^3 + 12 a^3 c d^2 f^3 g^8 x^3 - 12 a^2 b c^3 f^2 g^9 x^3 - 12 a^3 c^2 d f^2 g^9 x^3 + 4 a^3 c^3 f g^{10} x^3 + 6 b^3 d^3 f^8 g^3 x^2 - 18 b^3 c d^2 f^7 g^4 x^2 - 18 a b^2 d^3 f^7 g^4 x^2 + 18 b^3 c^2 d f^6 g^5 x^2 + 54 a b^2 c d^2 f^6 g^5 x^2 + 18 a^2 b d^3 f^6 g^5 x^2 - 6 b^3 c^3 f^5 g^6 x^2 - 54 a b^2 c^2 d f^5 g^6 x^2 - 54 a^2 b c d^2 f^5 g^6 x^2 - 6 a^3 d^3 f^5 g^6 x^2 + 18 a b^2 c^3 f^4 g^7 x^2 + 54 a^2 b c^2 d f^4 g^7 x^2 + 18 a^3 c d^2 f^4 g^7 x^2 - 18 a^2 b c^3 f^3 g^8 x^2 - 18 a^3 c^2 d f^3 g^8 x^2 + 6 a^3 c^3 f^2 g^9 x^2 + 4 b^3 d^3 f^9 g^2 x - 12 b^3 c d^2 f^8 g^3 x - 12 a b^2 d^3 f^8 g^3 x + 12 b^3 c^2 d f^7 g^4 x + 36 a b^2 c d^2 f^7 g^4 x + 12 a^2 b d^3 f^7 g^4 x - 4 b^3 c^3 f^6 g^5 x - 36 a b^2 c^2 d f^6 g^5 x - 36 a^2 b c d^2 f^6 g^5 x - 4 a^3 d^3 f^6 g^5 x + 12 a b^2 c^3 f^5 g^6 x + 36 a^2 b c^2 d f^5 g^6 x + 12 a^3 c d^2 f^5 g^6 x - 12 a^2 b c^3 f^4 g^7 x - 12 a^3 c^2 d f^4 g^7 x + 4 a^3 c^3 f^3 g^8 x + b^3 d^3 f^{10} g - 3 b^3 c d^2 f^9 g^2 - 3 a b^2 d^3 f^9 g^2 + 3 b^3 c^2 d f^8 g^3 + 9 a b^2 c d^2 f^8 g^3 + 3 a^2 b d^3 f^8 g^3 - b^3 c^3 f^7 g^4 - 9 a b^2 c^2 d f^7 g^4 - 9 a^2 b c d^2 f^7 g^4 - a^3 d^3 f^7 g^4 + 3 a b^2 c^3 f^6 g^5 + 9 a^2 b c^2 d f^6 g^5 + 3 a^3 c d^2 f^6 g^5 - 3 a^2 b c^3 f^5 g^6 - 3 a^3 c^2 d f^5 g^6 + a^3 c^3 f^4 g^7 + 1/8 * (2 B b^5 c d^4 f^4 - 2 B a b^4 d^5 f^4 - 4 B b^5 c^2 d^3 f^3 g + 4 B a^2 b^3 d^5 f^3 g + 6 B b^5 c^3 d^2 f^2 g^2 - 6 B a a b^4 c^2 d^3 f^2 g^2 + 6 B a^2 b^3 c d^4 f^2 g^2 - 6 B a^3 b^2 d^5 f^2 g^2 - 4 B b^5 c^4 d f g^3 + 4 B a a b^4 c^3 d^2 f g^3 - 4 B a^3 b^2 c d^4 f g^3 + 4 B a^4 b d^5 f g^3 + B b^5 c^5 g^4 - B a a b^4 c^4 d g^4 + B a^4 b c d^4 g^4 - B a^5 d^5 g^4) * log(abs((2 b d x + b c + a d - abs(-b c + a d)) / (2 b d x + b c + a d + abs(-b c + a d)))) / ((b^4 d^4 f^8 g - 4 b^4 c d^3 f^7 g^2 - 4 a b^3 d^4 f^7 g^2 + 6 b^4 c^2 d^2 f^6 g^3 + 16 a b^3 c d^3 f^6 g^3 + 6 a^2 b^2 d^4 f^6 g^3 - 4 b^4 c^3 d f^5 g^4 - 24 a b^3 c^2 d^2 f^5 g^4 - 24 a^2 b^2 c d^3 f^5 g^4 - 4 a^3 b d^4 f^5 g^4 + b^4 c^4 f^4 g^5 + 16 a b^3 c^3 d f^4 g^5 + 36 a^2 b^2 c^2 d^2 f^4 g^5 + 16 a^3 b c d^3 f^4 g^5 + a^4 d^4 f^4 g^5 - 4 a b^3 c^4 f^3 g^6 - 24 a^2 b^2 c^3 d f^3 g^6 - 24 a^3 b c^2 d^2 f^3 g^6 - 4 a^4 c d^3 f^3 g^6 + 6 a^2 b^2 c^4 f^2 g^7 + 16 a^3 b c^3 d f^2 g^7 + 6 a^4 c^2 d^2 f^2 g^7 - 4 a^3 b c^4 f g^8 - 4 a^4 c^3 d f g^8 + a^4 c^4 g^9) * abs(-b c + a d))
\end{aligned}$$

**3.240**  $\int (f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx$

**Optimal.** Leaf size=874

$$\frac{B^2 g^3 \log \left( \frac{a+bx}{c+dx} \right) (bc - ad)^4}{6b^4 d^4} + \frac{B^2 g^3 \log(c + dx)(bc - ad)^4}{6b^4 d^4} + \frac{B^2 g^3 x(bc - ad)^3}{6b^3 d^3} + \frac{B^2 g^2(4bdf - 3bcg - adg) \log \left( \frac{a+bx}{c+dx} \right) (bc - ad)^4}{4b^4 d^4}$$

[Out]  $(B^2*(b*c - a*d)^3*g^3*x)/(6*b^3*d^3) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*x)/(4*b^3*d^3) + (B^2*(b*c - a*d)^2*g^3*(c + d*x)^2)/(12*b^2*d^4) + (B^2*(b*c - a*d)^4*g^3*Log[(a + b*x)/(c + d*x)])/(6*b^4*d^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*Log[(a + b*x)/(c + d*x)])/(4*b^4*d^4) - (B*(b*c - a*d)*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b^4*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*b^2*d^4) - (B*(b*c - a*d)*g^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*b*d^4) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b^4*d^4) - ((b*f - a*g)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]^2)/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]^2)/(4*g) + (B^2*(b*c - a*d)^4*g^3*Log[c + d*x])/(6*b^4*d^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*Log[c + d*x])/(4*b^4*d^4) + (B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*Log[c + d*x])/(2*b^4*d^4) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(2*b^4*d^4)$

**Rubi [A]** time = 1.74393, antiderivative size = 994, normalized size of antiderivative = 1.14, number of steps used = 33, number of rules used = 13, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$ , Rules used = {2525, 12, 2528, 2486, 31, 72, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2 \log^2(a + bx)(bf - ag)^4}{4b^4 g} - \frac{B \log(a + bx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right) (bf - ag)^4}{2b^4 g} - \frac{B^2 \log(a + bx) \log \left( \frac{b(c+dx)}{bc-ad} \right) (bf - ag)^4}{2b^4 g} - B^2$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2,x]

[Out]  $-(B^2*(b*c - a*d)^2*(b*c + a*d)*g^3*x)/(6*b^3*d^3) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - b*c*g - a*d*g)*x)/(4*b^3*d^3) - (A*B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*x)/(2*b^3*d^3) + (B^2*(b*c - a*d)^2*g^3*x^2)/(12*b^2*d^2) - (a^3*B^2*(b*c - a*d)*g^3*Log[a + b*x])/(6*b^4*d) + (a^2*B^2*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*Log[a + b*x])/(4*b^4*d^2) + (B^2*(b*f - a*g)^4*Log[a + b*x]^2)/(4*b^4*g) - (B^2*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)]/(2*b^4*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*b*d) - (B*(b*f - a*g)^4*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]^2)/(4*g) + (B^2*c^3*(b*c - a*d)*g^3*Log[c + d*x])/(6*b*d^4) - (B^2*c^2*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*Log[c + d*x])/(4*b^2*d^4) + (B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4$

$$\begin{aligned} & *c*d*f*g + c^2*g^2)*\text{Log}[c + d*x]]/(2*b^4*d^4) - (B^2*(d*f - c*g)^4*\text{Log}[-(( \\ & d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x]]/(2*d^4*g) + (B*(d*f - c*g)^4*(A + \\ & B*\text{Log}[(e*(a + b*x))/(c + d*x)])*\text{Log}[c + d*x]]/(2*d^4*g) + (B^2*(d*f - c*g)^ \\ & 4*\text{Log}[c + d*x]^2)/(4*d^4*g) - (B^2*(b*f - a*g)^4*\text{Log}[a + b*x]*\text{Log}[(b*(c + d \\ & *x))/(b*c - a*d))]/(2*b^4*g) - (B^2*(b*f - a*g)^4*\text{PolyLog}[2, -((d*(a + b*x) \\ & )/(b*c - a*d))]/(2*b^4*g) - (B^2*(d*f - c*g)^4*\text{PolyLog}[2, (b*(c + d*x))/(b \\ & *c - a*d))]/(2*d^4*g) \end{aligned}$$
Rule 2525

$$\begin{aligned} & \text{Int}[(a_.) + \text{Log}[(c_.)*(Rfx_)^(p_.)]*(b_.)]^(n_.)*((d_.) + (e_.)*(x_.))^(m_. \\ & ), x\_Symbol] \text{:>} \text{Simp}[(d + e*x)^(m + 1)*(a + b*\text{Log}[c*Rfx^p])^n]/(e*(m + 1)) \\ & , x] - \text{Dist}[(b*n*p)/(e*(m + 1)), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^(m + 1)*( \\ & a + b*\text{Log}[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; \text{FreeQ}\{a, b, c, d \\ & , e, m, p\}, x] \&\& \text{RationalFunctionQ}[Rfx, x] \&\& \text{IGtQ}[n, 0] \&\& (\text{EqQ}[n, 1] || \\ & \text{IntegerQ}[m]) \&\& \text{NeQ}[m, -1] \end{aligned}$$
Rule 12

$$\text{Int}[(a_)*(u_), x\_Symbol] \text{:>} \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \&\& !\text{Match} \\ \text{Q}[u, (b_)*(v_)] /; \text{FreeQ}[b, x]$$
Rule 2528

$$\begin{aligned} & \text{Int}[(a_.) + \text{Log}[(c_.)*(Rfx_)^(p_.)]*(b_.)]^(n_.)*(RGx_), x\_Symbol] \text{:>} \text{With} \\ & [\{u = \text{ExpandIntegrand}[(a + b*\text{Log}[c*Rfx^p])^n, RGx, x]\}, \text{Int}[u, x] /; \text{SumQ}[u \\ & ]] /; \text{FreeQ}\{a, b, c, p\}, x] \&\& \text{RationalFunctionQ}[Rfx, x] \&\& \text{RationalFunci} \\ & \text{onQ}[RGx, x] \&\& \text{IGtQ}[n, 0] \end{aligned}$$
Rule 2486

$$\begin{aligned} & \text{Int}[\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.))*((c_.) + (d_.)*(x_.))^(q_.)) \\ & ^{(r_.)]^(s_.), x\_Symbol] \text{:>} \text{Simp}[(a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x) \\ & ^q)^r]^s/b, x] + \text{Dist}[(q*r*s*(b*c - a*d))/b, \text{Int}[\text{Log}[e*(f*(a + b*x)^p*(c + \\ & d*x)^q)^r]^(s - 1)/(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, p, q, r, s \\ & \}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{IGtQ}[s, 0] \end{aligned}$$
Rule 31

$$\text{Int}[(a_ + (b_.)*(x_.))^{-1}, x\_Symbol] \text{:>} \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, \\ x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$$
Rule 72

$$\begin{aligned} & \text{Int}[(e_.) + (f_.)*(x_.)]^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), \\ & x\_Symbol] \text{:>} \text{Int}[\text{ExpandIntegrand}[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] \\ & /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IntegerQ}[p] \end{aligned}$$
Rule 2524

$$\begin{aligned} & \text{Int}[(a_.) + \text{Log}[(c_.)*(Rfx_)^(p_.)]*(b_.)]^(n_.)/((d_.) + (e_.)*(x_.)), x\_S \\ & ymbol] \text{:>} \text{Simp}[(\text{Log}[d + e*x]*(a + b*\text{Log}[c*Rfx^p])^n)/e, x] - \text{Dist}[(b*n*p)/e \\ & , \text{Int}[(\text{Log}[d + e*x]*(a + b*\text{Log}[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; \\ & \text{FreeQ}\{a, b, c, d, e, p\}, x] \&\& \text{RationalFunctionQ}[Rfx, x] \&\& \text{IGtQ}[n, 0] \end{aligned}$$
Rule 2418

$$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.)]^(p_.)*(Rfx_), x\_Sy \\ \text{mbol}] \text{:>} \text{With}[\{u = \text{ExpandIntegrand}[(a + b*\text{Log}[c*(d + e*x)^n])^p, Rfx, x]\},$$

Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[  
RFX, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.  
)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^  
n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E  
qQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Lo  
g[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_  
)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x  
)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x)  
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_  
Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x  
, x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*  
(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2  
, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
\int (f + gx)^3 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2 dx &= \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{4g} - \frac{B \int \frac{(bc - ad)(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{(a + bx)(c + dx)} dx}{2g} \\
&= \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{4g} - \frac{(B(bc - ad)) \int \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{(a + bx)(c + dx)} dx}{2g} \\
&= \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{4g} - \frac{(B(bc - ad)) \int \left( \frac{g^2(a^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))}{(a + bx)(c + dx)} \right) dx}{2g} \\
&= \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{4g} - \frac{(B(bc - ad)g^3) \int x^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{2bd} \\
&= -\frac{AB(bc - ad)g(a^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{2b^3 d^3} \\
&= -\frac{AB(bc - ad)g(a^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{2b^3 d^3} \\
&= -\frac{AB(bc - ad)g(a^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{2b^3 d^3} \\
&= -\frac{B^2(bc - ad)^2(bc + ad)g^3 x}{6b^3 d^3} + \frac{B^2(bc - ad)^2 g^2(4bdf - bcg - adg)x}{4b^3 d^3} - \frac{AB(b^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{4b^3 d^3} \\
&= -\frac{B^2(bc - ad)^2(bc + ad)g^3 x}{6b^3 d^3} + \frac{B^2(bc - ad)^2 g^2(4bdf - bcg - adg)x}{4b^3 d^3} - \frac{AB(b^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{4b^3 d^3} \\
&= -\frac{B^2(bc - ad)^2(bc + ad)g^3 x}{6b^3 d^3} + \frac{B^2(bc - ad)^2 g^2(4bdf - bcg - adg)x}{4b^3 d^3} - \frac{AB(b^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{4b^3 d^3} \\
&= -\frac{B^2(bc - ad)^2(bc + ad)g^3 x}{6b^3 d^3} + \frac{B^2(bc - ad)^2 g^2(4bdf - bcg - adg)x}{4b^3 d^3} - \frac{AB(b^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{4b^3 d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.972346, size = 733, normalized size = 0.84

$$\frac{(f + gx)^4 \left( B \log \left( \frac{e(a + bx)}{c + dx} \right) + A \right)^2 - \frac{B(3b^4 B(df - cg)^4 (2 \text{PolyLog} \left( 2, \frac{b(c + dx)}{bc - ad} \right) + \log(c + dx) (2 \log \left( \frac{d(a + bx)}{ad - bc} \right) - \log(c + dx))) - 3Bd^4 (bf - ag)^4 (\log(a + bx) (\log(a + bx) + \log(c + dx)) + \log(bx + c) (\log(bx + c) + \log(c + dx)))}{(a + bx)(c + dx)}}{(a + bx)(c + dx)}}{(a + bx)(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])^2, x]

[Out] ((f + g\*x)^4\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])^2 - (B\*(6\*A\*b\*d\*(b\*c - a\*d)\*g^2\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(-4\*d\*f + c\*g) + b^2\*(6\*d^2\*f^2 - 4\*c\*d\*f\*g + c^2\*g^2))\*x + 6\*B\*d\*(b\*c - a\*d)\*g^2\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(-4\*d\*f + c\*g) + b^2\*(6\*d^2\*f^2 - 4\*c\*d\*f\*g + c^2\*g^2))\*(a + b\*x)\*Log[(e\*(a + b\*x))/(c + d\*x)] + 3\*b^2\*d^2\*(b\*c - a\*d)\*g^3\*(4\*b\*d\*f - b\*c\*g - a\*d\*g)\*x^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]]) + 2\*b^3\*d^3\*(b\*c - a\*d)\*g^4\*x^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]]) + 6\*d^4\*(b\*f - a\*g)^4\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]]) - 6\*B\*(b\*c - a\*d)^2\*g^2\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(-4\*d\*f + c\*g) + b^2\*(6\*d^2\*f^2 - 4\*c\*d\*f\*g + c^2\*g^2))\*x)/(a + b\*x)(c + d\*x)

g) + b^2\*(6\*d^2\*f^2 - 4\*c\*d\*f\*g + c^2\*g^2))\*Log[c + d\*x] - 6\*b^4\*(d\*f - c\*g)^4\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])\*Log[c + d\*x] + B\*(b\*c - a\*d)\*g^4\*(b\*d\*(b\*c - a\*d)\*x\*(2\*b\*c + 2\*a\*d - b\*d\*x) + 2\*a^3\*d^3\*Log[a + b\*x] - 2\*b^3\*c^3\*Log[c + d\*x]) - 3\*B\*(b\*c - a\*d)\*g^3\*(-4\*b\*d\*f + b\*c\*g + a\*d\*g)\*(-(a^2\*d^2\*Log[a + b\*x]) + b\*(d\*(-(b\*c) + a\*d)\*x + b\*c^2\*Log[c + d\*x])) - 3\*B\*d^4\*(b\*f - a\*g)^4\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-(b\*c) + a\*d)]) + 3\*b^4\*B\*(d\*f - c\*g)^4\*((2\*Log[(d\*(a + b\*x))/(-(b\*c) + a\*d)] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/(3\*b^4\*d^4)/(4\*g)

**Maple [F]** time = 2.677, size = 0, normalized size = 0.

$$\int (gx + f)^3 \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^3\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

[Out] int((g\*x+f)^3\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

**Maxima [B]** time = 1.80513, size = 2889, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="maxima")

[Out] 1/4\*A^2\*g^3\*x^4 + A^2\*f\*g^2\*x^3 + 3/2\*A^2\*f^2\*g\*x^2 + 2\*(x\*log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c)) + a\*log(b\*x + a)/b - c\*log(d\*x + c)/d)\*A\*B\*f^3 + 3\*(x^2\*log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c)) - a^2\*log(b\*x + a)/b^2 + c^2\*log(d\*x + c)/d^2 - (b\*c - a\*d)\*x/(b\*d))\*A\*B\*f^2\*g + (2\*x^3\*log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c)) + 2\*a^3\*log(b\*x + a)/b^3 - 2\*c^3\*log(d\*x + c)/d^3 - ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2))\*A\*B\*f\*g^2 + 1/12\*(6\*x^4\*log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c)) - 6\*a^4\*log(b\*x + a)/b^4 + 6\*c^4\*log(d\*x + c)/d^4 - (2\*(b^3\*c\*d^2 - a\*b^2\*d^3)\*x^3 - 3\*(b^3\*c^2\*d - a^2\*b\*d^3)\*x^2 + 6\*(b^3\*c^3 - a^3\*d^3)\*x)/(b^3\*d^3))\*A\*B\*g^3 + A^2\*f^3\*x - 1/12\*(6\*a^3\*c\*d^3\*g^3 - 3\*(8\*c\*d^3\*f\*g^2 - c^2\*d^2\*g^3)\*a^2\*b + 2\*(18\*c\*d^3\*f^2\*g - 6\*c^2\*d^2\*f\*g^2 + c^3\*d\*g^3)\*a\*b^2 + (24\*c\*d^3\*f^3\*log(e) - (6\*g^3\*log(e) + 11\*g^3)\*c^4 + 12\*(2\*f\*g^2\*log(e) + 3\*f\*g^2)\*c^3\*d - 36\*(f^2\*g\*log(e) + f^2\*g)\*c^2\*d^2)\*b^3)\*B^2\*log(d\*x + c)/(b^3\*d^4) + 1/2\*(4\*a\*b^3\*d^4\*f^3 - 6\*a^2\*b^2\*d^4\*f^2\*g + 4\*a^3\*b\*d^4\*f\*g^2 - a^4\*d^4\*g^3 - (4\*c\*d^3\*f^3 - 6\*c^2\*d^2\*f^2\*g + 4\*c^3\*d\*f\*g^2 - c^4\*g^3)\*b^4)\*(log(b\*x + a)\*log((b\*d\*x + a\*d)/(b\*c - a\*d) + 1) + dilog(-(b\*d\*x + a\*d)/(b\*c - a\*d)))\*B^2/(b^4\*d^4) + 1/12\*(3\*B^2\*b^4\*d^4\*g^3\*x^4\*log(e)^2 + 2\*(a\*b^3\*d^4\*g^3\*log(e) + (6\*d^4\*f\*g^2\*log(e)^2 - c\*d^3\*g^3\*log(e))\*b^4)\*B^2\*x^3 - ((3\*g^3\*log(e) - g^3)\*a^2\*b^2\*d^4 - 2\*(6\*d^4\*f\*g^2\*log(e) - c\*d^3\*g^3)\*a\*b^3 - (18\*d^4\*f^2\*g\*log(e)^2 - 12\*c\*d^3\*f\*g^2\*log(e) + (3\*g^3\*log(e) + g^3)\*c^2\*d^2)\*b^4)\*B^2\*x^2 + ((6\*g^3\*log(e) - 5\*g^3)\*a^3\*b\*d^4 + (5\*c\*d^3\*g^3 - 12\*(2\*f\*g^2\*log(e) - f\*g^2)\*d^4)\*a^2\*b^2 + (36\*d^4\*f^2\*g\*log(e) - 24\*c\*d^3\*f\*g^2 + 5\*c^2\*d^2\*g^3)\*a\*b^3 + (12\*d^4\*f^3\*log(e)^2 - 36\*c\*d^3\*f^2\*g\*log(e) - (6\*g^3\*log(e) + 5\*g^3)\*c^3\*d + 12\*(2\*f\*g^2\*log(e) + f\*g^2)\*c^2\*d^2)\*b^4)\*B^2\*x + 3\*(B^2\*b^4\*d^4\*g^3\*x^4 + 4\*B^2\*b^4\*d^4\*f\*g^2\*x^3 + 6\*B^2\*b^4\*d^4\*f^2\*g\*x^2 + 4\*B^2\*b^4\*d^4\*f^3\*x + (4\*a\*b

$$\begin{aligned} &^3d^4f^3 - 6a^2b^2d^4f^2g + 4a^3bd^4f^2g^2 - a^4d^4g^3)B^2) \log(bx + a)^2 + 3(B^2b^4d^4g^3x^4 + 4B^2b^4d^4f^2g^2x^3 + 6B^2b^4d^4f^2g^2x^2 + 4B^2b^4d^4f^3x + (4cd^3f^3 - 6c^2d^2f^2g + 4c^3d^2f^2g^2 - c^4g^3)B^2b^4) \log(dx + c)^2 + (6B^2b^4d^4g^3x^4 \log(e) + 2(a^3bd^4g^3 + (12d^4f^2g^2 \log(e) - cd^3g^3)b^4)B^2x^3 + 3(4a^3bd^4f^2g^2 - a^2b^2d^4g^3 + (12d^4f^2g^2 \log(e) - 4cd^3f^2g^2 + c^2d^2g^3)b^4)B^2x^2 + 6(6a^3bd^4f^2g - 4a^2b^2d^4f^2g^2 + a^3bd^4g^3 + (4d^4f^3 \log(e) - 6cd^3f^2g + 4c^2d^2f^2g^2 - c^3d^2g^3)b^4)B^2x - ((6g^3 \log(e) - 11g^3)a^4d^4 + 2(cd^3g^3 - 6(2f^2g^2 \log(e) - 3f^2g^2)d^4)a^3b - 3(4cd^3f^2g^2 - c^2d^2g^3 - 12(f^2g \log(e) - f^2g)d^4)a^2b^2 - 6(4d^4f^3 \log(e) - 6cd^3f^2g + 4c^2d^2f^2g^2 - c^3d^2g^3)a^3b)B^2) \log(bx + a) - (6B^2b^4d^4g^3x^4 \log(e) + 2(a^3bd^4g^3 + (12d^4f^2g^2 \log(e) - cd^3g^3)b^4)B^2x^3 + 3(4a^3bd^4f^2g^2 - a^2b^2d^4g^3 + (12d^4f^2g^2 \log(e) - 4cd^3f^2g^2 + c^2d^2g^3)b^4)B^2x^2 + 6(6a^3bd^4f^2g - 4a^2b^2d^4f^2g^2 + a^3bd^4g^3 + (4d^4f^3 \log(e) - 6cd^3f^2g + 4c^2d^2f^2g^2 - c^3d^2g^3)b^4)B^2x + 6(B^2b^4d^4g^3x^4 + 4B^2b^4d^4f^2g^2x^3 + 6B^2b^4d^4f^2g^2x^2 + 4B^2b^4d^4f^3x + (4a^3bd^4f^3 - 6a^2b^2d^4f^2g + 4a^3bd^4f^2g^2 - a^4d^4g^3)B^2) \log(bx + a)) \log(dx + c)) / (b^4d^4) \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(A^2g^3x^3 + 3A^2fg^2x^2 + 3A^2f^2gx + A^2f^3 + (B^2g^3x^3 + 3B^2fg^2x^2 + 3B^2f^2gx + B^2f^3) \log\left(\frac{bex + ae}{dx + c}\right)^2 + 2(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out] integral(A^2\*g^3\*x^3 + 3\*A^2\*f\*g^2\*x^2 + 3\*A^2\*f^2\*g\*x + A^2\*f^3 + (B^2\*g^3\*x^3 + 3\*B^2\*f\*g^2\*x^2 + 3\*B^2\*f^2\*g\*x + B^2\*f^3)\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*(A\*B\*g^3\*x^3 + 3\*A\*B\*f\*g^2\*x^2 + 3\*A\*B\*f^2\*g\*x + A\*B\*f^3)\*log((b\*e\*x + a\*e)/(d\*x + c)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*3\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (gx + f)^3 \left( B \log\left(\frac{bx + a}{dx + c}\right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```



$$3.241 \quad \int (f + gx)^2 \left( A + B \log \left( \frac{e^{(a+bx)}}{c+dx} \right) \right)^2 dx$$

**Optimal.** Leaf size=532

$$\frac{2B^2(bc - ad)(a^2d^2g^2 - abdg(3df - cg) + b^2(c^2g^2 - 3cdfg + 3d^2f^2)) \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3} + \frac{2B(bc - ad)(a^2d^2g^2 - abdg(3df - cg) + b^2(c^2g^2 - 3cdfg + 3d^2f^2)) \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$

```
[Out] (B^2*(b*c - a*d)^2*g^2*x)/(3*b^2*d^2) + (B^2*(b*c - a*d)^3*g^2*Log[(a + b*x)/(c + d*x)])/(3*b^3*d^3) - (2*B*(b*c - a*d)*g*(3*b*d*f - 2*b*c*g - a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x]]))/(3*b^3*d^2) - (B*(b*c - a*d)*g^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x]]))/(3*b*d^3) + (2*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x]]))/(3*b^3*d^3) - ((b*f - a*g)^3*(A + B*Log[(e*(a + b*x))/(c + d*x]]^2)/(3*b^3*g) + ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x]]^2)/(3*g) + (B^2*(b*c - a*d)^3*g^2*Log[c + d*x])/(3*b^3*d^3) + (2*B^2*(b*c - a*d)^2*g*(3*b*d*f - 2*b*c*g - a*d*g)*Log[c + d*x])/(3*b^3*d^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b^3*d^3)
```

**Rubi [A]** time = 1.09353, antiderivative size = 649, normalized size of antiderivative = 1.22, number of steps used = 29, number of rules used = 13, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.448$ , Rules used = {2525, 12, 2528, 2486, 31, 72, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2B^2(bf - ag)^3 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{3b^3g} - \frac{2B^2(df - cg)^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3d^3g} + \frac{a^2B^2g^2(bc - ad) \log(a + bx)}{3b^3d} - \frac{2ABgx}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x]]^2, x]
```

```
[Out] (B^2*(b*c - a*d)^2*g^2*x)/(3*b^2*d^2) - (2*A*B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*x)/(3*b^2*d^2) + (a^2*B^2*(b*c - a*d)*g^2*Log[a + b*x])/(3*b^3*d) + (B^2*(b*f - a*g)^3*Log[a + b*x]^2)/(3*b^3*g) - (2*B^2*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x]])/(3*b^3*d^2) - (B*(b*c - a*d)*g^2*x^2*(A + B*Log[(e*(a + b*x))/(c + d*x]]))/(3*b*d) - (2*B*(b*f - a*g)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x]]))/(3*b^3*g) + ((f + g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x]]^2)/(3*g) - (B^2*c^2*(b*c - a*d)*g^2*Log[c + d*x])/(3*b*d^3) + (2*B^2*(b*c - a*d)^2*g*(3*b*d*f - b*c*g - a*d*g)*Log[c + d*x])/(3*b^3*d^3) - (2*B^2*(d*f - c*g)^3*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*d^3*g) + (2*B*(d*f - c*g)^3*(A + B*Log[(e*(a + b*x))/(c + d*x]])*Log[c + d*x])/(3*d^3*g) + (B^2*(d*f - c*g)^3*Log[c + d*x]^2)/(3*d^3*g) - (2*B^2*(b*f - a*g)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]/(3*b^3*g) - (2*B^2*(b*f - a*g)^3*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(3*b^3*g) - (2*B^2*(d*f - c*g)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]/(3*d^3*g)
```

**Rule 2525**

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d
```

, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] :> With[{u = ExpandIntegrand[(a + b\*Log[c\*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.), x\_Symbol] :> Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 72

Int[((e\_.) + (f\_.)\*(x\_))^(p\_.)/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x]]/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(RFx\_), x\_Symbol] :> With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int (f + gx)^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx &= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{3g} - \frac{(2B) \int \frac{(bc-ad)(f+gx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{(a+bx)(c+dx)} dx}{3g} \\
&= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{3g} - \frac{(2B(bc - ad)) \int \frac{(f+gx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{(a+bx)(c+dx)} dx}{3g} \\
&= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{3g} - \frac{(2B(bc - ad)) \int \left( \frac{g^2(3bdf - bcg - adg) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{b^2 d^2} \right) dx}{3g} \\
&= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2}{3g} - \frac{(2B(bc - ad)g^2) \int x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{3bd} \\
&= -\frac{2AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} - \frac{B(bc - ad)g^2 x^2 \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{3bd} \\
&= -\frac{2AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} - \frac{2B^2(bc - ad)g(3bdf - bcg - adg)(a + bx)}{3b^3 d^2} \\
&= -\frac{2AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} - \frac{2B^2(bc - ad)g(3bdf - bcg - adg)(a + bx)}{3b^3 d^2} \\
&= \frac{B^2(bc - ad)^2 g^2 x}{3b^2 d^2} - \frac{2AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} + \frac{a^2 B^2(bc - ad)g^2 \log \left( \frac{e(a+bx)}{c+dx} \right)}{3b^3 d} \\
&= \frac{B^2(bc - ad)^2 g^2 x}{3b^2 d^2} - \frac{2AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} + \frac{a^2 B^2(bc - ad)g^2 \log \left( \frac{e(a+bx)}{c+dx} \right)}{3b^3 d} \\
&= \frac{B^2(bc - ad)^2 g^2 x}{3b^2 d^2} - \frac{2AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} + \frac{a^2 B^2(bc - ad)g^2 \log \left( \frac{e(a+bx)}{c+dx} \right)}{3b^3 d} \\
&= \frac{B^2(bc - ad)^2 g^2 x}{3b^2 d^2} - \frac{2AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} + \frac{a^2 B^2(bc - ad)g^2 \log \left( \frac{e(a+bx)}{c+dx} \right)}{3b^3 d}
\end{aligned}$$

**Mathematica [A]** time = 0.530899, size = 486, normalized size = 0.91

$$(f + gx)^3 \left( B \log \left( \frac{e(a+bx)}{c+dx} \right) + A \right)^2 - \frac{B \left( b^3 B(df - cg)^3 \left( 2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \left( 2 \log \left( \frac{d(a+bx)}{ad-bc} \right) - \log(c+dx) \right) \right) - Bd^3 (bf - ag)^3 \left( \log(a+bx) \left( \log(a+bx) \right) \right)}{3b^2 d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2,x]

[Out] ((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2 - (B\*(2\*A\*b\*d\*(b\*c - a\*d)\*g^2\*(3\*b\*d\*f - b\*c\*g - a\*d\*g)\*x + 2\*B\*d\*(b\*c - a\*d)\*g^2\*(3\*b\*d\*f - b\*c\*g - a\*d\*g)\*(a + b\*x)\*Log[(e\*(a + b\*x))/(c + d\*x)] + b^2\*d^2\*(b\*c - a\*d)\*g^3\*x^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])) + 2\*d^3\*(b\*f - a\*g)^3\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])) + 2\*B\*(b\*c - a\*d)^2\*g^2\*(-3\*b\*d\*f + b\*c\*g + a\*d\*g)\*Log[c + d\*x] - 2\*b^3\*(d\*f - c\*g)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))\*Log[c + d\*x] - B\*(b\*c - a\*d)\*g^3\*(a^2\*d^2\*Log[a + b\*x] - b\*(d\*(-(b\*c) + a\*d)\*x + b\*c^2\*Log[c + d\*x])) - B\*d^3\*(b\*f - a\*g)^3\*(Log[a + b\*x]\*

$\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)] - 2*\text{PolyLog}[2, (d*(a + b*x))/(-b*c + a*d)] + b^3*B*(d*f - c*g)^3*((2*\text{Log}[(d*(a + b*x))/(-b*c + a*d)] - \text{Log}[c + d*x])* \text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(b^3*d^3))/(3*g)$

**Maple [F]** time = 2.257, size = 0, normalized size = 0.

$$\int (gx + f)^2 \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

[Out] int((g\*x+f)^2\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

**Maxima [B]** time = 1.69195, size = 1755, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2\*(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="maxima")

[Out]  $\frac{1}{3}A^2g^2x^3 + A^2f*gx^2 + 2*(x*\log(b*ex/(d*x + c) + a*e/(d*x + c)) + a*\log(b*x + a)/b - c*\log(d*x + c)/d)*A*B*f^2 + 2*(x^2*\log(b*ex/(d*x + c) + a*e/(d*x + c)) - a^2*\log(b*x + a)/b^2 + c^2*\log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*f*g + \frac{1}{3}*(2*x^3*\log(b*ex/(d*x + c) + a*e/(d*x + c)) + 2*a^3*\log(b*x + a)/b^3 - 2*c^3*\log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*g^2 + A^2*f^2*x + \frac{1}{3}*(2*a^2*c*d^2*g^2 - (6*c*d^2*f*g - c^2*d*g^2)*a*b - (6*c*d^2*f^2*\log(e) + (2*g^2*\log(e) + 3*g^2)*c^3 - 6*(f*g*\log(e) + f*g)*c^2*d)*b^2)*B^2*\log(d*x + c)/(b^2*d^3) + \frac{2}{3}*(3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2 - (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*b^3)*( \log(b*x + a)*\log((b*d*x + a*d)/(b*c - a*d) + 1) + \text{dilog}(-b*d*x + a*d)/(b*c - a*d))*B^2/(b^3*d^3) + \frac{1}{3}*(B^2*b^3*d^3*g^2*x^3*\log(e)^2 + (a*b^2*d^3*g^2*\log(e) + (3*d^3*f*g*\log(e)^2 - c*d^2*g^2*\log(e))*b^3)*B^2*x^2 - ((2*g^2*\log(e) - g^2)*a^2*b*d^3 - 2*(3*d^3*f*g*\log(e) - c*d^2*g^2)*a*b^2 - (3*d^3*f^2*\log(e)^2 - 6*c*d^2*f*g*\log(e) + (2*g^2*\log(e) + g^2)*c^2*d)*b^3)*B^2*x + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2)*B^2)*\log(b*x + a)^2 + (B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*B^2*b^3)*\log(d*x + c)^2 + (2*B^2*b^3*d^3*g^2*x^3*\log(e) + (a*b^2*d^3*g^2 + (6*d^3*f*g*\log(e) - c*d^2*g^2)*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*f*g - a^2*b*d^3*g^2 + (3*d^3*f^2*\log(e) - 3*c*d^2*f*g + c^2*d*g^2)*a*b^2)*B^2)*\log(b*x + a) - (2*B^2*b^3*d^3*g^2*x^3*\log(e) + (a*b^2*d^3*g^2 + (6*d^3*f*g*\log(e) - c*d^2*g^2)*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*f*g - a^2*b*d^3*g^2 + (3*d^3*f^2*\log(e) - 3*c*d^2*f*g + c^2*d*g^2)*b^3)*B^2*x + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2)*B^2)*\log(b*x + a))*\log(d*x + c)/(b^3*d^3)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2 g^2 x^2 + 2 A^2 f g x + A^2 f^2 + (B^2 g^2 x^2 + 2 B^2 f g x + B^2 f^2) \log \left( \frac{b e x + a e}{d x + c} \right)^2 + 2 (A B g^2 x^2 + 2 A B f g x + A B f^2) \log \left( \frac{b e x + a e}{d x + c} \right), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2\*(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out] integral(A^2\*g^2\*x^2 + 2\*A^2\*f\*g\*x + A^2\*f^2 + (B^2\*g^2\*x^2 + 2\*B^2\*f\*g\*x + B^2\*f^2)\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*(A\*B\*g^2\*x^2 + 2\*A\*B\*f\*g\*x + A\*B\*f^2)\*log((b\*e\*x + a\*e)/(d\*x + c)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (g x + f)^2 \left( B \log \left( \frac{b x + a e}{d x + c} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2\*(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="giac")

[Out] integrate((g\*x + f)^2\*(B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2, x)

$$3.242 \quad \int (f + gx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx$$

**Optimal.** Leaf size=270

$$\frac{B^2(bc - ad)(-adg - bcg + 2bdf)\text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2} + \frac{B(bc - ad)(-adg - bcg + 2bdf) \log\left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + \right)}{b^2d^2}$$

```
[Out] -((B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*d))
+ (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[(b*c - a*d)/(b*(c + d*x))]*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*d^2) - ((b*f - a*g)^2*(A + B*Log
[(e*(a + b*x))/(c + d*x)]^2)/(2*b^2*g) + ((f + g*x)^2*(A + B*Log[(e*(a + b
*x))/(c + d*x)]^2)/(2*g) + (B^2*(b*c - a*d)^2*g*Log[c + d*x])/(b^2*d^2) +
(B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2, (d*(a + b*x))/(b*(c +
d*x))])/(b^2*d^2)
```

**Rubi [A]** time = 0.820035, antiderivative size = 444, normalized size of antiderivative = 1.64, number of steps used = 25, number of rules used = 12, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$ , Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2(bf - ag)^2\text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{b^2g} - \frac{B^2(df - cg)^2\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{d^2g} - \frac{B(bf - ag)^2 \log(a + bx) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + \right)}{b^2g}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

```
[Out] -((A*B*(b*c - a*d)*g*x)/(b*d)) + (B^2*(b*f - a*g)^2*Log[a + b*x]^2)/(2*b^2*g)
- (B^2*(b*c - a*d)*g*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)]/(b^2*d) - (
B*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*g)
+ ((f + g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]^2)/(2*g) + (B^2*(b*c -
a*d)^2*g*Log[c + d*x])/(b^2*d^2) - (B^2*(d*f - c*g)^2*Log[-((d*(a + b*x))/(
b*c - a*d))]*Log[c + d*x])/(d^2*g) + (B*(d*f - c*g)^2*(A + B*Log[(e*(a + b
x))/(c + d*x)]*Log[c + d*x])/(d^2*g) + (B^2*(d*f - c*g)^2*Log[c + d*x]^2)/
(2*d^2*g) - (B^2*(b*f - a*g)^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])
/(b^2*g) - (B^2*(b*f - a*g)^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b^
2*g) - (B^2*(d*f - c*g)^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]/(d^2*g)
```

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[ {u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
] ] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

#### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391



Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
 \int (f + gx) \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2 dx &= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{2g} - \frac{B \int \frac{(bc - ad)(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{(a + bx)(c + dx)} dx}{g} \\
 &= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{2g} - \frac{(B(bc - ad)) \int \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{(a + bx)(c + dx)} dx}{g} \\
 &= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{2g} - \frac{(B(bc - ad)) \int \left( \frac{g^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{bd} + \frac{(bf - ag)^2 \log(a + bx)}{b^2d} \right) dx}{g} \\
 &= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{2g} - \frac{(B(bc - ad)g) \int \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right) dx}{bd} \\
 &= -\frac{AB(bc - ad)gx}{bd} - \frac{B(bf - ag)^2 \log(a + bx) \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)}{b^2g} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{2g} \\
 &= -\frac{AB(bc - ad)gx}{bd} - \frac{B^2(bc - ad)g(a + bx) \log \left( \frac{e(a + bx)}{c + dx} \right)}{b^2d} - \frac{B(bf - ag)^2 \log(a + bx)}{b^2d} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{2g} \\
 &= -\frac{AB(bc - ad)gx}{bd} - \frac{B^2(bc - ad)g(a + bx) \log \left( \frac{e(a + bx)}{c + dx} \right)}{b^2d} - \frac{B(bf - ag)^2 \log(a + bx)}{b^2d} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{2g} \\
 &= -\frac{AB(bc - ad)gx}{bd} - \frac{B^2(bc - ad)g(a + bx) \log \left( \frac{e(a + bx)}{c + dx} \right)}{b^2d} - \frac{B(bf - ag)^2 \log(a + bx)}{b^2d} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{2g} \\
 &= -\frac{AB(bc - ad)gx}{bd} - \frac{B^2(bc - ad)g(a + bx) \log \left( \frac{e(a + bx)}{c + dx} \right)}{b^2d} - \frac{B(bf - ag)^2 \log(a + bx)}{b^2d} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{2g} \\
 &= -\frac{AB(bc - ad)gx}{bd} + \frac{B^2(bf - ag)^2 \log^2(a + bx)}{2b^2g} - \frac{B^2(bc - ad)g(a + bx) \log \left( \frac{e(a + bx)}{c + dx} \right)}{b^2d} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{2g} \\
 &= -\frac{AB(bc - ad)gx}{bd} + \frac{B^2(bf - ag)^2 \log^2(a + bx)}{2b^2g} - \frac{B^2(bc - ad)g(a + bx) \log \left( \frac{e(a + bx)}{c + dx} \right)}{b^2d} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)}{c + dx} \right) \right)^2}{2g}
 \end{aligned}$$

**Mathematica [A]** time = 0.314106, size = 346, normalized size = 1.28

$$\frac{(f + gx)^2 \left( B \log \left( \frac{e(a + bx)}{c + dx} \right) + A \right)^2 - \frac{B \left( b^2 B (df - cg)^2 \left( 2 \text{PolyLog} \left( 2, \frac{b(c + dx)}{bc - ad} \right) + \log(c + dx) \left( 2 \log \left( \frac{d(a + bx)}{ad - bc} \right) - \log(c + dx) \right) \right) - Bd^2 (bf - ag)^2 \left( \log(a + bx) \left( \log(a + bx) \right) \right)}{2b^2g}}{2g}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2,x]

[Out] ((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2 - (B\*(2\*A\*b\*d\*(b\*c - a\*d)\*g^2\*x + 2\*B\*d\*(b\*c - a\*d)\*g^2\*(a + b\*x)\*Log[(e\*(a + b\*x))/(c + d\*x)] + 2\*d^2\*(b\*f - a\*g)^2\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]) - 2\*B\*

$$\frac{(b*c - a*d)^2*g^2*\text{Log}[c + d*x] - 2*b^2*(d*f - c*g)^2*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)])*\text{Log}[c + d*x] - B*d^2*(b*f - a*g)^2*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b^2*B*(d*f - c*g)^2*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \text{Log}[c + d*x])* \text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])}{(b^2*d^2)/(2*g)}$$

**Maple [F]** time = 1.851, size = 0, normalized size = 0.

$$\int (gx + f) \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

[Out] int((g\*x+f)\*(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

**Maxima [B]** time = 1.58545, size = 909, normalized size = 3.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="maxima")

[Out]  $\frac{1}{2}A^2gx^2 + 2(x*\log(b*ex/(d*x + c) + a*e/(d*x + c)) + a*\log(b*x + a)/b - c*\log(d*x + c)/d)*A*B*f + (x^2*\log(b*ex/(d*x + c) + a*e/(d*x + c)) - a^2*\log(b*x + a)/b^2 + c^2*\log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*g + A^2*f*x - (a*c*d*g + (2*c*d*f*\log(e) - (g*\log(e) + g)*c^2)*b)*B^2*\log(d*x + c)/(b*d^2) + (2*a*b*d^2*f - a^2*d^2*g - (2*c*d*f - c^2*g)*b^2)*( \log(b*x + a) * \log((b*d*x + a*d)/(b*c - a*d) + 1) + \text{dilog}(-(b*d*x + a*d)/(b*c - a*d))) * B^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*\log(e)^2 + 2*(a*b*d^2*g*\log(e) + (d^2*f*\log(e)^2 - c*d*g*\log(e))*b^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*\log(b*x + a)^2 + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*c*d*f - c^2*g)*B^2*b^2)*\log(d*x + c)^2 + 2*(B^2*b^2*d^2*g*x^2*\log(e) + (a*b*d^2*g + (2*d^2*f*\log(e) - c*d*g)*b^2)*B^2*x - ((g*\log(e) - g)*a^2*d^2 - (2*d^2*f*\log(e) - c*d*g)*a*b)*B^2)*\log(b*x + a) - 2*(B^2*b^2*d^2*g*x^2*\log(e) + (a*b*d^2*g + (2*d^2*f*\log(e) - c*d*g)*b^2)*B^2*x + (B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*\log(b*x + a))*\log(d*x + c))/(b^2*d^2)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2gx + A^2f + (B^2gx + B^2f) \log \left( \frac{bex + ae}{dx + c} \right)^2 + 2(ABgx + ABf) \log \left( \frac{bex + ae}{dx + c} \right), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out]  $\text{integral}(A^2g*x + A^2*f + (B^2g*x + B^2*f)*\log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*g*x + A*B*f)*\log((b*e*x + a*e)/(d*x + c)), x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((g*x+f)*(A+B*\ln(e*(b*x+a)/(d*x+c)))^2, x)$

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (gx + f) \left( B \log \left( \frac{(bx + a)e}{dx + c} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((g*x+f)*(A+B*\log(e*(b*x+a)/(d*x+c)))^2, x, \text{algorithm}="giac")$

[Out]  $\text{integrate}((g*x + f)*(B*\log((b*x + a)*e/(d*x + c)) + A)^2, x)$

$$3.243 \quad \int \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx$$

**Optimal.** Leaf size=125

$$\frac{2B^2(bc-ad)\text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd} + \frac{2B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{bd} + \frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)^2}{b}$$

[Out] (2\*B\*(b\*c - a\*d)\*Log[(b\*c - a\*d)/(b\*(c + d\*x))]\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]))/(b\*d) + ((a + b\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2)/b + (2\*B^2\*(b\*c - a\*d)\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*d)

**Rubi [A]** time = 0.637131, antiderivative size = 246, normalized size of antiderivative = 1.97, number of steps used = 22, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {2523, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2aB^2\text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{b} + \frac{2B^2c\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{d} + \frac{2aB\log(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{b} - \frac{2Bc\log(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2,x]

[Out] -((a\*B^2\*Log[a + b\*x]^2)/b) + (2\*a\*B\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]]))/b + x\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]]^2 + (2\*B^2\*c\*Log[-(d\*(a + b\*x))/(b\*c - a\*d)]\*Log[c + d\*x])/d - (2\*B\*c\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]]\*Log[c + d\*x])/d - (B^2\*c\*Log[c + d\*x]^2)/d + (2\*a\*B^2\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/b + (2\*a\*B^2\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/b + (2\*B^2\*c\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]/d

### Rule 2523

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*Rfx^p])^n, x] - Dist[b\*n\*p, Int[SimplifyIntegrand[(x\*(a + b\*Log[c\*Rfx^p])^(n-1)\*D[Rfx, x])/Rfx, x], x], x] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^(n-1)\*D[Rfx, x])/Rfx, x], x] /;

FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(RFx\_), x\_Symbol] :> With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n]^p, RFx, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] :> Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] :> Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
\int \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 dx &= x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 - (2B) \int \frac{(bc-ad)x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{(a+bx)(c+dx)} dx \\
&= x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 - (2B(bc-ad)) \int \frac{x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{(a+bx)(c+dx)} dx \\
&= x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 - (2B(bc-ad)) \int \left( -\frac{a \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{(bc-ad)(a+bx)} + \frac{c \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{(bc-ad)(c+dx)} \right) dx \\
&= x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 + (2aB) \int \frac{A + B \log \left( \frac{e(a+bx)}{c+dx} \right)}{a+bx} dx - (2Bc) \int \frac{A + B \log \left( \frac{e(a+bx)}{c+dx} \right)}{c+dx} dx \\
&= \frac{2aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 - \frac{2Bc \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{b} \\
&= \frac{2aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 - \frac{2Bc \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{b} \\
&= \frac{2aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 - \frac{2Bc \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{b} \\
&= \frac{2aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 - \frac{2Bc \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{b} \\
&= \frac{2aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 + \frac{2B^2c \log \left( -\frac{d(a+bx)}{bc-ad} \right)}{d} \\
&= -\frac{aB^2 \log^2(a+bx)}{b} + \frac{2aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2 \\
&= -\frac{aB^2 \log^2(a+bx)}{b} + \frac{2aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right)^2
\end{aligned}$$

**Mathematica [A]** time = 0.164023, size = 214, normalized size = 1.71

$$\frac{B \left( -aBd \left( \log(a+bx) \left( \log(a+bx) - 2 \log \left( \frac{b(c+dx)}{bc-ad} \right) \right) - 2 \text{PolyLog} \left( 2, \frac{d(a+bx)}{ad-bc} \right) \right) + bBc \left( 2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \right) \right)}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2,x]

[Out] x\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2 + (B\*(2\*a\*d\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])) - 2\*b\*c\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))\*Log[c + d\*x] - a\*B\*d\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c) + a\*d])) + b\*B\*c\*((2\*Log[(d\*(a + b\*x))/(-b\*c) + a\*d] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/(b\*d)

**Maple [F]** time = 1.722, size = 0, normalized size = 0.

$$\int \left( A + B \ln \left( \frac{e(bx+a)}{dx+c} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

[Out] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2 \left( x \log \left( \frac{(bx+a)e}{dx+c} \right) + \frac{ae \log(bx+a)}{b} - \frac{ce \log(dx+c)}{d} \right) AB + A^2 x + B^2 \left( \frac{bdx \log(bx+a)^2 + (bdx+bc) \log(dx+c)^2 - 2(bdx \log(bx+a) \log(dx+c))}{bd} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="maxima")

[Out] 2\*(x\*log((b\*x + a)\*e/(d\*x + c)) + (a\*e\*log(b\*x + a)/b - c\*e\*log(d\*x + c)/d)/e)\*A\*B + A^2\*x + B^2\*((b\*d\*x\*log(b\*x + a)^2 + (b\*d\*x + b\*c)\*log(d\*x + c)^2 - 2\*(b\*d\*x\*log(e) + (b\*d\*x + a\*d)\*log(b\*x + a))\*log(d\*x + c))/(b\*d) + integrate(((log(e)^2 + 2\*log(e))\*b^2\*d\*x^2 + a\*b\*c\*log(e)^2 + (b^2\*c\*log(e)^2 + (log(e)^2 + 2\*log(e))\*a\*b\*d)\*x + 2\*(b^2\*d\*x^2\*log(e) + a\*b\*c\*log(e) + a^2\*d + (a\*b\*d\*(log(e) + 2) + b^2\*c\*(log(e) - 1))\*x)\*log(b\*x + a))/(b^2\*d\*x^2 + a\*b\*c + (b^2\*c + a\*b\*d)\*x), x))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( B^2 \log \left( \frac{bex+ae}{dx+c} \right)^2 + 2AB \log \left( \frac{bex+ae}{dx+c} \right) + A^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out] integral(B^2\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*A\*B\*log((b\*e\*x + a\*e)/(d\*x + c)) + A^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( B \log \left( \frac{(bx+a)e}{dx+c} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```



$$3.244 \quad \int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx} dx$$

**Optimal.** Leaf size=277

$$\frac{2B \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{g} - \frac{2B \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right) \left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{g} - \frac{2B^2 \text{PolyLog}\left(3, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{g}$$

```
[Out] -((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x]))^2)/g
) + ((A + B*Log[(e*(a + b*x))/(c + d*x]))^2*Log[1 - ((d*f - c*g)*(a + b*x))
/((b*f - a*g)*(c + d*x))])/g - (2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Po
lyLog[2, (d*(a + b*x))/(b*(c + d*x))])/g + (2*B*(A + B*Log[(e*(a + b*x))/(c
+ d*x)])*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/g +
(2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/g - (2*B^2*PolyLog[3, ((d*f
- c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/g
```

**Rubi [B]** time = 4.90048, antiderivative size = 1998, normalized size of antiderivative = 7.21, number of steps used = 41, number of rules used = 21, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.724$ , Rules used = {2524, 12, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 6742, 2500, 2433, 2375, 2317, 2374, 6589, 2440, 2437, 2435, 2315}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x]))^2/(f + g*x), x]
```

```
[Out] -((B^2*Log[a + b*x]^2*Log[f + g*x])/g) - (2*A*B*Log[-((g*(a + b*x))/(b*f -
a*g))]*Log[f + g*x])/g - (B^2*Log[(c + d*x)^(-1)]^2*Log[f + g*x])/g + (2*B^
2*Log[-((g*(a + b*x))/(b*f - a*g))]*(Log[a + b*x] + Log[(c + d*x)^(-1)] - L
og[(e*(a + b*x))/(c + d*x)])*Log[f + g*x])/g + ((A + B*Log[(e*(a + b*x))/(c
+ d*x]))^2*Log[f + g*x])/g + (2*B^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[
c + d*x]*Log[f + g*x])/g - (2*B^2*Log[-((g*(a + b*x))/(b*f - a*g))]*(Log[(c
+ d*x)^(-1)] + Log[c + d*x])*Log[f + g*x])/g + (2*B^2*Log[a + b*x]*Log[(b*
(c + d*x))/(b*c - a*d)]*Log[f + g*x])/g + (2*A*B*Log[-((g*(c + d*x))/(d*f -
c*g))]*Log[f + g*x])/g - (2*B^2*(Log[a + b*x] + Log[(c + d*x)^(-1)] - Log[
(e*(a + b*x))/(c + d*x)])*Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/g
+ (B^2*Log[a + b*x]^2*Log[(b*(f + g*x))/(b*f - a*g)))/g + (B^2*Log[(c + d*
x)^(-1)]^2*Log[(d*(f + g*x))/(d*f - c*g)))/g + (B^2*(Log[(b*(c + d*x))/(b*c
- a*d)] + Log[(b*f - a*g)/(b*(f + g*x))] - Log[((b*f - a*g)*(c + d*x))/((b
*c - a*d)*(f + g*x))])*Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)
))]^2)/g - (B^2*(Log[(b*(c + d*x))/(b*c - a*d)] - Log[-((g*(c + d*x))/(d*f
- c*g))])*Log[a + b*x] + Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b
*x)))]^2)/g + (B^2*(Log[-((d*(a + b*x))/(b*c - a*d))] + Log[(d*f - c*g)/(d
*(f + g*x))] - Log[-(((d*f - c*g)*(a + b*x))/((b*c - a*d)*(f + g*x)))])*Log
[(((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))]^2)/g - (B^2*(Log[-((d*(a
+ b*x))/(b*c - a*d))] - Log[-((g*(a + b*x))/(b*f - a*g))])*Log[c + d*x] +
Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))]^2)/g + (2*B^2*(Log[f
+ g*x] - Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))])*PolyLog[2
, -((d*(a + b*x))/(b*c - a*d))])/g + (2*B^2*Log[a + b*x]*PolyLog[2, -((g*(a
+ b*x))/(b*f - a*g))])/g + (2*B^2*(Log[f + g*x] - Log[((b*c - a*d)*(f + g*
x))/((b*f - a*g)*(c + d*x))])*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]/g - (2
*B^2*Log[(c + d*x)^(-1)]*PolyLog[2, -((g*(c + d*x))/(d*f - c*g))])/g - (2*B
^2*Log[-(((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x)))]*PolyLog[2, (g*(a
+ b*x))/(b*(f + g*x))])/g + (2*B^2*Log[-(((b*c - a*d)*(f + g*x))/((d*f - c
```

```

*g)*(a + b*x)))]*PolyLog[2, -(((d*f - c*g)*(a + b*x))/((b*c - a*d)*(f + g*x
))))]/g - (2*B^2*Log[((b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))]*PolyLo
g[2, (g*(c + d*x))/(d*(f + g*x))]/g + (2*B^2*Log[((b*c - a*d)*(f + g*x))/
((b*f - a*g)*(c + d*x))]*PolyLog[2, ((b*f - a*g)*(c + d*x))/((b*c - a*d)*(f
+ g*x))]/g - (2*A*B*PolyLog[2, (b*(f + g*x))/(b*f - a*g)]/g + (2*B^2*(Lo
g[a + b*x] + Log[(c + d*x)^(-1)] - Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2,
(b*(f + g*x))/(b*f - a*g)]/g - (2*B^2*(Log[(c + d*x)^(-1)] + Log[c + d*x]
)*PolyLog[2, (b*(f + g*x))/(b*f - a*g)]/g + (2*B^2*(Log[c + d*x] + Log[((b
*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))])*PolyLog[2, (b*(f + g*x))/(b*
f - a*g)]/g + (2*A*B*PolyLog[2, (d*(f + g*x))/(d*f - c*g)]/g - (2*B^2*(Lo
g[a + b*x] + Log[(c + d*x)^(-1)] - Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2,
(d*(f + g*x))/(d*f - c*g)]/g + (2*B^2*(Log[a + b*x] + Log[-(((b*c - a*d)*
(f + g*x))/((d*f - c*g)*(a + b*x))])*PolyLog[2, (d*(f + g*x))/(d*f - c*g)]
)/g - (2*B^2*PolyLog[3, -((d*(a + b*x))/(b*c - a*d))]/g - (2*B^2*PolyLog[3
, -((g*(a + b*x))/(b*f - a*g))]/g - (2*B^2*PolyLog[3, (b*(c + d*x))/(b*c -
a*d)]/g - (2*B^2*PolyLog[3, -((g*(c + d*x))/(d*f - c*g))]/g - (2*B^2*Pol
yLog[3, (g*(a + b*x))/(b*(f + g*x))]/g + (2*B^2*PolyLog[3, -(((d*f - c*g)*
(a + b*x))/((b*c - a*d)*(f + g*x)))]/g - (2*B^2*PolyLog[3, (g*(c + d*x))/(
d*(f + g*x))]/g + (2*B^2*PolyLog[3, ((b*f - a*g)*(c + d*x))/((b*c - a*d)*(
f + g*x))]/g - (2*B^2*PolyLog[3, (b*(f + g*x))/(b*f - a*g)]/g - (2*B^2*Po
lyLog[3, (d*(f + g*x))/(d*f - c*g)]/g

```

#### Rule 2524

```

Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]

```

#### Rule 12

```

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]

```

#### Rule 2528

```

Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]

```

#### Rule 2418

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]

```

#### Rule 2390

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]

```

#### Rule 2301

```

Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]

```

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]

Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 2500

Int[(Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))\*((c\_.) + (d\_.)\*(x\_)^(q\_.))^(r\_.)]\*(s\_.) + Log[(i\_.)\*((g\_.) + (h\_.)\*(x\_)^(n\_.))]\*(t\_.))/((j\_.) + (k\_.)\*(x\_)), x\_Symbol] := Dist[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] - Log[(a + b\*x)^(p\*r)] - Log[(c + d\*x)^(q\*r)], Int[(s + t\*Log[i\*(g + h\*x)^n])/(j + k\*x), x], x] + (Int[(Log[(a + b\*x)^(p\*r)]\*(s + t\*Log[i\*(g + h\*x)^n]))/(j + k\*x), x] + Int[(Log[(c + d\*x)^(q\*r)]\*(s + t\*Log[i\*(g + h\*x)^n]))/(j + k\*x), x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ[b\*c - a\*d, 0]

Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))]\*(g\_.))\*((k\_.) + (l\_.)\*(x\_)^(r\_.)), x\_Symbol] := Dist[1/e, Subst[Int[(k\*x)/d]^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

Rule 2375

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))^(r\_.)]\*(a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Simp[(Log[d\*(e + f\*x^m)^r]\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[(f\*m\*r)/(b\*n\*(p + 1)), Int[(x^(m - 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(e + f\*x^m), x], x] /; FreeQ[{a, b, c, d, e, f, r, m, n}, x] && IGtQ[p, 0] && NeQ[d\*e, 1]

Rule 2317

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^p)/e, x] - Dist[(b*n*p)/e,
Int[(Log[1 + (e*x)/d]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b,
c, d, e, n}, x] && IGtQ[p, 0]
```

#### Rule 2374

```
Int[(Log[(d_.)*((e_.) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.))/(x_), x_Symbol]
:> -Simp[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^p)/m, x] + Dist[(b*n*p)/m, Int[(PolyLog[2, -(d*f*x^m)]*(a + b*Log[c*x^n])^(p - 1))/x, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0] && EqQ[d*e, 1]
```

#### Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_)^(p_.))]/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[b*d, a*e]
```

#### Rule 2440

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]*(b_.))*((f_.) + Log[(h_.)*((i_.) + (j_.)*(x_)^(m_.)]*(g_.))*((k_.) + (l_.)*(x_)^(r_.)), x_Symbol]
:> Dist[1/l, Subst[Int[x^r*(a + b*Log[c*(-((e*k - d*1)/1) + (e*x)/1)^n])*(f + g*Log[h*(-((j*k - i*1)/1) + (j*x)/1)^m]), x], x, k + l*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, m, n}, x] && IntegerQ[r]
```

#### Rule 2437

```
Int[(Log[(c_.)*((d_.) + (e_.)*(x_)^(n_.))]*Log[(h_.)*((i_.) + (j_.)*(x_)^(m_.))]/(x_), x_Symbol]
:> Dist[m, Int[(Log[i + j*x]*Log[c*(d + e*x)^n])/x, x], x] - Dist[m*Log[i + j*x] - Log[h*(i + j*x)^m], Int[Log[c*(d + e*x)^n]/x, x], x] /; FreeQ[{c, d, e, h, i, j, m, n}, x] && NeQ[e*i - d*j, 0] && NeQ[i + j*x, h*(i + j*x)^m]
```

#### Rule 2435

```
Int[(Log[(a_) + (b_.)*(x_)])*Log[(c_) + (d_.)*(x_)])/(x_), x_Symbol]
:> Simp[Log[-((b*x)/a)]*Log[a + b*x]*Log[c + d*x], x] + (Simp[(1*(Log[-((b*x)/a)] - Log[-((b*c - a*d)*x]/(a*(c + d*x)))] + Log[(b*c - a*d)/(b*(c + d*x))])*Log[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - Simp[(1*(Log[-((b*x)/a)] - Log[-((d*x)/c]))*(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x)]))^2)/2, x] + Simp[(Log[c + d*x] - Log[(a*(c + d*x))/(c*(a + b*x)])]*PolyLog[2, 1 + (b*x)/a], x] + Simp[(Log[a + b*x] + Log[(a*(c + d*x))/(c*(a + b*x)])]*PolyLog[2, 1 + (d*x)/c], x] + Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (c*(a + b*x))/(a*(c + d*x))], x] - Simp[Log[(a*(c + d*x))/(c*(a + b*x))]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))], x] - Simp[PolyLog[3, 1 + (b*x)/a], x] - Simp[PolyLog[3, 1 + (d*x)/c], x] + Simp[PolyLog[3, (c*(a + b*x))/(a*(c + d*x))], x] - Simp[PolyLog[3, (d*(a + b*x))/(b*(c + d*x))], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_.) + (e_.)*(x_)), x_Symbol]
:> -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{f+gx} dx &= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(f+gx)}{g} - \frac{(2B) \int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2} + \frac{be}{c+dx}\right)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f+gx)}{e(a+bx)} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(f+gx)}{g} - \frac{(2B) \int \frac{(c+dx)\left(-\frac{de(a+bx)}{(c+dx)^2} + \frac{be}{c+dx}\right)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f+gx)}{a+bx} dx}{eg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(f+gx)}{g} - \frac{(2B) \int \frac{(bc-ad)e\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f+gx)}{(a+bx)(c+dx)} dx}{eg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(f+gx)}{g} - \frac{(2B(bc-ad)) \int \frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f+gx)}{(a+bx)(c+dx)} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(f+gx)}{g} - \frac{(2B(bc-ad)) \int \left(\frac{b\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f+gx)}{(bc-ad)(a+bx)} - \frac{d\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f+gx)}{g}\right) dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(f+gx)}{g} - \frac{(2bB) \int \frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f+gx)}{a+bx} dx}{g} + \frac{(2Bd) \int \frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f+gx)}{a+bx} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(f+gx)}{g} - \frac{(2bB) \int \left(\frac{A \log(f+gx)}{a+bx} + \frac{B \log\left(\frac{e(a+bx)}{c+dx}\right) \log(f+gx)}{a+bx}\right) dx}{g} + \frac{(2Bd) \int \frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) \log(f+gx)}{a+bx} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(f+gx)}{g} - \frac{(2AbB) \int \frac{\log(f+gx)}{a+bx} dx}{g} - \frac{(2bB^2) \int \frac{\log\left(\frac{e(a+bx)}{c+dx}\right) \log(f+gx)}{a+bx} dx}{g} \\
&= -\frac{2AB \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \log(f+gx)}{g} + \frac{2AB \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} \\
&= -\frac{2AB \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} + \frac{2B^2 \log\left(-\frac{g(a+bx)}{bf-ag}\right) \left(\log(a+bx) + \log\left(\frac{1}{c+dx}\right) - \log\left(\frac{1}{bf-ag}\right)\right)}{g} \\
&= -\frac{B^2 \log^2(a+bx) \log(f+gx)}{g} - \frac{2AB \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} - \frac{B^2 \log^2\left(\frac{1}{c+dx}\right) \log(f+gx)}{g} \\
&= -\frac{B^2 \log^2(a+bx) \log(f+gx)}{g} - \frac{2AB \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} - \frac{B^2 \log^2\left(\frac{1}{c+dx}\right) \log(f+gx)}{g} \\
&= -\frac{B^2 \log^2(a+bx) \log(f+gx)}{g} - \frac{2AB \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} - \frac{B^2 \log^2\left(\frac{1}{c+dx}\right) \log(f+gx)}{g} \\
&= -\frac{B^2 \log^2(a+bx) \log(f+gx)}{g} - \frac{2AB \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} - \frac{B^2 \log^2\left(\frac{1}{c+dx}\right) \log(f+gx)}{g}
\end{aligned}$$

**Mathematica [A]** time = 0.857124, size = 431, normalized size = 1.56

$$2AB \text{PolyLog}\left(2, \frac{g(a+bx)}{ag-bf}\right) + 2B^2 \log\left(\frac{e(a+bx)}{c+dx}\right) \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right) - 2B^2 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right) \log\left(\frac{e(a+bx)}{c+dx}\right) - 2B^2 \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x),x]
```

```
[Out] (-B^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log[(b*c - a*d)/(b*c + b*d*x)] + A^2
*Log[f + g*x] - 2*A*B*Log[a/b + x]*Log[f + g*x] + 2*A*B*Log[c/d + x]*Log[f
+ g*x] + 2*A*B*Log[(e*(a + b*x))/(c + d*x)]*Log[f + g*x] + 2*A*B*Log[a/b +
x]*Log[(b*(f + g*x))/(b*f - a*g)] - 2*A*B*Log[c/d + x]*Log[(d*(f + g*x))/(d
*f - c*g)] + B^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log[((b*c - a*d)*(f + g*x))
/((b*f - a*g)*(c + d*x))] + 2*A*B*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)]
- 2*B^2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))
] + 2*B^2*Log[(e*(a + b*x))/(c + d*x)]*PolyLog[2, ((d*f - c*g)*(a + b*x))/
(b*f - a*g)*(c + d*x)] - 2*A*B*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)] +
2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))] - 2*B^2*PolyLog[3, ((d*f - c*
g)*(a + b*x))/((b*f - a*g)*(c + d*x))]/g
```

---

**Maple [B]** time = 0.336, size = 2428, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x)
```

```
[Out] -2*B^2/g/(a*d-b*c)*polylog(3,1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b*c-A^2
/g/(a*d-b*c)*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*c*g-d*f*(b*e/d+(a*d-b*c)*e/d/
(d*x+c))-a*e*g+b*e*f)*b*c+A^2/g/(a*d-b*c)*ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-b*e)*b*c+2*B^2/g/(a*d-b*c)*polylog(3,-(c*g-d*f)/(-a*e*g+b*e*f)*(b*e/d+(a
d-b*c)*e/d/(d*x+c))*b*c-d*A^2/g/(a*d-b*c)*ln(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-b*e)*a-2*d*B^2/g/(a*d-b*c)*polylog(3,-(c*g-d*f)/(-a*e*g+b*e*f)*(b*e/d+(a
d-b*c)*e/d/(d*x+c))*a+2*d*B^2/g/(a*d-b*c)*polylog(3,1/b/e*d*(b*e/d+(a*d-b
c)*e/d/(d*x+c))*a+d*A^2/g/(a*d-b*c)*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*c*g-
d*f*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)*a-2*A*B/(a*d-b*c)*ln(b*e/d+(
a*d-b*c)*e/d/(d*x+c))*ln(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*
e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*c^2*b+2*A*B/g/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))*ln(-(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)/b/e)*b*c-2*d*A*B/g/(a
d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-b*e)/b/e)*a+2*d*A*B/(a*d-b*c)*dilog(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*c*a-B^2/g/(a*d-b*c)*ln(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^2*ln(1+(c*g-d*f)/(-a*e*g+b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*
x+c))*b*c-2*B^2/g/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2,-(c*
g-d*f)/(-a*e*g+b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b*c-2*d*A*B/g/(a*d-b*c
)*dilog(-(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)/b/e)*a-2*A*B/(a*d-b*c)*dilog
(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g
-d*f)*c^2*b+2*A*B/g/(a*d-b*c)*dilog(-(d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-b*e)/
b/e)*b*c+2*d*B^2/g/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2,-(c*
g-d*f)/(-a*e*g+b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a-d*B^2/g/(a*d-b*c)*ln
(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(1-1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))
)*a-2*d*B^2/g/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2,1/b/e*d*(b
e/d+(a*d-b*c)*e/d/(d*x+c))*a+d*B^2/g/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^2*ln(1+(c*g-d*f)/(-a*e*g+b*e*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*a+B^2/
g/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(1-1/b/e*d*(b*e/d+(a*d-b*c)
*e/d/(d*x+c))*b*c+2*B^2/g/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylo
g(2,1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*b*c-2*d^2*A*B/g/(a*d-b*c)*dilog(
((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-
d*f)*f*a+2*d*A*B/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(((c*g-d*f)*(b
e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*c*a+2*d*
A*B/g/(a*d-b*c)*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(((c*g-d*f)*(b*e/d+(a*d-b
c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*f*b*c+2*d*A*B/g/(a
```

$$d-b*c)*\text{dilog}(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*f*b*c-2*d^2*A*B/g/(a*d-b*c)*\ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)))*\ln(((c*g-d*f)*(b*e/d+(a*d-b*c)*e/d/(d*x+c))-a*e*g+b*e*f)/(-a*e*g+b*e*f))/(c*g-d*f)*f*a$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{A^2 \log(gx + f)}{g} - \int \frac{B^2 \log(bx + a)^2 + B^2 \log(e)^2 + 2AB \log(e) + 2(B^2 \log(e) + AB) \log(bx + a) - 2(B^2 \log(bx + a) + A^2) \log(d*x + c)}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f),x, algorithm="maxima")

[Out] A^2\*log(g\*x + f)/g - integrate(-(B^2\*log(b\*x + a)^2 + B^2\*log(e)^2 + 2\*A\*B\*log(e) + 2\*(B^2\*log(e) + A\*B)\*log(b\*x + a) - 2\*(B^2\*log(b\*x + a) + B^2\*log(e) + A\*B)\*log(d\*x + c))/(g\*x + f), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B^2 \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2AB \log\left(\frac{bex+ae}{dx+c}\right) + A^2}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f),x, algorithm="fricas")

[Out] integral((B^2\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*A\*B\*log((b\*e\*x + a\*e)/(d\*x + c)) + A^2)/(g\*x + f), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2/(g\*x+f),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+ae)}{dx+c}\right) + A\right)^2}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(g*x+f),x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(g*x + f), x)
```



$$3.245 \quad \int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^2} dx$$

**Optimal.** Leaf size=196

$$\frac{2B^2(bc-ad)\text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{(bf-ag)(df-cg)} + \frac{2B(bc-ad)\log\left(1 - \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{(bf-ag)(df-cg)} + \frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{(f+gx)(bf-ag)}$$

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x]))^2)/((b*f - a*g)*(f + g*x))
+ (2*B*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x]])*Log[1 - ((d*f - c*g)
)*(a + b*x))/((b*f - a*g)*(c + d*x)))/((b*f - a*g)*(d*f - c*g)) + (2*B^2*(
b*c - a*d)*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b
*f - a*g)*(d*f - c*g))
```

**Rubi [B]** time = 1.1194, antiderivative size = 612, normalized size of antiderivative = 3.12, number of steps used = 32, number of rules used = 10, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{2bB^2\text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{g(bf-ag)} + \frac{2B^2d\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{g(df-cg)} - \frac{2B^2(bc-ad)\text{PolyLog}\left(2, \frac{b(f+gx)}{bf-ag}\right)}{(bf-ag)(df-cg)} + \frac{2B^2(bc-ad)\text{PolyLog}\left(2, \frac{b(f+gx)}{bf-ag}\right)}{(bf-ag)(df-cg)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x]))^2/(f + g*x)^2, x]
```

```
[Out] -((b*B^2*Log[a + b*x]^2)/(g*(b*f - a*g))) + (2*b*B*Log[a + b*x]*(A + B*Log[
(e*(a + b*x))/(c + d*x]]))/(g*(b*f - a*g)) - (A + B*Log[(e*(a + b*x))/(c +
d*x]]^2/(g*(f + g*x)) + (2*B^2*d*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c +
d*x])/(g*(d*f - c*g)) - (2*B*d*(A + B*Log[(e*(a + b*x))/(c + d*x]])*Log[c
+ d*x])/(g*(d*f - c*g)) - (B^2*d*Log[c + d*x]^2)/(g*(d*f - c*g)) + (2*b*B^2
*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)))/(g*(b*f - a*g)) - (2*B^2*(b*c
- a*d)*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/((b*f - a*g)*(d*f -
c*g)) + (2*B*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x]])*Log[f + g*x]
)/((b*f - a*g)*(d*f - c*g)) + (2*B^2*(b*c - a*d)*Log[-((g*(c + d*x))/(d*f -
c*g))]*Log[f + g*x])/((b*f - a*g)*(d*f - c*g)) + (2*b*B^2*PolyLog[2, -((d*
(a + b*x))/(b*c - a*d))])/((g*(b*f - a*g)) + (2*B^2*d*PolyLog[2, (b*(c + d*x)
))/(b*c - a*d)))/(g*(d*f - c*g)) - (2*B^2*(b*c - a*d)*PolyLog[2, (b*(f + g*
x))/(b*f - a*g)))/((b*f - a*g)*(d*f - c*g)) + (2*B^2*(b*c - a*d)*PolyLog[2,
(d*(f + g*x))/(d*f - c*g)))/((b*f - a*g)*(d*f - c*g))
```

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f + gx)^2} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f + gx)} + \frac{(2B) \int \frac{(bc-ad)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)(f+gx)} dx}{g} \\
 &= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f + gx)} + \frac{(2B(bc - ad)) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)(c+dx)(f+gx)} dx}{g} \\
 &= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f + gx)} + \frac{(2B(bc - ad)) \int \left(\frac{b^2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(-df+cg)(c+dx)} + \frac{g}{(bc-ad)(-df+cg)(c+dx)}\right) dx}{g} \\
 &= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f + gx)} + \frac{(2b^2B) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx} dx}{g(bf - ag)} - \frac{(2Bd^2) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} dx}{g(df - cg)} + \frac{2Bd \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} dx}{g(df - cg)} \\
 &= \frac{2bB \log(a + bx) \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf - ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f + gx)} - \frac{2Bd \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(df - cg)} \\
 &= \frac{2bB \log(a + bx) \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf - ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f + gx)} - \frac{2Bd \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(df - cg)} \\
 &= \frac{2bB \log(a + bx) \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf - ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f + gx)} - \frac{2Bd \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(df - cg)} \\
 &= \frac{2bB \log(a + bx) \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf - ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f + gx)} - \frac{2Bd \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(df - cg)} \\
 &= \frac{2bB \log(a + bx) \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf - ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f + gx)} + \frac{2B^2d \log\left(-\frac{d(a+bx)}{bc-ad}\right)}{g(df - cg)} \\
 &= -\frac{bB^2 \log^2(a + bx)}{g(bf - ag)} + \frac{2bB \log(a + bx) \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf - ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f + gx)} + \frac{2B^2d \log\left(-\frac{d(a+bx)}{bc-ad}\right)}{g(df - cg)} \\
 &= -\frac{bB^2 \log^2(a + bx)}{g(bf - ag)} + \frac{2bB \log(a + bx) \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf - ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{g(f + gx)} + \frac{2B^2d \log\left(-\frac{d(a+bx)}{bc-ad}\right)}{g(df - cg)}
 \end{aligned}$$

**Mathematica [B]** time = 0.958893, size = 402, normalized size = 2.05

$$\frac{B\left(-bB(df-cg)\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)+Bd(bf-ag)\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\right)-2B^2d\log\left(-\frac{d(a+bx)}{bc-ad}\right)}{g(bf-ag)g(df-cg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^2,x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)) + (B*(2*b*(d*f - c*g)*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(b*f - a*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*(b*c - a*d)*g*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[f + g*x] - b*B*(d*f - c*g)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*(b*f - a*g)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b*c - a*d)*g*((Log[(g*(a + b*x))/(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f)
```

+ c\*g]))\*Log[f + g\*x] + PolyLog[2, (b\*(f + g\*x))/(b\*f - a\*g)] - PolyLog[2, (d\*(f + g\*x))/(d\*f - c\*g]])))/((b\*f - a\*g)\*(d\*f - c\*g)))/g

**Maple [F]** time = 2.067, size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^2} \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^2,x)

[Out] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2AB \left( \frac{b \log(bx + a)}{bfg - ag^2} - \frac{d \log(dx + c)}{dfg - cg^2} + \frac{(bc - ad) \log(gx + f)}{bdf^2 + acg^2 - (bc + ad)fg} - \frac{\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right)}{g^2x + fg} \right) - B^2 \left( \frac{\log(dx + c)^2}{g^2x + fg} + \int -\frac{dgx \log}{g^2x + fg} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^2,x, algorithm="maxima")

[Out] 2\*A\*B\*(b\*log(b\*x + a)/(b\*f\*g - a\*g^2) - d\*log(d\*x + c)/(d\*f\*g - c\*g^2) + (b\*c - a\*d)\*log(g\*x + f)/(b\*d\*f^2 + a\*c\*g^2 - (b\*c + a\*d)\*f\*g) - log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c))/(g^2\*x + f\*g)) - B^2\*(log(d\*x + c)^2/(g^2\*x + f\*g) + integrate(-(d\*g\*x\*log(e)^2 + c\*g\*log(e)^2 + (d\*g\*x + c\*g)\*log(b\*x + a)^2 + 2\*(d\*g\*x\*log(e) + c\*g\*log(e))\*log(b\*x + a) - 2\*((g\*log(e) - g)\*d\*x + c\*g\*log(e) - d\*f + (d\*g\*x + c\*g)\*log(b\*x + a))\*log(d\*x + c))/(d\*g^3\*x^3 + c\*f^2\*g + (2\*d\*f\*g^2 + c\*g^3)\*x^2 + (d\*f^2\*g + 2\*c\*f\*g^2)\*x), x)) - A^2/(g^2\*x + f\*g)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2AB \log\left(\frac{bex+ae}{dx+c}\right) + A^2}{g^2x^2 + 2fgx + f^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^2,x, algorithm="fricas")

[Out] integral((B^2\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*A\*B\*log((b\*e\*x + a\*e)/(d\*x + c)) + A^2)/(g^2\*x^2 + 2\*f\*g\*x + f^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2/(g\*x+f)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{(gx+f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^2,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2/(g\*x + f)^2, x)

$$3.246 \quad \int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3} dx$$

**Optimal.** Leaf size=369

$$\frac{B^2(bc-ad)(-adg-bcg+2bdf)\text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{(bf-ag)^2(df-cg)^2} + \frac{b^2\left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)^2}{2g(bf-ag)^2} + \frac{Bg(a+bx)(bc-ad)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{(f+gx)(bf-ag)^2(df-cg)^2}$$

[Out] (B\*(b\*c - a\*d)\*g\*(a + b\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])))/((b\*f - a\*g)^2\*(d\*f - c\*g)\*(f + g\*x)) + (b^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2)/(2\*g\*(b\*f - a\*g)^2) - (A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2/(2\*g\*(f + g\*x)^2) + (B^2\*(b\*c - a\*d)^2\*g\*Log[(f + g\*x)/(c + d\*x)])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (B\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))\*Log[1 - ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*PolyLog[2, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2)

**Rubi [B]** time = 1.48104, antiderivative size = 883, normalized size of antiderivative = 2.39, number of steps used = 36, number of rules used = 11, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

$$-\frac{B^2 \log^2(a+bx)b^2}{2g(bf-ag)^2} + \frac{B \log(a+bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{g(bf-ag)^2} + \frac{B^2 \log(a+bx) \log\left(\frac{b(c+dx)}{bc-ad}\right)b^2}{g(bf-ag)^2} + \frac{B^2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{g(bf-ag)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2/(f + g\*x)^3, x]

[Out] (b\*B^2\*(b\*c - a\*d)\*Log[a + b\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)) - (b^2\*B^2\*Log[a + b\*x]^2)/(2\*g\*(b\*f - a\*g)^2) - (B\*(b\*c - a\*d)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])))/((b\*f - a\*g)\*(d\*f - c\*g)\*(f + g\*x)) + (b^2\*B\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])))/(g\*(b\*f - a\*g)^2) - (A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2/(2\*g\*(f + g\*x)^2) - (B^2\*d\*(b\*c - a\*d)\*Log[c + d\*x])/((b\*f - a\*g)\*(d\*f - c\*g)^2) + (B^2\*d^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x])/((g\*(d\*f - c\*g)^2) - (B\*d^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))\*Log[c + d\*x])/((g\*(d\*f - c\*g)^2) - (B^2\*d^2\*Log[c + d\*x]^2)/(2\*g\*(d\*f - c\*g)^2) + (b^2\*B^2\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(g\*(b\*f - a\*g)^2) + (B^2\*(b\*c - a\*d)^2\*g\*Log[f + g\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) - (B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*Log[f + g\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (B\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))\*Log[f + g\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*Log[f + g\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (b^2\*B^2\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))]/(g\*(b\*f - a\*g)^2) + (B^2\*d^2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]/(g\*(d\*f - c\*g)^2) - (B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*PolyLog[2, (b\*(f + g\*x))/(b\*f - a\*g)]/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*PolyLog[2, (d\*(f + g\*x))/(d\*f - c\*g)]/((b\*f - a\*g)^2\*(d\*f - c\*g)^2)

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1))

, x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2528

Int[((a\_) + Log[(c\_)\*(RFx\_)^(p\_)]\*(b\_))^(n\_)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

### Rule 2524

Int[((a\_) + Log[(c\_)\*(RFx\_)^(p\_)]\*(b\_))^(n\_)/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

### Rule 2418

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)]\*(b\_))^(p\_)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

### Rule 2390

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)]\*(b\_))^(p\_)\*((f\_) + (g\_)\*(x\_))^(q\_), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

Int[((a\_) + Log[(c\_)\*(x\_)^(n\_)]\*(b\_))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

### Rule 2394

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)]\*(b\_))/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))]\*(b\_))/((f\_) + (g\_)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

### Rule 72

```
Int[((e_.) + (f_.)*(x_)^(p_.))/((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^3} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{B \int \frac{(bc-ad)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)(f+gx)^2} dx}{g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{(B(bc-ad)) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)(c+dx)(f+gx)^2} dx}{g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{(B(bc-ad)) \int \left(\frac{b^3\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d^3\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(-df+cg)^2(c+dx)} + \frac{g^2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)}\right) dx}{g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2g(f+gx)^2} + \frac{(b^3B) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx} dx}{g(bf-ag)^2} - \frac{(Bd^3) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} dx}{g(df-cg)^2} + \frac{(Bbc) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx} dx}{g} \\
&= -\frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B \log(a+bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)^2} - \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g(f+gx)} \\
&= -\frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B \log(a+bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)^2} - \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g(f+gx)} \\
&= -\frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B \log(a+bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)^2} - \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2g(f+gx)} \\
&= \frac{bB^2(bc-ad) \log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B \log(a+bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)^2} \\
&= \frac{bB^2(bc-ad) \log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B \log(a+bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)^2} \\
&= \frac{bB^2(bc-ad) \log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{b^2B^2 \log^2(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B \log(a+bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)^2} \\
&= \frac{bB^2(bc-ad) \log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{b^2B^2 \log^2(a+bx)}{2g(bf-ag)^2} - \frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{b^2B \log(a+bx)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{g(bf-ag)^2}
\end{aligned}$$

**Mathematica [A]** time = 2.0669, size = 595, normalized size = 1.61

$$\frac{B(f+gx)\left(b^2B(f+gx)(df-cg)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)-Bd^2(f+gx)(bf-ag)^2\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)\right)\right)\right)}{g(bf-ag)^2(df-cg)}$$

Antiderivative was successfully verified.



[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2/(f + g\*x)^3,x]

[Out] -((A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2 + (B\*(f + g\*x)\*(2\*(b\*c - a\*d)\*g\*(b\*f - a\*g)\*(d\*f - c\*g)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]) - 2\*b^2\*(d\*f - c\*g)^2\*(f + g\*x)\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]) + 2\*d^2\*(b\*f - a\*g)^2\*(f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])\*Log[c + d\*x] + 2\*(b\*c - a\*d)\*g\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g)\*(f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])\*Log[f + g\*x] - 2\*B\*(b\*c - a\*d)\*g\*(f + g\*x)\*(b\*(d\*f - c\*g)\*Log[a + b\*x] + (-b\*d\*f) + a\*d\*g)\*Log[c + d\*x] + (b\*c - a\*d)\*g\*Log[f + g\*x]) + b^2\*B\*(d\*f - c\*g)^2\*(f + g\*x)\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c) + a\*d]) - B\*d^2\*(b\*f - a\*g)^2\*(f + g\*x)\*((2\*Log[(d\*(a + b\*x))/(-b\*c) + a\*d] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)] - 2\*B\*(b\*c - a\*d)\*g\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g)\*(f + g\*x)\*((Log[(g\*(a + b\*x))/(-b\*f) + a\*g] - Log[(g\*(c + d\*x))/(-d\*f) + c\*g])\*Log[f + g\*x] + PolyLog[2, (b\*(f + g\*x))/(b\*f - a\*g)] - PolyLog[2, (d\*(f + g\*x))/(d\*f - c\*g)])))/(b\*f - a\*g)^2\*(d\*f - c\*g)^2)/(2\*g\*(f + g\*x)^2)

**Maple [F]** time = 2.909, size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^3} \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^3,x)

[Out] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left( \frac{b^2 \log(bx + a)}{b^2 f^2 g - 2 abf g^2 + a^2 g^3} - \frac{d^2 \log(dx + c)}{d^2 f^2 g - 2 cdf g^2 + c^2 g^3} + \frac{(2(b^2 cd - abd^2)f - (b^2 c^2 - a^2 d^2)g) \log(gx + f)}{b^2 d^2 f^4 + a^2 c^2 g^4 - 2(b^2 cd + abd^2)f^3 g + (b^2 c^2 + 4 abcd + a^2 d^2)g^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^3,x, algorithm="maxima")

[Out] (b^2\*log(b\*x + a)/(b^2\*f^2\*g - 2\*a\*b\*f\*g^2 + a^2\*g^3) - d^2\*log(d\*x + c)/(d^2\*f^2\*g - 2\*c\*d\*f\*g^2 + c^2\*g^3) + (2\*(b^2\*c\*d - a\*b\*d^2)\*f - (b^2\*c^2 - a^2\*d^2)\*g)\*log(g\*x + f)/(b^2\*d^2\*f^4 + a^2\*c^2\*g^4 - 2\*(b^2\*c\*d + a\*b\*d^2)\*f^3\*g + (b^2\*c^2 + 4\*a\*b\*c\*d + a^2\*d^2)\*f^2\*g^2 - 2\*(a\*b\*c^2 + a^2\*c\*d)\*f\*g^3 - (b\*c - a\*d)/(b\*d\*f^3 + a\*c\*f\*g^2 - (b\*c + a\*d)\*f^2\*g + (b\*d\*f^2\*g + a\*c\*g^3 - (b\*c + a\*d)\*f\*g^2)\*x) - log(b\*e\*x/(d\*x + c) + a\*e/(d\*x + c))/(g^3\*x^2 + 2\*f\*g^2\*x + f^2\*g)\*A\*B - 1/2\*B^2\*(log(d\*x + c)^2/(g^3\*x^2 + 2\*f\*g^2\*x + f^2\*g) + 2\*integrate(-(d\*g\*x\*log(e)^2 + c\*g\*log(e)^2 + (d\*g\*x + c\*g)\*log(b\*x + a)^2 + 2\*(d\*g\*x\*log(e) + c\*g\*log(e))\*log(b\*x + a) - ((2\*g\*log(e) - g)\*d\*x + 2\*c\*g\*log(e) - d\*f + 2\*(d\*g\*x + c\*g)\*log(b\*x + a))\*log(d\*x + c))/(d\*g^4\*x^4 + c\*f^3\*g + (3\*d\*f\*g^3 + c\*g^4)\*x^3 + 3\*(d\*f^2\*g^2 + c\*f\*g^3)\*x^2 + (d\*f^3\*g + 3\*c\*f^2\*g^2)\*x), x)) - 1/2\*A^2/(g^3\*x^2 + 2\*f\*g^2\*x + f^2\*g)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2AB \log\left(\frac{bex+ae}{dx+c}\right) + A^2}{g^3x^3 + 3fg^2x^2 + 3f^2gx + f^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^3,x, algorithm="fricas")

[Out] integral((B^2\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*A\*B\*log((b\*e\*x + a\*e)/(d\*x + c)) + A^2)/(g^3\*x^3 + 3\*f\*g^2\*x^2 + 3\*f^2\*g\*x + f^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2/(g\*x+f)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{(gx+f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^3,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2/(g\*x + f)^3, x)

$$3.247 \quad \int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4} dx$$

**Optimal.** Leaf size=714

$$\frac{2B^2(bc - ad) \left(a^2 d^2 g^2 - abdg(3df - cg) + b^2(c^2 g^2 - 3cdfg + 3d^2 f^2)\right) \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right) + 2B(bc - ad) \left(a^2 d^2 g^2 - abdg(3df - cg) + b^2(c^2 g^2 - 3cdfg + 3d^2 f^2)\right)}{3(bf - ag)^3(df - cg)^3}$$

```
[Out] (B^2*(b*c - a*d)^2*g^2*(c + d*x))/(3*(b*f - a*g)^2*(d*f - c*g)^3*(f + g*x))
+ (B^2*(b*c - a*d)^3*g^2*Log[(a + b*x)/(c + d*x)]/(3*(b*f - a*g)^3*(d*f -
c*g)^3) - (B*(b*c - a*d)*g^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)])))/(3*(b*f - a*g)*(d*f - c*g)^3*(f + g*x)^2) + (2*B*(b*c - a*d)*g*(3*b*d*
f - b*c*g - 2*a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(b*
f - a*g)^3*(d*f - c*g)^2*(f + g*x)) + (b^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2)/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(3*g*(
f + g*x)^3) - (B^2*(b*c - a*d)^3*g^2*Log[(f + g*x)/(c + d*x)]/(3*(b*f - a*
g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)^2*g*(3*b*d*f - b*c*g - 2*a*d*g)*Lo
g[(f + g*x)/(c + d*x)]/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B*(b*c - a*d)*
(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2
))*Log[(e*(a + b*x))/(c + d*x)]*Log[1 - ((d*f - c*g)*(a + b*x))/((b*
f - a*g)*(c + d*x))]/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)
*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^
2))*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))]/(3*(b*f -
a*g)^3*(d*f - c*g)^3)
```

**Rubi [A]** time = 2.38124, antiderivative size = 1356, normalized size of antiderivative = 1.9, number of steps used = 40, number of rules used = 11, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

$$-\frac{B^2 \log^2(a + bx) b^3}{3g(bf - ag)^3} + \frac{2B \log(a + bx) \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right) b^3}{3g(bf - ag)^3} + \frac{2B^2 \log(a + bx) \log\left(\frac{b(c+dx)}{bc-ad}\right) b^3}{3g(bf - ag)^3} + \frac{2B^2 \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right) b^3}{3g(bf - ag)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^4, x]
```

```
[Out] -(B^2*(b*c - a*d)^2*g)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^2*B^2
*(b*c - a*d)*Log[a + b*x]/(3*(b*f - a*g)^3*(d*f - c*g)) + (2*b*B^2*(b*c -
a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[a + b*x]/(3*(b*f - a*g)^3*(d*f - c*g)^2
) - (b^3*B^2*Log[a + b*x]^2)/(3*g*(b*f - a*g)^3) - (B*(b*c - a*d)*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]))/(3*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (2*B
*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (2*b^3*B*Log[a + b*x]*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x
))/(c + d*x)])^2/(3*g*(f + g*x)^3) - (B^2*d^2*(b*c - a*d)*Log[c + d*x]/(3*
(b*f - a*g)*(d*f - c*g)^3) - (2*B^2*d*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)
*Log[c + d*x]/(3*(b*f - a*g)^2*(d*f - c*g)^3) + (2*B^2*d^3*Log[-((d*(a + b
*x))/(b*c - a*d))]*Log[c + d*x]/(3*g*(d*f - c*g)^3) - (2*B*d^3*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*Log[c + d*x]/(3*g*(d*f - c*g)^3) - (B^2*d^3*Log[
c + d*x]^2)/(3*g*(d*f - c*g)^3) + (2*b^3*B^2*Log[a + b*x]*Log[(b*(c + d*x))
/(b*c - a*d)]/(3*g*(b*f - a*g)^3) + (B^2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g
- a*d*g)*Log[f + g*x]/((b*f - a*g)^3*(d*f - c*g)^3) - (2*B^2*(b*c - a*d)*
(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2
))*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x]/(3*(b*f - a*g)^3*(d*f - c
```

```
*g)^3) + (2*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*b^3*B^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))]/(3*g*(b*f - a*g)^3) + (2*B^2*d^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]/(3*g*(d*f - c*g)^3) - (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, (b*(f + g*x))/(b*f - a*g)]/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (2*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, (d*(f + g*x))/(d*f - c*g)]/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_)^(p_.))/((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)),
x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3} + \frac{(2B) \int \frac{(bc-ad)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)(f+gx)^3} dx}{3g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3} + \frac{(2B(bc-ad)) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)(c+dx)(f+gx)^3} dx}{3g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3} + \frac{(2B(bc-ad)) \int \left(\frac{b^4\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(bf-ag)^3(a+bx)} + \frac{d^4\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(-df+cg)^3(c+dx)} + \frac{g^2}{(bf-ag)^3}\right) dx}{3g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3g(f+gx)^3} + \frac{(2b^4B) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx} dx}{3g(bf-ag)^3} - \frac{(2Bd^4) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} dx}{3g(df-cg)^3} + \frac{(2Bg^2) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{f+gx} dx}{3g} \\
&= -\frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)^2(df-cg)^2(f+gx)} \\
&= -\frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)^2(df-cg)^2(f+gx)} \\
&= -\frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bf-ag)^2(df-cg)^2(f+gx)} \\
&= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-ad)(2bdf-bcg)}{3(bf-ag)^3(df-cg)} \\
&= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-ad)(2bdf-bcg)}{3(bf-ag)^3(df-cg)} \\
&= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-ad)(2bdf-bcg)}{3(bf-ag)^3(df-cg)} \\
&= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{2bB^2(bc-ad)(2bdf-bcg)}{3(bf-ag)^3(df-cg)}
\end{aligned}$$

**Mathematica [A]** time = 4.224, size = 894, normalized size = 1.25

$$\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^4} + \frac{B(f+gx)\left(2d^3(f+gx)^2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)(bf-ag)^3-Bd^3(f+gx)^2\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\log(c+dx)+2\text{PolyLog}\right)}{3g^2(f+gx)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2/(f + g\*x)^4, x]

[Out] -((A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2 + (B\*(f + g\*x)\*((b\*c - a\*d)\*g\*(b\*f - a\*g)^2\*(d\*f - c\*g)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])) + 2\*(b\*c - a\*d)\*g\*(b\*f - a\*g)\*(-(d\*f) + c\*g)\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g)\*(f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x])) - 2\*b^3\*(d\*f - c\*g)^3\*(f + g\*x)^2\*Log[a + b\*x]\* (A + B\*Log[(e\*(a + b\*x))/(c + d\*x])) + 2\*d^3\*(b\*f - a\*g)^3\*(f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))\*Log[c + d\*x] - 2\*(b\*c - a\*d)\*g\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(-3\*d\*f + c\*g) + b^2\*(3\*d^2\*f^2 - 3\*c\*d\*f\*g + c^2\*g^2))\*(f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))\*Log[f + g\*x] - 2\*B\*(b\*c - a\*d)\*

$$g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*(b*(d*f - c*g)*\text{Log}[a + b*x] + (-(b*d*f) + a*d*g)*\text{Log}[c + d*x] + (b*c - a*d)*g*\text{Log}[f + g*x]) + B*(b*c - a*d)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g) - b^2*(d*f - c*g)^2*(f + g*x)*\text{Log}[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)*\text{Log}[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*\text{Log}[f + g*x]) + b^3*B*(d*f - c*g)^3*(f + g*x)^2*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]) - B*d^3*(b*f - a*g)^3*(f + g*x)^2*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)]) + 2*B*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(f + g*x)^2*((\text{Log}[g*(a + b*x))/(-(b*f) + a*g)] - \text{Log}[g*(c + d*x))/(-(d*f) + c*g))*\text{Log}[f + g*x] + \text{PolyLog}[2, (b*(f + g*x))/(b*f - a*g)] - \text{PolyLog}[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)^3*(d*f - c*g)^3)/(3*g*(f + g*x)^3)$$

**Maple [F]** time = 4.183, size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^4} \left( A + B \ln \left( \frac{e^{(bx + a)}}{dx + c} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^4,x)

[Out] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^4,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^4,x, algorithm="maxima")

[Out]  $\frac{1}{3}*(2*b^3*\log(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3*g^4) - 2*d^3*\log(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g + (b^3*c^3 - a^3*d^3)*g^2)*\log(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5) - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2*c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2*d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 + a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2*f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - 2*\log(b*e*x/(d*x + c) + a*e/(d*x + c))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)*A*B - 1/3*B^2*(\log(d*x + c)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*\integrate(-1/3*(3*d*g*x*\log(e)^2 + 3*c*g*\log(e)^2 + 3*(d*g*x + c*g)*\log(b*x + a)^2 + 6*(d*g*x*\log(e) + c*g*\log(e))*\log(b*x + a) - 2*((3*g*\log(e) - g)*d*x + 3*c*g*\log(e) - d*f + 3*(d*g*x + c*g)*\log(b*x + a))*\log(d*x + c))/(d*g^5*x^5 + c*f^4*g + (4*d*f*g^4 + c*g^5)*x^4 + 2*(3*d*f^2*g^3 + 2*c*f*g^4)*x^3 + 2*(2*d*f^3*$

$$g^2 + 3cf^2g^3)x^2 + (df^4g + 4cf^3g^2)x, x) - 1/3A^2/(g^4x^3 + 3f^3g^3x^2 + 3f^2g^2x + f^3g)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2AB \log\left(\frac{bex+ae}{dx+c}\right) + A^2}{g^4x^4 + 4fg^3x^3 + 6f^2g^2x^2 + 4f^3gx + f^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^4,x, algorithm="fricas")

[Out] integral((B^2\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*A\*B\*log((b\*e\*x + a\*e)/(d\*x + c)) + A^2)/(g^4\*x^4 + 4\*f\*g^3\*x^3 + 6\*f^2\*g^2\*x^2 + 4\*f^3\*g\*x + f^4), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2/(g\*x+f)\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{(gx+f)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^4,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2/(g\*x + f)^4, x)



$$3.248 \quad \int \frac{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5} dx$$

**Optimal.** Leaf size=1159

result too large to display

```
[Out] -(B^2*(b*c - a*d)^2*g^3*(c + d*x)^2)/(12*(b*f - a*g)^2*(d*f - c*g)^4*(f + g*x)^2) - (B^2*(b*c - a*d)^3*g^3*(c + d*x))/(6*(b*f - a*g)^3*(d*f - c*g)^4*(f + g*x)) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*(c + d*x))/(4*(b*f - a*g)^3*(d*f - c*g)^4*(f + g*x)) - (B^2*(b*c - a*d)^4*g^3*Log[(a + b*x)/(c + d*x)])/(6*(b*f - a*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*Log[(a + b*x)/(c + d*x)])/(4*(b*f - a*g)^4*(d*f - c*g)^4) + (B*(b*c - a*d)*g^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*(b*f - a*g)*(d*f - c*g)^4*(f + g*x)^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(b*f - a*g)^2*(d*f - c*g)^4*(f + g*x)^2) + (B*(b*c - a*d)*g*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*f - a*g)^4*(d*f - c*g)^3*(f + g*x)) + (b^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(4*g*(f + g*x)^4) + (B^2*(b*c - a*d)^4*g^3*Log[(f + g*x)/(c + d*x)])/(6*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*Log[(f + g*x)/(c + d*x)])/(4*(b*f - a*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^2*g*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[(f + g*x)/(c + d*x)])/(2*(b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(2*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/(2*(b*f - a*g)^4*(d*f - c*g)^4)
```

---

**Rubi [A]** time = 3.4031, antiderivative size = 1881, normalized size of antiderivative = 1.62, number of steps used = 44, number of rules used = 11, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.379$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(f + g*x)^5,x]
```

```
[Out] -(B^2*(b*c - a*d)^2*g)/(12*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (5*B^2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g))/(12*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^3*B^2*(b*c - a*d)*Log[a + b*x])/(6*(b*f - a*g)^4*(d*f - c*g)) + (b^2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[a + b*x])/(4*(b*f - a*g)^4*(d*f - c*g)^2) + (b*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[a + b*x])/(2*(b*f - a*g)^4*(d*f - c*g)^3) - (b^4*B^2*Log[a + b*x]^2)/(4*g*(b*f - a*g)^4) - (B*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*
```

$$\begin{aligned}
& g*(b*f - a*g)^4 - (A + B*\text{Log}[(e*(a + b*x))/(c + d*x)])^2/(4*g*(f + g*x)^4) \\
& - (B^2*d^3*(b*c - a*d)*\text{Log}[c + d*x])/(6*(b*f - a*g)*(d*f - c*g)^4) - (B^2*d^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*\text{Log}[c + d*x])/(4*(b*f - a*g)^2*(d*f - c*g)^4) \\
& - (B^2*d*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*\text{Log}[c + d*x])/(2*(b*f - a*g)^3*(d*f - c*g)^4) \\
& + (B^2*d^4*\text{Log}[-((d*(a + b*x))/(b*c - a*d))]*\text{Log}[c + d*x])/(2*g*(d*f - c*g)^4) - (B*d^4*(A + B*\text{Log}[(e*(a + b*x))/(c + d*x)])*\text{Log}[c + d*x])/(2*g*(d*f - c*g)^4) \\
& - (B^2*d^4*\text{Log}[c + d*x]^2)/(4*g*(d*f - c*g)^4) + (b^4*B^2*\text{Log}[a + b*x]*\text{Log}[(b*(c + d*x))/(b*c - a*d)])/(2*g*(b*f - a*g)^4) + (B^2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g)^2*\text{Log}[f + g*x])/(4*(b*f - a*g)^4*(d*f - c*g)^4) \\
& + (2*B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*\text{Log}[f + g*x])/(3*(b*f - a*g)^4*(d*f - c*g)^4) \\
& + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*\text{Log}[-((g*(a + b*x))/(b*f - a*g))]*\text{Log}[f + g*x])/(2*(b*f - a*g)^4*(d*f - c*g)^4) \\
& - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*\text{Log}[-((g*(a + b*x))/(b*f - a*g))]*\text{Log}[f + g*x])/(2*(b*f - a*g)^4*(d*f - c*g)^4) \\
& - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*\text{Log}[-((g*(c + d*x))/(d*f - c*g))]*\text{Log}[f + g*x])/(2*(b*f - a*g)^4*(d*f - c*g)^4) \\
& + (b^4*B^2*\text{PolyLog}[2, -((d*(a + b*x))/(b*c - a*d))])/(2*g*(b*f - a*g)^4) + (B^2*d^4*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(2*g*(d*f - c*g)^4) \\
& + (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*\text{PolyLog}[2, (b*(f + g*x))/(b*f - a*g)])/(2*(b*f - a*g)^4*(d*f - c*g)^4) \\
& - (B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*\text{PolyLog}[2, (d*(f + g*x))/(d*f - c*g)])/(2*(b*f - a*g)^4*(d*f - c*g)^4)
\end{aligned}$$

### Rule 2525

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

```

### Rule 12

```

Int[(a_)*(u_), x_Symbol]
:> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

```

### Rule 2528

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol]
:> With[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

```

### Rule 2524

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

```

### Rule 2418

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy

```

```
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFX, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFX, x] && IntegerQ[p]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(f+gx)^5} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4} + \frac{B \int \frac{(bc-ad)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)(c+dx)(f+gx)^4} dx}{2g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad)) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{(a+bx)(c+dx)(f+gx)^4} dx}{2g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad)) \int \left(\frac{b^5\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)(-df+cg)^4(c+dx)} + \frac{g^2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^4}\right) dx}{2g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4g(f+gx)^4} + \frac{(b^5B) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx} dx}{2g(bf-ag)^4} - \frac{(Bd^5) \int \frac{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx} dx}{2g(df-cg)^4} + \frac{(B(bc-ad)) \int \frac{g^2\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bf-ag)^4} dx}{2g} \\
&= \frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&= \frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&= \frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&= -\frac{B^2(bc-ad)^2g}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3B^2(bc-ad) \log\left(\frac{e(a+bx)}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4} \\
&= -\frac{B^2(bc-ad)^2g}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3B^2(bc-ad) \log\left(\frac{e(a+bx)}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4} \\
&= -\frac{B^2(bc-ad)^2g}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3B^2(bc-ad) \log\left(\frac{e(a+bx)}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4} \\
&= -\frac{B^2(bc-ad)^2g}{12(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{12(bf-ag)^3(df-cg)^3(f+gx)} + \frac{b^3B^2(bc-ad) \log\left(\frac{e(a+bx)}{c+dx}\right)}{6(bf-ag)^4(df-cg)^4}
\end{aligned}$$

**Mathematica [A]** time = 7.31149, size = 1448, normalized size = 1.25

$$B(bc-ad) \left( \frac{\log(a+bx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^4}{(bc-ad)(bf-ag)^4} - \frac{B\left(\log^2(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\log(a+bx)-2\text{PolyLog}\left(2,-\frac{d(a+bx)}{bc-ad}\right)\right)b^4}{2(bc-ad)(bf-ag)^4} - \frac{g\left(\left(3d^2f^2-3cdgf+c^2g^2\right)b^2-adg(3df-cg)\right)}{(bf-ag)^3(df-cg)^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2/(f + g\*x)^5,x]

[Out] -(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2/(4\*g\*(f + g\*x)^4) + (B\*(b\*c - a\*d)\*(-g\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]))/(3\*(b\*f - a\*g)\*(d\*f - c\*g)\*(f + g\*x)^3) - (g\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]))/(2\*(b\*f - a\*g)^2\*(d\*f - c\*g)^2\*(f + g\*x)^2) - (g\*(a^2\*d^2\*g^2 - a\*b\*d\*g\*(3\*d\*f - c\*g) + b^2\*(3\*d^2\*f^2 - 3\*c\*d\*f\*g + c^2\*g^2))\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])/((b\*f - a\*g)^3\*(d\*f - c\*g)^3\*(f + g\*x)) + (b^4\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])/((b\*c - a\*d)\*(b\*f - a\*g)^4) - (d^4\*

$$\begin{aligned} & (A + B \cdot \text{Log}[(e \cdot (a + b \cdot x)) / (c + d \cdot x)]) \cdot \text{Log}[c + d \cdot x] / ((b \cdot c - a \cdot d) \cdot (d \cdot f - c \cdot g)^4) \\ & + (g \cdot (2 \cdot b \cdot d \cdot f - b \cdot c \cdot g - a \cdot d \cdot g) \cdot (2 \cdot b^2 \cdot d^2 \cdot f^2 - 2 \cdot b^2 \cdot c \cdot d \cdot f \cdot g - 2 \cdot a \cdot b \cdot d^2 \cdot f \cdot g \\ & + b^2 \cdot c^2 \cdot g^2 + a^2 \cdot d^2 \cdot g^2) \cdot (A + B \cdot \text{Log}[(e \cdot (a + b \cdot x)) / (c + d \cdot x)]) \cdot \text{Log}[f + g \cdot x]) / ((b \cdot f - a \cdot g)^4 \cdot (d \cdot f - c \cdot g)^4) \\ & + (B \cdot (b \cdot c - a \cdot d) \cdot g \cdot (a^2 \cdot d^2 \cdot g^2 - a \cdot b \cdot d \cdot g \cdot (3 \cdot d \cdot f - c \cdot g) + b^2 \cdot (3 \cdot d^2 \cdot f^2 - 3 \cdot c \cdot d \cdot f \cdot g + c^2 \cdot g^2)) \cdot ((b \cdot \text{Log}[a + b \cdot x]) / ((b \cdot c - a \cdot d) \cdot (b \cdot f - a \cdot g)) - (d \cdot \text{Log}[c + d \cdot x]) / ((b \cdot c - a \cdot d) \cdot (d \cdot f - c \cdot g))) + (g \cdot \text{Log}[f + g \cdot x]) / ((b \cdot f - a \cdot g) \cdot (d \cdot f - c \cdot g))) / ((b \cdot f - a \cdot g)^3 \cdot (d \cdot f - c \cdot g)^3) \\ & - (B \cdot (b \cdot c - a \cdot d) \cdot g \cdot (2 \cdot b \cdot d \cdot f - b \cdot c \cdot g - a \cdot d \cdot g) \cdot (g / ((b \cdot f - a \cdot g) \cdot (d \cdot f - c \cdot g)) \cdot (f + g \cdot x)) - (b^2 \cdot \text{Log}[a + b \cdot x]) / ((b \cdot c - a \cdot d) \cdot (b \cdot f - a \cdot g)^2) + (d^2 \cdot \text{Log}[c + d \cdot x]) / ((b \cdot c - a \cdot d) \cdot (d \cdot f - c \cdot g)^2) - (g \cdot (2 \cdot b \cdot d \cdot f - b \cdot c \cdot g - a \cdot d \cdot g) \cdot \text{Log}[f + g \cdot x]) / ((b \cdot f - a \cdot g)^2 \cdot (d \cdot f - c \cdot g)^2)) / (2 \cdot (b \cdot f - a \cdot g)^2 \cdot (d \cdot f - c \cdot g)^2) \\ & - (B \cdot (b \cdot c - a \cdot d) \cdot g \cdot (g / ((b \cdot f - a \cdot g) \cdot (d \cdot f - c \cdot g)) \cdot (f + g \cdot x)^2) + (2 \cdot g \cdot (2 \cdot b \cdot d \cdot f - b \cdot c \cdot g - a \cdot d \cdot g)) / ((b \cdot f - a \cdot g)^2 \cdot (d \cdot f - c \cdot g)^2 \cdot (f + g \cdot x)) - (2 \cdot b^3 \cdot \text{Log}[a + b \cdot x]) / ((b \cdot c - a \cdot d) \cdot (b \cdot f - a \cdot g)^3) + (2 \cdot d^3 \cdot \text{Log}[c + d \cdot x]) / ((b \cdot c - a \cdot d) \cdot (d \cdot f - c \cdot g)^3) \\ & - (2 \cdot g \cdot (a^2 \cdot d^2 \cdot g^2 - a \cdot b \cdot d \cdot g \cdot (3 \cdot d \cdot f - c \cdot g) + b^2 \cdot (3 \cdot d^2 \cdot f^2 - 3 \cdot c \cdot d \cdot f \cdot g + c^2 \cdot g^2)) \cdot \text{Log}[f + g \cdot x]) / ((b \cdot f - a \cdot g)^3 \cdot (d \cdot f - c \cdot g)^3)) / (6 \cdot (b \cdot f - a \cdot g) \cdot (d \cdot f - c \cdot g)) \\ & - (b^4 \cdot B \cdot (\text{Log}[a + b \cdot x]^2 - 2 \cdot \text{Log}[a + b \cdot x] \cdot \text{Log}[(b \cdot (c + d \cdot x)) / (b \cdot c - a \cdot d)] - 2 \cdot \text{PolyLog}[2, -((d \cdot (a + b \cdot x)) / (b \cdot c - a \cdot d))]) / (2 \cdot (b \cdot c - a \cdot d) \cdot (b \cdot f - a \cdot g)^4) \\ & + (B \cdot d^4 \cdot (2 \cdot \text{Log}[-((d \cdot (a + b \cdot x)) / (b \cdot c - a \cdot d))] \cdot \text{Log}[c + d \cdot x] - \text{Log}[c + d \cdot x]^2 + 2 \cdot \text{PolyLog}[2, (b \cdot (c + d \cdot x)) / (b \cdot c - a \cdot d)])) / (2 \cdot (b \cdot c - a \cdot d) \cdot (d \cdot f - c \cdot g)^4) \\ & - (B \cdot g \cdot (2 \cdot b \cdot d \cdot f - b \cdot c \cdot g - a \cdot d \cdot g) \cdot (2 \cdot b^2 \cdot d^2 \cdot f^2 - 2 \cdot b^2 \cdot c \cdot d \cdot f \cdot g - 2 \cdot a \cdot b \cdot d^2 \cdot f \cdot g + b^2 \cdot c^2 \cdot g^2 + a^2 \cdot d^2 \cdot g^2) \cdot (\text{Log}[-((g \cdot (a + b \cdot x)) / (b \cdot f - a \cdot g))] \cdot \text{Log}[f + g \cdot x] - \text{Log}[-((g \cdot (c + d \cdot x)) / (d \cdot f - c \cdot g))] \cdot \text{Log}[f + g \cdot x] + \text{PolyLog}[2, (b \cdot (f + g \cdot x)) / (b \cdot f - a \cdot g)] - \text{PolyLog}[2, (d \cdot (f + g \cdot x)) / (d \cdot f - c \cdot g)])) / ((b \cdot f - a \cdot g)^4 \cdot (d \cdot f - c \cdot g)^4)) / (2 \cdot g) \end{aligned}$$

**Maple [F]** time = 6.561, size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^5} \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^5,x)

[Out] int((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^5,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^5,x, algorithm="maxima")

[Out]  $\frac{1}{12} \cdot (6 \cdot b^4 \cdot \log(b \cdot x + a) / (b^4 \cdot f^4 \cdot g - 4 \cdot a \cdot b^3 \cdot f^3 \cdot g^2 + 6 \cdot a^2 \cdot b^2 \cdot f^2 \cdot g^3 - 4 \cdot a^3 \cdot b \cdot f \cdot g^4 + a^4 \cdot g^5) - 6 \cdot d^4 \cdot \log(d \cdot x + c) / (d^4 \cdot f^4 \cdot g - 4 \cdot c \cdot d^3 \cdot f^3 \cdot g^2 + 6 \cdot c^2 \cdot d^2 \cdot f^2 \cdot g^3 - 4 \cdot c^3 \cdot d \cdot f \cdot g^4 + c^4 \cdot g^5) + 6 \cdot (4 \cdot (b^4 \cdot c \cdot d^3 - a \cdot b^3 \cdot d^4) \cdot f^3 - 6 \cdot (b^4 \cdot c^2 \cdot d^2 - a^2 \cdot b^2 \cdot d^4) \cdot f^2 \cdot g + 4 \cdot (b^4 \cdot c^3 \cdot d - a^3 \cdot b \cdot d^4) \cdot f \cdot g^2 - (b^4 \cdot c^4 - a^4 \cdot d^4) \cdot g^3) \cdot \log(g \cdot x + f) / (b^4 \cdot d^4 \cdot f^8 + a^4 \cdot c^4 \cdot g^8 - 4 \cdot (b^4 \cdot c \cdot d^3 + a \cdot b^3 \cdot d^4) \cdot f^7 \cdot g + 2 \cdot (3 \cdot b^4 \cdot c^2 \cdot d^2 + 8 \cdot a \cdot b^3 \cdot c \cdot d^3 + 3 \cdot a^2 \cdot b^2 \cdot d^4) \cdot f^6 \cdot g^2 - 4 \cdot (b^4 \cdot c^3 \cdot d + 6 \cdot a \cdot b^3 \cdot c^2 \cdot d^2 + 6 \cdot a^2 \cdot b^2 \cdot c \cdot d^3 + a^3 \cdot b \cdot d^4) \cdot f^5 \cdot g^3 + (b^4 \cdot c^4 + 16 \cdot a \cdot b^3 \cdot c^3 \cdot d + 36 \cdot a^2 \cdot b^2 \cdot c^2 \cdot d^2 + 16 \cdot a^3 \cdot b \cdot c \cdot d^3 + a^4 \cdot d^4) \cdot f^4 \cdot g^4 - 4 \cdot (a \cdot b^3 \cdot c^4 + 6 \cdot a^2 \cdot b^2 \cdot c^3 \cdot d + 6 \cdot a^3 \cdot b \cdot c^2 \cdot d^2 +$

$$\begin{aligned}
& a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2 \\
& *g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 - \\
& 31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b \\
& *c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c \\
& ^3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d - \\
& a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2 \\
& *d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2 \\
& *d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^ \\
& 3*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*d \\
& + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b \\
& *c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g \\
& ^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3 \\
& *(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3 \\
& )*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3 \\
& *(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d \\
& )*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^ \\
& 3)*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 + \\
& 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b* \\
& c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^ \\
& 3*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3* \\
& c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a \\
& ^2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f \\
& ^4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 6*log(b*e*x/(d*x + c) + a \\
& e/(d*x + c))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g) \\
& )*A*B - 1/4*B^2*(log(d*x + c))^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f \\
& ^3*g^2*x + f^4*g) + 4*integrate(-1/2*(2*d*g*x*log(e)^2 + 2*c*g*log(e)^2 + 2 \\
& *(d*g*x + c*g)*log(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) \\
& - ((4*g*log(e) - g)*d*x + 4*c*g*log(e) - d*f + 4*(d*g*x + c*g)*log(b*x + a) \\
& )*log(d*x + c))/(d*g^6*x^6 + c*f^5*g + (5*d*f*g^5 + c*g^6)*x^5 + 5*(2*d*f^2 \\
& *g^4 + c*f*g^5)*x^4 + 10*(d*f^3*g^3 + c*f^2*g^4)*x^3 + 5*(d*f^4*g^2 + 2*c*f \\
& ^3*g^3)*x^2 + (d*f^5*g + 5*c*f^4*g^2)*x), x) - 1/4*A^2/(g^5*x^4 + 4*f*g^4* \\
& x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
\end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2AB \log\left(\frac{bex+ae}{dx+c}\right) + A^2}{g^5x^5 + 5fg^4x^4 + 10f^2g^3x^3 + 10f^3g^2x^2 + 5f^4gx + f^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^5,x, algorithm="fricas")

[Out] integral((B^2\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*A\*B\*log((b\*e\*x + a\*e)/(d\*x + c)) + A^2)/(g^5\*x^5 + 5\*f\*g^4\*x^4 + 10\*f^2\*g^3\*x^3 + 10\*f^3\*g^2\*x^2 + 5\*f^4\*g\*x + f^5), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2/(g\*x+f)\*\*5,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{(gx+f)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2/(g\*x+f)^5,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2/(g\*x + f)^5, x)

$$3.249 \quad \int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2} dx$$

**Optimal.** Leaf size=35

$$2 \log\left(-\frac{x}{1-x}\right) - \frac{(x+1) \log\left(-\frac{x+1}{1-x}\right)}{x}$$

[Out] 2\*Log[-(x/(1 - x))] - ((1 + x)\*Log[-((1 + x)/(1 - x))])/x

**Rubi [A]** time = 0.0142757, antiderivative size = 34, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 14,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {2490, 36, 29, 31}

$$2 \log(x) - 2 \log(x+1) - \frac{(1-x) \log\left(-\frac{x+1}{1-x}\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[Log[(1 + x)/(-1 + x)]/x^2,x]

[Out] 2\*Log[x] - 2\*Log[1 + x] - ((1 - x)\*Log[-((1 + x)/(1 - x))])/x

#### Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_))^2, x_Symbol] := Simp[((a + b*x)*Log[e*(f
*(a + b*x)^p*(c + d*x)^q]^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(
b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(
(c + d*x)*(g + h*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s},
x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0
]
```

#### Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rubi steps



$$\begin{aligned}
\int \frac{\log\left(\frac{1+x}{-1+x}\right)}{x^2} dx &= -\frac{(1-x)\log\left(\frac{-1+x}{1-x}\right)}{x} + 2 \int \frac{1}{x(1+x)} dx \\
&= -\frac{(1-x)\log\left(\frac{-1+x}{1-x}\right)}{x} + 2 \int \frac{1}{x} dx - 2 \int \frac{1}{1+x} dx \\
&= 2 \log(x) - 2 \log(1+x) - \frac{(1-x)\log\left(\frac{-1+x}{1-x}\right)}{x}
\end{aligned}$$

**Mathematica [A]** time = 0.0053786, size = 30, normalized size = 0.86

$$-\log(1-x^2) + 2 \log(x) - \frac{\log\left(\frac{x+1}{x-1}\right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Log[(1 + x)/(-1 + x)]/x^2,x]

[Out] 2\*Log[x] - Log[(1 + x)/(-1 + x)]/x - Log[1 - x^2]

**Maple [A]** time = 0.131, size = 46, normalized size = 1.3

$$2 \ln\left(2(x-1)^{-1}+2\right) - 2 \ln\left(1+2(x-1)^{-1}\right)\left(1+2(x-1)^{-1}\right)\left(2(x-1)^{-1}+2\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(ln((1+x)/(x-1))/x^2,x)

[Out] 2\*ln(2/(x-1)+2)-2\*ln(1+2/(x-1))\*(1+2/(x-1))/(2/(x-1)+2)

**Maxima [A]** time = 1.12857, size = 43, normalized size = 1.23

$$-\frac{\log\left(\frac{x+1}{x-1}\right)}{x} - \log(x+1) - \log(x-1) + 2 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(log((1+x)/(-1+x))/x^2,x, algorithm="maxima")

[Out] -log((x + 1)/(x - 1))/x - log(x + 1) - log(x - 1) + 2\*log(x)

**Fricas [A]** time = 0.947874, size = 77, normalized size = 2.2

$$-\frac{x \log(x^2 - 1) - 2x \log(x) + \log\left(\frac{x+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1+x)/(-1+x))/x^2,x, algorithm="fricas")
```

```
[Out] -(x*log(x^2 - 1) - 2*x*log(x) + log((x + 1)/(x - 1)))/x
```

**Sympy [A]** time = 0.140983, size = 20, normalized size = 0.57

$$2 \log(x) - \log(x^2 - 1) - \frac{\log\left(\frac{x+1}{x-1}\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(ln((1+x)/(-1+x))/x**2,x)
```

```
[Out] 2*log(x) - log(x**2 - 1) - log((x + 1)/(x - 1))/x
```

**Giac [A]** time = 1.26509, size = 39, normalized size = 1.11

$$-\frac{\log\left(\frac{x+1}{x-1}\right)}{x} + \log(x^2) - \log(|x^2 - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(log((1+x)/(-1+x))/x^2,x, algorithm="giac")
```

```
[Out] -log((x + 1)/(x - 1))/x + log(x^2) - log(abs(x^2 - 1))
```

$$3.250 \quad \int \frac{(f+gx)^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{(f+gx)^2}{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}, x\right)$$

[Out] Unintegrable[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

**Rubi [A]** time = 0.168348, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(f+gx)^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

[Out] f^2\*Defer[Int][(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]^(-1), x] + 2\*f\*g\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x] + g^2\*Defer[Int][x^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

Rubi steps

$$\begin{aligned} \int \frac{(f+gx)^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx &= \int \left( \frac{f^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} + \frac{2fgx}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} + \frac{g^2x^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} \right) dx \\ &= f^2 \int \frac{1}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx + (2fg) \int \frac{x}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx + g^2 \int \frac{x^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.420802, size = 0, normalized size = 0.

$$\int \frac{(f+gx)^2}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

[Out] Integrate[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

**Maple [A]** time = 1.279, size = 0, normalized size = 0.

$$\int (gx+f)^2 \left( A + B \ln\left(\frac{e(bx+a)}{dx+c}\right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] `int((g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^2}{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")`

[Out] `integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{g^2x^2 + 2fgx + f^2}{B \log\left(\frac{bex+ae}{dx+c}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")`

[Out] `integral((g^2*x^2 + 2*f*g*x + f^2)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^2}{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")`

```
[Out] integrate((g*x + f)^2/(B*log((b*x + a)*e/(d*x + c)) + A), x)
```

$$3.251 \quad \int \frac{f+gx}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

**Optimal.** Leaf size=29

$$\text{Unintegrable}\left(\frac{f+gx}{B \log\left(\frac{e(a+bx)}{c+dx}\right)+A}, x\right)$$

[Out] Unintegrable[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

**Rubi [A]** time = 0.0903596, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f+gx}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

[Out] f\*Defer[Int][(A + B\*Log[(e\*(a + b\*x))/(c + d\*x))]^(-1), x] + g\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

Rubi steps

$$\begin{aligned} \int \frac{f+gx}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx &= \int \left( \frac{f}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} + \frac{gx}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} \right) dx \\ &= f \int \frac{1}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx + g \int \frac{x}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.244547, size = 0, normalized size = 0.

$$\int \frac{f+gx}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

[Out] Integrate[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)]), x]

**Maple [A]** time = 1.067, size = 0, normalized size = 0.

$$\int (gx + f) \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] `int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")`

[Out] `integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{gx + f}{B \log\left(\frac{bex+ae}{dx+c}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")`

[Out] `integral((g*x + f)/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")`

[Out] `integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A), x)`

$$3.252 \quad \int \frac{1}{A+B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{B \log\left(\frac{e(a+bx)}{c+dx}\right) + A}, x\right)$$

[Out] Unintegrable[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^(-1), x]

**Rubi [A]** time = 0.0136068, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^(-1), x]

[Out] Defer[Int] [(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^(-1), x]

Rubi steps

$$\int \frac{1}{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx = \int \frac{1}{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

**Mathematica [A]** time = 0.0231281, size = 0, normalized size = 0.

$$\int \frac{1}{A + B \log\left(\frac{e(a+bx)}{c+dx}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^(-1), x]

[Out] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^(-1), x]

**Maple [A]** time = 1.074, size = 0, normalized size = 0.

$$\int \left(A + B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(A+B\*ln(e\*(b\*x+a)/(d\*x+c))), x)



[Out] `int(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")`

[Out] `integrate(1/(B*log((b*x + a)*e/(d*x + c)) + A), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{B \log\left(\frac{bex+ae}{dx+c}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")`

[Out] `integral(1/(B*log((b*e*x + a*e)/(d*x + c)) + A), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{B \log\left(\frac{(bx+a)e}{dx+c}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")`

[Out] `integrate(1/(B*log((b*x + a)*e/(d*x + c)) + A), x)`

$$3.253 \quad \int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{1}{(f+gx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}, x\right)$$

[Out] Unintegrable[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))), x]

**Rubi [A]** time = 0.0650778, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])), x]

[Out] Defer[Int][1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])), x]

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)} dx = \int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)} dx$$

**Mathematica [A]** time = 0.905712, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])), x]

[Out] Integrate[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])), x]

**Maple [A]** time = 1.409, size = 0, normalized size = 0.

$$\int \frac{1}{gx+f}\left(A+B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(g\*x+f)/(A+B\*ln(e\*(b\*x+a)/(d\*x+c))), x)

[Out] `int(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")`

[Out] `integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{Agx + Af + (Bgx + Bf) \log\left(\frac{bex+ae}{dx+c}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")`

[Out] `integral(1/(A*g*x + A*f + (B*g*x + B*f)*log((b*e*x + a*e)/(d*x + c))), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")`

[Out] `integrate(1/((g*x + f)*(B*log((b*x + a)*e/(d*x + c)) + A)), x)`

$$3.254 \quad \int \frac{1}{(f+gx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{1}{(f+gx)^2 \left( B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}, x\right)$$

[Out] Unintegrable[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])), x]

**Rubi [A]** time = 0.065483, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])), x]

[Out] Defer[Int][1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])), x]

Rubi steps

$$\int \frac{1}{(f+gx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx = \int \frac{1}{(f+gx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

**Mathematica [A]** time = 0.883091, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])), x]

[Out] Integrate[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])), x]

**Maple [A]** time = 1.366, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^2 \left( A+B \ln\left(\frac{e(bx+a)}{dx+c}\right) \right)^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(g\*x+f)^2/(A+B\*ln(e\*(b\*x+a)/(d\*x+c))), x)

[Out] `int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^2 \left( B \log\left(\frac{(bx+a)e}{dx+c}\right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")`

[Out] `integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{Ag^2x^2 + 2Afgx + Af^2 + (Bg^2x^2 + 2Bfgx + Bf^2) \log\left(\frac{bex+ae}{dx+c}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")`

[Out] `integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2)*log((b*e*x + a*e)/(d*x + c))), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^2 \left( B \log\left(\frac{(bx+a)e}{dx+c}\right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")`

[Out] `integrate(1/((g*x + f)^2*(B*log((b*x + a)*e/(d*x + c)) + A)), x)`

$$3.255 \quad \int \frac{1}{(f+gx)^3 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{1}{(f+gx)^3 \left( B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)}, x\right)$$

[Out] Unintegrable[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])), x]

**Rubi [A]** time = 0.0614351, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)^3 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])), x]

[Out] Defer[Int][1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])), x]

Rubi steps

$$\int \frac{1}{(f+gx)^3 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx = \int \frac{1}{(f+gx)^3 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

**Mathematica [A]** time = 8.46788, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)^3 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])), x]

[Out] Integrate[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]])), x]

**Maple [A]** time = 1.424, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^3 \left( A+B \ln\left(\frac{e(bx+a)}{dx+c}\right) \right)^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(g\*x+f)^3/(A+B\*ln(e\*(b\*x+a)/(d\*x+c))), x)

[Out] `int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^3 \left( B \log\left(\frac{(bx+a)e}{dx+c}\right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")`

[Out] `integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{Ag^3x^3 + 3Afg^2x^2 + 3Af^2gx + Af^3 + (Bg^3x^3 + 3Bfg^2x^2 + 3Bf^2gx + Bf^3)\log\left(\frac{bex+ae}{dx+c}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")`

[Out] `integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 + 3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log((b*e*x + a*e)/(d*x + c))), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)/(d*x+c))),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^3 \left( B \log\left(\frac{(bx+a)e}{dx+c}\right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")`

[Out] `integrate(1/((g*x + f)^3*(B*log((b*x + a)*e/(d*x + c)) + A)), x)`

$$3.256 \quad \int \frac{(f+gx)^2}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{(f+gx)^2}{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2, x]

**Rubi [A]** time = 0.184068, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(f+gx)^2}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2,x]

[Out] f^2\*Defer[Int][(A + B\*Log[(e\*(a + b\*x))/(c + d\*x))]^(-2), x] + 2\*f\*g\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x))]^2, x] + g^2\*Defer[Int][x^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x))]^2, x]

Rubi steps

$$\begin{aligned} \int \frac{(f+gx)^2}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx &= \int \left( \frac{f^2}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} + \frac{2fgx}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} + \frac{g^2x^2}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} \right) dx \\ &= f^2 \int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx + (2fg) \int \frac{x}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx + g^2 \int \frac{x^2}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 1.31223, size = 0, normalized size = 0.

$$\int \frac{(f+gx)^2}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2,x]

[Out] Integrate[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2, x]



**Maple [A]** time = 1.112, size = 0, normalized size = 0.

$$\int (gx + f)^2 \left( A + B \ln \left( \frac{e(bx + a)}{dx + c} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

[Out] int((g\*x+f)^2/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bdg^2x^4 + acf^2 + (adg^2 + (2dfg + cg^2)b)x^3 + ((2dfg + cg^2)a + (df^2 + 2cfcg)b)x^2 + (bcf^2 + (df^2 + 2cfcg)a)x}{(bc - ad)B^2 \log(bx + a) - (bc - ad)B^2 \log(dx + c) + (bc - ad)AB + (bc \log(e) - ad \log(e))B^2} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="maxima")

[Out]  $-(b*d*g^2*x^4 + a*c*f^2 + (a*d*g^2 + (2*d*f*g + c*g^2)*b)*x^3 + ((2*d*f*g + c*g^2)*a + (d*f^2 + 2*c*f*g)*b)*x^2 + (b*c*f^2 + (d*f^2 + 2*c*f*g)*a)*x / ((b*c - a*d)*B^2*\log(b*x + a) - (b*c - a*d)*B^2*\log(d*x + c) + (b*c - a*d)*A*B + (b*c*\log(e) - a*d*\log(e))*B^2) + \int$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{g^2x^2 + 2fgx + f^2}{B^2 \log \left( \frac{bex+ae}{dx+c} \right)^2 + 2AB \log \left( \frac{bex+ae}{dx+c} \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out] integral((g^2\*x^2 + 2\*f\*g\*x + f^2)/(B^2\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*A\*B\*log((b\*e\*x + a\*e)/(d\*x + c)) + A^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^2}{\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="giac")

[Out] integrate((g\*x + f)^2/(B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2, x)

$$3.257 \quad \int \frac{f+gx}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

**Optimal.** Leaf size=29

$$\text{Unintegrable}\left(\frac{f+gx}{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2, x]

**Rubi [A]** time = 0.0996355, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f+gx}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2, x]

[Out] f\*Defer[Int][(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^(-2), x] + g\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{f+gx}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx &= \int \left( \frac{f}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} + \frac{gx}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} \right) dx \\ &= f \int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx + g \int \frac{x}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.882346, size = 0, normalized size = 0.

$$\int \frac{f+gx}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2, x]

[Out] Integrate[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^2, x]

**Maple [A]** time = 1.115, size = 0, normalized size = 0.

$$\int (gx + f) \left( A + B \ln\left(\frac{e(bx + a)}{dx + c}\right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)`

[Out] `int((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bdgx^3 + acf + (adg + (df + cg)b)x^2 + (bcf + (df + cg)a)x}{(bc - ad)B^2 \log(bx + a) - (bc - ad)B^2 \log(dx + c) + (bc - ad)AB + (bc \log(e) - ad \log(e))B^2} + \int \frac{dx}{(bc - ad)B^2 \log(bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima")`

[Out] `-(b*d*g*x^3 + a*c*f + (a*d*g + (d*f + c*g)*b)*x^2 + (b*c*f + (d*f + c*g)*a)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2) + integrate((3*b*d*g*x^2 + b*c*f + (d*f + c*g)*a + 2*(a*d*g + (d*f + c*g)*b)*x)/((b*c - a*d)*B^2*log(b*x + a) - (b*c - a*d)*B^2*log(d*x + c) + (b*c - a*d)*A*B + (b*c*log(e) - a*d*log(e))*B^2), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{gx + f}{B^2 \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2AB \log\left(\frac{bex+ae}{dx+c}\right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas")`

[Out] `integral((g*x + f)/(B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*x + c)) + A^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{\left(B \log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

$$3.258 \quad \int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

**Optimal.** Leaf size=23

$$\text{Unintegrable}\left(\frac{1}{\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^(-2), x]

**Rubi [A]** time = 0.0141137, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^(-2), x]

[Out] Defer[Int][(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^(-2), x]

Rubi steps

$$\int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx = \int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

**Mathematica [A]** time = 0.478802, size = 0, normalized size = 0.

$$\int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^(-2), x]

[Out] Integrate[(A + B\*Log[(e\*(a + b\*x))/(c + d\*x)])^(-2), x]

**Maple [A]** time = 1.079, size = 0, normalized size = 0.

$$\int \left(A+B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

[Out] int(1/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx^2 + ac + (bc + ad)x}{(bc - ad)B^2 \log(bx + a) - (bc - ad)B^2 \log(dx + c) + (bc - ad)AB + (bc \log(e) - ad \log(e))B^2} + \int \frac{1}{(bc - ad)B^2 \log(bx + a) - (bc - ad)B^2 \log(dx + c) + (bc - ad)AB + (bc \log(e) - ad \log(e))B^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="maxima")

[Out] -(b\*d\*x^2 + a\*c + (b\*c + a\*d)\*x)/((b\*c - a\*d)\*B^2\*log(b\*x + a) - (b\*c - a\*d)\*B^2\*log(d\*x + c) + (b\*c - a\*d)\*A\*B + (b\*c\*log(e) - a\*d\*log(e))\*B^2) + integrate((2\*b\*d\*x + b\*c + a\*d)/((b\*c - a\*d)\*B^2\*log(b\*x + a) - (b\*c - a\*d)\*B^2\*log(d\*x + c) + (b\*c - a\*d)\*A\*B + (b\*c\*log(e) - a\*d\*log(e))\*B^2), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{B^2 \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2AB \log\left(\frac{bex+ae}{dx+c}\right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out] integral(1/(B^2\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*A\*B\*log((b\*e\*x + a\*e)/(d\*x + c)) + A^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(B \log\left(\frac{bx+a}{dx+c}\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)*e/(d*x + c)) + A)^(-2), x)
```



$$3.259 \quad \int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{1}{(f+gx)\left(B \log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

**Rubi [A]** time = 0.0696734, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

[Out] Defer[Int][1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx = \int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

**Mathematica [A]** time = 2.50478, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

[Out] Integrate[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

**Maple [A]** time = 1.425, size = 0, normalized size = 0.

$$\int \frac{1}{gx+f} \left(A+B \ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(g\*x+f)/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

[Out] int(1/(g\*x+f)/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx^2 + ac + (bc + ad)x}{(bcf - adf)AB + (bcf \log(e) - adf \log(e))B^2 + ((bcg - adg)AB + (bcg \log(e) - adg \log(e))B^2)x + ((bcg - adg)B^2x +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="maxima")

[Out]  $-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f - a*d*f)*A*B + (b*c*f*\log(e) - a*d*f*\log(e))*B^2 + ((b*c*g - a*d*g)*A*B + (b*c*g*\log(e) - a*d*g*\log(e))*B^2)*x + ((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*\log(b*x + a) - ((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*\log(d*x + c)) + \text{integrate}((b*d*g*x^2 + 2*b*d*f*x + b*c*f + (d*f - c*g)*a)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*\log(e) - a*d*f^2*\log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*\log(e) - a*d*g^2*\log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*\log(e) - a*d*f*g*\log(e))*B^2)*x + ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*\log(b*x + a) - ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*\log(d*x + c)), x$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2gx + A^2f + (B^2gx + B^2f) \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2(ABgx + ABf) \log\left(\frac{bex+ae}{dx+c}\right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out] integral(1/(A^2\*g\*x + A^2\*f + (B^2\*g\*x + B^2\*f)\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*(A\*B\*g\*x + A\*B\*f)\*log((b\*e\*x + a\*e)/(d\*x + c))), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f) \left( B \log \left( \frac{(bx+a)e}{dx+c} \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="giac")

[Out] integrate(1/((g\*x + f)\*(B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2), x)

$$3.260 \quad \int \frac{1}{(f+gx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable} \left( \frac{1}{(f+gx)^2 \left( B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)^2}, x \right)$$

[Out] Unintegrable[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

**Rubi [A]** time = 0.0747316, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

[Out] Defer[Int][1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

Rubi steps

$$\int \frac{1}{(f+gx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx = \int \frac{1}{(f+gx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx$$

**Mathematica [A]** time = 4.41417, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)^2 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

[Out] Integrate[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

**Maple [A]** time = 1.58, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^2 \left( A+B \ln\left(\frac{e(bx+a)}{dx+c}\right) \right)^{-2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(g\*x+f)^2/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

[Out] int(1/(g\*x+f)^2/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)^2/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="maxima")

[Out] 
$$-(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*\log(e) - a*d*f^2*\log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*\log(e) - a*d*g^2*\log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*\log(e) - a*d*f*g*\log(e))*B^2)*x + ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*\log(b*x + a) - ((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*\log(d*x + c) - \int \text{egrate}(- (b*c*f + (d*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*x)/((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*\log(e) - a*d*g^3*\log(e))*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*\log(e) - a*d*f^3*\log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*\log(e) - a*d*f*g^2*\log(e))*B^2)*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*g*\log(e) - a*d*f^2*g*\log(e))*B^2)*x + ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*\log(b*x + a) - ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*\log(d*x + c), x)$$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

integral 
$$\left( \frac{1}{A^2g^2x^2 + 2A^2fgx + A^2f^2 + (B^2g^2x^2 + 2B^2fgx + B^2f^2)\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2(ABg^2x^2 + 2ABfgx + ABf^2)\log\left(\frac{bex+ae}{dx+c}\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)^2/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out] integral(1/(A^2\*g^2\*x^2 + 2\*A^2\*f\*g\*x + A^2\*f^2 + (B^2\*g^2\*x^2 + 2\*B^2\*f\*g\*x + B^2\*f^2)\*log((b\*e\*x + a\*e)/(d\*x + c))^2 + 2\*(A\*B\*g^2\*x^2 + 2\*A\*B\*f\*g\*x + A\*B\*f^2)\*log((b\*e\*x + a\*e)/(d\*x + c))), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)\*\*2/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^2 \left( B \log\left(\frac{(bx+a)e}{dx+c}\right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)^2/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="giac")

[Out] integrate(1/((g\*x + f)^2\*(B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2), x)

$$3.261 \quad \int \frac{1}{(f+gx)^3 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable} \left( \frac{1}{(f+gx)^3 \left( B \log\left(\frac{e(a+bx)}{c+dx}\right) + A \right)^2}, x \right)$$

[Out] Unintegrable[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

**Rubi [A]** time = 0.0697515, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)^3 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

[Out] Defer[Int][1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

Rubi steps

$$\int \frac{1}{(f+gx)^3 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx = \int \frac{1}{(f+gx)^3 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx$$

**Mathematica [A]** time = 30.1188, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)^3 \left( A+B \log\left(\frac{e(a+bx)}{c+dx}\right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

[Out] Integrate[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x))/(c + d\*x]))^2), x]

**Maple [A]** time = 1.841, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^3 \left( A+B \ln\left(\frac{e(bx+a)}{dx+c}\right) \right)^{-2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(g\*x+f)^3/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

[Out] int(1/(g\*x+f)^3/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)^3/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="maxima")

[Out] 
$$-(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*\log(e) - a*d*g^3*\log(e))*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*\log(e) - a*d*f^3*\log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*\log(e) - a*d*f*g^2*\log(e))*B^2)*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*g*\log(e) - a*d*f^2*g*\log(e))*B^2)*x + ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*\log(b*x + a) - ((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*\log(d*x + c) - \text{integrate}((b*d*g*x^2 - b*c*f - (d*f - 3*c*g)*a + 2*(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4 - a*d*g^4)*A*B + (b*c*g^4*\log(e) - a*d*g^4*\log(e))*B^2)*x^4 + 4*((b*c*f*g^3 - a*d*f*g^3)*A*B + (b*c*f*g^3*\log(e) - a*d*f*g^3*\log(e))*B^2)*x^3 + (b*c*f^4 - a*d*f^4)*A*B + (b*c*f^4*\log(e) - a*d*f^4*\log(e))*B^2 + 6*((b*c*f^2*g^2 - a*d*f^2*g^2)*A*B + (b*c*f^2*g^2*\log(e) - a*d*f^2*g^2*\log(e))*B^2)*x^2 + 4*((b*c*f^3*g - a*d*f^3*g)*A*B + (b*c*f^3*g*\log(e) - a*d*f^3*g*\log(e))*B^2)*x + ((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)*\log(b*x + a) - ((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)*\log(d*x + c)), x)$$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

integral 
$$\left( \frac{1}{A^2 g^3 x^3 + 3 A^2 f g^2 x^2 + 3 A^2 f^2 g x + A^2 f^3 + (B^2 g^3 x^3 + 3 B^2 f g^2 x^2 + 3 B^2 f^2 g x + B^2 f^3) \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2 (ABg^3 x^3 + 3 B^2 f g^2 x^2 + 3 B^2 f^2 g x + B^2 f^3) \log\left(\frac{bex+ae}{dx+c}\right) + 2 (ABg^3 x^3 + 3 B^2 f g^2 x^2 + 3 B^2 f^2 g x + B^2 f^3) \log\left(\frac{bex+ae}{dx+c}\right)^2 + 2 (ABg^3 x^3 + 3 B^2 f g^2 x^2 + 3 B^2 f^2 g x + B^2 f^3) \log\left(\frac{bex+ae}{dx+c}\right)}{1} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)^3/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="fricas")

[Out] integral(1/(A^2\*g^3\*x^3 + 3\*A^2\*f\*g^2\*x^2 + 3\*A^2\*f^2\*g\*x + A^2\*f^3 + (B^2\*g^3\*x^3 + 3\*B^2\*f\*g^2\*x^2 + 3\*B^2\*f^2\*g\*x + B^2\*f^3)\*log((b\*e\*x + a\*e)/(d\*x + c)))^2 + 2\*(A\*B\*g^3\*x^3 + 3\*A\*B\*f\*g^2\*x^2 + 3\*A\*B\*f^2\*g\*x + A\*B\*f^3)\*log((b\*e\*x + a\*e)/(d\*x + c))), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)\*\*3/(A+B\*ln(e\*(b\*x+a)/(d\*x+c)))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^3 \left( B \log\left(\frac{(bx+a)e}{dx+c}\right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)^3/(A+B\*log(e\*(b\*x+a)/(d\*x+c)))^2,x, algorithm="giac")

[Out] integrate(1/((g\*x + f)^3\*(B\*log((b\*x + a)\*e/(d\*x + c)) + A)^2), x)

### 3.262 $\int (f + gx)^4 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx$

**Optimal.** Leaf size=357

$$\frac{Bg^2x^2(bc - ad)(a^2d^2g^2 - abdg(5df - cg) + b^2(c^2g^2 - 5cdfg + 10d^2f^2))}{5b^3d^3} + \frac{2Bgx(bc - ad)(-a^2bd^2g^2(5df - cg) + a^3d^3g^3)}{5b^3d^3}$$

```
[Out] (2*B*(b*c - a*d)*g*(a^3*d^3*g^3 - a^2*b*d^2*g^2*(5*d*f - c*g) + a*b^2*d*g*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2) - b^3*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*d*f*g^2 - c^3*g^3))*x)/(5*b^4*d^4) - (B*(b*c - a*d)*g^2*(a^2*d^2*g^2 - a*b*d*g*(5*d*f - c*g) + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(5*b^3*d^3) - (2*B*(b*c - a*d)*g^3*(5*b*d*f - b*c*g - a*d*g)*x^3)/(15*b^2*d^2) - (B*(b*c - a*d)*g^4*x^4)/(10*b*d) - (2*B*(b*f - a*g)^5*Log[a + b*x])/(5*b^5*g) + ((f + g*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*g) + (2*B*(d*f - c*g)^5*Log[c + d*x])/(5*d^5*g)
```

**Rubi [A]** time = 0.50101, antiderivative size = 341, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2525, 12, 72}

$$\frac{Bg^2x^2(bc - ad)(a^2d^2g^2 - abdg(5df - cg) + b^2(c^2g^2 - 5cdfg + 10d^2f^2))}{5b^3d^3} + \frac{2Bgx(-10a^2b^2d^4f^2g + 5a^3bd^4fg^2 - a^4d^4g^3)}{5b^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

```
[Out] (2*B*g*(10*a*b^3*d^4*f^3 - 10*a^2*b^2*d^4*f^2*g + 5*a^3*b*d^4*f*g^2 - a^4*d^4*g^3 - b^4*c*(10*d^3*f^3 - 10*c*d^2*f^2*g + 5*c^2*d*f*g^2 - c^3*g^3))*x)/(5*b^4*d^4) - (B*(b*c - a*d)*g^2*(a^2*d^2*g^2 - a*b*d*g*(5*d*f - c*g) + b^2*(10*d^2*f^2 - 5*c*d*f*g + c^2*g^2))*x^2)/(5*b^3*d^3) - (2*B*(b*c - a*d)*g^3*(5*b*d*f - b*c*g - a*d*g)*x^3)/(15*b^2*d^2) - (B*(b*c - a*d)*g^4*x^4)/(10*b*d) - (2*B*(b*f - a*g)^5*Log[a + b*x])/(5*b^5*g) + ((f + g*x)^5*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(5*g) + (2*B*(d*f - c*g)^5*Log[c + d*x])/(5*d^5*g)
```

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
```

/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

### Rubi steps

$$\begin{aligned} \int (f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right) dx &= \frac{(f + gx)^5 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{5g} - \frac{B \int \frac{2(bc - ad)(f + gx)^5}{(a + bx)(c + dx)} dx}{5g} \\ &= \frac{(f + gx)^5 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{5g} - \frac{(2B(bc - ad)) \int \frac{(f + gx)^5}{(a + bx)(c + dx)} dx}{5g} \\ &= \frac{(f + gx)^5 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{5g} - \frac{(2B(bc - ad)) \int \left( \frac{g^2(-a^3 d^3 g^3 + a^2 b d^2 g^2 (5df - cg) + ab^2 dg (10d^2 f^2 - 5cdfg + c^2 g^2))}{(a + bx)(c + dx)} \right) dx}{5g} \\ &= \frac{2B(bc - ad)g \left( a^3 d^3 g^3 - a^2 b d^2 g^2 (5df - cg) + ab^2 dg (10d^2 f^2 - 5cdfg + c^2 g^2) \right)}{5b^4 d^4} \end{aligned}$$

**Mathematica [A]** time = 0.587325, size = 282, normalized size = 0.79

$$\frac{B g^2 x (a d - b c) (6 a^2 b d^2 g^2 (-2 c g + 10 d f + d g x) - 12 a^3 d^3 g^3 - 2 a b^2 d g (6 c^2 g^2 - 3 c d g (10 f + g x) + d^2 (60 f^2 + 15 f g x + 2 g^2 x^2))) + b^3 (6 c^2 d g^2 (10 f + g x) - 12 c^3 g^3 - 2 c d^2 g (60 f^2 + 15 f g x + 2 g^2 x^2))}{6 b^4 d^4}$$

5g

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^4\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]),x]

[Out] ((B\*(-(b\*c) + a\*d)\*g^2\*x\*(-12\*a^3\*d^3\*g^3 + 6\*a^2\*b\*d^2\*g^2\*(10\*d\*f - 2\*c\*g + d\*g\*x) - 2\*a\*b^2\*d\*g\*(6\*c^2\*g^2 - 3\*c\*d\*g\*(10\*f + g\*x) + d^2\*(60\*f^2 + 15\*f\*g\*x + 2\*g^2\*x^2)) + b^3\*(-12\*c^3\*g^3 + 6\*c^2\*d\*g^2\*(10\*f + g\*x) - 2\*c\*d^2\*g\*(60\*f^2 + 15\*f\*g\*x + 2\*g^2\*x^2) + d^3\*(120\*f^3 + 60\*f^2\*g\*x + 20\*f\*g^2\*x^2 + 3\*g^3\*x^3)))/(6\*b^4\*d^4) - (2\*B\*(b\*f - a\*g)^5\*Log[a + b\*x])/b^5 + (f + g\*x)^5\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) + (2\*B\*(d\*f - c\*g)^5\*Log[c + d\*x])/d^5)/(5\*g)

**Maple [B]** time = 0.318, size = 2438, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^4\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2)),x)

[Out] -2/d^4\*B\*g^3\*c^4\*ln(1/(d\*x+c))\*f+4\*B\*g^2\*a^3/b^3\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*f^2-4\*B\*g^2\*a^3/b^3\*ln(1/(d\*x+c))\*f^2+4\*B\*g/b^2\*ln(1/(d\*x+c))\*a^2\*f^3-4\*B\*g/b^2\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a^2\*f^3-4\*B/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a\*c\*f^4+4/d^2\*B\*c^2\*f^2\*g^2\*x-2/d\*B\*g^2\*c\*f^2\*x^2+2\*B\*g^3\*a^3/b^3\*f\*x+4\*B\*g/b\*a\*f^3\*x-2\*B/b\*ln(1/(d\*x+c))\*a\*f^4+2/5\*B\*g^4\*a^5/b^5\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)-2/5\*B\*g^4\*a^5/b^5\*ln(1/(d\*x+c))+1/5\*B\*g^4\*a^3/b^3\*x^2+2/15/d^2\*B\*g^4\*c^2\*x^3-1/5/d^3\*B\*g^4\*c^3\*x^2+2/5/d^4\*B\*g^4\*c^4\*x-1/10/d\*B\*g^4\*c\*x^4+2\*B\*g^2\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*f^2\*x^3-2/5\*B\*g^4\*a^4/b^4\*x+1/10\*B\*g^4\*a/b\*x^4-2/15\*B\*g^4\*a^2/b^2\*x^3+2\*B\*g\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*f^3\*x^2+1/5/d^5\*B\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*c^5\*g^4-1/d^4\*B\*g^3\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)

) + b)^2/d^2)\*f\*c^4-2/d^2\*B\*g\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*f^3\*c^2+8/d^3\*B\*g^2\*c^3\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*f^2-4/d^2\*B\*g\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*c^2\*f^3-4/d^2\*B\*ln(1/(d\*x+c))\*c^2\*f^3\*g-6/d^4\*B\*g^3\*c^4\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*f-1/5/d^2\*B\*g^4\*a^3/b^3\*c^2+B\*g^3\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*f\*x^4-1/10/d^4\*B\*g^4\*a/b\*c^4-1/d^4\*A\*c^4\*f\*g^3+2/d^3\*A\*c^3\*f^2\*g^2-2/d^2\*A\*c^2\*f^3\*g+6/d^3\*B\*g^2\*c^3\*f^2-4/d^2\*B\*g\*c^2\*f^3-11/3/d^4\*B\*g^3\*c^4\*f-2\*B\*g^3\*a^4/b^4\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*f+2\*B\*g^3\*a^4/b^4\*ln(1/(d\*x+c))\*f-4/d\*B\*c\*f^3\*g\*x-2/3/d\*B\*g^3\*c\*f\*x^3+1/d^2\*B\*g^3\*c^2\*f\*x^2-2/d^3\*B\*g^3\*c^3\*f\*x+4/d^3\*B\*g^2\*c^3\*ln(1/(d\*x+c))\*f^2+2/d^3\*B\*g^2\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*f^2\*c^3-2/5/d\*B\*g^4\*a^4/b^4\*c+12/d\*B/b/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a^2\*c^2\*f^2\*g^2-8/d^2\*B/b/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a^2\*c^3\*f\*g^3+2/5/d^5\*B\*g^4\*c^5\*ln(1/(d\*x+c))+8/5/d^5\*B\*g^4\*c^5\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)+1/d\*B\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*c\*f^4+2/d\*B\*ln(1/(d\*x+c))\*c\*f^4+5/6/d^5\*B\*g^4\*c^5+1/5/d^5\*A\*c^5\*g^4+1/d\*A\*c\*f^4+A\*x^4\*f\*g^3+2\*A\*x^3\*f^2\*g^2+2\*A\*x^2\*f^3\*g+1/5\*B\*g^4\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*x^5+B\*ln(e\*(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*x\*f^4+2/d^3\*B/b/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a^2\*c^4\*g^4-8/d^4\*B/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*c^5\*b\*f\*g^3-8\*B/b/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a^2\*c\*f^3\*g+12/d^3\*B/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*c^4\*b\*f^2\*g^2+16/d^3\*B/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a\*c^4\*f\*g^3+8/d\*B\*g/b\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a\*c\*f^3+16/d\*B/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a\*c^2\*f^3\*g-12/d^2\*B\*g^2\*a/b\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*c^2\*f^2-8/d^2\*B/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*c^3\*b\*f^3\*g+8/d^3\*B\*g^3\*a/b\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*c^3\*f-24/d^2\*B/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a\*c^3\*f^2\*g^2+A\*x\*f^4+1/5\*A\*x^5\*g^4-2/15/d^3\*B\*g^4\*a^2/b^2\*c^3+2/3\*B\*g^3\*a/b\*f\*x^3-B\*g^3\*a^2/b^2\*f\*x^2+2\*B\*g^2\*a/b\*f^2\*x^2-4\*B\*g^2\*a^2/b^2\*f^2\*x+2/d^5\*B/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*c^6\*b\*g^4-2/d^4\*B\*g^4\*a/b\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*c^4+2\*d\*B/b/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a^2\*f^4-4/d^4\*B/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a\*c^5\*g^4+2/d\*B/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*c^2\*b\*f^4+2/d\*B\*g^3\*a^3/b^3\*f\*c+4/d\*B\*g/b\*a\*f^3\*c-4/d\*B\*g^2\*a^2/b^2\*f^2\*c+1/d^2\*B\*g^3\*a^2/b^2\*c^2\*f+2/3/d^3\*B\*g^3\*a/b\*c^3\*f-2/d^2\*B\*g^2\*a/b\*c^2\*f^2

**Maxima [B]** time = 1.42897, size = 1154, normalized size = 3.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^4\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="maxima")

[Out] 1/5\*A\*g^4\*x^5 + A\*f\*g^3\*x^4 + 2\*A\*f^2\*g^2\*x^3 + 2\*A\*f^3\*g\*x^2 + (x\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 2\*a\*log(b\*x + a)/b - 2\*c\*log(d\*x + c)/d)\*B\*f^4 + 2\*(x^2\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) - 2\*a^2\*log(b\*x + a)/b^2 + 2\*c^2\*log(d\*x + c)/d^2 - 2\*(b\*c - a\*d)\*x/(b\*d))\*B\*f^3\*g + 2\*(x^3\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 2\*a^3\*log(b\*x + a)/b^3 - 2\*c^3\*log(d\*x + c)/d^3 - ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2))\*B\*f^2\*g^2 + 1/3\*(3\*x^4\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) - 6\*a^4\*log(b\*x + a)/b^4 + 6\*c^4\*log(d\*x + c)/d^4 - (2\*(b^3\*c\*d^2 - a\*b^2\*d^3)\*x^3 - 3\*(b^3\*c^2\*d - a^2\*b\*d^3)\*x^2 + 6\*(b^3\*c^3 - a^3\*d^3)\*x)/(b^3\*d^3))\*B\*f\*g^3 + 1/30\*(6\*x^5\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 12\*a^5\*log(b\*x + a)/b^5 - 12

$$\frac{c^5 \log(dx + c)/d^5 - (3(b^4cd^3 - ab^3d^4)x^4 - 4(b^4c^2d^2 - a^2b^2d^4)x^3 + 6(b^4c^3d - a^3bd^4)x^2 - 12(b^4c^4 - a^4d^4)x)/(b^4d^4) * B * g^4 + A * f^4 * x}{}$$

**Fricas [A]** time = 3.49111, size = 1328, normalized size = 3.72

$$6 Ab^5 d^5 g^4 x^5 + 3 (10 Ab^5 d^5 f g^3 - (Bb^5 cd^4 - Bab^4 d^5) g^4) x^4 + 4 (15 Ab^5 d^5 f^2 g^2 - 5 (Bb^5 cd^4 - Bab^4 d^5) f g^3 + (Bb^5 c^2 d^3 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^4\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="fricas")

[Out] 1/30\*(6\*A\*b^5\*d^5\*g^4\*x^5 + 3\*(10\*A\*b^5\*d^5\*f\*g^3 - (B\*b^5\*c\*d^4 - B\*a\*b^4\*d^5)\*g^4)\*x^4 + 4\*(15\*A\*b^5\*d^5\*f^2\*g^2 - 5\*(B\*b^5\*c\*d^4 - B\*a\*b^4\*d^5)\*f\*g^3 + (B\*b^5\*c^2\*d^3 - B\*a^2\*b^3\*d^5)\*g^4)\*x^3 + 6\*(10\*A\*b^5\*d^5\*f^3\*g - 10\*(B\*b^5\*c\*d^4 - B\*a\*b^4\*d^5)\*f^2\*g^2 + 5\*(B\*b^5\*c^2\*d^3 - B\*a^2\*b^3\*d^5)\*f\*g^3 - (B\*b^5\*c^3\*d^2 - B\*a^3\*b^2\*d^5)\*g^4)\*x^2 + 6\*(5\*A\*b^5\*d^5\*f^4 - 20\*(B\*b^5\*c\*d^4 - B\*a\*b^4\*d^5)\*f^3\*g + 20\*(B\*b^5\*c^2\*d^3 - B\*a^2\*b^3\*d^5)\*f^2\*g^2 - 10\*(B\*b^5\*c^3\*d^2 - B\*a^3\*b^2\*d^5)\*f\*g^3 + 2\*(B\*b^5\*c^4\*d - B\*a^4\*b\*d^5)\*g^4)\*x + 12\*(5\*B\*a\*b^4\*d^5\*f^4 - 10\*B\*a^2\*b^3\*d^5\*f^3\*g + 10\*B\*a^3\*b^2\*d^5\*f^2\*g^2 - 5\*B\*a^4\*b\*d^5\*f\*g^3 + B\*a^5\*d^5\*g^4)\*log(b\*x + a) - 12\*(5\*B\*b^5\*c\*d^4\*f^4 - 10\*B\*b^5\*c^2\*d^3\*f^3\*g + 10\*B\*b^5\*c^3\*d^2\*f^2\*g^2 - 5\*B\*b^5\*c^4\*d\*f\*g^3 + B\*b^5\*c^5\*g^4)\*log(d\*x + c) + 6\*(B\*b^5\*d^5\*g^4\*x^5 + 5\*B\*b^5\*d^5\*f\*g^3\*x^4 + 10\*B\*b^5\*d^5\*f^2\*g^2\*x^3 + 10\*B\*b^5\*d^5\*f^3\*g\*x^2 + 5\*B\*b^5\*d^5\*f^4\*x)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))/(b^5\*d^5)

**Sympy [B]** time = 35.7938, size = 1562, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*4\*(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2)),x)

[Out] A\*g\*\*4\*x\*\*5/5 + 2\*B\*a\*(a\*\*4\*g\*\*4 - 5\*a\*\*3\*b\*f\*g\*\*3 + 10\*a\*\*2\*b\*\*2\*f\*\*2\*g\*\*2 - 10\*a\*b\*\*3\*f\*\*3\*g + 5\*b\*\*4\*f\*\*4)\*log(x + (2\*B\*a\*\*5\*c\*d\*\*4\*g\*\*4 - 10\*B\*a\*\*4\*b\*c\*d\*\*4\*f\*g\*\*3 + 20\*B\*a\*\*3\*b\*\*2\*c\*d\*\*4\*f\*\*2\*g\*\*2 - 20\*B\*a\*\*2\*b\*\*3\*c\*d\*\*4\*f\*\*3\*g + 2\*B\*a\*\*2\*d\*\*5\*(a\*\*4\*g\*\*4 - 5\*a\*\*3\*b\*f\*g\*\*3 + 10\*a\*\*2\*b\*\*2\*f\*\*2\*g\*\*2 - 10\*a\*b\*\*3\*f\*\*3\*g + 5\*b\*\*4\*f\*\*4))/b + 2\*B\*a\*b\*\*4\*c\*\*5\*g\*\*4 - 10\*B\*a\*b\*\*4\*c\*\*4\*d\*f\*g\*\*3 + 20\*B\*a\*b\*\*4\*c\*\*3\*d\*\*2\*f\*\*2\*g\*\*2 - 20\*B\*a\*b\*\*4\*c\*\*2\*d\*\*3\*f\*\*3\*g + 20\*B\*a\*b\*\*4\*c\*d\*\*4\*f\*\*4 - 2\*B\*a\*c\*d\*\*4\*(a\*\*4\*g\*\*4 - 5\*a\*\*3\*b\*f\*g\*\*3 + 10\*a\*\*2\*b\*\*2\*f\*\*2\*g\*\*2 - 10\*a\*b\*\*3\*f\*\*3\*g + 5\*b\*\*4\*f\*\*4))/(2\*B\*a\*\*5\*d\*\*5\*g\*\*4 - 10\*B\*a\*\*4\*b\*d\*\*5\*f\*g\*\*3 + 20\*B\*a\*\*3\*b\*\*2\*d\*\*5\*f\*\*2\*g\*\*2 - 20\*B\*a\*\*2\*b\*\*3\*d\*\*5\*f\*\*3\*g + 10\*B\*a\*b\*\*4\*d\*\*5\*f\*\*4 + 2\*B\*b\*\*5\*c\*\*5\*g\*\*4 - 10\*B\*b\*\*5\*c\*\*4\*d\*f\*g\*\*3 + 20\*B\*b\*\*5\*c\*\*3\*d\*\*2\*f\*\*2\*g\*\*2 - 20\*B\*b\*\*5\*c\*\*2\*d\*\*3\*f\*\*3\*g + 10\*B\*b\*\*5\*c\*d\*\*4\*f\*\*4))/(5\*b\*\*5) - 2\*B\*c\*(c\*\*4\*g\*\*4 - 5\*c\*\*3\*d\*f\*g\*\*3 + 10\*c\*\*2\*d\*\*2\*f\*\*2\*g\*\*2 - 10\*c\*d\*\*3\*f\*\*3\*g + 5\*d\*\*4\*f\*\*4)\*log(x + (2\*B\*a\*\*5\*c\*d\*\*4\*g\*\*4 - 10\*B\*a\*\*4\*b\*c\*d\*\*4\*f\*g\*\*3 + 20\*B\*a\*\*3\*b\*\*2\*c\*d\*\*4\*f\*\*2\*g\*\*2 - 20\*B\*a\*\*2\*b\*\*3\*c\*d\*\*4\*f\*\*3\*g + 2\*B\*a\*\*2\*d\*\*5\*(a\*\*4\*g\*\*4 - 5\*a\*\*3\*b\*f\*g\*\*3 + 10\*a\*\*2\*b\*\*2\*f\*\*2\*g\*\*2 - 10\*a\*b\*\*3\*f\*\*3\*g + 5\*b\*\*4\*f\*\*4))/b + 2\*B\*a\*b\*\*4\*c\*\*5\*g\*\*4 - 10\*B\*a\*b\*\*4\*c\*\*4\*d\*f\*g\*\*3 + 20\*B\*a\*b\*\*4\*c\*\*3\*d\*\*2\*f\*\*2\*g\*\*2 - 20\*B\*a\*b\*\*4\*c\*\*2\*d\*\*3\*f\*\*3\*g + 20\*B\*a\*b\*\*4\*c\*d\*\*4\*f\*\*4 - 2\*B\*a\*b\*\*4\*c\*(c\*\*4\*g\*\*4 - 5\*c\*\*3\*d\*f\*g\*\*3 + 10\*c\*\*2\*d\*\*2\*f\*\*2\*g\*\*2 - 10\*c\*d\*\*3\*f\*\*3\*g + 5\*d\*\*4\*f\*\*4) + 2\*B\*b\*\*5\*c\*\*2\*(c\*\*4\*g\*\*4

$$\begin{aligned}
& - 5c^{3d}fg^3 + 10c^2d^2f^2g^2 - 10cd^3f^3g + 5d^4f^4 \\
& )/d)/(2Ba^5d^5g^4 - 10Ba^4bd^5f^3g + 20Ba^3b^2d^5f^2g^2 - 20Ba^2b^3d^5f^3g + 10Bab^4d^5f^4 + 2Bb^5c^5g^4 - 10Bb^5c^4d^5fg^3 + 20Bb^5c^3d^2f^2g^2 - 20Bb^5c^2d^3f^3g + 10Bb^5cd^4f^4)/(5d^5) + (Bf^4x + 2Bf^3g^2x^2 + 2Bf^2g^2x^3 + Bfg^3x^4 + Bg^4x^5/5)\log(e(a + bx)^2/(c + dx)^2) + x^4(10Abd^5fg^3 + Ba^4d^5g^4 - Bb^4c^4g^4)/(10abd) - x^3(-30Ab^2d^2f^2g^2 + 2Ba^2d^2g^4 - 10Bab^2d^2f^3g - 2Bb^2c^2g^4 + 10Bb^2cd^2fg^3)/(15b^2d^2) + x^2(10Aab^3d^3f^3g + Ba^3d^3g^4 - 5Ba^2bd^3fg^3 + 10Bab^2d^3f^2g^2 - Bb^3c^3g^4 + 5Bb^3c^2d^2fg^3 - 10Bb^3cd^2f^2g^2)/(5b^3d^3) - x(-5Aab^4d^4f^4 + 2Ba^4d^4g^4 - 10Ba^3bd^4fg^3 + 20Ba^2b^2d^4f^2g^2 - 20Bab^3d^4f^3g - 2Bb^4c^4g^4 + 10Bb^4c^3d^2fg^3 - 20Bb^4c^2d^2f^2g^2 + 20Bb^4cd^3f^3g)/(5b^4d^4)
\end{aligned}$$

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^4\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="giac")

[Out] Timed out

### 3.263 $\int (f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx$

**Optimal.** Leaf size=229

$$\frac{Bgx(bc - ad) \left( a^2 d^2 g^2 - abdg(4df - cg) + b^2 (c^2 g^2 - 4cdfg + 6d^2 f^2) \right)}{2b^3 d^3} + \frac{(f + gx)^4 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}{4g} - \frac{Bg^2 x^2 (bc - ad)}{2b^3 d^3}$$

```
[Out] -(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*x)/(2*b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*x^2)/(4*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3)/(6*b*d) - (B*(b*f - a*g)^4*Log[a + b*x])/(2*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2]/(c + d*x)^2]))/(4*g) + (B*(d*f - c*g)^4*Log[c + d*x])/(2*d^4*g)
```

**Rubi [A]** time = 0.323511, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2525, 12, 72}

$$\frac{Bgx(bc - ad) \left( a^2 d^2 g^2 - abdg(4df - cg) + b^2 (c^2 g^2 - 4cdfg + 6d^2 f^2) \right)}{2b^3 d^3} + \frac{(f + gx)^4 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}{4g} - \frac{Bg^2 x^2 (bc - ad)}{2b^3 d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2]/(c + d*x)^2]),x]
```

```
[Out] -(B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*x)/(2*b^3*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*x^2)/(4*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3)/(6*b*d) - (B*(b*f - a*g)^4*Log[a + b*x])/(2*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2]/(c + d*x)^2]))/(4*g) + (B*(d*f - c*g)^4*Log[c + d*x])/(2*d^4*g)
```

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rubi steps

$$\int (f + gx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right) dx = \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{4g} - \frac{B \int \frac{2(bc - ad)(f + gx)^4}{(a + bx)(c + dx)} dx}{4g}$$

$$= \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{4g} - \frac{(B(bc - ad)) \int \frac{(f + gx)^4}{(a + bx)(c + dx)} dx}{2g}$$

$$= \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{4g} - \frac{(B(bc - ad)) \int \left( \frac{g^2(a^2d^2g^2 - abdg(4df - cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2))}{b^3d^3} \right) dx}{2b^3d^3}$$

$$= - \frac{B(bc - ad)g \left( a^2d^2g^2 - abdg(4df - cg) + b^2(6d^2f^2 - 4cdfg + c^2g^2) \right) x}{2b^3d^3} - \frac{B(bc - ad)}{2b^3d^3}$$

**Mathematica [A]** time = 0.257244, size = 217, normalized size = 0.95

$$\frac{(f + gx)^4 \left( B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) + A \right) - \frac{B(6bdg^2x(bc - ad)(a^2d^2g^2 + abdg(cg - 4df) + b^2(c^2g^2 - 4cdfg + 6d^2f^2)) + 3b^2d^2g^3x^2(bc - ad)(-adg - bcg + 4bdf) + 2b^3d^3g^4x)}{3b^4d^4}}{4g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]), x]
```

```
[Out] ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(6*b*d*(b*c - a*d)*g^2*(a^2*d^2*g^2 + a*b*d*g*(-4*d*f + c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*x + 3*b^2*d^2*(b*c - a*d)*g^3*(4*b*d*f - b*c*g - a*d*g)*x^2 + 2*b^3*d^3*(b*c - a*d)*g^4*x^3 + 6*d^4*(b*f - a*g)^4*Log[a + b*x] - 6*b^4*(d*f - c*g)^4*Log[c + d*x]))/(3*b^4*d^4))/(4*g)
```

**Maple [B]** time = 0.266, size = 1783, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^3*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)), x)
```

```
[Out] A*x^3*f*g^2+3/2*A*x^2*f^2*g-1/4/d^4*A*c^4*g^3+1/d*A*c*f^3-11/12/d^4*B*g^3*c^4+1/4*B*g^3*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*x^4+B*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*x*f^3-3/d^2*B*g*ln(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*c^2*f^2-3/d^2*B*ln(1/(d*x+c))*c^2*f^2*g+4/d^3*B*g^2*c^3*ln(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*f+1/d^3*B*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*c^3*f*g^2+2*B*g^2*a^3/b^3*ln(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*f-4*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*a*c*f^3+3*B*g/b^2*ln(1/(d*x+c))*a^2*f^2-3*B*g/b^2*ln(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*a^2*f^2-2*B*g^2*a^3/b^3*ln(1/(d*x+c))*f+2/d^2*B*g^2*c^2*f*x+2/d^3*B*g^2*c^3*ln(1/(d*x+c))*f-1/2/d^3*B*g^3*c^3*x+1/4/d^2*B*g^3*c^2*x^2+1/2*B*g^3*a^4/b^4*ln(1/(d*x+c))-1/2*B*g^3*a^4/b^4*ln(1/(d*x+c)*a*d-b*c/(d*x+c)+b)-2*B/b*ln(1/(d*x+c))*a*f^3+1/d*B*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*c*f^3+2/d*B*ln(1/(d*x+c))*c*f^3-1/4/d^4*B*g^3*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*c^4-1/2/d^4*B*g^3*c^4*ln(1/(d*x+c))-3/2/d^4*B*g^3*c^4*ln(1/(d*x+c)*a*d-b*c/(d*x+c)+b)+B*g^2*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*f*x^3+3/2*B*g*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*f^2*x^2+1/6*B*g^3*a/b*x^3-1/4*B*g^3*a^2/b^2*x^2+1/2*B*g^3*a^3/b^3*x-1/6/d*B*g^3*c*x^3-1/d*B*g^2*c*f*x^2-2*B*g^2*a^2/b^2*f*x+6/d*B/b/(a*d-b*c)*ln(1/(d*x+c)
```



) \* a \* d - b \* c / (d \* x + c) + b) \* a ^ 2 \* c ^ 2 \* f \* g ^ 2 + 1 / d ^ 3 \* A \* c ^ 3 \* f \* g ^ 2 - 3 / 2 / d ^ 2 \* A \* c ^ 2 \* f ^ 2 \* g + 3 / d ^ 3 \* B \* g ^ 2 \* c ^ 3 \* f - 3 / d ^ 2 \* B \* g \* c ^ 2 \* f ^ 2 - 3 / 2 / d ^ 2 \* B \* ln(e \* (1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) ^ 2 / d ^ 2) \* c ^ 2 \* f ^ 2 \* g - 3 / d \* B \* g \* c \* f ^ 2 \* x + 3 \* B \* g / b \* a \* f ^ 2 \* x + B \* g ^ 2 \* a / b \* f \* x ^ 2 + 1 / 4 \* A \* x ^ 4 \* g ^ 3 + A \* x \* f ^ 3 + 12 / d \* B / (a \* d - b \* c) \* ln(1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) \* a \* c ^ 2 \* f ^ 2 \* g + 6 / d \* B \* g / b \* ln(1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) \* a \* c \* f ^ 2 + 6 / d ^ 3 \* B / (a \* d - b \* c) \* ln(1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) \* c ^ 4 \* b \* f \* g ^ 2 - 6 / d ^ 2 \* B / (a \* d - b \* c) \* ln(1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) \* c ^ 3 \* b \* f ^ 2 \* g - 12 / d ^ 2 \* B / (a \* d - b \* c) \* ln(1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) \* a \* c ^ 3 \* f \* g ^ 2 - 2 / d ^ 2 \* B / b / (a \* d - b \* c) \* ln(1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) \* a ^ 2 \* c ^ 3 \* g ^ 3 - 6 / d ^ 2 \* B \* g ^ 2 \* a / b \* ln(1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) \* c ^ 2 \* f - 6 \* B / b / (a \* d - b \* c) \* ln(1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) \* a ^ 2 \* c \* f ^ 2 \* g + 1 / 6 / d ^ 3 \* B \* g ^ 3 \* a / b \* c ^ 3 + 1 / 4 / d ^ 2 \* B \* g ^ 3 \* a ^ 2 / b ^ 2 \* c ^ 2 + 1 / 2 / d \* B \* g ^ 3 \* a ^ 3 / b ^ 3 \* c + 3 / d \* B \* g / b \* a \* f ^ 2 \* c - 2 / d \* B \* g ^ 2 \* a ^ 2 / b ^ 2 \* f \* c + 2 / d ^ 3 \* B \* g ^ 3 \* a / b \* ln(1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) \* c ^ 3 + 2 \* d \* B / b / (a \* d - b \* c) \* ln(1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) \* a ^ 2 \* f ^ 3 + 2 / d \* B / (a \* d - b \* c) \* ln(1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) \* c ^ 2 \* b \* f ^ 3 - 2 / d ^ 4 \* B / (a \* d - b \* c) \* ln(1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) \* c ^ 5 \* b \* g ^ 3 + 4 / d ^ 3 \* B / (a \* d - b \* c) \* ln(1 / (d \* x + c) \* a \* d - b \* c / (d \* x + c) + b) \* a \* c ^ 4 \* g ^ 3 - 1 / d ^ 2 \* B \* g ^ 2 \* a / b \* c ^ 2 \* f

**Maxima [B]** time = 1.34343, size = 841, normalized size = 3.67

$$\frac{1}{4} A g^3 x^4 + A f g^2 x^3 + \frac{3}{2} A f^2 g x^2 + \left( x \log \left( \frac{b^2 e x^2}{d^2 x^2 + 2 c d x + c^2} + \frac{2 a b e x}{d^2 x^2 + 2 c d x + c^2} + \frac{a^2 e}{d^2 x^2 + 2 c d x + c^2} \right) + \frac{2 a \log(b x + a)}{b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="maxima")

[Out] 1/4\*A\*g^3\*x^4 + A\*f\*g^2\*x^3 + 3/2\*A\*f^2\*g\*x^2 + (x\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 2\*a\*log(b\*x + a)/b - 2\*c\*log(d\*x + c)/d)\*B\*f^3 + 3/2\*(x^2\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) - 2\*a^2\*log(b\*x + a)/b^2 + 2\*c^2\*log(d\*x + c)/d^2 - 2\*(b\*c - a\*d)\*x/(b\*d))\*B\*f^2\*g + (x^3\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + 2\*a^3\*log(b\*x + a)/b^3 - 2\*c^3\*log(d\*x + c)/d^3 - ((b^2\*c\*d - a\*b\*d^2)\*x^2 - 2\*(b^2\*c^2 - a^2\*d^2)\*x)/(b^2\*d^2))\*B\*f\*g^2 + 1/12\*(3\*x^4\*log(b^2\*e\*x^2/(d^2\*x^2 + 2\*c\*d\*x + c^2) + 2\*a\*b\*e\*x/(d^2\*x^2 + 2\*c\*d\*x + c^2) + a^2\*e/(d^2\*x^2 + 2\*c\*d\*x + c^2)) - 6\*a^4\*log(b\*x + a)/b^4 + 6\*c^4\*log(d\*x + c)/d^4 - (2\*(b^3\*c\*d^2 - a\*b^2\*d^3)\*x^3 - 3\*(b^3\*c^2\*d - a^2\*b\*d^3)\*x^2 + 6\*(b^3\*c^3 - a^3\*d^3)\*x)/(b^3\*d^3))\*B\*g^3 + A\*f^3\*x

**Fricas [B]** time = 1.91492, size = 938, normalized size = 4.1

$$3 A b^4 d^4 g^3 x^4 + 2 (6 A b^4 d^4 f g^2 - (B b^4 c d^3 - B a b^3 d^4) g^3) x^3 + 3 (6 A b^4 d^4 f^2 g - 4 (B b^4 c d^3 - B a b^3 d^4) f g^2 + (B b^4 c^2 d^2 - B a^2 b^3 d^4) f^2 g + 4 (B b^4 c^2 d^2 - B a^2 b^3 d^4) f g^2 - (B b^4 c^3 d - B a^3 b^3 d^4) g^3) x + 6 (4 B a^2 b^3 d^4 f^3 - 6 B a^2 b^3 d^4 f^2 g$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="fricas")

[Out] 1/12\*(3\*A\*b^4\*d^4\*g^3\*x^4 + 2\*(6\*A\*b^4\*d^4\*f\*g^2 - (B\*b^4\*c\*d^3 - B\*a\*b^3\*d^4)\*g^3)\*x^3 + 3\*(6\*A\*b^4\*d^4\*f^2\*g - 4\*(B\*b^4\*c\*d^3 - B\*a\*b^3\*d^4)\*f\*g^2 + (B\*b^4\*c^2\*d^2 - B\*a^2\*b^3\*d^4)\*g^3)\*x^2 + 6\*(2\*A\*b^4\*d^4\*f^3 - 6\*(B\*b^4\*c\*d^3 - B\*a\*b^3\*d^4)\*f^2\*g + 4\*(B\*b^4\*c^2\*d^2 - B\*a^2\*b^3\*d^4)\*f\*g^2 - (B\*b^4\*c^3\*d - B\*a^3\*b^3\*d^4)\*g^3)\*x + 6\*(4\*B\*a^2\*b^3\*d^4\*f^3 - 6\*B\*a^2\*b^3\*d^4\*f^2 g

$$g + 4*B*a^3*b*d^4*f*g^2 - B*a^4*d^4*g^3)*\log(b*x + a) - 6*(4*B*b^4*c*d^3*f^3 - 6*B*b^4*c^2*d^2*f^2*g + 4*B*b^4*c^3*d*f*g^2 - B*b^4*c^4*g^3)*\log(d*x + c) + 3*(B*b^4*d^4*g^3*x^4 + 4*B*b^4*d^4*f*g^2*x^3 + 6*B*b^4*d^4*f^2*g*x^2 + 4*B*b^4*d^4*f^3*x)*\log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b^4*d^4)$$

**Sympy [B]** time = 18.9458, size = 1052, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g**3*x**4/4 - B*a*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)*log(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B*a**2*d**4*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)/b + B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g - 8*B*a*b**3*c*d**3*f**3 - B*a*c*d**3*(a*g - 2*b*f)*(a**2*g**2 - 2*a*b*f*g + 2*b**2*f**2)))/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(2*b**4) + B*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)*log(x + (B*a**4*c*d**3*g**3 - 4*B*a**3*b*c*d**3*f*g**2 + 6*B*a**2*b**2*c*d**3*f**2*g + B*a*b**3*c**4*g**3 - 4*B*a*b**3*c**3*d*f*g**2 + 6*B*a*b**3*c**2*d**2*f**2*g - 8*B*a*b**3*c*d**3*f**3 - B*a*b**3*c*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2) + B*b**4*c**2*(c*g - 2*d*f)*(c**2*g**2 - 2*c*d*f*g + 2*d**2*f**2)/d)/(B*a**4*d**4*g**3 - 4*B*a**3*b*d**4*f*g**2 + 6*B*a**2*b**2*d**4*f**2*g - 4*B*a*b**3*d**4*f**3 + B*b**4*c**4*g**3 - 4*B*b**4*c**3*d*f*g**2 + 6*B*b**4*c**2*d**2*f**2*g - 4*B*b**4*c*d**3*f**3))/(2*d**4) + (B*f**3*x + 3*B*f**2*g*x**2/2 + B*f*g**2*x**3 + B*g**3*x**4/4)*log(e*(a + b*x)**2/(c + d*x)**2) + x**3*(6*A*b*d*f*g**2 + B*a*d*g**3 - B*b*c*g**3)/(6*b*d) - x**2*(-6*A*b**2*d**2*f**2*g + B*a**2*d**2*g**3 - 4*B*a*b*d**2*f*g**2 - B*b**2*c**2*g**3 + 4*B*b**2*c*d*f*g**2)/(4*b**2*d**2) + x*(2*A*b**3*d**3*f**3 + B*a**3*d**3*g**3 - 4*B*a**2*b*d**3*f*g**2 + 6*B*a*b**2*d**3*f**2*g - B*b**3*c**3*g**3 + 4*B*b**3*c**2*d*f*g**2 - 6*B*b**3*c*d**2*f**2*g)/(2*b**3*d**3)
```

**Giac [B]** time = 137.852, size = 603, normalized size = 2.63

$$\frac{1}{4} (Ag^3 + Bg^3)x^4 + \frac{(6Abdfg^2 + 6Bbdfg^2 - Bbcg^3 + Badg^3)x^3}{6bd} + \frac{1}{4} (Bg^3x^4 + 4Bfg^2x^3 + 6Bf^2gx^2 + 4Bf^3x) \log\left(\frac{b^2x^2}{d^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^3*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] 1/4*(A*g^3 + B*g^3)*x^4 + 1/6*(6*A*b*d*f*g^2 + 6*B*b*d*f*g^2 - B*b*c*g^3 + B*a*d*g^3)*x^3/(b*d) + 1/4*(B*g^3*x^4 + 4*B*f*g^2*x^3 + 6*B*f^2*g*x^2 + 4*B*f^3*x)*log((b^2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/4*(6*A*b^2*d^2*f^2*g + 6*B*b^2*d^2*f^2*g - 4*B*b^2*c*d*f*g^2 + 4*B*a*b*d^2*f*g^2 + B*b^2*c^2*g^3 - B*a^2*d^2*g^3)*x^2/(b^2*d^2) + 1/2*(4*B*a*b^3*f^3 - 6*B*a^2*b^2*f^2*g + 4*B*a^3*b*f*g^2 - B*a^4*g^3)*log(b*x + a)/b^4 - 1/2*(4*B*c*d^3*f^3 - 6*B*c^2*d^2*f^2*g + 4*B*c^3*d*f*g^2 - B*c^4*g^3)*log(-d*x - c)/d^4 + 1/2*(2*A*b^3*d^3*f^3 + 2*B*b^3*d^3*f^3 - 6*B*b^3*c*d^2*f^2*g + 6*B*a*b^2
```

$$\frac{*d^3*f^2*g + 4*B*b^3*c^2*d*f*g^2 - 4*B*a^2*b*d^3*f*g^2 - B*b^3*c^3*g^3 + B*a^3*d^3*g^3)*x}{(b^3*d^3)}$$

$$3.264 \quad \int (f + gx)^2 \left( A + B \log \left( \frac{e^{(a+bx)^2}}{(c+dx)^2} \right) \right) dx$$

**Optimal.** Leaf size=152

$$\frac{(f + gx)^3 \left( B \log \left( \frac{e^{(a+bx)^2}}{(c+dx)^2} \right) + A \right)}{3g} - \frac{2Bgx(bc - ad)(-adg - bcg + 3bdf)}{3b^2d^2} - \frac{2B(bf - ag)^3 \log(a + bx)}{3b^3g} - \frac{Bg^2x^2(bc - ad)}{3bd} + \frac{2B(bf - ag)^3 \log(a + bx)}{3b^3g}$$

[Out]  $(-2*B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*x)/(3*b^2*d^2) - (B*(b*c - a*d)*g^2*x^2)/(3*b*d) - (2*B*(b*f - a*g)^3*\text{Log}[a + b*x])/(3*b^3*g) + ((f + g*x)^3*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*g) + (2*B*(d*f - c*g)^3*\text{Log}[c + d*x])/(3*d^3*g)$

**Rubi [A]** time = 0.160903, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2525, 12, 72}

$$\frac{(f + gx)^3 \left( B \log \left( \frac{e^{(a+bx)^2}}{(c+dx)^2} \right) + A \right)}{3g} - \frac{2Bgx(bc - ad)(-adg - bcg + 3bdf)}{3b^2d^2} - \frac{2B(bf - ag)^3 \log(a + bx)}{3b^3g} - \frac{Bg^2x^2(bc - ad)}{3bd} + \frac{2B(bf - ag)^3 \log(a + bx)}{3b^3g}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)^2*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]), x]$

[Out]  $(-2*B*(b*c - a*d)*g*(3*b*d*f - b*c*g - a*d*g)*x)/(3*b^2*d^2) - (B*(b*c - a*d)*g^2*x^2)/(3*b*d) - (2*B*(b*f - a*g)^3*\text{Log}[a + b*x])/(3*b^3*g) + ((f + g*x)^3*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*g) + (2*B*(d*f - c*g)^3*\text{Log}[c + d*x])/(3*d^3*g)$

### Rule 2525

$\text{Int}[(a + \text{Log}[c*\text{RFX}^p])*(b + (d + e*x)^m)^n], x\_Symbol] :> \text{Simp}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^n/(e^{m+1}), x] - \text{Dist}[(b*n*p)/(e^{m+1}), \text{Int}[\text{SimplifyIntegrand}[(d + e*x)^{m+1}*(a + b*\text{Log}[c*\text{RFX}^p])^{n-1}*D[\text{RFX}, x])/RFX, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

$\text{Int}[(a + (b + v)^m)^n], x\_Symbol] :> \text{Dist}[a, \text{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b + v)^m] /; FreeQ[b, x]

### Rule 72

$\text{Int}[(e + f*x)^p/((a + b*x)*(c + d*x))], x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /;$  FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

### Rubi steps

$$\begin{aligned} \int (f + gx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx &= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{3g} - \frac{B \int \frac{2(bc-ad)(f+gx)^3}{(a+bx)(c+dx)} dx}{3g} \\ &= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{3g} - \frac{(2B(bc - ad)) \int \frac{(f+gx)^3}{(a+bx)(c+dx)} dx}{3g} \\ &= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{3g} - \frac{(2B(bc - ad)) \int \left( \frac{g^2(3bdf - bcg - adg)}{b^2d^2} + \frac{g^3x}{bd} \right) dx}{3g} \\ &= -\frac{2B(bc - ad)g(3bdf - bcg - adg)x}{3b^2d^2} - \frac{B(bc - ad)g^2x^2}{3bd} - \frac{2B(bf - ag)^3 \log(c+dx)}{3b^3g} \end{aligned}$$

**Mathematica [A]** time = 0.131152, size = 142, normalized size = 0.93

$$\frac{(f + gx)^3 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right) - \frac{B(b^2d^2g^3x^2(bc-ad)+2bdg^2x(bc-ad)(-adg-bcg+3bdf)+2d^3(bf-ag)^3 \log(a+bx)-2b^3(df-cg)^3 \log(c+dx))}{b^3d^3}}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]),x]
```

```
[Out] ((f + g*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - (B*(2*b*d*(b*c - a*d)*g^2*(3*b*d*f - b*c*g - a*d*g)*x + b^2*d^2*(b*c - a*d)*g^3*x^2 + 2*d^3*(b*f - a*g)^3*Log[a + b*x] - 2*b^3*(d*f - c*g)^3*Log[c + d*x]))/(b^3*d^3)/(3*g)
```

**Maple [B]** time = 0.237, size = 1188, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)
```

```
[Out] -2/3/d*B*g^2*a^2/b^2*c-2/d*B*g*c*f*x-1/d^2*B*g*ln(e*(1/(d*x+c))*a*d-b*c/(d*x+c)+b)^2/d^2)*f*c^2-2/d^2*B*g*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*c^2*f-2*B*g/b^2*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*a^2*f-2/d^2*B*ln(1/(d*x+c))*c^2*f*g-4*B/(a*d-b*c)*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*a*c*f^2+2*B*g/b^2*ln(1/(d*x+c))*a^2*f+2*B*g/b*a*f*x+A*x^2*f*g+B*ln(e*(1/(d*x+c))*a*d-b*c/(d*x+c)+b)^2/d^2)*x*f^2+1/3/d^3*A*c^3*g^2+1/d*A*c*f^2+1/d^3*B*c^3*g^2+1/3*B*g^2*ln(e*(1/(d*x+c))*a*d-b*c/(d*x+c)+b)^2/d^2)*x^3-2/3*B*g^2*a^3/b^3*ln(1/(d*x+c))+2/3*B*g^2*a^3/b^3*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)+B*g*ln(e*(1/(d*x+c))*a*d-b*c/(d*x+c)+b)^2/d^2)*f*x^2+1/d*B*ln(e*(1/(d*x+c))*a*d-b*c/(d*x+c)+b)^2/d^2)*c*f^2+2/d*B*ln(1/(d*x+c))*c*f^2-1/3/d*B*c*g^2*x^2+2/3/d^2*B*c^2*g^2*x+1/3*B*g^2*a/b*x^2+1/3/d^3*B*g^2*ln(e*(1/(d*x+c))*a*d-b*c/(d*x+c)+b)^2/d^2)*c^3+2/3/d^3*B*g^2*c^3*ln(1/(d*x+c))+4/3/d^3*B*g^2*c^3*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)-2*B/b*ln(1/(d*x+c))*a*f^2+2/d*B*g/b*a*f*c+1/3*A*x^3*g^2+A*x*f^2+2/d*B/b/(a*d-b*c)*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*a^2*c^2*g^2-4*B/b/(a*d-b*c)*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*a^2*c*f*g+8/d*B/(a*d-b*c)*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*a*c^2*f*g-4/d^2*B/(a*d-b*c)*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*c^3*b*f*g+4/d*B*g/b*ln(1/(d*x+c))*a*d-b*c/(d*x+c)+b)*a*c*f-2/3*B*g^2*a^2/b^2*x-2/d^2*B*g*c^2*f-1/d^2*A*c^2*f*g-1/3/d^2*B*g^2*a/b*c^2+2/d^3*B/(a*d-b*c)*ln(1/(d*x+c))*a*
```

$$d-b*c/(d*x+c)+b)*c^4*b*g^2+2*d*B/b/(a*d-b*c)*ln(1/(d*x+c)*a*d-b*c/(d*x+c)+b) *a^2*f^2+2/d*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*c^2*b*f^2-4/d^2*B/(a*d-b*c)*ln(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*a*c^3*g^2-2/d^2*B*g^2*a/b*ln(1/(d*x+c)*a*d-b*c/(d*x+c)+b)*c^2$$

**Maxima [B]** time = 1.32247, size = 566, normalized size = 3.72

$$\frac{1}{3} Ag^2x^3 + Afgx^2 + \left( x \log \left( \frac{b^2ex^2}{d^2x^2 + 2cdx + c^2} + \frac{2abex}{d^2x^2 + 2cdx + c^2} + \frac{a^2e}{d^2x^2 + 2cdx + c^2} \right) + \frac{2a \log(bx + a)}{b} - \frac{2c \log(dx + c)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/3*A*g^2*x^3 + A*f*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*f^2 + (x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*f*g + 1/3*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*g^2 + A*f^2*x
```

**Fricas [B]** time = 1.33308, size = 621, normalized size = 4.09

$$Ab^3d^3g^2x^3 + (3Ab^3d^3fg - (Bb^3cd^2 - Bab^2d^3)g^2)x^2 + (3Ab^3d^3f^2 - 6(Bb^3cd^2 - Bab^2d^3)fg + 2(Bb^3c^2d - Ba^2bd^3)g^2)x -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] 1/3*(A*b^3*d^3*g^2*x^3 + (3*A*b^3*d^3*f*g - (B*b^3*c*d^2 - B*a*b^2*d^3)*g^2)*x^2 + (3*A*b^3*d^3*f^2 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*f*g + 2*(B*b^3*c^2*d - B*a^2*b*d^3)*g^2)*x + 2*(3*B*a*b^2*d^3*f^2 - 3*B*a^2*b*d^3*f*g + B*a^3*d^3*g^2)*log(b*x + a) - 2*(3*B*b^3*c*d^2*f^2 - 3*B*b^3*c^2*d*f*g + B*b^3*c^3*g^2)*log(d*x + c) + (B*b^3*d^3*g^2*x^3 + 3*B*b^3*d^3*f*g*x^2 + 3*B*b^3*d^3*f^2*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b^3*d^3)
```

**Sympy [B]** time = 10.3726, size = 719, normalized size = 4.73

$$\frac{Ag^2x^3}{3} + \frac{2Ba(a^2g^2 - 3abfg + 3b^2f^2) \log \left( x + \frac{2Ba^3cd^2g^2 - 6Ba^2bcd^2fg + \frac{2Ba^2d^3(a^2g^2 - 3abfg + 3b^2f^2)}{b} + 2Bab^2c^3g^2 - 6Bab^2c^2dfg + 12Bab^2cd^2f^2 - 2Ba^3d^3g^2 - 6Ba^2bd^3fg + 6Bab^2d^3f^2 + 2Bb^3c^3g^2 - 6Bb^3c^2dfg + 6Bb^3cd^2f^2}{2Ba^3d^3g^2 - 6Ba^2bd^3fg + 6Bab^2d^3f^2 + 2Bb^3c^3g^2 - 6Bb^3c^2dfg + 6Bb^3cd^2f^2} \right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g**2*x**3/3 + 2*B*a*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2)*log(x + (2*B*a*
*3*c*d**2*g**2 - 6*B*a**2*b*c*d**2*f*g + 2*B*a**2*d**3*(a**2*g**2 - 3*a*b*f
*g + 3*b**2*f**2))/b + 2*B*a*b**2*c**3*g**2 - 6*B*a*b**2*c**2*d*f*g + 12*B*a
*b**2*c*d**2*f**2 - 2*B*a*c*d**2*(a**2*g**2 - 3*a*b*f*g + 3*b**2*f**2))/(2*
B*a**3*d**3*g**2 - 6*B*a**2*b*d**3*f*g + 6*B*a*b**2*d**3*f**2 + 2*B*b**3*c*
*3*g**2 - 6*B*b**3*c**2*d*f*g + 6*B*b**3*c*d**2*f**2))/(3*b**3) - 2*B*c*(c*
*2*g**2 - 3*c*d*f*g + 3*d**2*f**2)*log(x + (2*B*a**3*c*d**2*g**2 - 6*B*a**2
*b*c*d**2*f*g + 2*B*a*b**2*c**3*g**2 - 6*B*a*b**2*c**2*d*f*g + 12*B*a*b**2*
c*d**2*f**2 - 2*B*a*b**2*c*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2) + 2*B*b**3
*c**2*(c**2*g**2 - 3*c*d*f*g + 3*d**2*f**2)/d)/(2*B*a**3*d**3*g**2 - 6*B*a*
*2*b*d**3*f*g + 6*B*a*b**2*d**3*f**2 + 2*B*b**3*c**3*g**2 - 6*B*b**3*c**2*d
*f*g + 6*B*b**3*c*d**2*f**2))/(3*d**3) + (B*f**2*x + B*f*g*x**2 + B*g**2*x*
*3/3)*log(e*(a + b*x)**2/(c + d*x)**2) + x**2*(3*A*b*d*f*g + B*a*d*g**2 - B
*b*c*g**2)/(3*b*d) - x*(-3*A*b**2*d**2*f**2 + 2*B*a**2*d**2*g**2 - 6*B*a*b*
d**2*f*g - 2*B*b**2*c**2*g**2 + 6*B*b**2*c*d*f*g)/(3*b**2*d**2)
```

---

**Giac [A]** time = 14.6037, size = 377, normalized size = 2.48

$$\frac{1}{3}(Ag^2 + Bg^2)x^3 + \frac{1}{3}(Bg^2x^3 + 3Bfgx^2 + 3Bf^2x) \log\left(\frac{b^2x^2 + 2abx + a^2}{d^2x^2 + 2cdx + c^2}\right) + \frac{(3Abdfg + 3Bbdfg - Bbcg^2 + Badg^2)}{3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] 1/3*(A*g^2 + B*g^2)*x^3 + 1/3*(B*g^2*x^3 + 3*B*f*g*x^2 + 3*B*f^2*x)*log((b^
2*x^2 + 2*a*b*x + a^2)/(d^2*x^2 + 2*c*d*x + c^2)) + 1/3*(3*A*b*d*f*g + 3*B*
b*d*f*g - B*b*c*g^2 + B*a*d*g^2)*x^2/(b*d) + 2/3*(3*B*a*b^2*f^2 - 3*B*a^2*b
*f*g + B*a^3*g^2)*log(b*x + a)/b^3 - 2/3*(3*B*c*d^2*f^2 - 3*B*c^2*d*f*g + B
*c^3*g^2)*log(-d*x - c)/d^3 + 1/3*(3*A*b^2*d^2*f^2 + 3*B*b^2*d^2*f^2 - 6*B*
b^2*c*d*f*g + 6*B*a*b*d^2*f*g + 2*B*b^2*c^2*g^2 - 2*B*a^2*d^2*g^2)*x/(b^2*d
^2)
```

$$3.265 \quad \int (f + gx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx$$

**Optimal.** Leaf size=104

$$\frac{(f + gx)^2 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}{2g} - \frac{B(bf - ag)^2 \log(a + bx)}{b^2g} - \frac{Bgx(bc - ad)}{bd} + \frac{B(df - cg)^2 \log(c + dx)}{d^2g}$$

[Out] -((B\*(b\*c - a\*d)\*g\*x)/(b\*d)) - (B\*(b\*f - a\*g)^2\*Log[a + b\*x])/(b^2\*g) + ((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2]/(c + d\*x)^2]))/(2\*g) + (B\*(d\*f - c\*g)^2\*Log[c + d\*x])/(d^2\*g)

**Rubi [A]** time = 0.0870162, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$ , Rules used = {2525, 12, 72}

$$\frac{(f + gx)^2 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}{2g} - \frac{B(bf - ag)^2 \log(a + bx)}{b^2g} - \frac{Bgx(bc - ad)}{bd} + \frac{B(df - cg)^2 \log(c + dx)}{d^2g}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2]/(c + d\*x)^2]), x]

[Out] -((B\*(b\*c - a\*d)\*g\*x)/(b\*d)) - (B\*(b\*f - a\*g)^2\*Log[a + b\*x])/(b^2\*g) + ((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2]/(c + d\*x)^2]))/(2\*g) + (B\*(d\*f - c\*g)^2\*Log[c + d\*x])/(d^2\*g)

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol]
:> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol]
:> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

### Rubi steps



$$\begin{aligned}
\int (f + gx) \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right) dx &= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{2g} - \frac{B \int \frac{2(bc - ad)(f + gx)^2}{(a + bx)(c + dx)} dx}{2g} \\
&= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{2g} - \frac{(B(bc - ad)) \int \frac{(f + gx)^2}{(a + bx)(c + dx)} dx}{g} \\
&= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{2g} - \frac{(B(bc - ad)) \int \left( \frac{g^2}{bd} + \frac{(bf - ag)^2}{b(bc - ad)(a + bx)} + \frac{g}{d(-bc - ad)} \right) dx}{g} \\
&= -\frac{B(bc - ad)gx}{bd} - \frac{B(bf - ag)^2 \log(a + bx)}{b^2g} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{2g}
\end{aligned}$$

**Mathematica [A]** time = 0.104722, size = 118, normalized size = 1.13

$$\frac{b \left( d \left( 2Bg^2x(ad - bc) + Abd(f + gx)^2 \right) + bBd^2(f + gx)^2 \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) + 2bB(df - cg)^2 \log(c + dx) \right) - 2Bd^2(bf - ag)^2 \log(a + bx)}{2b^2d^2g}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]),x]

[Out] (-2\*B\*d^2\*(b\*f - a\*g)^2\*Log[a + b\*x] + b\*(d\*(2\*B\*(-b\*c) + a\*d)\*g^2\*x + A\*b\*d\*(f + g\*x)^2) + b\*B\*d^2\*(f + g\*x)^2\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2] + 2\*b\*B\*(d\*f - c\*g)^2\*Log[c + d\*x])/(2\*b^2\*d^2\*g)

**Maple [B]** time = 0.244, size = 656, normalized size = 6.3

$$\frac{Bgac}{bd} - 4 \frac{Bacf}{ad - bc} \ln \left( \frac{ad}{dx + c} - \frac{bc}{dx + c} + b \right) - \frac{Ac^2g}{2d^2} + \frac{Acf}{d} - \frac{Bc^2g}{d^2} + B \ln \left( \frac{e}{d^2} \left( \frac{ad}{dx + c} - \frac{bc}{dx + c} + b \right)^2 \right) xf + \frac{Bgx^2}{2} \ln \left( \frac{e}{d^2} \left( \frac{ad}{dx + c} - \frac{bc}{dx + c} + b \right)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2)),x)

[Out] 1/d\*B\*g/b\*a\*c-4\*B/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a\*c\*f-1/2/d^2\*A\*c^2\*g+1/d\*A\*c\*f-1/d^2\*B\*c^2\*g+B\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*x\*f+1/2\*B\*g\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*x^2-2\*B/b\*ln(1/(d\*x+c))\*a\*f+1/d\*B\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*c\*f+2/d\*B\*ln(1/(d\*x+c))\*c\*f-1/d^2\*B\*g\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*c^2-1/2/d^2\*B\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*c^2\*g-1/d^2\*B\*ln(1/(d\*x+c))\*c^2\*g-1/d\*B\*c\*g\*x+2/d\*B\*g/b\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a\*c+2/d\*B/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*c^2\*b\*f+4/d\*B/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a\*c^2\*g+2\*d\*B/b/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a^2\*f+B\*g/b\*a\*x-2/d^2\*B/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*c^3\*b\*g-2\*B/b/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a^2\*c\*g-B\*g/b^2\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a^2+B\*g/b^2\*ln(1/(d\*x+c))\*a^2+1/2\*A\*x^2\*g+A\*f\*x

**Maxima [B]** time = 1.16359, size = 332, normalized size = 3.19

$$\frac{1}{2} Agx^2 + \left( x \log \left( \frac{b^2ex^2}{d^2x^2 + 2cdx + c^2} + \frac{2abex}{d^2x^2 + 2cdx + c^2} + \frac{a^2e}{d^2x^2 + 2cdx + c^2} \right) + \frac{2a \log(bx + a)}{b} - \frac{2c \log(dx + c)}{d} \right) B$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")
```

```
[Out] 1/2*A*g*x^2 + (x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*B*f + 1/2*(x^2*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 - 2*(b*c - a*d)*x/(b*d))*B*g + A*f*x
```

**Fricas [A]** time = 1.04507, size = 373, normalized size = 3.59

$$\frac{Ab^2d^2gx^2 + 2(Ab^2d^2f - (Bb^2cd - Babd^2)g)x + 2(2Babd^2f - Ba^2d^2g)\log(bx + a) - 2(2Bb^2cdf - Bb^2c^2g)\log(dx + c)}{2b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")
```

```
[Out] 1/2*(A*b^2*d^2*g*x^2 + 2*(A*b^2*d^2*f - (B*b^2*c*d - B*a*b*d^2)*g)*x + 2*(2*B*a*b*d^2*f - B*a^2*d^2*g)*log(b*x + a) - 2*(2*B*b^2*c*d*f - B*b^2*c^2*g)*log(d*x + c) + (B*b^2*d^2*g*x^2 + 2*B*b^2*d^2*f*x)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)))/(b^2*d^2)
```

**Sympy [B]** time = 4.26136, size = 321, normalized size = 3.09

$$\frac{Agx^2}{2} - \frac{Ba(ag - 2bf)\log\left(x + \frac{Ba^2cdg + \frac{Ba^2d^2(ag-2bf)}{b} + Babc^2g - 4Babcdf - Bacd(ag-2bf)}{Ba^2d^2g - 2Babd^2f + Bb^2c^2g - 2Bb^2cdf}\right)}{b^2} + \frac{Bc(cg - 2df)\log\left(x + \frac{Ba^2cdg + Babc^2g - 4Babcdf - Babcd^2g}{Ba^2d^2g - 2Babd^2f + Bb^2c^2g - 2Bb^2cdf}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)
```

```
[Out] A*g*x**2/2 - B*a*(a*g - 2*b*f)*log(x + (B*a**2*c*d*g + B*a**2*d**2*(a*g - 2*b*f)/b + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*c*d*(a*g - 2*b*f))/(B*a**2*d**2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/b**2 + B*c*(c*g - 2*d*f)*log(x + (B*a**2*c*d*g + B*a*b*c**2*g - 4*B*a*b*c*d*f - B*a*b*c*(c*g - 2*d*f) + B*b**2*c**2*(c*g - 2*d*f)/d)/(B*a**2*d**2*g - 2*B*a*b*d**2*f + B*b**2*c**2*g - 2*B*b**2*c*d*f))/d**2 + (B*f*x + B*g*x**2/2)*log(e*(a + b*x)**2/(c + d*x)**2) + x*(A*b*d*f + B*a*d*g - B*b*c*g)/(b*d)
```

**Giac [A]** time = 2.10539, size = 196, normalized size = 1.88

$$\frac{1}{2}(Ag + Bg)x^2 + \frac{1}{2}(Bgx^2 + 2Bfx)\log\left(\frac{b^2x^2 + 2abx + a^2}{d^2x^2 + 2cdx + c^2}\right) + \frac{(Abdf + Bbdf - Bbcg + Badg)x}{bd} + \frac{(2Babf - Ba^2g)\log(dx + c)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] 1/2*(A*g + B*g)*x^2 + 1/2*(B*g*x^2 + 2*B*f*x)*log((b^2*x^2 + 2*a*b*x + a^2)
/(d^2*x^2 + 2*c*d*x + c^2)) + (A*b*d*f + B*b*d*f - B*b*c*g + B*a*d*g)*x/(b*
d) + (2*B*a*b*f - B*a^2*g)*log(b*x + a)/b^2 - (2*B*c*d*f - B*c^2*g)*log(-d*
x - c)/d^2
```

$$3.266 \quad \int \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx$$

**Optimal.** Leaf size=54

$$\frac{B(a+bx) \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{b} - \frac{2B(bc-ad) \log(c+dx)}{bd} + Ax$$

[Out] A\*x + (B\*(a + b\*x)\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/b - (2\*B\*(b\*c - a\*d)\*Log[c + d\*x])/(b\*d)

**Rubi [A]** time = 0.0271851, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {2486, 31}

$$\frac{B(a+bx) \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{b} - \frac{2B(bc-ad) \log(c+dx)}{bd} + Ax$$

Antiderivative was successfully verified.

[In] Int[A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2], x]

[Out] A\*x + (B\*(a + b\*x)\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/b - (2\*B\*(b\*c - a\*d)\*Log[c + d\*x])/(b\*d)

#### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] :> Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned} \int \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) dx &= Ax + B \int \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) dx \\ &= Ax + \frac{B(a+bx) \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{b} - \frac{(2B(bc-ad)) \int \frac{1}{c+dx} dx}{b} \\ &= Ax + \frac{B(a+bx) \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{b} - \frac{2B(bc-ad) \log(c+dx)}{bd} \end{aligned}$$

**Mathematica [A]** time = 0.0232233, size = 54, normalized size = 1.

$$\frac{B(a+bx) \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{b} - \frac{2B(bc-ad) \log(c+dx)}{bd} + Ax$$

Antiderivative was successfully verified.

[In] Integrate[A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2], x]

[Out] A\*x + (B\*(a + b\*x)\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/b - (2\*B\*(b\*c - a\*d)\*Log[c + d\*x])/(b\*d)

**Maple [B]** time = 0.224, size = 233, normalized size = 4.3

$$Ax + B \ln \left( \frac{e}{d^2} \left( \frac{ad}{dx+c} - \frac{bc}{dx+c} + b \right)^2 \right) x + \frac{Bc}{d} \ln \left( \frac{e}{d^2} \left( \frac{ad}{dx+c} - \frac{bc}{dx+c} + b \right)^2 \right) - 2 \frac{B \ln((dx+c)^{-1}) a}{b} + 2 \frac{B \ln((dx+c)^{-1})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2), x)

[Out] A\*x+B\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*x+B/d\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*c-2\*B/b\*ln(1/(d\*x+c))\*a+2\*B/d\*ln(1/(d\*x+c))\*c+2\*B\*d/b/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*a^2-4\*B/(a\*d-b\*c)\*ln(1/(d\*x+c))\*a\*d-b\*c/(d\*x+c)+b)\*a\*c+2\*B/d/(a\*d-b\*c)\*ln(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)\*c^2\*b

**Maxima [A]** time = 1.23, size = 77, normalized size = 1.43

$$\left( x \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + \frac{2 \left( \frac{ae \log(bx+a)}{b} - \frac{ce \log(dx+c)}{d} \right)}{e} \right) B + Ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2), x, algorithm="maxima")

[Out] (x\*log((b\*x + a)^2\*e/(d\*x + c)^2) + 2\*(a\*e\*log(b\*x + a)/b - c\*e\*log(d\*x + c)/d)/e)\*B + A\*x

**Fricas [A]** time = 1.01892, size = 184, normalized size = 3.41

$$\frac{Bbdx \log \left( \frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2} \right) + Abdx + 2Bad \log(bx+a) - 2Bbc \log(dx+c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2), x, algorithm="fricas")

[Out] (B\*b\*d\*x\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + A\*b\*d\*x + 2\*B\*a\*d\*log(b\*x + a) - 2\*B\*b\*c\*log(d\*x + c))/(b\*d)

**Sympy [B]** time = 1.14493, size = 104, normalized size = 1.93

$$Ax + \frac{2Ba \log\left(x + \frac{\frac{2Ba^2d}{b} + 2Bac}{2Bad + 2Bbc}\right)}{b} - \frac{2Bc \log\left(x + \frac{2Bac + \frac{2Bbc^2}{d}}{2Bad + 2Bbc}\right)}{d} + Bx \log\left(\frac{e(a + bx)^2}{(c + dx)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2),x)

[Out] A\*x + 2\*B\*a\*log(x + (2\*B\*a\*\*2\*d/b + 2\*B\*a\*c)/(2\*B\*a\*d + 2\*B\*b\*c))/b - 2\*B\*c\*log(x + (2\*B\*a\*c + 2\*B\*b\*c\*\*2/d)/(2\*B\*a\*d + 2\*B\*b\*c))/d + B\*x\*log(e\*(a + b\*x)\*\*2/(c + d\*x)\*\*2)

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.267 \quad \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx} dx$$

**Optimal.** Leaf size=144

$$\frac{2BPolyLog\left(2, \frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{2BPolyLog\left(2, \frac{d(f+gx)}{df-cg}\right)}{g} + \frac{\log(f+gx)\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{g} - \frac{2B \log(f+gx) \log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g}$$

[Out] (-2\*B\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*Log[f + g\*x])/g + ((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[f + g\*x])/g + (2\*B\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*Log[f + g\*x])/g - (2\*B\*PolyLog[2, (b\*(f + g\*x))/(b\*f - a\*g)])/g + (2\*B\*PolyLog[2, (d\*(f + g\*x))/(d\*f - c\*g)])/g

**Rubi [A]** time = 0.308878, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$ , Rules used = {2524, 12, 2418, 2394, 2393, 2391}

$$\frac{2BPolyLog\left(2, \frac{b(f+gx)}{bf-ag}\right)}{g} + \frac{2BPolyLog\left(2, \frac{d(f+gx)}{df-cg}\right)}{g} + \frac{\log(f+gx)\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{g} - \frac{2B \log(f+gx) \log\left(-\frac{g(a+bx)}{bf-ag}\right)}{g}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(f + g\*x), x]

[Out] (-2\*B\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*Log[f + g\*x])/g + ((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[f + g\*x])/g + (2\*B\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*Log[f + g\*x])/g - (2\*B\*PolyLog[2, (b\*(f + g\*x))/(b\*f - a\*g)])/g + (2\*B\*PolyLog[2, (d\*(f + g\*x))/(d\*f - c\*g)])/g

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_.)), x\_Symbol] :> Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))^(p\_.)\*(RFx\_), x\_Symbol] :> With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_.))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_.)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))])\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rubi steps

$$\begin{aligned}
 \int \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx} dx &= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{g} - \frac{B \int \frac{(c+dx)^2 \left(-\frac{2de(a+bx)^2}{(c+dx)^3} + \frac{2be(a+bx)}{(c+dx)^2}\right) \log(f+gx)}{e(a+bx)^2} dx}{g} \\
 &= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{g} - \frac{B \int \frac{(c+dx)^2 \left(-\frac{2de(a+bx)^2}{(c+dx)^3} + \frac{2be(a+bx)}{(c+dx)^2}\right) \log(f+gx)}{(a+bx)^2} dx}{eg} \\
 &= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{g} - \frac{B \int \left(\frac{2be \log(f+gx)}{a+bx} - \frac{2de \log(f+gx)}{c+dx}\right) dx}{eg} \\
 &= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{g} - \frac{(2bB) \int \frac{\log(f+gx)}{a+bx} dx}{g} + \frac{(2Bd) \int \frac{\log(f+gx)}{c+dx} dx}{g} \\
 &= -\frac{2B \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{g} + \frac{2B \log\left(-\frac{g(c+dx)}{df-cg}\right) \log(f+gx)}{g} \\
 &= -\frac{2B \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{g} + \frac{2B \log\left(-\frac{g(c+dx)}{df-cg}\right) \log(f+gx)}{g} \\
 &= -\frac{2B \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{g} + \frac{2B \log\left(-\frac{g(c+dx)}{df-cg}\right) \log(f+gx)}{g}
 \end{aligned}$$

**Mathematica [A]** time = 0.0591387, size = 119, normalized size = 0.83

$$\frac{-2BPolyLog\left(2, \frac{b(f+gx)}{bf-ag}\right) + 2BPolyLog\left(2, \frac{d(f+gx)}{df-cg}\right) + \log(f+gx) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) - 2B \log\left(\frac{g(a+bx)}{ag-bf}\right)\right) + A + 2B \log\left(\frac{g(c+dx)}{cg-df}\right)}{g}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(f + g\*x), x]

[Out] ((A - 2\*B\*Log[(g\*(a + b\*x))/(-(b\*f) + a\*g)] + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2] + 2\*B\*Log[(g\*(c + d\*x))/(-(d\*f) + c\*g)])\*Log[f + g\*x] - 2\*B\*PolyLog[2, (b\*(f + g\*x))/(b\*f - a\*g)] + 2\*B\*PolyLog[2, (d\*(f + g\*x))/(d\*f - c\*g)]/g

**Maple [B]** time = 0.445, size = 1143, normalized size = 7.9

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x)`

[Out] 
$$-A/g*\ln(1/(d*x+c))+A/g*\ln(1/(d*x+c)*c*g-d*f/(d*x+c)-g)-B/g*\ln(1/(d*x+c))*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)+2*d*B/g*\operatorname{dilog}((1/(d*x+c)*(a*d-b*c)+b)/b)/(a*d-b*c)*a-2*B/g*\operatorname{dilog}((1/(d*x+c)*(a*d-b*c)+b)/b)/(a*d-b*c)*b*c+2*d*B/g*\ln(1/(d*x+c))*\ln((1/(d*x+c)*(a*d-b*c)+b)/b)/(a*d-b*c)*a-2*B/g*\ln(1/(d*x+c))*\ln((1/(d*x+c)*(a*d-b*c)+b)/b)/(a*d-b*c)*b*c+B*\ln((c*g-d*f)/(d*x+c)-g)/(c*g-d*f)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*c-d*B/g*\ln((c*g-d*f)/(d*x+c)-g)/(c*g-d*f)*\ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*f-2*d*B/(c*g-d*f)*\operatorname{dilog}(((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*a*c+2*d^2*B/g/(c*g-d*f)*\operatorname{dilog}(((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*a*f+2*B/(c*g-d*f)*\operatorname{dilog}(((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*b*c^2-2*d*B/g/(c*g-d*f)*\operatorname{dilog}(((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*b*c*f-2*d*B/(c*g-d*f)*\ln((c*g-d*f)/(d*x+c)-g)*\ln(((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*a*c+2*d^2*B/g/(c*g-d*f)*\ln((c*g-d*f)/(d*x+c)-g)*\ln(((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*a*f+2*B/(c*g-d*f)*\ln((c*g-d*f)/(d*x+c)-g)*\ln(((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*b*c^2-2*d*B/g/(c*g-d*f)*\ln((c*g-d*f)/(d*x+c)-g)*\ln(((c*g-d*f)/(d*x+c)-g)*(a*d-b*c)+a*d*g-b*d*f)/(a*d*g-b*d*f))/(a*d-b*c)*b*c*f$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-B \int -\frac{2 \log(bx + a) - 2 \log(dx + c) + \log(e)}{gx + f} dx + \frac{A \log(gx + f)}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="maxima")`

[Out] `-B*integrate(-(2*log(b*x + a) - 2*log(d*x + c) + log(e))/(g*x + f), x) + A*log(g*x + f)/g`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{B \log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right) + A}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f),x, algorithm="fricas")`

[Out] `integral((B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A)/(g*x + f), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))/(g\*x+f),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f),x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)/(g\*x + f), x)

$$3.268 \quad \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2} dx$$

**Optimal.** Leaf size=90

$$\frac{(a+bx)\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{(f+gx)(bf-ag)} + \frac{2B(bc-ad) \log\left(\frac{f+gx}{c+dx}\right)}{(bf-ag)(df-cg)}$$

[Out] ((a + b\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x]^2]))/((b\*f - a\*g)\*(f + g\*x)) + (2\*B\*(b\*c - a\*d)\*Log[(f + g\*x)/(c + d\*x]])/((b\*f - a\*g)\*(d\*f - c\*g))

**Rubi [A]** time = 0.092491, antiderivative size = 117, normalized size of antiderivative = 1.3, number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2525, 12, 72}

$$-\frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{g(f+gx)} + \frac{2B(bc-ad) \log(f+gx)}{(bf-ag)(df-cg)} + \frac{2bB \log(a+bx)}{g(bf-ag)} - \frac{2Bd \log(c+dx)}{g(df-cg)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x]^2)]/(f + g\*x)^2, x]

[Out] (2\*b\*B\*Log[a + b\*x])/(g\*(b\*f - a\*g)) - (A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x]^2)]/(g\*(f + g\*x)) - (2\*B\*d\*Log[c + d\*x])/(g\*(d\*f - c\*g)) + (2\*B\*(b\*c - a\*d)\*Log[f + g\*x])/((b\*f - a\*g)\*(d\*f - c\*g))

#### Rule 2525

```
Int[((a_.) + Log[(c_.)*(RFX_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFX^p])^(n - 1)*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

#### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^2} dx &= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{g(f+gx)} + \frac{B \int \frac{2(bc-ad)}{(a+bx)(c+dx)(f+gx)} dx}{g} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{g(f+gx)} + \frac{(2B(bc-ad)) \int \frac{1}{(a+bx)(c+dx)(f+gx)} dx}{g} \\
&= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{g(f+gx)} + \frac{(2B(bc-ad)) \int \left(\frac{b^2}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2}{(bc-ad)(-df+cg)(c+dx)} + \frac{g^2}{(bf-ag)(df-cg)}\right) dx}{g} \\
&= \frac{2bB \log(a+bx)}{g(bf-ag)} - \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{g(f+gx)} - \frac{2Bd \log(c+dx)}{g(df-cg)} + \frac{2B(bc-ad) \log(f+gx)}{(bf-ag)(df-cg)}
\end{aligned}$$

**Mathematica [A]** time = 0.164156, size = 108, normalized size = 1.2

$$\frac{\frac{2B(b \log(a+bx)(df-cg) + \log(c+dx)(adg-bdf) + g(bc-ad) \log(f+gx))}{(bf-ag)(df-cg)} - \frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{f+gx}}{g}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(f + g\*x)^2,x]

[Out] (-((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(f + g\*x)) + (2\*B\*(b\*(d\*f - c\*g)\*Log[a + b\*x] + (-b\*d\*f) + a\*d\*g)\*Log[c + d\*x] + (b\*c - a\*d)\*g\*Log[f + g\*x]))/((b\*f - a\*g)\*(d\*f - c\*g))/g

**Maple [B]** time = 0.09, size = 388, normalized size = 4.3

$$\frac{dA}{cg-df} \left( \frac{cg}{dx+c} - \frac{df}{dx+c} - g \right)^{-1} + \frac{Bb}{ag-bf} \ln \left( \frac{e}{d^2} \left( \frac{ad}{dx+c} - \frac{bc}{dx+c} + b \right)^2 \right) \left( \frac{cg}{dx+c} - \frac{df}{dx+c} - g \right)^{-1} + \frac{Bda}{(ag-bf)(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f)^2,x)

[Out] d\*A/(1/(d\*x+c)\*c\*g-d\*f/(d\*x+c)-g)/(c\*g-d\*f)+1/(1/(d\*x+c)\*c\*g-d\*f/(d\*x+c)-g)\*b\*B/(a\*g-b\*f)\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)+d/(1/(d\*x+c)\*c\*g-d\*f/(d\*x+c)-g)\*B/(a\*g-b\*f)/(d\*x+c)\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*a-1/(1/(d\*x+c)\*c\*g-d\*f/(d\*x+c)-g)\*B/(a\*g-b\*f)/(d\*x+c)\*ln(e\*(1/(d\*x+c)\*a\*d-b\*c/(d\*x+c)+b)^2/d^2)\*b\*c-2\*d\*B/(a\*c\*g^2-a\*d\*f\*g-b\*c\*f\*g+b\*d\*f^2)\*ln(1/(d\*x+c)\*c\*g-d\*f/(d\*x+c)-g)\*a+2\*B/(a\*c\*g^2-a\*d\*f\*g-b\*c\*f\*g+b\*d\*f^2)\*ln(1/(d\*x+c)\*c\*g-d\*f/(d\*x+c)-g)\*b\*c

**Maxima [B]** time = 1.17778, size = 259, normalized size = 2.88

$$B \left( \frac{2b \log(bx+a)}{bfg-ag^2} - \frac{2d \log(dx+c)}{dfg-cg^2} + \frac{2(bc-ad) \log(gx+f)}{bdf^2+acg^2-(bc+ad)fg} - \frac{\log\left(\frac{b^2ex^2}{d^2x^2+2cdx+c^2} + \frac{2abex}{d^2x^2+2cdx+c^2} + \frac{a^2e}{d^2x^2+2cdx+c^2}\right)}{g^2x+fg} \right) - \frac{A}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f)^2,x, algorithm="maxima")

[Out]  $B*(2*b*log(b*x + a)/(b*f*g - a*g^2) - 2*d*log(d*x + c)/(d*f*g - c*g^2) + 2*(b*c - a*d)*log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)))/(g^2*x + f*g) - A/(g^2*x + f*g)$

**Fricas [B]** time = 22.2962, size = 613, normalized size = 6.81

$$\frac{Abdf^2 + Aacg^2 - (Abc + Aad)fg - 2(Bbdf^2 - Bbcfg + (Bbdfg - Bbcg^2)x) \log(bx + a) + 2(Bbdf^2 - Badfg + (Bbdfg - Bbcg^2)x) \log(gx + f) + (Bbdf^2 - Bbcfg + (Bbdfg - Bbcg^2)x) \log\left(\frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2}\right)}{bdf^3g + acfg^3 - (bc + ad)fg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f)^2,x, algorithm="fricas")

[Out]  $-(A*b*d*f^2 + A*a*c*g^2 - (A*b*c + A*a*d)*f*g - 2*(B*b*d*f^2 - B*b*c*f*g + (B*b*d*f*g - B*b*c*g^2)*x)*log(b*x + a) + 2*(B*b*d*f^2 - B*a*d*f*g + (B*b*d*f*g - B*a*d*g^2)*x)*log(d*x + c) - 2*((B*b*c - B*a*d)*g^2*x + (B*b*c - B*a*d)*f*g)*log(g*x + f) + (B*b*d*f^2 + B*a*c*g^2 - (B*b*c + B*a*d)*f*g)*log\left(\frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2}\right)/(b*d*f^3*g + a*c*f*g^3 - (b*c + a*d)*f^2*g^2 + (b*d*f^2*g^2 + a*c*g^4 - (b*c + a*d)*f*g^3)*x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))/(g\*x+f)\*\*2,x)

[Out] Timed out

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f)^2,x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.269 \quad \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^3} dx$$

**Optimal.** Leaf size=175

$$-\frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{2g(f+gx)^2} + \frac{b^2 B \log(a+bx)}{g(bf-ag)^2} - \frac{B(bc-ad)}{(f+gx)(bf-ag)(df-cg)} + \frac{B(bc-ad) \log(f+gx)(-adg-bcg+2bdf)}{(bf-ag)^2(df-cg)^2} - \frac{B(bc-ad) \log(f+gx)(-adg-bcg+2bdf)}{(bf-ag)^2(df-cg)^2}$$

[Out]  $-\left(\frac{B(b*c - a*d)}{(b*f - a*g)*(d*f - c*g)*(f + g*x)}\right) + \frac{(b^2*B*\text{Log}[a + b*x])}{(g*(b*f - a*g)^2)} - \frac{(A + B*\text{Log}[(e*(a + b*x)^2]/(c + d*x)^2])}{(2*g*(f + g*x)^2)} - \frac{(B*d^2*\text{Log}[c + d*x])}{(g*(d*f - c*g)^2)} + \frac{(B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*\text{Log}[f + g*x])}{((b*f - a*g)^2*(d*f - c*g)^2)}$

**Rubi [A]** time = 0.169762, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2525, 12, 72}

$$-\frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{2g(f+gx)^2} + \frac{b^2 B \log(a+bx)}{g(bf-ag)^2} - \frac{B(bc-ad)}{(f+gx)(bf-ag)(df-cg)} + \frac{B(bc-ad) \log(f+gx)(-adg-bcg+2bdf)}{(bf-ag)^2(df-cg)^2} - \frac{B(bc-ad) \log(f+gx)(-adg-bcg+2bdf)}{(bf-ag)^2(df-cg)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(f + g\*x)^3,x]

[Out]  $-\left(\frac{B(b*c - a*d)}{(b*f - a*g)*(d*f - c*g)*(f + g*x)}\right) + \frac{(b^2*B*\text{Log}[a + b*x])}{(g*(b*f - a*g)^2)} - \frac{(A + B*\text{Log}[(e*(a + b*x)^2]/(c + d*x)^2])}{(2*g*(f + g*x)^2)} - \frac{(B*d^2*\text{Log}[c + d*x])}{(g*(d*f - c*g)^2)} + \frac{(B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*\text{Log}[f + g*x])}{((b*f - a*g)^2*(d*f - c*g)^2)}$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 72

Int[((e\_.) + (f\_.)\*(x\_.))^(p\_.)/(((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))), x\_Symbol] :> Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

### Rubi steps

$$\begin{aligned} \int \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f + gx)^3} dx &= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f + gx)^2} + \frac{B \int \frac{2(bc-ad)}{(a+bx)(c+dx)(f+gx)^2} dx}{2g} \\ &= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f + gx)^2} + \frac{(B(bc - ad)) \int \frac{1}{(a+bx)(c+dx)(f+gx)^2} dx}{g} \\ &= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f + gx)^2} + \frac{(B(bc - ad)) \int \left(\frac{b^3}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d^3}{(bc-ad)(-df+cg)^2(c+dx)} + \frac{1}{(bf-ag)(f+gx)^2}\right) dx}{g} \\ &= -\frac{B(bc - ad)}{(bf - ag)(df - cg)(f + gx)} + \frac{b^2 B \log(a + bx)}{g(bf - ag)^2} - \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{2g(f + gx)^2} - \frac{Bd^2 \log(c + dx)}{g(df - cg)^2} \end{aligned}$$

**Mathematica [A]** time = 0.655836, size = 172, normalized size = 0.98

$$\frac{2B(bc - ad) \left( \frac{b^2 \log(a+bx)}{(bc-ad)(bf-ag)^2} + \frac{\frac{d^2 \log(c+dx)}{ad-bc} + \frac{g(cg-df)}{(f+gx)(bf-ag)} - \frac{g \log(f+gx)(adg+bcg-2bdf)}{(bf-ag)^2} \right) - \frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{(f+gx)^2}}{2g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3, x]
```

```
[Out] -(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^2 + 2*B*(b*c - a*d)*((b^2*Log[a + b*x])/((b*c - a*d)*(b*f - a*g)^2) + ((g*(-d*f) + c*g))/((b*f - a*g)*(f + g*x)) + (d^2*Log[c + d*x])/(-(b*c) + a*d) - (g*(-2*b*d*f + b*c*g + a*d*g)*Log[f + g*x])/(b*f - a*g)^2)/(d*f - c*g)^2)/(2*g)
```

**Maple [B]** time = 0.174, size = 1554, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^3, x)
```

```
[Out] -1/2*d^2*A*g/(c*g-d*f)^2/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^2-d^2*A/(c*g-d*f)^2/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)-d^2/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^2/(a*g-b*f)/(d*x+c)^2*B*a+d/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^2/(a*g-b*f)/(d*x+c)^2*B*b*c-1/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^2*b^2*B/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*c*g+d/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^2*b^2*B/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*f+d^2/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^2*g/(a*c*g^2-a*d*f*g-b*c*f*g+b*d*f^2)/(d*x+c)*B*a-d/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^2*g/(a*c*g^2-a*d*f*g-b*c*f*g+b*d*f^2)/(d*x+c)*B*b*c-1/2*d^2/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^2*B/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*a^2*g+d^2/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^2*B/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*a*b*f+1/2/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^2*B/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*b^2*c^2*g-d/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^2*B/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)*b^2*c*f+1/2/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^2*b^2*g*B/(a^2*g^2-2*a*b*f*g+b^2*f^2)*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)+d^2*B/(a^2*c^2*g^4-2*a
```

$$\begin{aligned} &^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)*\ln(1/(d*x+c)*c*g-d*f/(d*x+c)-g)*a^2*g-2*d^2*B/(a^2*c^2*g^4-2*a^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)*\ln(1/(d*x+c)*c*g-d*f/(d*x+c)-g)*a*b*f-B/(a^2*c^2*g^4-2*a^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)*\ln(1/(d*x+c)*c*g-d*f/(d*x+c)-g)*b^2*c^2*g+2*d*B/(a^2*c^2*g^4-2*a^2*c*d*f*g^3+a^2*d^2*f^2*g^2-2*a*b*c^2*f*g^3+4*a*b*c*d*f^2*g^2-2*a*b*d^2*f^3*g+b^2*c^2*f^2*g^2-2*b^2*c*d*f^3*g+b^2*d^2*f^4)*\ln(1/(d*x+c)*c*g-d*f/(d*x+c)-g)*b^2*c*f \end{aligned}$$

**Maxima [B]** time = 1.31545, size = 547, normalized size = 3.13

$$\frac{1}{2} \left( \frac{2b^2 \log(bx+a)}{b^2 f^2 g - 2abfg^2 + a^2 g^3} - \frac{2d^2 \log(dx+c)}{d^2 f^2 g - 2cdfg^2 + c^2 g^3} + \frac{2(2(b^2 cd - abd^2)f - (b^2 c^2 - a^2 d^2)g) \log(gx+f)}{b^2 d^2 f^4 + a^2 c^2 g^4 - 2(b^2 cd + abd^2)f^3 g + (b^2 c^2 + 4abcd + a^2 d^2)f^2 g} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f)^3,x, algorithm="maxima")

[Out]  $\frac{1}{2} * (2 * b^2 * \log(b * x + a) / (b^2 * f^2 * g - 2 * a * b * f * g^2 + a^2 * g^3) - 2 * d^2 * \log(d * x + c) / (d^2 * f^2 * g - 2 * c * d * f * g^2 + c^2 * g^3) + 2 * (2 * (b^2 * c * d - a * b * d^2) * f - (b^2 * c^2 - a^2 * d^2) * g) * \log(g * x + f) / (b^2 * d^2 * f^4 + a^2 * c^2 * g^4 - 2 * (b^2 * c * d + a * b * d^2) * f^3 * g + (b^2 * c^2 + 4 * a * b * c * d + a^2 * d^2) * f^2 * g^2 - 2 * (a * b * c^2 + a^2 * c * d) * f * g^3) - 2 * (b * c - a * d) / (b * d * f^3 + a * c * f * g^2 - (b * c + a * d) * f^2 * g + (b * d * f^2 * g + a * c * g^3 - (b * c + a * d) * f * g^2) * x) - \log(b^2 * e * x^2 / (d^2 * x^2 + 2 * c * d * x + c^2)) + 2 * a * b * e * x / (d^2 * x^2 + 2 * c * d * x + c^2) + a^2 * e / (d^2 * x^2 + 2 * c * d * x + c^2)) / (g^3 * x^2 + 2 * f * g^2 * x + f^2 * g) * B - 1/2 * A / (g^3 * x^2 + 2 * f * g^2 * x + f^2 * g)$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f)^3,x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))/(g\*x+f)\*\*3,x)

[Out] Timed out



**Giac [B]** time = 1.83874, size = 1148, normalized size = 6.56

$$\frac{(2Bb^2cdf - 2Babd^2f - Bb^2c^2g + Ba^2d^2g)\log(gx + f)}{b^2d^2f^4 - 2b^2cdf^3g - 2abd^2f^3g + b^2c^2f^2g^2 + 4abcdf^2g^2 + a^2d^2f^2g^2 - 2abc^2fg^3 - 2a^2cdfg^3 + a^2c^2g^4} - \frac{B\log\left(\frac{b^2}{d^2}\right)}{2(g^3x^2 + 2g^2x + g)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f)^3,x, algorithm="giac")

[Out] (2\*B\*b^2\*c\*d\*f - 2\*B\*a\*b\*d^2\*f - B\*b^2\*c^2\*g + B\*a^2\*d^2\*g)\*log(g\*x + f)/(b^2\*d^2\*f^4 - 2\*b^2\*c\*d\*f^3\*g - 2\*a\*b\*d^2\*f^3\*g + b^2\*c^2\*f^2\*g^2 + 4\*a\*b\*c\*d\*f^2\*g^2 + a^2\*d^2\*f^2\*g^2 - 2\*a\*b\*c^2\*f\*g^3 - 2\*a^2\*c\*d\*f\*g^3 + a^2\*c^2\*g^4) - 1/2\*B\*log((b^2\*x^2 + 2\*a\*b\*x + a^2)/(d^2\*x^2 + 2\*c\*d\*x + c^2))/(g^3\*x^2 + 2\*f\*g^2\*x + f^2\*g) - 1/2\*(2\*B\*b^2\*c\*d\*f - 2\*B\*a\*b\*d^2\*f - B\*b^2\*c^2\*g + B\*a^2\*d^2\*g)\*log(abs(b\*d\*x^2 + b\*c\*x + a\*d\*x + a\*c))/(b^2\*d^2\*f^4 - 2\*b^2\*c\*d\*f^3\*g - 2\*a\*b\*d^2\*f^3\*g + b^2\*c^2\*f^2\*g^2 + 4\*a\*b\*c\*d\*f^2\*g^2 + a^2\*d^2\*f^2\*g^2 - 2\*a\*b\*c^2\*f\*g^3 - 2\*a^2\*c\*d\*f\*g^3 + a^2\*c^2\*g^4) - 1/2\*(2\*B\*b\*c\*g^2\*x - 2\*B\*a\*d\*g^2\*x + A\*b\*d\*f^2 + B\*b\*d\*f^2 - A\*b\*c\*f\*g + B\*b\*c\*f\*g - A\*a\*d\*f\*g - 3\*B\*a\*d\*f\*g + A\*a\*c\*g^2 + B\*a\*c\*g^2)/(b\*d\*f^2\*g^3\*x^2 - b\*c\*f\*g^4\*x^2 - a\*d\*f\*g^4\*x^2 + a\*c\*g^5\*x^2 + 2\*b\*d\*f^3\*g^2\*x - 2\*b\*c\*f^2\*g^3\*x - 2\*a\*d\*f^2\*g^3\*x + 2\*a\*c\*f\*g^4\*x + b\*d\*f^4\*g - b\*c\*f^3\*g^2 - a\*d\*f^3\*g^2 + a\*c\*f^2\*g^3) + 1/2\*(2\*B\*b^3\*c\*d^2\*f^2 - 2\*B\*a\*b^2\*d^3\*f^2 - 2\*B\*b^3\*c^2\*d\*f\*g + 2\*B\*a^2\*b\*d^3\*f\*g + B\*b^3\*c^3\*g^2 - B\*a\*b^2\*c^2\*d\*g^2 + B\*a^2\*b\*c\*d^2\*g^2 - B\*a^3\*d^3\*g^2)\*log(abs((2\*b\*d\*x + b\*c + a\*d - abs(-b\*c + a\*d))/(2\*b\*d\*x + b\*c + a\*d + abs(-b\*c + a\*d))))/(b^2\*d^2\*f^4\*g - 2\*b^2\*c\*d\*f^3\*g^2 - 2\*a\*b\*d^2\*f^3\*g^2 + b^2\*c^2\*f^2\*g^3 + 4\*a\*b\*c\*d\*f^2\*g^3 + a^2\*d^2\*f^2\*g^3 - 2\*a\*b\*c^2\*f\*g^4 - 2\*a^2\*c\*d\*f\*g^4 + a^2\*c^2\*g^5)\*abs(-b\*c + a\*d)

$$3.270 \quad \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4} dx$$

**Optimal.** Leaf size=277

$$\frac{2B(bc-ad)\log(f+gx)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2))}{3(bf-ag)^3(df-cg)^3} - \frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{3g(f+gx)^3} + \frac{2b^3B \log(a+bx)}{3g(bf-ag)^3}$$

[Out]  $-(B*(b*c - a*d))/(3*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (2*B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (2*b^3*B*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(3*g*(f + g*x)^3) - (2*B*d^3*Log[c + d*x])/(3*g*(d*f - c*g)^3) + (2*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)$

**Rubi [A]** time = 0.327443, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2525, 12, 72}

$$\frac{2B(bc-ad)\log(f+gx)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2))}{3(bf-ag)^3(df-cg)^3} - \frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}{3g(f+gx)^3} + \frac{2b^3B \log(a+bx)}{3g(bf-ag)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(f + g\*x)^4, x]

[Out]  $-(B*(b*c - a*d))/(3*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (2*B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (2*b^3*B*Log[a + b*x])/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(3*g*(f + g*x)^3) - (2*B*d^3*Log[c + d*x])/(3*g*(d*f - c*g)^3) + (2*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[f + g*x])/(3*(b*f - a*g)^3*(d*f - c*g)^3)$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 72

Int[((e\_.) + (f\_.)\*(x\_.))^(p\_.)/(((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))), x\_Symbol] :> Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

### Rubi steps

$$\int \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^4} dx = -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3} + \frac{B \int \frac{2(bc-ad)}{(a+bx)(c+dx)(f+gx)^3} dx}{3g}$$

$$= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3} + \frac{(2B(bc-ad)) \int \frac{1}{(a+bx)(c+dx)(f+gx)^3} dx}{3g}$$

$$= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{3g(f+gx)^3} + \frac{(2B(bc-ad)) \int \left(\frac{b^4}{(bc-ad)(bf-ag)^3(a+bx)} + \frac{d^4}{(bc-ad)(-df+cg)^3(c+dx)} + \frac{1}{(bf-ag)^3}\right) dx}{3g}$$

$$= -\frac{B(bc-ad)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{2B(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{2b^3B \log(a+bx)}{3g(bf-ag)^3} - \frac{2d^3B \log(c+dx)}{3g(bf-ag)^3}$$

**Mathematica [A]** time = 0.891529, size = 263, normalized size = 0.95

$$\frac{2B(bc-ad) \left( \frac{g \log(f+gx)(a^2d^2g^2+abdgcg-3df)+b^2(c^2g^2-3cdfg+3d^2f^2)}{(bf-ag)^3(df-cg)^3} + \frac{b^3 \log(a+bx)}{(bc-ad)(bf-ag)^3} + \frac{d^3 \log(c+dx)}{(bc-ad)(cg-df)^3} + \frac{g(adg+bcg-2bdf)}{(f+gx)(bf-ag)^2(df-cg)^2} - \frac{2d^3 \log(c+dx)}{(bf-ag)^3} \right)}{3g}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^4, x]
```

```
[Out] (-((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(f + g*x)^3) + 2*B*(b*c - a*d)*(-g/(2*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) + (g*(-2*b*d*f + b*c*g + a*d*g))/(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (b^3*Log[a + b*x])/(b*c - a*d)*(b*f - a*g)^3) + (d^3*Log[c + d*x])/(b*c - a*d)*(-(d*f) + c*g)^3) + (g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[f + g*x])/(b*f - a*g)^3*(d*f - c*g)^3))/(3*g)
```

**Maple [B]** time = 0.277, size = 4421, normalized size = 16.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^4, x)
```

```
[Out] d^3*A/(c*g-d*f)^3/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)-2/3*d^3*B/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^3-3*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d^3*f^4*g^2+3*a*b^2*c^3*f^2*g^4-9*a*b^2*c^2*d*f^3*g^3+9*a*b^2*c*d^2*f^4*g^2-3*a*b^2*d^3*f^5*g-b^3*c^3*f^3*g^3+3*b^3*c^2*d*f^4*g^2-3*b^3*c*d^2*f^5*g+b^3*d^3*f^6)*ln(1/(d*x+c)*c*g-d*f/(d*x+c)-g)*a^3*g^2+2/3*B/(a^3*c^3*g^6-3*a^3*c^2*d*f*g^5+3*a^3*c*d^2*f^2*g^4-a^3*d^3*f^3*g^3-3*a^2*b*c^3*f*g^5+9*a^2*b*c^2*d*f^2*g^4-9*a^2*b*c*d^2*f^3*g^3+3*a^2*b*d^3*f^4*g^2+3*a*b^2*c^3*f^2*g^4-9*a*b^2*c^2*d*f^3*g^3+9*a*b^2*c*d^2*f^4*g^2-3*a*b^2*d^3*f^5*g-b^3*c^3*f^3*g^3+3*b^3*c^2*d*f^4*g^2-3*b^3*c*d^2*f^5*g+b^3*d^3*f^6)*ln(1/(d*x+c)*c*g-d*f/(d*x+c)-g)*b^3*c^3*g^2+1/3/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^3*b^3*g^2*B/(a^3*g^3-3*a^2*b*f*g^2+3*a*b^2*f^2*g-b^3*f^3)*ln(e*(1/(d*x+c)*a*d-b*c/(d*x+c)+b)^2/d^2)+1/3*d^2/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^3/(c*g-d*f)*g^2/(a^2*g^2-2*a*b*f*g+b^2*f^2)/(d*x+c)^2*B*a*b*c+d^3*A*g/(c*g-d*f)^3/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^2+1/3*d^3*A*g^2/(c*g-d*f)^3/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)^3+5/3*d^2/(1/(d*x+c)*c*g-d*f/(d*x+c)-g)
```

$$\begin{aligned}
&^3/(a^2g^2-2abfg+b^2f^2)/(dx+c)^3Bb^2cf-5/3d^3/(1/(dx+c)*cg-d \\
&f/(dx+c)-g)^3/(a^2g^2-2abfg+b^2f^2)/(dx+c)^3Babf-2/3d/(1/(dx \\
&+c)*cg-df/(dx+c)-g)^3g^3/(a^2c^2g^4-2a^2c*d*fg^3+a^2d^2f^2g^2-2 \\
&ab*c^2f*g^3+4ab*c*d*f^2g^2-2ab*d^2f^3g+b^2c^2f^2g^2-2b^2c*d* \\
&f^3g+b^2d^2f^4)/(dx+c)*Bb^2c^2-2/3d/(1/(dx+c)*cg-df/(dx+c)-g)^3/ \\
&(a^2g^2-2abfg+b^2f^2)*g/(dx+c)^3Bb^2c^2-5/3d^3/(1/(dx+c)*cg-d* \\
&f/(dx+c)-g)^3/(cg-df)*g^2/(a^2g^2-2abfg+b^2f^2)/(dx+c)^2Bab^2-1/ \\
&3/(1/(dx+c)*cg-df/(dx+c)-g)^3B/(a^3g^3-3a^2b*fg^2+3ab^2f^2g-b^ \\
&3f^3)/(dx+c)^3*ln(e*(1/(dx+c)*ad-bc/(dx+c)+b)^2/d^2)*b^3c^3g^2+1/(1 \\
&/dx+c)*cg-df/(dx+c)-g)^3b^3B/(a^3g^3-3a^2b*fg^2+3ab^2f^2g-b^ \\
&3f^3)/(dx+c)^2*ln(e*(1/(dx+c)*ad-bc/(dx+c)+b)^2/d^2)*c^2g^2-1/(1/(d* \\
&x+c)*cg-df/(dx+c)-g)^3b^3g^2B/(a^3g^3-3a^2b*fg^2+3ab^2f^2g-b^ \\
&3f^3)/(dx+c)*ln(e*(1/(dx+c)*ad-bc/(dx+c)+b)^2/d^2)*c+1/3d^3/(1/(dx+ \\
&c)*cg-df/(dx+c)-g)^3B/(a^3g^3-3a^2b*fg^2+3ab^2f^2g-b^3f^3)/(d* \\
&x+c)^3*ln(e*(1/(dx+c)*ad-bc/(dx+c)+b)^2/d^2)*a^3g^2+d^2/(1/(dx+c)*cg \\
&-df/(dx+c)-g)^3b^3B/(a^3g^3-3a^2b*fg^2+3ab^2f^2g-b^3f^3)/(dx+ \\
&c)^2*ln(e*(1/(dx+c)*ad-bc/(dx+c)+b)^2/d^2)*f^2+2d^3B/(a^3c^3g^6-3a \\
&^3c^2*d*fg^5+3a^3c*d^2f^2g^4-a^3d^3f^3g^3-3a^2b*c^3f*fg^5+9a^2* \\
&b*c^2*d*f^2g^4-9a^2b*c*d^2f^3g^3+3a^2b*d^3f^4g^2+3ab^2c^3f^2g \\
&^4-9ab^2c^2*d*f^3g^3+9ab^2c*d^2f^4g^2-3ab^2d^3f^5g-b^3c^3f^ \\
&3g^3+3b^3c^2*d*f^4g^2-3b^3c*d^2f^5g+b^3d^3f^6)*ln(1/(dx+c)*cg-d \\
&f/(dx+c)-g)*a^2b*fg-2*d*B/(a^3c^3g^6-3a^3c^2*d*fg^5+3a^3c*d^2f^ \\
&2g^4-a^3d^3f^3g^3-3a^2b*c^3f*fg^5+9a^2b*c^2*d*f^2g^4-9a^2b*c*d^2 \\
&f^3g^3+3a^2b*d^3f^4g^2+3ab^2c^3f^2g^4-9ab^2c^2*d*f^3g^3+9a* \\
&b^2c*d^2f^4g^2-3ab^2d^3f^5g-b^3c^3f^3g^3+3b^3c^2*d*f^4g^2-3b \\
&^3c*d^2f^5g+b^3d^3f^6)*ln(1/(dx+c)*cg-df/(dx+c)-g)*b^3c^2*f*g+d/( \\
&1/(dx+c)*cg-df/(dx+c)-g)^3b^3g*B/(a^3g^3-3a^2b*fg^2+3ab^2f^2g \\
&-b^3f^3)/(dx+c)*ln(e*(1/(dx+c)*ad-bc/(dx+c)+b)^2/d^2)*f+2/3d^3/(1/(d \\
&*x+c)*cg-df/(dx+c)-g)^3g^3/(a^2c^2g^4-2a^2c*d*fg^3+a^2d^2f^2g^2 \\
&-2ab*c^2f*g^3+4ab*c*d*f^2g^2-2ab*d^2f^3g+b^2c^2f^2g^2-2b^2c* \\
&d*f^3g+b^2d^2f^4)/(dx+c)*Bab^2+d^3/(1/(dx+c)*cg-df/(dx+c)-g)^3/(a^2 \\
&g^2-2abfg+b^2f^2)*g/(dx+c)^3Bab^2+2d^2B/(a^3c^3g^6-3a^3c^2*d* \\
&fg^5+3a^3c*d^2f^2g^4-a^3d^3f^3g^3-3a^2b*c^3f*fg^5+9a^2b*c^2*d*f \\
&^2g^4-9a^2b*c*d^2f^3g^3+3a^2b*d^3f^4g^2+3ab^2c^3f^2g^4-9ab^ \\
&2c^2*d*f^3g^3+9ab^2c*d^2f^4g^2-3ab^2d^3f^5g-b^3c^3f^3g^3+3b \\
&^3c^2*d*f^4g^2-3b^3c*d^2f^5g+b^3d^3f^6)*ln(1/(dx+c)*cg-df/(dx+c \\
&)-g)*b^3c*f^2-2d^3B/(a^3c^3g^6-3a^3c^2*d*fg^5+3a^3c*d^2f^2g^4-a \\
&^3d^3f^3g^3-3a^2b*c^3f*fg^5+9a^2b*c^2*d*f^2g^4-9a^2b*c*d^2f^3g^ \\
&3+3a^2b*d^3f^4g^2+3ab^2c^3f^2g^4-9ab^2c^2*d*f^3g^3+9ab^2c*d \\
&^2f^4g^2-3ab^2d^3f^5g-b^3c^3f^3g^3+3b^3c^2*d*f^4g^2-3b^3c*d^ \\
&2f^5g+b^3d^3f^6)*ln(1/(dx+c)*cg-df/(dx+c)-g)*ab^2f^2+4/3d/(1/(d* \\
&x+c)*cg-df/(dx+c)-g)^3/(cg-df)*g^2/(a^2g^2-2abfg+b^2f^2)/(dx+c) \\
&^2Bb^2c^2+d^3/(1/(dx+c)*cg-df/(dx+c)-g)^3B/(a^3g^3-3a^2b*fg^2+3 \\
&ab^2f^2g-b^3f^3)/(dx+c)^3*ln(e*(1/(dx+c)*ad-bc/(dx+c)+b)^2/d^2)*a \\
&b^2f^2-d^2/(1/(dx+c)*cg-df/(dx+c)-g)^3B/(a^3g^3-3a^2b*fg^2+3ab \\
&^2f^2g-b^3f^3)/(dx+c)^3*ln(e*(1/(dx+c)*ad-bc/(dx+c)+b)^2/d^2)*b^3c \\
&f^2+4/3d^2/(1/(dx+c)*cg-df/(dx+c)-g)^3g^2/(a^2c^2g^4-2a^2c*d*fg \\
&^3+a^2d^2f^2g^2-2ab*c^2f*g^3+4ab*c*d*f^2g^2-2ab*d^2f^3g+b^2c^ \\
&2f^2g^2-2b^2c*d*f^3g+b^2d^2f^4)/(dx+c)*Bb^2cf+3d^3/(1/(dx+c)*c \\
&g-df/(dx+c)-g)^3/(cg-df)*g/(a^2g^2-2abfg+b^2f^2)/(dx+c)^2Bab \\
&f-3d^2/(1/(dx+c)*cg-df/(dx+c)-g)^3/(cg-df)*g/(a^2g^2-2abfg+b^2 \\
&f^2)/(dx+c)^2Bb^2cf-d^3/(1/(dx+c)*cg-df/(dx+c)-g)^3B/(a^3g^3-3* \\
&a^2b*fg^2+3ab^2f^2g-b^3f^3)/(dx+c)^3*ln(e*(1/(dx+c)*ad-bc/(dx+c \\
&)+b)^2/d^2)*a^2b*fg+d/(1/(dx+c)*cg-df/(dx+c)-g)^3B/(a^3g^3-3a^2b* \\
&fg^2+3ab^2f^2g-b^3f^3)/(dx+c)^3*ln(e*(1/(dx+c)*ad-bc/(dx+c)+b)^2 \\
&/d^2)*b^3c^2*f*g-2d/(1/(dx+c)*cg-df/(dx+c)-g)^3b^3B/(a^3g^3-3a^2* \\
&b*fg^2+3ab^2f^2g-b^3f^3)/(dx+c)^2*ln(e*(1/(dx+c)*ad-bc/(dx+c)+b) \\
&^2/d^2)*c*f*g-1/3d^2/(1/(dx+c)*cg-df/(dx+c)-g)^3/(a^2g^2-2abfg+b^ \\
&2f^2)*g/(dx+c)^3Bab*c-4/3d^3/(1/(dx+c)*cg-df/(dx+c)-g)^3g^2/(a^2
\end{aligned}$$

$$\frac{c^2 g^4 - 2 a^2 c d f g^3 + a^2 d^2 f^2 g^2 - 2 a b c^2 f g^3 + 4 a b c d f^2 g^2 - 2 a b d^2 f^3 g + b^2 c^2 f^2 g^2 - 2 b^2 c d f^3 g + b^2 d^2 f^4}{(d x + c) B a b f}$$

**Maxima [B]** time = 1.53729, size = 1215, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f)^4,x, algorithm="maxima")

[Out]  $\frac{1}{3} \cdot (2 b^3 \log(b x + a) / (b^3 f^3 g - 3 a b^2 f^2 g^2 + 3 a^2 b f g^3 - a^3 g^4) - 2 d^3 \log(d x + c) / (d^3 f^3 g - 3 c d^2 f^2 g^2 + 3 c^2 d f g^3 - c^3 g^4) + 2 \cdot (3 (b^3 c d^2 - a b^2 d^3) f^2 - 3 (b^3 c^2 d - a^2 b d^3) f g + (b^3 c^3 - a^3 d^3) g^2) \log(g x + f) / (b^3 d^3 f^6 + a^3 c^3 g^6 - 3 (b^3 c d^2 + a b^2 d^3) f^5 g + 3 (b^3 c^2 d + 3 a b^2 c d^2 + a^2 b d^3) f^4 g^2 - (b^3 c^3 + 9 a b^2 c^2 d + 9 a^2 b c d^2 + a^3 d^3) f^3 g^3 + 3 (a b^2 c^3 + 3 a^2 b c^2 d + a^3 c d^2) f^2 g^4 - 3 (a^2 b c^3 + a^3 c^2 d) f g^5) - (5 (b^2 c d - a b d^2) f^2 - 3 (b^2 c^2 - a^2 d^2) f g + (a b c^2 - a^2 c d) g^2 + 2 (2 (b^2 c d - a b d^2) f g - (b^2 c^2 - a^2 d^2) g^2) x) / (b^2 d^2 f^6 + a^2 c^2 f^2 g^4 - 2 (b^2 c d + a b d^2) f^5 g + (b^2 c^2 + 4 a b c d + a^2 d^2) f^4 g^2 - 2 (a b c^2 + a^2 c d) f^3 g^3 + (b^2 d^2 f^4 g^2 + a^2 c^2 g^6 - 2 (b^2 c d + a b d^2) f^3 g^3 + (b^2 c^2 + 4 a b c d + a^2 d^2) f^2 g^4 - 2 (a b c^2 + a^2 c d) f g^5) x^2 + 2 (b^2 d^2 f^5 g + a^2 c^2 f g^5 - 2 (b^2 c d + a b d^2) f^4 g^2 + (b^2 c^2 + 4 a b c d + a^2 d^2) f^3 g^3 - 2 (a b c^2 + a^2 c d) f^2 g^4) x) - \log(b^2 e x^2 / (d^2 x^2 + 2 c d x + c^2)) / (g^4 x^3 + 3 f g^3 x^2 + 3 f^2 g^2 x + f^3 g) \cdot B - 1/3 A / (g^4 x^3 + 3 f g^3 x^2 + 3 f^2 g^2 x + f^3 g)$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f)^4,x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))/(g\*x+f)\*\*4,x)

[Out] Timed out

**Giac [B]** time = 3.94971, size = 2788, normalized size = 10.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f)^4,x, algorithm="giac")

[Out] 
$$\frac{2}{3} \cdot (3Bb^3cd^2f^2 - 3Bab^2d^3f^2 - 3Bb^3c^2d^2f^2g + 3Ba^2b^2d^3f^2g + Bb^3c^3g^2 - Ba^3d^3g^2) \cdot \log(gx + f) / (b^3d^3f^6 - 3b^3cd^2f^5g - 3ab^2d^3f^5g + 3b^3c^2d^2f^4g^2 + 9ab^2cd^2f^4g^2 + 3a^2b^2d^3f^4g^2 - b^3c^3f^3g^3 - 9ab^2c^2d^2f^3g^3 - 9a^2b^2cd^2f^3g^3 - a^3d^3f^3g^3 + 3ab^2c^3f^2g^4 + 9a^2b^2cd^2f^2g^4 + 3a^3c^3d^2f^2g^4 - 3a^2b^2c^3f^2g^4 - 3a^3c^2d^2f^2g^4 + a^3c^3g^6) - \frac{1}{3} B \log\left(\frac{b^2x^2 + 2abx + a^2}{d^2x^2 + 2cdx + c^2}\right) / (g^4x^3 + 3f^3g^3x^2 + 3f^2g^2x + f^3g) - \frac{1}{3} \cdot (3Bb^3cd^2f^2 - 3Bab^2d^3f^2 - 3Bb^3c^2d^2f^2g + 3Ba^2b^2d^3f^2g + Bb^3c^3g^2 - Ba^3d^3g^2) \cdot \log(\text{abs}(b^2d^2x^2 + b^2cx + a^2dx + a^2c)) / (b^3d^3f^6 - 3b^3cd^2f^5g - 3ab^2d^3f^5g + 3b^3c^2d^2f^4g^2 + 9ab^2cd^2f^4g^2 + 3a^2b^2d^3f^4g^2 - b^3c^3f^3g^3 - 9ab^2c^2d^2f^3g^3 - 9a^2b^2cd^2f^3g^3 - a^3d^3f^3g^3 + 3ab^2c^3f^2g^4 + 9a^2b^2cd^2f^2g^4 + 3a^3c^3d^2f^2g^4 - 3a^2b^2c^3f^2g^4 - 3a^3c^2d^2f^2g^4 + a^3c^3g^6) - \frac{1}{3} \cdot (4Bb^2cd^2f^2g^3x^2 - 4Bab^2d^2f^2g^3x^2 - 2Bb^2c^2g^4x^2 + 2Ba^2d^2g^4x^2 + 9Bb^2cd^2f^2g^2x - 9Bab^2d^2f^2g^2x - 5Bb^2c^2f^2g^3x + 5Ba^2d^2f^2g^3x + B^2ab^2c^2g^4x - B^2a^2cd^2g^4x + Ab^2d^2f^4 + B^2b^2d^2f^4 - 2A^2b^2cd^2f^3g + 3Bb^2cd^2f^3g - 2A^2ab^2d^2f^3g - 7B^2ab^2d^2f^3g + A^2b^2c^2f^2g^2 - 2Bb^2c^2f^2g^2 + 4A^2ab^2cd^2f^2g^2 + 4B^2ab^2cd^2f^2g^2 + A^2a^2d^2f^2g^2 + 4B^2a^2d^2f^2g^2 - 2A^2ab^2c^2f^2g^3 - B^2ab^2c^2f^2g^3 - 2A^2a^2cd^2f^2g^3 - 3B^2a^2cd^2f^2g^3 + A^2a^2c^2g^4 + B^2a^2c^2g^4) / (b^2d^2f^4g^4x^3 - 2b^2cd^2f^3g^5x^3 - 2ab^2d^2f^3g^5x^3 + b^2c^2f^2g^6x^3 + 4ab^2cd^2f^2g^6x^3 + a^2d^2f^2g^6x^3 - 2ab^2c^2f^2g^7x^3 - 2a^2cd^2f^2g^7x^3 + a^2c^2g^8x^3 + 3b^2d^2f^5g^3x^2 - 6b^2cd^2f^4g^4x^2 - 6ab^2d^2f^4g^4x^2 + 3b^2c^2f^3g^5x^2 + 12ab^2cd^2f^3g^5x^2 + 3a^2d^2f^3g^5x^2 - 6ab^2c^2f^2g^6x^2 - 6a^2cd^2f^2g^6x^2 + 3a^2c^2f^2g^7x^2 + 3b^2d^2f^6g^2x - 6b^2cd^2f^5g^3x - 6ab^2d^2f^5g^3x + 3b^2c^2f^4g^4x + 12ab^2cd^2f^4g^4x + 3a^2d^2f^4g^4x - 6ab^2c^2f^3g^5x - 6a^2cd^2f^3g^5x + 3a^2c^2f^2g^6x + b^2d^2f^7g - 2b^2cd^2f^6g^2 - 2ab^2d^2f^6g^2 + b^2c^2f^5g^3 + 4ab^2cd^2f^5g^3 + a^2d^2f^5g^3 - 2ab^2c^2f^4g^4 - 2a^2cd^2f^4g^4 + a^2c^2f^3g^5) + \frac{1}{3} \cdot (2Bb^4cd^3f^3 - 2Bab^3d^4f^3 - 3Bb^4c^2d^2f^2g + 3Ba^2b^2d^4f^2g + 3Bb^4c^3d^2f^2g - 3B^2ab^3c^2d^2f^2g + 3B^2a^2b^2cd^3f^2g - 3B^2a^3b^2d^4f^2g - B^2b^4c^4g^3 + B^2ab^3c^3d^2g^3 - B^2a^3b^2cd^3g^3 + B^2a^4d^4g^3) \cdot \log(\text{abs}((2b^2dx + bc + ad - \text{abs}(-bc + ad)) / (2b^2dx + bc + ad + \text{abs}(-bc + ad)))) / ((b^3d^3f^6g - 3b^3cd^2f^5g^2 - 3ab^2d^3f^5g^2 + 3b^3c^2d^2f^4g^3 + 9ab^2cd^2f^4g^3 + 3a^2b^2d^3f^4g^3 - b^3c^3f^3g^4 - 9ab^2c^2d^2f^3g^4 - 9a^2b^2cd^2f^3g^4 - a^3d^3f^3g^4 + 3ab^2c^3f^2g^5 + 9a^2b^2cd^2f^2g^5 + 3a^3c^3d^2f^2g^5 - 3a^2b^2c^3f^2g^6 - 3a^3c^2d^2f^2g^6 + a^3c^3g^7) \cdot \text{abs}(-bc + ad))$$

$$3.271 \quad \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5} dx$$

**Optimal.** Leaf size=381

$$\frac{B(bc-ad)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2))}{2(f+gx)(bf-ag)^3(df-cg)^3} - \frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf)}{2(bf-ag)^3(df-cg)^3}$$

[Out]  $-(B*(b*c - a*d))/(6*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g))/(4*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/(2*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*Log[a + b*x])/(2*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(4*g*(f + g*x)^4) - (B*d^4*Log[c + d*x])/(2*g*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log[f + g*x])/(2*(b*f - a*g)^4*(d*f - c*g)^4)$

**Rubi [A]** time = 0.553283, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {2525, 12, 72}

$$\frac{B(bc-ad)(a^2d^2g^2-abdg(3df-cg)+b^2(c^2g^2-3cdfg+3d^2f^2))}{2(f+gx)(bf-ag)^3(df-cg)^3} - \frac{B(bc-ad)\log(f+gx)(-adg-bcg+2bdf)}{2(bf-ag)^3(df-cg)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(f + g\*x)^5, x]

[Out]  $-(B*(b*c - a*d))/(6*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g))/(4*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2)))/(2*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (b^4*B*Log[a + b*x])/(2*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(4*g*(f + g*x)^4) - (B*d^4*Log[c + d*x])/(2*g*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log[f + g*x])/(2*(b*f - a*g)^4*(d*f - c*g)^4)$

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*Rfx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

### Rule 72

Int[((e\_.) + (f\_.)\*(x\_.))^(p\_.)/(((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))), x\_Symbol] := Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x]

;/ FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rubi steps

$$\int \frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(f+gx)^5} dx = -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4} + \frac{B \int \frac{2(bc-ad)}{(a+bx)(c+dx)(f+gx)^4} dx}{4g}$$

$$= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4} + \frac{(B(bc-ad)) \int \frac{1}{(a+bx)(c+dx)(f+gx)^4} dx}{2g}$$

$$= -\frac{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{4g(f+gx)^4} + \frac{(B(bc-ad)) \int \left(\frac{b^5}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5}{(bc-ad)(-df+cg)^4(c+dx)} + \frac{g}{(bf-ag)(df-cg)(f+gx)}\right) dx}{2g}$$

$$= -\frac{B(bc-ad)}{6(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)}{4(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{B(bc-ad)(a^2d^2g^2-abdf)}{2(bf-ag)(df-cg)(f+gx)}$$

**Mathematica [A]** time = 1.13547, size = 358, normalized size = 0.94

$$\frac{2B(bc-ad) \left( -\frac{g(a^2d^2g^2+abdgcg-3df)+b^2(c^2g^2-3cdfg+3d^2f^2)}{(f+gx)(bf-ag)^3(df-cg)^3} - \frac{g \log(f+gx)(adg+bcg-2bdf)(a^2d^2g^2-2abd^2fg+b^2(c^2g^2-2cdfg+2d^2f^2))}{(bf-ag)^4(df-cg)^4} + \frac{b^4 \log(c+dx)}{(bc-ad)(df-cg)(f+gx)} \right)}{4g}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(f + g\*x)^5,x]

[Out] (-((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(f + g\*x)^4) + 2\*B\*(b\*c - a\*d)\*(-g/(3\*(b\*f - a\*g)\*(d\*f - c\*g)\*(f + g\*x)^3) + (g\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g))/(2\*(b\*f - a\*g)^2\*(d\*f - c\*g)^2\*(f + g\*x)^2) - (g\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(d\*f - c\*g) + b^2\*(3\*d^2\*f^2 - 3\*c\*d\*f\*g + c^2\*g^2)))/((b\*f - a\*g)^3\*(d\*f - c\*g)^3\*(f + g\*x)) + (b^4\*Log[a + b\*x])/((b\*c - a\*d)\*(b\*f - a\*g)^4) - (d^4\*Log[c + d\*x])/((b\*c - a\*d)\*(d\*f - c\*g)^4) - (g\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g)\*(-2\*a\*b\*d^2\*f\*g + a^2\*d^2\*g^2 + b^2\*(2\*d^2\*f^2 - 2\*c\*d\*f\*g + c^2\*g^2))\*Log[f + g\*x])/((b\*f - a\*g)^4\*(d\*f - c\*g)^4))/(4\*g)

**Maple [B]** time = 0.438, size = 10401, normalized size = 27.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f)^5,x)

[Out] result too large to display

**Maxima [B]** time = 1.92675, size = 2442, normalized size = 6.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f)^5,x, algorithm="maxima")

[Out] 
$$\frac{1}{12} \cdot \frac{(6b^4 \log(bx+a) / (b^4 f^4 g - 4ab^3 f^3 g^2 + 6a^2 b^2 f^2 g^3 - 4a^3 b f g^4 + a^4 g^5) - 6d^4 \log(dx+c) / (d^4 f^4 g - 4cd^3 f^3 g^2 + 6c^2 d^2 f^2 g^3 - 4c^3 d f g^4 + c^4 g^5) + 6(4(b^4 c d^3 - ab^3 d^4) f^3 - 6(b^4 c^2 d^2 - a^2 b^2 d^4) f^2 g + 4(b^4 c^3 d - a^3 b d^4) f g^2 - (b^4 c^4 - a^4 d^4) g^3) \log(gx+f) / (b^4 d^4 f^8 + a^4 c^4 g^8 - 4(b^4 c d^3 + ab^3 d^4) f^7 g + 2(3b^4 c^2 d^2 + 8a^2 b^3 c d^3 + 3a^2 b^2 d^4) f^6 g^2 - 4(b^4 c^3 d + 6a^2 b^3 c^2 d^2 + 6a^2 b^2 c d^3 + a^3 b d^4) f^5 g^3 + (b^4 c^4 + 16a^2 b^3 c^3 d + 36a^2 b^2 c^2 d^2 + 16a^3 b c d^3 + a^4 d^4) f^4 g^4 - 4(ab^3 c^4 + 6a^2 b^2 c^3 d + 6a^3 b c^2 d^2 + a^4 c d^3) f^3 g^5 + 2(3a^2 b^2 c^4 + 8a^3 b c^3 d + 3a^4 c^2 d^2) f^2 g^6 - 4(a^3 b c^4 + a^4 c^3 d) f g^7) - (26(b^3 c d^2 - ab^2 d^3) f^4 - 31(b^3 c^2 d - a^2 b d^3) f^3 g + (11b^3 c^3 + 15a^2 b^2 c^2 d - 15a^2 b c d^2 - 11a^3 d^3) f^2 g^2 - 7(ab^2 c^3 - a^3 c d^2) f g^3 + 2(a^2 b c^3 - a^3 c^2 d) g^4 + 6(3(b^3 c d^2 - ab^2 d^3) f^2 g^2 - 3(b^3 c^2 d - a^2 b d^3) f g^3 + (b^3 c^3 - a^3 d^3) g^4) x^2 + 3(14(b^3 c d^2 - ab^2 d^3) f^3 g - 15(b^3 c^2 d - a^2 b d^3) f^2 g^2 + (5b^3 c^3 + 3a^2 b^2 c^2 d - 3a^2 b c d^2 - 5a^3 d^3) f g^3 - (ab^2 c^3 - a^3 c d^2) g^4) x) / (b^3 d^3 f^9 + a^3 c^3 f^3 g^6 - 3(b^3 c d^2 + ab^2 d^3) f^8 g + 3(b^3 c^2 d + 3a^2 b^2 c d^2 + a^2 b d^3) f^7 g^2 - (b^3 c^3 + 9a^2 b^2 c^2 d + 9a^2 b c d^2 + a^3 d^3) f^6 g^3 + 3(ab^2 c^3 + 3a^2 b c^2 d + a^3 c d^2) f^5 g^4 - 3(a^2 b c^3 + a^3 c^2 d) f^4 g^5 + (b^3 d^3 f^6 g^3 + a^3 c^3 g^9 - 3(b^3 c d^2 + ab^2 d^3) f^5 g^4 + 3(b^3 c^2 d + 3a^2 b^2 c d^2 + a^2 b d^3) f^4 g^5 - (b^3 c^3 + 9a^2 b^2 c^2 d + 9a^2 b c d^2 + a^3 d^3) f^3 g^6 + 3(ab^2 c^3 + 3a^2 b c^2 d + a^3 c d^2) f^2 g^7 - 3(a^2 b c^3 + a^3 c^2 d) f g^8) x^3 + 3(b^3 d^3 f^7 g^2 + a^3 c^3 f g^8 - 3(b^3 c d^2 + ab^2 d^3) f^6 g^3 + 3(b^3 c^2 d + 3a^2 b^2 c d^2 + a^2 b d^3) f^5 g^4 - (b^3 c^3 + 9a^2 b^2 c^2 d + 9a^2 b c d^2 + a^3 d^3) f^4 g^5 + 3(ab^2 c^3 + 3a^2 b c^2 d + a^3 c d^2) f^3 g^6 - 3(a^2 b c^3 + a^3 c^2 d) f^2 g^7) x^2 + 3(b^3 d^3 f^8 g + a^3 c^3 f^2 g^7 - 3(b^3 c d^2 + ab^2 d^3) f^7 g^2 + 3(b^3 c^2 d + 3a^2 b^2 c d^2 + a^2 b d^3) f^6 g^3 - (b^3 c^3 + 9a^2 b^2 c^2 d + 9a^2 b c d^2 + a^3 d^3) f^5 g^4 + 3(ab^2 c^3 + 3a^2 b c^2 d + a^3 c d^2) f^4 g^5 - 3(a^2 b c^3 + a^3 c^2 d) f^3 g^6) x) - 3 \log(b^2 e x^2 / (d^2 x^2 + 2c d x + c^2)) + 2 a b e x / (d^2 x^2 + 2c d x + c^2) + a^2 e / (d^2 x^2 + 2c d x + c^2)) / (g^5 x^4 + 4f g^4 x^3 + 6f^2 g^3 x^2 + 4f^3 g^2 x + f^4 g) ) B - 1/4 A / (g^5 x^4 + 4f g^4 x^3 + 6f^2 g^3 x^2 + 4f^3 g^2 x + f^4 g)$$

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))/(g\*x+f)^5,x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**2/(d*x+c)**2))/(g*x+f)**5,x)
```

```
[Out] Timed out
```

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))/(g*x+f)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.272 \quad \int (f + gx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=869

$$\frac{2B^2g^3 \log\left(\frac{a+bx}{c+dx}\right)(bc-ad)^4}{3b^4d^4} + \frac{2B^2g^3 \log(c+dx)(bc-ad)^4}{3b^4d^4} + \frac{2B^2g^3x(bc-ad)^3}{3b^3d^3} + \frac{B^2g^2(4bdf-3bcg-adg) \log\left(\frac{a+bx}{c+dx}\right)}{b^4d^4}$$

[Out]  $(2*B^2*(b*c - a*d)^3*g^3*x)/(3*b^3*d^3) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*x)/(b^3*d^3) + (B^2*(b*c - a*d)^2*g^3*(c + d*x)^2)/(3*b^2*d^4) - (B*(b*c - a*d)*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b^4*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*b^2*d^4) - (B*(b*c - a*d)*g^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b*d^4) - ((b*f - a*g)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*g) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c - a*d)/(b*(c + d*x)))]/(b^4*d^4) + (2*B^2*(b*c - a*d)^4*g^3*Log[(a + b*x)/(c + d*x)])/(3*b^4*d^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*Log[(a + b*x)/(c + d*x)])/(b^4*d^4) + (2*B^2*(b*c - a*d)^4*g^3*Log[c + d*x])/(3*b^4*d^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - 3*b*c*g - a*d*g)*Log[c + d*x])/(b^4*d^4) + (2*B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - 2*a*b*d*g*(2*d*f - c*g) + b^2*(6*d^2*f^2 - 8*c*d*f*g + 3*c^2*g^2))*Log[c + d*x])/(b^4*d^4) - (2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^4*d^4)$

**Rubi [A]** time = 1.80806, antiderivative size = 973, normalized size of antiderivative = 1.12, number of steps used = 33, number of rules used = 13, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$ , Rules used = {2525, 12, 2528, 2486, 31, 72, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{B^2 \log^2(a + bx)(bf - ag)^4}{b^4g} - \frac{B \log(a + bx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) (bf - ag)^4}{b^4g} - \frac{2B^2 \log(a + bx) \log \left( \frac{b(c+dx)}{bc-ad} \right) (bf - ag)^4}{b^4g}$$

Antiderivative was successfully verified.

[In] Int[(f + gx)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2,x]

[Out]  $(-2*B^2*(b*c - a*d)^2*(b*c + a*d)*g^3*x)/(3*b^3*d^3) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - b*c*g - a*d*g)*x)/(b^3*d^3) - (A*B*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*x)/(b^3*d^3) + (B^2*(b*c - a*d)^2*g^3*x^2)/(3*b^2*d^2) - (2*a^3*B^2*(b*c - a*d)*g^3*Log[a + b*x])/(3*b^4*d) + (a^2*B^2*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*Log[a + b*x])/(b^4*d^2) + (B^2*(b*f - a*g)^4*Log[a + b*x]^2)/(b^4*g) - (B^2*(b*c - a*d)*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b^4*d^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*x^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*b^2*d^2) - (B*(b*c - a*d)*g^3*x^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*b*d) - (B*(b*f - a*g)^4*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b^4*g) + ((f + g*x)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(4*g) + (2*B^2*c^3*(b*c - a*d)*g^3*Log[c + d*x])/(3*b*d^4) - (B^2*c^2*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - a*d*g)*Log[c + d*x])/(b^2$

```
*d^4) + (2*B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - a*b*d*g*(4*d*f - c*g) + b^2*(
6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[c + d*x])/(b^4*d^4) - (2*B^2*(d*f - c
*g)^4*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(d^4*g) + (B*(d*f - c
*g)^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(d^4*g) + (B^2
*(d*f - c*g)^4*Log[c + d*x]^2)/(d^4*g) - (2*B^2*(b*f - a*g)^4*Log[a + b*x]*
Log[(b*(c + d*x))/(b*c - a*d)])/(b^4*g) - (2*B^2*(b*f - a*g)^4*PolyLog[2, -
((d*(a + b*x))/(b*c - a*d))])/(b^4*g) - (2*B^2*(d*f - c*g)^4*PolyLog[2, (b*
(c + d*x))/(b*c - a*d)])/(d^4*g)
```

### Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*
(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.)
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)
^q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^(s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

### Rule 31

```
Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
```

```
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFX, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFX, x] && IntegerQ[p]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps

$$\begin{aligned}
\int (f + gx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2 dx &= \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{4g} - \frac{B \int \frac{2(bc - ad)(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{(a + bx)(c + dx)} dx}{2g} \\
&= \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{4g} - \frac{(B(bc - ad)) \int \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{(a + bx)(c + dx)} dx}{g} \\
&= \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{4g} - \frac{(B(bc - ad)) \int \left( \frac{g^2(a^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))}{(a + bx)(c + dx)} \right) dx}{b^3 d^3} \\
&= \frac{(f + gx)^4 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{4g} - \frac{(B(bc - ad)g^3) \int x^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{bd} \\
&= -\frac{AB(bc - ad)g(a^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{b^3 d^3} \\
&= -\frac{AB(bc - ad)g(a^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{b^3 d^3} \\
&= -\frac{AB(bc - ad)g(a^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{b^3 d^3} \\
&= -\frac{2B^2(bc - ad)^2(bc + ad)g^3 x}{3b^3 d^3} + \frac{B^2(bc - ad)^2 g^2(4bdf - bcg - adg)x}{b^3 d^3} - \frac{AB(bc - ad)g(a^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{b^3 d^3} \\
&= -\frac{2B^2(bc - ad)^2(bc + ad)g^3 x}{3b^3 d^3} + \frac{B^2(bc - ad)^2 g^2(4bdf - bcg - adg)x}{b^3 d^3} - \frac{AB(bc - ad)g(a^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{b^3 d^3} \\
&= -\frac{2B^2(bc - ad)^2(bc + ad)g^3 x}{3b^3 d^3} + \frac{B^2(bc - ad)^2 g^2(4bdf - bcg - adg)x}{b^3 d^3} - \frac{AB(bc - ad)g(a^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{b^3 d^3} \\
&= -\frac{2B^2(bc - ad)^2(bc + ad)g^3 x}{3b^3 d^3} + \frac{B^2(bc - ad)^2 g^2(4bdf - bcg - adg)x}{b^3 d^3} - \frac{AB(bc - ad)g(a^2 d^2 g^2 - abd g(4df - cg) + b^2(6d^2 f^2 - 4cdfg + c^2 g^2))x}{b^3 d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.966501, size = 746, normalized size = 0.86

$$\frac{(f + gx)^4 \left( B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) + A \right)^2 - \frac{2B \left( 6b^4 B(df - cg)^4 \left( 2 \text{PolyLog} \left( 2, \frac{b(c + dx)}{bc - ad} \right) + \log(c + dx) \left( 2 \log \left( \frac{d(a + bx)}{ad - bc} \right) - \log(c + dx) \right) \right) - 6Bd^4(bf - ag)^4 \left( \log(a + bx) \left( \log(a + bx) \right) \right)}{b^3 d^3}}{b^3 d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x]^2)]^2,x]

[Out] ((f + g\*x)^4\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x]^2) - (2\*B\*(6\*A\*b\*d\*(b\*c - a\*d)\*g^2\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(-4\*d\*f + c\*g) + b^2\*(6\*d^2\*f^2 - 4\*c\*d\*f\*g + c^2\*g^2))\*x + 6\*B\*d\*(b\*c - a\*d)\*g^2\*(a^2\*d^2\*g^2 + a\*b\*d\*g\*(-4\*d\*f + c\*g) + b^2\*(6\*d^2\*f^2 - 4\*c\*d\*f\*g + c^2\*g^2))\*(a + b\*x)\*Log[(e\*(a + b\*x)^2)/(c + d\*x]^2) + 3\*b^2\*d^2\*(b\*c - a\*d)\*g^3\*(4\*b\*d\*f - b\*c\*g - a\*d\*g)\*x^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x]^2) + 2\*b^3\*d^3\*(b\*c - a\*d)\*g^4\*x^3\*(A +

$$\begin{aligned}
& B \cdot \text{Log}\left[\frac{e \cdot (a + b \cdot x)^2}{(c + d \cdot x)^2}\right] + 6 \cdot d^4 \cdot (b \cdot f - a \cdot g)^4 \cdot \text{Log}[a + b \cdot x] \cdot (A \\
& + B \cdot \text{Log}\left[\frac{e \cdot (a + b \cdot x)^2}{(c + d \cdot x)^2}\right]) - 12 \cdot B \cdot (b \cdot c - a \cdot d)^2 \cdot g^2 \cdot (a^2 \cdot d^2 \cdot g^2 \\
& + a \cdot b \cdot d \cdot g \cdot (-4 \cdot d \cdot f + c \cdot g) + b^2 \cdot (6 \cdot d^2 \cdot f^2 - 4 \cdot c \cdot d \cdot f \cdot g + c^2 \cdot g^2)) \cdot \text{Log}[c + \\
& d \cdot x] - 6 \cdot b^4 \cdot (d \cdot f - c \cdot g)^4 \cdot (A + B \cdot \text{Log}\left[\frac{e \cdot (a + b \cdot x)^2}{(c + d \cdot x)^2}\right]) \cdot \text{Log}[c + \\
& d \cdot x] + 2 \cdot B \cdot (b \cdot c - a \cdot d) \cdot g^4 \cdot (b \cdot d \cdot (b \cdot c - a \cdot d) \cdot x \cdot (2 \cdot b \cdot c + 2 \cdot a \cdot d - b \cdot d \cdot x) + 2 \cdot \\
& a^3 \cdot d^3 \cdot \text{Log}[a + b \cdot x] - 2 \cdot b^3 \cdot c^3 \cdot \text{Log}[c + d \cdot x]) - 6 \cdot B \cdot (b \cdot c - a \cdot d) \cdot g^3 \cdot (-4 \cdot b \cdot \\
& d \cdot f + b \cdot c \cdot g + a \cdot d \cdot g) \cdot (-a^2 \cdot d^2 \cdot \text{Log}[a + b \cdot x]) + b \cdot (d \cdot (-b \cdot c) + a \cdot d) \cdot x + b \cdot c \\
& ^2 \cdot \text{Log}[c + d \cdot x]) - 6 \cdot B \cdot d^4 \cdot (b \cdot f - a \cdot g)^4 \cdot (\text{Log}[a + b \cdot x] \cdot (\text{Log}[a + b \cdot x] - 2 \cdot \text{Log} \\
& \text{og}\left[\frac{b \cdot (c + d \cdot x)}{b \cdot c - a \cdot d}\right]) - 2 \cdot \text{PolyLog}[2, \frac{d \cdot (a + b \cdot x)}{-(b \cdot c) + a \cdot d}]) \\
& ) + 6 \cdot b^4 \cdot B \cdot (d \cdot f - c \cdot g)^4 \cdot ((2 \cdot \text{Log}\left[\frac{d \cdot (a + b \cdot x)}{-(b \cdot c) + a \cdot d}\right] - \text{Log}[c + d \\
& \cdot x]) \cdot \text{Log}[c + d \cdot x] + 2 \cdot \text{PolyLog}[2, \frac{b \cdot (c + d \cdot x)}{b \cdot c - a \cdot d}])) / (3 \cdot b^4 \cdot d^4) \\
& / (4 \cdot g)
\end{aligned}$$

**Maple [F]** time = 1.77, size = 0, normalized size = 0.

$$\int (gx + f)^3 \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^3\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int((g\*x+f)^3\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [B]** time = 1.9495, size = 3174, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out]  $\begin{aligned}
& 1/4 \cdot A^2 \cdot g^3 \cdot x^4 + A^2 \cdot f \cdot g^2 \cdot x^3 + 3/2 \cdot A^2 \cdot f^2 \cdot g \cdot x^2 + 2 \cdot (x \cdot \log(b^2 \cdot e \cdot x^2 / (d \\
& ^2 \cdot x^2 + 2 \cdot c \cdot d \cdot x + c^2) + 2 \cdot a \cdot b \cdot e \cdot x / (d^2 \cdot x^2 + 2 \cdot c \cdot d \cdot x + c^2) + a^2 \cdot e / (d^2 \cdot \\
& x^2 + 2 \cdot c \cdot d \cdot x + c^2)) + 2 \cdot a \cdot \log(b \cdot x + a) / b - 2 \cdot c \cdot \log(d \cdot x + c) / d) \cdot A \cdot B \cdot f^3 + \\
& 3 \cdot (x^2 \cdot \log(b^2 \cdot e \cdot x^2 / (d^2 \cdot x^2 + 2 \cdot c \cdot d \cdot x + c^2) + 2 \cdot a \cdot b \cdot e \cdot x / (d^2 \cdot x^2 + 2 \cdot c \cdot d \\
& \cdot x + c^2) + a^2 \cdot e / (d^2 \cdot x^2 + 2 \cdot c \cdot d \cdot x + c^2)) - 2 \cdot a^2 \cdot \log(b \cdot x + a) / b^2 + 2 \cdot c \\
& ^2 \cdot \log(d \cdot x + c) / d^2 - 2 \cdot (b \cdot c - a \cdot d) \cdot x / (b \cdot d)) \cdot A \cdot B \cdot f^2 \cdot g + 2 \cdot (x^3 \cdot \log(b^2 \cdot e \cdot x \\
& ^2 / (d^2 \cdot x^2 + 2 \cdot c \cdot d \cdot x + c^2) + 2 \cdot a \cdot b \cdot e \cdot x / (d^2 \cdot x^2 + 2 \cdot c \cdot d \cdot x + c^2) + a^2 \cdot e / \\
& (d^2 \cdot x^2 + 2 \cdot c \cdot d \cdot x + c^2)) + 2 \cdot a^3 \cdot \log(b \cdot x + a) / b^3 - 2 \cdot c^3 \cdot \log(d \cdot x + c) / d^3 \\
& - ((b^2 \cdot c \cdot d - a \cdot b \cdot d^2) \cdot x^2 - 2 \cdot (b^2 \cdot c^2 - a^2 \cdot d^2) \cdot x) / (b^2 \cdot d^2)) \cdot A \cdot B \cdot f \cdot g^2 \\
& + 1/6 \cdot (3 \cdot x^4 \cdot \log(b^2 \cdot e \cdot x^2 / (d^2 \cdot x^2 + 2 \cdot c \cdot d \cdot x + c^2) + 2 \cdot a \cdot b \cdot e \cdot x / (d^2 \cdot x^2 \\
& + 2 \cdot c \cdot d \cdot x + c^2) + a^2 \cdot e / (d^2 \cdot x^2 + 2 \cdot c \cdot d \cdot x + c^2)) - 6 \cdot a^4 \cdot \log(b \cdot x + a) / b \\
& ^4 + 6 \cdot c^4 \cdot \log(d \cdot x + c) / d^4 - (2 \cdot (b^3 \cdot c \cdot d^2 - a \cdot b^2 \cdot d^3) \cdot x^3 - 3 \cdot (b^3 \cdot c^2 \cdot d \\
& - a^2 \cdot b \cdot d^3) \cdot x^2 + 6 \cdot (b^3 \cdot c^3 - a^3 \cdot d^3) \cdot x) / (b^3 \cdot d^3)) \cdot A \cdot B \cdot g^3 + A^2 \cdot f^3 \cdot x \\
& - 1/3 \cdot (6 \cdot a^3 \cdot c \cdot d^3 \cdot g^3 - 3 \cdot (8 \cdot c \cdot d^3 \cdot f \cdot g^2 - c^2 \cdot d^2 \cdot g^3)) \cdot a^2 \cdot b + 2 \cdot (18 \cdot c \cdot d \\
& ^3 \cdot f^2 \cdot g - 6 \cdot c^2 \cdot d^2 \cdot f \cdot g^2 + c^3 \cdot d \cdot g^3) \cdot a \cdot b^2 + (12 \cdot c \cdot d^3 \cdot f^3 \cdot \log(e) - (3 \cdot g \\
& ^3 \cdot \log(e) + 11 \cdot g^3) \cdot c^4 + 12 \cdot (f \cdot g^2 \cdot \log(e) + 3 \cdot f \cdot g^2) \cdot c^3 \cdot d - 18 \cdot (f^2 \cdot g \cdot \log \\
& (e) + 2 \cdot f^2 \cdot g) \cdot c^2 \cdot d^2) \cdot b^3) \cdot B^2 \cdot \log(d \cdot x + c) / (b^3 \cdot d^4) + 2 \cdot (4 \cdot a \cdot b^3 \cdot d^4 \cdot f^3 \\
& - 6 \cdot a^2 \cdot b^2 \cdot d^4 \cdot f^2 \cdot g + 4 \cdot a^3 \cdot b \cdot d^4 \cdot f \cdot g^2 - a^4 \cdot d^4 \cdot g^3 - (4 \cdot c \cdot d^3 \cdot f^3 - \\
& 6 \cdot c^2 \cdot d^2 \cdot f^2 \cdot g + 4 \cdot c^3 \cdot d \cdot f \cdot g^2 - c^4 \cdot g^3) \cdot b^4) \cdot (\log(b \cdot x + a) \cdot \log((b \cdot d \cdot x + \\
& a \cdot d) / (b \cdot c - a \cdot d) + 1) + \text{dilog}(-(b \cdot d \cdot x + a \cdot d) / (b \cdot c - a \cdot d))) \cdot B^2 / (b^4 \cdot d^4) + \\
& 1/12 \cdot (3 \cdot B^2 \cdot b^4 \cdot d^4 \cdot g^3 \cdot x^4 \cdot \log(e)^2 + 4 \cdot (a \cdot b^3 \cdot d^4 \cdot g^3 \cdot \log(e) + (3 \cdot d^4 \cdot f \cdot g
\end{aligned}$

$$\begin{aligned} &^2*\log(e)^2 - c*d^3*g^3*\log(e))*b^4)*B^2*x^3 - 2*((3*g^3*\log(e) - 2*g^3)*a^ \\ &2*b^2*d^4 - 4*(3*d^4*f*g^2*\log(e) - c*d^3*g^3)*a*b^3 - (9*d^4*f^2*g*\log(e)^ \\ &2 - 12*c*d^3*f*g^2*\log(e) + (3*g^3*\log(e) + 2*g^3)*c^2*d^2)*b^4)*B^2*x^2 + \\ &4*((3*g^3*\log(e) - 5*g^3)*a^3*b*d^4 + (5*c*d^3*g^3 - 12*(f*g^2*\log(e) - f*g \\ &^2)*d^4)*a^2*b^2 + (18*d^4*f^2*g*\log(e) - 24*c*d^3*f*g^2 + 5*c^2*d^2*g^3)*a \\ &*b^3 + (3*d^4*f^3*\log(e)^2 - 18*c*d^3*f^2*g*\log(e) - (3*g^3*\log(e) + 5*g^3) \\ &*c^3*d + 12*(f*g^2*\log(e) + f*g^2)*c^2*d^2)*b^4)*B^2*x + 12*(B^2*b^4*d^4*g^ \\ &3*x^4 + 4*B^2*b^4*d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f \\ &^3*x + (4*a*b^3*d^4*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4 \\ &*g^3)*B^2)*\log(b*x + a)^2 + 12*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4*d^4*f*g^2*x \\ &^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*c*d^3*f^3 - 6*c^2*d \\ &^2*f^2*g + 4*c^3*d*f*g^2 - c^4*g^3)*B^2*b^4)*\log(d*x + c)^2 + 4*(3*B^2*b^4* \\ &d^4*g^3*x^4*\log(e) + 2*(a*b^3*d^4*g^3 + (6*d^4*f*g^2*\log(e) - c*d^3*g^3)*b^ \\ &4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^3 + (6*d^4*f^2*g*\log(e) - \\ &4*c*d^3*f*g^2 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g - 4*a^2*b \\ &^2*d^4*f*g^2 + a^3*b*d^4*g^3 + (2*d^4*f^3*\log(e) - 6*c*d^3*f^2*g + 4*c^2*d^ \\ &2*f*g^2 - c^3*d*g^3)*b^4)*B^2*x - ((3*g^3*\log(e) - 11*g^3)*a^4*d^4 + 2*(c*d \\ &^3*g^3 - 6*(f*g^2*\log(e) - 3*f*g^2)*d^4)*a^3*b - 3*(4*c*d^3*f*g^2 - c^2*d^2 \\ &*g^3 - 6*(f^2*g*\log(e) - 2*f^2*g)*d^4)*a^2*b^2 - 6*(2*d^4*f^3*\log(e) - 6*c* \\ &d^3*f^2*g + 4*c^2*d^2*f*g^2 - c^3*d*g^3)*a*b^3)*B^2)*\log(b*x + a) - 4*(3*B^ \\ &2*b^4*d^4*g^3*x^4*\log(e) + 2*(a*b^3*d^4*g^3 + (6*d^4*f*g^2*\log(e) - c*d^3*g \\ &^3)*b^4)*B^2*x^3 + 3*(4*a*b^3*d^4*f*g^2 - a^2*b^2*d^4*g^3 + (6*d^4*f^2*g*\log \\ &(e) - 4*c*d^3*f*g^2 + c^2*d^2*g^3)*b^4)*B^2*x^2 + 6*(6*a*b^3*d^4*f^2*g - 4 \\ &*a^2*b^2*d^4*f*g^2 + a^3*b*d^4*g^3 + (2*d^4*f^3*\log(e) - 6*c*d^3*f^2*g + 4* \\ &c^2*d^2*f*g^2 - c^3*d*g^3)*b^4)*B^2*x + 6*(B^2*b^4*d^4*g^3*x^4 + 4*B^2*b^4* \\ &d^4*f*g^2*x^3 + 6*B^2*b^4*d^4*f^2*g*x^2 + 4*B^2*b^4*d^4*f^3*x + (4*a*b^3*d^ \\ &4*f^3 - 6*a^2*b^2*d^4*f^2*g + 4*a^3*b*d^4*f*g^2 - a^4*d^4*g^3)*B^2)*\log(b*x \\ &+ a))*\log(d*x + c))/(b^4*d^4) \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2 g^3 x^3 + 3 A^2 f g^2 x^2 + 3 A^2 f^2 g x + A^2 f^3 + (B^2 g^3 x^3 + 3 B^2 f g^2 x^2 + 3 B^2 f^2 g x + B^2 f^3) \log \left( \frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(A^2\*g^3\*x^3 + 3\*A^2\*f\*g^2\*x^2 + 3\*A^2\*f^2\*g\*x + A^2\*f^3 + (B^2\*g^3\*x^3 + 3\*B^2\*f\*g^2\*x^2 + 3\*B^2\*f^2\*g\*x + B^2\*f^3)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*(A\*B\*g^3\*x^3 + 3\*A\*B\*f\*g^2\*x^2 + 3\*A\*B\*f^2\*g\*x + A\*B\*f^3)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*3\*(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)

[Out] Timed out



---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (gx + f)^3 \left( B \log \left( \frac{(bx + a)^2 e}{(dx + c)^2} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^3\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="giac")

[Out] integrate((g\*x + f)^3\*(B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2, x)

$$3.273 \quad \int (f + gx)^2 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=542

$$\frac{8B^2(bc - ad)(a^2d^2g^2 - abdg(3df - cg) + b^2(c^2g^2 - 3cdfg + 3d^2f^2)) \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right) + 4B(bc - ad)(a^2d^2g^2 - abdg}{3b^3d^3}$$

[Out] (4\*B^2\*(b\*c - a\*d)^2\*g^2\*x)/(3\*b^2\*d^2) - (4\*B\*(b\*c - a\*d)\*g\*(3\*b\*d\*f - 2\*b\*c\*g - a\*d\*g)\*(a + b\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/(3\*b^3\*d^2) - (2\*B\*(b\*c - a\*d)\*g^2\*(c + d\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/(3\*b\*d^3) - ((b\*f - a\*g)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2)/(3\*b^3\*g) + ((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2)/(3\*g) + (4\*B\*(b\*c - a\*d)\*(a^2\*d^2\*g^2 - a\*b\*d\*g\*(3\*d\*f - c\*g) + b^2\*(3\*d^2\*f^2 - 3\*c\*d\*f\*g + c^2\*g^2))\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[(b\*c - a\*d)/(b\*(c + d\*x)))]/(3\*b^3\*d^3) + (4\*B^2\*(b\*c - a\*d)^3\*g^2\*Log[(a + b\*x)/(c + d\*x)])/(3\*b^3\*d^3) + (4\*B^2\*(b\*c - a\*d)^3\*g^2\*Log[c + d\*x]/(3\*b^3\*d^3) + (8\*B^2\*(b\*c - a\*d)^2\*g\*(3\*b\*d\*f - 2\*b\*c\*g - a\*d\*g)\*Log[c + d\*x]/(3\*b^3\*d^3) + (8\*B^2\*(b\*c - a\*d)\*(a^2\*d^2\*g^2 - a\*b\*d\*g\*(3\*d\*f - c\*g) + b^2\*(3\*d^2\*f^2 - 3\*c\*d\*f\*g + c^2\*g^2))\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x)))]/(3\*b^3\*d^3)

**Rubi [A]** time = 1.19867, antiderivative size = 659, normalized size of antiderivative = 1.22, number of steps used = 29, number of rules used = 13, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$ , Rules used = {2525, 12, 2528, 2486, 31, 72, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$-\frac{8B^2(bf - ag)^3 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{3b^3g} - \frac{8B^2(df - cg)^3 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{3d^3g} + \frac{4a^2B^2g^2(bc - ad) \log(a + bx)}{3b^3d} - \frac{4ABgx(bc - ad)}{3b^3d}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2,x]

[Out] (4\*B^2\*(b\*c - a\*d)^2\*g^2\*x)/(3\*b^2\*d^2) + (4\*A\*B\*(b\*c - a\*d)\*g\*(3\*b\*d\*f - b\*c\*g - a\*d\*g)\*x)/(3\*b^2\*d^2) + (4\*a^2\*B^2\*(b\*c - a\*d)\*g^2\*Log[a + b\*x]/(3\*b^3\*d) + (4\*B^2\*(b\*f - a\*g)^3\*Log[a + b\*x]^2)/(3\*b^3\*g) - (4\*B^2\*(b\*c - a\*d)\*g\*(3\*b\*d\*f - b\*c\*g - a\*d\*g)\*(a + b\*x)\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/(3\*b^3\*d^2) - (2\*B\*(b\*c - a\*d)\*g^2\*x^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/(3\*b\*d) - (4\*B\*(b\*f - a\*g)^3\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/(3\*b^3\*g) + ((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2)/(3\*g) - (4\*B^2\*c^2\*(b\*c - a\*d)\*g^2\*Log[c + d\*x]/(3\*b\*d^3) + (8\*B^2\*(b\*c - a\*d)^2\*g\*(3\*b\*d\*f - b\*c\*g - a\*d\*g)\*Log[c + d\*x]/(3\*b^3\*d^3) - (8\*B^2\*(d\*f - c\*g)^3\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x]/(3\*d^3\*g) + (4\*B\*(d\*f - c\*g)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[c + d\*x])/(3\*d^3\*g) + (4\*B^2\*(d\*f - c\*g)^3\*Log[c + d\*x]^2)/(3\*d^3\*g) - (8\*B^2\*(b\*f - a\*g)^3\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]/(3\*b^3\*g) - (8\*B^2\*(b\*f - a\*g)^3\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))]/(3\*b^3\*g) - (8\*B^2\*(d\*f - c\*g)^3\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]/(3\*d^3\*g)

**Rule 2525**

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] :> Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[(d + e\*x)^(m + 1)\*(

$a + b \cdot \log[c \cdot \text{RFx}^p]^{(n-1)} \cdot D[\text{RFx}, x] / \text{RFx}, x], x] /;$  FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 2528

Int[((a\_) + Log[(c\_)\*(RFx\_)^(p\_)])\*(b\_)^(n\_)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

### Rule 2486

Int[Log[(e\_)\*((f\_)\*((a\_) + (b\_)\*(x\_))^(p\_))\*((c\_) + (d\_)\*(x\_))^(q\_))^(r\_)]^(s\_), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

### Rule 31

Int[((a\_) + (b\_)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 72

Int[((e\_) + (f\_)\*(x\_))^(p\_)/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

### Rule 2524

Int[((a\_) + Log[(c\_)\*(RFx\_)^(p\_)])\*(b\_)^(n\_)/((d\_) + (e\_)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x]/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

### Rule 2418

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_)^(p\_)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

### Rule 2390

Int[((a\_) + Log[(c\_)\*((d\_) + (e\_)\*(x\_))^(n\_)])\*(b\_)^(p\_)\*((f\_) + (g\_)\*(x\_))^(q\_), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rubi steps

$$\begin{aligned}
\int (f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2 dx &= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{3g} - \frac{(2B) \int \frac{2(bc - ad)(f + gx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{(a + bx)(c + dx)}}{3g} \\
&= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{3g} - \frac{(4B(bc - ad)) \int \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{(a + bx)(c + dx)}}{3g} \\
&= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{3g} - \frac{(4B(bc - ad)) \int \left( \frac{g^2(3bdf - bcg - adg)(A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right))}{b^2 d^2} \right)}{3g} \\
&= \frac{(f + gx)^3 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{3g} - \frac{(4B(bc - ad)g^2) \int x \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{3bd} \\
&= -\frac{4AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} - \frac{2B(bc - ad)g^2 x^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{3bd} \\
&= -\frac{4AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} - \frac{4B^2(bc - ad)g(3bdf - bcg - adg)}{3b^3 d^2} \\
&= -\frac{4AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} - \frac{4B^2(bc - ad)g(3bdf - bcg - adg)}{3b^3 d^2} \\
&= \frac{4B^2(bc - ad)^2 g^2 x}{3b^2 d^2} - \frac{4AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} + \frac{4a^2 B^2(bc - ad)}{3b^3 d^2} \\
&= \frac{4B^2(bc - ad)^2 g^2 x}{3b^2 d^2} - \frac{4AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} + \frac{4a^2 B^2(bc - ad)}{3b^3 d^2} \\
&= \frac{4B^2(bc - ad)^2 g^2 x}{3b^2 d^2} - \frac{4AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} + \frac{4a^2 B^2(bc - ad)}{3b^3 d^2} \\
&= \frac{4B^2(bc - ad)^2 g^2 x}{3b^2 d^2} - \frac{4AB(bc - ad)g(3bdf - bcg - adg)x}{3b^2 d^2} + \frac{4a^2 B^2(bc - ad)}{3b^3 d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.53153, size = 497, normalized size = 0.92

$$\frac{(f + gx)^3 \left( B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) + A \right)^2 - \frac{2B \left( 2b^3 B(df - cg)^3 \left( 2 \text{PolyLog} \left( 2, \frac{b(c + dx)}{bc - ad} \right) + \log(c + dx) \left( 2 \log \left( \frac{d(a + bx)}{ad - bc} \right) - \log(c + dx) \right) \right) - 2Bd^3 (bf - ag)^3 \left( \log(a + bx) \right)}{3b^2 d^2}}{3b^2 d^2}}{3b^2 d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2,x]

[Out] ((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2 - (2\*B\*(2\*A\*b\*d\*(b\*c - a\*d)\*g^2\*(3\*b\*d\*f - b\*c\*g - a\*d\*g)\*x + 2\*B\*d\*(b\*c - a\*d)\*g^2\*(3\*b\*d\*f - b\*c\*g - a\*d\*g)\*(a + b\*x)\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2] + b^2\*d^2\*(b\*c - a\*d)\*g^3\*x^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) + 2\*d^3\*(b\*f - a\*g)^3\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) + 4\*B\*(b\*c - a\*d)^2\*g^2\*(-3\*b\*d\*f + b\*c\*g + a\*d\*g)\*Log[c + d\*x] - 2\*b^3\*(d\*f - c\*g)^3\*(A + B\*L

```
og[(e*(a + b*x)^2)/(c + d*x)^2]*Log[c + d*x] - 2*B*(b*c - a*d)*g^3*(a^2*d^
2*Log[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 2*B*d^3*(b*
f - a*g)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
- 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^3*B*(d*f - c*g)^3*((2*L
og[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])))/(b^3*d^3)/(3*g)
```

**Maple [F]** time = 1.569, size = 0, normalized size = 0.

$$\int (gx + f)^2 \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

```
[Out] int((g*x+f)^2*(A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2,x)
```

**Maxima [B]** time = 1.84261, size = 1968, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)^2*(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="maxima
")
```

```
[Out] 1/3*A^2*g^2*x^3 + A^2*f*g*x^2 + 2*(x*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2
) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))
+ 2*a*log(b*x + a)/b - 2*c*log(d*x + c)/d)*A*B*f^2 + 2*(x^2*log(b^2*e*x^2/(
d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2
*x^2 + 2*c*d*x + c^2)) - 2*a^2*log(b*x + a)/b^2 + 2*c^2*log(d*x + c)/d^2 -
2*(b*c - a*d)*x/(b*d))*A*B*f*g + 2/3*(x^3*log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x
+ c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c
^2)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^
2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*g^2 + A^2*f^2*x + 4/3*(2*a
^2*c*d^2*g^2 - (6*c*d^2*f*g - c^2*d*g^2)*a*b - (3*c*d^2*f^2*log(e) + (g^2*1
og(e) + 3*g^2)*c^3 - 3*(f*g*log(e) + 2*f*g)*c^2*d)*b^2)*B^2*log(d*x + c)/(b
^2*d^3) + 8/3*(3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2 - (3*c*d^2*f
^2 - 3*c^2*d*f*g + c^3*g^2)*b^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d
) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^3) + 1/3*(B^2*b^3*d^
3*g^2*x^3*log(e)^2 + (2*a*b^2*d^3*g^2*log(e) + (3*d^3*f*g*log(e)^2 - 2*c*d^
2*g^2*log(e))*b^3)*B^2*x^2 - (4*(g^2*log(e) - g^2)*a^2*b*d^3 - 4*(3*d^3*f*g
*log(e) - 2*c*d^2*g^2)*a*b^2 - (3*d^3*f^2*log(e)^2 - 12*c*d^2*f*g*log(e) +
4*(g^2*log(e) + g^2)*c^2*d)*b^3)*B^2*x + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3
*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a
^3*d^3*g^2)*B^2)*log(b*x + a)^2 + 4*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*
g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*c*d^2*f^2 - 3*c^2*d*f*g + c^3*g^2)*B^2*b^3
)*log(d*x + c)^2 + 4*(B^2*b^3*d^3*g^2*x^3*log(e) + (a*b^2*d^3*g^2 + (3*d^3*
f*g*log(e) - c*d^2*g^2)*b^3)*B^2*x^2 + (6*a*b^2*d^3*f*g - 2*a^2*b*d^3*g^2 +
(3*d^3*f^2*log(e) - 6*c*d^2*f*g + 2*c^2*d*g^2)*b^3)*B^2*x + ((g^2*log(e) -
3*g^2)*a^3*d^3 + (c*d^2*g^2 - 3*(f*g*log(e) - 2*f*g)*d^3)*a^2*b + (3*d^3*f
^2*log(e) - 6*c*d^2*f*g + 2*c^2*d*g^2)*a*b^2)*B^2)*log(b*x + a) - 4*(B^2*b^
3*d^3*g^2*x^3*log(e) + (a*b^2*d^3*g^2 + (3*d^3*f*g*log(e) - c*d^2*g^2)*b^3)
```

$$*B^2*x^2 + (6*a*b^2*d^3*f*g - 2*a^2*b*d^3*g^2 + (3*d^3*f^2*\log(e) - 6*c*d^2*f*g + 2*c^2*d*g^2)*b^3)*B^2*x + 2*(B^2*b^3*d^3*g^2*x^3 + 3*B^2*b^3*d^3*f*g*x^2 + 3*B^2*b^3*d^3*f^2*x + (3*a*b^2*d^3*f^2 - 3*a^2*b*d^3*f*g + a^3*d^3*g^2)*B^2)*\log(b*x + a)*\log(d*x + c))/(b^3*d^3)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(A^2g^2x^2 + 2A^2fgx + A^2f^2 + (B^2g^2x^2 + 2B^2fgx + B^2f^2)\log\left(\frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2}\right)^2 + 2(ABg^2x^2 + 2ABf^2)\log\left(\frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2}\right)\right), x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(A^2\*g^2\*x^2 + 2\*A^2\*f\*g\*x + A^2\*f^2 + (B^2\*g^2\*x^2 + 2\*B^2\*f\*g\*x + B^2\*f^2)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*(A\*B\*g^2\*x^2 + 2\*A\*B\*f\*g\*x + A\*B\*f^2)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2\*(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (gx + f)^2 \left( B \log\left(\frac{(bx + a)^2 e}{(dx + c)^2}\right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="giac")

[Out] integrate((g\*x + f)^2\*(B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2, x)

$$3.274 \quad \int (f + gx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=281

$$\frac{4B^2(bc - ad)(-adg - bcg + 2bdf)\text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{b^2d^2} + \frac{2B(bc - ad)(-adg - bcg + 2bdf) \log\left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + \right)}{b^2d^2}$$

[Out]  $(-2*B*(b*c - a*d)*g*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b^2*d) - ((b*f - a*g)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*b^2*g) + ((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*g) + (2*B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[(b*c - a*d)/(b*(c + d*x))])/(b^2*d^2) + (4*B^2*(b*c - a*d)^2*g*Log[c + d*x])/(b^2*d^2) + (4*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)$

**Rubi [A]** time = 0.959411, antiderivative size = 450, normalized size of antiderivative = 1.6, number of steps used = 25, number of rules used = 12, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.414$ , Rules used = {2525, 12, 2528, 2486, 31, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{4B^2(bf - ag)^2\text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{b^2g} - \frac{4B^2(df - cg)^2\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{d^2g} - \frac{2B(bf - ag)^2 \log(a + bx) \left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + \right)}{b^2g}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

[Out]  $(-2*A*B*(b*c - a*d)*g*x)/(b*d) + (2*B^2*(b*f - a*g)^2*Log[a + b*x]^2)/(b^2*g) - (2*B^2*(b*c - a*d)*g*(a + b*x)*Log[(e*(a + b*x)^2)/(c + d*x)^2])/(b^2*d) - (2*B*(b*f - a*g)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(b^2*g) + ((f + g*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(2*g) + (4*B^2*(b*c - a*d)^2*g*Log[c + d*x])/(b^2*d^2) - (4*B^2*(d*f - c*g)^2*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(d^2*g) + (2*B*(d*f - c*g)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(d^2*g) + (2*B^2*(d*f - c*g)^2*Log[c + d*x]^2)/(d^2*g) - (4*B^2*(b*f - a*g)^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(b^2*g) - (4*B^2*(b*f - a*g)^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(b^2*g) - (4*B^2*(d*f - c*g)^2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(d^2*g)$

#### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFx^p])^(n - 1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2528



Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(Rfx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*Rfx^p])^(n - 1)\*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(Rfx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))])\*(b\_.)/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*(d\_) + (e\_.)\*(x\_)^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rubi steps

$$\begin{aligned}
 \int (f + gx) \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2 dx &= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{2g} - \frac{B \int \frac{2(bc - ad)(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{(a + bx)(c + dx)} dx}{g} \\
 &= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{2g} - \frac{(2B(bc - ad)) \int \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{(a + bx)(c + dx)} dx}{g} \\
 &= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{2g} - \frac{(2B(bc - ad)) \int \left( \frac{g^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{bd} + \frac{(bf - ag)^2 \log^2(a + bx)}{b^2d} \right) dx}{g} \\
 &= \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{2g} - \frac{(2B(bc - ad)g) \int \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right) dx}{bd} \\
 &= -\frac{2AB(bc - ad)gx}{bd} - \frac{2B(bf - ag)^2 \log(a + bx) \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)}{b^2g} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{2g} \\
 &= -\frac{2AB(bc - ad)gx}{bd} - \frac{2B^2(bc - ad)g(a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{b^2d} - \frac{2B(bf - ag)^2 \log^2(a + bx)}{b^2d} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{2g} \\
 &= -\frac{2AB(bc - ad)gx}{bd} - \frac{2B^2(bc - ad)g(a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{b^2d} - \frac{2B(bf - ag)^2 \log^2(a + bx)}{b^2d} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{2g} \\
 &= -\frac{2AB(bc - ad)gx}{bd} - \frac{2B^2(bc - ad)g(a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{b^2d} - \frac{2B(bf - ag)^2 \log^2(a + bx)}{b^2d} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{2g} \\
 &= -\frac{2AB(bc - ad)gx}{bd} - \frac{2B^2(bc - ad)g(a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{b^2d} - \frac{2B(bf - ag)^2 \log^2(a + bx)}{b^2d} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{2g} \\
 &= -\frac{2AB(bc - ad)gx}{bd} + \frac{2B^2(bf - ag)^2 \log^2(a + bx)}{b^2g} - \frac{2B^2(bc - ad)g(a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{b^2d} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{2g} \\
 &= -\frac{2AB(bc - ad)gx}{bd} + \frac{2B^2(bf - ag)^2 \log^2(a + bx)}{b^2g} - \frac{2B^2(bc - ad)g(a + bx) \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right)}{b^2d} + \frac{(f + gx)^2 \left( A + B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) \right)^2}{2g}
 \end{aligned}$$

**Mathematica [A]** time = 0.306174, size = 351, normalized size = 1.25

$$\frac{(f + gx)^2 \left( B \log \left( \frac{e(a + bx)^2}{(c + dx)^2} \right) + A \right)^2 - \frac{4B \left( b^2 B(df - cg)^2 \left( 2 \text{PolyLog} \left( 2, \frac{b(c + dx)}{bc - ad} \right) + \log(c + dx) \left( 2 \log \left( \frac{d(a + bx)}{ad - bc} \right) - \log(c + dx) \right) \right) - Bd^2(bf - ag)^2 \left( \log(a + bx) \left( \log(a + bx) + \log(c + dx) \right) \right)}{b^2d}}{2g}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x]^2)]^2,x]

[Out] ((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x]^2)]^2 - (4\*B\*(A\*b\*d\*(b\*c - a\*d)\*g^2\*x + B\*d\*(b\*c - a\*d)\*g^2\*(a + b\*x)\*Log[(e\*(a + b\*x)^2)/(c + d\*x]^2)

$$2] + d^2*(b*f - a*g)^2*\text{Log}[a + b*x]*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2]) - 2*B*(b*c - a*d)^2*g^2*\text{Log}[c + d*x] - b^2*(d*f - c*g)^2*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])*\text{Log}[c + d*x] - B*d^2*(b*f - a*g)^2*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Log}[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-b*c) + a*d]) + b^2*B*(d*f - c*g)^2*((2*\text{Log}[(d*(a + b*x))/(-b*c) + a*d]) - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)])/(b^2*d^2)/(2*g)$$

**Maple [F]** time = 1.416, size = 0, normalized size = 0.

$$\int (gx + f) \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int((g\*x+f)\*(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [B]** time = 1.59674, size = 1061, normalized size = 3.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out]  $\frac{1}{2}A^2g^2x^2 + 2*(x*\log(b^2*ex^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*ex/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2)) + 2*a*\log(b*x + a)/b - 2*c*\log(dx + c)/d)*A*B*f + (x^2*\log(b^2*ex^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*ex/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))) - 2*a^2*\log(b*x + a)/b^2 + 2*c^2*\log(dx + c)/d^2 - 2*(b*c - a*d)*x/(b*d)) * A*B*g + A^2*f*x - 2*(2*a*c*d*g + (2*c*d*f*\log(e) - (g*\log(e) + 2*g)*c^2)*b)*B^2*\log(dx + c)/(b*d^2) + 4*(2*a*b*d^2*f - a^2*d^2*g - (2*c*d*f - c^2*g)*b^2)*(\log(b*x + a)*\log((b*d*x + a*d)/(b*c - a*d) + 1) + \text{dilog}(-(b*d*x + a*d)/(b*c - a*d))) * B^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*g*x^2*\log(e)^2 + 2*(2*a*b*d^2*g*\log(e) + (d^2*f*\log(e)^2 - 2*c*d*g*\log(e))*b^2)*B^2*x + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*\log(b*x + a)^2 + 4*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*c*d*f - c^2*g)*B^2*b^2)*\log(dx + c)^2 + 4*(B^2*b^2*d^2*g*x^2*\log(e) + 2*(a*b*d^2*g + (d^2*f*\log(e) - c*d*g)*b^2)*B^2*x - ((g*\log(e) - 2*g)*a^2*d^2 - 2*(d^2*f*\log(e) - c*d*g)*a*b)*B^2)*\log(b*x + a) - 4*(B^2*b^2*d^2*g*x^2*\log(e) + 2*(a*b*d^2*g + (d^2*f*\log(e) - c*d*g)*b^2)*B^2*x + 2*(B^2*b^2*d^2*g*x^2 + 2*B^2*b^2*d^2*f*x + (2*a*b*d^2*f - a^2*d^2*g)*B^2)*\log(b*x + a))*\log(dx + c))/(b^2*d^2)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^2gx + A^2f + (B^2gx + B^2f) \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right)^2 + 2(ABgx + ABf) \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(A^2\*g\*x + A^2\*f + (B^2\*g\*x + B^2\*f)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*(A\*B\*g\*x + A\*B\*f)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (gx + f) \left( B \log \left( \frac{(bx + a)^2 e}{(dx + c)^2} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="giac")

[Out] integrate((g\*x + f)\*(B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2, x)

$$3.275 \quad \int \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx$$

**Optimal.** Leaf size=129

$$\frac{8B^2(bc-ad)\text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd} + \frac{4B(bc-ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{bd} + \frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}{b}$$

[Out] ((a + b\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])/b + (4\*B\*(b\*c - a\*d)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[(b\*c - a\*d)/(b\*(c + d\*x))]))/(b\*d) + (8\*B^2\*(b\*c - a\*d)\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*d)

**Rubi [A]** time = 0.773903, antiderivative size = 252, normalized size of antiderivative = 1.95, number of steps used = 22, number of rules used = 10, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$ , Rules used = {2523, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{8aB^2\text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{b} + \frac{8B^2c\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{d} + \frac{4aB\log(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{b} - \frac{4Bc\log(c+dx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

[Out] (-4\*a\*B^2\*Log[a + b\*x]^2)/b + (4\*a\*B\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/b + x\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2 + (8\*B^2\*c\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x])/d - (4\*B\*c\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[c + d\*x])/d - (4\*B^2\*c\*Log[c + d\*x]^2)/d + (8\*a\*B^2\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/b + (8\*a\*B^2\*PolyLog[2, -(d\*(a + b\*x))/(b\*c - a\*d)])/b + (8\*B^2\*c\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/d

#### Rule 2523

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.), x\_Symbol] := Simp[x\*(a + b\*Log[c\*RFx^p])^n, x] - Dist[b\*n\*p, Int[SimplifyIntegrand[(x\*(a + b\*Log[c\*RFx^p])^(n-1)\*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 2528

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)\*(RGx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

#### Rule 2524

Int[((a\_.) + Log[(c\_.)\*(RFx\_)^(p\_.)]\*(b\_.))^(n\_.)/((d\_.) + (e\_.)\*(x\_)), x\_Symbol] := Simp[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^n)/e, x] - Dist[(b\*n\*p)/e, Int[(Log[d + e\*x]\*(a + b\*Log[c\*RFx^p])^(n-1)\*D[RFx, x])/RFx, x], x] /;

FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

#### Rule 2418

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*(RFx\_), x\_Symbol] := With[{u = ExpandIntegrand[(a + b\*Log[c\*(d + e\*x)^n])^p, RFx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x] && IntegerQ[p]

#### Rule 2390

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_) + (g\_.)\*(x\_))^(q\_.), x\_Symbol] := Dist[1/e, Subst[Int[((f\*x)/d)^q\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e\*f - d\*g, 0]

#### Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rubi steps

$$\begin{aligned}
\int \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 dx &= x \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 - (2B) \int \frac{2(bc-ad)x \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{(a+bx)(c+dx)} dx \\
&= x \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 - (4B(bc-ad)) \int \frac{x \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{(a+bx)(c+dx)} dx \\
&= x \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 - (4B(bc-ad)) \int \left( -\frac{a \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{(bc-ad)(a+bx)} + \frac{c \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{(bc-ad)(c+dx)} \right) dx \\
&= x \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 + (4aB) \int \frac{A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{a+bx} dx - (4Bc) \int \frac{A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{c+dx} dx \\
&= \frac{4aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 - \frac{4Bc \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{b} \\
&= \frac{4aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 - \frac{4Bc \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{b} \\
&= \frac{4aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 - \frac{4Bc \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{b} \\
&= \frac{4aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 - \frac{4Bc \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{b} \\
&= \frac{4aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 + \frac{8B^2c \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right)}{b} \\
&= -\frac{4aB^2 \log^2(a+bx)}{b} + \frac{4aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2 \\
&= -\frac{4aB^2 \log^2(a+bx)}{b} + \frac{4aB \log(a+bx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)}{b} + x \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2
\end{aligned}$$

**Mathematica [A]** time = 0.162735, size = 220, normalized size = 1.71

$$\frac{4B \left( -aBd \left( \log(a+bx) \left( \log(a+bx) - 2 \log \left( \frac{b(c+dx)}{bc-ad} \right) \right) - 2 \text{PolyLog} \left( 2, \frac{d(a+bx)}{ad-bc} \right) \right) + bBc \left( 2 \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) + \log(c+dx) \right) \right)}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

[Out] x\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2 + (4\*B\*(a\*d\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) - b\*c\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[c + d\*x] - a\*B\*d\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c + a\*d)]) + b\*B\*c\*((2\*Log[(d\*(a + b\*x))/(-b\*c + a\*d)] - Log[c + d\*x])\*Log[c + d\*x] + 2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])))/(b\*d)

**Maple [F]** time = 1.319, size = 0, normalized size = 0.

$$\int \left( A + B \ln \left( \frac{e (bx + a)^2}{(dx + c)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2 \left( x \log \left( \frac{(bx + a)^2 e}{(dx + c)^2} \right) + \frac{2 \left( \frac{ae \log(bx+a)}{b} - \frac{ce \log(dx+c)}{d} \right)}{e} \right) AB + A^2 x + B^2 \left( \frac{4 (bdx \log(bx + a)^2 + (bdx + bc) \log(dx + c)^2 - (bdx + c)/d)/e}{bd} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out] 2\*(x\*log((b\*x + a)^2\*e/(d\*x + c)^2) + 2\*(a\*e\*log(b\*x + a)/b - c\*e\*log(d\*x + c)/d)/e)\*A\*B + A^2\*x + B^2\*(4\*(b\*d\*x\*log(b\*x + a)^2 + (b\*d\*x + b\*c)\*log(d\*x + c)^2 - (b\*d\*x\*log(e) + 2\*(b\*d\*x + a\*d)\*log(b\*x + a))\*log(d\*x + c))/(b\*d) + integrate(((log(e)^2 + 4\*log(e))\*b^2\*d\*x^2 + a\*b\*c\*log(e)^2 + (b^2\*c\*log(e)^2 + (log(e)^2 + 4\*log(e))\*a\*b\*d)\*x + 4\*(b^2\*d\*x^2\*log(e) + a\*b\*c\*log(e) + 2\*a^2\*d + (a\*b\*d\*(log(e) + 4) + b^2\*c\*(log(e) - 2))\*x)\*log(b\*x + a))/(b^2\*d\*x^2 + a\*b\*c + (b^2\*c + a\*b\*d)\*x), x))

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( B^2 \log \left( \frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2} \right)^2 + 2 A B \log \left( \frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2} \right) + A^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(B^2\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*A\*B\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + A^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)



[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2, x)

$$3.276 \quad \int \frac{\left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{f+gx} dx$$

**Optimal.** Leaf size=285

$$\frac{4B \text{PolyLog} \left( 2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)} \right) \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}{g} - \frac{4B \text{PolyLog} \left( 2, \frac{d(a+bx)}{b(c+dx)} \right) \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}{g} - \frac{8B^2 \text{PolyLog} \left( 3, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)} \right)}{g}$$

[Out] -(((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2\*Log[(b\*c - a\*d)/(b\*(c + d\*x))])/g) + ((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2\*Log[1 - ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/g - (4\*B\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/g + (4\*B\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*PolyLog[2, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/g + (8\*B^2\*PolyLog[3, (d\*(a + b\*x))/(b\*(c + d\*x))])/g - (8\*B^2\*PolyLog[3, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/g

**Rubi [B]** time = 5.84608, antiderivative size = 2126, normalized size of antiderivative = 7.46, number of steps used = 44, number of rules used = 21, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.677$ , Rules used = {2524, 12, 2528, 2418, 2390, 2301, 2394, 2393, 2391, 6688, 6742, 2500, 2433, 2375, 2317, 2374, 6589, 2440, 2437, 2435, 2315}

result too large to display

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(f + g\*x), x]

[Out] (-4\*A\*B\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*Log[f + g\*x])/g - (B^2\*Log[(a + b\*x)^2]^2\*Log[f + g\*x])/g - (B^2\*Log[(c + d\*x)^(-2)]^2\*Log[f + g\*x])/g + (4\*B^2\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*(Log[(a + b\*x)^2] + Log[(c + d\*x)^(-2)] - Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[f + g\*x])/g + ((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2\*Log[f + g\*x])/g + (8\*B^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x]\*Log[f + g\*x])/g - (4\*B^2\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*(Log[(c + d\*x)^(-2)] + 2\*Log[c + d\*x])\*Log[f + g\*x])/g + (8\*B^2\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]\*Log[f + g\*x])/g + (4\*A\*B\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*Log[f + g\*x])/g - (4\*B^2\*(2\*Log[a + b\*x] - Log[(a + b\*x)^2])\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*Log[f + g\*x])/g - (4\*B^2\*(Log[(a + b\*x)^2] + Log[(c + d\*x)^(-2)] - Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*Log[f + g\*x])/g + (B^2\*Log[(a + b\*x)^2]^2\*Log[(b\*(f + g\*x))/(b\*f - a\*g)]/g + (B^2\*Log[(c + d\*x)^(-2)]^2\*Log[(d\*(f + g\*x))/(d\*f - c\*g)]/g + (4\*B^2\*(Log[(b\*(c + d\*x))/(b\*c - a\*d)] + Log[(b\*f - a\*g)/(b\*(f + g\*x))]) - Log[((b\*f - a\*g)\*(c + d\*x))/((b\*c - a\*d)\*(f + g\*x))])\*Log[-((b\*c - a\*d)\*(f + g\*x))/((d\*f - c\*g)\*(a + b\*x))]^2)/g - (4\*B^2\*(Log[(b\*(c + d\*x))/(b\*c - a\*d)] - Log[-((g\*(c + d\*x))/(d\*f - c\*g))])\*Log[a + b\*x] + Log[-((b\*c - a\*d)\*(f + g\*x))/((d\*f - c\*g)\*(a + b\*x))]^2)/g + (4\*B^2\*(Log[-((d\*(a + b\*x))/(b\*c - a\*d))] + Log[(d\*f - c\*g)/(d\*(f + g\*x))] - Log[-((d\*f - c\*g)\*(a + b\*x))/((b\*c - a\*d)\*(f + g\*x))])\*Log[((b\*c - a\*d)\*(f + g\*x))/((b\*f - a\*g)\*(c + d\*x))]^2)/g - (4\*B^2\*(Log[-((d\*(a + b\*x))/(b\*c - a\*d))] - Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*(Log[c + d\*x] + Log[(b\*c - a\*d)\*(f + g\*x)]/((b\*f - a\*g)\*(c + d\*x))]^2)/g + (8\*B^2\*(Log[f + g\*x] - Log[-((b\*c - a\*d)\*(f + g\*x))/((d\*f - c\*g)\*(a + b\*x))])\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/g + (4\*B^2\*Log[(a + b\*x)^2]\*PolyLog[2, -((g\*(a + b\*x))/(b\*f - a\*g))])/g + (8\*B^2\*(Log[f + g\*x] - Log[(b\*c - a\*d)\*(f + g\*x)]/((b\*f - a\*g)\*(c + d\*x)))\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)]/g - (4\*B^2\*Log[(c + d

$$\begin{aligned}
& *x)^{-2}] * \text{PolyLog}[2, -((g*(c + d*x))/(d*f - c*g))]/g - (8*B^2 * \text{Log}[-((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x))]) * \text{PolyLog}[2, (g*(a + b*x))/(b*(f + g*x))]/g + (8*B^2 * \text{Log}[-((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x))]) * \text{PolyLog}[2, -((d*f - c*g)*(a + b*x))/((b*c - a*d)*(f + g*x))]/g - (8*B^2 * \text{Log}[(b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))] * \text{PolyLog}[2, (g*(c + d*x))/(d*(f + g*x))]/g + (8*B^2 * \text{Log}[(b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))] * \text{PolyLog}[2, ((b*f - a*g)*(c + d*x))/((b*c - a*d)*(f + g*x))]/g - (4*A*B * \text{PolyLog}[2, (b*(f + g*x))/(b*f - a*g)]/g + (4*B^2 * (\text{Log}[(a + b*x)^2] + \text{Log}[(c + d*x)^{-2}] - \text{Log}[(e*(a + b*x)^2]/(c + d*x)^2]) * \text{PolyLog}[2, (b*(f + g*x))/(b*f - a*g)]/g - (4*B^2 * (\text{Log}[(c + d*x)^{-2}] + 2 * \text{Log}[c + d*x]) * \text{PolyLog}[2, (b*(f + g*x))/(b*f - a*g)]/g + (8*B^2 * (\text{Log}[c + d*x] + \text{Log}[(b*c - a*d)*(f + g*x))/((b*f - a*g)*(c + d*x))]) * \text{PolyLog}[2, (b*(f + g*x))/(b*f - a*g)]/g + (4*A*B * \text{PolyLog}[2, (d*(f + g*x))/(d*f - c*g)]/g - (4*B^2 * (2 * \text{Log}[a + b*x] - \text{Log}[(a + b*x)^2]) * \text{PolyLog}[2, (d*(f + g*x))/(d*f - c*g)]/g - (4*B^2 * (\text{Log}[(a + b*x)^2] + \text{Log}[(c + d*x)^{-2}] - \text{Log}[(e*(a + b*x)^2]/(c + d*x)^2]) * \text{PolyLog}[2, (d*(f + g*x))/(d*f - c*g)]/g + (8*B^2 * (\text{Log}[a + b*x] + \text{Log}[-((b*c - a*d)*(f + g*x))/((d*f - c*g)*(a + b*x))]) * \text{PolyLog}[2, (d*(f + g*x))/(d*f - c*g)]/g - (8*B^2 * \text{PolyLog}[3, -((d*(a + b*x))/(b*c - a*d))]/g - (8*B^2 * \text{PolyLog}[3, -((g*(a + b*x))/(b*f - a*g))]/g - (8*B^2 * \text{PolyLog}[3, (b*(c + d*x))/(b*c - a*d)]/g - (8*B^2 * \text{PolyLog}[3, -((g*(c + d*x))/(d*f - c*g))]/g - (8*B^2 * \text{PolyLog}[3, (g*(a + b*x))/(b*(f + g*x))]/g + (8*B^2 * \text{PolyLog}[3, -((d*f - c*g)*(a + b*x))/((b*c - a*d)*(f + g*x))]/g - (8*B^2 * \text{PolyLog}[3, (g*(c + d*x))/(d*(f + g*x))]/g + (8*B^2 * \text{PolyLog}[3, ((b*f - a*g)*(c + d*x))/((b*c - a*d)*(f + g*x))]/g - (8*B^2 * \text{PolyLog}[3, (b*(f + g*x))/(b*f - a*g)]/g - (8*B^2 * \text{PolyLog}[3, (d*(f + g*x))/(d*f - c*g)]/g
\end{aligned}$$

#### Rule 2524

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_Symbol]
:> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e,
Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

```

#### Rule 12

```

Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]

```

#### Rule 2528

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] :> With
[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /;
FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x]
&& IGtQ[n, 0]

```

#### Rule 2418

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Symbol]
:> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[RFx, x]
&& IntegerQ[p]

```

#### Rule 2390

```

Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.)
*(x_.))^(q_.), x_Symbol] :> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x,
d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]

```

Rule 2301

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/(x\_), x\_Symbol] := Simp[(a + b\*Log[c\*x^n])^2/(2\*b\*n), x] /; FreeQ[{a, b, c, n}, x]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n])/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 6688

Int[u\_, x\_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]

Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rule 2500

Int[(Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))\*((c\_.) + (d\_.)\*(x\_)^(q\_.))]^(r\_.))\*((s\_.) + Log[(i\_.)\*((g\_.) + (h\_.)\*(x\_)^(n\_.))\*(t\_.)))/((j\_.) + (k\_.)\*(x\_)), x\_Symbol] := Dist[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r] - Log[(a + b\*x)^(p\*r)] - Log[(c + d\*x)^(q\*r)], Int[(s + t\*Log[i\*(g + h\*x)^n])/(j + k\*x), x], x] + (Int[(Log[(a + b\*x)^(p\*r)]\*(s + t\*Log[i\*(g + h\*x)^n])/(j + k\*x), x] + Int[(Log[(c + d\*x)^(q\*r)]\*(s + t\*Log[i\*(g + h\*x)^n])/(j + k\*x), x]) /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, s, t, n, p, q, r}, x] && NeQ[b\*c - a\*d, 0]

Rule 2433

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]\*(b\_.))^(p\_.)\*((f\_.) + Log[(h\_.)\*((i\_.) + (j\_.)\*(x\_)^(m\_.))\*(g\_.))\*((k\_.) + (l\_.)\*(x\_)^(r\_.)), x\_Symbol] := Dist[1/e, Subst[Int[(k\*x)/d]^r\*(a + b\*Log[c\*x^n])^p\*(f + g\*Log[h\*(e\*i - d\*j)/e + (j\*x)/e]^m), x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, j, k, l, n, p, r}, x] && EqQ[e\*k - d\*l, 0]

Rule 2375

Int[(Log[(d\_.)\*((e\_) + (f\_.)\*(x\_)^(m\_.))]^(r\_.))\*((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)/(x\_), x\_Symbol] := Simp[(Log[d\*(e + f\*x^m)^r]\*(a + b\*Log[c\*x^n])^(p + 1))/(b\*n\*(p + 1)), x] - Dist[(f\*m\*r)/(b\*n\*(p + 1)), Int[(x^(m - 1)\*(a + b\*Log[c\*x^n])^(p + 1))/(e + f\*x^m), x], x] /; FreeQ[{a, b, c, d,

$e, f, r, m, n\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{NeQ}[d*e, 1]$

### Rule 2317

$\text{Int}[(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}/((d_.) + (e_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[1 + (e*x)/d]*(a + b*\text{Log}[c*x^n])^p)/e, x] - \text{Dist}[(b*n*p)/e, \text{Int}[(\text{Log}[1 + (e*x)/d]*(a + b*\text{Log}[c*x^n])^{(p-1)})/x, x], x] /; \text{FreeQ}\{a, b, c, d, e, n\}, x] \&\& \text{IGtQ}[p, 0]$

### Rule 2374

$\text{Int}[(\text{Log}[(d_.)*((e_.) + (f_.)*(x_.)^{(m_.)})]*(a_.) + \text{Log}[(c_.)*(x_.)^{(n_.)}]*(b_.)]^{(p_.)}/(x_.), x\_Symbol] \rightarrow -\text{Simp}[(\text{PolyLog}[2, -(d*f*x^m)]*(a + b*\text{Log}[c*x^n])^p)/m, x] + \text{Dist}[(b*n*p)/m, \text{Int}[(\text{PolyLog}[2, -(d*f*x^m)]*(a + b*\text{Log}[c*x^n])^{(p-1)})/x, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x] \&\& \text{IGtQ}[p, 0] \&\& \text{EqQ}[d*e, 1]$

### Rule 6589

$\text{Int}[\text{PolyLog}[n_, (c_.)*((a_.) + (b_.)*(x_.)^{(p_.)})]/((d_.) + (e_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[\text{PolyLog}[n + 1, c*(a + b*x)^p]/(e*p), x] /; \text{FreeQ}\{a, b, c, d, e, n, p\}, x] \&\& \text{EqQ}[b*d, a*e]$

### Rule 2440

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.)^{(n_.)})]*(b_.)*((f_.) + \text{Log}[(h_.)*((i_.) + (j_.)*(x_.)^{(m_.)})*(g_.)*((k_.) + (l_.)*(x_.)^{(r_.)})], x\_Symbol] \rightarrow \text{Dist}[1/l, \text{Subst}[\text{Int}[x^r*(a + b*\text{Log}[c*(-((e*k - d*l)/l) + (e*x)/l)^n])*(f + g*\text{Log}[h*(-((j*k - i*l)/l) + (j*x)/l)^m]), x], x, k + l*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, i, j, k, l, m, n\}, x] \&\& \text{IntegerQ}[r]$

### Rule 2437

$\text{Int}[(\text{Log}[(c_.)*((d_.) + (e_.)*(x_.)^{(n_.)})]*\text{Log}[(h_.)*((i_.) + (j_.)*(x_.)^{(m_.)})]/(x_.), x\_Symbol] \rightarrow \text{Dist}[m, \text{Int}[(\text{Log}[i + j*x]*\text{Log}[c*(d + e*x)^n])/x, x], x] - \text{Dist}[m*\text{Log}[i + j*x] - \text{Log}[h*(i + j*x)^m], \text{Int}[\text{Log}[c*(d + e*x)^n]/x, x], x] /; \text{FreeQ}\{c, d, e, h, i, j, m, n\}, x] \&\& \text{NeQ}[e*i - d*j, 0] \&\& \text{NeQ}[i + j*x, h*(i + j*x)^m]$

### Rule 2435

$\text{Int}[(\text{Log}[(a_.) + (b_.)*(x_.)]*\text{Log}[(c_.) + (d_.)*(x_.)]/(x_.), x\_Symbol] \rightarrow \text{Simp}[\text{Log}[-((b*x)/a)]*\text{Log}[a + b*x]*\text{Log}[c + d*x], x] + (\text{Simp}[(1*(\text{Log}[-((b*x)/a)] - \text{Log}[-((b*c - a*d)*x)/(a*(c + d*x))]) + \text{Log}[(b*c - a*d)/(b*(c + d*x))])* \text{Log}[(a*(c + d*x))/(c*(a + b*x))]^2)/2, x] - \text{Simp}[(1*(\text{Log}[-((b*x)/a)] - \text{Log}[-((d*x)/c]))*(\text{Log}[a + b*x] + \text{Log}[(a*(c + d*x))/(c*(a + b*x))])^2)/2, x] + \text{Simp}[(\text{Log}[c + d*x] - \text{Log}[(a*(c + d*x))/(c*(a + b*x))])* \text{PolyLog}[2, 1 + (b*x)/a], x] + \text{Simp}[(\text{Log}[a + b*x] + \text{Log}[(a*(c + d*x))/(c*(a + b*x))])* \text{PolyLog}[2, 1 + (d*x)/c], x] + \text{Simp}[\text{Log}[(a*(c + d*x))/(c*(a + b*x))]* \text{PolyLog}[2, (c*(a + b*x))/(a*(c + d*x))], x] - \text{Simp}[\text{Log}[(a*(c + d*x))/(c*(a + b*x))]* \text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))], x] - \text{Simp}[\text{PolyLog}[3, 1 + (b*x)/a], x] - \text{Simp}[\text{PolyLog}[3, 1 + (d*x)/c], x] + \text{Simp}[\text{PolyLog}[3, (c*(a + b*x))/(a*(c + d*x))], x] - \text{Simp}[\text{PolyLog}[3, (d*(a + b*x))/(b*(c + d*x))], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

### Rule 2315

$\text{Int}[\text{Log}[(c_.)*(x_.)]/((d_.) + (e_.)*(x_.)), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{EqQ}[e + c*d, 0]$

Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f+gx} dx &= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(f+gx)}{g} - \frac{(2B) \int \frac{(c+dx)^2 \left(-\frac{2de(a+bx)^2}{(c+dx)^3} + \frac{2be(a+bx)}{(c+dx)^2}\right) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{e(a+bx)^2} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(f+gx)}{g} - \frac{(2B) \int \frac{(c+dx)^2 \left(-\frac{2de(a+bx)^2}{(c+dx)^3} + \frac{2be(a+bx)}{(c+dx)^2}\right) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{(a+bx)^2} dx}{eg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(f+gx)}{g} - \frac{(2B) \int \frac{2(bc-ad)e \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{(a+bx)(c+dx)} dx}{eg} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(f+gx)}{g} - \frac{(4B(bc-ad)) \int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{(a+bx)(c+dx)} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(f+gx)}{g} - \frac{(4B(bc-ad)) \int \left( \frac{b \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{(bc-ad)(a+bx)} - \frac{d \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{(bc-ad)(a+bx)} \right) dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(f+gx)}{g} - \frac{(4bB) \int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{a+bx} dx}{g} + \frac{(4Bd) \int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{a+bx} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(f+gx)}{g} - \frac{(4bB) \int \left( \frac{A \log(f+gx)}{a+bx} + \frac{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) \log(f+gx)}{a+bx} \right) dx}{g} + \frac{(4Bd) \int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right) \log(f+gx)}{a+bx} dx}{g} \\
&= \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(f+gx)}{g} - \frac{(4AbB) \int \frac{\log(f+gx)}{a+bx} dx}{g} - \frac{(4bB^2) \int \frac{\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) \log(f+gx)}{a+bx} dx}{g} \\
&= -\frac{4AB \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} + \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2 \log(f+gx)}{g} + \frac{4AB \log\left(-\frac{g(c+dx)}{df-ag}\right) \log(f+gx)}{g} \\
&= -\frac{4AB \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} + \frac{4B^2 \log\left(-\frac{g(a+bx)}{bf-ag}\right) \left(\log((a+bx)^2) + \log\left(\frac{1}{(c+dx)^2}\right) - \log\left(-\frac{g(c+dx)}{df-ag}\right)\right) \log(f+gx)}{g} \\
&= -\frac{4AB \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} - \frac{B^2 \log^2((a+bx)^2) \log(f+gx)}{g} - \frac{B^2 \log^2\left(\frac{1}{(c+dx)^2}\right) \log(f+gx)}{g} \\
&= -\frac{4AB \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} - \frac{B^2 \log^2((a+bx)^2) \log(f+gx)}{g} - \frac{B^2 \log^2\left(\frac{1}{(c+dx)^2}\right) \log(f+gx)}{g} \\
&= -\frac{4AB \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} - \frac{B^2 \log^2((a+bx)^2) \log(f+gx)}{g} - \frac{B^2 \log^2\left(\frac{1}{(c+dx)^2}\right) \log(f+gx)}{g} \\
&= -\frac{4AB \log\left(-\frac{g(a+bx)}{bf-ag}\right) \log(f+gx)}{g} - \frac{B^2 \log^2((a+bx)^2) \log(f+gx)}{g} - \frac{B^2 \log^2\left(\frac{1}{(c+dx)^2}\right) \log(f+gx)}{g}
\end{aligned}$$

**Mathematica [F]** time = 2.19976, size = 0, normalized size = 0.

$$\int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{f + gx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(f + g\*x), x]

[Out] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(f + g\*x), x]

**Maple [F]** time = 1.629, size = 0, normalized size = 0.

$$\int \frac{1}{gx + f} \left( A + B \ln\left(\frac{e(bx + a)^2}{(dx + c)^2}\right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f), x)

[Out] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{A^2 \log(gx + f)}{g} - \int \frac{4B^2 \log(bx + a)^2 + B^2 \log(e)^2 + 2AB \log(e) + 4(B^2 \log(e) + AB) \log(bx + a) - 4(2B^2 \log(e) + AB) \log(dx + c)}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f), x, algorithm="maxima")

[Out] A^2\*log(g\*x + f)/g - integrate(-(4\*B^2\*log(b\*x + a)^2 + B^2\*log(e)^2 + 2\*A\*B\*log(e) + 4\*(B^2\*log(e) + A\*B)\*log(b\*x + a) - 4\*(2\*B^2\*log(b\*x + a) + B^2\*log(e) + A\*B)\*log(d\*x + c))/(g\*x + f), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B^2 \log\left(\frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2}\right) + 2 A B \log\left(\frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2}\right) + A^2}{g x + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f), x, algorithm="fricas")

[Out] integral((B^2\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*A\*B\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + A^2)/(g\*x + f), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2/(g\*x+f), x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log\left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)^2}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f), x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2/(g\*x + f), x)



$$3.277 \quad \int \frac{\left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{(f+gx)^2} dx$$

**Optimal.** Leaf size=200

$$\frac{8B^2(bc-ad)\text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{(bf-ag)(df-cg)} + \frac{4B(bc-ad)\log\left(1 - \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{(bf-ag)(df-cg)} + \frac{(a+bx)\left(B\log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{(f+gx)(bf-ag)}$$

[Out] ((a + b\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2)/((b\*f - a\*g)\*(f + g\*x)) + (4\*B\*(b\*c - a\*d)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[1 - ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/((b\*f - a\*g)\*(d\*f - c\*g)) + (8\*B^2\*(b\*c - a\*d)\*PolyLog[2, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/((b\*f - a\*g)\*(d\*f - c\*g))

**Rubi [B]** time = 1.30136, antiderivative size = 620, normalized size of antiderivative = 3.1, number of steps used = 32, number of rules used = 10, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391}

$$\frac{8bB^2\text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right)}{g(bf-ag)} + \frac{8B^2d\text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right)}{g(df-cg)} - \frac{8B^2(bc-ad)\text{PolyLog}\left(2, \frac{b(f+gx)}{bf-ag}\right)}{(bf-ag)(df-cg)} + \frac{8B^2(bc-ad)\text{PolyLog}\left(2, \frac{b(f+gx)}{bf-ag}\right)}{(bf-ag)(df-cg)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(f + g\*x)^2, x]

[Out] (-4\*b\*B^2\*Log[a + b\*x]^2)/(g\*(b\*f - a\*g)) + (4\*b\*B\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/(g\*(b\*f - a\*g)) - (A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(g\*(f + g\*x)) + (8\*B^2\*d\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x])/(g\*(d\*f - c\*g)) - (4\*B\*d\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[c + d\*x])/(g\*(d\*f - c\*g)) - (4\*B^2\*d\*Log[c + d\*x]^2)/(g\*(d\*f - c\*g)) + (8\*b\*B^2\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(g\*(b\*f - a\*g)) - (8\*B^2\*(b\*c - a\*d)\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*Log[f + g\*x])/((b\*f - a\*g)\*(d\*f - c\*g)) + (4\*B\*(b\*c - a\*d)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[f + g\*x])/((b\*f - a\*g)\*(d\*f - c\*g)) + (8\*B^2\*(b\*c - a\*d)\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*Log[f + g\*x])/((b\*f - a\*g)\*(d\*f - c\*g)) + (8\*b\*B^2\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/((b\*f - a\*g)\*(d\*f - c\*g)) + (8\*B^2\*d\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/(g\*(d\*f - c\*g)) - (8\*B^2\*(b\*c - a\*d)\*PolyLog[2, (b\*(f + g\*x))/(b\*f - a\*g)])/((b\*f - a\*g)\*(d\*f - c\*g)) + (8\*B^2\*(b\*c - a\*d)\*PolyLog[2, (d\*(f + g\*x))/(d\*f - c\*g)])/((b\*f - a\*g)\*(d\*f - c\*g))

### Rule 2525

Int[((a\_.) + Log[(c\_.)\*(RFX\_)^(p\_.)]\*(b\_.))^(n\_.)\*((d\_.) + (e\_.)\*(x\_.))^(m\_.), x\_Symbol] := Simp[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFX^p])^n)/(e\*(m + 1)), x] - Dist[(b\*n\*p)/(e\*(m + 1)), Int[SimplifyIntegrand[((d + e\*x)^(m + 1)\*(a + b\*Log[c\*RFX^p])^(n - 1)\*D[RFX, x])/RFX, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFX, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u
]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunci
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x)
, x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] := -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^2} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{(2B) \int \frac{2(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)(f+gx)} dx}{g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{(4B(bc-ad)) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)(c+dx)(f+gx)} dx}{g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{(4B(bc-ad)) \int \left[ \frac{b^2\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(bf-ag)(a+bx)} + \frac{d^2\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(-df+cg)(c+dx)} \right] dx}{g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{(4b^2B) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx} dx}{g(bf-ag)} - \frac{(4Bd^2) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{c+dx} dx}{g(df-cg)} \\
&= \frac{4bB \log(a+bx) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4Bd \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(df-cg)} \\
&= \frac{4bB \log(a+bx) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4Bd \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(df-cg)} \\
&= \frac{4bB \log(a+bx) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4Bd \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(df-cg)} \\
&= \frac{4bB \log(a+bx) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4Bd \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(df-cg)} \\
&= \frac{4bB \log(a+bx) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4Bd \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(df-cg)} \\
&= \frac{4bB \log(a+bx) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} + \frac{8B^2d \log\left(-\frac{d(a+bx)}{bc-ad}\right)}{g(df-cg)} \\
&= -\frac{4bB^2 \log^2(a+bx)}{g(bf-ag)} + \frac{4bB \log(a+bx) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} \\
&= -\frac{4bB^2 \log^2(a+bx)}{g(bf-ag)} + \frac{4bB \log(a+bx) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} - \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)}
\end{aligned}$$

**Mathematica [B]** time = 0.852957, size = 409, normalized size = 2.04

$$\frac{4B \left( -bB(df-cg) \left( \log(a+bx) \left( \log(a+bx) - 2 \log\left(\frac{b(c+dx)}{bc-ad}\right) \right) - 2 \text{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right) \right) + Bd(bf-ag) \left( 2 \text{PolyLog}\left(2, \frac{b(c+dx)}{bc-ad}\right) + \log(c+dx) \left( 2 \log\left(\frac{d(a+bx)}{ad-bc}\right) - \log(c+dx) \right) \right) \right)}{g(bf-ag) \left( \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{g(f+gx)} - \frac{4bB \log(a+bx) \left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)} + \frac{8B^2d \log\left(-\frac{d(a+bx)}{bc-ad}\right)}{g(df-cg)} \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(f + g\*x)^2, x]

[Out] (-(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(f + g\*x) + (4\*B\*(b\*(d\*f - c\*g)\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) - d\*(b\*f - a\*g)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[c + d\*x] + (b\*c - a\*d)\*g\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[f + g\*x] - b\*B\*(d\*f - c\*g)\*(Log[a + b\*x]\*(Log[a + b\*x] - 2\*Log[(b\*(c + d\*x))/(b\*c - a\*d)]) - 2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c) + a\*d])) + B\*d\*(b\*f - a\*g)\*((2\*Log[(d\*(a + b\*x))/(-b\*c) + a\*d]))

$a*d)] - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)] - 2*B*(b*c - a*d)*g*((\text{Log}[(g*(a + b*x))/(-(b*f) + a*g)] - \text{Log}[(g*(c + d*x))/(-(d*f) + c*g)])*\text{Log}[f + g*x] + \text{PolyLog}[2, (b*(f + g*x))/(b*f - a*g)] - \text{PolyLog}[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)*(d*f - c*g))/g$

**Maple [F]** time = 1.794, size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^2} \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^2,x)

[Out] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2AB \left( \frac{2b \log(bx + a)}{bfg - ag^2} - \frac{2d \log(dx + c)}{dfg - cg^2} + \frac{2(bc - ad) \log(gx + f)}{bdf^2 + acg^2 - (bc + ad)fg} - \frac{\log\left(\frac{b^2ex^2}{d^2x^2 + 2cdx + c^2} + \frac{2abex}{d^2x^2 + 2cdx + c^2} + \frac{a^2e}{d^2x^2 + 2cdx + c^2}\right)}{g^2x + fg} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^2,x, algorithm="maxima")

[Out]  $2*A*B*(2*b*\log(b*x + a)/(b*f*g - a*g^2) - 2*d*\log(d*x + c)/(d*f*g - c*g^2) + 2*(b*c - a*d)*\log(g*x + f)/(b*d*f^2 + a*c*g^2 - (b*c + a*d)*f*g) - \log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^2))/(g^2*x + f*g) - B^2*(4*\log(d*x + c)^2/(g^2*x + f*g) + \text{integrate}(-d*g*x*\log(e)^2 + c*g*\log(e)^2 + 4*(d*g*x + c*g)*\log(b*x + a)^2 + 4*(d*g*x*\log(e) + c*g*\log(e))*\log(b*x + a) - 4*((g*\log(e) - 2*g)*d*x + c*g*\log(e) - 2*d*f + 2*(d*g*x + c*g)*\log(b*x + a))*\log(d*x + c))/(d*g^3*x^3 + c*f^2*g + (2*d*f*g^2 + c*g^3)*x^2 + (d*f^2*g + 2*c*f*g^2)*x), x) - A^2/(g^2*x + f*g)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log\left(\frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2}\right)^2 + 2AB \log\left(\frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2}\right) + A^2}{g^2x^2 + 2fgx + f^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^2,x, algorithm="fricas")

[Out] integral((B^2\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*A\*B\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)))

+ A<sup>2</sup>)/(g<sup>2</sup>\*x<sup>2</sup> + 2\*f\*g\*x + f<sup>2</sup>), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2/(g\*x+f)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log\left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)^2}{(gx+f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^2,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2/(g\*x + f)^2, x)

$$3.278 \quad \int \frac{\left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{(f+gx)^3} dx$$

**Optimal.** Leaf size=381

$$\frac{4B^2(bc-ad)(-adg-bcg+2bdf)\text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right)}{(bf-ag)^2(df-cg)^2} + \frac{b^2\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)^2}{2g(bf-ag)^2} + \frac{2Bg(a+bx)(bc-ad)\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A\right)}{(f+gx)(bf-ag)^2(df-cg)}$$

[Out] (2\*B\*(b\*c - a\*d)\*g\*(a + b\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/((b\*f - a\*g)^2\*(d\*f - c\*g)\*(f + g\*x)) + (b^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2)/(2\*g\*(b\*f - a\*g)^2 - (A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(2\*g\*(f + g\*x)^2) + (4\*B^2\*(b\*c - a\*d)^2\*g\*Log[(f + g\*x)/(c + d\*x)])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (2\*B\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[1 - ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (4\*B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*PolyLog[2, ((d\*f - c\*g)\*(a + b\*x))/((b\*f - a\*g)\*(c + d\*x))])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2)

**Rubi [B]** time = 1.64416, antiderivative size = 899, normalized size of antiderivative = 2.36, number of steps used = 36, number of rules used = 11, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

$$-\frac{2B^2 \log^2(a+bx)b^2}{g(bf-ag)^2} + \frac{2B \log(a+bx)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)b^2}{g(bf-ag)^2} + \frac{4B^2 \log(a+bx) \log\left(\frac{b(c+dx)}{bc-ad}\right)b^2}{g(bf-ag)^2} + \frac{4B^2 \text{PolyLog}\left(2, -\frac{d}{b}\right)}{g(bf-ag)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(f + g\*x)^3,x]

[Out] (4\*b\*B^2\*(b\*c - a\*d)\*Log[a + b\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)) - (2\*b^2\*B^2\*Log[a + b\*x]^2)/(g\*(b\*f - a\*g)^2 - (2\*B\*(b\*c - a\*d)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/((b\*f - a\*g)\*(d\*f - c\*g)\*(f + g\*x)) + (2\*b^2\*B\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/((b\*f - a\*g)^2) - (A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(2\*g\*(f + g\*x)^2) - (4\*B^2\*d\*(b\*c - a\*d)\*Log[c + d\*x])/((b\*f - a\*g)\*(d\*f - c\*g)^2) + (4\*B^2\*d^2\*Log[-((d\*(a + b\*x))/(b\*c - a\*d))]\*Log[c + d\*x])/((b\*f - a\*g)\*(d\*f - c\*g)^2) - (2\*B\*d^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[c + d\*x])/((b\*f - a\*g)\*(d\*f - c\*g)^2) - (2\*B^2\*d^2\*Log[c + d\*x]^2)/(g\*(d\*f - c\*g)^2) + (4\*b^2\*B^2\*Log[a + b\*x]\*Log[(b\*(c + d\*x))/(b\*c - a\*d)])/(g\*(b\*f - a\*g)^2) + (4\*B^2\*(b\*c - a\*d)^2\*g\*Log[f + g\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) - (4\*B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*Log[-((g\*(a + b\*x))/(b\*f - a\*g))]\*Log[f + g\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (2\*B\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[f + g\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (4\*B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*Log[-((g\*(c + d\*x))/(d\*f - c\*g))]\*Log[f + g\*x])/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (4\*b^2\*B^2\*PolyLog[2, -((d\*(a + b\*x))/(b\*c - a\*d))])/((b\*f - a\*g)^2) + (4\*B^2\*d^2\*PolyLog[2, (b\*(c + d\*x))/(b\*c - a\*d)])/(g\*(d\*f - c\*g)^2) - (4\*B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*PolyLog[2, (b\*(f + g\*x))/(b\*f - a\*g)]/((b\*f - a\*g)^2\*(d\*f - c\*g)^2) + (4\*B^2\*(b\*c - a\*d)\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*PolyLog[2, (d\*(f + g\*x))/(d\*f - c\*g)]/((b\*f - a\*g)^2\*(d\*f - c\*g)^2)

Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]
```

### Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

### Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]
```

### Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_Symbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Symbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[Rfx, x] && IntegerQ[p]
```

### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_.))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && EqQ[e*f - d*g, 0]
```

### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Log[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_.)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))]*(b_.))/((f_.) + (g_.)*(x_.)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]
```

Rule 2391

Int[Log[(c\_.)\*(d\_) + (e\_.)\*(x\_)^(n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 72

Int[((e\_.) + (f\_.)\*(x\_)^(p\_.))/((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_)), x\_Symbol] := Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^3} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2} + \frac{B \int \frac{2(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)(f+gx)^2} dx}{g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2} + \frac{(2B(bc-ad)) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)(c+dx)(f+gx)^2} dx}{g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2} + \frac{(2B(bc-ad)) \int \left(\frac{b^3\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(bf-ag)^2(a+bx)} - \frac{d^3\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(-df+cg)^2(c+dx)} + \frac{g}{(bf-ag)(df-cg)(f+gx)}\right) dx}{g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{2g(f+gx)^2} + \frac{(2b^3B) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx} dx}{g(bf-ag)^2} - \frac{(2Bd^3) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{c+dx} dx}{g(df-cg)^2} + \frac{(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right))}{2g} \\
&= -\frac{2B(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B \log(a+bx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)^2} - \frac{(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right))}{2g} \\
&= -\frac{2B(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B \log(a+bx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)^2} - \frac{(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right))}{2g} \\
&= -\frac{2B(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B \log(a+bx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)^2} - \frac{(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right))}{2g} \\
&= \frac{4bB^2(bc-ad) \log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{2B(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B \log(a+bx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)^2} \\
&= \frac{4bB^2(bc-ad) \log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{2B(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B \log(a+bx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)^2} \\
&= \frac{4bB^2(bc-ad) \log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{2b^2B^2 \log^2(a+bx)}{g(bf-ag)^2} - \frac{2B(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B \log(a+bx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)^2} \\
&= \frac{4bB^2(bc-ad) \log(a+bx)}{(bf-ag)^2(df-cg)} - \frac{2b^2B^2 \log^2(a+bx)}{g(bf-ag)^2} - \frac{2B(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bf-ag)(df-cg)(f+gx)} + \frac{2b^2B \log(a+bx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{g(bf-ag)^2}
\end{aligned}$$

**Mathematica [A]** time = 1.9621, size = 603, normalized size = 1.58

$$\frac{4B(f+gx)\left(b^2B(f+gx)(df-cg)^2\left(\log(a+bx)\left(\log(a+bx)-2\log\left(\frac{b(c+dx)}{bc-ad}\right)\right)\right)-2\text{PolyLog}\left(2,\frac{d(a+bx)}{ad-bc}\right)\right)-Bd^2(f+gx)(bf-ag)^2\left(2\text{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)+\log(c+dx)\right)\left(2\log\left(\frac{d(c+dx)}{ad-bc}\right)\right)}{g^2(bf-ag)^2(df-cg)}$$



Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^3,x]
```

```
[Out] -((A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2 + (4*B*(f + g*x)*((b*c - a*d)*
g*(b*f - a*g)*(d*f - c*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]) - b^2*(d
*f - c*g)^2*(f + g*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])
+ d^2*(b*f - a*g)^2*(f + g*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log
[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*(A + B*Log[(
e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x] - 2*B*(b*c - a*d)*g*(f + g*x)*(b*
(d*f - c*g)*Log[a + b*x] + (-(b*d*f) + a*d*g)*Log[c + d*x] + (b*c - a*d)*g*
Log[f + g*x]) + b^2*B*(d*f - c*g)^2*(f + g*x)*(Log[a + b*x]*(Log[a + b*x] -
2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a
*d)]) - B*d^2*(b*f - a*g)^2*(f + g*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
- Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) -
2*B*(b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*((Log[(g*(a + b*x))/
(-(b*f) + a*g)] - Log[(g*(c + d*x))/(-(d*f) + c*g)])*Log[f + g*x] + PolyLog
[2, (b*(f + g*x))/(b*f - a*g)] - PolyLog[2, (d*(f + g*x))/(d*f - c*g)])))/(
(b*f - a*g)^2*(d*f - c*g)^2)/(2*g*(f + g*x)^2)
```

**Maple [F]** time = 2.223, size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^3} \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\left( \frac{2b^2 \log(bx + a)}{b^2 f^2 g - 2abfg^2 + a^2 g^3} - \frac{2d^2 \log(dx + c)}{d^2 f^2 g - 2cdfg^2 + c^2 g^3} + \frac{2(2(b^2 cd - abd^2)f - (b^2 c^2 - a^2 d^2)g) \log(g)}{b^2 d^2 f^4 + a^2 c^2 g^4 - 2(b^2 cd + abd^2)f^3 g + (b^2 c^2 + 4abcd + a^2 d^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^3,x, algorithm="maxima
")
```

```
[Out] (2*b^2*log(b*x + a)/(b^2*f^2*g - 2*a*b*f*g^2 + a^2*g^3) - 2*d^2*log(d*x + c
)/(d^2*f^2*g - 2*c*d*f*g^2 + c^2*g^3) + 2*(2*(b^2*c*d - a*b*d^2)*f - (b^2*c
^2 - a^2*d^2)*g)*log(g*x + f)/(b^2*d^2*f^4 + a^2*c^2*g^4 - 2*(b^2*c*d + a*b
*d^2)*f^3*g + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^2*g^2 - 2*(a*b*c^2 + a^2*c*
d)*f*g^3) - 2*(b*c - a*d)/(b*d*f^3 + a*c*f*g^2 - (b*c + a*d)*f^2*g + (b*d*f
^2*g + a*c*g^3 - (b*c + a*d)*f*g^2)*x) - log(b^2*e*x^2/(d^2*x^2 + 2*c*d*x +
c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + c^
2))/(g^3*x^2 + 2*f*g^2*x + f^2*g)*A*B - B^2*(2*log(d*x + c)^2/(g^3*x^2 + 2
*f*g^2*x + f^2*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + 4*(d*g*x +
c*g)*log(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 4*((g*log
(e) - g)*d*x + c*g*log(e) - d*f + 2*(d*g*x + c*g)*log(b*x + a))*log(d*x +
```

c))/(d\*g^4\*x^4 + c\*f^3\*g + (3\*d\*f\*g^3 + c\*g^4)\*x^3 + 3\*(d\*f^2\*g^2 + c\*f\*g^3)\*x^2 + (d\*f^3\*g + 3\*c\*f^2\*g^2)\*x), x) - 1/2\*A^2/(g^3\*x^2 + 2\*f\*g^2\*x + f^2\*g)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log \left( \frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2} \right)^2 + 2 A B \log \left( \frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2} \right) + A^2}{g^3 x^3 + 3 f g^2 x^2 + 3 f^2 g x + f^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^3,x, algorithm="fricas")

[Out] integral((B^2\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*A\*B\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + A^2)/(g^3\*x^3 + 3\*f\*g^2\*x^2 + 3\*f^2\*g\*x + f^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2/(g\*x+f)\*\*3,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( \frac{(b x + a)^2 e}{(d x + c)^2} \right) + A \right)^2}{(g x + f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^3,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2/(g\*x + f)^3, x)

$$3.279 \quad \int \frac{\left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{(f+gx)^4} dx$$

**Optimal.** Leaf size=724

$$\frac{8B^2(bc - ad)(a^2d^2g^2 - abdg(3df - cg) + b^2(c^2g^2 - 3cdfg + 3d^2f^2)) \text{PolyLog}\left(2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)}\right) + 4B(bc - ad)(a^2d^2g^2)}{3(bf - ag)^3(df - cg)^3}$$

```
[Out] (4*B^2*(b*c - a*d)^2*g^2*(c + d*x))/(3*(b*f - a*g)^2*(d*f - c*g)^3*(f + g*x)) - (2*B*(b*c - a*d)*g^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*f - a*g)*(d*f - c*g)^3*(f + g*x)^2) + (4*B*(b*c - a*d)*g*(3*b*d*f - b*c*g - 2*a*d*g)*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*f - a*g)^3*(d*f - c*g)^2*(f + g*x)) + (b^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2)/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(3*g*(f + g*x)^3) + (4*B^2*(b*c - a*d)^3*g^2*Log[(a + b*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g)^3) - (4*B^2*(b*c - a*d)^3*g^2*Log[(f + g*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (8*B^2*(b*c - a*d)^2*g*(3*b*d*f - b*c*g - 2*a*d*g)*Log[(f + g*x)/(c + d*x)])/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (4*B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x)))]/(3*(b*f - a*g)^3*(d*f - c*g)^3) + (8*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x)))]/(3*(b*f - a*g)^3*(d*f - c*g)^3)
```

**Rubi [A]** time = 2.53975, antiderivative size = 1369, normalized size of antiderivative = 1.89, number of steps used = 40, number of rules used = 11, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

$$\frac{4B^2 \log^2(a + bx)b^3}{3g(bf - ag)^3} + \frac{4B \log(a + bx) \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right) b^3}{3g(bf - ag)^3} + \frac{8B^2 \log(a + bx) \log \left( \frac{b(c+dx)}{bc-ad} \right) b^3}{3g(bf - ag)^3} + \frac{8B^2 \text{PolyLog} \left( 2, \frac{(a+bx)(df-cg)}{(c+dx)(bf-ag)} \right) b^3}{3g(bf - ag)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^4, x]
```

```
[Out] (-4*B^2*(b*c - a*d)^2*g)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (4*b^2*B^2*(b*c - a*d)*Log[a + b*x])/(3*(b*f - a*g)^3*(d*f - c*g)) + (8*b*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[a + b*x])/(3*(b*f - a*g)^3*(d*f - c*g)^2) - (4*b^3*B^2*Log[a + b*x]^2)/(3*g*(b*f - a*g)^3) - (2*B*(b*c - a*d)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*f - a*g)*(d*f - c*g)*(f + g*x)^2) - (4*B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)) + (4*b^3*B*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*g*(b*f - a*g)^3) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(3*g*(f + g*x)^3) - (4*B^2*d^2*(b*c - a*d)*Log[c + d*x])/(3*(b*f - a*g)*(d*f - c*g)^3) - (8*B^2*d*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[c + d*x])/(3*(b*f - a*g)^2*(d*f - c*g)^3) + (8*B^2*d^3*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(3*g*(d*f - c*g)^3) - (4*B*d^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(3*g*(d*f - c*g)^3) - (4*B^2*d^3*Log[c + d*x]^2)/(3*g*(d*f - c*g)^3) + (8*b^3*B^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)]/(3*g*(b*f - a*g)^3) + (4*B^2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g)*Log[f + g*x])/((b*f - a*g)^3*(d*f - c*g)^3) - (8*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f
```

$$\begin{aligned} & + g*x] / (3*(b*f - a*g)^3*(d*f - c*g)^3) + (4*B*(b*c - a*d)*(a^2*d^2*g^2 - \\ & a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*\text{Log}[( \\ & e*(a + b*x)^2/(c + d*x)^2])*\text{Log}[f + g*x]) / (3*(b*f - a*g)^3*(d*f - c*g)^3) \\ & + (8*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 \\ & - 3*c*d*f*g + c^2*g^2))*\text{Log}[-((g*(c + d*x))/(d*f - c*g))]*\text{Log}[f + g*x]) / (3* \\ & (b*f - a*g)^3*(d*f - c*g)^3) + (8*b^3*B^2*\text{PolyLog}[2, -((d*(a + b*x))/(b*c - \\ & a*d))]) / (3*g*(b*f - a*g)^3) + (8*B^2*d^3*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a \\ & *d)]) / (3*g*(d*f - c*g)^3) - (8*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d* \\ & f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*\text{PolyLog}[2, (b*(f + g*x))/ \\ & (b*f - a*g)]) / (3*(b*f - a*g)^3*(d*f - c*g)^3) + (8*B^2*(b*c - a*d)*(a^2*d^2 \\ & *g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*\text{PolyL} \\ & \text{og}[2, (d*(f + g*x))/(d*f - c*g)]) / (3*(b*f - a*g)^3*(d*f - c*g)^3) \end{aligned}$$
Rule 2525

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_.))^(m_.
), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*Log[c*Rfx^p])^n)/(e*(m + 1))
, x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(
a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0] && (EqQ[n, 1] ||
IntegerQ[m]) && NeQ[m, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 2528

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol] := With
[{u = ExpandIntegrand[(a + b*Log[c*Rfx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]]
/; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[Rfx, x] && RationalFuncti
onQ[RGx, x] && IGtQ[n, 0]
```

Rule 2524

```
Int[((a_.) + Log[(c_.)*(Rfx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_.)), x_S
ymbol] := Simp[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^n)/e, x] - Dist[(b*n*p)/e
, Int[(Log[d + e*x]*(a + b*Log[c*Rfx^p])^(n - 1)*D[Rfx, x])/Rfx, x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[Rfx, x] && IGtQ[n, 0]
```

Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*(Rfx_), x_Sy
mbol] := With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, Rfx, x]},
Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
Rfx, x] && IntegerQ[p]
```

Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_.))^(q_.), x_Symbol] := Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] := Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_)^(p_.))/((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)),
x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^4} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{(2B) \int \frac{2(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)(f+gx)^3} dx}{3g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{(4B(bc-ad)) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)(c+dx)(f+gx)^3} dx}{3g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{(4B(bc-ad)) \int \left(\frac{b^4\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(bf-ag)^3(a+bx)} + \frac{d^4\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(-df+cg)^3(c+dx)} + \frac{g^4\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(f+gx)^3}\right) dx}{3g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{(4b^4B) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx} dx}{3g(bf-ag)^3} - \frac{(4Bd^4) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{c+dx} dx}{3g(df-cg)^3} + \frac{(4g^4B) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{f+gx} dx}{3g} \\
&= -\frac{2B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{4B(bc-ad)(2bdf-bcg-adg)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)^2(df-cg)^2(f+gx)} \\
&= -\frac{2B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{4B(bc-ad)(2bdf-bcg-adg)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)^2(df-cg)^2(f+gx)} \\
&= -\frac{2B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^2} - \frac{4B(bc-ad)(2bdf-bcg-adg)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)^2(df-cg)^2(f+gx)} \\
&= -\frac{4B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{4b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{8bB^2(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^3} \\
&= -\frac{4B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{4b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{8bB^2(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^3} \\
&= -\frac{4B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{4b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{8bB^2(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^3} \\
&= -\frac{4B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)} + \frac{4b^2B^2(bc-ad)\log(a+bx)}{3(bf-ag)^3(df-cg)} + \frac{8bB^2(bc-ad)(2bdf-bcg-adg)}{3(bf-ag)^3}
\end{aligned}$$

**Mathematica [A]** time = 4.11075, size = 909, normalized size = 1.26

$$-\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{3g(f+gx)^3} + \frac{2B(f+gx)\left(2d^3(f+gx)^2\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)\log(c+dx)(bf-ag)^3-2Bd^3(f+gx)^2\left(\left(2\log\left(\frac{d(a+bx)}{ad-bc}\right)-\log(c+dx)\right)\log(c+dx)+2\text{Poly}\left(\frac{d(a+bx)}{ad-bc}, c+dx\right)\right)\right)}{3g(bf-ag)^3(df-cg)^2(f+gx)}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(f + g\*x)^4, x]

[Out] -((A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2 + (2\*B\*(f + g\*x)\*((b\*c - a\*d)\*g\*(b\*f - a\*g)^2\*(d\*f - c\*g)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) + 2\*(b\*c - a\*d)\*g\*(b\*f - a\*g)\*(-(d\*f) + c\*g)\*(-2\*b\*d\*f + b\*c\*g + a\*d\*g)\*(f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) - 2\*b^3\*(d\*f - c\*g)^3\*(f + g\*x)^2\*Log[a + b\*x]\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]) + 2\*d^3\*(b\*f - a\*g)^3\*(f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])\*Log[c + d\*x] - 2\*(

$$\begin{aligned}
& b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3*c*d \\
& *f*g + c^2*g^2))*(f + g*x)^2*(A + B*\text{Log}[(e*(a + b*x)^2)/(c + d*x)^2])* \text{Log}[f \\
& + g*x] - 4*B*(b*c - a*d)*g*(2*b*d*f - b*c*g - a*d*g)*(f + g*x)^2*(b*(d*f - \\
& c*g)*\text{Log}[a + b*x] + (-(b*d*f) + a*d*g)*\text{Log}[c + d*x] + (b*c - a*d)*g*\text{Log}[f \\
& + g*x]) + 2*B*(b*c - a*d)*g*(f + g*x)*((b*c - a*d)*g*(b*f - a*g)*(d*f - c*g \\
& ) - b^2*(d*f - c*g)^2*(f + g*x)*\text{Log}[a + b*x] + d^2*(b*f - a*g)^2*(f + g*x)* \\
& \text{Log}[c + d*x] + (b*c - a*d)*g*(-2*b*d*f + b*c*g + a*d*g)*(f + g*x)*\text{Log}[f + g \\
& *x]) + 2*b^3*B*(d*f - c*g)^3*(f + g*x)^2*(\text{Log}[a + b*x]*(\text{Log}[a + b*x] - 2*\text{Lo} \\
& g[(b*(c + d*x))/(b*c - a*d)]) - 2*\text{PolyLog}[2, (d*(a + b*x))/(-(b*c) + a*d)]) \\
& - 2*B*d^3*(b*f - a*g)^3*(f + g*x)^2*((2*\text{Log}[(d*(a + b*x))/(-(b*c) + a*d)] \\
& - \text{Log}[c + d*x])*\text{Log}[c + d*x] + 2*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)]) + 4 \\
& *B*(b*c - a*d)*g*(a^2*d^2*g^2 + a*b*d*g*(-3*d*f + c*g) + b^2*(3*d^2*f^2 - 3 \\
& *c*d*f*g + c^2*g^2))*(f + g*x)^2*((\text{Log}[(g*(a + b*x))/(-(b*f) + a*g)] - \text{Log} \\
& (g*(c + d*x))/(-(d*f) + c*g)))*\text{Log}[f + g*x] + \text{PolyLog}[2, (b*(f + g*x))/(b*f \\
& - a*g)] - \text{PolyLog}[2, (d*(f + g*x))/(d*f - c*g)])))/((b*f - a*g)^3*(d*f - c \\
& *g)^3)/(3*g*(f + g*x)^3)
\end{aligned}$$

**Maple [F]** time = 2.948, size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^4} \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^4,x)

[Out] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^4,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^4,x, algorithm="maxima")

[Out] 
$$\begin{aligned}
& 2/3*(2*b^3*\text{log}(b*x + a)/(b^3*f^3*g - 3*a*b^2*f^2*g^2 + 3*a^2*b*f*g^3 - a^3* \\
& g^4) - 2*d^3*\text{log}(d*x + c)/(d^3*f^3*g - 3*c*d^2*f^2*g^2 + 3*c^2*d*f*g^3 - c^ \\
& 3*g^4) + 2*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2 - 3*(b^3*c^2*d - a^2*b*d^3)*f*g + \\
& (b^3*c^3 - a^3*d^3)*g^2)*\text{log}(g*x + f)/(b^3*d^3*f^6 + a^3*c^3*g^6 - 3*(b^3* \\
& c*d^2 + a*b^2*d^3)*f^5*g + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^4*g^ \\
& 2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^3 + 3*(a*b^2* \\
& c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f*g^5) \\
& - (5*(b^2*c*d - a*b*d^2)*f^2 - 3*(b^2*c^2 - a^2*d^2)*f*g + (a*b*c^2 - a^2* \\
& c*d)*g^2 + 2*(2*(b^2*c*d - a*b*d^2)*f*g - (b^2*c^2 - a^2*d^2)*g^2)*x)/(b^2* \\
& d^2*f^6 + a^2*c^2*f^2*g^4 - 2*(b^2*c*d + a*b*d^2)*f^5*g + (b^2*c^2 + 4*a*b* \\
& c*d + a^2*d^2)*f^4*g^2 - 2*(a*b*c^2 + a^2*c*d)*f^3*g^3 + (b^2*d^2*f^4*g^2 + \\
& a^2*c^2*g^6 - 2*(b^2*c*d + a*b*d^2)*f^3*g^3 + (b^2*c^2 + 4*a*b*c*d + a^2*d \\
& ^2)*f^2*g^4 - 2*(a*b*c^2 + a^2*c*d)*f*g^5)*x^2 + 2*(b^2*d^2*f^5*g + a^2*c^2 \\
& *f*g^5 - 2*(b^2*c*d + a*b*d^2)*f^4*g^2 + (b^2*c^2 + 4*a*b*c*d + a^2*d^2)*f^ \\
& 3*g^3 - 2*(a*b*c^2 + a^2*c*d)*f^2*g^4)*x) - \text{log}(b^2*e*x^2/(d^2*x^2 + 2*c*d*x \\
& + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c*d*x + \\
& c^2))/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)*A*B - 1/3*B^2*(4*\text{log}(
\end{aligned}$$

$$d*x + c)^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g) + 3*\text{integrate}(-1/3*(3*d*g*x*\log(e)^2 + 3*c*g*\log(e)^2 + 12*(d*g*x + c*g)*\log(b*x + a)^2 + 12*(d*g*x*\log(e) + c*g*\log(e))*\log(b*x + a) - 4*((3*g*\log(e) - 2*g)*d*x + 3*c*g*\log(e) - 2*d*f + 6*(d*g*x + c*g)*\log(b*x + a))*\log(d*x + c))/(d*g^5*x^5 + c*f^4*g + (4*d*f*g^4 + c*g^5)*x^4 + 2*(3*d*f^2*g^3 + 2*c*f*g^4)*x^3 + 2*(2*d*f^3*g^2 + 3*c*f^2*g^3)*x^2 + (d*f^4*g + 4*c*f^3*g^2)*x), x) - 1/3*A^2/(g^4*x^3 + 3*f*g^3*x^2 + 3*f^2*g^2*x + f^3*g)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log\left(\frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2}\right)^2 + 2 A B \log\left(\frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2}\right) + A^2}{g^4 x^4 + 4 f g^3 x^3 + 6 f^2 g^2 x^2 + 4 f^3 g x + f^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^4,x, algorithm="fricas")

[Out] integral((B^2\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*A\*B\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + A^2)/(g^4\*x^4 + 4\*f\*g^3\*x^3 + 6\*f^2\*g^2\*x^2 + 4\*f^3\*g\*x + f^4), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2/(g\*x+f)\*\*4,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(b x+a)^2 e}{(d x+c)^2}\right) + A\right)^2}{(g x+f)^4} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^4,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2/(g\*x + f)^4, x)



$$3.280 \quad \int \frac{\left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2}{(f+gx)^5} dx$$

**Optimal.** Leaf size=1154

result too large to display

```
[Out] -(B^2*(b*c - a*d)^2*g^3*(c + d*x)^2)/(3*(b*f - a*g)^2*(d*f - c*g)^4*(f + g*x)^2) - (2*B^2*(b*c - a*d)^3*g^3*(c + d*x))/(3*(b*f - a*g)^3*(d*f - c*g)^4*(f + g*x)) + (B^2*(b*c - a*d)^2*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*(c + d*x))/((b*f - a*g)^3*(d*f - c*g)^4*(f + g*x)) + (B*(b*c - a*d)*g^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*f - a*g)*(d*f - c*g)^4*(f + g*x)^3) - (B*(b*c - a*d)*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*(b*f - a*g)^2*(d*f - c*g)^4*(f + g*x)^2) + (B*(b*c - a*d)*g*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*(a + b*x)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*f - a*g)^4*(d*f - c*g)^3*(f + g*x)) + (b^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))^2/(4*g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(4*g*(f + g*x)^4) - (2*B^2*(b*c - a*d)^4*g^3*Log[(a + b*x)/(c + d*x)])/(3*(b*f - a*g)^4*(d*f - c*g)^4) + (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*Log[(a + b*x)/(c + d*x)])/((b*f - a*g)^4*(d*f - c*g)^4) + (2*B^2*(b*c - a*d)^4*g^3*Log[(f + g*x)/(c + d*x)])/(3*(b*f - a*g)^4*(d*f - c*g)^4) - (B^2*(b*c - a*d)^3*g^2*(4*b*d*f - b*c*g - 3*a*d*g)*Log[(f + g*x)/(c + d*x)])/((b*f - a*g)^4*(d*f - c*g)^4) + (2*B^2*(b*c - a*d)^2*g*(3*a^2*d^2*g^2 - 2*a*b*d*g*(4*d*f - c*g) + b^2*(6*d^2*f^2 - 4*c*d*f*g + c^2*g^2))*Log[(f + g*x)/(c + d*x)])/((b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[1 - ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^4*(d*f - c*g)^4) - (2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*PolyLog[2, ((d*f - c*g)*(a + b*x))/((b*f - a*g)*(c + d*x))])/((b*f - a*g)^4*(d*f - c*g)^4)
```

**Rubi [A]** time = 3.54306, antiderivative size = 1854, normalized size of antiderivative = 1.61, number of steps used = 44, number of rules used = 11, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$ , Rules used = {2525, 12, 2528, 2524, 2418, 2390, 2301, 2394, 2393, 2391, 72}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(f + g*x)^5,x]
```

```
[Out] -(B^2*(b*c - a*d)^2*g)/(3*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (5*B^2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g))/(3*(b*f - a*g)^3*(d*f - c*g)^3*(f + g*x)) + (2*b^3*B^2*(b*c - a*d)*Log[a + b*x])/(3*(b*f - a*g)^4*(d*f - c*g)) + (b^2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[a + b*x])/((b*f - a*g)^4*(d*f - c*g)^2) + (2*b*B^2*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[a + b*x])/((b*f - a*g)^4*(d*f - c*g)^3) - (b^4*B^2*Log[a + b*x]^2)/(g*(b*f - a*g)^4) - (B*(b*c - a*d)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(3*(b*f - a*g)*(d*f - c*g)*(f + g*x)^3) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(2*(b*f - a*g)^2*(d*f - c*g)^2*(f + g*x)^2) - (B*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/((b*f - a*g)^3*(d*f - c*g)^2)
```

$$\begin{aligned}
& c*g)^3*(f + g*x)) + (b^4*B*Log[a + b*x]*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2]))/(g*(b*f - a*g)^4) - (A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])^2/(4*g*(f + g*x)^4) - (2*B^2*d^3*(b*c - a*d)*Log[c + d*x])/(3*(b*f - a*g)*(d*f - c*g)^4) - (B^2*d^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*Log[c + d*x])/((b*f - a*g)^2*(d*f - c*g)^4) - (2*B^2*d*(b*c - a*d)*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[c + d*x])/((b*f - a*g)^3*(d*f - c*g)^4) + (2*B^2*d^4*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(g*(d*f - c*g)^4) - (B*d^4*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[c + d*x])/(g*(d*f - c*g)^4) - (B^2*d^4*Log[c + d*x]^2)/(g*(d*f - c*g)^4) + (2*b^4*B^2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)])/(g*(b*f - a*g)^4) + (B^2*(b*c - a*d)^2*g*(2*b*d*f - b*c*g - a*d*g)^2*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4) + (8*B^2*(b*c - a*d)^2*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*Log[f + g*x])/(3*(b*f - a*g)^4*(d*f - c*g)^4) + (2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log[-((g*(a + b*x))/(b*f - a*g))]*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4) - (B*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*(A + B*Log[(e*(a + b*x)^2)/(c + d*x)^2])*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4) - (2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*Log[-((g*(c + d*x))/(d*f - c*g))]*Log[f + g*x])/((b*f - a*g)^4*(d*f - c*g)^4) + (2*b^4*B^2*PolyLog[2, -((d*(a + b*x))/(b*c - a*d))])/(g*(b*f - a*g)^4) + (2*B^2*d^4*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(g*(d*f - c*g)^4) + (2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*PolyLog[2, (b*(f + g*x))/(b*f - a*g)])/(b*f - a*g)^4*(d*f - c*g)^4) - (2*B^2*(b*c - a*d)*(2*b*d*f - b*c*g - a*d*g)*(2*a*b*d^2*f*g - a^2*d^2*g^2 - b^2*(2*d^2*f^2 - 2*c*d*f*g + c^2*g^2))*PolyLog[2, (d*(f + g*x))/(d*f - c*g)])/(b*f - a*g)^4*(d*f - c*g)^4)
\end{aligned}$$

### Rule 2525

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x_Symbol]
:> Simp[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^n)/(e*(m + 1)), x] - Dist[(b*n*p)/(e*(m + 1)), Int[SimplifyIntegrand[((d + e*x)^(m + 1)*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0] && (EqQ[n, 1] || IntegerQ[m]) && NeQ[m, -1]

```

### Rule 12

```

Int[(a_)*(u_), x_Symbol]
:> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

```

### Rule 2528

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)*(RGx_), x_Symbol]
:> With[{u = ExpandIntegrand[(a + b*Log[c*RFx^p])^n, RGx, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, p}, x] && RationalFunctionQ[RFx, x] && RationalFunctionQ[RGx, x] && IGtQ[n, 0]

```

### Rule 2524

```

Int[((a_.) + Log[(c_.)*(RFx_)^(p_.)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbol]
:> Simp[(Log[d + e*x]*(a + b*Log[c*RFx^p])^n)/e, x] - Dist[(b*n*p)/e, Int[(Log[d + e*x]*(a + b*Log[c*RFx^p])^(n - 1)*D[RFx, x])/RFx, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && RationalFunctionQ[RFx, x] && IGtQ[n, 0]

```

### Rule 2418

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*(RFx_), x_Sy
mbol] :=> With[{u = ExpandIntegrand[(a + b*Log[c*(d + e*x)^n])^p, RFx, x]},
Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, n}, x] && RationalFunctionQ[
RFx, x] && IntegerQ[p]
```

#### Rule 2390

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_) + (g_.
)*(x_))^(q_.), x_Symbol] :=> Dist[1/e, Subst[Int[((f*x)/d)^q*(a + b*Log[c*x^
n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, n, p, q}, x] && E
qQ[e*f - d*g, 0]
```

#### Rule 2301

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :=> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

#### Rule 2394

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_
)), x_Symbol] :=> Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x
)^n]))/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x
), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]
```

#### Rule 2393

```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_
Symbol] :=> Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x
], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*
(e*f - d*g), 0]
```

#### Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :=> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :=> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{(f+gx)^5} dx &= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{B \int \frac{2(bc-ad)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(a+bx)(c+dx)(f+gx)^4} dx}{2g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad)) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)(c+dx)(f+gx)^4} dx}{g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{(B(bc-ad)) \int \left(\frac{b^5\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(bf-ag)^4(a+bx)} - \frac{d^5\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{(bc-ad)(-df+cg)^4(c+dx)} + \frac{g^2}{(bf-ag)^3}\right) dx}{g} \\
&= -\frac{\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2}{4g(f+gx)^4} + \frac{(b^5B) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{a+bx} dx}{g(bf-ag)^4} - \frac{(Bd^5) \int \frac{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)}{c+dx} dx}{g(df-cg)^4} + \frac{B}{g} \\
&= -\frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&= -\frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&= -\frac{B(bc-ad)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{3(bf-ag)(df-cg)(f+gx)^3} - \frac{B(bc-ad)(2bdf-bcg-adg)\left(A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)}{2(bf-ag)^2(df-cg)^2(f+gx)^2} \\
&= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{3(bf-ag)^3(df-cg)^3(f+gx)} + \frac{2b^3B^2(bc-ad)}{3(bf-ag)^4} \\
&= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{3(bf-ag)^3(df-cg)^3(f+gx)} + \frac{2b^3B^2(bc-ad)}{3(bf-ag)^4} \\
&= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{3(bf-ag)^3(df-cg)^3(f+gx)} + \frac{2b^3B^2(bc-ad)}{3(bf-ag)^4} \\
&= -\frac{B^2(bc-ad)^2g}{3(bf-ag)^2(df-cg)^2(f+gx)^2} - \frac{5B^2(bc-ad)^2g(2bdf-bcg-adg)}{3(bf-ag)^3(df-cg)^3(f+gx)} + \frac{2b^3B^2(bc-ad)}{3(bf-ag)^4}
\end{aligned}$$

**Mathematica [A]** time = 7.2527, size = 1453, normalized size = 1.26

$$B(bc-ad) \left( \frac{\log(a+bx) \left( A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) \right) b^4}{(bc-ad)(bf-ag)^4} - \frac{B \left( \log^2(a+bx) - 2 \log\left(\frac{b(c+dx)}{bc-ad}\right) \log(a+bx) - 2 \text{PolyLog}\left(2, -\frac{d(a+bx)}{bc-ad}\right) \right) b^4}{(bc-ad)(bf-ag)^4} - \frac{g \left( (3d^2f^2 - 3cdgf + c^2g^2)b^2 - adg(3df - cg) \right)}{(bf-ag)^3(df-cg)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(f + g\*x)^5, x]

[Out] -(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2/(4\*g\*(f + g\*x)^4) + (B\*(b\*c - a\*d)\*(-(g\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/(3\*(b\*f - a\*g)\*(d\*f - c\*g)\*(f + g\*x)^3) - (g\*(2\*b\*d\*f - b\*c\*g - a\*d\*g)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]))/(2\*(b\*f - a\*g)^2\*(d\*f - c\*g)^2\*(f + g\*x)^2) - (g\*(a^2\*d^2\*g^2 - a\*b\*d\*g\*(3\*d\*f - c\*g) + b^2\*(3\*d^2\*f^2 - 3\*c\*d\*f\*g + c^2\*g^2))\*(A + B\*Lo

$$g\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]/\left((bf-ag)^3(df-cg)^3(f+gx)\right) + (b^4\text{Log}[a+bx]*(A+B\text{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right]))/\left((b^4c-ad)*(bf-ag)^4\right) - (d^4*(A+B\text{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right])\text{Log}[c+dx])/((b^4c-ad)*(df-cg)^4) + (g*(2*b*d*f - b*c*g - a*d*g)*(2*b^2*d^2*f^2 - 2*b^2*c*d*f*g - 2*a*b*d^2*f*g + b^2*c^2*g^2 + a^2*d^2*g^2)*(A+B\text{Log}\left[\frac{e^{(a+bx)^2}}{(c+dx)^2}\right])\text{Log}[f+gx])/((bf-ag)^4*(df-cg)^4) + (2*B*(b^4c-ad)*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*((b\text{Log}[a+bx]))/((b^4c-ad)*(bf-ag)) - (d*\text{Log}[c+dx])/((b^4c-ad)*(df-cg)) + (g*\text{Log}[f+gx])/((bf-ag)*(df-cg)))/((bf-ag)^3*(df-cg)^3) - (B*(b^4c-ad)*g*(2*b*d*f - b*c*g - a*d*g)*(g/((bf-ag)*(df-cg)*(f+gx)) - (b^2*\text{Log}[a+bx])/((b^4c-ad)*(bf-ag)^2) + (d^2*\text{Log}[c+dx])/((b^4c-ad)*(df-cg)^2) - (g*(2*b*d*f - b*c*g - a*d*g)*\text{Log}[f+gx])/((bf-ag)^2*(df-cg)^2)))/((bf-ag)^2*(df-cg)^2) - (B*(b^4c-ad)*g*(g/((bf-ag)*(df-cg)*(f+gx)^2) + (2*g*(2*b*d*f - b*c*g - a*d*g))/((bf-ag)^2*(df-cg)^2*(f+gx)) - (2*b^3*\text{Log}[a+bx])/((b^4c-ad)*(bf-ag)^3) + (2*d^3*\text{Log}[c+dx])/((b^4c-ad)*(df-cg)^3) - (2*g*(a^2*d^2*g^2 - a*b*d*g*(3*d*f - c*g) + b^2*(3*d^2*f^2 - 3*c*d*f*g + c^2*g^2))*\text{Log}[f+gx])/((bf-ag)^3*(df-cg)^3))/((3*(bf-ag)*(df-cg)) - (b^4*B*(\text{Log}[a+bx]^2 - 2*\text{Log}[a+bx]*\text{Log}[(b*(c+dx))/(b^4c-ad)] - 2*\text{PolyLog}[2, -((d*(a+bx))/(b^4c-ad)])))/((b^4c-ad)*(bf-ag)^4) + (B*d^4*(2*\text{Log}[-((d*(a+bx))/(b^4c-ad))]*\text{Log}[c+dx] - \text{Log}[c+dx]^2 + 2*\text{PolyLog}[2, (b*(c+dx))/(b^4c-ad)]))/((b^4c-ad)*(df-cg)^4) - (2*B*g*(2*b*d*f - b*c*g - a*d*g)*(2*b^2*d^2*f^2 - 2*b^2*c*d*f*g - 2*a*b*d^2*f*g + b^2*c^2*g^2 + a^2*d^2*g^2)*(\text{Log}[-((g*(a+bx))/(bf-ag))]*\text{Log}[f+gx] - \text{Log}[-((g*(c+dx))/(df-cg))]*\text{Log}[f+gx] + \text{PolyLog}[2, (b*(f+gx))/(bf-ag)] - \text{PolyLog}[2, (d*(f+gx))/(df-cg)]))/((bf-ag)^4*(df-cg)^4))/g$$

**Maple [F]** time = 4.122, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^5} \left( A + B \ln \left( \frac{e^{(bx+a)^2}}{(dx+c)^2} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^5,x)

[Out] int((A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^5,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^5,x, algorithm="maxima")

[Out] 1/6\*(6\*b^4\*log(b\*x + a)/(b^4\*f^4\*g - 4\*a\*b^3\*f^3\*g^2 + 6\*a^2\*b^2\*f^2\*g^3 - 4\*a^3\*b\*f\*g^4 + a^4\*g^5) - 6\*d^4\*log(d\*x + c)/(d^4\*f^4\*g - 4\*c\*d^3\*f^3\*g^2 + 6\*c^2\*d^2\*f^2\*g^3 - 4\*c^3\*d\*f\*g^4 + c^4\*g^5) + 6\*(4\*(b^4\*c\*d^3 - a\*b^3\*d^4)\*f^3 - 6\*(b^4\*c^2\*d^2 - a^2\*b^2\*d^4)\*f^2\*g + 4\*(b^4\*c^3\*d - a^3\*b\*d^4)\*f\*g^2 - (b^4\*c^4 - a^4\*d^4)\*g^3)\*log(g\*x + f)/(b^4\*d^4\*f^8 + a^4\*c^4\*g^8 - 4\*(b^4\*c\*d^3 + a\*b^3\*d^4)\*f^7\*g + 2\*(3\*b^4\*c^2\*d^2 + 8\*a\*b^3\*c\*d^3 + 3\*a^2\*b^4

```

2*d^4)*f^6*g^2 - 4*(b^4*c^3*d + 6*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 + a^3*b*d
^4)*f^5*g^3 + (b^4*c^4 + 16*a*b^3*c^3*d + 36*a^2*b^2*c^2*d^2 + 16*a^3*b*c*d
^3 + a^4*d^4)*f^4*g^4 - 4*(a*b^3*c^4 + 6*a^2*b^2*c^3*d + 6*a^3*b*c^2*d^2 +
a^4*c*d^3)*f^3*g^5 + 2*(3*a^2*b^2*c^4 + 8*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f^2*
g^6 - 4*(a^3*b*c^4 + a^4*c^3*d)*f*g^7) - (26*(b^3*c*d^2 - a*b^2*d^3)*f^4 -
31*(b^3*c^2*d - a^2*b*d^3)*f^3*g + (11*b^3*c^3 + 15*a*b^2*c^2*d - 15*a^2*b*
c*d^2 - 11*a^3*d^3)*f^2*g^2 - 7*(a*b^2*c^3 - a^3*c*d^2)*f*g^3 + 2*(a^2*b*c^
3 - a^3*c^2*d)*g^4 + 6*(3*(b^3*c*d^2 - a*b^2*d^3)*f^2*g^2 - 3*(b^3*c^2*d -
a^2*b*d^3)*f*g^3 + (b^3*c^3 - a^3*d^3)*g^4)*x^2 + 3*(14*(b^3*c*d^2 - a*b^2*
d^3)*f^3*g - 15*(b^3*c^2*d - a^2*b*d^3)*f^2*g^2 + (5*b^3*c^3 + 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 - 5*a^3*d^3)*f*g^3 - (a*b^2*c^3 - a^3*c*d^2)*g^4)*x)/(b^3
*d^3*f^9 + a^3*c^3*f^3*g^6 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^8*g + 3*(b^3*c^2*d
+ 3*a*b^2*c*d^2 + a^2*b*d^3)*f^7*g^2 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*
c*d^2 + a^3*d^3)*f^6*g^3 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^5*g^
4 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^4*g^5 + (b^3*d^3*f^6*g^3 + a^3*c^3*g^9 - 3*
(b^3*c*d^2 + a*b^2*d^3)*f^5*g^4 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)
*f^4*g^5 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^3*g^6 + 3*
(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^2*g^7 - 3*(a^2*b*c^3 + a^3*c^2*d)
*f*g^8)*x^3 + 3*(b^3*d^3*f^7*g^2 + a^3*c^3*f*g^8 - 3*(b^3*c*d^2 + a*b^2*d^3)
*f^6*g^3 + 3*(b^3*c^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^5*g^4 - (b^3*c^3 +
9*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*f^4*g^5 + 3*(a*b^2*c^3 + 3*a^2*b*
c^2*d + a^3*c*d^2)*f^3*g^6 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^2*g^7)*x^2 + 3*(b^3
*d^3*f^8*g + a^3*c^3*f^2*g^7 - 3*(b^3*c*d^2 + a*b^2*d^3)*f^7*g^2 + 3*(b^3*c
^2*d + 3*a*b^2*c*d^2 + a^2*b*d^3)*f^6*g^3 - (b^3*c^3 + 9*a*b^2*c^2*d + 9*a^
2*b*c*d^2 + a^3*d^3)*f^5*g^4 + 3*(a*b^2*c^3 + 3*a^2*b*c^2*d + a^3*c*d^2)*f^
4*g^5 - 3*(a^2*b*c^3 + a^3*c^2*d)*f^3*g^6)*x) - 3*log(b^2*e*x^2/(d^2*x^2 +
2*c*d*x + c^2) + 2*a*b*e*x/(d^2*x^2 + 2*c*d*x + c^2) + a^2*e/(d^2*x^2 + 2*c
*d*x + c^2))/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g^2*x + f^4*g)
)*A*B - B^2*(log(d*x + c)^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*g^3*x^2 + 4*f^3*g
^2*x + f^4*g) + integrate(-(d*g*x*log(e)^2 + c*g*log(e)^2 + 4*(d*g*x + c*g)
*log(b*x + a)^2 + 4*(d*g*x*log(e) + c*g*log(e))*log(b*x + a) - 2*((2*g*log(
e) - g)*d*x + 2*c*g*log(e) - d*f + 4*(d*g*x + c*g)*log(b*x + a))*log(d*x +
c))/(d*g^6*x^6 + c*f^5*g + (5*d*f*g^5 + c*g^6)*x^5 + 5*(2*d*f^2*g^4 + c*f*g
^5)*x^4 + 10*(d*f^3*g^3 + c*f^2*g^4)*x^3 + 5*(d*f^4*g^2 + 2*c*f^3*g^3)*x^2
+ (d*f^5*g + 5*c*f^4*g^2)*x), x)) - 1/4*A^2/(g^5*x^4 + 4*f*g^4*x^3 + 6*f^2*
g^3*x^2 + 4*f^3*g^2*x + f^4*g)

```

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log \left( \frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2} \right)^2 + 2 A B \log \left( \frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2} \right) + A^2}{g^5 x^5 + 5 f g^4 x^4 + 10 f^2 g^3 x^3 + 10 f^3 g^2 x^2 + 5 f^4 g x + f^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2/(g*x+f)^5,x, algorithm="fricas")
```

```
[Out] integral((B^2*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*A*B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A^2)/(g^5*x^5 + 5*f*g^4*x^4 + 10*f^2*g^3*x^3 + 10*f^3*g^2*x^2 + 5*f^4*g*x + f^5), x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2/(g\*x+f)\*\*5,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A\right)^2}{(gx+f)^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2/(g\*x+f)^5,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2/(g\*x + f)^5, x)

$$3.281 \quad \int \frac{(f+gx)^2}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

**Optimal.** Leaf size=33

$$\text{Unintegrable}\left(\frac{(f+gx)^2}{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}, x\right)$$

[Out] Unintegrable[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

**Rubi [A]** time = 0.174391, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{(f+gx)^2}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

[Out] f^2\*Defer[Int][(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-1), x] + 2\*f\*g\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x] + g^2\*Defer[Int][x^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

Rubi steps

$$\begin{aligned} \int \frac{(f+gx)^2}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx &= \int \left( \frac{f^2}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} + \frac{2fgx}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} + \frac{g^2x^2}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} \right) dx \\ &= f^2 \int \frac{1}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx + (2fg) \int \frac{x}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx + g^2 \int \frac{x^2}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.176417, size = 0, normalized size = 0.

$$\int \frac{(f+gx)^2}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

[Out] Integrate[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]



**Maple [A]** time = 1.148, size = 0, normalized size = 0.

$$\int (gx + f)^2 \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2)),x)

[Out] int((g\*x+f)^2/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2)),x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^2}{B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="maxima")

[Out] integrate((g\*x + f)^2/(B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{g^2 x^2 + 2 f g x + f^2}{B \log \left( \frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2} \right) + A}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="fricas")

[Out] integral((g^2\*x^2 + 2\*f\*g\*x + f^2)/(B\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + A), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2/(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2)),x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^2}{B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2)),x, algorithm="giac")

[Out] integrate((g\*x + f)^2/(B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A), x)

$$3.282 \quad \int \frac{f+gx}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{f+gx}{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A}, x\right)$$

[Out] Unintegrable[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

**Rubi [A]** time = 0.0918307, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f+gx}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

[Out] f\*Defer[Int][(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-1), x] + g\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

Rubi steps

$$\begin{aligned} \int \frac{f+gx}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx &= \int \left( \frac{f}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} + \frac{gx}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} \right) dx \\ &= f \int \frac{1}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx + g \int \frac{x}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx \end{aligned}$$

**Mathematica [A]** time = 0.126028, size = 0, normalized size = 0.

$$\int \frac{f+gx}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

[Out] Integrate[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2]), x]

**Maple [A]** time = 0.936, size = 0, normalized size = 0.

$$\int (gx + f) \left( A + B \ln\left(\frac{e(bx + a)^2}{(dx + c)^2}\right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

[Out] `int((g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")`

[Out] `integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{gx + f}{B \log\left(\frac{b^2 ex^2 + 2 abex + a^2 e}{d^2 x^2 + 2 cdx + c^2}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")`

[Out] `integral((g*x + f)/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2)) + A), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)
```

$$3.283 \quad \int \frac{1}{A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right) + A}, x\right)$$

[Out] Unintegrable[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-1), x]

**Rubi [A]** time = 0.0141488, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-1), x]

[Out] Defer[Int] [(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-1), x]

Rubi steps

$$\int \frac{1}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx = \int \frac{1}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

**Mathematica [A]** time = 0.0354575, size = 0, normalized size = 0.

$$\int \frac{1}{A + B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-1), x]

[Out] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-1), x]

**Maple [A]** time = 0.885, size = 0, normalized size = 0.

$$\int \left( A + B \ln\left(\frac{e(bx + a)^2}{(dx + c)^2}\right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

[Out] `int(1/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")`

[Out] `integrate(1/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{B \log\left(\frac{b^2 e x^2 + 2 a b e x + a^2 e}{d^2 x^2 + 2 c d x + c^2}\right) + A}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")`

[Out] `integral(1/(B*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2) + A), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")`

[Out] `integrate(1/(B*log((b*x + a)^2*e/(d*x + c)^2) + A), x)`

$$3.284 \quad \int \frac{1}{(f+gx) \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

**Optimal.** Leaf size=33

$$\text{Unintegrable} \left( \frac{1}{(f+gx) \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}, x \right)$$

[Out] Unintegrable[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

**Rubi [A]** time = 0.0643133, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx) \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

[Out] Defer[Int][1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

Rubi steps

$$\int \frac{1}{(f+gx) \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx = \int \frac{1}{(f+gx) \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

**Mathematica [A]** time = 0.0823876, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx) \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

[Out] Integrate[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

**Maple [A]** time = 1.248, size = 0, normalized size = 0.

$$\int \frac{1}{gx+f} \left( A+B \ln \left( \frac{e(bx+a)^2}{(dx+c)^2} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

[Out] `int(1/(g*x+f)/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f) \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")`

[Out] `integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{Agx + Af + (Bgx + Bf) \log \left( \frac{b^2 ex^2 + 2 abex + a^2 e}{d^2 x^2 + 2 cdx + c^2} \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")`

[Out] `integral(1/(A*g*x + A*f + (B*g*x + B*f)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f) \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")`

[Out] `integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)`

$$3.285 \quad \int \frac{1}{(f+gx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

**Optimal.** Leaf size=33

$$\text{Unintegrable} \left( \frac{1}{(f+gx)^2 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}, x \right)$$

[Out] Unintegrable[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

**Rubi [A]** time = 0.0678718, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0$ ., Rules used = {}

$$\int \frac{1}{(f+gx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

[Out] Defer[Int][1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

Rubi steps

$$\int \frac{1}{(f+gx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx = \int \frac{1}{(f+gx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

**Mathematica [A]** time = 0.0852005, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

[Out] Integrate[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

**Maple [A]** time = 1.284, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^2 \left( A+B \ln \left( \frac{e(bx+a)^2}{(dx+c)^2} \right) \right)^{-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

[Out] `int(1/(g*x+f)^2/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^2 \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")`

[Out] `integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{Ag^2x^2 + 2Afgx + Af^2 + (Bg^2x^2 + 2Bfgx + Bf^2) \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")`

[Out] `integral(1/(A*g^2*x^2 + 2*A*f*g*x + A*f^2 + (B*g^2*x^2 + 2*B*f*g*x + B*f^2)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^2 \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

$$3.286 \quad \int \frac{1}{(f+gx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

**Optimal.** Leaf size=33

$$\text{Unintegrable} \left( \frac{1}{(f+gx)^3 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)}, x \right)$$

[Out] Unintegrable[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

**Rubi [A]** time = 0.0615744, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])),x]

[Out] Defer[Int][1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

Rubi steps

$$\int \frac{1}{(f+gx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx = \int \frac{1}{(f+gx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

**Mathematica [A]** time = 0.0891913, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)^3 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])),x]

[Out] Integrate[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])), x]

**Maple [A]** time = 1.363, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^3} \left( A+B \ln \left( \frac{e(bx+a)^2}{(dx+c)^2} \right) \right)^{-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

[Out] `int(1/(g*x+f)^3/(A+B*ln(e*(b*x+a)^2/(d*x+c)^2)),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^3 \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="maxima")`

[Out] `integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{Ag^3x^3 + 3Afg^2x^2 + 3Af^2gx + Af^3 + (Bg^3x^3 + 3Bfg^2x^2 + 3Bf^2gx + Bf^3) \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="fricas")`

[Out] `integral(1/(A*g^3*x^3 + 3*A*f*g^2*x^2 + 3*A*f^2*g*x + A*f^3 + (B*g^3*x^3 + 3*B*f*g^2*x^2 + 3*B*f^2*g*x + B*f^3)*log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)**3/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2)),x)`

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^3 \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^3/(A+B*log(e*(b*x+a)^2/(d*x+c)^2)),x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^3*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)), x)
```

$$3.287 \quad \int \frac{(f+gx)^2}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

**Optimal.** Leaf size=33

$$\text{Unintegrable}\left(\frac{(f+gx)^2}{\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

**Rubi [A]** time = 0.191497, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0$ , Rules used = {}

$$\int \frac{(f+gx)^2}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

[Out] f^2\*Defer[Int][(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-2), x] + 2\*f\*g\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x] + g^2\*Defer[Int][x^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{(f+gx)^2}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx &= \int \left( \frac{f^2}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} + \frac{2fgx}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} + \frac{g^2x^2}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} \right) dx \\ &= f^2 \int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx + (2fg) \int \frac{x}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx + g^2 \int \frac{x^2}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.694762, size = 0, normalized size = 0.

$$\int \frac{(f+gx)^2}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

[Out] Integrate[(f + g\*x)^2/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]



**Maple [A]** time = 1.006, size = 0, normalized size = 0.

$$\int (gx + f)^2 \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int((g\*x+f)^2/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bdg^2x^4 + acf^2 + (adg^2 + (2dfg + cg^2)b)x^3 + ((2dfg + cg^2)a + (df^2 + 2cfcg)b)x^2 + (bcf^2 + (df^2 + 2cfcg)a)x}{2((bc - ad)B^2 \log(bx + a) - 2(bc - ad)B^2 \log(dx + c) + (bc - ad)AB + (bc \log(e) - ad \log(e))B^2)} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out] -1/2\*(b\*d\*g^2\*x^4 + a\*c\*f^2 + (a\*d\*g^2 + (2\*d\*f\*g + c\*g^2)\*b)\*x^3 + ((2\*d\*f\*g + c\*g^2)\*a + (d\*f^2 + 2\*c\*f\*g)\*b)\*x^2 + (b\*c\*f^2 + (d\*f^2 + 2\*c\*f\*g)\*a)\*x)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) + (b\*c - a\*d)\*A\*B + (b\*c\*log(e) - a\*d\*log(e))\*B^2) + integrate(1/2\*(4\*b\*d\*g^2\*x^3 + b\*c\*f^2 + 3\*(a\*d\*g^2 + (2\*d\*f\*g + c\*g^2)\*b)\*x^2 + (d\*f^2 + 2\*c\*f\*g)\*a + 2\*((2\*d\*f\*g + c\*g^2)\*a + (d\*f^2 + 2\*c\*f\*g)\*b)\*x)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) + (b\*c - a\*d)\*A\*B + (b\*c\*log(e) - a\*d\*log(e))\*B^2), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{g^2x^2 + 2fgx + f^2}{B^2 \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right)^2 + 2AB \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral((g^2\*x^2 + 2\*f\*g\*x + f^2)/(B^2\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*A\*B\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + A^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2/(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^2}{\left(B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="giac")

[Out] integrate((g\*x + f)^2/(B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2, x)

$$3.288 \quad \int \frac{f+gx}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

**Optimal.** Leaf size=31

$$\text{Unintegrable}\left(\frac{f+gx}{\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

**Rubi [A]** time = 0.102734, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{f+gx}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

[Out] f\*Defer[Int][(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-2), x] + g\*Defer[Int][x/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

Rubi steps

$$\begin{aligned} \int \frac{f+gx}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx &= \int \left( \frac{f}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} + \frac{gx}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} \right) dx \\ &= f \int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx + g \int \frac{x}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx \end{aligned}$$

**Mathematica [A]** time = 0.410425, size = 0, normalized size = 0.

$$\int \frac{f+gx}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

[Out] Integrate[(f + g\*x)/(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2, x]

**Maple [A]** time = 1.032, size = 0, normalized size = 0.

$$\int (gx + f) \left( A + B \ln \left( \frac{e(bx + a)^2}{(dx + c)^2} \right) \right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int((g\*x+f)/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bdgx^3 + acf + (adg + (df + cg)b)x^2 + (bcf + (df + cg)a)x}{2(2(bc - ad)B^2 \log(bx + a) - 2(bc - ad)B^2 \log(dx + c) + (bc - ad)AB + (bc \log(e) - ad \log(e))B^2)} + \int \frac{1}{2(2(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out] -1/2\*(b\*d\*g\*x^3 + a\*c\*f + (a\*d\*g + (d\*f + c\*g)\*b)\*x^2 + (b\*c\*f + (d\*f + c\*g)\*a)\*x)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) + (b\*c - a\*d)\*A\*B + (b\*c\*log(e) - a\*d\*log(e))\*B^2) + integrate(1/2\*(3\*b\*d\*g\*x^2 + b\*c\*f + (d\*f + c\*g)\*a + 2\*(a\*d\*g + (d\*f + c\*g)\*b)\*x)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) + (b\*c - a\*d)\*A\*B + (b\*c\*log(e) - a\*d\*log(e))\*B^2), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{gx + f}{B^2 \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right)^2 + 2AB \log \left( \frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2} \right) + A^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral((g\*x + f)/(B^2\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*A\*B\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + A^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)/(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{gx + f}{\left(B \log\left(\frac{(bx+a)^2 e}{(dx+c)^2}\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="giac")

[Out] integrate((g\*x + f)/(B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2, x)

$$3.289 \quad \int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

**Optimal.** Leaf size=25

$$\text{Unintegrable}\left(\frac{1}{\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-2), x]

**Rubi [A]** time = 0.0139057, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-2), x]

[Out] Defer[Int][(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-2), x]

Rubi steps

$$\int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx = \int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

**Mathematica [A]** time = 0.213572, size = 0, normalized size = 0.

$$\int \frac{1}{\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-2), x]

[Out] Integrate[(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^(-2), x]

**Maple [A]** time = 1.013, size = 0, normalized size = 0.

$$\int \left(A + B \ln\left(\frac{e(bx + a)^2}{(dx + c)^2}\right)\right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int(1/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx^2 + ac + (bc + ad)x}{2\left(2(bc - ad)B^2 \log(bx + a) - 2(bc - ad)B^2 \log(dx + c) + (bc - ad)AB + (bc \log(e) - ad \log(e))B^2\right)} + \int \frac{1}{2\left(2(bc - ad)B^2 \log(bx + a) - 2(bc - ad)B^2 \log(dx + c) + (bc - ad)AB + (bc \log(e) - ad \log(e))B^2\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out] -1/2\*(b\*d\*x^2 + a\*c + (b\*c + a\*d)\*x)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) + (b\*c - a\*d)\*A\*B + (b\*c\*log(e) - a\*d\*log(e))\*B^2) + integrate(1/2\*(2\*b\*d\*x + b\*c + a\*d)/(2\*(b\*c - a\*d)\*B^2\*log(b\*x + a) - 2\*(b\*c - a\*d)\*B^2\*log(d\*x + c) + (b\*c - a\*d)\*A\*B + (b\*c\*log(e) - a\*d\*log(e))\*B^2), x)

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{B^2 \log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)^2 + 2AB \log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right) + A^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(1/(B^2\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*A\*B\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2)) + A^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(B \log\left(\frac{(bx+a)^2e}{(dx+c)^2}\right) + A\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^2*e/(d*x + c)^2) + A)^(-2), x)
```



$$3.290 \quad \int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

**Optimal.** Leaf size=33

$$\text{Unintegrable}\left(\frac{1}{(f+gx)\left(B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)+A\right)^2}, x\right)$$

[Out] Unintegrable[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

**Rubi [A]** time = 0.071707, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

[Out] Defer[Int][1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

Rubi steps

$$\int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx = \int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

**Mathematica [A]** time = 0.485656, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)\left(A+B \log\left(\frac{e(a+bx)^2}{(c+dx)^2}\right)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

[Out] Integrate[1/((f + g\*x)\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

**Maple [A]** time = 1.44, size = 0, normalized size = 0.

$$\int \frac{1}{gx+f}\left(A+B \ln\left(\frac{e(bx+a)^2}{(dx+c)^2}\right)\right)^{-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(g\*x+f)/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int(1/(g\*x+f)/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\frac{bdx^2 + ac + (bc + ad)x}{2((bcf - adf)AB + (bcf \log(e) - adf \log(e))B^2 + ((bcg - adg)AB + (bcg \log(e) - adg \log(e))B^2)x + 2((bcg - adg)B^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out] 
$$-1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f - a*d*f)*A*B + (b*c*f*\log(e) - a*d*f*\log(e))*B^2 + ((b*c*g - a*d*g)*A*B + (b*c*g*\log(e) - a*d*g*\log(e))*B^2)*x + 2*((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*\log(b*x + a) - 2*((b*c*g - a*d*g)*B^2*x + (b*c*f - a*d*f)*B^2)*\log(d*x + c) + \text{integrate}(1/2*(b*d*g*x^2 + 2*b*d*f*x + b*c*f + (d*f - c*g)*a)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*\log(e) - a*d*f^2*\log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*\log(e) - a*d*g^2*\log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*\log(e) - a*d*f*g*\log(e))*B^2)*x + 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*\log(b*x + a) - 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*\log(d*x + c)), x)$$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{A^2gx + A^2f + (B^2gx + B^2f) \log\left(\frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2}\right)^2 + 2(ABgx + ABf) \log\left(\frac{b^2ex^2 + 2abex + a^2e}{d^2x^2 + 2cdx + c^2}\right)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] 
$$\text{integral}(1/(A^2*g*x + A^2*f + (B^2*g*x + B^2*f)*\log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))^2 + 2*(A*B*g*x + A*B*f)*\log((b^2*e*x^2 + 2*a*b*e*x + a^2*e)/(d^2*x^2 + 2*c*d*x + c^2))), x)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)/(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)

[Out] Timed out

---

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f) \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

$$3.291 \quad \int \frac{1}{(f+gx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

**Optimal.** Leaf size=33

$$\text{Unintegrable} \left( \frac{1}{(f+gx)^2 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)^2}, x \right)$$

[Out] Unintegrable[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

**Rubi [A]** time = 0.0757691, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

[Out] Defer[Int][1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

Rubi steps

$$\int \frac{1}{(f+gx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx = \int \frac{1}{(f+gx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

**Mathematica [A]** time = 0.607418, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)^2 \left( A+B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

[Out] Integrate[1/((f + g\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

**Maple [A]** time = 1.641, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^2 \left( A+B \ln \left( \frac{e(bx+a)^2}{(dx+c)^2} \right) \right)^{-2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(g\*x+f)^2/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int(1/(g\*x+f)^2/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out] 
$$-1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/((b*c*f^2 - a*d*f^2)*A*B + (b*c*f^2*\log(e) - a*d*f^2*\log(e))*B^2 + ((b*c*g^2 - a*d*g^2)*A*B + (b*c*g^2*\log(e) - a*d*g^2*\log(e))*B^2)*x^2 + 2*((b*c*f*g - a*d*f*g)*A*B + (b*c*f*g*\log(e) - a*d*f*g*\log(e))*B^2)*x + 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*\log(b*x + a) - 2*((b*c*g^2 - a*d*g^2)*B^2*x^2 + 2*(b*c*f*g - a*d*f*g)*B^2*x + (b*c*f^2 - a*d*f^2)*B^2)*\log(d*x + c)) - \text{integrate}(-1/2*(b*c*f + (d*f - 2*c*g)*a - (a*d*g - (2*d*f - c*g)*b)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*\log(e) - a*d*g^3*\log(e))*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*\log(e) - a*d*f^3*\log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*\log(e) - a*d*f*g^2*\log(e))*B^2)*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*g*\log(e) - a*d*f^2*g*\log(e))*B^2)*x + 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*\log(b*x + a) - 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*\log(d*x + c)), x)$$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

integral 
$$\left( \frac{1}{A^2g^2x^2 + 2A^2fgx + A^2f^2 + (B^2g^2x^2 + 2B^2fgx + B^2f^2) \log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)^2 + 2(ABg^2x^2 + 2ABfgx + A^2f^2) \log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)^2/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(1/(A^2\*g^2\*x^2 + 2\*A^2\*f\*g\*x + A^2\*f^2 + (B^2\*g^2\*x^2 + 2\*B^2\*f\*g\*x + B^2\*f^2)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*(A\*B\*g^2\*x^2 + 2\*A\*B\*f\*g\*x + A\*B\*f^2)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)**2/(A+B*ln(e*(b*x+a)**2/(d*x+c)**2))**2,x)
```

```
[Out] Timed out
```

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^2 \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(g*x+f)^2/(A+B*log(e*(b*x+a)^2/(d*x+c)^2))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((g*x + f)^2*(B*log((b*x + a)^2*e/(d*x + c)^2) + A)^2), x)
```

$$3.292 \quad \int \frac{1}{(f+gx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

**Optimal.** Leaf size=33

$$\text{Unintegrable} \left[ \frac{1}{(f+gx)^3 \left( B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) + A \right)^2}, x \right]$$

[Out] Unintegrable[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

**Rubi [A]** time = 0.0713971, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.$ , Rules used = {}

$$\int \frac{1}{(f+gx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2),x]

[Out] Defer[Int][1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

Rubi steps

$$\int \frac{1}{(f+gx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx = \int \frac{1}{(f+gx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

**Mathematica [A]** time = 0.627948, size = 0, normalized size = 0.

$$\int \frac{1}{(f+gx)^3 \left( A + B \log \left( \frac{e(a+bx)^2}{(c+dx)^2} \right) \right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2),x]

[Out] Integrate[1/((f + g\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^2)/(c + d\*x)^2])^2), x]

**Maple [A]** time = 2.162, size = 0, normalized size = 0.

$$\int \frac{1}{(gx+f)^3 \left( A + B \ln \left( \frac{e(bx+a)^2}{(dx+c)^2} \right) \right)^{-2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(g\*x+f)^3/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

[Out] int(1/(g\*x+f)^3/(A+B\*ln(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x)

**Maxima [A]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)^3/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="maxima")

[Out] 
$$-1/2*(b*d*x^2 + a*c + (b*c + a*d)*x)/(((b*c*g^3 - a*d*g^3)*A*B + (b*c*g^3*\log(e) - a*d*g^3*\log(e))*B^2)*x^3 + (b*c*f^3 - a*d*f^3)*A*B + (b*c*f^3*\log(e) - a*d*f^3*\log(e))*B^2 + 3*((b*c*f*g^2 - a*d*f*g^2)*A*B + (b*c*f*g^2*\log(e) - a*d*f*g^2*\log(e))*B^2)*x^2 + 3*((b*c*f^2*g - a*d*f^2*g)*A*B + (b*c*f^2*g*\log(e) - a*d*f^2*g*\log(e))*B^2)*x + 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*\log(b*x + a) - 2*((b*c*g^3 - a*d*g^3)*B^2*x^3 + 3*(b*c*f*g^2 - a*d*f*g^2)*B^2*x^2 + 3*(b*c*f^2*g - a*d*f^2*g)*B^2*x + (b*c*f^3 - a*d*f^3)*B^2)*\log(d*x + c) - \text{integrate}(1/2*(b*d*g*x^2 - b*c*f - (d*f - 3*c*g)*a + 2*(a*d*g - (d*f - c*g)*b)*x)/(((b*c*g^4 - a*d*g^4)*A*B + (b*c*g^4*\log(e) - a*d*g^4*\log(e))*B^2)*x^4 + 4*((b*c*f*g^3 - a*d*f*g^3)*A*B + (b*c*f*g^3*\log(e) - a*d*f*g^3*\log(e))*B^2)*x^3 + (b*c*f^4 - a*d*f^4)*A*B + (b*c*f^4*\log(e) - a*d*f^4*\log(e))*B^2 + 6*((b*c*f^2*g^2 - a*d*f^2*g^2)*A*B + (b*c*f^2*g^2*\log(e) - a*d*f^2*g^2*\log(e))*B^2)*x^2 + 4*((b*c*f^3*g - a*d*f^3*g)*A*B + (b*c*f^3*g*\log(e) - a*d*f^3*g*\log(e))*B^2)*x + 2*((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)*\log(b*x + a) - 2*((b*c*g^4 - a*d*g^4)*B^2*x^4 + 4*(b*c*f*g^3 - a*d*f*g^3)*B^2*x^3 + 6*(b*c*f^2*g^2 - a*d*f^2*g^2)*B^2*x^2 + 4*(b*c*f^3*g - a*d*f^3*g)*B^2*x + (b*c*f^4 - a*d*f^4)*B^2)*\log(d*x + c)), x)$$

**Fricas [A]** time = 0., size = 0, normalized size = 0.

integral  $\left( \frac{1}{A^2g^3x^3 + 3A^2fg^2x^2 + 3A^2f^2gx + A^2f^3 + (B^2g^3x^3 + 3B^2fg^2x^2 + 3B^2f^2gx + B^2f^3) \log\left(\frac{b^2ex^2+2abex+a^2e}{d^2x^2+2cdx+c^2}\right)^2} + 2 \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)^3/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="fricas")

[Out] integral(1/(A^2\*g^3\*x^3 + 3\*A^2\*f\*g^2\*x^2 + 3\*A^2\*f^2\*g\*x + A^2\*f^3 + (B^2\*g^3\*x^3 + 3\*B^2\*f\*g^2\*x^2 + 3\*B^2\*f^2\*g\*x + B^2\*f^3)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))^2 + 2\*(A\*B\*g^3\*x^3 + 3\*A\*B\*f\*g^2\*x^2 + 3\*A\*B\*f^2\*g\*x + A\*B\*f^3)\*log((b^2\*e\*x^2 + 2\*a\*b\*e\*x + a^2\*e)/(d^2\*x^2 + 2\*c\*d\*x + c^2))), x)



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)\*\*3/(A+B\*ln(e\*(b\*x+a)\*\*2/(d\*x+c)\*\*2))\*\*2,x)

[Out] Timed out

**Giac [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(gx + f)^3 \left( B \log \left( \frac{(bx+a)^2 e}{(dx+c)^2} \right) + A \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(g\*x+f)^3/(A+B\*log(e\*(b\*x+a)^2/(d\*x+c)^2))^2,x, algorithm="giac")

[Out] integrate(1/((g\*x + f)^3\*(B\*log((b\*x + a)^2\*e/(d\*x + c)^2) + A)^2), x)

### 3.293 $\int (g+hx)^4 (A + B \log(e(a + bx)^n(c + dx)^{-n})) dx$

**Optimal.** Leaf size=365

$$\frac{Bh^2nx^2(bc - ad)(a^2d^2h^2 - abdh(5dg - ch) + b^2(c^2h^2 - 5cdgh + 10d^2g^2))}{10b^3d^3} + \frac{Bhnx(bc - ad)(-a^2bd^2h^2(5dg - ch) + a^3d^3)}{10b^3d^3}$$

```
[Out] (B*(b*c - a*d)*h*(a^3*d^3*h^3 - a^2*b*d^2*h^2*(5*d*g - c*h) + a*b^2*d*h*(10*d^2*g^2 - 5*c*d*g*h + c^2*h^2) - b^3*(10*d^3*g^3 - 10*c*d^2*g^2*h + 5*c^2*d*g*h^2 - c^3*h^3))*n*x)/(5*b^4*d^4) - (B*(b*c - a*d)*h^2*(a^2*d^2*h^2 - a*b*d*h*(5*d*g - c*h) + b^2*(10*d^2*g^2 - 5*c*d*g*h + c^2*h^2))*n*x^2)/(10*b^3*d^3) - (B*(b*c - a*d)*h^3*(5*b*d*g - b*c*h - a*d*h)*n*x^3)/(15*b^2*d^2) - (B*(b*c - a*d)*h^4*n*x^4)/(20*b*d) - (B*(b*g - a*h)^5*n*Log[a + b*x])/(5*b^5*h) + (B*(d*g - c*h)^5*n*Log[c + d*x])/(5*d^5*h) + ((g + h*x)^5*(A + B*Log[(e*(a + b*x)^n]/(c + d*x)^n]))/(5*h)
```

**Rubi [A]** time = 0.712262, antiderivative size = 377, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6742, 2492, 72}

$$\frac{Bh^2nx^2(bc - ad)(a^2d^2h^2 - abdh(5dg - ch) + b^2(c^2h^2 - 5cdgh + 10d^2g^2))}{10b^3d^3} + \frac{Bhnx(bc - ad)(-a^2bd^2h^2(5dg - ch) + a^3d^3)}{10b^3d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^4*(A + B*Log[(e*(a + b*x)^n]/(c + d*x)^n]), x]
```

```
[Out] (B*(b*c - a*d)*h*(a^3*d^3*h^3 - a^2*b*d^2*h^2*(5*d*g - c*h) + a*b^2*d*h*(10*d^2*g^2 - 5*c*d*g*h + c^2*h^2) - b^3*(10*d^3*g^3 - 10*c*d^2*g^2*h + 5*c^2*d*g*h^2 - c^3*h^3))*n*x)/(5*b^4*d^4) - (B*(b*c - a*d)*h^2*(a^2*d^2*h^2 - a*b*d*h*(5*d*g - c*h) + b^2*(10*d^2*g^2 - 5*c*d*g*h + c^2*h^2))*n*x^2)/(10*b^3*d^3) - (B*(b*c - a*d)*h^3*(5*b*d*g - b*c*h - a*d*h)*n*x^3)/(15*b^2*d^2) - (B*(b*c - a*d)*h^4*n*x^4)/(20*b*d) + (A*(g + h*x)^5)/(5*h) - (B*(b*g - a*h)^5*n*Log[a + b*x])/(5*b^5*h) + (B*(d*g - c*h)^5*n*Log[c + d*x])/(5*d^5*h) + (B*(g + h*x)^5*Log[(e*(a + b*x)^n]/(c + d*x)^n]))/(5*h)
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.))*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0] && NeQ[m, -1]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int (g + hx)^4 (A + B \log(e(a + bx)^n(c + dx)^{-n})) dx &= \int (A(g + hx)^4 + B(g + hx)^4 \log(e(a + bx)^n(c + dx)^{-n})) dx \\
&= \frac{A(g + hx)^5}{5h} + B \int (g + hx)^4 \log(e(a + bx)^n(c + dx)^{-n}) dx \\
&= \frac{A(g + hx)^5}{5h} + \frac{B(g + hx)^5 \log(e(a + bx)^n(c + dx)^{-n})}{5h} - \frac{(B(bc - ad))}{5h} \\
&= \frac{A(g + hx)^5}{5h} + \frac{B(g + hx)^5 \log(e(a + bx)^n(c + dx)^{-n})}{5h} - \frac{(B(bc - ad))}{5h} \\
&= \frac{B(bc - ad)h (a^3 d^3 h^3 - a^2 b d^2 h^2 (5dg - ch) + ab^2 dh (10d^2 g^2 - 5cdg))}{5b^4 d^4}
\end{aligned}$$

**Mathematica [A]** time = 0.952097, size = 463, normalized size = 1.27

$$\frac{bdx \left( Bhn(bc - ad) \left( -6a^2bd^2h^2(-2ch + 10dg + dhx) + 12a^3d^3h^3 + 2ab^2dh(6c^2h^2 - 3cdh(10g + hx) + d^2(60g^2 + 15ghx) \right) \right)}{5b^4d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)^4\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]),x]

[Out] (b\*d\*x\*(12\*A\*b^4\*d^4\*(5\*g^4 + 10\*g^3\*h\*x + 10\*g^2\*h^2\*x^2 + 5\*g\*h^3\*x^3 + h^4\*x^4) + B\*(b\*c - a\*d)\*h\*n\*(12\*a^3\*d^3\*h^3 - 6\*a^2\*b\*d^2\*h^2\*(10\*d\*g - 2\*c\*h + d\*h\*x) + 2\*a\*b^2\*d\*h\*(6\*c^2\*h^2 - 3\*c\*d\*h\*(10\*g + h\*x) + d^2\*(60\*g^2 + 15\*g\*h\*x + 2\*h^2\*x^2)) - b^3\*(-12\*c^3\*h^3 + 6\*c^2\*d\*h^2\*(10\*g + h\*x) - 2\*c\*d^2\*h\*(60\*g^2 + 15\*g\*h\*x + 2\*h^2\*x^2) + d^3\*(120\*g^3 + 60\*g^2\*h\*x + 20\*g\*h^2\*x^2 + 3\*h^3\*x^3))) + 12\*a^2\*B\*d^5\*h\*(-10\*b^3\*g^3 + 10\*a\*b^2\*g^2\*h - 5\*a^2\*b\*g\*h^2 + a^3\*h^3)\*n\*Log[a + b\*x] - 12\*b^4\*B\*(-5\*a\*d^5\*g^4 + b\*c\*(5\*d^4\*g^4 - 10\*c\*d^3\*g^3\*h + 10\*c^2\*d^2\*g^2\*h^2 - 5\*c^3\*d\*g\*h^3 + c^4\*h^4))\*n\*Log[c + d\*x] + 12\*b^4\*B\*d^5\*(5\*a\*g^4 + b\*x\*(5\*g^4 + 10\*g^3\*h\*x + 10\*g^2\*h^2\*x^2 + 5\*g\*h^3\*x^3 + h^4\*x^4))\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]/(60\*b^5\*d^5)

**Maple [C]** time = 0.689, size = 2576, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)^4\*(A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n))),x)

[Out] B\*g^4\*x\*ln((b\*x+a)^n)+B\*ln(e)\*g^4\*x+1/5\*h^4\*B\*ln(e)\*x^5+1/5\*h^4\*B\*x^5\*ln((b\*x+a)^n)-1/5\*(h\*x+g)^5\*B/h\*ln((d\*x+c)^n)+2\*h\*B\*ln(e)\*g^3\*x^2+2\*h^2\*B\*g^2\*x^3\*ln((b\*x+a)^n)+2\*h\*B\*g^3\*x^2\*ln((b\*x+a)^n)+1/5/h\*B\*ln(d\*x+c)\*g^5\*n+h^3\*B\*g\*x^4\*ln((b\*x+a)^n)+h^3\*B\*ln(e)\*g\*x^4+2\*h^2\*B\*ln(e)\*g^2\*x^3+1/5\*h^4\*A\*x^5+2\*h^2/b^3\*B\*ln(-b\*x-a)\*a^3\*g^2\*n-2\*h/b^2\*B\*ln(-b\*x-a)\*a^2\*g^3\*n-I\*h\*B\*Pi\*g^3\*x^2\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3+1/2\*I\*B\*Pi\*g^4\*x\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2-1/2\*I\*h^3\*B\*Pi\*g\*x^4\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3+1/10\*I\*h^4\*B\*Pi\*x^5\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2-1/2\*I\*h^3\*B\*Pi\*g\*x^4\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3-I\*h\*B\*Pi\*g^3\*x^2\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3+1/2\*I\*B\*Pi\*g^4\*x\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+1/10\*I\*h^4\*B\*Pi\*x^5\*csgn(I\*e)\*csgn(I\*

$$\begin{aligned}
& e/((d*x+c)^n)*(b*x+a)^n)^2+1/10*I*h^4*B*Pi*x^5*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/10*I*h^4*B*Pi*x^5*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-I*h^2*B*Pi*g^2*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I*h^2*B*Pi*g^2*x^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+1/2*I*B*Pi*g^4*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/d^4*h^3*B*ln(d*x+c)*c^4*g^n-2/d^3*h^2*B*ln(d*x+c)*c^3*g^2*n+2/d^2*h*B*ln(d*x+c)*c^2*g^3*n-h^3/b^4*B*ln(-b*x-a)*a^4*g^n+1/15/d^2*h^4*B*c^2*n*x^3+1/10*h^4/b^3*B*a^3*n*x^2-1/10/d^3*h^4*B*c^3*n*x^2-1/5*h^4/b^4*B*a^4*n*x+1/5/d^4*h^4*B*c^4*n*x+I*h*B*Pi*g^3*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*h*B*Pi*g^3*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*h^2*B*Pi*g^2*x^3*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*h^2*B*Pi*g^2*x^3*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*h^2*B*Pi*g^2*x^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*h*B*Pi*g^3*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I*B*Pi*g^4*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2*I*B*Pi*g^4*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/10*I*h^4*B*Pi*x^5*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/10*I*h^4*B*Pi*x^5*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+1/2*I*h^3*B*Pi*g*x^4*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*h^3*B*Pi*g*x^4*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*h^3*B*Pi*g*x^4*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*h^3*B*Pi*g*x^4*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+h^3*A*g*x^4+2*h^2*A*g^2*x^3+2*h*A*g^3*x^2+A*g^4*x-1/d*B*ln(d*x+c)*c*g^4*n+1/b*B*ln(-b*x-a)*a*g^4*n-1/5/d^5*h^4*B*ln(d*x+c)*c^5*n+1/5*h^4/b^5*B*ln(-b*x-a)*a^5*n-1/10*I*h^4*B*Pi*x^5*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/10*I*h^4*B*Pi*x^5*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I*B*Pi*g^4*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I*B*Pi*g^4*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+2*h/b*B*a*g^3*n*x-1/d^3*h^3*B*c^3*g^n*x+2/d^2*h^2*B*c^2*g^2*n*x-2/d*h*B*c*g^3*n*x+1/3*h^3/b*B*a*g^n*x^3-1/3/d*h^3*B*c*g^n*x^3-1/2*h^3/b^2*B*a^2*g^n*x^2+h^2/b*B*a*g^2*n*x^2+1/2/d^2*h^3*B*c^2*g^n*x^2-1/d*h^2*B*c*g^2*n*x^2+h^3/b^3*B*a^3*g^n*x-2*h^2/b^2*B*a^2*g^2*n*x+I*h^2*B*Pi*g^2*x^3*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*h*B*Pi*g^3*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/20*h^4/b*B*a*n*x^4-1/20/d*h^4*B*c*n*x^4-1/15*h^4/b^2*B*a^2*n*x^3-1/2*I*h^3*B*Pi*g*x^4*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-I*h*B*Pi*g^3*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*h*B*Pi*g^3*x^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-I*h^2*B*Pi*g^2*x^3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*h^2*B*Pi*g^2*x^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2*I*h^3*B*Pi*g*x^4*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)
\end{aligned}$$

**Maxima [A]** time = 1.40192, size = 906, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^4\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="maxima")

[Out]  $1/5*B*h^4*x^5*\log((b*x + a)^n*e/(d*x + c)^n) + 1/5*A*h^4*x^5 + B*g*h^3*x^4*\log((b*x + a)^n*e/(d*x + c)^n) + A*g*h^3*x^4 + 2*B*g^2*h^2*x^3*\log((b*x + a)^n*e/(d*x + c)^n) + 2*A*g^2*h^2*x^3 + 2*B*g^3*h*x^2*\log((b*x + a)^n*e/(d*x + c)^n) + 2*A*g^3*h*x^2 + B*g^4*x*\log((b*x + a)^n*e/(d*x + c)^n) + A*g^4*x + (a*e*n*\log(b*x + a)/b - c*e*n*\log(d*x + c)/d)*B*g^4/e - 2*(a^2*e*n*\log(b$

$$\begin{aligned} & *x + a)/b^2 - c^2*e^n*\log(d*x + c)/d^2 + (b*c*e^n - a*d*e^n)*x/(b*d))*B*g^3 \\ & *h/e + (2*a^3*e^n*\log(b*x + a)/b^3 - 2*c^3*e^n*\log(d*x + c)/d^3 - ((b^2*c*d \\ & *e^n - a*b*d^2*e^n)*x^2 - 2*(b^2*c^2*e^n - a^2*d^2*e^n)*x)/(b^2*d^2))*B*g^2 \\ & *h^2/e - 1/6*(6*a^4*e^n*\log(b*x + a)/b^4 - 6*c^4*e^n*\log(d*x + c)/d^4 + (2* \\ & (b^3*c*d^2*e^n - a*b^2*d^3*e^n)*x^3 - 3*(b^3*c^2*d*e^n - a^2*b*d^3*e^n)*x^2 \\ & + 6*(b^3*c^3*e^n - a^3*d^3*e^n)*x)/(b^3*d^3))*B*g*h^3/e + 1/60*(12*a^5*e^n \\ & *\log(b*x + a)/b^5 - 12*c^5*e^n*\log(d*x + c)/d^5 - (3*(b^4*c*d^3*e^n - a*b^3 \\ & *d^4*e^n)*x^4 - 4*(b^4*c^2*d^2*e^n - a^2*b^2*d^4*e^n)*x^3 + 6*(b^4*c^3*d*e \\ & n - a^3*b*d^4*e^n)*x^2 - 12*(b^4*c^4*e^n - a^4*d^4*e^n)*x)/(b^4*d^4))*B*h^4 \\ & /e \end{aligned}$$

**Fricas [B]** time = 1.1267, size = 1623, normalized size = 4.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas")
```

```
[Out] 1/60*(12*A*b^5*d^5*h^4*x^5 + 3*(20*A*b^5*d^5*g*h^3 - (B*b^5*c*d^4 - B*a*b^4*d^5)*d^5)*h^4*n)*x^4 + 4*(30*A*b^5*d^5*g^2*h^2 - (5*(B*b^5*c*d^4 - B*a*b^4*d^5)*g*h^3 - (B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*h^4)*n)*x^3 + 6*(20*A*b^5*d^5*g^3*h - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*g^2*h^2 - 5*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g*h^3 + (B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*h^4)*n)*x^2 + 12*(5*A*b^5*d^5*g^4 - (10*(B*b^5*c*d^4 - B*a*b^4*d^5)*g^3*h - 10*(B*b^5*c^2*d^3 - B*a^2*b^3*d^5)*g^2*h^2 + 5*(B*b^5*c^3*d^2 - B*a^3*b^2*d^5)*g*h^3 - (B*b^5*c^4*d - B*a^4*b*d^5)*h^4)*n)*x + 12*(B*b^5*d^5*h^4*n*x^5 + 5*B*b^5*d^5*g*h^3*n*x^4 + 10*B*b^5*d^5*g^2*h^2*n*x^3 + 10*B*b^5*d^5*g^3*h*n*x^2 + 5*B*b^5*d^5*g^4*n*x + (5*B*a*b^4*d^5*g^4 - 10*B*a^2*b^3*d^5*g^3*h + 10*B*a^3*b^2*d^5*g^2*h^2 - 5*B*a^4*b*d^5*g*h^3 + B*a^5*d^5*h^4)*n)*log(b*x + a) - 12*(B*b^5*d^5*h^4*n*x^5 + 5*B*b^5*d^5*g*h^3*n*x^4 + 10*B*b^5*d^5*g^2*h^2*n*x^3 + 10*B*b^5*d^5*g^3*h*n*x^2 + 5*B*b^5*d^5*g^4*n*x + (5*B*b^5*c*d^4*g^4 - 10*B*b^5*c^2*d^3*g^3*h + 10*B*b^5*c^3*d^2*g^2*h^2 - 5*B*b^5*c^4*d*g*h^3 + B*b^5*c^5*h^4)*n)*log(d*x + c) + 12*(B*b^5*d^5*h^4*x^5 + 5*B*b^5*d^5*g*h^3*x^4 + 10*B*b^5*d^5*g^2*h^2*x^3 + 10*B*b^5*d^5*g^3*h*x^2 + 5*B*b^5*d^5*g^4*x)*log(e))/(b^5*d^5)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**4*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^4*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.294 $\int (g+hx)^3 (A + B \log(e(a + bx)^n(c + dx)^{-n})) dx$

**Optimal.** Leaf size=236

$$\frac{Bhx(bc - ad)(a^2d^2h^2 - abdh(4dg - ch) + b^2(c^2h^2 - 4cdgh + 6d^2g^2))}{4b^3d^3} + \frac{(g + hx)^4(B \log(e(a + bx)^n(c + dx)^{-n})) + A}{4h}$$

[Out]  $-(B*(b*c - a*d)*h*(a^2*d^2*h^2 - a*b*d*h*(4*d*g - c*h) + b^2*(6*d^2*g^2 - 4*c*d*g*h + c^2*h^2))*n*x)/(4*b^3*d^3) - (B*(b*c - a*d)*h^2*(4*b*d*g - b*c*h - a*d*h)*n*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*h^3*n*x^3)/(12*b*d) - (B*(b*g - a*h)^4*n*Log[a + b*x])/(4*b^4*h) + (B*(d*g - c*h)^4*n*Log[c + d*x])/(4*d^4*h) + ((g + h*x)^4*(A + B*Log[(e*(a + b*x)^n]/(c + d*x)^n]))/(4*h)$

**Rubi [A]** time = 0.455644, antiderivative size = 248, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6742, 2492, 72}

$$\frac{Bhx(bc - ad)(a^2d^2h^2 - abdh(4dg - ch) + b^2(c^2h^2 - 4cdgh + 6d^2g^2))}{4b^3d^3} - \frac{Bh^2nx^2(bc - ad)(-adh - bch + 4bdg)}{8b^2d^2} - \frac{Bn}{4h}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(g + hx)^3*(A + B*Log[(e*(a + b*x)^n]/(c + d*x)^n]), x]$

[Out]  $-(B*(b*c - a*d)*h*(a^2*d^2*h^2 - a*b*d*h*(4*d*g - c*h) + b^2*(6*d^2*g^2 - 4*c*d*g*h + c^2*h^2))*n*x)/(4*b^3*d^3) - (B*(b*c - a*d)*h^2*(4*b*d*g - b*c*h - a*d*h)*n*x^2)/(8*b^2*d^2) - (B*(b*c - a*d)*h^3*n*x^3)/(12*b*d) + (A*(g + h*x)^4)/(4*h) - (B*(b*g - a*h)^4*n*Log[a + b*x])/(4*b^4*h) + (B*(d*g - c*h)^4*n*Log[c + d*x])/(4*d^4*h) + (B*(g + h*x)^4*Log[(e*(a + b*x)^n]/(c + d*x)^n))/(4*h)$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

#### Rule 2492

$\text{Int}[\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.))*((c_.) + (d_.)*(x_.))^(q_.))]^(r_.)]^(s_.)*((g_.) + (h_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[((g + hx)^(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - \text{Dist}[(p*r*s*(b*c - a*d))/(h*(m + 1)), \text{Int}[(g + hx)^(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{NeQ}[m, -1]$

#### Rule 72

$\text{Int}[(e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{IntegerQ}[p]$

#### Rubi steps

$$\begin{aligned}
\int (g + hx)^3 (A + B \log(e(a + bx)^n(c + dx)^{-n})) dx &= \int (A(g + hx)^3 + B(g + hx)^3 \log(e(a + bx)^n(c + dx)^{-n})) dx \\
&= \frac{A(g + hx)^4}{4h} + B \int (g + hx)^3 \log(e(a + bx)^n(c + dx)^{-n}) dx \\
&= \frac{A(g + hx)^4}{4h} + \frac{B(g + hx)^4 \log(e(a + bx)^n(c + dx)^{-n})}{4h} - \frac{(B(bc - ad)n)}{4h} \\
&= \frac{A(g + hx)^4}{4h} + \frac{B(g + hx)^4 \log(e(a + bx)^n(c + dx)^{-n})}{4h} - \frac{(B(bc - ad)n)}{4h} \\
&= -\frac{B(bc - ad)h(a^2d^2h^2 - abdh(4dg - ch) + b^2(6d^2g^2 - 4cdgh + c^2h^2))}{4b^3d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.566985, size = 314, normalized size = 1.33

$$\frac{bdx(6Ab^3d^3(6g^2hx + 4g^3 + 4gh^2x^2 + h^3x^3) - Bhn(bc - ad)(6a^2d^2h^2 - 3abdh(-2ch + 8dg + dhx) + b^2(6c^2h^2 - 3cdh(8g$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)^3\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]),x]

[Out] (b\*d\*x\*(6\*A\*b^3\*d^3\*(4\*g^3 + 6\*g^2\*h\*x + 4\*g\*h^2\*x^2 + h^3\*x^3) - B\*(b\*c - a\*d)\*h\*n\*(6\*a^2\*d^2\*h^2 - 3\*a\*b\*d\*h\*(8\*d\*g - 2\*c\*h + d\*h\*x) + b^2\*(6\*c^2\*h^2 - 3\*c\*d\*h\*(8\*g + h\*x) + 2\*d^2\*(18\*g^2 + 6\*g\*h\*x + h^2\*x^2)))) - 6\*a^2\*B\*d^4\*h\*(6\*b^2\*g^2 - 4\*a\*b\*g\*h + a^2\*h^2)\*n\*Log[a + b\*x] + 6\*b^3\*B\*(4\*a\*d^4\*g^3 + b\*c\*(-4\*d^3\*g^3 + 6\*c\*d^2\*g^2\*h - 4\*c^2\*d\*g\*h^2 + c^3\*h^3))\*n\*Log[c + d\*x] + 6\*b^3\*B\*d^4\*(4\*a\*g^3 + b\*x\*(4\*g^3 + 6\*g^2\*h\*x + 4\*g\*h^2\*x^2 + h^3\*x^3))\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]/(24\*b^4\*d^4)

**Maple [C]** time = 0.613, size = 1967, normalized size = 8.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)^3\*(A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n))),x)

[Out] B\*g^3\*x\*ln((b\*x+a)^n)+B\*ln(e)\*g^3\*x+1/4\*h^3\*B\*x^4\*ln((b\*x+a)^n)+1/4\*h^3\*B\*ln(e)\*x^4-1/4\*(h\*x+g)^4\*B/h\*ln((d\*x+c)^n)+1/4\*h^3\*A\*x^4+3/2\*h\*B\*ln(e)\*g^2\*x^2+1/4/h\*B\*ln(-d\*x-c)\*g^4\*n+h^2\*B\*g\*x^3\*ln((b\*x+a)^n)+3/2\*h\*B\*g^2\*x^2\*ln((b\*x+a)^n)+h^2\*B\*ln(e)\*g\*x^3+1/12\*h^3/b\*B\*a\*n\*x^3-1/12/d\*h^3\*B\*c\*n\*x^3-1/8\*h^3/b^2\*B\*a^2\*n\*x^2+1/8/d^2\*h^3\*B\*c^2\*n\*x^2+1/4\*h^3/b^3\*B\*a^3\*n\*x-1/4/d^3\*h^3\*B\*c^3\*n\*x-1/d^3\*h^2\*B\*ln(-d\*x-c)\*c^3\*g\*n+3/2/d^2\*h\*B\*ln(-d\*x-c)\*c^2\*g^2\*n+h^2/b^3\*B\*ln(b\*x+a)\*a^3\*g\*n-3/2\*h/b^2\*B\*ln(b\*x+a)\*a^2\*g^2\*n+1/2\*I\*B\*Pi\*g^3\*x\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+1/2\*I\*B\*Pi\*g^3\*x\*csgn(I\*e)\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+1/8\*I\*h^3\*B\*Pi\*x^4\*csgn(I\*e)\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2-1/2\*I\*h^2\*B\*Pi\*g\*x^3\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-1/2\*I\*h^2\*B\*Pi\*g\*x^3\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3-3/4\*I\*h\*B\*Pi\*g^2\*x^2\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-3/4\*I\*h\*B\*Pi\*g^2\*x^2\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3+1/8\*I\*h^3\*B\*Pi\*x^4\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+1/8\*I\*h^3\*B\*Pi\*x^4\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+1/2\*I\*B\*Pi\*g^3\*x\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+1/2\*I\*B\*Pi\*g^3\*x\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+



$a^n / ((dx+c)^n)^{2+1/8} I^h B^3 \pi x^4 \operatorname{csgn}(I(bx+a)^n) \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n)^{2-1/2} I B \pi g^3 x \operatorname{csgn}(I e / ((dx+c)^n) (bx+a)^n)^{3-1/8} I^h B^3 \pi x^4 \operatorname{csgn}(I e / ((dx+c)^n) (bx+a)^n)^{3-1/8} I^h B^3 \pi x^4 \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n)^{3-1/2} I B \pi g^3 x \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n)^{3+1/2} h^2 / b B a g^n x^2 - 1/2 / d h^2 B c g^n x^2 - h^2 / b^2 B a^2 g^n x + 3/2 h / b B a g^2 n x + 1/d^2 h^2 B c^2 g^n x - 3/2 / d h B c g^2 n x + 1/2 I^h B \pi g x^3 \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n) \operatorname{csgn}(I e / ((dx+c)^n) (bx+a)^n)^{2+3/4} I^h B \pi g^2 x^2 \operatorname{csgn}(I(bx+a)^n) \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n)^{2-1/8} I^h B^3 \pi x^4 \operatorname{csgn}(I(bx+a)^n) \operatorname{csgn}(I / ((dx+c)^n) \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n)) + 3/4 I^h B \pi g^2 x^2 \operatorname{csgn}(I e) \operatorname{csgn}(I e / ((dx+c)^n) (bx+a)^n)^{2-1/2} I B \pi g^3 x \operatorname{csgn}(I e) \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n) \operatorname{csgn}(I e / ((dx+c)^n) (bx+a)^n) - 1/2 I B \pi g^3 x \operatorname{csgn}(I(bx+a)^n) \operatorname{csgn}(I / ((dx+c)^n) \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n)) + 1/2 I^h B \pi g x^3 \operatorname{csgn}(I(bx+a)^n) \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n)^{2+1/2} I^h B \pi g x^3 \operatorname{csgn}(I / ((dx+c)^n) \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n)^{2+3/4} I^h B \pi g^2 x^2 \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n) \operatorname{csgn}(I e / ((dx+c)^n) (bx+a)^n)^{2+1/2} I^h B \pi g x^3 \operatorname{csgn}(I e) \operatorname{csgn}(I e / ((dx+c)^n) (bx+a)^n)^{2+3/4} I^h B \pi g^2 x^2 \operatorname{csgn}(I / ((dx+c)^n) \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n)^{2-1/8} I^h B^3 \pi x^4 \operatorname{csgn}(I e) \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n) \operatorname{csgn}(I e / ((dx+c)^n) (bx+a)^n) + h^2 A g x^3 + 3/2 h A g^2 x^2 + A g^3 x + 1/4 / d^4 h^3 B \ln(-dx-c) c^4 n - 1/4 h^3 / b^4 B \ln(bx+a) a^4 n - 1/d B \ln(-dx-c) c g^3 n + 1/b B \ln(bx+a) a g^3 n - 3/4 I^h B \pi g^2 x^2 \operatorname{csgn}(I e) \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n) \operatorname{csgn}(I e / ((dx+c)^n) (bx+a)^n) - 1/2 I^h B \pi g x^3 \operatorname{csgn}(I(bx+a)^n) \operatorname{csgn}(I / ((dx+c)^n) \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n)) - 3/4 I^h B \pi g^2 x^2 \operatorname{csgn}(I(bx+a)^n) \operatorname{csgn}(I / ((dx+c)^n) \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n)) - 1/2 I^h B \pi g x^3 \operatorname{csgn}(I e) \operatorname{csgn}(I(bx+a)^n / ((dx+c)^n) \operatorname{csgn}(I e / ((dx+c)^n) (bx+a)^n)$

**Maxima [B]** time = 1.20409, size = 630, normalized size = 2.67

$$\frac{1}{4} B h^3 x^4 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{1}{4} A h^3 x^4 + B g h^2 x^3 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A g h^2 x^3 + \frac{3}{2} B g^2 h x^2 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + \frac{3}{2} A g^2 h x^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(A+B*log(e*(bx+a)^n/((dx+c)^n))),x, algorithm="maxima")
```

```
[Out] 1/4*B*h^3*x^4*log((bx+a)^n*e/(dx+c)^n) + 1/4*A*h^3*x^4 + B*g*h^2*x^3*log((bx+a)^n*e/(dx+c)^n) + A*g*h^2*x^3 + 3/2*B*g^2*h*x^2*log((bx+a)^n*e/(dx+c)^n) + 3/2*A*g^2*h*x^2 + B*g^3*x*log((bx+a)^n*e/(dx+c)^n) + A*g^3*x + (a*e*n*log(bx+a)/b - c*e*n*log(dx+c)/d)*B*g^3/e - 3/2*(a^2*e*n*log(bx+a)/b^2 - c^2*e*n*log(dx+c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*B*g^2*h/e + 1/2*(2*a^3*e*n*log(bx+a)/b^3 - 2*c^3*e*n*log(dx+c)/d^3 - ((b^2*c*d*e*n - a*b*d^2*e*n)*x^2 - 2*(b^2*c^2*e*n - a^2*d^2*e*n)*x)/(b^2*d^2))*B*g*h^2/e - 1/24*(6*a^4*e*n*log(bx+a)/b^4 - 6*c^4*e*n*log(dx+c)/d^4 + (2*(b^3*c*d^2*e*n - a*b^2*d^3*e*n)*x^3 - 3*(b^3*c^2*d*e*n - a^2*b*d^3*e*n)*x^2 + 6*(b^3*c^3*e*n - a^3*d^3*e*n)*x)/(b^3*d^3))*B*h^3/e
```

**Fricas [B]** time = 1.10268, size = 1149, normalized size = 4.87

$$6 A b^4 d^4 h^3 x^4 + 2 (12 A b^4 d^4 g h^2 - (B b^4 c d^3 - B a b^3 d^4) h^3 n) x^3 + 3 (12 A b^4 d^4 g^2 h - (4 (B b^4 c d^3 - B a b^3 d^4) g h^2 - (B b^4 c^2 d^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="fricas")
```

```
[Out] 1/24*(6*A*b^4*d^4*h^3*x^4 + 2*(12*A*b^4*d^4*g*h^2 - (B*b^4*c*d^3 - B*a*b^3*d^4)*h^3*n)*x^3 + 3*(12*A*b^4*d^4*g^2*h - (4*(B*b^4*c*d^3 - B*a*b^3*d^4)*g*h^2 - (B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*h^3)*n)*x^2 + 6*(4*A*b^4*d^4*g^3 - (6*(B*b^4*c*d^3 - B*a*b^3*d^4)*g^2*h - 4*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*g*h^2 + (B*b^4*c^3*d - B*a^3*b*d^4)*h^3)*n)*x + 6*(B*b^4*d^4*h^3*n*x^4 + 4*B*b^4*d^4*g*h^2*n*x^3 + 6*B*b^4*d^4*g^2*h*n*x^2 + 4*B*b^4*d^4*g^3*n*x + (4*B*a*b^3*d^4*g^3 - 6*B*a^2*b^2*d^4*g^2*h + 4*B*a^3*b*d^4*g*h^2 - B*a^4*d^4*h^3)*n)*log(b*x + a) - 6*(B*b^4*d^4*h^3*n*x^4 + 4*B*b^4*d^4*g*h^2*n*x^3 + 6*B*b^4*d^4*g^2*h*n*x^2 + 4*B*b^4*d^4*g^3*n*x + (4*B*b^4*c*d^3*g^3 - 6*B*b^4*c^2*d^2*g^2*h + 4*B*b^4*c^3*d*g*h^2 - B*b^4*c^4*h^3)*n)*log(d*x + c) + 6*(B*b^4*d^4*h^3*x^4 + 4*B*b^4*d^4*g*h^2*x^3 + 6*B*b^4*d^4*g^2*h*x^2 + 4*B*b^4*d^4*g^3*x)*log(e))/(b^4*d^4)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)**3*(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

```
[Out] Timed out
```

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((h*x+g)^3*(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorithm="giac")
```

```
[Out] Timed out
```

### 3.295 $\int (g+hx)^2 (A + B \log(e(a+bx)^n(c+dx)^{-n})) dx$

**Optimal.** Leaf size=158

$$\frac{(g+hx)^3 (B \log(e(a+bx)^n(c+dx)^{-n}) + A)}{3h} - \frac{Bhnx(bc-ad)(-adh-bch+3bdg)}{3b^2d^2} - \frac{Bn(bg-ah)^3 \log(a+bx)}{3b^3h} - \frac{Bh^2n}{6bd}$$

[Out]  $-(B*(b*c - a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*x)/(3*b^2*d^2) - (B*(b*c - a*d)*h^2*n*x^2)/(6*b*d) - (B*(b*g - a*h)^3*n*Log[a + b*x])/(3*b^3*h) + (B*(d*g - c*h)^3*n*Log[c + d*x])/(3*d^3*h) + ((g + h*x)^3*(A + B*Log[(e*(a + b*x)^n]/(c + d*x)^n]))/(3*h)$

**Rubi [A]** time = 0.240267, antiderivative size = 170, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6742, 2492, 72}

$$\frac{Bhnx(bc-ad)(-adh-bch+3bdg)}{3b^2d^2} - \frac{Bn(bg-ah)^3 \log(a+bx)}{3b^3h} + \frac{B(g+hx)^3 \log(e(a+bx)^n(c+dx)^{-n})}{3h} - \frac{Bh^2nx^2(bd)}{6bd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(g + hx)^2*(A + B*Log[(e*(a + b*x)^n]/(c + d*x)^n]), x]$

[Out]  $-(B*(b*c - a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*x)/(3*b^2*d^2) - (B*(b*c - a*d)*h^2*n*x^2)/(6*b*d) + (A*(g + h*x)^3)/(3*h) - (B*(b*g - a*h)^3*n*Log[a + b*x])/(3*b^3*h) + (B*(d*g - c*h)^3*n*Log[c + d*x])/(3*d^3*h) + (B*(g + h*x)^3*Log[(e*(a + b*x)^n]/(c + d*x)^n))/(3*h)$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

#### Rule 2492

$\text{Int}[\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.))*((c_.) + (d_.)*(x_.))^(q_.))^(r_.)]^(s_.)*((g_.) + (h_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(g + hx)^(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s/(h*(m + 1)), x] - \text{Dist}[(p*r*s*(b*c - a*d))/(h*(m + 1)), \text{Int}[(g + hx)^(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s/(a + b*x)*(c + d*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, m, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{NeQ}[m, -1]$

#### Rule 72

$\text{Int}[(e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{IntegerQ}[p]$

#### Rubi steps



$(x+c)^n * (b*x+a)^n)^{2+1/6} * I * h^2 * B * \pi * x^3 * \text{csgn}(I * (b*x+a)^n) * \text{csgn}(I * (b*x+a)^n / ((d*x+c)^n))^{2+1/6} * I * h^2 * B * \pi * x^3 * \text{csgn}(I / ((d*x+c)^n)) * \text{csgn}(I * (b*x+a)^n / ((d*x+c)^n))^{2-1/6} * I * h^2 * B * \pi * x^3 * \text{csgn}(I * e) * \text{csgn}(I * (b*x+a)^n / ((d*x+c)^n)) * \text{csgn}(I * e / ((d*x+c)^n) * (b*x+a)^n) - 1/6 * I * h^2 * B * \pi * x^3 * \text{csgn}(I * (b*x+a)^n) * \text{csgn}(I / ((d*x+c)^n)) * \text{csgn}(I * (b*x+a)^n / ((d*x+c)^n)) + 1/2 * I * h * B * \pi * g * x^2 * \text{csgn}(I * e) * \text{csgn}(I * e / ((d*x+c)^n) * (b*x+a)^n)^{2+1/2} * I * h * B * \pi * g * x^2 * \text{csgn}(I * (b*x+a)^n / ((d*x+c)^n)) * \text{csgn}(I * e / ((d*x+c)^n) * (b*x+a)^n)^{2+1/2} * I * h * B * \pi * g * x^2 * \text{csgn}(I * (b*x+a)^n) * \text{csgn}(I * (b*x+a)^n / ((d*x+c)^n))^{2+1/2} * I * h * B * \pi * g * x^2 * \text{csgn}(I / ((d*x+c)^n)) * \text{csgn}(I * (b*x+a)^n / ((d*x+c)^n))^{2-1/2} * I * B * \pi * g^2 * x * \text{csgn}(I * e) * \text{csgn}(I * (b*x+a)^n / ((d*x+c)^n)) * \text{csgn}(I * e / ((d*x+c)^n) * (b*x+a)^n) - 1/2 * I * B * \pi * g^2 * x * \text{csgn}(I * (b*x+a)^n) * \text{csgn}(I / ((d*x+c)^n)) * \text{csgn}(I * (b*x+a)^n / ((d*x+c)^n)) + h / b * B * a * g * n * x - 1 / d * h * B * c * g * n * x$

**Maxima [A]** time = 1.19188, size = 397, normalized size = 2.51

$$\frac{1}{3} B h^2 x^3 \log\left(\frac{(b x + a)^n e}{(d x + c)^n}\right) + \frac{1}{3} A h^2 x^3 + B g h x^2 \log\left(\frac{(b x + a)^n e}{(d x + c)^n}\right) + A g h x^2 + B g^2 x \log\left(\frac{(b x + a)^n e}{(d x + c)^n}\right) + A g^2 x + \frac{\left(a e n \log(b x + a)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="maxima")

[Out] 1/3\*B\*h^2\*x^3\*log((b\*x + a)^n\*e/(d\*x + c)^n) + 1/3\*A\*h^2\*x^3 + B\*g\*h\*x^2\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A\*g\*h\*x^2 + B\*g^2\*x\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A\*g^2\*x + (a\*e\*n\*log(b\*x + a)/b - c\*e\*n\*log(d\*x + c)/d)\*B\*g^2/e - (a^2\*e\*n\*log(b\*x + a)/b^2 - c^2\*e\*n\*log(d\*x + c)/d^2 + (b\*c\*e\*n - a\*d\*e\*n)\*x/(b\*d))\*B\*g\*h/e + 1/6\*(2\*a^3\*e\*n\*log(b\*x + a)/b^3 - 2\*c^3\*e\*n\*log(d\*x + c)/d^3 - ((b^2\*c\*d\*e\*n - a\*b\*d^2\*e\*n)\*x^2 - 2\*(b^2\*c^2\*e\*n - a^2\*d^2\*e\*n)\*x)/(b^2\*d^2))\*B\*h^2/e

**Fricas [B]** time = 1.08998, size = 753, normalized size = 4.77

$$2 A b^3 d^3 h^2 x^3 + (6 A b^3 d^3 g h - (B b^3 c d^2 - B a b^2 d^3) h^2 n) x^2 + 2 (3 A b^3 d^3 g^2 - (3 (B b^3 c d^2 - B a b^2 d^3) g h - (B b^3 c^2 d - B a^2 b d^3) h^2 n) x + 2 (3 B b^3 d^3 h^2 n x^3 + 3 B b^3 d^3 g h n x^2 + 3 B b^3 d^3 g^2 n x + (3 B a b^2 d^3 g^2 - 3 B a^2 b d^3 g h + B a^3 d^3 h^2 n) * \log(b x + a) - 2 (B b^3 d^3 h^2 n x^3 + 3 B b^3 d^3 g h n x^2 + 3 B b^3 d^3 g^2 n x + (3 B b^3 c d^2 g^2 - 3 B b^3 c^2 d g h + B b^3 c^3 h^2 n) * \log(d x + c) + 2 (B b^3 d^3 h^2 x^3 + 3 B b^3 d^3 g h x^2 + 3 B b^3 d^3 g^2 x) * \log(e)) / (b^3 d^3))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="fricas")

[Out] 1/6\*(2\*A\*b^3\*d^3\*h^2\*x^3 + (6\*A\*b^3\*d^3\*g\*h - (B\*b^3\*c\*d^2 - B\*a\*b^2\*d^3)\*h^2\*n)\*x^2 + 2\*(3\*A\*b^3\*d^3\*g^2 - (3\*(B\*b^3\*c\*d^2 - B\*a\*b^2\*d^3)\*g\*h - (B\*b^3\*c^2\*d - B\*a^2\*b\*d^3)\*h^2)\*n)\*x + 2\*(B\*b^3\*d^3\*h^2\*n\*x^3 + 3\*B\*b^3\*d^3\*g\*h\*n\*x^2 + 3\*B\*b^3\*d^3\*g^2\*n\*x + (3\*B\*a\*b^2\*d^3\*g^2 - 3\*B\*a^2\*b\*d^3\*g\*h + B\*a^3\*d^3\*h^2)\*n)\*log(b\*x + a) - 2\*(B\*b^3\*d^3\*h^2\*n\*x^3 + 3\*B\*b^3\*d^3\*g\*h\*n\*x^2 + 3\*B\*b^3\*d^3\*g^2\*n\*x + (3\*B\*b^3\*c\*d^2\*g^2 - 3\*B\*b^3\*c^2\*d\*g\*h + B\*b^3\*c^3\*h^2)\*n)\*log(d\*x + c) + 2\*(B\*b^3\*d^3\*h^2\*x^3 + 3\*B\*b^3\*d^3\*g\*h\*x^2 + 3\*B\*b^3\*d^3\*g^2\*x)\*log(e))/(b^3\*d^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*2\*(A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n))),x)

[Out] Timed out

**Giac [B]** time = 77.4189, size = 402, normalized size = 2.54

$$\frac{1}{3}(Ah^2 + Bh^2)x^3 + \frac{1}{3}(Bh^2nx^3 + 3Bghnx^2 + 3Bg^2nx)\log(bx + a) - \frac{1}{3}(Bh^2nx^3 + 3Bghnx^2 + 3Bg^2nx)\log(dx + c) - \frac{(Bh^2nx^3 + 3Bghnx^2 + 3Bg^2nx)\log(dx + c)}{d^3} - \frac{1}{6}\frac{(B^2c^2h^2n - B^2adh^2n - 6A^2bdgh - 6B^2bdgh)x^2}{bd} + \frac{1}{3}\frac{(3B^2ab^2g^2n - 3B^2a^2bdgh + B^2a^3h^2n)\log(bx + a)}{b^3} - \frac{1}{3}\frac{(3B^2cd^2g^2n - 3B^2c^2dgh + B^2c^3h^2n)\log(-dx - c)}{d^3} - \frac{1}{3}\frac{(3B^2b^2cdgh - 3B^2abd^2gh - B^2b^2c^2h^2n + B^2a^2d^2h^2n - 3A^2bd^2g^2 - 3B^2bd^2g^2)x}{b^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="giac")

[Out] 1/3\*(A\*h^2 + B\*h^2)\*x^3 + 1/3\*(B\*h^2\*n\*x^3 + 3\*B\*g\*h\*n\*x^2 + 3\*B\*g^2\*n\*x)\*log(b\*x + a) - 1/3\*(B\*h^2\*n\*x^3 + 3\*B\*g\*h\*n\*x^2 + 3\*B\*g^2\*n\*x)\*log(d\*x + c) - 1/6\*(B\*b\*c\*h^2\*n - B\*a\*d\*h^2\*n - 6\*A\*b\*d\*g\*h - 6\*B\*b\*d\*g\*h)\*x^2/(b\*d) + 1/3\*(3\*B\*a\*b^2\*g^2\*n - 3\*B\*a^2\*b\*g\*h\*n + B\*a^3\*h^2\*n)\*log(b\*x + a)/b^3 - 1/3\*(3\*B\*c\*d^2\*g^2\*n - 3\*B\*c^2\*d\*g\*h\*n + B\*c^3\*h^2\*n)\*log(-d\*x - c)/d^3 - 1/3\*(3\*B\*b^2\*c\*d\*g\*h\*n - 3\*B\*a\*b\*d^2\*g\*h\*n - B\*b^2\*c^2\*h^2\*n + B\*a^2\*d^2\*h^2\*n - 3\*A\*b^2\*d^2\*g^2 - 3\*B\*b^2\*d^2\*g^2)\*x/(b^2\*d^2)

### 3.296 $\int (g + hx) (A + B \log (e(a + bx)^n (c + dx)^{-n})) dx$

**Optimal.** Leaf size=116

$$\frac{(g + hx)^2 (B \log (e(a + bx)^n (c + dx)^{-n}) + A)}{2h} - \frac{Bn(bg - ah)^2 \log(a + bx)}{2b^2h} - \frac{Bhnx(bc - ad)}{2bd} + \frac{Bn(dg - ch)^2 \log(c + dx)}{2d^2h}$$

[Out]  $-(B*(b*c - a*d)*h*n*x)/(2*b*d) - (B*(b*g - a*h)^2*n*Log[a + b*x])/(2*b^2*h) + (B*(d*g - c*h)^2*n*Log[c + d*x])/(2*d^2*h) + ((g + h*x)^2*(A + B*Log[(e*(a + b*x)^n]/(c + d*x)^n]))/(2*h)$

**Rubi [A]** time = 0.148869, antiderivative size = 128, normalized size of antiderivative = 1.1, number of steps used = 5, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {6742, 2492, 72}

$$-\frac{Bn(bg - ah)^2 \log(a + bx)}{2b^2h} + \frac{B(g + hx)^2 \log (e(a + bx)^n (c + dx)^{-n})}{2h} - \frac{Bhnx(bc - ad)}{2bd} + \frac{A(g + hx)^2}{2h} + \frac{Bn(dg - ch)^2 \log(c + dx)}{2d^2h}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(g + h*x)*(A + B*Log[(e*(a + b*x)^n]/(c + d*x)^n]), x]$

[Out]  $-(B*(b*c - a*d)*h*n*x)/(2*b*d) + (A*(g + h*x)^2)/(2*h) - (B*(b*g - a*h)^2*n*Log[a + b*x])/(2*b^2*h) + (B*(d*g - c*h)^2*n*Log[c + d*x])/(2*d^2*h) + (B*(g + h*x)^2*Log[(e*(a + b*x)^n]/(c + d*x)^n))/(2*h)$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

#### Rule 2492

$\text{Int}[\text{Log}[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.))*((c_.) + (d_.)*(x_.))^(q_.)]^(r_.)]^(s_.)*((g_.) + (h_.)*(x_.))^(m_.), x\_Symbol] \rightarrow \text{Simp}[(g + h*x)^(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s]/(h*(m + 1)), x] - \text{Dist}[(p*r*s*(b*c - a*d))/(h*(m + 1)), \text{Int}[(g + h*x)^(m + 1)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s]/((a + b*x)*(c + d*x)), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, m, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{NeQ}[m, -1]$

#### Rule 72

$\text{Int}[(e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{IntegerQ}[p]$

#### Rubi steps

$$\begin{aligned} \int (g + hx)(A + B \log(e(a + bx)^n(c + dx)^{-n})) dx &= \int (A(g + hx) + B(g + hx) \log(e(a + bx)^n(c + dx)^{-n})) dx \\ &= \frac{A(g + hx)^2}{2h} + B \int (g + hx) \log(e(a + bx)^n(c + dx)^{-n}) dx \\ &= \frac{A(g + hx)^2}{2h} + \frac{B(g + hx)^2 \log(e(a + bx)^n(c + dx)^{-n})}{2h} - \frac{(B(bc - ad)n)}{2} \int \frac{1}{c + dx} dx \\ &= \frac{A(g + hx)^2}{2h} + \frac{B(g + hx)^2 \log(e(a + bx)^n(c + dx)^{-n})}{2h} - \frac{(B(bc - ad)n)}{2} \ln(c + dx) \\ &= -\frac{B(bc - ad)hnx}{2bd} + \frac{A(g + hx)^2}{2h} - \frac{B(bg - ah)^2 n \log(a + bx)}{2b^2 h} + \frac{B(dg - ah)}{2b^2} \ln(c + dx) \end{aligned}$$

**Mathematica [A]** time = 0.167751, size = 124, normalized size = 1.07

$$\frac{-a^2 B d^2 h n \log(a + b x) + b d (x (B h n (a d - b c) + A b d (2 g + h x)) + B d (2 a g + b x (2 g + h x)) \log(e(a + b x)^n(c + d x)^{-n})) + b B n}{2 b^2 d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]), x]
```

```
[Out] -(a^2*B*d^2*h*n*Log[a + b*x]) + b*B*(2*a*d^2*g + b*c*(-2*d*g + c*h))*n*Log[c + d*x] + b*d*(x*(B*(-b*c) + a*d)*h*n + A*b*d*(2*g + h*x)) + B*d*(2*a*g + b*x*(2*g + h*x))*Log[(e*(a + b*x)^n)/(c + d*x)^n]/(2*b^2*d^2)
```

**Maple [C]** time = 0.541, size = 839, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))), x)
```

```
[Out] 1/2*A*h*x^2+A*g*x+1/2*B*h*x^2*ln((b*x+a)^n)-1/2*B*x*(h*x+2*g)*ln((d*x+c)^n)+B*g*x*ln((b*x+a)^n)+1/2*B*ln(e)*h*x^2+B*ln(e)*g*x-1/2*I*B*Pi*g*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I*B*Pi*g*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+1/2*I*B*Pi*g*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I*B*Pi*g*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2/b^2*B*ln(b*x+a)*a^2*h*n+1/b*B*ln(b*x+a)*a*g*n+1/2/d^2*B*ln(-d*x-c)*c^2*h*n-1/d*B*ln(-d*x-c)*c*g*n-1/4*I*B*Pi*h*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/4*I*B*Pi*h*x^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/4*I*B*Pi*h*x^2*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/4*I*B*Pi*h*x^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I*B*Pi*g*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))-1/2*I*B*Pi*g*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+1/2/b*B*a*h*n*x-1/2/d*B*c*h*n*x+1/4*I*B*Pi*h*x^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/4*I*B*Pi*h*x^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/4*I*B*Pi*h*x^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/4*I*B*Pi*h*x^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I*B*Pi*g*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2
```



**Maxima [A]** time = 1.16413, size = 208, normalized size = 1.79

$$\frac{1}{2} B h x^2 \log\left(\frac{(b x + a)^n e}{(d x + c)^n}\right) + \frac{1}{2} A h x^2 + B g x \log\left(\frac{(b x + a)^n e}{(d x + c)^n}\right) + A g x + \frac{\left(\frac{a e n \log(b x + a)}{b} - \frac{c e n \log(d x + c)}{d}\right) B g}{e} - \frac{\left(\frac{a^2 e n \log(b x + a)}{b^2} - \frac{c^2 e n \log(d x + c)}{d^2}\right) B g}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="maxima")

[Out] 1/2\*B\*h\*x^2\*log((b\*x + a)^n\*e/(d\*x + c)^n) + 1/2\*A\*h\*x^2 + B\*g\*x\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A\*g\*x + (a\*e\*n\*log(b\*x + a)/b - c\*e\*n\*log(d\*x + c)/d)\*B\*g/e - 1/2\*(a^2\*e\*n\*log(b\*x + a)/b^2 - c^2\*e\*n\*log(d\*x + c)/d^2 + (b\*c\*e\*n - a\*d\*e\*n)\*x/(b\*d))\*B\*h/e

**Fricas [A]** time = 1.01071, size = 413, normalized size = 3.56

$$\frac{A b^2 d^2 h x^2 + (2 A b^2 d^2 g - (B b^2 c d - B a b d^2) h n) x + (B b^2 d^2 h m x^2 + 2 B b^2 d^2 g n x + (2 B a b d^2 g - B a^2 d^2 h) n) \log(b x + a) - (B b^2 d^2 h n x^2 + 2 B b^2 d^2 g n x + (2 B b^2 c d g - B b^2 c^2 h) n) \log(d x + c) + (B b^2 d^2 h x^2 + 2 B b^2 d^2 g x) \log(e)}{2 b^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="fricas")

[Out] 1/2\*(A\*b^2\*d^2\*h\*x^2 + (2\*A\*b^2\*d^2\*g - (B\*b^2\*c\*d - B\*a\*b\*d^2)\*h\*n)\*x + (B\*b^2\*d^2\*h\*n\*x^2 + 2\*B\*b^2\*d^2\*g\*n\*x + (2\*B\*a\*b\*d^2\*g - B\*a^2\*d^2\*h)\*n)\*log(b\*x + a) - (B\*b^2\*d^2\*h\*n\*x^2 + 2\*B\*b^2\*d^2\*g\*n\*x + (2\*B\*b^2\*c\*d\*g - B\*b^2\*c^2\*h)\*n)\*log(d\*x + c) + (B\*b^2\*d^2\*h\*x^2 + 2\*B\*b^2\*d^2\*g\*x)\*log(e))/(b^2\*d^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n))),x)

[Out] Timed out

**Giac [A]** time = 5.78446, size = 201, normalized size = 1.73

$$\frac{1}{2} (A h + B h) x^2 + \frac{1}{2} (B h n x^2 + 2 B g n x) \log(b x + a) - \frac{1}{2} (B h n x^2 + 2 B g n x) \log(d x + c) - \frac{(B b c h n - B a d h n - 2 A b d g - 2 B b^2 c d g) x}{2 b d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n))),x, algorithm="giac")

[Out] 1/2\*(A\*h + B\*h)\*x^2 + 1/2\*(B\*h\*n\*x^2 + 2\*B\*g\*n\*x)\*log(b\*x + a) - 1/2\*(B\*h\*n\*x^2 + 2\*B\*g\*n\*x)\*log(d\*x + c) - 1/2\*(B\*b\*c\*h\*n - B\*a\*d\*h\*n - 2\*A\*b\*d\*g - 2\*B\*b\*d\*g)\*x/(b\*d) + 1/2\*(2\*B\*a\*b\*g\*n - B\*a^2\*h\*n)\*log(b\*x + a)/b^2 - 1/2\*(2\*B\*c\*d\*g\*n - B\*c^2\*h\*n)\*log(-d\*x - c)/d^2

### 3.297 $\int (A + B \log(e(a + bx)^n(c + dx)^{-n})) dx$

**Optimal.** Leaf size=57

$$\frac{B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{Bn(bc - ad) \log(c + dx)}{bd} + Ax$$

[Out] A\*x - (B\*(b\*c - a\*d)\*n\*Log[c + d\*x])/(b\*d) + (B\*(a + b\*x)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/b

**Rubi [A]** time = 0.0299171, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {2486, 31}

$$\frac{B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{Bn(bc - ad) \log(c + dx)}{bd} + Ax$$

Antiderivative was successfully verified.

[In] Int[A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n], x]

[Out] A\*x - (B\*(b\*c - a\*d)\*n\*Log[c + d\*x])/(b\*d) + (B\*(a + b\*x)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/b

#### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] :> Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps

$$\begin{aligned} \int (A + B \log(e(a + bx)^n(c + dx)^{-n})) dx &= Ax + B \int \log(e(a + bx)^n(c + dx)^{-n}) dx \\ &= Ax + \frac{B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{(B(bc - ad)n) \int \frac{1}{c + dx} dx}{b} \\ &= Ax - \frac{B(bc - ad)n \log(c + dx)}{bd} + \frac{B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} \end{aligned}$$

**Mathematica [A]** time = 0.0107576, size = 57, normalized size = 1.

$$\frac{B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{Bn(bc - ad) \log(c + dx)}{bd} + Ax$$

Antiderivative was successfully verified.

[In] Integrate[A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n], x]

[Out]  $A*x - (B*(b*c - a*d)*n*\text{Log}[c + d*x])/(b*d) + (B*(a + b*x)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/b$

**Maple [B]** time = 0.058, size = 123, normalized size = 2.2

$$Ax + B \ln\left(\frac{e(bx+a)^n}{(dx+c)^n}\right) x + \frac{Bna^2 \ln(bx+a)d}{b(ad-bc)} - \frac{Bna \ln(bx+a)c}{ad-bc} - \frac{Bnc \ln(dx+c)a}{ad-bc} + \frac{Bnc^2 \ln(dx+c)b}{d(ad-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)),x)`

[Out]  $A*x+B*\ln(e*(b*x+a)^n/((d*x+c)^n))*x+B*n*a^2/(a*d-b*c)/b*\ln(b*x+a)*d-B*n*a/(a*d-b*c)*\ln(b*x+a)*c-B*n*c/(a*d-b*c)*\ln(d*x+c)*a+B*n*c^2/(a*d-b*c)/d*\ln(d*x+c)*b$

**Maxima [A]** time = 1.12728, size = 80, normalized size = 1.4

$$Bx \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + Ax + \frac{\left(\frac{aen \log(bx+a)}{b} - \frac{cen \log(dx+c)}{d}\right) B}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="maxima")`

[Out]  $B*x*\log((b*x + a)^n*e/(d*x + c)^n) + A*x + (a*e*n*\log(b*x + a)/b - c*e*n*\log(d*x + c)/d)*B/e$

**Fricas [A]** time = 1.03051, size = 146, normalized size = 2.56

$$\frac{Bbdx \log(e) + Abdx + (Bbdnx + Badn) \log(bx + a) - (Bbdnx + Bbcn) \log(dx + c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(A+B*log(e*(b*x+a)^n/((d*x+c)^n)),x, algorithm="fricas")`

[Out]  $(B*b*d*x*\log(e) + A*b*d*x + (B*b*d*n*x + B*a*d*n)*\log(b*x + a) - (B*b*d*n*x + B*b*c*n)*\log(d*x + c))/(b*d)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)),x)`

[Out] Timed out

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**Giac [A]** time = 1.32551, size = 74, normalized size = 1.3

$$\left( nx \log (bx + a) - nx \log (dx + c) + \frac{an \log (bx + a)}{b} - \frac{cn \log (-dx - c)}{d} + x \right) B + Ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)),x, algorithm="giac")

[Out] (n\*x\*log(b\*x + a) - n\*x\*log(d\*x + c) + a\*n\*log(b\*x + a)/b - c\*n\*log(-d\*x - c)/d + x)\*B + A\*x

$$3.298 \quad \int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{g+hx} dx$$

**Optimal.** Leaf size=148

$$\frac{Bn \operatorname{PolyLog}\left(2, \frac{b(g+hx)}{bg-ah}\right)}{h} + \frac{Bn \operatorname{PolyLog}\left(2, \frac{d(g+hx)}{dg-ch}\right)}{h} + \frac{\log(g+hx)(B \log(e(a+bx)^n(c+dx)^{-n}) + A)}{h} - \frac{Bn \log(g+hx)}{h}$$

[Out]  $-\left(\frac{Bn \operatorname{Log}\left[-\left(\frac{h(a+bx)}{bg-ah}\right)\right] \operatorname{Log}[g+hx]}{h}\right) + \left(\frac{Bn \operatorname{Log}\left[-\left(\frac{h(c+dx)}{dg-ch}\right)\right] \operatorname{Log}[g+hx]}{h}\right) + \left(\frac{(A+B \operatorname{Log}\left[\frac{e(a+bx)^n}{(c+dx)^n}\right]) \operatorname{Log}[g+hx]}{h}\right) - \left(\frac{Bn \operatorname{PolyLog}\left[2, \frac{b(g+hx)}{bg-ah}\right]}{h}\right) + \left(\frac{Bn \operatorname{PolyLog}\left[2, \frac{d(g+hx)}{dg-ch}\right]}{h}\right)$

**Rubi [A]** time = 0.188928, antiderivative size = 156, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {6742, 2494, 2394, 2393, 2391}

$$\frac{Bn \operatorname{PolyLog}\left(2, \frac{b(g+hx)}{bg-ah}\right)}{h} + \frac{Bn \operatorname{PolyLog}\left(2, \frac{d(g+hx)}{dg-ch}\right)}{h} + \frac{B \log(g+hx) \log(e(a+bx)^n(c+dx)^{-n})}{h} - \frac{Bn \log(g+hx) \log(e(a+bx)^n(c+dx)^{-n})}{h}$$

Antiderivative was successfully verified.

[In] `Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x), x]`

[Out]  $(A \operatorname{Log}[g+hx])/h - \left(\frac{Bn \operatorname{Log}\left[-\left(\frac{h(a+bx)}{bg-ah}\right)\right] \operatorname{Log}[g+hx]}{h}\right) + \left(\frac{Bn \operatorname{Log}\left[-\left(\frac{h(c+dx)}{dg-ch}\right)\right] \operatorname{Log}[g+hx]}{h}\right) + \left(\frac{B \operatorname{Log}\left[\frac{e(a+bx)^n}{(c+dx)^n}\right] \operatorname{Log}[g+hx]}{h}\right) - \left(\frac{Bn \operatorname{PolyLog}\left[2, \frac{b(g+hx)}{bg-ah}\right]}{h}\right) + \left(\frac{Bn \operatorname{PolyLog}\left[2, \frac{d(g+hx)}{dg-ch}\right]}{h}\right)$

#### Rule 6742

`Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]`

#### Rule 2494

`Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.)]^(r_.)]/((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(Log[g + h*x]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/h, x] + (-Dist[(b*p*r)/h, Int[Log[g + h*x]/(a + b*x), x], x] - Dist[(d*q*r)/h, Int[Log[g + h*x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b*c - a*d, 0]`

#### Rule 2394

`Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Simp[(Log[(e*(f + g*x))/(e*f - d*g)]*(a + b*Log[c*(d + e*x)^n])/g, x] - Dist[(b*e*n)/g, Int[Log[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e*f - d*g, 0]`

#### Rule 2393

`Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))]*(b_.))/((f_.) + (g_.)*(x_)), x_Symbol] := Dist[1/g, Subst[Int[(a + b*Log[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && EqQ[g + c*(e*f - d*g), 0]`

**Rule 2391**

$\text{Int}[\text{Log}[(c\_)*(d\_)+(e\_)*(x\_)^{(n\_)}]]/(x\_), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x\} \ \&\& \ \text{EqQ}[c*d, 1]$

**Rubi steps**

$$\begin{aligned} \int \frac{A + B \log(e(a + bx)^n(c + dx)^{-n})}{g + hx} dx &= \int \left( \frac{A}{g + hx} + \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{g + hx} \right) dx \\ &= \frac{A \log(g + hx)}{h} + B \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{g + hx} dx \\ &= \frac{A \log(g + hx)}{h} + \frac{B \log(e(a + bx)^n(c + dx)^{-n}) \log(g + hx)}{h} - \frac{(bBn) \int \frac{\log(g + hx)}{a + bx} dx}{h} \\ &= \frac{A \log(g + hx)}{h} - \frac{Bn \log\left(-\frac{h(a + bx)}{bg - ah}\right) \log(g + hx)}{h} + \frac{Bn \log\left(-\frac{h(c + dx)}{dg - ch}\right) \log(g + hx)}{h} \\ &= \frac{A \log(g + hx)}{h} - \frac{Bn \log\left(-\frac{h(a + bx)}{bg - ah}\right) \log(g + hx)}{h} + \frac{Bn \log\left(-\frac{h(c + dx)}{dg - ch}\right) \log(g + hx)}{h} \\ &= \frac{A \log(g + hx)}{h} - \frac{Bn \log\left(-\frac{h(a + bx)}{bg - ah}\right) \log(g + hx)}{h} + \frac{Bn \log\left(-\frac{h(c + dx)}{dg - ch}\right) \log(g + hx)}{h} \end{aligned}$$

**Mathematica [A]** time = 0.0949328, size = 150, normalized size = 1.01

$$\frac{Bn \left( \text{PolyLog}\left(2, \frac{h(a+bx)}{ah-bg}\right) + \log(a+bx) \log\left(\frac{b(g+hx)}{bg-ah}\right) \right) - Bn \left( \text{PolyLog}\left(2, \frac{h(c+dx)}{ch-dg}\right) + \log(c+dx) \log\left(\frac{d(g+hx)}{dg-ch}\right) \right) + \log(g+hx)}{h}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(g + h\*x), x]

[Out] ((A + B\*(-(n\*Log[a + b\*x]) + n\*Log[c + d\*x] + Log[(e\*(a + b\*x)^n)/(c + d\*x]^n)))\*Log[g + h\*x] + B\*n\*(Log[a + b\*x]\*Log[(b\*(g + h\*x))/(b\*g - a\*h)] + PolyLog[2, (h\*(a + b\*x))/(-b\*g) + a\*h]) - B\*n\*(Log[c + d\*x]\*Log[(d\*(g + h\*x))/(d\*g - c\*h)] + PolyLog[2, (h\*(c + d\*x))/(-d\*g) + c\*h]))/h

**Maple [C]** time = 0.783, size = 597, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g), x)

[Out] 1/2\*I\*ln(h\*x+g)/h\*B\*Pi\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+1/2\*I\*ln(h\*x+g)/h\*B\*Pi\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+1/2\*I\*ln(h\*x+g)/h\*B\*Pi\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2-1/2\*I\*ln(h\*x+g)/h\*B\*Pi\*csgn(I\*e)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)+1/2\*I\*ln(h\*x+g)/h\*B\*Pi\*csgn(I\*e)\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2-1/2\*I\*ln(h\*x+g)/h\*B\*Pi\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3-1/2\*I\*ln(h\*x+g)/h\*B\*Pi\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-1/2\*I\*ln(h\*x+g)/

$$h*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+$$

$$\ln(h*x+g)/h*B*\ln(e)+A*\ln(h*x+g)/h+B*\ln(h*x+g)/h*\ln((b*x+a)^n)-B/h*n*dilog(($$

$$b*(h*x+g)+a*h-b*g)/(a*h-b*g))-B/h*n*\ln(h*x+g)*\ln((b*(h*x+g)+a*h-b*g)/(a*h-b$$

$$*g))-B*\ln(h*x+g)/h*\ln((d*x+c)^n)+B/h*n*dilog((d*(h*x+g)+c*h-d*g)/(c*h-d*g))$$

$$+B/h*n*\ln(h*x+g)*\ln((d*(h*x+g)+c*h-d*g)/(c*h-d*g))$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-B \int -\frac{\log((bx+a)^n) - \log((dx+c)^n) + \log(e)}{hx+g} dx + \frac{A \log(hx+g)}{h}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g),x, algorithm="maxima")

[Out] -B\*integrate(-(log((b\*x + a)^n) - log((d\*x + c)^n) + log(e))/(h\*x + g), x) + A\*log(h\*x + g)/h

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A}{hx+g}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g),x, algorithm="fricas")

[Out] integral((B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)/(h\*x + g), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))/(h\*x+g),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A}{hx+g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g),x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)/(h\*x + g), x)

$$3.299 \quad \int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^2} dx$$

**Optimal.** Leaf size=120

$$-\frac{B \log(e(a+bx)^n(c+dx)^{-n}) + A}{h(g+hx)} + \frac{Bn(bc-ad) \log(g+hx)}{(bg-ah)(dg-ch)} + \frac{bBn \log(a+bx)}{h(bg-ah)} - \frac{Bdn \log(c+dx)}{h(dg-ch)}$$

[Out] (b\*B\*n\*Log[a + b\*x])/(h\*(b\*g - a\*h)) - (B\*d\*n\*Log[c + d\*x])/(h\*(d\*g - c\*h)) - (A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(h\*(g + h\*x)) + (B\*(b\*c - a\*d)\*n\*Log[g + h\*x])/((b\*g - a\*h)\*(d\*g - c\*h))

**Rubi [A]** time = 0.119829, antiderivative size = 132, normalized size of antiderivative = 1.1, number of steps used = 6, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {6742, 2490, 36, 31}

$$\frac{B(a+bx) \log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)(bg-ah)} - \frac{Bn(bc-ad) \log(c+dx)}{(bg-ah)(dg-ch)} + \frac{Bn(bc-ad) \log(g+hx)}{(bg-ah)(dg-ch)} - \frac{A}{h(g+hx)}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(g + h\*x)^2, x]

[Out] -(A/(h\*(g + h\*x))) - (B\*(b\*c - a\*d)\*n\*Log[c + d\*x])/((b\*g - a\*h)\*(d\*g - c\*h)) + (B\*(a + b\*x)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/((b\*g - a\*h)\*(g + h\*x)) + (B\*(b\*c - a\*d)\*n\*Log[g + h\*x])/((b\*g - a\*h)\*(d\*g - c\*h))

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2490

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_))^2, x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/((b\*g - a\*h)\*(g + h\*x)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(b\*g - a\*h), Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x)\*(g + h\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && NeQ[b\*g - a\*h, 0] && IGtQ[s, 0]

#### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rubi steps



$$\begin{aligned} \int \frac{A + B \log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^2} dx &= \int \left( \frac{A}{(g + hx)^2} + \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^2} \right) dx \\ &= -\frac{A}{h(g + hx)} + B \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^2} dx \\ &= -\frac{A}{h(g + hx)} + \frac{B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} - \frac{(B(bc - ad)n) \int \frac{1}{(c+dx)(g+hx)}}{bg - ah} \\ &= -\frac{A}{h(g + hx)} + \frac{B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} - \frac{(Bd(bc - ad)n) \int \frac{1}{c+dx}}{(bg - ah)(dg - ch)} \\ &= -\frac{A}{h(g + hx)} - \frac{B(bc - ad)n \log(c + dx)}{(bg - ah)(dg - ch)} + \frac{B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} \end{aligned}$$

**Mathematica [A]** time = 0.224986, size = 117, normalized size = 0.98

$$\frac{-\frac{B \log(e(a+bx)^n(c+dx)^{-n})}{g+hx} + \frac{Bn(b \log(a+bx)(dg-ch)+\log(c+dx)(adh-bdg)+h(bc-ad) \log(g+hx))}{(bg-ah)(dg-ch)} - \frac{A}{g+hx}}{h}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2,x]
```

```
[Out] (-A/(g + h*x)) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x) + (B*n*(b*(d*g - c*h)*Log[a + b*x] + (-b*d*g) + a*d*h)*Log[c + d*x] + (b*c - a*d)*h*Log[g + h*x])/((b*g - a*h)*(d*g - c*h))/h
```

**Maple [C]** time = 0.486, size = 1796, normalized size = 15.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^2,x)
```

```
[Out] B/h/(h*x+g)*ln((d*x+c)^n)-1/2*(I*B*Pi*a*d*h*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)*g-2*A*a*d*h*g-2*A*b*c*h*g+2*A*b*d*g^2+2*A*a*c*h^2-I*B*Pi*b*c*h*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*g-I*B*Pi*b*c*h*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*g-I*B*Pi*b*c*h*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2*g-I*B*Pi*a*c*h^2*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)+I*B*Pi*b*d*g^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-2*B*ln(e)*a*d*h*g-2*B*ln(e)*b*c*h*g+2*B*a*c*h^2*ln((b*x+a)^n)+2*B*b*d*g^2*ln((b*x+a)^n)+2*B*ln(e)*a*c*h^2+2*B*ln(e)*b*d*g^2+I*B*Pi*a*d*h*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*g+I*B*Pi*b*c*h*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)*g-2*B*a*d*g*h*ln((b*x+a)^n)-2*B*b*c*g*h*ln((b*x+a)^n)-2*B*ln(-b*x-a)*b*d*g*h*n*x-I*B*Pi*b*d*g^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I*B*Pi*b*d*g^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3+I*B*Pi*b*d*g^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*a*c*h^2*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*a*c*h^2*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*a*c*h^2*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+I*B*Pi*a*c*h^2*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*b*c*h*csgn(I*(b*x+a)^n/((d*x+c)^n))^3*g+I*B*Pi*b*c*h*csgn(I*e/((d*x+c
```

)^n\*(b\*x+a)^n)^3\*g+I\*B\*Pi\*b\*d\*g^2\*csgn(I\*e)\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2+I\*B\*Pi\*b\*d\*g^2\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2+I\*B\*Pi\*b\*c\*h\*csgn(I\*(b\*x+a)^n)\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*g-I\*B\*Pi\*a\*d\*h\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2\*g-I\*B\*Pi\*a\*d\*h\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2\*g-I\*B\*Pi\*b\*c\*h\*csgn(I\*e)\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2\*g-I\*B\*Pi\*b\*d\*g^2\*csgn(I\*e)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)-I\*B\*Pi\*b\*d\*g^2\*csgn(I\*(b\*x+a)^n)\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))-I\*B\*Pi\*a\*c\*h^2\*csgn(I\*(b\*x+a)^n)\*csgn(I/((d\*x+c)^n))\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))-I\*B\*Pi\*a\*d\*h\*csgn(I\*e)\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^2\*g-I\*B\*Pi\*a\*d\*h\*csgn(I\*(b\*x+a)^n)\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^2\*g-2\*B\*ln(-b\*x-a)\*b\*d\*g^2\*n+2\*B\*ln(-d\*x-c)\*b\*d\*g^2\*n+I\*B\*Pi\*a\*d\*h\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3\*g+I\*B\*Pi\*a\*d\*h\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3\*g+2\*B\*ln(-b\*x-a)\*b\*c\*h^2\*n\*x+2\*B\*ln(h\*x+g)\*a\*d\*h^2\*n\*x-2\*B\*ln(h\*x+g)\*b\*c\*h^2\*n\*x-2\*B\*ln(-d\*x-c)\*a\*d\*h^2\*n\*x+2\*B\*ln(-b\*x-a)\*b\*c\*g\*h\*n+2\*B\*ln(h\*x+g)\*a\*d\*g\*h\*n-2\*B\*ln(h\*x+g)\*b\*c\*g\*h\*n-2\*B\*ln(-d\*x-c)\*a\*d\*g\*h\*n-I\*B\*Pi\*a\*c\*h^2\*csgn(I\*(b\*x+a)^n/((d\*x+c)^n))^3-I\*B\*Pi\*a\*c\*h^2\*csgn(I\*e/((d\*x+c)^n)\*(b\*x+a)^n)^3+2\*B\*ln(-d\*x-c)\*b\*d\*g\*h\*n\*x)/(h\*x+g)/(a\*c\*h^2-a\*d\*g\*h-b\*c\*g\*h+b\*d\*g^2)/h

**Maxima [A]** time = 1.1812, size = 204, normalized size = 1.7

$$\frac{\left(\frac{ben \log(bx+a)}{bgh-ah^2} - \frac{den \log(dx+c)}{dgh-ch^2} - \frac{(bcen-aden) \log(hx+g)}{(dgh-ch^2)a-(dg^2-cgh)b}\right)B}{e} - \frac{B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{h^2x + gh} - \frac{A}{h^2x + gh}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g)^2,x, algorithm="maxima")

[Out] (b\*e\*n\*log(b\*x + a)/(b\*g\*h - a\*h^2) - d\*e\*n\*log(d\*x + c)/(d\*g\*h - c\*h^2) - (b\*c\*e\*n - a\*d\*e\*n)\*log(h\*x + g)/((d\*g\*h - c\*h^2)\*a - (d\*g^2 - c\*g\*h)\*b))\*B /e - B\*log((b\*x + a)^n\*e/(d\*x + c)^n)/(h^2\*x + g\*h) - A/(h^2\*x + g\*h)

**Fricas [B]** time = 18.5281, size = 551, normalized size = 4.59

$$\frac{Abdg^2 + Aach^2 - (Abc + Aad)gh - ((Bbdgh - Bbch^2)nx + (Badgh - Bach^2)n) \log(bx + a) + ((Bbdgh - Badh^2)nx + (Bbdg^3h + acgh^3 - (bc + ad)g^2h$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g)^2,x, algorithm="fricas")

[Out] -(A\*b\*d\*g^2 + A\*a\*c\*h^2 - (A\*b\*c + A\*a\*d)\*g\*h - ((B\*b\*d\*g\*h - B\*b\*c\*h^2)\*n\*x + (B\*a\*d\*g\*h - B\*a\*c\*h^2)\*n)\*log(b\*x + a) + ((B\*b\*d\*g\*h - B\*a\*d\*h^2)\*n\*x + (B\*b\*c\*g\*h - B\*a\*c\*h^2)\*n)\*log(d\*x + c) - ((B\*b\*c - B\*a\*d)\*h^2\*n\*x + (B\*b\*c - B\*a\*d)\*g\*h\*n)\*log(h\*x + g) + (B\*b\*d\*g^2 + B\*a\*c\*h^2 - (B\*b\*c + B\*a\*d)\*g\*h)\*log(e))/(b\*d\*g^3\*h + a\*c\*g\*h^3 - (b\*c + a\*d)\*g^2\*h^2 + (b\*d\*g^2\*h^2 + a\*c\*h^4 - (b\*c + a\*d)\*g\*h^3)\*x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))/(h\*x+g)\*\*2,x)

[Out] Timed out

**Giac [A]** time = 1.47081, size = 224, normalized size = 1.87

$$\frac{Bb^2n \log(|-bx - a|)}{b^2gh - abh^2} - \frac{Bd^2n \log(|dx + c|)}{d^2gh - cdh^2} - \frac{Bn \log(bx + a)}{h^2x + gh} + \frac{Bn \log(dx + c)}{h^2x + gh} + \frac{(Bbcn - Badn) \log(hx + g)}{bdg^2 - bcgh - adgh + ach^2} - \frac{A + B}{h^2x + gh}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g)^2,x, algorithm="giac")

[Out] B\*b^2\*n\*log(abs(-b\*x - a))/(b^2\*g\*h - a\*b\*h^2) - B\*d^2\*n\*log(abs(d\*x + c))/(d^2\*g\*h - c\*d\*h^2) - B\*n\*log(b\*x + a)/(h^2\*x + g\*h) + B\*n\*log(d\*x + c)/(h^2\*x + g\*h) + (B\*b\*c\*n - B\*a\*d\*n)\*log(h\*x + g)/(b\*d\*g^2 - b\*c\*g\*h - a\*d\*g\*h + a\*c\*h^2) - (A + B)/(h^2\*x + g\*h)

$$3.300 \quad \int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^3} dx$$

**Optimal.** Leaf size=191

$$-\frac{B \log(e(a+bx)^n(c+dx)^{-n}) + A}{2h(g+hx)^2} + \frac{b^2 B n \log(a+bx)}{2h(bg-ah)^2} - \frac{Bn(bc-ad)}{2(g+hx)(bg-ah)(dg-ch)} + \frac{Bn(bc-ad) \log(g+hx)(-adh-bch+2)}{2(bg-ah)^2(dg-ch)^2}$$

[Out]  $-(B*(b*c - a*d)*n)/(2*(b*g - a*h)*(d*g - c*h)*(g + h*x)) + (b^2*B*n*Log[a + b*x])/(2*h*(b*g - a*h)^2) - (B*d^2*n*Log[c + d*x])/(2*h*(d*g - c*h)^2) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*h*(g + h*x)^2) + (B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*Log[g + h*x])/(2*(b*g - a*h)^2*(d*g - c*h)^2)$

**Rubi [A]** time = 0.300838, antiderivative size = 203, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6742, 2492, 72}

$$\frac{b^2 B n \log(a+bx)}{2h(bg-ah)^2} - \frac{B \log(e(a+bx)^n(c+dx)^{-n})}{2h(g+hx)^2} - \frac{Bn(bc-ad)}{2(g+hx)(bg-ah)(dg-ch)} + \frac{Bn(bc-ad) \log(g+hx)(-adh-bch+2)}{2(bg-ah)^2(dg-ch)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(g + h\*x)^3, x]

[Out]  $-A/(2*h*(g + h*x)^2) - (B*(b*c - a*d)*n)/(2*(b*g - a*h)*(d*g - c*h)*(g + h*x)) + (b^2*B*n*Log[a + b*x])/(2*h*(b*g - a*h)^2) - (B*d^2*n*Log[c + d*x])/(2*h*(d*g - c*h)^2) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(2*h*(g + h*x)^2) + (B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*Log[g + h*x])/(2*(b*g - a*h)^2*(d*g - c*h)^2)$

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2492

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*((g\_.) + (h\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(h\*(m + 1)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(h\*(m + 1)), Int[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0] && NeQ[m, -1]

#### Rule 72

Int[((e\_.) + (f\_.)\*(x\_))^(p\_.)/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

#### Rubi steps

$$\int \frac{A + B \log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^3} dx = \int \left( \frac{A}{(g + hx)^3} + \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^3} \right) dx$$

$$= -\frac{A}{2h(g + hx)^2} + B \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^3} dx$$

$$= -\frac{A}{2h(g + hx)^2} - \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{2h(g + hx)^2} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)(c+dx)(g+hx)^2} dx}{2h}$$

$$= -\frac{A}{2h(g + hx)^2} - \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{2h(g + hx)^2} + \frac{(B(bc - ad)n) \int \left( \frac{b^3}{(bc-ad)(bg-ah)} \right)}{2h}$$

$$= -\frac{A}{2h(g + hx)^2} - \frac{B(bc - ad)n}{2(bg - ah)(dg - ch)(g + hx)} + \frac{b^2 Bn \log(a + bx)}{2h(bg - ah)^2} - \frac{Bd^2 n \log(c + dx)}{2h(dg - ch)^2}$$

**Mathematica [A]** time = 0.733684, size = 178, normalized size = 0.93

$$Bn(bc - ad) \left( \frac{\frac{d^2 \log(c+dx)}{bc-ad} + \frac{h \left( \frac{(bg-ah)(dg-ch)}{g+hx} + \log(g+hx)(adh+bch-2bdg) \right)}{(bg-ah)^2}}{(dg-ch)^2} - \frac{b^2 \log(a+bx)}{(bc-ad)(bg-ah)^2} \right) + \frac{B \log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^2} + \frac{A}{(g+hx)^2}$$


---

2h

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^3, x]
```

```
[Out] -(A/(g + h*x)^2 + (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^2 + B*(b*c - a*d)*n*(-((b^2*Log[a + b*x])/(b*c - a*d)*(b*g - a*h)^2)) + ((d^2*Log[c + d*x])/(b*c - a*d) + (h*((b*g - a*h)*(d*g - c*h))/(g + h*x) + (-2*b*d*g + b*c*h + a*d*h)*Log[g + h*x]))/(b*g - a*h)^2/(d*g - c*h)^2)/(2*h)
```

**Maple [C]** time = 0.69, size = 4925, normalized size = 25.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^3, x)
```

```
[Out] 1/2*B/h/(h*x+g)^2*ln((d*x+c)^n)-1/4*(-2*I*B*Pi*a^2*c*d*g*h^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-2*I*B*Pi*a*b*c^2*g*h^3*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+2*I*B*Pi*a*b*c^2*g*h^3*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))+2*I*B*Pi*a^2*c*d*g*h^3*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-I*B*Pi*a^2*c^2*h^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+2*B*ln(e)*a^2*c^2*h^4+2*B*ln(e)*b^2*d^2*g^4+2*A*a^2*c^2*h^4+2*A*b^2*d^2*g^4-I*B*Pi*b^2*d^2*g^4*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-I*B*Pi*b^2*d^2*g^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-I*B*Pi*a^2*c^2*h^4*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-2*I*B*Pi*a*b*c^2*g*h^3*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+2*B*a^2*d^2*g^2*h^2*n-2*B*b^2*c^2*g^2*h^2*n-4*B*ln(e)*a^2*c*d*g*h^3-4*B*ln(e)*a*b*c^2*g*h^3-4*B*ln(e)*a*b*d^2*g^3*h-4*B*ln(e)*b^2*c*d*g^3*h+2*A*a^2*d^2*g^2*h^2+2*A*b^2*c^2*g^2*h^2+2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-2*I*B*Pi*b^2*c*d*g^3*h*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+I*B*Pi*a^2*c^2*h^4*csgn(I*(b*x+a)^n/((d*x+c)^n))^2
```

$$\begin{aligned} & \operatorname{gn}(Ie) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n)^2 + I*B*Pi*a^2*c^2*h^4 * \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n))^2 + I*B*Pi*a^2*c^2*h^4 * \operatorname{csgn}(I / ((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n))^2 + I*B*Pi*a^2*c^2*h^4 * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n)^2 + I*B*Pi*b^2*d^2*g^4 * \operatorname{csgn}(Ie) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n)^2 - 2*I*B*Pi*a*b*c^2*g^h^3 * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n)^2 - 2*I*B*Pi*a^2*c*d*g^h^3 * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n)^2 + 8*A*a*b*c*d*g^2*h^2 - 2*I*B*Pi*a^2*c*d*g^h^3 * \operatorname{csgn}(I / ((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n))^2 + 2*I*B*Pi*a*b*d^2*g^3*h * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n)^3 + 2*I*B*Pi*b^2*c*d*g^3*h * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n))^3 + 2*B*a^2*c^2*h^4 * \ln((b*x+a)^n) + 2*B*b^2*d^2*g^4 * \ln((b*x+a)^n) - 2*B*\ln(b*x+a) * b^2*d^2*g^4 * n + 2*B*\ln(-d*x-c) * b^2*d^2*g^4 * n - I*B*Pi*b^2*d^2*g^4 * \operatorname{csgn}(Ie) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n) - I*B*Pi*b^2*d^2*g^4 * \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn}(I / ((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) + 2*I*B*Pi*a^2*c*d*g^h^3 * \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn}(I / ((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) + I*B*Pi*a^2*d^2*g^2*h^2 * \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n))^2 + I*B*Pi*a^2*d^2*g^2*h^2 * \operatorname{csgn}(I / ((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n))^2 + 2*I*B*Pi*a*b*d^2*g^3*h * \operatorname{csgn}(Ie) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n) + I*B*Pi*b^2*d^2*g^4 * \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n))^2 + I*B*Pi*b^2*d^2*g^4 * \operatorname{csgn}(I / ((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n))^2 + I*B*Pi*b^2*d^2*g^4 * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n)^2 - I*B*Pi*a^2*d^2*g^2*h^2 * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n))^3 - 2*B*a^2*c*d*g^h^3 * n + 2*B*a*b*c^2*g^h^3 * n - 2*B*a*b*d^2*g^3*h * n + 2*B*b^2*c*d*g^3*h * n + 8*B*a*b*c*d*g^2*h^2 * \ln((b*x+a)^n) + 4*B*\ln(b*x+a) * b^2*c*d*g^h^3 * n * x^2 + 4*B*\ln(-h*x-g) * a*b*d^2*g^h^3 * n * x^2 - 4*B*\ln(-h*x-g) * b^2*c*d*g^h^3 * n * x^2 - 4*B*\ln(-d*x-c) * a*b*d^2*g^h^3 * n * x^2 + 8*B*\ln(b*x+a) * b^2*c*d*g^2*h^2 * n * x + 8*B*\ln(-h*x-g) * a*b*d^2*g^2*h^2 * n * x - 8*B*\ln(-h*x-g) * b^2*c*d*g^2*h^2 * n * x - 8*B*\ln(-d*x-c) * a*b*d^2*g^2*h^2 * n * x - 2*B*a^2*c*d*h^4 * n * x + 2*B*a^2*d^2*g^h^3 * n * x + 2*B*a*b*c^2*h^4 * n * x - 2*B*b^2*c^2*g^h^3 * n * x - 4*I*B*Pi*a*b*c*d*g^2*h^2 * \operatorname{csgn}(Ie) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n) - 4*I*B*Pi*a*b*c*d*g^2*h^2 * \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn}(I / ((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) - 2*B*\ln(b*x+a) * b^2*c^2*h^4 * n * x^2 - 2*B*\ln(-h*x-g) * a^2*d^2*h^4 * n * x^2 + 2*B*\ln(-h*x-g) * b^2*c^2*h^4 * n * x^2 + 2*B*\ln(-d*x-c) * a^2*d^2*h^4 * n * x^2 - 2*B*\ln(b*x+a) * b^2*c^2*g^2*h^2 * n - 2*B*\ln(-h*x-g) * a^2*d^2*g^2*h^2 * n + 2*B*\ln(-h*x-g) * b^2*c^2*g^2*h^2 * n + 2*B*\ln(-d*x-c) * a^2*d^2*g^2*h^2 * n + 2*I*B*Pi*a*b*c^2*g^h^3 * \operatorname{csgn}(Ie) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n) + 2*B*a^2*d^2*g^2*h^2 * \ln((b*x+a)^n) + 2*B*b^2*c^2*g^2*h^2 * \ln((b*x+a)^n) + I*B*Pi*b^2*c^2*g^2*h^2 * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n)^2 + I*B*Pi*a^2*d^2*g^2*h^2 * \operatorname{csgn}(Ie) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n)^2 - I*B*Pi*b^2*c^2*g^2*h^2 * \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn}(I / ((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) - 2*I*B*Pi*b^2*c*d*g^3*h * \operatorname{csgn}(I / ((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n))^2 - I*B*Pi*a^2*d^2*g^2*h^2 * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n)^3 - 2*B*\ln(b*x+a) * b^2*d^2*g^2*h^2 * n * x^2 + 2*B*\ln(-d*x-c) * b^2*d^2*g^2*h^2 * n * x^2 - 4*B*\ln(b*x+a) * b^2*c^2*g^h^3 * n * x - 4*B*\ln(b*x+a) * b^2*d^2*g^3*h * n * x - 4*B*\ln(-h*x-g) * a^2*d^2*g^h^3 * n * x + 4*B*\ln(-h*x-g) * b^2*c^2*g^h^3 * n * x + 4*B*\ln(-d*x-c) * a^2*d^2*g^h^3 * n * x + 4*B*\ln(b*x+a) * b^2*c*d*g^3*h * n + 4*B*\ln(-h*x-g) * a*b*d^2*g^3*h * n - 4*B*\ln(-h*x-g) * b^2*c*d*g^3*h * n - 4*B*\ln(-d*x-c) * a*b*d^2*g^3*h * n - 4*A*a^2*c*d*g^h^3 - 4*A*a*b*c^2*g^h^3 - 4*A*a*b*d^2*g^3*h - 4*A*b^2*c*d*g^3*h - I*B*Pi*b^2*c^2*g^2*h^2 * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) ^3 - I*B*Pi*b^2*c^2*g^2*h^2 * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n) ^3 - 4*B*a*b*d^2*g^3*h * \ln((b*x+a)^n) - 4*B*b^2*c*d*g^3*h * \ln((b*x+a)^n) - 4*B*a^2*c*d*g^h^3 * \ln((b*x+a)^n) - 4*B*a*b*c^2*g^h^3 * \ln((b*x+a)^n) - I*B*Pi*a^2*c^2*h^4 * \operatorname{csgn}(Ie) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n) - I*B*Pi*a^2*c^2*h^4 * \operatorname{csgn}(I*(b*x+a)^n) * \operatorname{csgn}(I / ((d*x+c)^n)) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) + 2*I*B*Pi*a^2*c*d*g^h^3 * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) ^3 + 8*B*\ln(e) * a*b*c*d*g^2*h^2 + 2*I*B*Pi*b^2*c*d*g^3*h * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n) ^3 - 2*B*a*b*d^2*g^2*h^2 * n * x + 2*B*b^2*c*d*g^2*h^2 * n * x - I*B*Pi*b^2*c^2*g^2*h^2 * \operatorname{csgn}(Ie) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n) - I*B*Pi*a^2*d^2*g^2*h^2 * \operatorname{csgn}(Ie) * \operatorname{csgn}(I*(b*x+a)^n / ((d*x+c)^n)) * \operatorname{csgn}(Ie / ((d*x+c)^n) * (b*x+a)^n) + 2*B*\ln(e) * a^2*d^2*g^2*h^2 + 2*B*\ln(e) * b^2*c^2*g^2*h^2 - 4*I*B*Pi*a*b*c*d*g^2*h^2 * \operatorname{csgn}(I
\end{aligned}$$

$$\begin{aligned} & * (b*x+a)^n / ((d*x+c)^n) ^3 - 4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I*e / ((d*x+c)^n) * (b*x+a)^n) ^3 - 2*I*B*Pi*b^2*c*d*g^3*h*csgn(I*(b*x+a)^n) * csgn(I*(b*x+a)^n / ((d*x+c)^n)) ^2 - 2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*(b*x+a)^n / ((d*x+c)^n)) * csgn(I*e / ((d*x+c)^n) * (b*x+a)^n) ^2 - 2*I*B*Pi*a^2*c*d*g*h^3*csgn(I*e) * csgn(I*e / ((d*x+c)^n) * (b*x+a)^n) ^2 + I*B*Pi*a^2*d^2*g^2*h^2*csgn(I*(b*x+a)^n / ((d*x+c)^n)) * csgn(I*e / ((d*x+c)^n) * (b*x+a)^n) ^2 + I*B*Pi*b^2*c^2*g^2*h^2*csgn(I*e) * csgn(I*e / ((d*x+c)^n) * (b*x+a)^n) ^2 + I*B*Pi*b^2*c^2*g^2*h^2*csgn(I / ((d*x+c)^n)) * csgn(I*(b*x+a)^n / ((d*x+c)^n)) ^2 + 2*I*B*Pi*b^2*c*d*g^3*h*csgn(I*(b*x+a)^n) * csgn(I / ((d*x+c)^n)) * csgn(I*(b*x+a)^n / ((d*x+c)^n)) + 4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I*e) * csgn(I*e / ((d*x+c)^n) * (b*x+a)^n) ^2 + 4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I*(b*x+a)^n) * csgn(I*(b*x+a)^n / ((d*x+c)^n)) ^2 + 4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I / ((d*x+c)^n)) * csgn(I*(b*x+a)^n / ((d*x+c)^n)) ^2 + 2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*(b*x+a)^n) * csgn(I / ((d*x+c)^n)) * csgn(I*(b*x+a)^n / ((d*x+c)^n)) + 2*I*B*Pi*b^2*c*d*g^3*h*csgn(I*e) * csgn(I*(b*x+a)^n / ((d*x+c)^n)) * csgn(I*e / ((d*x+c)^n) * (b*x+a)^n) - 2*I*B*Pi*a*b*c^2*g*h^3*csgn(I*e) * csgn(I*e / ((d*x+c)^n) * (b*x+a)^n) ^2 - 2*I*B*Pi*b^2*c*d*g^3*h*csgn(I*(b*x+a)^n / ((d*x+c)^n)) * csgn(I*e / ((d*x+c)^n) * (b*x+a)^n) ^2 + 4*I*B*Pi*a*b*c*d*g^2*h^2*csgn(I*(b*x+a)^n / ((d*x+c)^n)) * csgn(I*e / ((d*x+c)^n) * (b*x+a)^n) ^2 - I*B*Pi*a^2*d^2*g^2*h^2*csgn(I*(b*x+a)^n) * csgn(I / ((d*x+c)^n)) * csgn(I*(b*x+a)^n / ((d*x+c)^n)) - 2*I*B*Pi*a*b*d^2*g^3*h*csgn(I*e) * csgn(I*e / ((d*x+c)^n) * (b*x+a)^n) ^2 - 2*I*B*Pi*a*b*d^2*g^3*h*csgn(I / ((d*x+c)^n)) * csgn(I*(b*x+a)^n / ((d*x+c)^n)) ^2 + 2*I*B*Pi*a^2*c*d*g*h^3*csgn(I*e / ((d*x+c)^n) * (b*x+a)^n) ^3 + 2*I*B*Pi*a*b*c^2*g*h^3*csgn(I*(b*x+a)^n / ((d*x+c)^n)) ^3 + 2*I*B*Pi*a*b*c^2*g*h^3*csgn(I*e / ((d*x+c)^n) * (b*x+a)^n) ^3) / (h*x+g)^2 / (a*c*h^2 - a*d*g*h - b*c*g*h + b*d*g^2) / (-c*h+d*g) / (-a*h+b*g) / h \end{aligned}$$

**Maxima [B]** time = 2.42682, size = 516, normalized size = 2.7

$$\frac{\left( \frac{b^2 e n \log(bx+a)}{b^2 g^2 h^2 - 2 a b g h^2 + a^2 h^3} - \frac{d^2 e n \log(dx+c)}{d^2 g^2 h^2 - 2 c d g h^2 + c^2 h^3} - \frac{(2 a b d^2 e g n - a^2 d^2 e h n - (2 c d e g n - c^2 e h n) b^2) \log(hx+g)}{(d^2 g^2 h^2 - 2 c d g h^3 + c^2 h^4) a^2 - 2 (d^2 g^3 h - 2 c d g^2 h^2 + c^2 g h^3) a b + (d^2 g^4 - 2 c d g^3 h + c^2 g^2 h^2) b^2} \right) + \frac{2 e}{(d g^2 h - c g h^2) a - (d g^2 h - c g h^2) a - (d g^2 h - c g h^2) a - (d g^2 h - c g h^2) a}}{2 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g)^3,x, algorithm="maxima")

[Out] 1/2\*(b^2\*e\*n\*log(b\*x + a)/(b^2\*g^2\*h - 2\*a\*b\*g\*h^2 + a^2\*h^3) - d^2\*e\*n\*log(d\*x + c)/(d^2\*g^2\*h - 2\*c\*d\*g\*h^2 + c^2\*h^3) - (2\*a\*b\*d^2\*e\*g\*n - a^2\*d^2\*e\*h\*n - (2\*c\*d\*e\*g\*n - c^2\*e\*h\*n)\*b^2)\*log(h\*x + g)/((d^2\*g^2\*h^2 - 2\*c\*d\*g\*h^3 + c^2\*h^4)\*a^2 - 2\*(d^2\*g^3\*h - 2\*c\*d\*g^2\*h^2 + c^2\*g\*h^3)\*a\*b + (d^2\*g^4 - 2\*c\*d\*g^3\*h + c^2\*g^2\*h^2)\*b^2) + (b\*c\*e\*n - a\*d\*e\*n)/((d\*g^2\*h - c\*g\*h^2)\*a - (d\*g^3 - c\*g^2\*h)\*b + ((d\*g\*h^2 - c\*h^3)\*a - (d\*g^2\*h - c\*g\*h^2)\*b)\*x)) \* B/e - 1/2\*B\*log((b\*x + a)^n\*e/(d\*x + c)^n)/(h^3\*x^2 + 2\*g\*h^2\*x + g^2\*h) - 1/2\*A/(h^3\*x^2 + 2\*g\*h^2\*x + g^2\*h)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g)^3,x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))/(h\*x+g)\*\*3,x)

[Out] Timed out

**Giac [B]** time = 1.87964, size = 1197, normalized size = 6.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g)^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/2*B*n*log(b*x + a)/(h^3*x^2 + 2*g*h^2*x + g^2*h) + 1/2*B*n*log(d*x + c)/ \\ & (h^3*x^2 + 2*g*h^2*x + g^2*h) + 1/2*(2*B*b^2*c*d*g^n - 2*B*a*b*d^2*g^n - B* \\ & b^2*c^2*h^n + B*a^2*d^2*h^n)*log(h*x + g)/(b^2*d^2*g^4 - 2*b^2*c*d*g^3*h - \\ & 2*a*b*d^2*g^3*h + b^2*c^2*g^2*h^2 + 4*a*b*c*d*g^2*h^2 + a^2*d^2*g^2*h^2 - \\ & 2*a*b*c^2*g*h^3 - 2*a^2*c*d*g*h^3 + a^2*c^2*h^4) - 1/4*(2*B*b^2*c*d*g^n - 2* \\ & B*a*b*d^2*g^n - B*b^2*c^2*h^n + B*a^2*d^2*h^n)*log(abs(b*d*x^2 + b*c*x + a* \\ & d*x + a*c))/(b^2*d^2*g^4 - 2*b^2*c*d*g^3*h - 2*a*b*d^2*g^3*h + b^2*c^2*g^2* \\ & h^2 + 4*a*b*c*d*g^2*h^2 + a^2*d^2*g^2*h^2 - 2*a*b*c^2*g*h^3 - 2*a^2*c*d*g*h \\ & ^3 + a^2*c^2*h^4) - 1/2*(B*b*c*h^2*n*x - B*a*d*h^2*n*x + B*b*c*g*h*n - B*a* \\ & d*g*h*n + A*b*d*g^2 + B*b*d*g^2 - A*b*c*g*h - B*b*c*g*h - A*a*d*g*h - B*a*d \\ & *g*h + A*a*c*h^2 + B*a*c*h^2)/(b*d*g^2*h^3*x^2 - b*c*g*h^4*x^2 - a*d*g*h^4* \\ & x^2 + a*c*h^5*x^2 + 2*b*d*g^3*h^2*x - 2*b*c*g^2*h^3*x - 2*a*d*g^2*h^3*x + 2 \\ & *a*c*g*h^4*x + b*d*g^4*h - b*c*g^3*h^2 - a*d*g^3*h^2 + a*c*g^2*h^3) + 1/4*( \\ & 2*B*b^3*c*d^2*g^2*n - 2*B*a*b^2*d^3*g^2*n - 2*B*b^3*c^2*d*g*h*n + 2*B*a^2*b \\ & *d^3*g*h*n + B*b^3*c^3*h^2*n - B*a*b^2*c^2*d*h^2*n + B*a^2*b*c*d^2*h^2*n - \\ & B*a^3*d^3*h^2*n)*log(abs((2*b*d*x + b*c + a*d - abs(-b*c + a*d))/(2*b*d*x + \\ & b*c + a*d + abs(-b*c + a*d)))/((b^2*d^2*g^4*h - 2*b^2*c*d*g^3*h^2 - 2*a*b \\ & *d^2*g^3*h^2 + b^2*c^2*g^2*h^3 + 4*a*b*c*d*g^2*h^3 + a^2*d^2*g^2*h^3 - 2*a* \\ & b*c^2*g*h^4 - 2*a^2*c*d*g*h^4 + a^2*c^2*h^5)*abs(-b*c + a*d)) \end{aligned}$$



$$3.301 \quad \int \frac{A+B \log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)^4} dx$$

**Optimal.** Leaf size=284

$$\frac{Bn(bc-ad) \log(g+hx) \left( a^2 d^2 h^2 - abdh(3dg-ch) + b^2 (c^2 h^2 - 3cdgh + 3d^2 g^2) \right)}{3(bg-ah)^3 (dg-ch)^3} - \frac{B \log(e(a+bx)^n(c+dx)^{-n}) + A}{3h(g+hx)^3} +$$

[Out]  $-(B*(b*c - a*d)*n)/(6*(b*g - a*h)*(d*g - c*h)*(g + h*x)^2) - (B*(b*c - a*d) * (2*b*d*g - b*c*h - a*d*h)*n)/(3*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)) + (b^3*B*n*Log[a + b*x])/(3*h*(b*g - a*h)^3) - (B*d^3*n*Log[c + d*x])/(3*h*(d*g - c*h)^3) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*h*(g + h*x)^3) + (B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*Log[g + h*x])/(3*(b*g - a*h)^3*(d*g - c*h)^3)$

**Rubi [A]** time = 0.536117, antiderivative size = 296, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6742, 2492, 72}

$$\frac{Bn(bc-ad) \log(g+hx) \left( a^2 d^2 h^2 - abdh(3dg-ch) + b^2 (c^2 h^2 - 3cdgh + 3d^2 g^2) \right)}{3(bg-ah)^3 (dg-ch)^3} + \frac{b^3 Bn \log(a+bx)}{3h(bg-ah)^3} - \frac{B \log(e(a+bx)^n(c+dx)^{-n}) + A}{3h(g+hx)^3} +$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(g + h\*x)^4, x]

[Out]  $-A/(3*h*(g + h*x)^3) - (B*(b*c - a*d)*n)/(6*(b*g - a*h)*(d*g - c*h)*(g + h*x)^2) - (B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n)/(3*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)) + (b^3*B*n*Log[a + b*x])/(3*h*(b*g - a*h)^3) - (B*d^3*n*Log[c + d*x])/(3*h*(d*g - c*h)^3) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*h*(g + h*x)^3) + (B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*Log[g + h*x])/(3*(b*g - a*h)^3*(d*g - c*h)^3)$

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2492

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*((g\_.) + (h\_.)\*(x\_))^(m\_.), x\_Symbol] := Simp[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(h\*(m + 1)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(h\*(m + 1)), Int[((g + h\*x)^(m + 1)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^(s - 1))/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0] && NeQ[m, -1]

#### Rule 72

Int[((e\_.) + (f\_.)\*(x\_))^(p\_.)/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Int[ExpandIntegrand[(e + f\*x)^p/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

#### Rubi steps

$$\begin{aligned}
\int \frac{A + B \log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^4} dx &= \int \left( \frac{A}{(g + hx)^4} + \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^4} \right) dx \\
&= -\frac{A}{3h(g + hx)^3} + B \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^4} dx \\
&= -\frac{A}{3h(g + hx)^3} - \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{3h(g + hx)^3} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)(c+dx)(g+hx)}}{3h} \\
&= -\frac{A}{3h(g + hx)^3} - \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{3h(g + hx)^3} + \frac{(B(bc - ad)n) \int \left( \frac{b^4}{(bc-ad)(bg-ah)^3(a+bx)} \right)}{3h} \\
&= -\frac{A}{3h(g + hx)^3} - \frac{B(bc - ad)n}{6(bg - ah)(dg - ch)(g + hx)^2} - \frac{B(bc - ad)(2bdg - bch - adh)n}{3(bg - ah)^2(dg - ch)^2(g + hx)}
\end{aligned}$$

**Mathematica [A]** time = 1.34277, size = 273, normalized size = 0.96

$$\frac{Bn(bc - ad) \left( -\frac{2h \log(g+hx)(a^2d^2h^2 + abdh(ch-3dg) + b^2(c^2h^2 - 3cdgh + 3d^2g^2))}{(bg-ah)^3(dg-ch)^3} - \frac{2b^3 \log(a+bx)}{(bc-ad)(bg-ah)^3} + \frac{2d^3 \log(c+dx)}{(bc-ad)(dg-ch)^3} - \frac{2h(adh+bch-2bdg)}{(g+hx)(bg-ah)^2(dg-ch)^2} + \frac{1}{(g+hx)(c+dx)(g+hx)} \right)}{6h}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(g + h\*x)^4,x]

[Out] -((2\*A)/(g + h\*x)^3 + (2\*B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(g + h\*x)^3 + B\*(b\*c - a\*d)\*n\*(h/((b\*g - a\*h)\*(d\*g - c\*h)\*(g + h\*x)^2) - (2\*h\*(-2\*b\*d\*g + b\*c\*h + a\*d\*h))/((b\*g - a\*h)^2\*(d\*g - c\*h)^2\*(g + h\*x)) - (2\*b^3\*Log[a + b\*x])/((b\*c - a\*d)\*(b\*g - a\*h)^3) + (2\*d^3\*Log[c + d\*x])/((b\*c - a\*d)\*(d\*g - c\*h)^3) - (2\*h\*(a^2\*d^2\*h^2 + a\*b\*d\*h\*(-3\*d\*g + c\*h) + b^2\*(3\*d^2\*g^2 - 3\*c\*d\*g\*h + c^2\*h^2))\*Log[g + h\*x])/((b\*g - a\*h)^3\*(d\*g - c\*h)^3))/(6\*h)

**Maple [C]** time = 0.925, size = 9645, normalized size = 34.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g)^4,x)

[Out] result too large to display

**Maxima [B]** time = 1.63639, size = 1242, normalized size = 4.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g)^4,x, algorithm="maxima")

[Out] 1/6\*(2\*b^3\*e\*n\*log(b\*x + a)/(b^3\*g^3\*h - 3\*a\*b^2\*g^2\*h^2 + 3\*a^2\*b\*g\*h^3 - a^3\*h^4) - 2\*d^3\*e\*n\*log(d\*x + c)/(d^3\*g^3\*h - 3\*c\*d^2\*g^2\*h^2 + 3\*c^2\*d\*g\*h^3))

$$h^3 - c^3 h^4) + 2*(3*a*b^2*d^3*e*g^2*n - 3*a^2*b*d^3*e*g*h*n + a^3*d^3*e*h^2*n - (3*c*d^2*e*g^2*n - 3*c^2*d*e*g*h*n + c^3*e*h^2*n)*b^3)*\log(h*x + g)/((d^3*g^3*h^3 - 3*c*d^2*g^2*h^4 + 3*c^2*d*g*h^5 - c^3*h^6)*a^3 - 3*(d^3*g^4*h^2 - 3*c*d^2*g^3*h^3 + 3*c^2*d*g^2*h^4 - c^3*g*h^5)*a^2*b + 3*(d^3*g^5*h - 3*c*d^2*g^4*h^2 + 3*c^2*d*g^3*h^3 - c^3*g^2*h^4)*a*b^2 - (d^3*g^6 - 3*c*d^2*g^5*h + 3*c^2*d*g^4*h^2 - c^3*g^3*h^3)*b^3) - ((3*d^2*e*g*h*n - c*d*e*h^2*n)*a^2 - (5*d^2*e*g^2*n - c^2*e*h^2*n)*a*b + (5*c*d*e*g^2*n - 3*c^2*e*g*h*n)*b^2 - 2*(2*a*b*d^2*e*g*h*n - a^2*d^2*e*h^2*n - (2*c*d*e*g*h*n - c^2*e*h^2*n)*b^2)*x)/((d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a^2 - 2*(d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*a*b + (d^2*g^6 - 2*c*d*g^5*h + c^2*g^4*h^2)*b^2 + ((d^2*g^2*h^4 - 2*c*d*g*h^5 + c^2*h^6)*a^2 - 2*(d^2*g^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a*b + (d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*b^2)*x^2 + 2*((d^2*g^3*h^3 - 2*c*d*g^2*h^4 + c^2*g*h^5)*a^2 - 2*(d^2*g^4*h^2 - 2*c*d*g^3*h^3 + c^2*g^2*h^4)*a*b + (d^2*g^5*h - 2*c*d*g^4*h^2 + c^2*g^3*h^3)*b^2)*x)*B/e - 1/3*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h^2*x + g^3*h) - 1/3*A/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h^2*x + g^3*h)$$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g)^4,x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))/(h\*x+g)\*\*4,x)

[Out] Timed out

**Giac [B]** time = 4.52414, size = 2041, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g)^4,x, algorithm="giac")

[Out]  $1/3*B*b^4*n*\log(\text{abs}(b*x + a))/(b^4*g^3*h - 3*a*b^3*g^2*h^2 + 3*a^2*b^2*g*h^3 - a^3*b*h^4) - 1/3*B*d^4*n*\log(\text{abs}(d*x + c))/(d^4*g^3*h - 3*c*d^3*g^2*h^2 + 3*c^2*d^2*g*h^3 - c^3*d*h^4) - 1/3*B*n*\log(b*x + a)/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h^2*x + g^3*h) + 1/3*B*n*\log(d*x + c)/(h^4*x^3 + 3*g*h^3*x^2 + 3*g^2*h^2*x + g^3*h) + 1/3*(3*B*b^3*c*d^2*g^2*n - 3*B*a*b^2*d^3*g^2*n - 3*B*$

$$\begin{aligned}
& b^3c^2d^2g^2h^n + 3Ba^2b^2d^3g^2h^n + Bb^3c^3h^2n - Ba^3d^3h^2n) * \\
& \log(hx + g) / (b^3d^3g^6 - 3b^3c^2d^2g^5h - 3a^2b^2d^3g^5h + 3b^3c^2d^2g^4h^2 + 9a^2b^2c^2d^2g^4h^2 + 3a^2b^2d^3g^4h^2 - b^3c^3g^3h^3 \\
& - 9a^2b^2c^2d^2g^3h^3 - 9a^2b^2c^2d^2g^3h^3 - a^3d^3g^3h^3 + 3a^2b^2c^3g^2h^4 + 9a^2b^2c^2d^2g^2h^4 + 3a^3c^2d^2g^2h^4 - 3a^2b^2c^3g^2h^5 \\
& - 3a^3c^2d^2g^2h^5 + a^3c^3h^6) - 1/6(4Bb^2c^2d^2g^2h^3n^2x^2 - 4Ba^2b^2d^2g^2h^3n^2x^2 - 2Bb^2c^2h^4n^2x^2 + 2Ba^2d^2h^4n^2x^2 + 9Bb^2c^2d^2g^2h^2n^2x \\
& - 9Ba^2b^2d^2g^2h^2n^2x - 5Bb^2c^2g^2h^3n^2x + 5Ba^2d^2g^2h^3n^2x + Ba^2b^2c^2h^4n^2x - Ba^2c^2d^2h^4n^2x + 5Bb^2c^2d^2g^3h^n \\
& - 5Ba^2b^2d^2g^3h^n - 3Bb^2c^2g^2h^2n + 3Ba^2d^2g^2h^2n + Ba^2b^2c^2g^2h^3n - Ba^2c^2d^2g^2h^3n + 2A^2b^2d^2g^4 + 2Bb^2d^2g^4 \\
& - 4A^2b^2c^2d^2g^3h - 4Bb^2c^2d^2g^3h - 4A^2a^2b^2d^2g^3h - 4Ba^2b^2d^2g^3h + 2A^2b^2c^2g^2h^2 + 2Bb^2c^2g^2h^2 + 8A^2a^2b^2c^2d^2g^2h^2 \\
& + 8B^2a^2b^2c^2d^2g^2h^2 + 2A^2a^2d^2g^2h^2 + 2Ba^2d^2g^2h^2 - 4A^2a^2b^2c^2g^2h^3 - 4B^2a^2b^2c^2g^2h^3 - 4A^2a^2c^2d^2g^2h^3 + 2A^2a^2c^2h^4 \\
& + 2B^2a^2c^2h^4) / (b^2d^2g^4h^4x^3 - 2b^2c^2d^2g^3h^5x^3 - 2a^2b^2d^2g^3h^5x^3 + b^2c^2g^2h^6x^3 + 4a^2b^2c^2d^2g^2h^6x^3 + a^2d^2g^2h^6x^3 \\
& - 2a^2b^2c^2g^2h^7x^3 - 2a^2c^2d^2g^2h^7x^3 + a^2c^2h^8x^3 + 3b^2d^2g^5h^3x^2 - 6b^2c^2d^2g^4h^4x^2 - 6a^2b^2d^2g^4h^4x^2 + 3b^2c^2g^3h^5x^2 + 12a^2b^2c^2d^2g^3h^5x^2 + 3a^2d^2g^3h^5x^2 \\
& - 6a^2b^2c^2g^2h^6x^2 - 6a^2c^2d^2g^2h^6x^2 + 3a^2c^2g^2h^7x^2 + 3b^2d^2g^6h^2x - 6b^2c^2d^2g^5h^3x - 6a^2b^2d^2g^5h^3x + 3b^2c^2g^4h^4x \\
& + 12a^2b^2c^2d^2g^4h^4x + 3a^2d^2g^4h^4x - 6a^2b^2c^2g^3h^5x - 6a^2c^2d^2g^3h^5x + 3a^2c^2g^2h^6x + b^2d^2g^7h - 2b^2c^2d^2g^6h^2 \\
& - 2a^2b^2d^2g^6h^2 + b^2c^2g^5h^3 + 4a^2b^2c^2d^2g^5h^3 + a^2d^2g^5h^3 - 2a^2b^2c^2g^4h^4 - 2a^2c^2d^2g^4h^4 + a^2c^2g^3h^5)
\end{aligned}$$

$$3.302 \quad \int \frac{A+B \log(e(ax+bx)^n(c+dx)^{-n})}{(g+hx)^5} dx$$

**Optimal.** Leaf size=389

$$\frac{Bn(bc-ad)(a^2d^2h^2-abdh(3dg-ch)+b^2(c^2h^2-3cdgh+3d^2g^2))}{4(g+hx)(bg-ah)^3(dg-ch)^3} - \frac{Bn(bc-ad)\log(g+hx)(-adh-bch+2bdg)}{4(bg-ah)^3(dg-ch)^3}$$

```
[Out] -(B*(b*c - a*d)*n)/(12*(b*g - a*h)*(d*g - c*h)*(g + h*x)^3) - (B*(b*c - a*d)
)*(2*b*d*g - b*c*h - a*d*h)*n)/(8*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)^2)
- (B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*
c*d*g*h + c^2*h^2))*n)/(4*(b*g - a*h)^3*(d*g - c*h)^3*(g + h*x)) + (b^4*B*n
*Log[a + b*x])/(4*h*(b*g - a*h)^4) - (B*d^4*n*Log[c + d*x])/(4*h*(d*g - c*h)
)^4) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(4*h*(g + h*x)^4) - (B*(b*c
- a*d)*(2*b*d*g - b*c*h - a*d*h)*(2*a*b*d^2*g*h - a^2*d^2*h^2 - b^2*(2*d^2
*g^2 - 2*c*d*g*h + c^2*h^2))*n*Log[g + h*x])/(4*(b*g - a*h)^4*(d*g - c*h)^4
)
```

**Rubi [A]** time = 0.820968, antiderivative size = 401, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {6742, 2492, 72}

$$\frac{Bn(bc-ad)(a^2d^2h^2-abdh(3dg-ch)+b^2(c^2h^2-3cdgh+3d^2g^2))}{4(g+hx)(bg-ah)^3(dg-ch)^3} - \frac{Bn(bc-ad)\log(g+hx)(-adh-bch+2bdg)}{4(bg-ah)^3(dg-ch)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^5, x]
```

```
[Out] -A/(4*h*(g + h*x)^4) - (B*(b*c - a*d)*n)/(12*(b*g - a*h)*(d*g - c*h)*(g + h
*x)^3) - (B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n)/(8*(b*g - a*h)^2*(d*g
- c*h)^2*(g + h*x)^2) - (B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h)
+ b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n)/(4*(b*g - a*h)^3*(d*g - c*h)^3
*(g + h*x)) + (b^4*B*n*Log[a + b*x])/(4*h*(b*g - a*h)^4) - (B*d^4*n*Log[c +
d*x])/(4*h*(d*g - c*h)^4) - (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(4*h*(g +
h*x)^4) - (B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*(2*a*b*d^2*g*h - a^2*d^
2*h^2 - b^2*(2*d^2*g^2 - 2*c*d*g*h + c^2*h^2))*n*Log[g + h*x])/(4*(b*g - a
h)^4*(d*g - c*h)^4)
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q]^r]^(s - 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
0] && NeQ[m, -1]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

### Rubi steps

$$\begin{aligned} \int \frac{A + B \log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^5} dx &= \int \left( \frac{A}{(g + hx)^5} + \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^5} \right) dx \\ &= -\frac{A}{4h(g + hx)^4} + B \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^5} dx \\ &= -\frac{A}{4h(g + hx)^4} - \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{4h(g + hx)^4} + \frac{(B(bc - ad)n) \int \frac{1}{(a+bx)(c+dx)(g+hx)}}{4h} \\ &= -\frac{A}{4h(g + hx)^4} - \frac{B \log(e(a + bx)^n(c + dx)^{-n})}{4h(g + hx)^4} + \frac{(B(bc - ad)n) \int \left( \frac{b^5}{(bc-ad)(bg-ah)^4(a+bx)} \right)}{4h} \\ &= -\frac{A}{4h(g + hx)^4} - \frac{B(bc - ad)n}{12(bg - ah)(dg - ch)(g + hx)^3} - \frac{B(bc - ad)(2bdg - bch - adh)}{8(bg - ah)^2(dg - ch)^2(g + hx)} \end{aligned}$$

**Mathematica [A]** time = 1.47082, size = 366, normalized size = 0.94

$$-Bn(bc - ad) \left( -\frac{h(a^2d^2h^2 + abdh(ch - 3dg) + b^2(c^2h^2 - 3cdgh + 3d^2g^2))}{(g + hx)(bg - ah)^3(dg - ch)^3} - \frac{h \log(g + hx)(adh + bch - 2bdg)(a^2d^2h^2 - 2abd^2gh + b^2(c^2h^2 - 2cdgh + 2d^2g^2))}{(bg - ah)^4(dg - ch)^4} + \frac{b^4 \log(a + bx)}{(bc - ad)(g + hx)} \right) / 4h$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^5, x]
```

```
[Out] -(A/(g + h*x)^4 + (B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(g + h*x)^4 - B*(b*c
- a*d)*n*(-h/(3*(b*g - a*h)*(d*g - c*h)*(g + h*x)^3) + (h*(-2*b*d*g + b*c*
h + a*d*h))/(2*(b*g - a*h)^2*(d*g - c*h)^2*(g + h*x)^2) - (h*(a^2*d^2*h^2 +
a*b*d*h*(-3*d*g + c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)))/((b*g - a
*h)^3*(d*g - c*h)^3*(g + h*x)) + (b^4*Log[a + b*x])/((b*c - a*d)*(b*g - a*h
)^4) - (d^4*Log[c + d*x])/((b*c - a*d)*(d*g - c*h)^4) - (h*(-2*b*d*g + b*c*
h + a*d*h)*(-2*a*b*d^2*g*h + a^2*d^2*h^2 + b^2*(2*d^2*g^2 - 2*c*d*g*h + c^2
*h^2))*Log[g + h*x])/(b*g - a*h)^4*(d*g - c*h)^4))/(4*h)
```

**Maple [C]** time = 1.286, size = 16077, normalized size = 41.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5, x)
```

```
[Out] result too large to display
```

**Maxima [B]** time = 2.13105, size = 2581, normalized size = 6.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="maxima")
```

```
[Out] 1/24*(6*b^4*e*n*log(b*x + a)/(b^4*g^4*h - 4*a*b^3*g^3*h^2 + 6*a^2*b^2*g^2*h^3 - 4*a^3*b*g*h^4 + a^4*h^5) - 6*d^4*e*n*log(d*x + c)/(d^4*g^4*h - 4*c*d^3*g^3*h^2 + 6*c^2*d^2*g^2*h^3 - 4*c^3*d*g*h^4 + c^4*h^5) - 6*(4*a*b^3*d^4*e*g^3*h^2 - 6*a^2*b^2*d^4*e*g^2*h^3 + 4*a^3*b*d^4*e*g*h^4 - a^4*d^4*e*h^3)*b^4*log(h*x + g)/((d^4*g^4*h^4 - 4*c*d^3*g^3*h^5 + 6*c^2*d^2*g^2*h^6 - 4*c^3*d*g*h^7 + c^4*h^8)*a^4 - 4*(d^4*g^5*h^3 - 4*c*d^3*g^4*h^4 + 6*c^2*d^2*g^3*h^5 - 4*c^3*d*g^2*h^6 + c^4*g*h^7)*a^3*b + 6*(d^4*g^6*h^2 - 4*c*d^3*g^5*h^3 + 6*c^2*d^2*g^4*h^4 - 4*c^3*d*g^3*h^5 + c^4*g^2*h^6)*a^2*b^2 - 4*(d^4*g^7*h - 4*c*d^3*g^6*h^2 + 6*c^2*d^2*g^5*h^3 - 4*c^3*d*g^4*h^4 + c^4*g^3*h^5)*a*b^3 + (d^4*g^8 - 4*c*d^3*g^7*h + 6*c^2*d^2*g^6*h^2 - 4*c^3*d*g^5*h^3 + c^4*g^4*h^4)*b^4) - ((11*d^3*e*g^2*h^2*n - 7*c*d^2*e*g*h^3*n + 2*c^2*d*e*h^4*n)*a^3 - (31*d^3*e*g^3*h*n - 15*c*d^2*e*g^2*h^2*n + 2*c^3*e*h^4*n)*a^2*b + (26*d^3*e*g^4*n - 15*c^2*d*e*g^2*h^2*n + 7*c^3*e*g*h^3*n)*a*b^2 - (26*c*d^2*e*g^4*n - 31*c^2*d*e*g^3*h*n + 11*c^3*e*g^2*h^2*n)*b^3 + 6*(3*a*b^2*d^3*e*g^2*h^2*n - 3*a^2*b*d^3*e*g*h^3*n + a^3*d^3*e*h^4*n - (3*c*d^2*e*g^2*h^2*n - 3*c^2*d*e*g*h^3*n + c^3*e*h^4*n)*b^3)*x^2 + 3*((5*d^3*e*g*h^3*n - c*d^2*e*h^4*n)*a^3 - 3*(5*d^3*e*g^2*h^2*n - c*d^2*e*g*h^3*n)*a^2*b + (14*d^3*e*g^3*h*n - 3*c^2*d*e*g*h^3*n + c^3*e*h^4*n)*a*b^2 - (14*c*d^2*e*g^3*h*n - 15*c^2*d*e*g^2*h^2*n + 5*c^3*e*g*h^3*n)*b^3)*x)/((d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*g^3*h^6)*a^3 - 3*(d^3*g^7*h^2 - 3*c*d^2*g^6*h^3 + 3*c^2*d*g^5*h^4 - c^3*g^4*h^5)*a^2*b + 3*(d^3*g^8*h - 3*c*d^2*g^7*h^2 + 3*c^2*d*g^6*h^3 - c^3*g^5*h^4)*a*b^2 - (d^3*g^9 - 3*c*d^2*g^8*h + 3*c^2*d*g^7*h^2 - c^3*g^6*h^3)*b^3 + ((d^3*g^3*h^6 - 3*c*d^2*g^2*h^7 + 3*c^2*d*g*h^8 - c^3*h^9)*a^3 - 3*(d^3*g^4*h^5 - 3*c*d^2*g^3*h^6 + 3*c^2*d*g^2*h^7 - c^3*g*h^8)*a^2*b + 3*(d^3*g^5*h^4 - 3*c*d^2*g^4*h^5 + 3*c^2*d*g^3*h^6 - c^3*g^2*h^7)*a*b^2 - (d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*g^3*h^6)*b^3)*x^3 + 3*((d^3*g^4*h^5 - 3*c*d^2*g^3*h^6 + 3*c^2*d*g^2*h^7 - c^3*g*h^8)*a^3 - 3*(d^3*g^5*h^4 - 3*c*d^2*g^4*h^5 + 3*c^2*d*g^3*h^6 - c^3*g^2*h^7)*a^2*b + 3*(d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*g^3*h^6)*a*b^2 - (d^3*g^7*h^2 - 3*c*d^2*g^6*h^3 + 3*c^2*d*g^5*h^4 - c^3*g^4*h^5)*b^3)*x^2 + 3*((d^3*g^5*h^4 - 3*c*d^2*g^4*h^5 + 3*c^2*d*g^3*h^6 - c^3*g^2*h^7)*a^3 - 3*(d^3*g^6*h^3 - 3*c*d^2*g^5*h^4 + 3*c^2*d*g^4*h^5 - c^3*g^3*h^6)*a^2*b + 3*(d^3*g^7*h^2 - 3*c*d^2*g^6*h^3 + 3*c^2*d*g^5*h^4 - c^3*g^4*h^5)*a*b^2 - (d^3*g^8*h - 3*c*d^2*g^7*h^2 + 3*c^2*d*g^6*h^3 - c^3*g^5*h^4)*b^3)*x))*B/e - 1/4*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x + g^4*h) - 1/4*A/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x + g^4*h)
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(h*x+g)^5,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))/(h\*x+g)\*\*5,x)

[Out] Timed out

**Giac [B]** time = 14.5972, size = 5925, normalized size = 15.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))/(h\*x+g)^5,x, algorithm="giac")

[Out] 
$$-1/4*B*n*log(b*x + a)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x + g^4*h) + 1/4*B*n*log(d*x + c)/(h^5*x^4 + 4*g*h^4*x^3 + 6*g^2*h^3*x^2 + 4*g^3*h^2*x + g^4*h) + 1/4*(4*B*b^4*c*d^3*g^3*n - 4*B*a*b^3*d^4*g^3*n - 6*B*b^4*c^2*d^2*g^2*h*n + 6*B*a^2*b^2*d^4*g^2*h*n + 4*B*b^4*c^3*d*g*h^2*n - 4*B*a^3*b*d^4*g*h^2*n - B*b^4*c^4*h^3*n + B*a^4*d^4*h^3*n)*log(h*x + g)/(b^4*d^4*g^8 - 4*b^4*c*d^3*g^7*h - 4*a*b^3*d^4*g^7*h + 6*b^4*c^2*d^2*g^6*h^2 + 16*a*b^3*c*d^3*g^6*h^2 + 6*a^2*b^2*d^4*g^6*h^2 - 4*b^4*c^3*d*g^5*h^3 - 24*a*b^3*c^2*d^2*g^5*h^3 - 24*a^2*b^2*c*d^3*g^5*h^3 - 4*a^3*b*d^4*g^5*h^3 + b^4*c^4*g^4*h^4 + 16*a*b^3*c^3*d*g^4*h^4 + 36*a^2*b^2*c^2*d^2*g^4*h^4 + 16*a^3*b*c*d^3*g^4*h^4 + a^4*d^4*g^4*h^4 - 4*a*b^3*c^4*g^3*h^5 - 24*a^2*b^2*c^3*d*g^3*h^5 - 24*a^3*b*c^2*d^2*g^3*h^5 - 4*a^4*c*d^3*g^3*h^5 + 6*a^2*b^2*c^4*g^2*h^6 + 16*a^3*b*c^3*d*g^2*h^6 + 6*a^4*c^2*d^2*g^2*h^6 - 4*a^3*b*c^4*g*h^7 - 4*a^4*c^3*d*g*h^7 + a^4*c^4*h^8) - 1/8*(4*B*b^4*c*d^3*g^3*n - 4*B*a*b^3*d^4*g^3*n - 6*B*b^4*c^2*d^2*g^2*h*n + 6*B*a^2*b^2*d^4*g^2*h*n + 4*B*b^4*c^3*d*g*h^2*n - 4*B*a^3*b*d^4*g*h^2*n - B*b^4*c^4*h^3*n + B*a^4*d^4*h^3*n)*log(abs(b*d*x^2 + b*c*x + a*d*x + a*c))/(b^4*d^4*g^8 - 4*b^4*c*d^3*g^7*h - 4*a*b^3*d^4*g^7*h + 6*b^4*c^2*d^2*g^6*h^2 + 16*a*b^3*c*d^3*g^6*h^2 + 6*a^2*b^2*d^4*g^6*h^2 - 4*b^4*c^3*d*g^5*h^3 - 24*a*b^3*c^2*d^2*g^5*h^3 - 24*a^2*b^2*c*d^3*g^5*h^3 - 4*a^3*b*d^4*g^5*h^3 + b^4*c^4*g^4*h^4 + 16*a*b^3*c^3*d*g^4*h^4 + 36*a^2*b^2*c^2*d^2*g^4*h^4 + 16*a^3*b*c*d^3*g^4*h^4 + a^4*d^4*g^4*h^4 - 4*a*b^3*c^4*g^3*h^5 - 24*a^2*b^2*c^3*d*g^3*h^5 - 24*a^3*b*c^2*d^2*g^3*h^5 - 4*a^4*c*d^3*g^3*h^5 + 6*a^2*b^2*c^4*g^2*h^6 + 16*a^3*b*c^3*d*g^2*h^6 + 6*a^4*c^2*d^2*g^2*h^6 - 4*a^3*b*c^4*g*h^7 - 4*a^4*c^3*d*g*h^7 + a^4*c^4*h^8) - 1/24*(18*B*b^3*c*d^2*g^2*h^4*n*x^3 - 18*B*a*b^2*d^3*g^2*h^4*n*x^3 - 18*B*b^3*c^2*d*g*h^5*n*x^3 + 18*B*a^2*b*d^3*g*h^5*n*x^3 + 6*B*b^3*c^3*h^6*n*x^3 - 6*B*a^3*d^3*h^6*n*x^3 + 60*B*b^3*c*d^2*g^3*h^3*n*x^2 - 60*B*a*b^2*d^3*g^3*h^3*n*x^2 - 63*B*b^3*c^2*d*g^2*h^4*n*x^2 + 63*B*a^2*b*d^3*g^2*h^4*n*x^2 + 21*B*b^3*c^3*g*h^5*n*x^2 + 9*B*a*b^2*c^2*d*g*h^5*n*x^2 - 9*B*a^2*b*c*d^2*g*h^5*n*x^2 - 21*B*a^3*d^3*g*h^5*n*x^2 - 3*B*a*b^2*c^3*h^6*n*x^2 + 3*B*a^3*c*d^2*h^6*n*x^2 + 68*B*b^3*c*d^2*g^4*h^2*n*x - 68*B*a*b^2*d^3*g^4*h^2*n*x - 76*B*b^3*c^2*d*g^3*h^3*n*x + 76*B*a^2*b*d^3*g^3*h^3*n*x + 26*B*b^3*c^3*g^2*h^4*n*x + 24*B*a*b^2*c^2*d*g^2*h^4*n*x - 24*B*a^2*b*c*d^2*g^2*h^4*n*x - 26*B*a^3*d^3*g^2*h^4*n*x - 10*B*a*b^2*c^3*g*h^5*n*x + 10*B*a^3*c*d^2*g*h^5*n*x + 2*B*a^2*b*c^3*h^6*n*x - 2*B*a^3*c^2*d*h^6*n*x + 26*B*b^3*c*d^2*g^5*h*n - 26*B*a*b^2*d^3*g^5*h*n - 31*B*b^3*c^2*d*g^4*h^2*n + 31*B*a^2*b*d^3*g^4*h^2*n + 11*B*b^3*c^3*g^3*h^3*n + 15*B*a*b^2*c^2*d*g^3*h^3*n - 15*B*a^2*b*c*d^2*g^3*h^3*n - 11*B*a^3*d^3*g^3*h^3*n - 7*B*a*b^2*c^3*g^2*h^4*n + 7*B*a^3*c*d^2*g^2*h^4*n + 2*B*a^2*b*c^3*g*h^5*n - 2*B*a^3*c^2*d*g*h^5*n + 6*A*b^3*d^3*g^$$



$$\begin{aligned}
& 6 + 6B^3b^3d^3g^6 - 18A^3b^3c^2d^2g^5h - 18B^3b^3c^2d^2g^5h - 18A^2a^* \\
& b^2d^3g^5h - 18B^2a^*b^2d^3g^5h + 18A^2b^3c^2d^2g^4h^2 + 18B^2b^3c^2 \\
& d^2g^4h^2 + 54A^2a^*b^2c^2d^2g^4h^2 + 54B^2a^*b^2c^2d^2g^4h^2 + 18A^2a^* \\
& b^2d^3g^4h^2 + 18B^2a^*b^2d^3g^4h^2 - 6A^2b^3c^3g^3h^3 - 6B^2b^3c^3 \\
& g^3h^3 - 54A^2a^*b^2c^2d^2g^3h^3 - 54B^2a^*b^2c^2d^2g^3h^3 - 54A^2a^* \\
& b^2c^2d^2g^3h^3 - 54B^2a^*b^2c^2d^2g^3h^3 - 6A^2a^3d^3g^3h^3 - 6B^2a^3 \\
& d^3g^3h^3 + 18A^2a^*b^2c^3g^2h^4 + 18B^2a^*b^2c^3g^2h^4 + 54A^2a^*b^2 \\
& c^2d^2g^2h^4 + 54B^2a^*b^2c^2d^2g^2h^4 + 18A^2a^3c^2d^2g^2h^4 + 18B^2a^3 \\
& c^2d^2g^2h^4 - 18A^2a^*b^2c^3g^2h^5 - 18B^2a^*b^2c^3g^2h^5 - 18A^2a^3c^2 \\
& d^2g^2h^5 - 18B^2a^3c^2d^2g^2h^5 + 6A^2a^3c^3h^6 + 6B^2a^3c^3h^6)/(b^3* \\
& d^3g^6h^5x^4 - 3b^3c^2d^2g^5h^6x^4 - 3a^2b^2d^3g^5h^6x^4 + 3b^3 \\
& c^2d^2g^4h^7x^4 + 9a^2b^2c^2d^2g^4h^7x^4 + 3a^2b^2d^3g^4h^7x^4 - \\
& b^3c^3g^3h^8x^4 - 9a^2b^2c^2d^2g^3h^8x^4 - 9a^2b^2c^2d^2g^3h^8x^4 \\
& - a^3d^3g^3h^8x^4 + 3a^2b^2c^3g^2h^9x^4 + 9a^2b^2c^2d^2g^2h^9x^4 \\
& + 3a^3c^2d^2g^2h^9x^4 - 3a^2b^2c^3g^2h^10x^4 - 3a^3c^2d^2g^2h^10x^4 \\
& + a^3c^3h^11x^4 + 4b^3d^3g^7h^4x^3 - 12b^3c^2d^2g^6h^5x^3 - \\
& 12a^2b^2d^3g^6h^5x^3 + 12b^3c^2d^2g^5h^6x^3 + 36a^2b^2c^2d^2g^5h^6 \\
& x^3 + 12a^2b^2d^3g^5h^6x^3 - 4b^3c^3g^4h^7x^3 - 36a^2b^2c^2d^2g^4 \\
& h^7x^3 - 36a^2b^2c^2d^2g^4h^7x^3 - 4a^3d^3g^4h^7x^3 + 12a^2b^2c^3 \\
& g^3h^8x^3 + 36a^2b^2c^2d^2g^3h^8x^3 + 12a^3c^2d^2g^3h^8x^3 - 1 \\
& 2a^2b^2c^3g^2h^9x^3 - 12a^3c^2d^2g^2h^9x^3 + 4a^3c^3g^2h^10x^3 + \\
& 6b^3d^3g^8h^3x^2 - 18b^3c^2d^2g^7h^4x^2 - 18a^2b^2d^3g^7h^4x^2 \\
& + 18b^3c^2d^2g^6h^5x^2 + 54a^2b^2c^2d^2g^6h^5x^2 + 18a^2b^2d^3g^6 \\
& h^5x^2 - 6b^3c^3g^5h^6x^2 - 54a^2b^2c^2d^2g^5h^6x^2 - 54a^2b^2c^2 \\
& d^2g^5h^6x^2 - 6a^3d^3g^5h^6x^2 + 18a^2b^2c^3g^4h^7x^2 + 54a^2 \\
& b^2c^2d^2g^4h^7x^2 + 18a^3c^2d^2g^4h^7x^2 - 18a^2b^2c^3g^3h^8x^2 - \\
& 18a^3c^2d^2g^3h^8x^2 + 6a^3c^3g^2h^9x^2 + 4b^3d^3g^9h^2x - \\
& 12b^3c^2d^2g^8h^3x - 12a^2b^2d^3g^8h^3x + 12b^3c^2d^2g^7h^4x + \\
& 36a^2b^2c^2d^2g^7h^4x + 12a^2b^2d^3g^7h^4x - 4b^3c^3g^6h^5x - \\
& 36a^2b^2c^2d^2g^6h^5x - 36a^2b^2c^2d^2g^6h^5x - 4a^3d^3g^6h^5x + \\
& 12a^2b^2c^3g^5h^6x + 36a^2b^2c^2d^2g^5h^6x + 12a^3c^2d^2g^5h^6x - \\
& 12a^2b^2c^3g^4h^7x - 12a^3c^2d^2g^4h^7x + 4a^3c^3g^3h^8x + \\
& b^3d^3g^10h - 3b^3c^2d^2g^9h^2 - 3a^2b^2d^3g^9h^2 + 3b^3c^2d^2g^8 \\
& h^3 + 9a^2b^2c^2d^2g^8h^3 + 3a^2b^2d^3g^8h^3 - b^3c^3g^7h^4 - 9a^2 \\
& b^2c^2d^2g^7h^4 - 9a^2b^2c^2d^2g^7h^4 - a^3d^3g^7h^4 + 3a^2b^2c^3g^6 \\
& h^5 + 9a^2b^2c^2d^2g^6h^5 + 3a^3c^2d^2g^6h^5 - 3a^2b^2c^3g^5h^6 - \\
& 3a^3c^2d^2g^5h^6 + a^3c^3g^4h^7) + 1/8*(2B^3b^5c^2d^4g^4n - 2B^2 \\
& a^*b^4d^5g^4n - 4B^3b^5c^2d^3g^3h^n + 4B^2a^*b^3d^5g^3h^n + 6B^2b^5 \\
& c^3d^2g^2h^2n - 6B^2a^*b^4c^2d^3g^2h^2n + 6B^2a^*b^3c^2d^4g^2h^2n - \\
& 6B^2a^3b^2d^5g^2h^2n - 4B^2b^5c^4d^2g^2h^3n + 4B^2a^*b^4c^3d^2 \\
& g^2h^3n - 4B^2a^3b^2c^2d^4g^2h^3n + 4B^2a^4b^2d^5g^2h^3n + B^2b^5c^5 \\
& h^4n - B^2a^*b^4c^4d^2h^4n + B^2a^4b^2c^2d^4h^4n - B^2a^5d^5h^4n)*log(ab \\
& s((2*b*d*x + b*c + a*d - abs(-b*c + a*d))/(2*b*d*x + b*c + a*d + abs(-b*c + \\
& a*d))))/((b^4*d^4*g^8h - 4b^4*c^2d^3g^7h^2 - 4a^2b^3d^4g^7h^2 + 6b^4 \\
& c^2d^2g^6h^3 + 16a^2b^3c^2d^3g^6h^3 + 6a^2b^2d^4g^6h^3 - 4b^4*c^3 \\
& d^2g^5h^4 - 24a^2b^3c^2d^2g^5h^4 - 24a^2b^2c^2d^3g^5h^4 - 4a^3 \\
& b^2d^4g^5h^4 + b^4*c^4g^4h^5 + 16a^2b^3c^3d^2g^4h^5 + 36a^2b^2c^2d^2 \\
& d^2g^4h^5 + 16a^3b^2c^2d^3g^4h^5 + a^4d^4g^4h^5 - 4a^2b^3c^4g^3h^6 - \\
& 24a^2b^2c^3d^2g^3h^6 - 24a^3b^2c^2d^2g^3h^6 - 4a^4c^2d^3g^3h^6 \\
& + 6a^2b^2c^4g^2h^7 + 16a^3b^2c^3d^2g^2h^7 + 6a^4c^2d^2g^2h^7 - \\
& 4a^3b^2c^4g^2h^8 - 4a^4c^3d^2g^2h^8 + a^4c^4h^9)*abs(-b*c + a*d))
\end{aligned}$$

### 3.303 $\int (g+hx)^2 (A + B \log(e(a + bx)^n(c + dx)^{-n}))^2 dx$

**Optimal.** Leaf size=570

$$\frac{2B^2n^2(bc - ad)(a^2d^2h^2 - abdh(3dg - ch) + b^2(c^2h^2 - 3cdgh + 3d^2g^2)) \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3} + \frac{2Bn(bc - ad)(a^2d^2h^2 - abdh(3dg - ch) + b^2(c^2h^2 - 3cdgh + 3d^2g^2)) \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3b^3d^3}$$

```
[Out] (B^2*(b*c - a*d)^2*h^2*n^2*x)/(3*b^2*d^2) + (B^2*(b*c - a*d)^3*h^2*n^2*Log[(a + b*x)/(c + d*x)])/(3*b^3*d^3) + (B^2*(b*c - a*d)^3*h^2*n^2*Log[c + d*x])/(3*b^3*d^3) + (2*B^2*(b*c - a*d)^2*h*(3*b*d*g - 2*b*c*h - a*d*h)*n^2*Log[c + d*x])/(3*b^3*d^3) - (2*B*(b*c - a*d)*h*(3*b*d*g - 2*b*c*h - a*d*h)*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*b^3*d^2) - (B*(b*c - a*d)*h^2*n*(c + d*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*b*d^3) + (2*B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(3*b^3*d^3) - ((b*g - a*h)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(3*b^3*h) + ((g + h*x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(3*h) + (2*B^2*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*b^3*d^3)
```

**Rubi [A]** time = 1.29385, antiderivative size = 697, normalized size of antiderivative = 1.22, number of steps used = 23, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 2492, 72, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315}

$$-\frac{2B^2n^2(bg - ah)^3 \text{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right)}{3b^3h} - \frac{2B^2n^2(dg - ch)^3 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3d^3h} + \frac{a^2B^2h^2n^2(bc - ad) \log(a + bx)}{3b^3d} - \frac{2B^2n^2(dg - ch)^3 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{3d^3h}$$

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2,x]
```

```
[Out] (-2*A*B*(b*c - a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*x)/(3*b^2*d^2) + (B^2*(b*c - a*d)^2*h^2*n^2*x)/(3*b^2*d^2) - (A*B*(b*c - a*d)*h^2*n*x^2)/(3*b*d) + (A^2*(g + h*x)^3)/(3*h) - (2*A*B*(b*g - a*h)^3*n*Log[a + b*x])/(3*b^3*h) + (a^2*B^2*(b*c - a*d)*h^2*n^2*Log[a + b*x])/(3*b^3*d) + (2*A*B*(d*g - c*h)^3*n*Log[c + d*x])/(3*d^3*h) - (B^2*c^2*(b*c - a*d)*h^2*n^2*Log[c + d*x])/(3*b*d^3) + (2*B^2*(b*c - a*d)^2*h*(3*b*d*g - b*c*h - a*d*h)*n^2*Log[c + d*x])/(3*b^3*d^3) - (B^2*(b*c - a*d)*h^2*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b*d) - (2*B^2*(b*c - a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b^3*d^2) + (2*A*B*(g + h*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*h) + (2*B^2*(b*g - a*h)^3*n*Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*b^3*h) - (2*B^2*(d*g - c*h)^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(3*d^3*h) + (B^2*(g + h*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(3*h) - (2*B^2*(d*g - c*h)^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(3*d^3*h) - (2*B^2*(b*g - a*h)^3*n^2*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(3*b^3*h)
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 2492

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_.))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
0] && NeQ[m, -1]
```

### Rule 72

```
Int[((e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

### Rule 2514

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))
^(r_.)]^(s_.)*(RFX_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]
```

### Rule 2486

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

### Rule 31

```
Int[((a_.) + (b_.)*(x_.))^(n_.), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

### Rule 2488

```
Int[Log[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_.)), x_Symbol] := -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/h, x] + Dist[(p*r*s*
(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

### Rule 2411

```
Int[((a_.) + Log[(c_.)*(x_.)^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)
*(x_.)^(q_.)*((h_.) + (i_.)*(x_.))^(r_.)), x_Symbol] := Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

### Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_.)^(n_.)]*(b_.))/((x_.)*((d_.) + (e_.)*(x_.)^(r_.))),
x_Symbol] := Dist[1/n, Subst[Int[(a + b*Log[c*x^n])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

### Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] := Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rubi steps

$$\begin{aligned}
 \int (g + hx)^2 (A + B \log(e(a + bx)^n(c + dx)^{-n}))^2 dx &= \int (A^2(g + hx)^2 + 2AB(g + hx)^2 \log(e(a + bx)^n(c + dx)^{-n}) + B^2(g + hx)^2) dx \\
 &= \frac{A^2(g + hx)^3}{3h} + (2AB) \int (g + hx)^2 \log(e(a + bx)^n(c + dx)^{-n}) dx + B^2 \int (g + hx)^2 dx \\
 &= \frac{A^2(g + hx)^3}{3h} + \frac{2AB(g + hx)^3 \log(e(a + bx)^n(c + dx)^{-n})}{3h} + \frac{B^2(g + hx)^3}{3h} \\
 &= \frac{A^2(g + hx)^3}{3h} + \frac{2AB(g + hx)^3 \log(e(a + bx)^n(c + dx)^{-n})}{3h} + \frac{B^2(g + hx)^3}{3h} \\
 &= -\frac{2AB(bc - ad)h(3bdg - bch - adh)nx}{3b^2d^2} - \frac{AB(bc - ad)h^2nx^2}{3bd} + \frac{A^2(g + hx)^3}{3h} \\
 &= -\frac{2AB(bc - ad)h(3bdg - bch - adh)nx}{3b^2d^2} - \frac{AB(bc - ad)h^2nx^2}{3bd} + \frac{A^2(g + hx)^3}{3h} \\
 &= -\frac{2AB(bc - ad)h(3bdg - bch - adh)nx}{3b^2d^2} - \frac{AB(bc - ad)h^2nx^2}{3bd} + \frac{A^2(g + hx)^3}{3h} \\
 &= -\frac{2AB(bc - ad)h(3bdg - bch - adh)nx}{3b^2d^2} + \frac{B^2(bc - ad)^2h^2n^2x}{3b^2d^2} - \frac{AB(bc - ad)h^2nx^2}{3bd} \\
 &= -\frac{2AB(bc - ad)h(3bdg - bch - adh)nx}{3b^2d^2} + \frac{B^2(bc - ad)^2h^2n^2x}{3b^2d^2} - \frac{AB(bc - ad)h^2nx^2}{3bd} \\
 &= -\frac{2AB(bc - ad)h(3bdg - bch - adh)nx}{3b^2d^2} + \frac{B^2(bc - ad)^2h^2n^2x}{3b^2d^2} - \frac{AB(bc - ad)h^2nx^2}{3bd}
 \end{aligned}$$

**Mathematica [A]** time = 1.80591, size = 906, normalized size = 1.59

---


$$-aB^2(3b^2g^2 - 3abhg + a^2h^2)n^2 \log^2(a + bx)d^3 + Bn \log(a + bx) \left( 2Bc(3d^2g^2 - 3cdhg + c^2h^2)n \log(c + dx)b^3 + 2B(-c + d) \right)$$


---

Antiderivative was successfully verified.

```
[In] Integrate[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)]^2), x]
```

```
[Out] -(a*B^2*d^3*(3*b^2*g^2 - 3*a*b*g*h + a^2*h^2)*n^2*Log[a + b*x]^2) + B*n*Lo
g[a + b*x]*(2*b^3*B*c*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2)*n*Log[c + d*x] + 2*
B*(3*a*b^2*d^3*g^2 - 3*a^2*b*d^3*g*h + a^3*d^3*h^2 - b^3*c*(3*d^2*g^2 - 3*c
*d*g*h + c^2*h^2))*n*Log[(b*(c + d*x))/(b*c - a*d)] + a*d*(2*A*d^2*(3*b^2*g
^2 - 3*a*b*g*h + a^2*h^2) + B*(-3*a^2*d^2*h^2 + a*b*d*h*(6*d*g + c*h) + 2*b
^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n + 2*B*d^2*(3*b^2*g^2 - 3*a*b*g*h +
```

$$a^2 h^2 \operatorname{Log}\left[\frac{e(a+bx)^n}{(c+dx)^n}\right] + b(-b^2 B^2 c(3d^2 g^2 - 3cdgh + c^2 h^2)n^2 \operatorname{Log}[c+dx]^2 + Bn \operatorname{Log}[c+dx](-2Ab^2 c(3d^2 g^2 - 3cdgh + c^2 h^2) + B(2a^2 cd^2 h^2 - 3b^2 c^2 h(-2d^2 g + ch) + a^2 b^2(-6d^2 g^2 - 6cdgh + c^2 h^2))n - 2b^2 B^2 c(3d^2 g^2 - 3cdgh + c^2 h^2) \operatorname{Log}\left[\frac{e(a+bx)^n}{(c+dx)^n}\right] + d(a^2 B^2 d^2 h^2 n(-2A + Bn)x + a^2 B^2 n(A^2 d^2(-6g^2 + 6ghx + h^2 x^2) - 2Bn(3d^2 g^2 + c^2 h^2 + cdh(-3g + hx))) + b^2 x(B^2 c^2 h^2 n^2 + A^2 d^2(3g^2 + 3ghx + h^2 x^2) + AB^2 chn(2ch - d(6g + hx))) + B(-2a^2 B^2 d^2 h^2 n^2 x + a^2 B^2 d^2 n(-6g^2 + 6ghx + h^2 x^2) + b^2 x(B^2 chn(-6dg + 2ch - d^2 hx) + 2A^2 d^2(3g^2 + 3ghx + h^2 x^2))) \operatorname{Log}\left[\frac{e(a+bx)^n}{(c+dx)^n}\right] + b^2 B^2 d^2 x(3g^2 + 3ghx + h^2 x^2) \operatorname{Log}\left[\frac{e(a+bx)^n}{(c+dx)^n}\right]^2) + 2B^2(3a^2 b^2 d^3 g^2 - 3a^2 b^2 d^3 gh + a^3 d^3 h^2 - b^3 c(3d^2 g^2 - 3cdgh + c^2 h^2))n^2 \operatorname{PolyLog}[2, (d(a+bx))/(-bc+ad)])/(3b^3 d^3)$$

**Maple [C]** time = 3.399, size = 22955, normalized size = 40.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((hx+g)^2(A+B\ln(e(bx+a)^n/((dx+c)^n)))^2, x)$

[Out] result too large to display

**Maxima [B]** time = 3.83006, size = 2256, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((hx+g)^2(A+B\log(e(bx+a)^n/((dx+c)^n)))^2, x, \operatorname{algorithm}="maxima")$

[Out]  $\frac{2}{3}ABh^2x^3\log((bx+a)^ne/(dx+c)^n) + \frac{1}{3}A^2h^2x^3 + 2ABghx^2\log((bx+a)^ne/(dx+c)^n) + A^2g^2x^2 + 2ABg^2x\log((bx+a)^ne/(dx+c)^n) + A^2g^2x + 2(a^2e\log(bx+a)/b - ce\log(dx+c)/d)ABg^2/e - 2(a^2e\log(bx+a)/b^2 - c^2e\log(dx+c)/d^2 + (b^2ce - a^2de)x/(bd))ABgh/e + \frac{1}{3}(2a^3e\log(bx+a)/b^3 - 2c^3e\log(dx+c)/d^3 - ((b^2cd^2e - a^2bd^2e)x^2 - 2(b^2c^2e - a^2d^2e)x)/(b^2d^2))ABh^2/e + \frac{1}{3}(2a^2cd^2h^2n^2 - (6cd^2ghn^2 - c^2dh^2n^2)ab - (6cd^2g^2n\log(e) + (3h^2n^2 + 2h^2n\log(e))c^3 - 6(ghn^2 + ghn\log(e))c^2d)b^2)B^2\log(dx+c)/(b^2d^3) + \frac{2}{3}(3a^2b^2d^3g^2n^2 - 3a^2b^2d^3ghn^2 + a^3d^3h^2n^2 - (3cd^2g^2n^2 - 3c^2dghn^2 + c^3h^2n^2)b^3)(\log(bx+a))\log((bdx+ad)/(bc-ad) + 1) + \operatorname{dilog}(-(bdx+ad)/(bc-ad))B^2/(b^3d^3) + \frac{1}{3}(B^2b^3d^3h^2x^3\log(e)^2 + 2(3cd^2g^2n^2 - 3c^2dghn^2 + c^3h^2n^2)B^2b^3\log(bx+a)\log(dx+c) - (3cd^2g^2n^2 - 3c^2dghn^2 + c^3h^2n^2)B^2b^3\log(dx+c)^2 + (a^2b^2d^3h^2n\log(e) - (cd^2h^2n\log(e) - 3d^3gh\log(e)^2)b^3)B^2x^2 - (3a^2b^2d^3g^2n^2 - 3a^2b^2d^3ghn^2 + a^3d^3h^2n^2)B^2\log(bx+a)^2 + ((h^2n^2 - 2h^2n\log(e))a^2bd^3 - 2(cd^2h^2n^2 - 3d^3ghn\log(e))ab^2 - (6cd^2ghn\log(e) - 3d^3g^2\log(e)^2 - (h^2n^2 + 2h^2n\log(e))c^2d)b^3)B^2x - ((3h^2n^2 - 2h^2n\log(e))a^3d^3 - ($

$$c*d^2*h^2*n^2 + 6*(g*h*n^2 - g*h*n*\log(e))*d^3)*a^2*b + 2*(3*c*d^2*g*h*n^2 - c^2*d*h^2*n^2 - 3*d^3*g^2*n*\log(e))*a*b^2)*B^2*\log(b*x + a) + (B^2*b^3*d^3*h^2*x^3 + 3*B^2*b^3*d^3*g*h*x^2 + 3*B^2*b^3*d^3*g^2*x)*\log((b*x + a)^n)^2 + (B^2*b^3*d^3*h^2*x^3 + 3*B^2*b^3*d^3*g*h*x^2 + 3*B^2*b^3*d^3*g^2*x)*\log((d*x + c)^n)^2 + (2*B^2*b^3*d^3*h^2*x^3*\log(e) - 2*(3*c*d^2*g^2*n - 3*c^2*d*g*h*n + c^3*h^2*n)*B^2*b^3*\log(d*x + c) + (a*b^2*d^3*h^2*n - (c*d^2*h^2*n - 6*d^3*g*h*\log(e))*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*g*h*n - a^2*b*d^3*h^2*n - (3*c*d^2*g*h*n - c^2*d*h^2*n - 3*d^3*g^2*\log(e))*b^3)*B^2*x + 2*(3*a*b^2*d^3*g^2*n - 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n)*B^2*\log(b*x + a))*\log((b*x + a)^n) - (2*B^2*b^3*d^3*h^2*x^3*\log(e) - 2*(3*c*d^2*g^2*n - 3*c^2*d*g*h*n + c^3*h^2*n)*B^2*b^3*\log(d*x + c) + (a*b^2*d^3*h^2*n - (c*d^2*h^2*n - 6*d^3*g*h*\log(e))*b^3)*B^2*x^2 + 2*(3*a*b^2*d^3*g*h*n - a^2*b*d^3*h^2*n - (3*c*d^2*g*h*n - c^2*d*h^2*n - 3*d^3*g^2*\log(e))*b^3)*B^2*x + 2*(3*a*b^2*d^3*g^2*n - 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n)*B^2*\log(b*x + a) + 2*(B^2*b^3*d^3*h^2*x^3 + 3*B^2*b^3*d^3*g*h*x^2 + 3*B^2*b^3*d^3*g^2*x)*\log((b*x + a)^n))*\log((d*x + c)^n))/(b^3*d^3)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(A^2h^2x^2 + 2A^2ghx + A^2g^2 + (B^2h^2x^2 + 2B^2ghx + B^2g^2)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 2(ABh^2x^2 + 2ABghx + ABg^2)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x, algorithm="fricas")

[Out] integral(A^2\*h^2\*x^2 + 2\*A^2\*g\*h\*x + A^2\*g^2 + (B^2\*h^2\*x^2 + 2\*B^2\*g\*h\*x + B^2\*g^2)\*log((b\*x + a)^n\*e/(d\*x + c)^n)^2 + 2\*(A\*B\*h^2\*x^2 + 2\*A\*B\*g\*h\*x + A\*B\*g^2)\*log((b\*x + a)^n\*e/(d\*x + c)^n), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*2\*(A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*2,x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x, algorithm="giac")

[Out] Timed out

### 3.304 $\int (g+hx) (A + B \log (e(a + bx)^n (c + dx)^{-n}))^2 dx$

**Optimal.** Leaf size=294

$$\frac{B^2 n^2 (bc - ad)(-adh - bch + 2bdg) \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{b^2 d^2} + \frac{Bn(bc - ad)(-adh - bch + 2bdg) \log\left(\frac{bc-ad}{b(c+dx)}\right) (B \log (e(a + bx)^n (c + dx)^{-n}))^2}{b^2 d^2}$$

[Out]  $(B^2(b*c - a*d)^2*h*n^2*\text{Log}[c + d*x])/(b^2*d^2) - (B*(b*c - a*d)*h*n*(a + b*x)*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x]^n)))/(b^2*d) + (B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*\text{Log}[(b*c - a*d)/(b*(c + d*x)])*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x]^n)))/(b^2*d^2) - ((b*g - a*h)^2*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x]^n))^2)/(2*b^2*h) + ((g + h*x)^2*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x]^n))^2)/(2*h) + (B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)$

**Rubi [A]** time = 0.955147, antiderivative size = 449, normalized size of antiderivative = 1.53, number of steps used = 20, number of rules used = 11, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$ , Rules used = {6742, 2492, 72, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315}

$$\frac{B^2 n^2 (bg - ah)^2 \text{PolyLog}\left(2, \frac{bc-ad}{d(a+bx)} + 1\right)}{b^2 h} - \frac{B^2 n^2 (dg - ch)^2 \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{d^2 h} - \frac{ABn(bg - ah)^2 \log(a + bx)}{b^2 h} + \frac{ABn^2 \log(a + bx)}{b^2 h}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(g + h*x)*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x]^n))^2, x]$

[Out]  $-((A*B*(b*c - a*d)*h*n*x)/(b*d)) + (A^2*(g + h*x)^2)/(2*h) - (A*B*(b*g - a*h)^2*n*\text{Log}[a + b*x])/(b^2*h) + (A*B*(d*g - c*h)^2*n*\text{Log}[c + d*x])/(d^2*h) + (B^2*(b*c - a*d)^2*h*n^2*\text{Log}[c + d*x])/(b^2*d^2) - (B^2*(b*c - a*d)*h*n*(a + b*x)*\text{Log}[(e*(a + b*x)^n)/(c + d*x]^n))/(b^2*d) + (A*B*(g + h*x)^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x]^n)/h + (B^2*(b*g - a*h)^2*n*\text{Log}[-((b*c - a*d)/(d*(a + b*x))])*\text{Log}[(e*(a + b*x)^n)/(c + d*x]^n))/(b^2*h) - (B^2*(d*g - c*h)^2*n*\text{Log}[(b*c - a*d)/(b*(c + d*x))]*\text{Log}[(e*(a + b*x)^n)/(c + d*x]^n))/(d^2*h) + (B^2*(g + h*x)^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x]^n)^2)/(2*h) - (B^2*(d*g - c*h)^2*n^2*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(d^2*h) - (B^2*(b*g - a*h)^2*n^2*\text{PolyLog}[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b^2*h)$

#### Rule 6742

$\text{Int}[u_, x\_Symbol] \rightarrow \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]]$

#### Rule 2492

$\text{Int}[\text{Log}[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^p_.)*((c_.) + (d_.)*(x_.))^q_.)^r_.]^{s_.}*((g_.) + (h_.)*(x_.))^m_.], x\_Symbol] \rightarrow \text{Simp}[(g + h*x)^{m+1}*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - \text{Dist}[(p*r*s*(b*c - a*d))/(h*(m + 1)), \text{Int}[(g + h*x)^{m+1}*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(a + b*x)*(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, m, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{NeQ}[m, -1]$

#### Rule 72

$\text{Int}[(e_.) + (f_.)*(x_.))^p_.]/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x\_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]$

;/ FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

#### Rule 2514

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)\*(RFX\_), x\_Symbol] := With[{u = ExpandIntegrand[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]

#### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s, x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 2488

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := -Simp[(Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d))/h, Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

#### Rule 2411

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_))^(q\_.)\*((h\_.) + (i\_.)\*(x\_))^(r\_.), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

#### Rule 2343

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/((x\_)\*((d\_) + (e\_.)\*(x\_))^(r\_.)), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*Log[c\*x])/((x\*(d + e\*x^(r/n))))], x], x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]

#### Rule 2333

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)/(x\_))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[(e + d\*x)^q\*(a + b\*Log[c\*x^n])^p, x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]

#### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

#### Rubi steps



$$\begin{aligned}
\int (g + hx) (A + B \log(e(a + bx)^n(c + dx)^{-n}))^2 dx &= \int (A^2(g + hx) + 2AB(g + hx) \log(e(a + bx)^n(c + dx)^{-n}) + B^2(g + hx) \log^2(e(a + bx)^n(c + dx)^{-n})) dx \\
&= \frac{A^2(g + hx)^2}{2h} + (2AB) \int (g + hx) \log(e(a + bx)^n(c + dx)^{-n}) dx + \frac{B^2}{2h} \int (g + hx) \log^2(e(a + bx)^n(c + dx)^{-n}) dx \\
&= \frac{A^2(g + hx)^2}{2h} + \frac{AB(g + hx)^2 \log(e(a + bx)^n(c + dx)^{-n})}{h} + \frac{B^2(g + hx)^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{2h} \\
&= \frac{A^2(g + hx)^2}{2h} + \frac{AB(g + hx)^2 \log(e(a + bx)^n(c + dx)^{-n})}{h} + \frac{B^2(g + hx)^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{2h} \\
&= -\frac{AB(bc - ad)hnx}{bd} + \frac{A^2(g + hx)^2}{2h} - \frac{AB(bg - ah)^2 n \log(a + bx)}{b^2 h} \\
&= -\frac{AB(bc - ad)hnx}{bd} + \frac{A^2(g + hx)^2}{2h} - \frac{AB(bg - ah)^2 n \log(a + bx)}{b^2 h} \\
&= -\frac{AB(bc - ad)hnx}{bd} + \frac{A^2(g + hx)^2}{2h} - \frac{AB(bg - ah)^2 n \log(a + bx)}{b^2 h} \\
&= -\frac{AB(bc - ad)hnx}{bd} + \frac{A^2(g + hx)^2}{2h} - \frac{AB(bg - ah)^2 n \log(a + bx)}{b^2 h} \\
&= -\frac{AB(bc - ad)hnx}{bd} + \frac{A^2(g + hx)^2}{2h} - \frac{AB(bg - ah)^2 n \log(a + bx)}{b^2 h} \\
&= -\frac{AB(bc - ad)hnx}{bd} + \frac{A^2(g + hx)^2}{2h} - \frac{AB(bg - ah)^2 n \log(a + bx)}{b^2 h} \\
&= -\frac{AB(bc - ad)hnx}{bd} + \frac{A^2(g + hx)^2}{2h} - \frac{AB(bg - ah)^2 n \log(a + bx)}{b^2 h} \\
&= -\frac{AB(bc - ad)hnx}{bd} + \frac{A^2(g + hx)^2}{2h} - \frac{AB(bg - ah)^2 n \log(a + bx)}{b^2 h}
\end{aligned}$$

**Mathematica [A]** time = 0.985289, size = 472, normalized size = 1.61

$$\frac{2B^2n^2(bc - ad)(adh + bch - 2bdg)\text{PolyLog}\left(2, \frac{d(a+bx)}{ad-bc}\right) - 2Bn \log(a + bx) \left(ad(Aadh - 2bdg) + Bd(ah - 2bg) \log(e(a + bx)^n(c + dx)^{-n})\right)}{2h}$$

Antiderivative was successfully verified.

[In] Integrate[(g + h\*x)\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2,x]

[Out] (a\*B^2\*d^2\*(-2\*b\*g + a\*h)\*n^2\*Log[a + b\*x]^2 - 2\*B\*n\*Log[a + b\*x]\*(b^2\*B\*c\*(-2\*d\*g + c\*h)\*n\*Log[c + d\*x] - B\*(b\*c - a\*d)\*(-2\*b\*d\*g + b\*c\*h + a\*d\*h)\*n\*Log[(b\*(c + d\*x))/(b\*c - a\*d)] + a\*d\*(A\*(-2\*b\*d\*g + a\*d\*h) + B\*(-2\*b\*d\*g + b\*c\*h - a\*d\*h)\*n + B\*d\*(-2\*b\*g + a\*h)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]) + b\*(b\*B^2\*c\*(-2\*d\*g + c\*h)\*n^2\*Log[c + d\*x]^2 + 2\*B\*n\*Log[c + d\*x]\*(A\*b\*c\*(-2\*d\*g + c\*h) + B\*(b\*c^2\*h - a\*d\*(2\*d\*g + c\*h))\*n + b\*B\*c\*(-2\*d\*g + c\*h)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n] + d\*(A\*b\*x\*(2\*A\*d\*g - 2\*B\*c\*h\*n + A\*d\*h\*x) + 2\*a\*B\*n\*(-2\*A\*d\*g - 2\*B\*d\*g\*n + B\*c\*h\*n + A\*d\*h\*x) + 2\*B\*(a\*B\*d\*n\*(-2\*g + h\*x) + b\*x\*(2\*A\*d\*g - B\*c\*h\*n + A\*d\*h\*x))\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n] + b\*B^2\*d\*x\*(2\*g + h\*x)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]^2) + 2\*B^2\*(b\*c - a\*d)\*(-2\*b\*d\*g + b\*c\*h + a\*d\*h)\*n^2\*PolyLog[2, (d\*(a + b\*x))/(-b\*c) + a\*d])/(2\*b^2\*d^2)

**Maple [C]** time = 2.088, size = 11007, normalized size = 37.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((h*x+g)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)`

[Out] result too large to display

**Maxima [B]** time = 3.61767, size = 1219, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="maxima")`

[Out] 
$$A*B*h*x^2*\log((b*x + a)^n*e/(d*x + c)^n) + 1/2*A^2*h*x^2 + 2*A*B*g*x*\log((b*x + a)^n*e/(d*x + c)^n) + A^2*g*x + 2*(a*e*n*\log(b*x + a)/b - c*e*n*\log(d*x + c)/d)*A*B*g/e - (a^2*e*n*\log(b*x + a)/b^2 - c^2*e*n*\log(d*x + c)/d^2 + (b*c*e*n - a*d*e*n)*x/(b*d))*A*B*h/e - (a*c*d*h*n^2 + (2*c*d*g*n*\log(e) - (h*n^2 + h*n*\log(e))*c^2)*b)*B^2*\log(d*x + c)/(b*d^2) + (2*a*b*d^2*g*n^2 - a^2*d^2*h*n^2 - (2*c*d*g*n^2 - c^2*h*n^2)*b^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/2*(B^2*b^2*d^2*h*x^2*log(e)^2 + 2*(2*c*d*g*n^2 - c^2*h*n^2)*B^2*b^2*log(b*x + a)*log(d*x + c) - (2*c*d*g*n^2 - c^2*h*n^2)*B^2*b^2*log(d*x + c)^2 - (2*a*b*d^2*g*n^2 - a^2*d^2*h*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d^2*h*n*log(e) - (c*d*h*n*log(e) - d^2*g*log(e)^2)*b^2)*B^2*x + 2*((h*n^2 - h*n*log(e))*a^2*d^2 - (c*d*h*n^2 - 2*d^2*g*n*log(e))*a*b)*B^2*log(b*x + a) + (B^2*b^2*d^2*h*x^2 + 2*B^2*b^2*d^2*g*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*h*x^2 + 2*B^2*b^2*d^2*g*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*h*x^2*log(e) - (2*c*d*g*n - c^2*h*n)*B^2*b^2*log(d*x + c) + (a*b*d^2*h*n - (c*d*h*n - 2*d^2*g*log(e))*b^2)*B^2*x + (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^2*log(b*x + a))*log((b*x + a)^n) - 2*(B^2*b^2*d^2*h*x^2*log(e) - (2*c*d*g*n - c^2*h*n)*B^2*b^2*log(d*x + c) + (a*b*d^2*h*n - (c*d*h*n - 2*d^2*g*log(e))*b^2)*B^2*x + (2*a*b*d^2*g*n - a^2*d^2*h*n)*B^2*log(b*x + a) + (B^2*b^2*d^2*h*x^2 + 2*B^2*b^2*d^2*g*x)*log((b*x + a)^n))*log((d*x + c)^n)/(b^2*d^2)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(A^2hx + A^2g + (B^2hx + B^2g)\log\left(\frac{(bx + a)^n e}{(dx + c)^n}\right)^2 + 2(ABhx + ABg)\log\left(\frac{(bx + a)^n e}{(dx + c)^n}\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algorithm="fricas")`

[Out] `integral(A^2*h*x + A^2*g + (B^2*h*x + B^2*g)*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*(A*B*h*x + A*B*g)*log((b*x + a)^n*e/(d*x + c)^n), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g) \left( B \log \left( \frac{(bx + a)^n e}{(dx + c)^n} \right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x, algorithm="giac")

[Out] integrate((h\*x + g)\*(B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^2, x)

### 3.305 $\int (A + B \log(e(a + bx)^n(c + dx)^{-n}))^2 dx$

**Optimal.** Leaf size=137

$$\frac{2B^2n^2(bc - ad)\text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd} + \frac{2Bn(bc - ad)\log\left(\frac{bc-ad}{b(c+dx)}\right)(B \log(e(a + bx)^n(c + dx)^{-n}) + A)}{bd} + \frac{(a + bx)(B \log(e(a + bx)^n(c + dx)^{-n}))^2}{bd}$$

[Out] (2\*B\*(b\*c - a\*d)\*n\*Log[(b\*c - a\*d)/(b\*(c + d\*x))]\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]))/(b\*d) + ((a + b\*x)\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2)/b + (2\*B^2\*(b\*c - a\*d)\*n^2\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*d)

**Rubi [A]** time = 0.320797, antiderivative size = 195, normalized size of antiderivative = 1.42, number of steps used = 10, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$ , Rules used = {6742, 2486, 31, 2488, 2411, 2343, 2333, 2315}

$$\frac{2B^2n^2(bc - ad)\text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd} + \frac{2AB(a + bx)\log(e(a + bx)^n(c + dx)^{-n})}{b} - \frac{2ABn(bc - ad)\log(c + dx)}{bd} + \frac{B^2(a + bx)(B \log(e(a + bx)^n(c + dx)^{-n}))^2}{bd}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2, x]

[Out] A^2\*x - (2\*A\*B\*(b\*c - a\*d)\*n\*Log[c + d\*x])/(b\*d) + (2\*A\*B\*(a + b\*x)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/b + (2\*B^2\*(b\*c - a\*d)\*n\*Log[(b\*c - a\*d)/(b\*(c + d\*x))]\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])/(b\*d) + (B^2\*(a + b\*x)\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n]^2)/b + (2\*B^2\*(b\*c - a\*d)\*n^2\*PolyLog[2, (d\*(a + b\*x))/(b\*(c + d\*x))])/(b\*d)

#### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

#### Rule 2486

Int[Log[(e\_.)\*((f\_.)\*(a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.), x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/b, x] + Dist[(q\*r\*s\*(b\*c - a\*d))/b, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 2488

Int[Log[(e\_.)\*((f\_.)\*(a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := -Simp[(Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d))/h, Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x))])\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ

[b\*g - a\*h, 0] && IGtQ[s, 0]

### Rule 2411

Int[((a\_.) + Log[(c\_.)\*(d\_) + (e\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((f\_.) + (g\_.)\*(x\_)^(q\_.))\*((h\_.) + (i\_.)\*(x\_)^(r\_.)), x\_Symbol] := Dist[1/e, Subst[Int[((g\*x)/e)^q\*((e\*h - d\*i)/e + (i\*x)/e)^r\*(a + b\*Log[c\*x^n])^p, x], x, d + e\*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e\*f - d\*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2\*r]

### Rule 2343

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))/((x\_)\*((d\_) + (e\_.)\*(x\_)^(r\_.))), x\_Symbol] := Dist[1/n, Subst[Int[(a + b\*Log[c\*x])/x\*(d + e\*x^(r/n))], x], x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]

### Rule 2333

Int[((a\_.) + Log[(c\_.)\*(x\_)^(n\_.)]\*(b\_.))^(p\_.)\*((d\_) + (e\_.)/(x\_))^(q\_.)\*(x\_)^(m\_.), x\_Symbol] := Int[(e + d\*x)^q\*(a + b\*Log[c\*x^n])^p, x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]

### Rule 2315

Int[Log[(c\_.)\*(x\_)]/((d\_) + (e\_.)\*(x\_)), x\_Symbol] := -Simp[PolyLog[2, 1 - c\*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c\*d, 0]

### Rubi steps

$$\begin{aligned}
 \int (A + B \log(e(a + bx)^n(c + dx)^{-n}))^2 dx &= \int (A^2 + 2AB \log(e(a + bx)^n(c + dx)^{-n}) + B^2 \log^2(e(a + bx)^n(c + dx)^{-n})) dx \\
 &= A^2x + (2AB) \int \log(e(a + bx)^n(c + dx)^{-n}) dx + B^2 \int \log^2(e(a + bx)^n(c + dx)^{-n}) dx \\
 &= A^2x + \frac{2AB(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{B^2(a + bx) \log^2(e(a + bx)^n(c + dx)^{-n})}{b} \\
 &= A^2x - \frac{2AB(bc - ad)n \log(c + dx)}{bd} + \frac{2AB(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} \\
 &= A^2x - \frac{2AB(bc - ad)n \log(c + dx)}{bd} + \frac{2AB(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} \\
 &= A^2x - \frac{2AB(bc - ad)n \log(c + dx)}{bd} + \frac{2AB(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} \\
 &= A^2x - \frac{2AB(bc - ad)n \log(c + dx)}{bd} + \frac{2AB(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} \\
 &= A^2x - \frac{2AB(bc - ad)n \log(c + dx)}{bd} + \frac{2AB(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b}
 \end{aligned}$$

**Mathematica [A]** time = 0.147173, size = 217, normalized size = 1.58

$$B^2n(bc - ad) \left( 2n \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) - \log \left( \frac{bc-ad}{bc+bdx} \right) \left( -2 \log(e(a + bx)^n(c + dx)^{-n}) + 2n \log \left( \frac{d(a+bx)}{ad-bc} \right) + n \log \left( \frac{bc-ad}{bc+bdx} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2,x]

[Out]  $(A^2*b*d*x - 2*A*B*(b*c - a*d)*n*\text{Log}[c + d*x] + 2*A*B*d*(a + b*x)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + B^2*d*(a + b*x)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2 + B^2*(b*c - a*d)*n*(-\text{Log}[(b*c - a*d)/(b*c + b*d*x)]*(2*n*\text{Log}[(d*(a + b*x))/(-b*c) + a*d]) - 2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] + n*\text{Log}[(b*c - a*d)/(b*c + b*d*x])) + 2*n*\text{PolyLog}[2, (b*(c + d*x))/(b*c - a*d)))/(b*d)$

**Maple [C]** time = 1.311, size = 4749, normalized size = 34.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x)

[Out]  $A^2*x^{-2}/d*n*B^2*\ln((b*x+a)^n)*c*\ln(d*x+c) - I*x*\ln((b*x+a)^n)*B^2*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3 - 1/4*B^2*Pi^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^6 + \ln(e)^{2*x}*B^{2+2}/b*A*B*\ln(b*x+a)*a^{n-1}/4*B^2*Pi^2*x*csgn(I/((d*x+c)^n))^2*csgn(I*(b*x+a)^n/((d*x+c)^n))^4 + 1/2*B^2*Pi^2*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^5 + 1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^4*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2 - 1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3 - 1/4*B^2*Pi^2*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^4 + B^2*x*\ln((d*x+c)^n)^2 - I*x*\ln((b*x+a)^n)*B^2*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3 - B^2*Pi^2*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^4 - 1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2 + 1/2*B^2*Pi^2*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3 - 1/2*B^2*Pi^2*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2 + B^2*x*\ln((b*x+a)^n)^2 - n^2*B^2*c/d*\ln(d*x+c)^2 - 1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2 + 1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^3*csgn(I*e/((d*x+c)^n)*(b*x+a)^n) + I*B^2*c*n/d*\ln(d*x+c)*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n)) - I*A*B*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)*x - I*A*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*x - I/b*B^2*\ln(b*x+a)*Pi*a^n*csgn(I*(b*x+a)^n/((d*x+c)^n))^3 + 2*\ln(e)*x*B*A + I/b*B^2*\ln(b*x+a)*Pi*a^n*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2 - I*B^2*c*n/d*\ln(d*x+c)*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2 - I*B^2*c*n/d*\ln(d*x+c)*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2 - 2*B*c*n/d*\ln(d*x+c)*A + 2/b*B^2*a^n^2*\ln(b*x+a)*\ln((-a*d+b*c+d*(b*x+a))/(-a*d+b*c)) - I*B^2*\ln(e)*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)*x - I*B^2*\ln(e)*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))*x + 2*B*x*\ln((b*x+a)^n)*A + B^2*a/b*\ln((b*x+a)^n)^2 + 2*x*\ln((b*x+a)^n)*B^2*\ln(e) + (-2*B^2*x*\ln((b*x+a)^n) - B*(-I*B*Pi*b*d*x*csgn(I*e)*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n) + I*B*Pi*b*d*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2 - I*B*Pi*b*d*x*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))) + I*B*Pi*b*d*x*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2 + I*B*Pi*b*d*x*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^n))^2 - I*B*Pi*b*d*x*csgn(I*(b*x+a)^n/((d*x+c)^n))^3 + I*B*Pi*b*d*x*csgn(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2 - I*B*Pi*b*d*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3 + 2*B*\ln(e)*b*d*x + 2*B*a*d*n*\ln(b*x+a) - 2*B*\ln(d*x+c)*b*c*n + 2*A*b*d*x/b/d*\ln((d*x+c)^n) - 1/4*B^2*Pi^2*x*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^6 - 2*B^2/b*a^n*\ln((b*x+a)^n) + 1/2*B^2*Pi^2*x*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)$



**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$2ABx \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^2x + B^2 \left( \frac{2bcn^2 \log(bx+a) \log(dx+c) - bcn^2 \log(dx+c)^2 + bdx \log((bx+a)^n)^2 + bdx \log}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x, algorithm="maxima")

[Out] 2\*A\*B\*x\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A^2\*x + B^2\*((2\*b\*c\*n^2\*log(b\*x + a)\*log(d\*x + c) - b\*c\*n^2\*log(d\*x + c)^2 + b\*d\*x\*log((b\*x + a)^n)^2 + b\*d\*x\*log((d\*x + c)^n)^2 + 2\*(a\*d\*n\*log(b\*x + a) - b\*c\*n\*log(d\*x + c) + b\*d\*x\*log(e))\*log((b\*x + a)^n) - 2\*(a\*d\*n\*log(b\*x + a) - b\*c\*n\*log(d\*x + c) + b\*d\*x\*log((b\*x + a)^n) + b\*d\*x\*log(e))\*log((d\*x + c)^n))/(b\*d) - integrate(-(b^2\*d\*x^2\*log(e)^2 + a\*b\*c\*log(e)^2 - ((2\*n\*log(e) - log(e)^2)\*b^2\*c - (2\*n\*log(e) + log(e)^2)\*a\*b\*d)\*x - 2\*(b^2\*c\*n^2\*x + 2\*a\*b\*c\*n^2 - a^2\*d\*n^2)\*log(b\*x + a))/(b^2\*d\*x^2 + a\*b\*c + (b^2\*c + a\*b\*d)\*x), x) + 2\*(a\*e\*n\*log(b\*x + a)/b - c\*e\*n\*log(d\*x + c)/d)\*A\*B/e

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(B^2 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 2AB \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x, algorithm="fricas")

[Out] integral(B^2\*log((b\*x + a)^n\*e/(d\*x + c)^n)^2 + 2\*A\*B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A^2, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^2, x)



$$3.306 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2}{g+hx} dx$$

**Optimal.** Leaf size=301

$$\frac{2Bn \text{PolyLog}\left(2, \frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right) (B \log(e(a+bx)^n(c+dx)^{-n}) + A)}{h} - \frac{2Bn \text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right) (B \log(e(a+bx)^n(c+dx)^{-n}))}{h}$$

[Out]  $-\left(\frac{\text{Log}[(b*c - a*d)/(b*(c + d*x))]}{h}\right) * (A + B * \text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^2 + \left(\frac{(A + B * \text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^2 * \text{Log}[1 - ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]}{h}\right) - \left(\frac{2*B*n*(A + B * \text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]) * \text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))]}{h}\right) + \left(\frac{2*B*n*(A + B * \text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]) * \text{PolyLog}[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]}{h}\right) + \left(\frac{2*B^2*n^2 * \text{PolyLog}[3, (d*(a + b*x))/(b*(c + d*x))]}{h}\right) - \left(\frac{2*B^2*n^2 * \text{PolyLog}[3, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]}{h}\right)$

**Rubi [A]** time = 0.815256, antiderivative size = 473, normalized size of antiderivative = 1.57, number of steps used = 16, number of rules used = 10, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$ , Rules used = {6742, 2494, 2394, 2393, 2391, 2489, 2488, 2506, 6610, 2503}

$$-\frac{2ABn \text{PolyLog}\left(2, \frac{b(g+hx)}{bg-ah}\right)}{h} + \frac{2B^2n \log(e(a+bx)^n(c+dx)^{-n}) \text{PolyLog}\left(2, 1 - \frac{(g+hx)(bc-ad)}{(c+dx)(bg-ah)}\right)}{h} - \frac{2B^2n \text{PolyLog}\left(2, 1 - \frac{(g+hx)(bc-ad)}{(c+dx)(bg-ah)}\right)}{h}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2/(g + h\*x), x]

[Out]  $-\left(\frac{B^2 * \text{Log}[(b*c - a*d)/(b*(c + d*x))]}{h}\right) * \text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2 + \left(\frac{A^2 * \text{Log}[g + h*x]}{h}\right) - \left(\frac{2*A*B*n * \text{Log}[-((h*(a + b*x))/(b*g - a*h))]}{h}\right) * \text{Log}[g + h*x] + \left(\frac{2*A*B*n * \text{Log}[-((h*(c + d*x))/(d*g - c*h))]}{h}\right) * \text{Log}[g + h*x] + \left(\frac{2*A*B * \text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] * \text{Log}[g + h*x]}{h}\right) + \left(\frac{B^2 * \text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2 * \text{Log}[(b*c - a*d)*(g + h*x)/((b*g - a*h)*(c + d*x))]}{h}\right) - \left(\frac{2*A*B*n * \text{PolyLog}[2, (b*(g + h*x))/(b*g - a*h)]}{h}\right) + \left(\frac{2*A*B*n * \text{PolyLog}[2, (d*(g + h*x))/(d*g - c*h)]}{h}\right) - \left(\frac{2*B^2*n * \text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] * \text{PolyLog}[2, 1 - (b*c - a*d)/(b*(c + d*x))]}{h}\right) + \left(\frac{2*B^2*n * \text{Log}[(e*(a + b*x)^n)/(c + d*x)^n] * \text{PolyLog}[2, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))]}{h}\right) + \left(\frac{2*B^2*n^2 * \text{PolyLog}[3, 1 - (b*c - a*d)/(b*(c + d*x))]}{h}\right) - \left(\frac{2*B^2*n^2 * \text{PolyLog}[3, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))]}{h}\right)$

**Rule 6742**

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

**Rule 2494**

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[(Log[g + h\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r)]/h, x] + (-Dist[(b\*p\*r)/h, Int[Log[g + h\*x]/(a + b\*x), x], x] - Dist[(d\*q\*r)/h, Int[Log[g + h\*x]/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b\*c - a\*d, 0]

**Rule 2394**

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[e\*(f + g\*x)]/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)

)^n))/g, x] - Dist[(b\*e^n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

### Rule 2489

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))\*((c\_.) + (d\_.)\*(x\_)^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Dist[d/h, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s/(c + d\*x), x], x] - Dist[(d\*g - c\*h)/h, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s/((c + d\*x)\*(g + h\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && NeQ[b\*g - a\*h, 0] && NeQ[d\*g - c\*h, 0] && IGtQ[s, 1]

### Rule 2488

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))\*((c\_.) + (d\_.)\*(x\_)^(q\_.))^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := -Simp[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d))/h, Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

### Rule 2506

Int[Log[v\_\*Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))\*((c\_.) + (d\_.)\*(x\_)^(q\_.))^(r\_.)]^(s\_.)\*(u\_), x\_Symbol] := With[{g = Simplify[((v - 1)\*(c + d\*x))/(a + b\*x)], h = Simplify[u\*(a + b\*x)\*(c + d\*x)]}, -Simp[(h\*PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s)/(b\*c - a\*d), x] + Dist[h\*p\*r\*s, Int[(PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]

### Rule 2503

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_)^(p\_.))\*((c\_.) + (d\_.)\*(x\_)^(q\_.))^(r\_.)]^(s\_.)\*(u\_), x\_Symbol] := With[{g = Coeff[Simplify[1/(u\*(a + b\*x))], x, 0], h = Coeff[Simplify[1/(u\*(a + b\*x))], x, 1]}, -Simp[(Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s\*Log[-((b\*c - a\*d)\*(g + h\*x))/((d\*g - c\*h)\*(a + b\*x)))]/(b\*g - a\*h), x] + Dist[(p\*r\*s\*(b\*c - a\*d))/(b\*g - a\*h), Int[(Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q)^r]^s - 1)\*Log[-((b\*c - a\*d)\*(g + h\*x))/((d\*g - c\*h)\*(a + b\*x)))]/((a + b\*x)\*(c + d\*x)), x], x] /; NeQ[b\*g - a\*h, 0] && NeQ[d\*g - c\*h, 0] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a

\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u\*(a + b\*x))], x]

Rubi steps

$$\begin{aligned} \int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^2}{g + hx} dx &= \int \left( \frac{A^2}{g + hx} + \frac{2AB \log(e(a + bx)^n(c + dx)^{-n})}{g + hx} + \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{g + hx} \right) dx \\ &= \frac{A^2 \log(g + hx)}{h} + (2AB) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{g + hx} dx + B^2 \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{g + hx} dx \\ &= \frac{A^2 \log(g + hx)}{h} + \frac{2AB \log(e(a + bx)^n(c + dx)^{-n}) \log(g + hx)}{h} + \frac{(B^2 d) \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{g + hx} dx}{h} \\ &= -\frac{B^2 \log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2(e(a + bx)^n(c + dx)^{-n})}{h} + \frac{A^2 \log(g + hx)}{h} - \frac{2ABn \log(e(a + bx)^n(c + dx)^{-n}) \log(g + hx)}{h} \\ &= -\frac{B^2 \log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2(e(a + bx)^n(c + dx)^{-n})}{h} + \frac{A^2 \log(g + hx)}{h} - \frac{2ABn \log(e(a + bx)^n(c + dx)^{-n}) \log(g + hx)}{h} \\ &= -\frac{B^2 \log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2(e(a + bx)^n(c + dx)^{-n})}{h} + \frac{A^2 \log(g + hx)}{h} - \frac{2ABn \log(e(a + bx)^n(c + dx)^{-n}) \log(g + hx)}{h} \end{aligned}$$

**Mathematica [B]** time = 0.462649, size = 1082, normalized size = 3.59

$$\frac{-2n(-n \log(a + bx) + n \log(c + dx) + \log(e(a + bx)^n(c + dx)^{-n})) \left( \log(c + dx) \log\left(\frac{d(g+hx)}{dg-ch}\right) + \text{PolyLog}\left(2, \frac{h(c+dx)}{ch-dg}\right) \right) B}{h}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]^2/(g + h\*x), x]

[Out] ((A + B\*(-(n\*Log[a + b\*x]) + n\*Log[c + d\*x] + Log[(e\*(a + b\*x)^n)/(c + d\*x]^n)))^2\*Log[g + h\*x] + 2\*B\*n\*(A + B\*(-(n\*Log[a + b\*x]) + n\*Log[c + d\*x] + Log[(e\*(a + b\*x)^n)/(c + d\*x]^n))\*(Log[a + b\*x]\*Log[(b\*(g + h\*x))/(b\*g - a\*h)] + PolyLog[2, (h\*(a + b\*x))/(-(b\*g) + a\*h)]) - 2\*A\*B\*n\*(Log[c + d\*x]\*Log[(d\*(g + h\*x))/(d\*g - c\*h)] + PolyLog[2, (h\*(c + d\*x))/(-(d\*g) + c\*h)]) - 2\*B^2\*n\*(-(n\*Log[a + b\*x]) + n\*Log[c + d\*x] + Log[(e\*(a + b\*x)^n)/(c + d\*x]^n))\*(Log[c + d\*x]\*Log[(d\*(g + h\*x))/(d\*g - c\*h)] + PolyLog[2, (h\*(c + d\*x))/(-(d\*g) + c\*h)]) + B^2\*n^2\*(Log[a + b\*x]^2\*Log[(b\*(g + h\*x))/(b\*g - a\*h)] + 2\*Log[a + b\*x]\*PolyLog[2, (h\*(a + b\*x))/(-(b\*g) + a\*h)] - 2\*PolyLog[3, (h\*(a + b\*x))/(-(b\*g) + a\*h)]) + B^2\*n^2\*(Log[c + d\*x]^2\*Log[(d\*(g + h\*x))/(d\*g - c\*h)] + 2\*Log[c + d\*x]\*PolyLog[2, (h\*(c + d\*x))/(-(d\*g) + c\*h)] - 2\*PolyLog[3, (h\*(c + d\*x))/(-(d\*g) + c\*h)]) - 2\*B^2\*n^2\*(Log[a + b\*x]\*Log[c + d\*x]\*Log[(b\*(g + h\*x))/(b\*g - a\*h)] + (Log[(h\*(c + d\*x))/(-(d\*g) + c\*h)]\*(-2\*Log[a + b\*x] + Log[(h\*(c + d\*x))/(-(d\*g) + c\*h)]\*(Log[(b\*(g + h\*x))/(b\*g - a\*h)] - Log[(d\*(g + h\*x))/(d\*g - c\*h)]))/2 + Log[(h\*(c + d\*x))/(-(d\*g) + c\*h)]\*Log[((b\*g - a\*h)\*(c + d\*x))/((d\*g - c\*h)\*(a + b\*x))]\*(-Log[(b\*(g + h\*x))/(b\*g - a\*h)] + Log[(d\*(g + h\*x))/(d\*g - c\*h)] + (Log[((b\*g - a\*h)\*(c + d\*x))/((d\*g - c\*h)\*(a + b\*x))]^2\*(Log[(-(b\*c) + a\*d)/(d\*(a + b\*x))] + Log[(b\*(g + h\*x))/(b\*g - a\*h)] - Log[(-(b\*c) + a\*d)\*(g + h\*x)/((d\*g - c\*h)\*(a + b\*x))]))/2 + (Log[c + d\*x] - Log[((b\*g - a\*h)\*(c + d\*x))/((d\*g - c\*h)\*(a + b\*x))])\*PolyLog[2, (h\*(a + b\*x))/(-(b\*g) + a\*h)] + (Log[a + b\*x] + Log[(b\*(g - a\*h)\*(c + d\*x))/((d\*g - c\*h)\*(a + b\*x))])\*PolyLog[2, (h\*(c + d\*x))/(-(d\*g) + c\*h)] + Log[((b\*g - a\*h)\*(c + d\*x))/((d\*g - c\*h)\*(a + b\*x))]\*(PolyLog[2, (b\*(c + d\*x))/(d\*(a + b\*x))] - PolyLog[2, ((b\*g - a\*h)\*(c + d\*x))/((

$d*g - c*h)*(a + b*x))]] - \text{PolyLog}[3, (h*(a + b*x))/(-(b*g) + a*h)] - \text{PolyLog}[3, (h*(c + d*x))/(-(d*g) + c*h)] - \text{PolyLog}[3, (b*(c + d*x))/(d*(a + b*x))] + \text{PolyLog}[3, ((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x)))]/h$

**Maple [F]** time = 2.559, size = 0, normalized size = 0.

$$\int \frac{1}{hx + g} \left( A + B \ln \left( \frac{e(bx + a)^n}{(dx + c)^n} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g),x)

[Out] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{A^2 \log(hx + g)}{h} + \int \frac{B^2 \log((bx + a)^n)^2 + B^2 \log((dx + c)^n)^2 + B^2 \log(e)^2 + 2AB \log(e) + 2(B^2 \log(e) + AB) \log((bx + a)^n)}{hx + g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g),x, algorithm="maxima")

[Out] A^2\*log(h\*x + g)/h + integrate((B^2\*log((b\*x + a)^n)^2 + B^2\*log((d\*x + c)^n)^2 + B^2\*log(e)^2 + 2\*A\*B\*log(e) + 2\*(B^2\*log(e) + A\*B)\*log((b\*x + a)^n) - 2\*(B^2\*log((b\*x + a)^n) + B^2\*log(e) + A\*B)\*log((d\*x + c)^n))/(h\*x + g), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right)^2 + 2AB \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A^2}{hx + g}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g),x, algorithm="fricas")

[Out] integral((B^2\*log((b\*x + a)^n\*e/(d\*x + c)^n)^2 + 2\*A\*B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A^2)/(h\*x + g), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*2/(h\*x+g),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log\left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)^2}{hx + g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g),x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^2/(h\*x + g), x)

$$3.307 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^2} dx$$

**Optimal.** Leaf size=208

$$\frac{2B^2n^2(bc-ad)\text{PolyLog}\left(2, \frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)}{(bg-ah)(dg-ch)} + \frac{2Bn(bc-ad) \log\left(1 - \frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right) (B \log(e(a+bx)^n(c+dx)^{-n}) + A)}{(bg-ah)(dg-ch)} + \frac{(a+bx)^2}{(g+hx)^2}$$

[Out] ((a + b\*x)\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n])^2)/((b\*g - a\*h)\*(g + h\*x)) + (2\*B\*(b\*c - a\*d)\*n\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n])\*Log[1 - ((d\*g - c\*h)\*(a + b\*x))/((b\*g - a\*h)\*(c + d\*x))])/((b\*g - a\*h)\*(d\*g - c\*h)) + (2\*B^2\*(b\*c - a\*d)\*n^2\*PolyLog[2, ((d\*g - c\*h)\*(a + b\*x))/((b\*g - a\*h)\*(c + d\*x))])/((b\*g - a\*h)\*(d\*g - c\*h))

**Rubi [A]** time = 0.406977, antiderivative size = 343, normalized size of antiderivative = 1.65, number of steps used = 10, number of rules used = 7, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$ , Rules used = {6742, 2490, 36, 31, 2503, 2502, 2315}

$$\frac{2B^2n^2(bc-ad)\text{PolyLog}\left(2, 1 - \frac{(g+hx)(bc-ad)}{(c+dx)(bg-ah)}\right)}{(bg-ah)(dg-ch)} + \frac{2AB(a+bx) \log(e(a+bx)^n(c+dx)^{-n})}{(g+hx)(bg-ah)} - \frac{2ABn(bc-ad) \log(c+dx)}{(bg-ah)(dg-ch)} + \frac{(a+bx)^2}{(g+hx)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n])^2/(g + h\*x)^2, x]

[Out] -(A^2/(h\*(g + h\*x))) - (2\*A\*B\*(b\*c - a\*d)\*n\*Log[c + d\*x])/((b\*g - a\*h)\*(d\*g - c\*h)) + (2\*A\*B\*(a + b\*x)\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n])/((b\*g - a\*h)\*(g + h\*x)) + (B^2\*(a + b\*x)\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n]^2)/((b\*g - a\*h)\*(g + h\*x)) + (2\*A\*B\*(b\*c - a\*d)\*n\*Log[g + h\*x])/((b\*g - a\*h)\*(d\*g - c\*h)) + (2\*B^2\*(b\*c - a\*d)\*n\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n]\*Log[((b\*c - a\*d)\*(g + h\*x))/((b\*g - a\*h)\*(c + d\*x))])/((b\*g - a\*h)\*(d\*g - c\*h)) + (2\*B^2\*(b\*c - a\*d)\*n^2\*PolyLog[2, 1 - ((b\*c - a\*d)\*(g + h\*x))/((b\*g - a\*h)\*(c + d\*x))])/((b\*g - a\*h)\*(d\*g - c\*h))

### Rule 6742

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

### Rule 2490

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_))^(p\_.))\*((c\_.) + (d\_.)\*(x\_))^(q\_.)]^(r\_.)]^(s\_.)/((g\_.) + (h\_.)\*(x\_))^2, x\_Symbol] := Simp[((a + b\*x)\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/((b\*g - a\*h)\*(g + h\*x)), x] - Dist[(p\*r\*s\*(b\*c - a\*d))/(b\*g - a\*h), Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(c + d\*x)\*(g + h\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && NeQ[b\*g - a\*h, 0] && IGtQ[s, 0]

### Rule 36

Int[1/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 31

$\text{Int}[(a + (b \cdot x))^{-1}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b \cdot x, x]]/b, x] \text{ ; FreeQ}\{a, b\}, x]$

Rule 2503

$\text{Int}[\text{Log}[(e \cdot (f \cdot (a + (b \cdot x))^{p \cdot (c + (d \cdot x))^{q \cdot (r \cdot (s \cdot u))})})^{(r \cdot (s \cdot u))}], x\_Symbol] \rightarrow \text{With}\{g = \text{Coeff}[\text{Simplify}[1/(u \cdot (a + b \cdot x))], x, 0], h = \text{Coeff}[\text{Simplify}[1/(u \cdot (a + b \cdot x))], x, 1]\}, -\text{Simp}[(\text{Log}[e \cdot (f \cdot (a + b \cdot x)^p \cdot (c + d \cdot x)^q \cdot r]^s \cdot \text{Log}[-((b \cdot c - a \cdot d) \cdot (g + h \cdot x))/((d \cdot g - c \cdot h) \cdot (a + b \cdot x))])]/(b \cdot g - a \cdot h), x] + \text{Dist}[(p \cdot r \cdot s \cdot (b \cdot c - a \cdot d))/(b \cdot g - a \cdot h), \text{Int}[(\text{Log}[e \cdot (f \cdot (a + b \cdot x)^p \cdot (c + d \cdot x)^q \cdot r]^s \cdot (s - 1) \cdot \text{Log}[-((b \cdot c - a \cdot d) \cdot (g + h \cdot x))/((d \cdot g - c \cdot h) \cdot (a + b \cdot x))])]/((a + b \cdot x) \cdot (c + d \cdot x)), x], x] \text{ ; NeQ}[b \cdot g - a \cdot h, 0] \&\& \text{NeQ}[d \cdot g - c \cdot h, 0] \text{ ; FreeQ}\{a, b, c, d, e, f, p, q, r, s\}, x] \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{LinearQ}[\text{Simplify}[1/(u \cdot (a + b \cdot x))], x]$

Rule 2502

$\text{Int}[\text{Log}[(e \cdot (c + (d \cdot x)))/(a + (b \cdot x))] \cdot u, x\_Symbol] \rightarrow \text{With}\{g = \text{Coeff}[\text{Simplify}[1/(u \cdot (a + b \cdot x))], x, 0], h = \text{Coeff}[\text{Simplify}[1/(u \cdot (a + b \cdot x))], x, 1]\}, -\text{Dist}[(b - d \cdot e)/(h \cdot (b \cdot c - a \cdot d)), \text{Subst}[\text{Int}[\text{Log}[e \cdot x]/(1 - e \cdot x), x], x, (c + d \cdot x)/(a + b \cdot x)], x] \text{ ; EqQ}[g \cdot (b - d \cdot e) - h \cdot (a - c \cdot e), 0] \text{ ; FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b \cdot c - a \cdot d, 0] \&\& \text{LinearQ}[\text{Simplify}[1/(u \cdot (a + b \cdot x))], x]$

Rule 2315

$\text{Int}[\text{Log}[(c \cdot x)]/((d + (e \cdot x)), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, 1 - c \cdot x]/e, x] \text{ ; FreeQ}\{c, d, e\}, x] \&\& \text{EqQ}[e + c \cdot d, 0]$

Rubi steps

$$\begin{aligned} \int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^2}{(g + hx)^2} dx &= \int \left( \frac{A^2}{(g + hx)^2} + \frac{2AB \log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^2} + \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^2} \right) dx \\ &= -\frac{A^2}{h(g + hx)} + (2AB) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^2} dx + B^2 \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^2} dx \\ &= -\frac{A^2}{h(g + hx)} + \frac{2AB(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} + \frac{B^2(a + bx) \log^2(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} \\ &= -\frac{A^2}{h(g + hx)} + \frac{2AB(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} + \frac{B^2(a + bx) \log^2(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} \\ &= -\frac{A^2}{h(g + hx)} - \frac{2AB(bc - ad)n \log(c + dx)}{(bg - ah)(dg - ch)} + \frac{2AB(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} \\ &= -\frac{A^2}{h(g + hx)} - \frac{2AB(bc - ad)n \log(c + dx)}{(bg - ah)(dg - ch)} + \frac{2AB(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} \end{aligned}$$

**Mathematica [B]** time = 2.28281, size = 3460, normalized size = 16.63

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2/(g + h\*x)^2,x]

[Out] 
$$\begin{aligned} & -(A^2*b*d*g^2) + A^2*b*c*g*h + a*A^2*d*g*h - a*A^2*c*h^2 + 2*A*b*B*d*g^2*n \\ & *Log[a + b*x] - 2*A*b*B*c*g*h*n*Log[a + b*x] + 2*A*b*B*d*g*h*n*x*Log[a + b* \\ & x] - 2*A*b*B*c*h^2*n*x*Log[a + b*x] - b*B^2*d*g^2*n^2*Log[a + b*x]^2 + b*B^2 \\ & *c*g*h*n^2*Log[a + b*x]^2 - b*B^2*d*g*h*n^2*x*Log[a + b*x]^2 + b*B^2*c*h^2 \\ & *n^2*x*Log[a + b*x]^2 - 2*A*b*B*d*g^2*n*Log[c + d*x] + 2*a*A*B*d*g*h*n*Log[ \\ & c + d*x] - 2*A*b*B*d*g*h*n*x*Log[c + d*x] + 2*a*A*B*d*h^2*n*x*Log[c + d*x] \\ & + 2*b*B^2*d*g^2*n^2*Log[a + b*x]*Log[c + d*x] - 2*a*B^2*d*g*h*n^2*Log[a + b \\ & *x]*Log[c + d*x] + 2*b*B^2*d*g*h*n^2*x*Log[a + b*x]*Log[c + d*x] - 2*a*B^2*d \\ & *h^2*n^2*x*Log[a + b*x]*Log[c + d*x] - b*B^2*d*g^2*n^2*Log[c + d*x]^2 + a* \\ & B^2*d*g*h*n^2*Log[c + d*x]^2 - b*B^2*d*g*h*n^2*x*Log[c + d*x]^2 + a*B^2*d*h \\ & ^2*n^2*x*Log[c + d*x]^2 - 2*b*B^2*c*g*h*n^2*Log[a + b*x]*Log[(h*(c + d*x))/ \\ & (-d*g) + c*h] + 2*a*B^2*d*g*h*n^2*Log[a + b*x]*Log[(h*(c + d*x))/(-d*g) \\ & + c*h] - 2*b*B^2*c*h^2*n^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-d*g) + c*h] \\ & ] + 2*a*B^2*d*h^2*n^2*x*Log[a + b*x]*Log[(h*(c + d*x))/(-d*g) + c*h] + b* \\ & B^2*c*g*h*n^2*Log[(h*(c + d*x))/(-d*g) + c*h]^2 - a*B^2*d*g*h*n^2*Log[(h* \\ & (c + d*x))/(-d*g) + c*h]^2 + b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x))/(-d*g) \\ & + c*h]^2 - a*B^2*d*h^2*n^2*x*Log[(h*(c + d*x))/(-d*g) + c*h]^2 - 2*b*B^2 \\ & *c*g*h*n^2*Log[(-b*c) + a*d)/(d*(a + b*x))] *Log[((b*g - a*h)*(c + d*x))/(( \\ & d*g - c*h)*(a + b*x))] + 2*a*B^2*d*g*h*n^2*Log[(-b*c) + a*d)/(d*(a + b*x)) \\ & ] *Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*b*B^2*c*h^2*n^2* \\ & x*Log[(-b*c) + a*d)/(d*(a + b*x))] *Log[((b*g - a*h)*(c + d*x))/((d*g - c*h) \\ & *(a + b*x))] + 2*a*B^2*d*h^2*n^2*x*Log[(-b*c) + a*d)/(d*(a + b*x))] *Log[( \\ & (b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*b*B^2*c*g*h*n^2*Log[(h* \\ & (c + d*x))/(-d*g) + c*h] *Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b* \\ & x))] + 2*a*B^2*d*g*h*n^2*Log[(h*(c + d*x))/(-d*g) + c*h] *Log[((b*g - a*h) \\ & *(c + d*x))/((d*g - c*h)*(a + b*x))] - 2*b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x) \\ & )/(-d*g) + c*h] *Log[((b*g - a*h)*(c + d*x))/((d*g - c*h)*(a + b*x))] + 2* \\ & a*B^2*d*h^2*n^2*x*Log[(h*(c + d*x))/(-d*g) + c*h] *Log[((b*g - a*h)*(c + d \\ & *x))/((d*g - c*h)*(a + b*x))] + b*B^2*c*g*h*n^2*Log[((b*g - a*h)*(c + d*x)) \\ & /((d*g - c*h)*(a + b*x))]^2 - a*B^2*d*g*h*n^2*Log[((b*g - a*h)*(c + d*x))/ \\ & (d*g - c*h)*(a + b*x))]^2 + b*B^2*c*h^2*n^2*x*Log[((b*g - a*h)*(c + d*x))/ \\ & (d*g - c*h)*(a + b*x))]^2 - a*B^2*d*h^2*n^2*x*Log[((b*g - a*h)*(c + d*x))/ \\ & (d*g - c*h)*(a + b*x))]^2 - 2*A*b*B*d*g^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] \\ & + 2*A*b*B*c*g*h*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*a*A*B*d*g*h*Log[(e*(a \\ & + b*x)^n)/(c + d*x)^n] - 2*a*A*B*c*h^2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2 \\ & *b*B^2*d*g^2*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b*B^2*c*g* \\ & h*n*Log[a + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*b*B^2*d*g*h*n*x*Log[a \\ & + b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b*B^2*c*h^2*n*x*Log[a + b*x]*L \\ & og[(e*(a + b*x)^n)/(c + d*x)^n] - 2*b*B^2*d*g^2*n*Log[c + d*x]*Log[(e*(a + \\ & b*x)^n)/(c + d*x)^n] + 2*a*B^2*d*g*h*n*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c \\ & + d*x)^n] - 2*b*B^2*d*g*h*n*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] \\ & + 2*a*B^2*d*h^2*n*x*Log[c + d*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] - b*B^2*d \\ & *g^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + b*B^2*c*g*h*Log[(e*(a + b*x)^n)/ \\ & (c + d*x)^n]^2 + a*B^2*d*g*h*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - a*B^2*c*h \\ & ^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 - 2*A*b*B*d*g^2*n*Log[(b*(g + h*x))/( \\ & b*g - a*h)] + 2*A*b*B*c*g*h*n*Log[(b*(g + h*x))/(b*g - a*h)] - 2*A*b*B*d*g* \\ & h*n*x*Log[(b*(g + h*x))/(b*g - a*h)] + 2*A*b*B*c*h^2*n*x*Log[(b*(g + h*x))/ \\ & (b*g - a*h)] + 2*b*B^2*d*g^2*n^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h) \\ & ] - 2*a*B^2*d*g*h*n^2*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*B^2 \\ & *d*g*h*n^2*x*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*a*B^2*d*h^2*n^ \\ & 2*x*Log[a + b*x]*Log[(b*(g + h*x))/(b*g - a*h)] - 2*b*B^2*d*g^2*n^2*Log[(h* \\ & (c + d*x))/(-d*g) + c*h] *Log[(b*(g + h*x))/(b*g - a*h)] + 2*b*B^2*c*g*h*n \\ & ^2*Log[(h*(c + d*x))/(-d*g) + c*h] *Log[(b*(g + h*x))/(b*g - a* \\ & h)] + 2*b*B^2*c*h^2*n^2*x*Log[(h*(c + d*x))/(-d*g) + c*h] *Log[(b*(g + h* \\ & x))/(b*g - a*h)] - 2*b*B^2*d*g^2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n] *Log[(b* \\ & (g + h*x))/(b*g - a*h)] + 2*b*B^2*c*g*h*n*Log[(e*(a + b*x)^n)/(c + d*x)^n] * \end{aligned}$$



$$\begin{aligned} & \text{Log}[(b*(g + h*x))/(b*g - a*h)] - 2*b*B^2*d*g*h*n*x*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]*\text{Log}[(b*(g + h*x))/(b*g - a*h)] + 2*b*B^2*c*h^2*n*x*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]*\text{Log}[(b*(g + h*x))/(b*g - a*h)] + 2*A*b*B*d*g^2*n*\text{Log}[(d*(g + h*x))/(d*g - c*h)] - 2*a*A*B*d*g*h*n*\text{Log}[(d*(g + h*x))/(d*g - c*h)] + 2*A*b*B*d*g*h*n*x*\text{Log}[(d*(g + h*x))/(d*g - c*h)] - 2*a*A*B*d*h^2*n*x*\text{Log}[(d*(g + h*x))/(d*g - c*h)] - 2*b*B^2*d*g^2*n^2*\text{Log}[a + b*x]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] + 2*a*B^2*d*g*h*n^2*\text{Log}[a + b*x]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] - 2*b*B^2*d*g*h*n^2*x*\text{Log}[a + b*x]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] + 2*a*B^2*d*h^2*n^2*x*\text{Log}[a + b*x]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] + 2*b*B^2*d*g^2*n^2*\text{Log}[(h*(c + d*x))/(-(d*g) + c*h)]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] - 2*b*B^2*c*g*h*n^2*\text{Log}[(h*(c + d*x))/(-(d*g) + c*h)]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] + 2*b*B^2*d*g*h*n^2*x*\text{Log}[(h*(c + d*x))/(-(d*g) + c*h)]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] - 2*b*B^2*c*h^2*n^2*x*\text{Log}[(h*(c + d*x))/(-(d*g) + c*h)]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] + 2*b*B^2*d*g^2*n*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] - 2*a*B^2*d*g*h*n*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] + 2*b*B^2*d*g*h*n*x*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] - 2*a*B^2*d*h^2*n*x*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]*\text{Log}[(d*(g + h*x))/(d*g - c*h)] + 2*B^2*(b*c - a*d)*h*n^2*(g + h*x)*\text{PolyLog}[2, (h*(a + b*x))/(-(b*g) + a*h)] - 2*B^2*(b*c - a*d)*h*n^2*(g + h*x)*\text{PolyLog}[2, (h*(c + d*x))/(-(d*g) + c*h)] - 2*b*B^2*c*g*h*n^2*\text{PolyLog}[2, (b*(c + d*x))/(d*(a + b*x))] + 2*a*B^2*d*g*h*n^2*\text{PolyLog}[2, (b*(c + d*x))/(d*(a + b*x))] - 2*b*B^2*c*h^2*n^2*x*\text{PolyLog}[2, (b*(c + d*x))/(d*(a + b*x))] + 2*a*B^2*d*h^2*n^2*x*\text{PolyLog}[2, (b*(c + d*x))/(d*(a + b*x))]/(h*(-(b*g) + a*h)*(-(d*g) + c*h)*(g + h*x)) \end{aligned}$$

**Maple [F]** time = 2.712, size = 0, normalized size = 0.

$$\int \frac{1}{(hx + g)^2} \left( A + B \ln \left( \frac{e(bx + a)^n}{(dx + c)^n} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g)^2,x)

[Out] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-B^2 \left( \frac{\log((dx + c)^n)^2}{h^2x + gh} + \int \frac{dhx \log(e)^2 + ch \log(e)^2 + (dhx + ch) \log((bx + a)^n)^2 + 2(dhx \log(e) + ch \log(e)) \log((dx + c)^n)}{dh^3x^3 + cg^2h + (2dgh^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g)^2,x, algorithm="maxima")

[Out] -B^2\*(log((d\*x + c)^n)^2/(h^2\*x + g\*h) + integrate(-(d\*h\*x\*log(e)^2 + c\*h\*log(e)^2 + (d\*h\*x + c\*h)\*log((b\*x + a)^n)^2 + 2\*(d\*h\*x\*log(e) + c\*h\*log(e))\*log((b\*x + a)^n) + 2\*(d\*g\*n + (h\*n - h\*log(e))\*d\*x - c\*h\*log(e) - (d\*h\*x + c\*h)\*log((b\*x + a)^n))\*log((d\*x + c)^n)/(d\*h^3\*x^3 + c\*g^2\*h + (2\*d\*g\*h^2 + c\*h^3)\*x^2 + (d\*g^2\*h + 2\*c\*g\*h^2)\*x), x)) + 2\*(b\*e\*n\*log(b\*x + a)/(b\*g\*h - a\*h^2) - d\*e\*n\*log(d\*x + c)/(d\*g\*h - c\*h^2) - (b\*c\*e\*n - a\*d\*e\*n)\*log(h\*

$x + g)/((d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*A*B/e - 2*A*B*log((b*x + a)^n*e/(d*x + c)^n)/(h^2*x + g*h) - A^2/(h^2*x + g*h)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^2 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 2AB \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^2}{h^2 x^2 + 2ghx + g^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g)^2,x, algorithm="fricas")

[Out] integral((B^2\*log((b\*x + a)^n\*e/(d\*x + c)^n))^2 + 2\*A\*B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A^2)/(h^2\*x^2 + 2\*g\*h\*x + g^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*2/(h\*x+g)\*\*2,x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(hx + g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g)^2,x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^2/(h\*x + g)^2, x)

$$3.308 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^2}{(g+hx)^3} dx$$

**Optimal.** Leaf size=393

$$\frac{B^2 n^2 (bc - ad)(-adh - bch + 2bdg) \text{PolyLog}\left(2, \frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)}{(bg - ah)^2 (dg - ch)^2} + \frac{b^2 (B \log(e(a + bx)^n(c + dx)^{-n}) + A)^2}{2h(bg - ah)^2} + \frac{Bhn(a + bx)(c + dx)^n}{(bg - ah)^2 (dg - ch)^2}$$

```
[Out] (B*(b*c - a*d)*h*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x]^n]))/((b*g - a*h)^2*(d*g - c*h)*(g + h*x)) + (b^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x]^n])^2)/(2*h*(b*g - a*h)^2) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x]^n])^2/(2*h*(g + h*x)^2) + (B^2*(b*c - a*d)^2*h*n^2*Log[(g + h*x)/(c + d*x]])/((b*g - a*h)^2*(d*g - c*h)^2) + (B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x]^n])*Log[1 - ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) + (B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2)
```

**Rubi [B]** time = 1.63057, antiderivative size = 968, normalized size of antiderivative = 2.46, number of steps used = 29, number of rules used = 16, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.485$ , Rules used = {6742, 2492, 72, 2514, 2488, 2411, 2343, 2333, 2315, 2490, 36, 31, 2494, 2394, 2393, 2391}

$$-\frac{A^2}{2h(g + hx)^2} + \frac{b^2 B n \log(a + bx) A}{h(bg - ah)^2} - \frac{B d^2 n \log(c + dx) A}{h(dg - ch)^2} - \frac{B \log(e(a + bx)^n(c + dx)^{-n}) A}{h(g + hx)^2} + \frac{B(bc - ad)(2bdg - bch - a^2)}{(bg - ah)^2 (dg - ch)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x]^n])^2/(g + h*x)^3, x]
```

```
[Out] -A^2/(2*h*(g + h*x)^2) - (A*B*(b*c - a*d)*n)/((b*g - a*h)*(d*g - c*h)*(g + h*x)) + (A*b^2*B*n*Log[a + b*x])/(h*(b*g - a*h)^2) - (A*B*d^2*n*Log[c + d*x])/(h*(d*g - c*h)^2) - (B^2*(b*c - a*d)^2*h*n^2*Log[c + d*x])/((b*g - a*h)^2*(d*g - c*h)^2) - (A*B*Log[(e*(a + b*x)^n)/(c + d*x]^n])/(h*(g + h*x)^2) + (B^2*(b*c - a*d)*h*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x]^n])/((b*g - a*h)^2*(d*g - c*h)*(g + h*x)) - (b^2*B^2*n*Log[-((b*c - a*d)/(d*(a + b*x))])*Log[(e*(a + b*x)^n)/(c + d*x]^n])/(h*(b*g - a*h)^2) + (B^2*d^2*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x]^n])/((b*g - a*h)^2*(d*g - c*h)^2) - (B^2*Log[(e*(a + b*x)^n)/(c + d*x]^n])^2/(2*h*(g + h*x)^2) + (A*B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2) + (B^2*(b*c - a*d)^2*h*n^2*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2) - (B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*Log[-((h*(a + b*x))/(b*g - a*h))]*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2) + (B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*Log[-((h*(c + d*x))/(d*g - c*h))]*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2) + (B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*Log[(e*(a + b*x)^n)/(c + d*x]^n]*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2) + (B^2*d^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) - (B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*PolyLog[2, (b*(g + h*x))/(b*g - a*h)])/((b*g - a*h)^2*(d*g - c*h)^2) + (B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*PolyLog[2, (d*(g + h*x))/(d*g - c*h)])/((b*g - a*h)^2*(d*g - c*h)^2) + (b^2*B^2*n^2*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/((b*g - a*h)^2)
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
0] && NeQ[m, -1]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFX_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]
```

#### Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/h, x] + Dist[(p*r*s*(
b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

#### Rule 2411

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)])*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] := Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x]^n)]^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

#### Rule 2343

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)])*(b_.))/((x_)*((d_.) + (e_.)*(x_)^(r_.))),
x_Symbol] := Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

#### Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_)])*(b_.))^(p_.)*((d_.) + (e_.)/(x_))^(q_.)*
(x_)^(m_.), x_Symbol] := Int[(e + d*x)^q*(a + b*Log[c*x]^n)]^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

#### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_.) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
```

$c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{EqQ}[e + c*d, 0]$

#### Rule 2490

$\text{Int}[\text{Log}[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^{(p_.)*((c_.) + (d_.)*(x_.))^{(q_.)})^{(r_.)}]^{(s_.)}/((g_.) + (h_.)*(x_.))^2, x\_Symbol] \rightarrow \text{Simp}[(a + b*x)*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s]/((b*g - a*h)*(g + h*x)), x] - \text{Dist}[(p*r*s*(b*c - a*d))/(b*g - a*h), \text{Int}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^{(s - 1)}/((c + d*x)*(g + h*x)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{NeQ}[b*g - a*h, 0] \&\& \text{IGtQ}[s, 0]$

#### Rule 36

$\text{Int}[1/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x\_Symbol] \rightarrow \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

#### Rule 31

$\text{Int}[(a_.) + (b_.)*(x_.))^{(-1)}, x\_Symbol] \rightarrow \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$

#### Rule 2494

$\text{Int}[\text{Log}[(e_.)*((f_.)*(a_.) + (b_.)*(x_.))^{(p_.)*((c_.) + (d_.)*(x_.))^{(q_.)})^{(r_.)}]^{(s_.)}/((g_.) + (h_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[g + h*x]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q)^r])/h, x] + (-\text{Dist}[(b*p*r)/h, \text{Int}[\text{Log}[g + h*x]/(a + b*x), x], x] - \text{Dist}[(d*q*r)/h, \text{Int}[\text{Log}[g + h*x]/(c + d*x), x], x]) /; \text{FreeQ}\{a, b, c, d, e, f, g, h, p, q, r\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

#### Rule 2394

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^{(n_.)}] * (b_.)] / ((f_.) + (g_.)*(x_.)), x\_Symbol] \rightarrow \text{Simp}[(\text{Log}[(e*(f + g*x))/(e*f - d*g)] * (a + b*\text{Log}[c*(d + e*x)^n]))/g, x] - \text{Dist}[(b*e*n)/g, \text{Int}[\text{Log}[(e*(f + g*x))/(e*f - d*g)]/(d + e*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, n\}, x] \&\& \text{NeQ}[e*f - d*g, 0]$

#### Rule 2393

$\text{Int}[(a_.) + \text{Log}[(c_.)*((d_.) + (e_.)*(x_.))] * (b_.)] / ((f_.) + (g_.)*(x_.)), x\_Symbol] \rightarrow \text{Dist}[1/g, \text{Subst}[\text{Int}[(a + b*\text{Log}[1 + (c*e*x)/g])/x, x], x, f + g*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g\}, x] \&\& \text{NeQ}[e*f - d*g, 0] \&\& \text{EqQ}[g + c*(e*f - d*g), 0]$

#### Rule 2391

$\text{Int}[\text{Log}[(c_.)*((d_.) + (e_.)*(x_.))^{(n_.)}] / (x_.), x\_Symbol] \rightarrow -\text{Simp}[\text{PolyLog}[2, -(c*e*x^n)]/n, x] /; \text{FreeQ}\{c, d, e, n\}, x] \&\& \text{EqQ}[c*d, 1]$

#### Rubi steps

$$\begin{aligned}
\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^2}{(g + hx)^3} dx &= \int \left( \frac{A^2}{(g + hx)^3} + \frac{2AB \log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^3} + \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^3} \right) dx \\
&= -\frac{A^2}{2h(g + hx)^2} + (2AB) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^3} dx + B^2 \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^3} dx \\
&= -\frac{A^2}{2h(g + hx)^2} - \frac{AB \log(e(a + bx)^n(c + dx)^{-n})}{h(g + hx)^2} - \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{2h(g + hx)^2} \\
&= -\frac{A^2}{2h(g + hx)^2} - \frac{AB \log(e(a + bx)^n(c + dx)^{-n})}{h(g + hx)^2} - \frac{B^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{2h(g + hx)^2} \\
&= -\frac{A^2}{2h(g + hx)^2} - \frac{AB(bc - ad)n}{(bg - ah)(dg - ch)(g + hx)} + \frac{Ab^2Bn \log(a + bx)}{h(bg - ah)^2} - \frac{ABd^2n \log^2(a + bx)}{h(dg - ch)^2} \\
&= -\frac{A^2}{2h(g + hx)^2} - \frac{AB(bc - ad)n}{(bg - ah)(dg - ch)(g + hx)} + \frac{Ab^2Bn \log(a + bx)}{h(bg - ah)^2} - \frac{ABd^2n \log^2(a + bx)}{h(dg - ch)^2} \\
&= -\frac{A^2}{2h(g + hx)^2} - \frac{AB(bc - ad)n}{(bg - ah)(dg - ch)(g + hx)} + \frac{Ab^2Bn \log(a + bx)}{h(bg - ah)^2} - \frac{ABd^2n \log^2(a + bx)}{h(dg - ch)^2} \\
&= -\frac{A^2}{2h(g + hx)^2} - \frac{AB(bc - ad)n}{(bg - ah)(dg - ch)(g + hx)} + \frac{Ab^2Bn \log(a + bx)}{h(bg - ah)^2} - \frac{ABd^2n \log^2(a + bx)}{h(dg - ch)^2} \\
&= -\frac{A^2}{2h(g + hx)^2} - \frac{AB(bc - ad)n}{(bg - ah)(dg - ch)(g + hx)} + \frac{Ab^2Bn \log(a + bx)}{h(bg - ah)^2} - \frac{ABd^2n \log^2(a + bx)}{h(dg - ch)^2} \\
&= -\frac{A^2}{2h(g + hx)^2} - \frac{AB(bc - ad)n}{(bg - ah)(dg - ch)(g + hx)} + \frac{Ab^2Bn \log(a + bx)}{h(bg - ah)^2} - \frac{ABd^2n \log^2(a + bx)}{h(dg - ch)^2}
\end{aligned}$$

**Mathematica [B]** time = 6.46763, size = 15422, normalized size = 39.24

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^2/(g + h\*x)^3,x]

[Out] Result too large to show

**Maple [F]** time = 1.788, size = 0, normalized size = 0.

$$\int \frac{1}{(hx + g)^3} \left( A + B \ln \left( \frac{e(bx + a)^n}{(dx + c)^n} \right) \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g)^3,x)

[Out] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g)^3,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{1}{2}B^2\left(\frac{\log((dx+c)^n)^2}{h^3x^2+2gh^2x+g^2h}+2\int-\frac{dhx\log(e)^2+ch\log(e)^2+(dhx+ch)\log((bx+a)^n)^2+2(dhx\log(e)+ch\log(e))\log((bx+a)^n)}{dh^4x^4+cg^3h+(3dgh^3-}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g)^3,x, algorithm="maxima")

[Out] -1/2\*B^2\*(log((d\*x + c)^n)^2/(h^3\*x^2 + 2\*g\*h^2\*x + g^2\*h) + 2\*integrate(-(d\*h\*x\*log(e)^2 + c\*h\*log(e)^2 + (d\*h\*x + c\*h)\*log((b\*x + a)^n)^2 + 2\*(d\*h\*x\*log(e) + c\*h\*log(e))\*log((b\*x + a)^n) + (d\*g\*n + (h\*n - 2\*h\*log(e))\*d\*x - 2\*c\*h\*log(e) - 2\*(d\*h\*x + c\*h)\*log((b\*x + a)^n))\*log((d\*x + c)^n))/(d\*h^4\*x^4 + c\*g^3\*h + (3\*d\*g\*h^3 + c\*h^4)\*x^3 + 3\*(d\*g^2\*h^2 + c\*g\*h^3)\*x^2 + (d\*g^3\*h + 3\*c\*g^2\*h^2)\*x), x) + (b^2\*e\*n\*log(b\*x + a)/(b^2\*g^2\*h - 2\*a\*b\*g\*h^2 + a^2\*h^3) - d^2\*e\*n\*log(d\*x + c)/(d^2\*g^2\*h - 2\*c\*d\*g\*h^2 + c^2\*h^3) - (2\*a\*b\*d^2\*e\*g\*n - a^2\*d^2\*e\*h\*n - (2\*c\*d\*e\*g\*n - c^2\*e\*h\*n)\*b^2)\*log(h\*x + g)/((d^2\*g^2\*h^2 - 2\*c\*d\*g\*h^3 + c^2\*h^4)\*a^2 - 2\*(d^2\*g^3\*h - 2\*c\*d\*g^2\*h^2 + c^2\*g\*h^3)\*a\*b + (d^2\*g^4 - 2\*c\*d\*g^3\*h + c^2\*g^2\*h^2)\*b^2) + (b\*c\*e\*n - a\*d\*e\*n)/((d\*g^2\*h - c\*g\*h^2)\*a - (d\*g^3 - c\*g^2\*h)\*b + ((d\*g\*h^2 - c\*h^3)\*a - (d\*g^2\*h - c\*g\*h^2)\*b)\*x)\*A\*B/e - A\*B\*log((b\*x + a)^n\*e/(d\*x + c)^n)/(h^3\*x^2 + 2\*g\*h^2\*x + g^2\*h) - 1/2\*A^2/(h^3\*x^2 + 2\*g\*h^2\*x + g^2\*h)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{B^2\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)^2+2AB\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A^2}{h^3x^3+3gh^2x^2+3g^2hx+g^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^2/(h\*x+g)^3,x, algorithm="fricas")

[Out] integral((B^2\*log((b\*x + a)^n\*e/(d\*x + c)^n)^2 + 2\*A\*B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A^2)/(h^3\*x^3 + 3\*g\*h^2\*x^2 + 3\*g^2\*h\*x + g^3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*2/(h\*x+g)\*\*3,x)

[Out] Timed out

---

**Giac [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(h*x+g)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```



### 3.309 $\int (g+hx)^2 (A + B \log (e(a + bx)^n(c + dx)^{-n}))^3 dx$

**Optimal.** Leaf size=875

$$\frac{B^3 h^2 n^3 \log(c + dx)(bc - ad)^3}{b^3 d^3} - \frac{B^2 h^2 n^2 (A + B \log (e(a + bx)^n(c + dx)^{-n})) \log\left(1 - \frac{b(c+dx)}{d(a+bx)}\right) (bc - ad)^3}{b^3 d^3} + \frac{B^3 h^2 n^3 \text{Poly}}{b^3 d^3}$$

```
[Out] -((B^3*(b*c - a*d)^3*h^2*n^3*Log[c + d*x])/(b^3*d^3)) + (B^2*(b*c - a*d)^2*
h^2*n^2*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b^3*d^2) - (2*
B^2*(b*c - a*d)^2*h*(3*b*d*g - 2*b*c*h - a*d*h)*n^2*Log[(b*c - a*d)/(b*(c +
d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))/(b^3*d^3) - (B*(b*c - a*d)
)*h*(3*b*d*g - 2*b*c*h - a*d*h)*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c +
d*x)^n])^2/(b^3*d^2) - (B*(b*c - a*d)*h^2*n*(c + d*x)^2*(A + B*Log[(e*(a
+ b*x)^n)/(c + d*x)^n])^2)/(2*b*d^3) + (B*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*
h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n*Log[(b*c - a*d)/
(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(b^3*d^3) - ((b*
g - a*h)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(3*b^3*h) + ((g + h*
x)^3*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(3*h) - (B^2*(b*c - a*d)^3
*h^2*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*Log[1 - (b*(c + d*x))/(d*
(a + b*x))])/(b^3*d^3) - (2*B^3*(b*c - a*d)^2*h*(3*b*d*g - 2*b*c*h - a*d*h)
*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b^3*d^3) + (2*B^2*(b*c - a*d)
)*(a^2*d^2*h^2 - a*b*d*h*(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h
^2))*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (d*(a + b*x))/
(b*(c + d*x))])/(b^3*d^3) + (B^3*(b*c - a*d)^3*h^2*n^3*PolyLog[2, (b*(c + d
*x))/(d*(a + b*x))])/(b^3*d^3) - (2*B^3*(b*c - a*d)*(a^2*d^2*h^2 - a*b*d*h*
(3*d*g - c*h) + b^2*(3*d^2*g^2 - 3*c*d*g*h + c^2*h^2))*n^3*PolyLog[3, (d*(a
+ b*x))/(b*(c + d*x))])/(b^3*d^3)
```

**Rubi [A]** time = 3.48124, antiderivative size = 1640, normalized size of antiderivative = 1.87, number of steps used = 53, number of rules used = 13, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.394$ , Rules used = {6742, 2492, 72, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(g + h*x)^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3,x]
```

```
[Out] -((A^2*B*(b*c - a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*x)/(b^2*d^2)) + (A*B^2*(
b*c - a*d)^2*h^2*n^2*x)/(b^2*d^2) - (A^2*B*(b*c - a*d)*h^2*n*x^2)/(2*b*d) +
(A^3*(g + h*x)^3)/(3*h) - (A^2*B*(b*g - a*h)^3*n*Log[a + b*x])/(b^3*h) + (
a^2*A*B^2*(b*c - a*d)*h^2*n^2*Log[a + b*x])/(b^3*d) + (A^2*B*(d*g - c*h)^3*
n*Log[c + d*x])/(d^3*h) - (A*B^2*c^2*(b*c - a*d)*h^2*n^2*Log[c + d*x])/(b*d
^3) + (2*A*B^2*(b*c - a*d)^2*h*(3*b*d*g - b*c*h - a*d*h)*n^2*Log[c + d*x])/(
b^3*d^3) - (B^3*(b*c - a*d)^3*h^2*n^3*Log[c + d*x])/(b^3*d^3) - (A*B^2*(b*
c - a*d)*h^2*n*x^2*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d) - (2*A*B^2*(b*c
- a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)
^2])/(b^3*d^2) + (B^3*(b*c - a*d)^2*h^2*n^2*(a + b*x)*Log[(e*(a + b*x)^n)/
(c + d*x)^n])/(b^3*d^2) + (A^2*B*(g + h*x)^3*Log[(e*(a + b*x)^n)/(c + d*x)^
n])/h + (2*A*B^2*(b*g - a*h)^3*n*Log[-((b*c - a*d)/(d*(a + b*x))])*Log[(e*(
a + b*x)^n)/(c + d*x)^n])/(b^3*h) - (a^2*B^3*(b*c - a*d)*h^2*n^2*Log[-((b*c
- a*d)/(d*(a + b*x))])*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b^3*d) - (2*A*B^
2*(d*g - c*h)^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d
*x)^n])/(d^3*h) + (B^3*c^2*(b*c - a*d)*h^2*n^2*Log[(b*c - a*d)/(b*(c + d*x)
)])*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d^3) - (2*B^3*(b*c - a*d)^2*h*(3*b*
```

$$\begin{aligned}
& d*g - b*c*h - a*d*h)*n^2*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n) / \\
& / (c + d*x)^n)] / (b^3*d^3) - (B^3*(b*c - a*d)*h^2*n*x^2*Log[(e*(a + b*x)^n) / ( \\
& c + d*x)^n]^2) / (2*b*d) - (B^3*(b*c - a*d)*h*(3*b*d*g - b*c*h - a*d*h)*n*(a \\
& + b*x)*Log[(e*(a + b*x)^n) / (c + d*x)^n]^2) / (b^3*d^2) + (A*B^2*(g + h*x)^3*L \\
& og[(e*(a + b*x)^n) / (c + d*x)^n]^2) / h + (B^3*(b*g - a*h)^3*n*Log[-((b*c - a* \\
& d)/(d*(a + b*x))])*Log[(e*(a + b*x)^n) / (c + d*x)^n]^2) / (b^3*h) - (B^3*(d*g \\
& - c*h)^3*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n) / (c + d*x)^n]^ \\
& 2) / (d^3*h) + (B^3*(g + h*x)^3*Log[(e*(a + b*x)^n) / (c + d*x)^n]^3) / (3*h) - ( \\
& 2*A*B^2*(d*g - c*h)^3*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))]) / (d^3*h) \\
& + (B^3*c^2*(b*c - a*d)*h^2*n^3*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))]) / (b* \\
& d^3) - (2*B^3*(b*c - a*d)^2*h*(3*b*d*g - b*c*h - a*d*h)*n^3*PolyLog[2, (d*( \\
& a + b*x))/(b*(c + d*x))]) / (b^3*d^3) - (2*A*B^2*(b*g - a*h)^3*n^2*PolyLog[2, \\
& 1 + (b*c - a*d)/(d*(a + b*x))]) / (b^3*h) + (a^2*B^3*(b*c - a*d)*h^2*n^3*Pol \\
& yLog[2, 1 + (b*c - a*d)/(d*(a + b*x))]) / (b^3*d) - (2*B^3*(b*g - a*h)^3*n^2* \\
& Log[(e*(a + b*x)^n) / (c + d*x)^n]*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))]) \\
& / (b^3*h) - (2*B^3*(d*g - c*h)^3*n^2*Log[(e*(a + b*x)^n) / (c + d*x)^n]*PolyLo \\
& g[2, 1 - (b*c - a*d)/(b*(c + d*x))]) / (d^3*h) - (2*B^3*(b*g - a*h)^3*n^3*Pol \\
& yLog[3, 1 + (b*c - a*d)/(d*(a + b*x))]) / (b^3*h) + (2*B^3*(d*g - c*h)^3*n^3*Pol \\
& yLog[3, 1 - (b*c - a*d)/(b*(c + d*x))]) / (d^3*h)
\end{aligned}$$

#### Rule 6742

```
Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] :=> Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
0] && NeQ[m, -1]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] :=> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFX_), x_Symbol] :=> With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]
```

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] :=> Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 31

```
Int[((a_.) + (b_.)*(x_))^(n_.), x_Symbol] :=> Simp[Log[RemoveContent[a + b*x,
```

$x]/b, x] /; \text{FreeQ}\{a, b\}, x]$

#### Rule 2488

$\text{Int}[\text{Log}[(e_{.})*((f_{.})*(a_{.}) + (b_{.})*(x_{.}))^{(p_{.})}*((c_{.}) + (d_{.})*(x_{.}))^{(q_{.})})^{(r_{.})}]^{(s_{.})}/((g_{.}) + (h_{.})*(x_{.})), x\_Symbol] :> -\text{Simp}[(\text{Log}[-((b*c - a*d)/(d*(a + b*x)))]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/h, x] + \text{Dist}[(p*r*s*(b*c - a*d))/h, \text{Int}[(\text{Log}[-((b*c - a*d)/(d*(a + b*x)))]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^{(s - 1)})/((a + b*x)*(c + d*x)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{EqQ}[b*g - a*h, 0] \&\& \text{IGtQ}[s, 0]$

#### Rule 2411

$\text{Int}[(a_{.}) + \text{Log}[(c_{.})*((d_{.}) + (e_{.})*(x_{.}))^{(n_{.})}*(b_{.})]^{(p_{.})}*((f_{.}) + (g_{.})*(x_{.}))^{(q_{.})}*((h_{.}) + (i_{.})*(x_{.}))^{(r_{.})}], x\_Symbol] :> \text{Dist}[1/e, \text{Subst}[\text{Int}[(g*x)/e]^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*\text{Log}[c*x^n])^p, x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, i, n, p, q, r\}, x] \&\& \text{EqQ}[e*f - d*g, 0] \&\& (\text{IGtQ}[p, 0] \mid \mid \text{IGtQ}[r, 0]) \&\& \text{IntegerQ}[2*r]$

#### Rule 2343

$\text{Int}[(a_{.}) + \text{Log}[(c_{.})*(x_{.})^{(n_{.})}*(b_{.})]/((x_{.})*((d_{.}) + (e_{.})*(x_{.})^{(r_{.})}))], x\_Symbol] :> \text{Dist}[1/n, \text{Subst}[\text{Int}[(a + b*\text{Log}[c*x])/x*(d + e*x^{(r/n)})], x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, e, n, r\}, x] \&\& \text{IntegerQ}[r/n]$

#### Rule 2333

$\text{Int}[(a_{.}) + \text{Log}[(c_{.})*(x_{.})^{(n_{.})}*(b_{.})]^{(p_{.})}*((d_{.}) + (e_{.})/(x_{.}))^{(q_{.})}*(x_{.})^{(m_{.})}], x\_Symbol] :> \text{Int}[(e + d*x)^q*(a + b*\text{Log}[c*x^n])^p, x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& \text{EqQ}[m, q] \&\& \text{IntegerQ}[q]$

#### Rule 2315

$\text{Int}[\text{Log}[(c_{.})*(x_{.})]/((d_{.}) + (e_{.})*(x_{.})), x\_Symbol] :> -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{EqQ}[e + c*d, 0]$

#### Rule 2506

$\text{Int}[\text{Log}[v_{.}]*\text{Log}[(e_{.})*((f_{.})*(a_{.}) + (b_{.})*(x_{.}))^{(p_{.})}*((c_{.}) + (d_{.})*(x_{.}))^{(q_{.})})^{(r_{.})}]^{(s_{.})}*(u_{.}), x\_Symbol] :> \text{With}\{g = \text{Simplify}[(v - 1)*(c + d*x)/(a + b*x)], h = \text{Simplify}[u*(a + b*x)*(c + d*x)]\}, -\text{Simp}[(h*\text{PolyLog}[2, 1 - v]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] + \text{Dist}[h*p*r*s, \text{Int}[(\text{PolyLog}[2, 1 - v]*\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^{(s - 1)})/((a + b*x)*(c + d*x)), x], x] /; \text{FreeQ}\{g, h\}, x] /; \text{FreeQ}\{a, b, c, d, e, f, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[s, 0] \&\& \text{EqQ}[p + q, 0]$

#### Rule 6610

$\text{Int}[(u_{.})*\text{PolyLog}[n_{.}, v_{.}], x\_Symbol] :> \text{With}\{w = \text{DerivativeDivides}[v, u*v, x]\}, \text{Simp}[w*\text{PolyLog}[n + 1, v], x] /; \text{!FalseQ}[w] /; \text{FreeQ}[n, x]$

#### Rubi steps

$$\begin{aligned}
\int (g + hx)^2 (A + B \log(e(a + bx)^n(c + dx)^{-n}))^3 dx &= \int (A^3(g + hx)^2 + 3A^2B(g + hx)^2 \log(e(a + bx)^n(c + dx)^{-n}) + 3AB^2(g + hx)^2 \log^2(e(a + bx)^n(c + dx)^{-n})) dx \\
&= \frac{A^3(g + hx)^3}{3h} + (3A^2B) \int (g + hx)^2 \log(e(a + bx)^n(c + dx)^{-n}) dx + \frac{3AB^2}{h} \int (g + hx)^2 \log^2(e(a + bx)^n(c + dx)^{-n}) dx \\
&= \frac{A^3(g + hx)^3}{3h} + \frac{A^2B(g + hx)^3 \log(e(a + bx)^n(c + dx)^{-n})}{h} + \frac{AB^2(g + hx)^3 \log^2(e(a + bx)^n(c + dx)^{-n})}{h} \\
&= \frac{A^3(g + hx)^3}{3h} + \frac{A^2B(g + hx)^3 \log(e(a + bx)^n(c + dx)^{-n})}{h} + \frac{AB^2(g + hx)^3 \log^2(e(a + bx)^n(c + dx)^{-n})}{h} \\
&= -\frac{A^2B(bc - ad)h(3bdg - bch - adh)nx}{b^2d^2} - \frac{A^2B(bc - ad)h^2nx^2}{2bd} + \frac{A^3(g + hx)^3}{3h} \\
&= -\frac{A^2B(bc - ad)h(3bdg - bch - adh)nx}{b^2d^2} - \frac{A^2B(bc - ad)h^2nx^2}{2bd} + \frac{A^3(g + hx)^3}{3h} \\
&= -\frac{A^2B(bc - ad)h(3bdg - bch - adh)nx}{b^2d^2} - \frac{A^2B(bc - ad)h^2nx^2}{2bd} + \frac{A^3(g + hx)^3}{3h} \\
&= -\frac{A^2B(bc - ad)h(3bdg - bch - adh)nx}{b^2d^2} - \frac{A^2B(bc - ad)h^2nx^2}{2bd} + \frac{A^3(g + hx)^3}{3h} \\
&= -\frac{A^2B(bc - ad)h(3bdg - bch - adh)nx}{b^2d^2} + \frac{AB^2(bc - ad)^2h^2n^2x}{b^2d^2} - \frac{A^2B(bc - ad)h^2nx^2}{2bd} + \frac{A^3(g + hx)^3}{3h} \\
&= -\frac{A^2B(bc - ad)h(3bdg - bch - adh)nx}{b^2d^2} + \frac{AB^2(bc - ad)^2h^2n^2x}{b^2d^2} - \frac{A^2B(bc - ad)h^2nx^2}{2bd} + \frac{A^3(g + hx)^3}{3h} \\
&= -\frac{A^2B(bc - ad)h(3bdg - bch - adh)nx}{b^2d^2} + \frac{AB^2(bc - ad)^2h^2n^2x}{b^2d^2} - \frac{A^2B(bc - ad)h^2nx^2}{2bd} + \frac{A^3(g + hx)^3}{3h} \\
&= -\frac{A^2B(bc - ad)h(3bdg - bch - adh)nx}{b^2d^2} + \frac{AB^2(bc - ad)^2h^2n^2x}{b^2d^2} - \frac{A^2B(bc - ad)h^2nx^2}{2bd} + \frac{A^3(g + hx)^3}{3h} \\
&= -\frac{A^2B(bc - ad)h(3bdg - bch - adh)nx}{b^2d^2} + \frac{AB^2(bc - ad)^2h^2n^2x}{b^2d^2} - \frac{A^2B(bc - ad)h^2nx^2}{2bd} + \frac{A^3(g + hx)^3}{3h} \\
&= -\frac{A^2B(bc - ad)h(3bdg - bch - adh)nx}{b^2d^2} + \frac{AB^2(bc - ad)^2h^2n^2x}{b^2d^2} - \frac{A^2B(bc - ad)h^2nx^2}{2bd} + \frac{A^3(g + hx)^3}{3h} \\
&= -\frac{A^2B(bc - ad)h(3bdg - bch - adh)nx}{b^2d^2} + \frac{AB^2(bc - ad)^2h^2n^2x}{b^2d^2} - \frac{A^2B(bc - ad)h^2nx^2}{2bd} + \frac{A^3(g + hx)^3}{3h}
\end{aligned}$$

**Mathematica [F]** time = 5.6802, size = 0, normalized size = 0.

$$\int (g + hx)^2 (A + B \log(e(a + bx)^n(c + dx)^{-n}))^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(g + h\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n)]^3,x]

[Out] Integrate[(g + h\*x)^2\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n)]^3, x]

**Maple [F]** time = 5.47, size = 0, normalized size = 0.

$$\int (hx + g)^2 \left( A + B \ln \left( \frac{e(bx + a)^n}{(dx + c)^n} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((h*x+g)^2*(A+B*\ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)$

[Out]  $\text{int}((h*x+g)^2*(A+B*\ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((h*x+g)^2*(A+B*\log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, \text{algorithm}="maxima")$

[Out]  $A^2*B*h^2*x^3*\log((b*x + a)^n*e/(d*x + c)^n) + 1/3*A^3*h^2*x^3 + 3*A^2*B*g*h*x^2*\log((b*x + a)^n*e/(d*x + c)^n) + A^3*g*h*x^2 + 3*A^2*B*g^2*x*\log((b*x + a)^n*e/(d*x + c)^n) + A^3*g^2*x + 3*(a*e^n*\log(b*x + a)/b - c*e^n*\log(d*x + c)/d)*A^2*B*g^2/e - 3*(a^2*e^n*\log(b*x + a)/b^2 - c^2*e^n*\log(d*x + c)/d^2 + (b*c*e^n - a*d*e^n)*x/(b*d))*A^2*B*g*h/e + 1/2*(2*a^3*e^n*\log(b*x + a)/b^3 - 2*c^3*e^n*\log(d*x + c)/d^3 - ((b^2*c*d*e^n - a*b*d^2*e^n)*x^2 - 2*(b^2*c^2*e^n - a^2*d^2*e^n)*x)/(b^2*d^2))*A^2*B*h^2/e - 1/6*(2*(B^3*b^3*d^3*h^2*x^3 + 3*B^3*b^3*d^3*g*h*x^2 + 3*B^3*b^3*d^3*g^2*x)*\log((d*x + c)^n)^3 + 3*(2*(3*c*d^2*g^2*n - 3*c^2*d*g*h*n + c^3*h^2*n)*B^3*b^3*\log(d*x + c) - 2*(3*a*b^2*d^3*g^2*n - 3*a^2*b*d^3*g*h*n + a^3*d^3*h^2*n)*B^3*\log(b*x + a) - 2*(B^3*b^3*d^3*h^2*\log(e) + A*B^2*b^3*d^3*h^2)*x^3 - (6*A*B^2*b^3*d^3*g*h + (a*b^2*d^3*h^2*n - (c*d^2*h^2*n - 6*d^3*g*h*\log(e))*b^3)*B^3)*x^2 - 2*(3*A*B^2*b^3*d^3*g^2 + (3*a*b^2*d^3*g*h*n - a^2*b*d^3*h^2*n - (3*c*d^2*g*h*n - c^2*d*h^2*n - 3*d^3*g^2*\log(e))*b^3)*B^3)*x - 2*(B^3*b^3*d^3*h^2*x^3 + 3*B^3*b^3*d^3*g*h*x^2 + 3*B^3*b^3*d^3*g^2*x)*\log((b*x + a)^n)*\log((d*x + c)^n)^2)/(b^3*d^3) - \text{integrate}(-(B^3*b^3*c*d^2*g^2*\log(e)^3 + 3*A*B^2*b^3*c*d^2*g^2*\log(e)^2 + (B^3*b^3*d^3*h^2*\log(e)^3 + 3*A*B^2*b^3*d^3*h^2*\log(e)^2)*x^3 + (B^3*b^3*d^3*h^2*x^3 + B^3*b^3*c*d^2*g^2 + (2*d^3*g*h + c*d^2*h^2)*B^3*b^3*x^2 + (d^3*g^2 + 2*c*d^2*g*h)*B^3*b^3*x)*\log((b*x + a)^n)^3 + (3*(2*d^3*g*h*\log(e)^2 + c*d^2*h^2*\log(e)^2)*A*B^2*b^3 + (2*d^3*g*h*\log(e)^3 + c*d^2*h^2*\log(e)^3)*B^3*b^3)*x^2 + 3*(B^3*b^3*c*d^2*g^2*\log(e) + A*B^2*b^3*c*d^2*g^2 + (B^3*b^3*d^3*h^2*\log(e) + A*B^2*b^3*d^3*h^2)*x^3 + ((2*d^3*g*h + c*d^2*h^2)*A*B^2*b^3 + (2*d^3*g*h*\log(e) + c*d^2*h^2*\log(e))*B^3*b^3)*x^2 + ((d^3*g^2 + 2*c*d^2*g*h)*A*B^2*b^3 + (d^3*g^2*\log(e) + 2*c*d^2*g*h*\log(e))*B^3*b^3)*x)*\log((b*x + a)^n)^2 + (3*(d^3*g^2*\log(e)^2 + 2*c*d^2*g*h*\log(e)^2)*A*B^2*b^3 + (d^3*g^2*\log(e)^3 + 2*c*d^2*g*h*\log(e)^3)*B^3*b^3)*x + 3*(B^3*b^3*c*d^2*g^2*\log(e)^2 + 2*A*B^2*b^3*c*d^2*g^2*\log(e) + (B^3*b^3*d^3*h^2*\log(e)^2 + 2*A*B^2*b^3*d^3*h^2*\log(e))*x^3 + (2*(2*d^3*g*h*\log(e) + c*d^2*h^2*\log(e))*A*B^2*b^3 + (2*d^3*g*h*\log(e)^2 + c*d^2*h^2*\log(e)^2)*B^3*b^3)*x^2 + (2*(d^3*g^2*\log(e) + 2*c*d^2*g*h*\log(e))*A*B^2*b^3 + (d^3*g^2*\log(e)^2 + 2*c*d^2*g*h*\log(e)^2)*B^3*b^3)*x)*\log((b*x + a)^n) - (3*B^3*b^3*c*d^2*g^2*\log(e)^2 + 6*A*B^2*b^3*c*d^2*g^2*\log(e) - 2*(3*c*d^2*g^2*n^2 - 3*c^2*d*g*h*n^2 + c^3*h^2*n^2)*B^3*b^3*\log(d*x + c) + 2*(3*a*b^2*d^3*g^2*n^2 - 3*a^2*b*d^3*g*h*n^2 + a^3*d^3*h^2*n^2)*B^3*\log(b*x + a) + (2*(h^2*n + 3*h^2*\log(e))*A*B^2*b^3*d^3 + (2*h^2*n*\log(e) + 3*h^2*\log(e)^2)*B^3*b^3*d^3)*x^3 + (6*(c*d^2*h^2*\log(e) + (g*h*n + 2*g*h*\log(e))*d^3)*A*B^2*b^3 + (a*b^2*d^3*h^2*n^2 - ((h^2*n^2 - 3*h^2*\log(e)^2)*c*d^2 - 6*(g*h*n*\log(e) + g*h*\log(e)^2)*d^3)*b^3)*B^3)*x^2 + 3*(B^3*b^3*d^3*h^2*x^3 + B^3*b^3*c*d^2*g^2 + (2*d^3*g*h + c*d^2*h^2)*B^3*b^3*x^2 + (d^3*g^2 + 2*c*d^2*g*h)*B^3*b^3*x)*\log((b*x + a)^n)^2 + (6*(2*c*d^2*g*h*\log(e) + (g^2*n + g^2*\log(e))*d^3)*A*B^2*b^3 + (6*a*b^2*d^3*g*h*n^2 - 2*a^2*b*d^3*h^2*n^2 + (2*c^2*d*h^2*n^2 - 6*(g*h*n^2 - g*h*\log(e)^2)*c*d^2 + 3*(2*g^2*n*\log(e) + g^2*\log(e)^2)*d^3)*b^3)*B^3)*x + 2*($

$$3B^3b^3cd^2g^2\log(e) + 3AB^2b^3cd^2g^2 + (3AB^2b^3d^3h^2 + h^{2n} + 3h^2\log(e))B^3b^3d^3x^3 + 3((2d^3gh + cd^2h^2)AB^2b^3 + (cd^2h^2\log(e) + (ghn + 2gh\log(e))d^3)B^3b^3)x^2 + 3((d^3g^2 + 2cd^2gh)AB^2b^3 + (2cd^2gh\log(e) + (g^2n + g^2\log(e))d^3)B^3b^3)x \cdot \log((bx + a)^n) \cdot \log((dx + c)^n) / (b^3d^3x + b^3cd^2), x)$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( A^3h^2x^2 + 2A^3ghx + A^3g^2 + (B^3h^2x^2 + 2B^3ghx + B^3g^2) \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right)^3 + 3(AB^2h^2x^2 + 2AB^2ghx + AB^2g^2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3,x, algorithm="fricas")

[Out] integral(A^3\*h^2\*x^2 + 2\*A^3\*g\*h\*x + A^3\*g^2 + (B^3\*h^2\*x^2 + 2\*B^3\*g\*h\*x + B^3\*g^2)\*log((b\*x + a)^n\*e/(d\*x + c)^n)^3 + 3\*(A\*B^2\*h^2\*x^2 + 2\*A\*B^2\*g\*h\*x + A\*B^2\*g^2)\*log((b\*x + a)^n\*e/(d\*x + c)^n)^2 + 3\*(A^2\*B\*h^2\*x^2 + 2\*A^2\*B\*g\*h\*x + A^2\*B\*g^2)\*log((b\*x + a)^n\*e/(d\*x + c)^n), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*\*2\*(A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*3,x)

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)^2\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3,x, algorithm="giac")

[Out] Timed out

### 3.310 $\int (g+hx) (A + B \log (e(a + bx)^n(c + dx)^{-n}))^3 dx$

**Optimal.** Leaf size=466

$$\frac{3B^2n^2(bc - ad)(-adh - bch + 2bdg)\text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)(B \log (e(a + bx)^n(c + dx)^{-n}) + A) - 3B^3n^3(bc - ad)(-adh - bch + 2bdg)}{b^2d^2}$$

[Out]  $(-3*B^2*(b*c - a*d)^2*h*n^2*\text{Log}[(b*c - a*d)/(b*(c + d*x))]*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]))/(b^2*d^2) - (3*B*(b*c - a*d)*h*n*(a + b*x)*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*b^2*d) + (3*B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*\text{Log}[(b*c - a*d)/(b*(c + d*x))]*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(2*b^2*d^2) - ((b*g - a*h)^2*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(2*b^2*h) + ((g + h*x)^2*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^3)/(2*h) - (3*B^3*(b*c - a*d)^2*h*n^3*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2) + (3*B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2) - (3*B^3*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^3*\text{PolyLog}[3, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2)$

**Rubi [B]** time = 2.11237, antiderivative size = 1030, normalized size of antiderivative = 2.21, number of steps used = 35, number of rules used = 13, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$ , Rules used = {6742, 2492, 72, 2514, 2486, 31, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

$$\frac{(g + hx)^2 A^3}{2h} - \frac{3B(bc - ad)hnxA^2}{2bd} - \frac{3B(bg - ah)^2 n \log(a + bx)A^2}{2b^2h} + \frac{3B(dg - ch)^2 n \log(c + dx)A^2}{2d^2h} + \frac{3B(g + hx)^2 \log}{2h}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(g + h*x)*(A + B*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^3, x]$

[Out]  $(-3*A^2*B*(b*c - a*d)*h*n*x)/(2*b*d) + (A^3*(g + h*x)^2)/(2*h) - (3*A^2*B*(b*g - a*h)^2*n*\text{Log}[a + b*x])/(2*b^2*h) + (3*A^2*B*(d*g - c*h)^2*n*\text{Log}[c + d*x])/(2*d^2*h) + (3*A*B^2*(b*c - a*d)^2*h*n^2*\text{Log}[c + d*x])/(b^2*d^2) - (3*A*B^2*(b*c - a*d)*h*n*(a + b*x)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(b^2*d) + (3*A^2*B*(g + h*x)^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(2*h) + (3*A*B^2*(b*g - a*h)^2*n*\text{Log}[-((b*c - a*d)/(d*(a + b*x))])*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(b^2*h) - (3*A*B^2*(d*g - c*h)^2*n*\text{Log}[(b*c - a*d)/(b*(c + d*x))]*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(d^2*h) - (3*B^3*(b*c - a*d)^2*h*n^2*\text{Log}[(b*c - a*d)/(b*(c + d*x))]*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])/(b^2*d^2) - (3*B^3*(b*c - a*d)*h*n*(a + b*x)*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^2/(2*b^2*d) + (3*A*B^2*(g + h*x)^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^2/(2*h) + (3*B^3*(b*g - a*h)^2*n*\text{Log}[-((b*c - a*d)/(d*(a + b*x))])*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^2/(2*b^2*h) - (3*B^3*(d*g - c*h)^2*n*\text{Log}[(b*c - a*d)/(b*(c + d*x))]*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^2/(2*d^2*h) + (B^3*(g + h*x)^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n])^3/(2*h) - (3*A*B^2*(d*g - c*h)^2*n^2*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(d^2*h) - (3*B^3*(b*c - a*d)^2*h*n^3*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))])/(b^2*d^2) - (3*A*B^2*(b*g - a*h)^2*n^2*\text{PolyLog}[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b^2*h) - (3*B^3*(b*g - a*h)^2*n^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]*\text{PolyLog}[2, 1 + (b*c - a*d)/(d*(a + b*x))])/(b^2*h) - (3*B^3*(d*g - c*h)^2*n^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]*\text{PolyLog}[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(d^2*h) - (3*B^3*(b*g - a*h)^2*n^3*\text{PolyLog}[3, 1 + (b*c - a*d)/(d*(a + b*x))])/(b^2*h) + (3*B^3*(d*g - c*h)^2*n^3*\text{PolyLog}[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(d^2*h)$

**Rule 6742**

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

#### Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s)/(a + b*x)*(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
0] && NeQ[m, -1]
```

#### Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

#### Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFX_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]
```

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^
q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c +
d*x)^q]^r]^s/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s
}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 31

```
Int[((a_.) + (b_.)*(x_))^(n_.), x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[(Log[-((b*c - a*d)/(
d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/h, x] + Dist[(p*r*s*(
b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p
*(c + d*x)^q]^r]^s)/(a + b*x)*(c + d*x), x], x] /; FreeQ[{a, b, c,
d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && EqQ
[b*g - a*h, 0] && IGtQ[s, 0]
```

#### Rule 2411

```
Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] := Dist[1/e, Subst[Int
[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e
*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f - d
*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

#### Rule 2343



```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Dist[1/n, Subst[Int[(a + b*Log[c*x])/x*(d + e*x^(r/n))], x],
x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

#### Rule 2333

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)/(x_))^(q_.)*(
x_)^(m_.), x_Symbol] :> Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{
a, b, c, d, e, m, n, p}, x] && EqQ[m, q] && IntegerQ[q]
```

#### Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

#### Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))
^(q_.))^(r_.)]^(s_.)*(u_), x_Symbol] :> With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

#### Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]
```

#### Rubi steps

$$\begin{aligned}
\int (g + hx) (A + B \log(e(a + bx)^n(c + dx)^{-n}))^3 dx &= \int (A^3(g + hx) + 3A^2B(g + hx) \log(e(a + bx)^n(c + dx)^{-n}) + 3AB^2(g \\
&= \frac{A^3(g + hx)^2}{2h} + (3A^2B) \int (g + hx) \log(e(a + bx)^n(c + dx)^{-n}) dx + (3 \\
&= \frac{A^3(g + hx)^2}{2h} + \frac{3A^2B(g + hx)^2 \log(e(a + bx)^n(c + dx)^{-n})}{2h} + \frac{3AB^2(g \\
&= \frac{A^3(g + hx)^2}{2h} + \frac{3A^2B(g + hx)^2 \log(e(a + bx)^n(c + dx)^{-n})}{2h} + \frac{3AB^2(g \\
&= -\frac{3A^2B(bc - ad)hnx}{2bd} + \frac{A^3(g + hx)^2}{2h} - \frac{3A^2B(bg - ah)^2n \log(a + bx)}{2b^2h} \\
&= -\frac{3A^2B(bc - ad)hnx}{2bd} + \frac{A^3(g + hx)^2}{2h} - \frac{3A^2B(bg - ah)^2n \log(a + bx)}{2b^2h} \\
&= -\frac{3A^2B(bc - ad)hnx}{2bd} + \frac{A^3(g + hx)^2}{2h} - \frac{3A^2B(bg - ah)^2n \log(a + bx)}{2b^2h} \\
&= -\frac{3A^2B(bc - ad)hnx}{2bd} + \frac{A^3(g + hx)^2}{2h} - \frac{3A^2B(bg - ah)^2n \log(a + bx)}{2b^2h} \\
&= -\frac{3A^2B(bc - ad)hnx}{2bd} + \frac{A^3(g + hx)^2}{2h} - \frac{3A^2B(bg - ah)^2n \log(a + bx)}{2b^2h} \\
&= -\frac{3A^2B(bc - ad)hnx}{2bd} + \frac{A^3(g + hx)^2}{2h} - \frac{3A^2B(bg - ah)^2n \log(a + bx)}{2b^2h} \\
&= -\frac{3A^2B(bc - ad)hnx}{2bd} + \frac{A^3(g + hx)^2}{2h} - \frac{3A^2B(bg - ah)^2n \log(a + bx)}{2b^2h} \\
&= -\frac{3A^2B(bc - ad)hnx}{2bd} + \frac{A^3(g + hx)^2}{2h} - \frac{3A^2B(bg - ah)^2n \log(a + bx)}{2b^2h} \\
&= -\frac{3A^2B(bc - ad)hnx}{2bd} + \frac{A^3(g + hx)^2}{2h} - \frac{3A^2B(bg - ah)^2n \log(a + bx)}{2b^2h} \\
&= -\frac{3A^2B(bc - ad)hnx}{2bd} + \frac{A^3(g + hx)^2}{2h} - \frac{3A^2B(bg - ah)^2n \log(a + bx)}{2b^2h}
\end{aligned}$$

**Mathematica [F]** time = 3.0359, size = 0, normalized size = 0.

$$\int (g + hx) (A + B \log(e(a + bx)^n(c + dx)^{-n}))^3 dx$$

Verification is Not applicable to the result.

[In] Integrate[(g + h\*x)\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n)]^3, x]

[Out] Integrate[(g + h\*x)\*(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x]^n)]^3, x]

**Maple [F]** time = 5.28, size = 0, normalized size = 0.

$$\int (hx + g) \left( A + B \ln \left( \frac{e(bx + a)^n}{(dx + c)^n} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((h\*x+g)\*(A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3, x)

[Out]  $\int ((h*x+g)*(A+B*\ln(e*(b*x+a)^n/((d*x+c)^n)))^3, x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="maxima")`

[Out] 
$$\begin{aligned} & \frac{3}{2}A^2Bhx^2 \log\left(\frac{b^2x^2 + a^2}{(dx+c)^2}\right) + \frac{1}{2}A^3hx^2 + 3A^2B^2g^2x \log\left(\frac{b^2x^2 + a^2}{(dx+c)^2}\right) + A^3g^2x + 3(a^2e^{2n} \log(bx+a)/b - c^2e^{2n} \log(dx+c)/d)A^2B^2g/e \\ & - \frac{3}{2}(a^2e^{2n} \log(bx+a)/b^2 - c^2e^{2n} \log(dx+c)/d^2 + (b^2c^2e^{2n} - a^2d^2e^{2n})x/(bd))A^2B^2h/e - \frac{1}{2}((B^3b^2d^2hx^2 + 2B^3b^2d^2g^2x) \log((dx+c)^n)^3 + 3((2c^2d^2g^2n - c^2h^2n)B^3b^2 \log(dx+c) - (2ab^2d^2g^2n - a^2d^2h^2n)B^3 \log(bx+a) - (B^3b^2d^2h \log(e) + AB^2b^2d^2h)x^2 - (2AB^2b^2d^2g + (abd^2h^2n - (cd^2h^2n - 2d^2g^2 \log(e))b^2)B^3)x - (B^3b^2d^2h^2x^2 + 2B^3b^2d^2g^2x) \log((bx+a)^n) \log((dx+c)^n)^2)/(b^2d^2) - \int (-(B^3b^2c^2d^2g^2 \log(e)^3 + 3AB^2b^2c^2d^2g^2 \log(e)^2 + (B^3b^2d^2h^2x^2 + B^3b^2c^2d^2g + (d^2g + cd^2h)B^3b^2x) \log((bx+a)^n)^3 + (B^3b^2d^2h \log(e)^3 + 3AB^2b^2d^2h \log(e)^2)x^2 + 3(B^3b^2c^2d^2g \log(e) + AB^2b^2c^2d^2g + (B^3b^2d^2h \log(e) + AB^2b^2d^2h)x^2 + ((d^2g + cd^2h)AB^2b^2 + (d^2g \log(e) + cd^2h \log(e))B^3b^2)x) \log((bx+a)^n)^2 + (3(d^2g \log(e)^2 + cd^2h \log(e)^2)AB^2b^2 + (d^2g \log(e)^3 + cd^2h \log(e)^3)B^3b^2)x + 3(B^3b^2c^2d^2g \log(e)^2 + 2AB^2b^2c^2d^2g \log(e) + (B^3b^2d^2h \log(e)^2 + 2AB^2b^2d^2h \log(e))x^2 + (2(d^2g \log(e) + cd^2h \log(e))AB^2b^2 + (d^2g \log(e)^2 + cd^2h \log(e)^2)B^3b^2)x) \log((bx+a)^n) - 3(B^3b^2c^2d^2g \log(e)^2 + 2AB^2b^2c^2d^2g \log(e) - (2c^2d^2g^2n^2 - c^2h^2n^2)B^3b^2 \log(dx+c) + (2abd^2g^2n^2 - a^2d^2h^2n^2)B^3 \log(bx+a) + ((hn + 2h \log(e))AB^2b^2d^2 + (hn \log(e) + h \log(e)^2)B^3b^2d^2)x^2 + (B^3b^2d^2h^2x^2 + B^3b^2c^2d^2g + (d^2g + cd^2h)B^3b^2x) \log((bx+a)^n)^2 + (2(cd^2h \log(e) + (gn + g \log(e))d^2)AB^2b^2 + (abd^2h^2n^2 - ((hn^2 - h \log(e)^2)cd - (2g^2n \log(e) + g \log(e)^2)d^2)b^2)B^3)x + (2B^3b^2c^2d^2g \log(e) + 2AB^2b^2c^2d^2g + ((hn + 2h \log(e))B^3b^2d^2 + 2AB^2b^2d^2h)x^2 + 2((d^2g + cd^2h)AB^2b^2 + (cd^2h \log(e) + (gn + g \log(e))d^2)B^3b^2)x) \log((bx+a)^n) \log((dx+c)^n))/(b^2d^2x + b^2cd), x) \end{aligned}$$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( A^3hx + A^3g + (B^3hx + B^3g) \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + 3(AB^2hx + AB^2g) \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 3(A^2Bhx + A^2Bg) \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((h*x+g)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="fricas")`

[Out] 
$$\int (A^3hx + A^3g + (B^3hx + B^3g) \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + 3(AB^2hx + AB^2g) \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 3(A^2Bhx + A^2Bg) \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3, x)$$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int (hx + g) \left( B \log \left( \frac{(bx + a)^n e}{(dx + c)^n} \right) + A \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((h\*x+g)\*(A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3,x, algorithm="giac")

[Out] integrate((h\*x + g)\*(B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^3, x)

### 3.311 $\int (A + B \log(e(a + bx)^n(c + dx)^{-n}))^3 dx$

**Optimal.** Leaf size=203

$$\frac{6B^2n^2(bc - ad)\text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)(B \log(e(a + bx)^n(c + dx)^{-n}) + A)}{bd} - \frac{6B^3n^3(bc - ad)\text{PolyLog}\left(3, \frac{d(a+bx)}{b(c+dx)}\right)}{bd} + \frac{3Bn^3}{bd}$$

```
[Out] (3*B*(b*c - a*d)*n*Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/(b*d) + ((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/b + (6*B^2*(b*c - a*d)*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d) - (6*B^3*(b*c - a*d)*n^3*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(b*d)
```

**Rubi [B]** time = 0.590136, antiderivative size = 408, normalized size of antiderivative = 2.01, number of steps used = 14, number of rules used = 10, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$ , Rules used = {6742, 2486, 31, 2488, 2411, 2343, 2333, 2315, 2506, 6610}

$$\frac{6AB^2n^2(bc - ad)\text{PolyLog}\left(2, \frac{d(a+bx)}{b(c+dx)}\right)}{bd} + \frac{6B^3n^2(bc - ad)\text{PolyLog}\left(2, 1 - \frac{bc-ad}{b(c+dx)}\right)\log(e(a + bx)^n(c + dx)^{-n})}{bd} - \frac{6B^3n^3}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3, x]
```

```
[Out] A^3*x - (3*A^2*B*(b*c - a*d)*n*Log[c + d*x])/(b*d) + (3*A^2*B*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/b + (6*A*B^2*(b*c - a*d)*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(b*d) + (3*A*B^2*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/b + (3*B^3*(b*c - a*d)*n*Log[(b*c - a*d)/(b*(c + d*x))]*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/(b*d) + (B^3*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3)/b + (6*A*B^2*(b*c - a*d)*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/(b*d) + (6*B^3*(b*c - a*d)*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*d) - (6*B^3*(b*c - a*d)*n^3*PolyLog[3, 1 - (b*c - a*d)/(b*(c + d*x))])/(b*d)
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 2486

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.), x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/b, x] + Dist[(q*r*s*(b*c - a*d))/b, Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s, 0]
```

#### Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

#### Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.)/((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[(Log[-((b*c - a*d)/(
```

$d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/h, x] + Dist[(p*r*s*(b*c - a*d))/h, Int[(Log[-((b*c - a*d)/(d*(a + b*x)))]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] \&\& NeQ[b*c - a*d, 0] \&\& EqQ[p + q, 0] \&\& EqQ[b*g - a*h, 0] \&\& IGtQ[s, 0]$

#### Rule 2411

$Int[((a_.) + Log[(c_.)*((d_.) + (e_.)*(x_.))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_.)*(x_.))^(q_.)*((h_.) + (i_.)*(x_.))^(r_.), x\_Symbol] := Dist[1/e, Subst[Int[((g*x)/e)^q*((e*h - d*i)/e + (i*x)/e)^r*(a + b*Log[c*x^n])^p, x], x, d + e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] \&\& EqQ[e*f - d*g, 0] \&\& (IGtQ[p, 0] || IGtQ[r, 0]) \&\& IntegerQ[2*r]$

#### Rule 2343

$Int[((a_.) + Log[(c_.)*(x_.)^(n_.)]*(b_.))/((x_.)*((d_.) + (e_.)*(x_.)^(r_.))), x\_Symbol] := Dist[1/n, Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] \&\& IntegerQ[r/n]$

#### Rule 2333

$Int[((a_.) + Log[(c_.)*(x_.)^(n_.)]*(b_.))^(p_.)*((d_.) + (e_.)/(x_.))^(q_.)*(x_.)^(m_.), x\_Symbol] := Int[(e + d*x)^q*(a + b*Log[c*x^n])^p, x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] \&\& EqQ[m, q] \&\& IntegerQ[q]$

#### Rule 2315

$Int[Log[(c_.)*(x_.)]/((d_.) + (e_.)*(x_.)), x\_Symbol] := -Simp[PolyLog[2, 1 - c*x]/e, x] /; FreeQ[{c, d, e}, x] \&\& EqQ[e + c*d, 0]$

#### Rule 2506

$Int[Log[v]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_.))^(p_.)*((c_.) + (d_.)*(x_.))^(q_.))^(r_.)]^s*(u_.), x\_Symbol] := With[{g = Simplify[((v - 1)*(c + d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s)/(b*c - a*d), x] + Dist[h*p*r*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q)^r]^s - 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] \&\& NeQ[b*c - a*d, 0] \&\& IGtQ[s, 0] \&\& EqQ[p + q, 0]$

#### Rule 6610

$Int[(u_.)*PolyLog[n_, v_], x\_Symbol] := With[{w = DerivativeDivides[v, u*v, x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]$

#### Rubi steps

$$\begin{aligned} \int (A + B \log(e(a + bx)^n(c + dx)^{-n}))^3 dx &= \int (A^3 + 3A^2B \log(e(a + bx)^n(c + dx)^{-n}) + 3AB^2 \log^2(e(a + bx)^n(c + dx)^{-n}) + B^3 \log^3(e(a + bx)^n(c + dx)^{-n})) dx \\ &= A^3x + (3A^2B) \int \log(e(a + bx)^n(c + dx)^{-n}) dx + (3AB^2) \int \log^2(e(a + bx)^n(c + dx)^{-n}) dx + B^3 \int \log^3(e(a + bx)^n(c + dx)^{-n}) dx \\ &= A^3x + \frac{3A^2B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{3AB^2(a + bx) \log^2(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{B^3(a + bx) \log^3(e(a + bx)^n(c + dx)^{-n})}{b} \\ &= A^3x - \frac{3A^2B(bc - ad)n \log(c + dx)}{bd} + \frac{3A^2B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{3AB^2(a + bx) \log^2(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{B^3(a + bx) \log^3(e(a + bx)^n(c + dx)^{-n})}{b} \\ &= A^3x - \frac{3A^2B(bc - ad)n \log(c + dx)}{bd} + \frac{3A^2B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{3AB^2(a + bx) \log^2(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{B^3(a + bx) \log^3(e(a + bx)^n(c + dx)^{-n})}{b} \\ &= A^3x - \frac{3A^2B(bc - ad)n \log(c + dx)}{bd} + \frac{3A^2B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{3AB^2(a + bx) \log^2(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{B^3(a + bx) \log^3(e(a + bx)^n(c + dx)^{-n})}{b} \\ &= A^3x - \frac{3A^2B(bc - ad)n \log(c + dx)}{bd} + \frac{3A^2B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{3AB^2(a + bx) \log^2(e(a + bx)^n(c + dx)^{-n})}{b} + \frac{B^3(a + bx) \log^3(e(a + bx)^n(c + dx)^{-n})}{b} \end{aligned}$$

**Mathematica [A]** time = 0.279521, size = 378, normalized size = 1.86

$$3AB^2n(bc - ad) \left( 2n \text{PolyLog} \left( 2, \frac{b(c+dx)}{bc-ad} \right) - \log \left( \frac{bc-ad}{bc+bdx} \right) \left( -2 \log(e(a + bx)^n(c + dx)^{-n}) + 2n \log \left( \frac{d(a+bx)}{ad-bc} \right) + n \log \left( \frac{bc-a}{bc+bdx} \right) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3, x]
```

```
[Out] (A^3*b*d*x - 3*A^2*B*(b*c - a*d)*n*Log[c + d*x] + 3*A^2*B*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 3*A*B^2*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2 + B^3*d*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3 + 3*A*B^2*(b*c - a*d)*n*(-(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*n*Log[(d*(a + b*x))/(b*c - a*d)] - 2*Log[(e*(a + b*x)^n)/(c + d*x)^n] + n*Log[(b*c - a*d)/(b*c + b*d*x]))) + 2*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] + 3*B^3*(b*c - a*d)*n*(Log[(e*(a + b*x)^n)/(c + d*x)^n]^2*Log[(b*c - a*d)/(b*c + b*d*x)] + 2*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(b*d)
```

**Maple [F]** time = 2.737, size = 0, normalized size = 0.

$$\int \left( A + B \ln \left( \frac{e(bx + a)^n}{(dx + c)^n} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3, x)
```

```
[Out] int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3, x)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$3 A^2 B x \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^3 x + \frac{3\left(\frac{aen \log(bx+a)}{b} - \frac{cen \log(dx+c)}{d}\right) A^2 B}{e} - \frac{B^3 b dx \log((dx+c)^n)^3 - 3(B^3 a d n \log(bx+a) - B^3 a d n \log(dx+c))}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3,x, algorithm="maxima")

[Out]  $3A^2Bx \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^3x + 3\left(\frac{aen \log(bx+a)}{b} - \frac{cen \log(dx+c)}{d}\right) \frac{A^2B}{e} - \frac{B^3 b dx \log((dx+c)^n)^3 - 3(B^3 a d n \log(bx+a) - B^3 a d n \log(dx+c))}{e}$

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(B^3 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 + 3AB^2 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 3A^2B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3,x, algorithm="fricas")

[Out]  $\text{integral}(B^3 \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3 + 3A^2B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2 + 3A^2B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A^3, x)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*3,x)

[Out] Timed out

---

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \left( B \log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A \right)^3 dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algorithm="giac")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3, x)
```

$$3.312 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^3}{g+hx} dx$$

**Optimal.** Leaf size=425

$$\frac{6B^2n^2 \text{PolyLog}\left(3, \frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right) (B \log(e(a+bx)^n(c+dx)^{-n}) + A)}{h} + \frac{6B^2n^2 \text{PolyLog}\left(3, \frac{d(a+bx)}{b(c+dx)}\right) (B \log(e(a+bx)^n(c+dx)^{-n}) + A)}{h}$$

[Out]  $-\left(\frac{\log\left(\frac{b^2c - a^2d}{b(c+dx)}\right) \left(A + B \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)\right)^3}{h} + \frac{\left(A + B \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)\right)^3 \log\left(1 - \frac{(dg - ch)(a+bx)}{(bg - ah)(c+dx)}\right)}{h} - \frac{3Bn \left(A + B \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)\right)^2 \text{PolyLog}\left[2, \frac{d(a+bx)}{b(c+dx)}\right]}{h} + \frac{3Bn \left(A + B \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)\right)^2 \text{PolyLog}\left[2, \frac{(dg - ch)(a+bx)}{(bg - ah)(c+dx)}\right]}{h} + \frac{6B^2n^2 \left(A + B \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)\right) \text{PolyLog}\left[3, \frac{d(a+bx)}{b(c+dx)}\right]}{h} - \frac{6B^2n^2 \left(A + B \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)\right) \text{PolyLog}\left[3, \frac{(dg - ch)(a+bx)}{(bg - ah)(c+dx)}\right]}{h} - \frac{6B^3n^3 \text{PolyLog}\left[4, \frac{d(a+bx)}{b(c+dx)}\right]}{h} + \frac{6B^3n^3 \text{PolyLog}\left[4, \frac{(dg - ch)(a+bx)}{(bg - ah)(c+dx)}\right]}{h}$

**Rubi [B]** time = 1.64183, antiderivative size = 921, normalized size of antiderivative = 2.17, number of steps used = 25, number of rules used = 11, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {6742, 2494, 2394, 2393, 2391, 2489, 2488, 2506, 6610, 2503, 2508}

$$\frac{\log(g+hx)A^3}{h} - \frac{3Bn \log\left(-\frac{h(a+bx)}{bg-ah}\right) \log(g+hx)A^2}{h} + \frac{3Bn \log\left(-\frac{h(c+dx)}{dg-ch}\right) \log(g+hx)A^2}{h} + \frac{3B \log(e(a+bx)^n(c+dx)^{-n})}{h}$$

Antiderivative was successfully verified.

[In] Int[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)]^3)/(g + h\*x), x]

[Out]  $\frac{-3AB^2 \log\left(\frac{b^2c - a^2d}{b(c+dx)}\right) \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^2}{h} - \frac{B^3 \log\left(\frac{b^2c - a^2d}{b(c+dx)}\right) \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^3}{h} + \frac{A^3 \log(g+hx)}{h} - \frac{3A^2Bn \log\left(-\frac{h(a+bx)}{bg-ah}\right) \log(g+hx)}{h} + \frac{3A^2Bn \log\left(-\frac{h(c+dx)}{dg-ch}\right) \log(g+hx)}{h} + \frac{3A^2B \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right) \log(g+hx)}{h} + \frac{3AB^2 \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^2 \log\left(\frac{(b^2c - a^2d)(g+hx)}{(bg - ah)(c+dx)}\right)}{h} + \frac{B^3 \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^3 \log\left(\frac{(b^2c - a^2d)(g+hx)}{(bg - ah)(c+dx)}\right)}{h} - \frac{3A^2Bn \text{PolyLog}\left[2, \frac{b(g+hx)}{bg-ah}\right]}{h} + \frac{3A^2Bn \text{PolyLog}\left[2, \frac{d(g+hx)}{dg-ch}\right]}{h} - \frac{6AB^2n \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right) \text{PolyLog}\left[2, 1 - \frac{b^2c - a^2d}{b(c+dx)}\right]}{h} - \frac{3B^3n \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^2 \text{PolyLog}\left[2, 1 - \frac{b^2c - a^2d}{b(c+dx)}\right]}{h} + \frac{6AB^2n \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right) \text{PolyLog}\left[2, 1 - \frac{(b^2c - a^2d)(g+hx)}{(bg - ah)(c+dx)}\right]}{h} + \frac{3B^3n \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^2 \text{PolyLog}\left[2, 1 - \frac{(b^2c - a^2d)(g+hx)}{(bg - ah)(c+dx)}\right]}{h} + \frac{6AB^2n^2 \text{PolyLog}\left[3, 1 - \frac{b^2c - a^2d}{b(c+dx)}\right]}{h} + \frac{6B^3n^2 \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right) \text{PolyLog}\left[3, 1 - \frac{b^2c - a^2d}{b(c+dx)}\right]}{h} - \frac{6AB^2n^2 \text{PolyLog}\left[3, 1 - \frac{(b^2c - a^2d)(g+hx)}{(bg - ah)(c+dx)}\right]}{h} - \frac{6B^3n^2 \log\left(\frac{e(a+bx)^n}{(c+dx)^n}\right) \text{PolyLog}\left[3, 1 - \frac{(b^2c - a^2d)(g+hx)}{(bg - ah)(c+dx)}\right]}{h} - \frac{6B^3n^3 \text{PolyLog}\left[4, 1 - \frac{b^2c - a^2d}{b(c+dx)}\right]}{h} + \frac{6B^3n^3 \text{PolyLog}\left[4, 1 - \frac{(b^2c - a^2d)(g+hx)}{(bg - ah)(c+dx)}\right]}{h}$

**Rule 6742**

Int[u\_, x\_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]

Rule 2494

Int[Log[(e\_.)\*((f\_.)\*(a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[g + h\*x]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r])/h, x] + (-Dist[(b\*p\*r)/h, Int[Log[g + h\*x]/(a + b\*x), x], x] - Dist[(d\*q\*r)/h, Int[Log[g + h\*x]/(c + d\*x), x], x]) /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b\*c - a\*d, 0]

Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))^(n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e\*n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] :> Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

Rule 2391

Int[Log[(c\_.)\*((d\_.) + (e\_.)\*(x\_)^(n\_.))]/(x\_), x\_Symbol] :> -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

Rule 2489

Int[Log[(e\_.)\*((f\_.)\*(a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] :> Dist[d/h, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s/(c + d\*x), x], x] - Dist[(d\*g - c\*h)/h, Int[Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s/((c + d\*x)\*(g + h\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && NeQ[b\*g - a\*h, 0] && NeQ[d\*g - c\*h, 0] && IGtQ[s, 1]

Rule 2488

Int[Log[(e\_.)\*((f\_.)\*(a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_)/((g\_.) + (h\_.)\*(x\_)), x\_Symbol] :> -Simp[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/h, x] + Dist[(p\*r\*s\*(b\*c - a\*d))/h, Int[(Log[-((b\*c - a\*d)/(d\*(a + b\*x)))]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && EqQ[p + q, 0] && EqQ[b\*g - a\*h, 0] && IGtQ[s, 0]

Rule 2506

Int[Log[v\_]\*Log[(e\_.)\*((f\_.)\*(a\_.) + (b\_.)\*(x\_))^(p\_.)\*((c\_.) + (d\_.)\*(x\_))^(q\_.))^(r\_.)]^(s\_)\*(u\_), x\_Symbol] :> With[{g = Simplify[(v - 1)\*(c + d\*x)/(a + b\*x)], h = Simplify[u\*(a + b\*x)\*(c + d\*x)]}, -Simp[(h\*PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(b\*c - a\*d), x] + Dist[h\*p\*r\*s, Int[(PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/((a + b\*x)\*(c + d\*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] :> With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] := With[{g = Coeff[Simplify[1/(u*(a + b*x))],
x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x)^p*(c + d*x)^q]^r]^s*Log[-(((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
))))]/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)*Log[-(((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x))))]/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
x]
```

Rule 2508

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_)*PolyLog[n_, v_], x_Symbol] := With[{g = Simplify[(v*(c +
d*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, Simp[(h*PolyLog[n
+ 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(b*c - a*d), x] - Dist[h*p*
r*s, Int[(PolyLog[n + 1, v]*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s - 1)/(
(a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e,
f, n, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^3}{g + hx} dx &= \int \left( \frac{A^3}{g + hx} + \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{g + hx} + \frac{3AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{g + hx} \right) dx \\
&= \frac{A^3 \log(g + hx)}{h} + (3A^2B) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{g + hx} dx + (3AB^2) \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{g + hx} dx \\
&= \frac{A^3 \log(g + hx)}{h} + \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n}) \log(g + hx)}{h} + \frac{(3AB^2d) \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{g + hx} dx}{h} \\
&= -\frac{3AB^2 \log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2(e(a + bx)^n(c + dx)^{-n})}{h} - \frac{B^3 \log\left(\frac{bc-ad}{b(c+dx)}\right) \log^3(e(a + bx)^n(c + dx)^{-n})}{h} \\
&= -\frac{3AB^2 \log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2(e(a + bx)^n(c + dx)^{-n})}{h} - \frac{B^3 \log\left(\frac{bc-ad}{b(c+dx)}\right) \log^3(e(a + bx)^n(c + dx)^{-n})}{h} \\
&= -\frac{3AB^2 \log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2(e(a + bx)^n(c + dx)^{-n})}{h} - \frac{B^3 \log\left(\frac{bc-ad}{b(c+dx)}\right) \log^3(e(a + bx)^n(c + dx)^{-n})}{h} \\
&= -\frac{3AB^2 \log\left(\frac{bc-ad}{b(c+dx)}\right) \log^2(e(a + bx)^n(c + dx)^{-n})}{h} - \frac{B^3 \log\left(\frac{bc-ad}{b(c+dx)}\right) \log^3(e(a + bx)^n(c + dx)^{-n})}{h}
\end{aligned}$$

**Mathematica [F]** time = 1.45157, size = 0, normalized size = 0.

$$\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^3}{g + hx} dx$$

Verification is Not applicable to the result.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^3/(g + h\*x), x]

[Out] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^3/(g + h\*x), x]

**Maple [F]** time = 3.127, size = 0, normalized size = 0.

$$\int \frac{1}{hx + g} \left( A + B \ln \left( \frac{e(bx + a)^n}{(dx + c)^n} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(h\*x+g), x)

[Out] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(h\*x+g), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{A^3 \log(hx + g)}{h} - \int \frac{B^3 \log((bx + a)^n)^3 - B^3 \log((dx + c)^n)^3 + B^3 \log(e)^3 + 3AB^2 \log(e)^2 + 3A^2B \log(e) + 3(B^3 \log(e) + A^3 \log(hx + g)) \log((bx + a)^n) \log((dx + c)^n)}{hx + g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(h\*x+g), x, algorithm="maxima")

[Out] A^3\*log(h\*x + g)/h - integrate(-(B^3\*log((b\*x + a)^n)^3 - B^3\*log((d\*x + c)^n)^3 + B^3\*log(e)^3 + 3\*A\*B^2\*log(e)^2 + 3\*A^2\*B\*log(e) + 3\*(B^3\*log(e) + A\*B^2)\*log((b\*x + a)^n)^2 + 3\*(B^3\*log((b\*x + a)^n) + B^3\*log(e) + A\*B^2)\*log((d\*x + c)^n)^2 + 3\*(B^3\*log(e)^2 + 2\*A\*B^2\*log(e) + A^2\*B)\*log((b\*x + a)^n) - 3\*(B^3\*log((b\*x + a)^n)^2 + B^3\*log(e)^2 + 2\*A\*B^2\*log(e) + A^2\*B + 2\*(B^3\*log(e) + A\*B^2)\*log((b\*x + a)^n))\*log((d\*x + c)^n)/(h\*x + g), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^3 \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right)^3 + 3AB^2 \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right)^2 + 3A^2B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A^3}{hx + g}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(h\*x+g), x, algorithm="fricas")

[Out] integral((B^3\*log((b\*x + a)^n\*e/(d\*x + c)^n)^3 + 3\*A\*B^2\*log((b\*x + a)^n\*e/(d\*x + c)^n)^2 + 3\*A^2\*B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A^3)/(h\*x + g), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*ln(e\*(b\*x+a)\*\*n/((d\*x+c)\*\*n)))\*\*3/(h\*x+g),x)

[Out] Timed out

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log\left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)^3}{hx + g} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(h\*x+g),x, algorithm="giac")

[Out] integrate((B\*log((b\*x + a)^n\*e/(d\*x + c)^n) + A)^3/(h\*x + g), x)

$$3.313 \quad \int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^2} dx$$

**Optimal.** Leaf size=302

$$\frac{6B^2n^2(bc-ad)\text{PolyLog}\left(2, \frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)(B \log(e(a+bx)^n(c+dx)^{-n})+A)}{(bg-ah)(dg-ch)} - \frac{6B^3n^3(bc-ad)\text{PolyLog}\left(3, \frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)}{(bg-ah)(dg-ch)}$$

```
[Out] ((a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3)/((b*g - a*h)*(g + h*x)) + (3*B*(b*c - a*d)*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*Log[1 - ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h)) + (6*B^2*(b*c - a*d)*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h)) - (6*B^3*(b*c - a*d)*n^3*PolyLog[3, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h))
```

**Rubi [B]** time = 0.807412, antiderivative size = 650, normalized size of antiderivative = 2.15, number of steps used = 14, number of rules used = 9, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$ , Rules used = {6742, 2490, 36, 31, 2503, 2502, 2315, 2506, 6610}

$$\frac{6AB^2n^2(bc-ad)\text{PolyLog}\left(2, 1 - \frac{(g+hx)(bc-ad)}{(c+dx)(bg-ah)}\right)}{(bg-ah)(dg-ch)} + \frac{6B^3n^2(bc-ad) \log(e(a+bx)^n(c+dx)^{-n}) \text{PolyLog}\left(2, 1 - \frac{(g+hx)(bc-ad)}{(c+dx)(bg-ah)}\right)}{(bg-ah)(dg-ch)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^2, x]
```

```
[Out] -(A^3/(h*(g + h*x))) - (3*A^2*B*(b*c - a*d)*n*Log[c + d*x])/((b*g - a*h)*(d*g - c*h)) + (3*A^2*B*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((b*g - a*h)*(g + h*x)) + (3*A*B^2*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/((b*g - a*h)*(g + h*x)) + (B^3*(a + b*x)*Log[(e*(a + b*x)^n)/(c + d*x)^n]^3)/((b*g - a*h)*(g + h*x)) + (3*A^2*B*(b*c - a*d)*n*Log[g + h*x])/((b*g - a*h)*(d*g - c*h)) + (6*A*B^2*(b*c - a*d)*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]*Log[((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h)) + (3*B^3*(b*c - a*d)*n*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2*Log[((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h)) + (6*A*B^2*(b*c - a*d)*n^2*PolyLog[2, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h)) + (6*B^3*(b*c - a*d)*n^2*Log[(e*(a + b*x)^n)/(c + d*x)^n]*PolyLog[2, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h)) - (6*B^3*(b*c - a*d)*n^3*PolyLog[3, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)*(d*g - c*h))
```

#### Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

#### Rule 2490

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))^(r_.)]^(s_.)/(g_.) + (h_.)*(x_)^2, x_Symbol] := Simp[((a + b*x)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/((b*g - a*h)*(g + h*x)), x] - Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^(s - 1)/(c + d*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && NeQ[b*g - a*h, 0] && IGtQ[s, 0]
```

]

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c
- a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2503

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(u_), x_Symbol] := With[{g = Coeff[Simplify[1/(u*(a + b*x))],
x, 0], h = Coeff[Simplify[1/(u*(a + b*x))], x, 1]}, -Simp[(Log[e*(f*(a + b
*x))^p*(c + d*x)^q]^r]^s*Log[-((b*c - a*d)*(g + h*x))/((d*g - c*h)*(a + b*x
)))]/(b*g - a*h), x] + Dist[(p*r*s*(b*c - a*d))/(b*g - a*h), Int[(Log[e*(f
*(a + b*x))^p*(c + d*x)^q]^r]^(s - 1)*Log[-((b*c - a*d)*(g + h*x))/((d*g -
c*h)*(a + b*x)))]/((a + b*x)*(c + d*x)), x], x] /; NeQ[b*g - a*h, 0] && Ne
Q[d*g - c*h, 0] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b*c - a
*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u*(a + b*x))],
x]
```

Rule 2502

```
Int[Log[((e_.)*((c_.) + (d_.)*(x_)))/((a_.) + (b_.)*(x_))]*(u_), x_Symbol]
:= With[{g = Coeff[Simplify[1/(u*(a + b*x))], x, 0], h = Coeff[Simplify[1/(
u*(a + b*x))], x, 1]}, -Dist[(b - d*e)/(h*(b*c - a*d)), Subst[Int[Log[e*x]/
(1 - e*x), x], x, (c + d*x)/(a + b*x)], x] /; EqQ[g*(b - d*e) - h*(a - c*e)
, 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LinearQ[Simplify
[1/(u*(a + b*x))], x]
```

Rule 2315

```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] := -Simp[PolyLog[2, 1 -
c*x]/e, x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

Rule 2506

```
Int[Log[v_]*Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_
))^(q_.)]^(r_.)]^(s_.)*(u_), x_Symbol] := With[{g = Simplify[((v - 1)*(c + d
*x))/(a + b*x)], h = Simplify[u*(a + b*x)*(c + d*x)]}, -Simp[(h*PolyLog[2,
1 - v]*Log[e*(f*(a + b*x))^p*(c + d*x)^q]^r]^s/(b*c - a*d), x] + Dist[h*p*r
*s, Int[(PolyLog[2, 1 - v]*Log[e*(f*(a + b*x))^p*(c + d*x)^q]^r]^(s - 1))/((
a + b*x)*(c + d*x)), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f
, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]
```

Rule 6610

```
Int[(u_)*PolyLog[n_, v_], x_Symbol] := With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /; !FalseQ[w] /; FreeQ[n, x]
```

Rubi steps



$$\begin{aligned}
\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^3}{(g + hx)^2} dx &= \int \left( \frac{A^3}{(g + hx)^2} + \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^2} + \frac{3AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^2} \right) dx \\
&= -\frac{A^3}{h(g + hx)} + (3A^2B) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^2} dx + (3AB^2) \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^2} dx \\
&= -\frac{A^3}{h(g + hx)} + \frac{3A^2B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} + \frac{3AB^2(a + bx) \log^2(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} \\
&= -\frac{A^3}{h(g + hx)} + \frac{3A^2B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} + \frac{3AB^2(a + bx) \log^2(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} \\
&= -\frac{A^3}{h(g + hx)} - \frac{3A^2B(bc - ad)n \log(c + dx)}{(bg - ah)(dg - ch)} + \frac{3A^2B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)} \\
&= -\frac{A^3}{h(g + hx)} - \frac{3A^2B(bc - ad)n \log(c + dx)}{(bg - ah)(dg - ch)} + \frac{3A^2B(a + bx) \log(e(a + bx)^n(c + dx)^{-n})}{(bg - ah)(g + hx)}
\end{aligned}$$

**Mathematica [F]** time = 3.62669, size = 0, normalized size = 0.

$$\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^3}{(g + hx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^3/(g + h\*x)^2, x]

[Out] Integrate[(A + B\*Log[(e\*(a + b\*x)^n)/(c + d\*x)^n])^3/(g + h\*x)^2, x]

**Maple [F]** time = 2.753, size = 0, normalized size = 0.

$$\int \frac{1}{(hx + g)^2} \left( A + B \ln \left( \frac{e(bx + a)^n}{(dx + c)^n} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(h\*x+g)^2, x)

[Out] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(h\*x+g)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(h\*x+g)^2, x, algorithm="maxima")

```
[Out] B^3*log((d*x + c)^n)^3/(h^2*x + g*h) + 3*(b*e*n*log(b*x + a)/(b*g*h - a*h^2)
) - d*e*n*log(d*x + c)/(d*g*h - c*h^2) - (b*c*e*n - a*d*e*n)*log(h*x + g)/(
(d*g*h - c*h^2)*a - (d*g^2 - c*g*h)*b))*A^2*B/e - 3*A^2*B*log((b*x + a)^n*e
/(d*x + c)^n)/(h^2*x + g*h) - A^3/(h^2*x + g*h) + integrate((B^3*c*h*log(e)
^3 + 3*A*B^2*c*h*log(e)^2 + (B^3*d*h*x + B^3*c*h)*log((b*x + a)^n)^3 + 3*(B
^3*c*h*log(e) + A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x + a)^n
)^2 + 3*(A*B^2*c*h - (d*g*n - c*h*log(e))*B^3 - ((h*n - h*log(e))*B^3*d - A
*B^2*d*h)*x + (B^3*d*h*x + B^3*c*h)*log((b*x + a)^n))*log((d*x + c)^n)^2 +
(B^3*d*h*log(e)^3 + 3*A*B^2*d*h*log(e)^2)*x + 3*(B^3*c*h*log(e)^2 + 2*A*B^2
*c*h*log(e) + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x)*log((b*x + a)^n) -
3*(B^3*c*h*log(e)^2 + 2*A*B^2*c*h*log(e) + (B^3*d*h*x + B^3*c*h)*log((b*x
+ a)^n)^2 + (B^3*d*h*log(e)^2 + 2*A*B^2*d*h*log(e))*x + 2*(B^3*c*h*log(e) +
A*B^2*c*h + (B^3*d*h*log(e) + A*B^2*d*h)*x)*log((b*x + a)^n))*log((d*x + c
)^n))/(d*h^3*x^3 + c*g^2*h + (2*d*g*h^2 + c*h^3)*x^2 + (d*g^2*h + 2*c*g*h^2
)*x), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^3 \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right)^3 + 3AB^2 \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right)^2 + 3A^2B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A^3}{h^2x^2 + 2ghx + g^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="fric
as")
```

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h^2*x^2 + 2
*g*h*x + g^2), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g)**2,x)
```

```
[Out] Timed out
```

**Giac [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\left( B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A \right)^3}{(hx + g)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^2,x, algorithm="giac
")
```

```
[Out] integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(h*x + g)^2, x)
```

**3.314**     
$$\int \frac{(A+B \log(e(a+bx)^n(c+dx)^{-n}))^3}{(g+hx)^3} dx$$

**Optimal.** Leaf size=629

$$\frac{3B^2n^2(bc - ad)(-adh - bch + 2bdg)\text{PolyLog}\left(2, \frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)(B \log(e(a + bx)^n(c + dx)^{-n}) + A)}{(bg - ah)^2(dg - ch)^2} + \frac{3B^3hn^3(bc - ad)^2\text{PolyLog}\left(3, \frac{(a+bx)(dg-ch)}{(c+dx)(bg-ah)}\right)}{(bg - ah)^2}$$

```
[Out] (3*B*(b*c - a*d)*h*n*(a + b*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)/
(2*(b*g - a*h)^2*(d*g - c*h)*(g + h*x)) + (b^2*(A + B*Log[(e*(a + b*x)^n)/(
c + d*x)^n])^3)/(2*h*(b*g - a*h)^2) - (A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])^3/(2*h*(g + h*x)^2) + (3*B^2*(b*c - a*d)^2*h*n^2*(A + B*Log[(e*(a + b*x)
)^n)/(c + d*x)^n])*Log[1 - ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]
)/((b*g - a*h)^2*(d*g - c*h)^2) + (3*B*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h
)*n*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2*Log[1 - ((d*g - c*h)*(a + b*
x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) + (3*B^3*(b*c
- a*d)^2*h*n^3*PolyLog[2, ((d*g - c*h)*(a + b*x))/((b*g - a*h)*(c + d*x))]
)/((b*g - a*h)^2*(d*g - c*h)^2) + (3*B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d
*h)*n^2*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])*PolyLog[2, ((d*g - c*h)*(a
+ b*x))/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2) - (3*B^3*(
b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^3*PolyLog[3, ((d*g - c*h)*(a + b*x))
/((b*g - a*h)*(c + d*x))])/((b*g - a*h)^2*(d*g - c*h)^2)
```

**Rubi [B]**     time = 3.7451, antiderivative size = 2207, normalized size of antiderivative = 3.51, number of steps used = 49, number of rules used = 21, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$ , Rules used = {6742, 2492, 72, 2514, 2488, 2411, 2343, 2333, 2315, 2490, 36, 31, 2494, 2394, 2393, 2391, 2506, 6610, 2503, 2502, 2489}

result too large to display

Antiderivative was successfully verified.

```
[In] Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3,x]
```

```
[Out] -A^3/(2*h*(g + h*x)^2) - (3*A^2*B*(b*c - a*d)*n)/(2*(b*g - a*h)*(d*g - c*h)
*(g + h*x)) + (3*A^2*b^2*B*n*Log[a + b*x])/(2*h*(b*g - a*h)^2) - (3*A^2*B*d
^2*n*Log[c + d*x])/(2*h*(d*g - c*h)^2) - (3*A*B^2*(b*c - a*d)^2*h*n^2*Log[c
+ d*x])/((b*g - a*h)^2*(d*g - c*h)^2) - (3*A^2*B*Log[(e*(a + b*x)^n)/(c +
d*x)^n])/(2*h*(g + h*x)^2) + (3*A*B^2*(b*c - a*d)*h*n*(a + b*x)*Log[(e*(a +
b*x)^n)/(c + d*x)^n])/((b*g - a*h)^2*(d*g - c*h)*(g + h*x)) - (3*A*b^2*B^2
*n*Log[-((b*c - a*d)/(d*(a + b*x))])*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(h*(
b*g - a*h)^2) + (3*A*B^2*d^2*n*Log[(b*c - a*d)/(b*(c + d*x))])*Log[(e*(a + b
*x)^n)/(c + d*x)^n])/(h*(d*g - c*h)^2) - (3*A*B^2*Log[(e*(a + b*x)^n)/(c +
d*x)^n]^2)/(2*h*(g + h*x)^2) + (3*B^3*(b*c - a*d)*h*n*(a + b*x)*Log[(e*(a +
b*x)^n)/(c + d*x)^n]^2)/(2*(b*g - a*h)^2*(d*g - c*h)*(g + h*x)) - (3*b^2*B
^3*n*Log[-((b*c - a*d)/(d*(a + b*x))])*Log[(e*(a + b*x)^n)/(c + d*x)^n]^2)/
(2*h*(b*g - a*h)^2) + (3*B^3*d^2*n*Log[(b*c - a*d)/(b*(c + d*x))])*Log[(e*(a
+ b*x)^n)/(c + d*x)^n]^2)/(2*h*(d*g - c*h)^2) - (3*B^3*(b*c - a*d)*(2*b*d*
g - b*c*h - a*d*h)*n*Log[(b*c - a*d)/(b*(c + d*x))])*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]^2)/(2*(b*g - a*h)^2*(d*g - c*h)^2) - (B^3*Log[(e*(a + b*x)^n)/(c
+ d*x)^n]^3)/(2*h*(g + h*x)^2) + (3*A^2*B*(b*c - a*d)*(2*b*d*g - b*c*h - a*
d*h)*n*Log[g + h*x])/(2*(b*g - a*h)^2*(d*g - c*h)^2) + (3*A*B^2*(b*c - a*d)
^2*h*n^2*Log[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2) - (3*A*B^2*(b*c - a*d)
*(2*b*d*g - b*c*h - a*d*h)*n^2*Log[-((h*(a + b*x))/(b*g - a*h))])*Log[g + h*
x])/((b*g - a*h)^2*(d*g - c*h)^2) + (3*A*B^2*(b*c - a*d)*(2*b*d*g - b*c*h -
a*d*h)*n^2*Log[-((h*(c + d*x))/(d*g - c*h))])*Log[g + h*x])/((b*g - a*h)^2*
```

$$\begin{aligned}
& (d*g - c*h)^2) + (3*A*B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*\text{Log}[(e*(a \\
& + b*x)^n)/(c + d*x)^n]*\text{Log}[g + h*x])/((b*g - a*h)^2*(d*g - c*h)^2) + (3*B^3 \\
& *(b*c - a*d)^2*h*n^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]*\text{Log}[(b*c - a*d)*(g \\
& + h*x))/((b*g - a*h)*(c + d*x))]/((b*g - a*h)^2*(d*g - c*h)^2) + (3*B^3*(b \\
& *c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]^2*\text{Lo} \\
& g[(b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))]/(2*(b*g - a*h)^2*(d*g - \\
& c*h)^2) + (3*A*B^2*d^2*n^2*\text{PolyLog}[2, (d*(a + b*x))/(b*(c + d*x))]/(h*(d \\
& g - c*h)^2) - (3*A*B^2*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*\text{PolyLog}[2, \\
& (b*(g + h*x))/(b*g - a*h)]/((b*g - a*h)^2*(d*g - c*h)^2) + (3*A*B^2*(b*c \\
& - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*\text{PolyLog}[2, (d*(g + h*x))/(d*g - c*h)]) \\
& /((b*g - a*h)^2*(d*g - c*h)^2) + (3*A*b^2*B^2*n^2*\text{PolyLog}[2, 1 + (b*c - a*d) \\
& ]/(d*(a + b*x))]/(h*(b*g - a*h)^2) + (3*b^2*B^3*n^2*\text{Log}[(e*(a + b*x)^n)/(c \\
& + d*x)^n]*\text{PolyLog}[2, 1 + (b*c - a*d)/(d*(a + b*x))]/(h*(b*g - a*h)^2) + ( \\
& 3*B^3*d^2*n^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]*\text{PolyLog}[2, 1 - (b*c - a*d)/( \\
& b*(c + d*x))]/(h*(d*g - c*h)^2) - (3*B^3*(b*c - a*d)*(2*b*d*g - b*c*h - a* \\
& d*h)*n^2*\text{Log}[(e*(a + b*x)^n)/(c + d*x)^n]*\text{PolyLog}[2, 1 - (b*c - a*d)/(b*(c \\
& + d*x))]/((b*g - a*h)^2*(d*g - c*h)^2) + (3*B^3*(b*c - a*d)^2*h*n^3*\text{PolyLo} \\
& g[2, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))]/((b*g - a*h)^2*( \\
& d*g - c*h)^2) + (3*B^3*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^2*\text{Log}[(e*(a \\
& + b*x)^n)/(c + d*x)^n]*\text{PolyLog}[2, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)* \\
& (c + d*x))]/((b*g - a*h)^2*(d*g - c*h)^2) + (3*b^2*B^3*n^3*\text{PolyLog}[3, 1 + \\
& (b*c - a*d)/(d*(a + b*x))]/(h*(b*g - a*h)^2) - (3*B^3*d^2*n^3*\text{PolyLog}[3, 1 \\
& - (b*c - a*d)/(b*(c + d*x))]/(h*(d*g - c*h)^2) + (3*B^3*(b*c - a*d)*(2*b* \\
& d*g - b*c*h - a*d*h)*n^3*\text{PolyLog}[3, 1 - (b*c - a*d)/(b*(c + d*x))]/((b*g - \\
& a*h)^2*(d*g - c*h)^2) - (3*B^3*(b*c - a*d)*(2*b*d*g - b*c*h - a*d*h)*n^3*P \\
& olyLog[3, 1 - ((b*c - a*d)*(g + h*x))/((b*g - a*h)*(c + d*x))]/((b*g - a*h) \\
& )^2*(d*g - c*h)^2)
\end{aligned}$$
Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 2492

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*((g_.) + (h_.)*(x_))^(m_.), x_Symbol] := Simp[((g + h*x)^(m +
1)*Log[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s)/(h*(m + 1)), x] - Dist[(p*r*s*(
b*c - a*d))/(h*(m + 1)), Int[((g + h*x)^(m + 1)*Log[e*(f*(a + b*x)^p*(c + d
*x)^q]^r]^s - 1)/(a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, m, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0] && IGtQ[s,
0] && NeQ[m, -1]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
/; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2514

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_.)*(RFX_), x_Symbol] := With[{u = ExpandIntegrand[Log[e*(f*(a +
b*x)^p*(c + d*x)^q]^r]^s, RFX, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c
, d, e, f, p, q, r, s}, x] && RationalFunctionQ[RFX, x] && IGtQ[s, 0]
```

Rule 2488

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
```

$$\int \frac{(r_.)^{(s_.)}}{(g_.) + (h_.)x}, x\_Symbol] := -\text{Simp}[(\text{Log}[-(b*c - a*d)/(d*(a + b*x))]) * \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s/h, x] + \text{Dist}[(p*r*s*(b*c - a*d))/h, \text{Int}[(\text{Log}[-(b*c - a*d)/(d*(a + b*x))]) * \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^{(s - 1)}/((a + b*x)*(c + d*x)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{EqQ}[b*g - a*h, 0] \&\& \text{IGtQ}[s, 0]$$

#### Rule 2411

$$\text{Int}[(a_.) + \text{Log}[(c_.) * ((d_.) + (e_.)x)^{(n_.)}] * (b_.)^{(p_.)} * ((f_.) + (g_.)x)^{(q_.)} * ((h_.) + (i_.)x)^{(r_.)}], x\_Symbol] := \text{Dist}[1/e, \text{Subst}[\text{Int}[(g*x)/e]^q * ((e*h - d*i)/e + (i*x)/e)^r * (a + b*\text{Log}[c*x]^n)^p, x], x, d + e*x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, i, n, p, q, r\}, x] \&\& \text{EqQ}[e*f - d*g, 0] \&\& (\text{IGtQ}[p, 0] || \text{IGtQ}[r, 0]) \&\& \text{IntegerQ}[2*r]$$

#### Rule 2343

$$\text{Int}[(a_.) + \text{Log}[(c_.) * (x_.)^{(n_.)}] * (b_.)] / ((x_.) * ((d_.) + (e_.)x^{(r_./n)})), x\_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[(a + b*\text{Log}[c*x]) / (x*(d + e*x^{(r/n)}))], x], x, x^n], x] /; \text{FreeQ}\{a, b, c, d, e, n, r\}, x] \&\& \text{IntegerQ}[r/n]$$

#### Rule 2333

$$\text{Int}[(a_.) + \text{Log}[(c_.) * (x_.)^{(n_.)}] * (b_.)^{(p_.)} * ((d_.) + (e_.)x)^{(q_.)} * x^{(m_.)}], x\_Symbol] := \text{Int}[(e + d*x)^q * (a + b*\text{Log}[c*x]^n)^p, x] /; \text{FreeQ}\{a, b, c, d, e, m, n, p\}, x] \&\& \text{EqQ}[m, q] \&\& \text{IntegerQ}[q]$$

#### Rule 2315

$$\text{Int}[\text{Log}[(c_.) * (x_.)] / ((d_.) + (e_.)x), x\_Symbol] := -\text{Simp}[\text{PolyLog}[2, 1 - c*x]/e, x] /; \text{FreeQ}\{c, d, e\}, x] \&\& \text{EqQ}[e + c*d, 0]$$

#### Rule 2490

$$\text{Int}[\text{Log}[(e_.) * ((f_.) * ((a_.) + (b_.)x)^{(p_.)} * ((c_.) + (d_.)x)^{(q_.)})^{(r_.)}]^{(s_.)} / ((g_.) + (h_.)x)^2, x\_Symbol] := \text{Simp}[(a + b*x) * \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^s / ((b*g - a*h)*(g + h*x)), x] - \text{Dist}[(p*r*s*(b*c - a*d))/(b*g - a*h), \text{Int}[\text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r]^{(s - 1)} / ((c + d*x)*(g + h*x)), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, p, q, r, s\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[p + q, 0] \&\& \text{NeQ}[b*g - a*h, 0] \&\& \text{IGtQ}[s, 0]$$

#### Rule 36

$$\text{Int}[1/((a_.) + (b_.)x) * ((c_.) + (d_.)x)], x\_Symbol] := \text{Dist}[b/(b*c - a*d), \text{Int}[1/(a + b*x), x], x] - \text{Dist}[d/(b*c - a*d), \text{Int}[1/(c + d*x), x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$$

#### Rule 31

$$\text{Int}[(a_.) + (b_.)x^{-1}], x\_Symbol] := \text{Simp}[\text{Log}[\text{RemoveContent}[a + b*x, x]]/b, x] /; \text{FreeQ}\{a, b\}, x]$$

#### Rule 2494

$$\text{Int}[\text{Log}[(e_.) * ((f_.) * ((a_.) + (b_.)x)^{(p_.)} * ((c_.) + (d_.)x)^{(q_.)})^{(r_.)}] / ((g_.) + (h_.)x), x\_Symbol] := \text{Simp}[(\text{Log}[g + h*x] * \text{Log}[e*(f*(a + b*x)^p*(c + d*x)^q]^r])/h, x] + (-\text{Dist}[(b*p*r)/h, \text{Int}[\text{Log}[g + h*x]/(a + b*x), x], x] - \text{Dist}[(d*q*r)/h, \text{Int}[\text{Log}[g + h*x]/(c + d*x), x], x]) /; \text{FreeQ}\{$$

a, b, c, d, e, f, g, h, p, q, r}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 2394

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^n\_.)]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Simp[(Log[(e\*(f + g\*x))/(e\*f - d\*g)]\*(a + b\*Log[c\*(d + e\*x)^n]))/g, x] - Dist[(b\*e^n)/g, Int[Log[(e\*(f + g\*x))/(e\*f - d\*g)]/(d + e\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, n}, x] && NeQ[e\*f - d\*g, 0]

#### Rule 2393

Int[((a\_.) + Log[(c\_.)\*((d\_) + (e\_.)\*(x\_))]\*(b\_.))/((f\_.) + (g\_.)\*(x\_)), x\_Symbol] := Dist[1/g, Subst[Int[(a + b\*Log[1 + (c\*e\*x)/g])/x, x], x, f + g\*x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e\*f - d\*g, 0] && EqQ[g + c\*(e\*f - d\*g), 0]

#### Rule 2391

Int[Log[(c\_.)\*((d\_) + (e\_.)\*(x\_)^n\_.)]/(x\_), x\_Symbol] := -Simp[PolyLog[2, -(c\*e\*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c\*d, 1]

#### Rule 2506

Int[Log[v\_]\*Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_)^p\_.))\*((c\_.) + (d\_.)\*(x\_)^q\_.)]^(r\_.)]^(s\_.)\*(u\_), x\_Symbol] := With[{g = Simplify[(v - 1)\*(c + d\*x)/(a + b\*x)], h = Simplify[u\*(a + b\*x)\*(c + d\*x)]}, -Simp[(h\*PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s)/(b\*c - a\*d), x] + Dist[h\*p\*r\*s, Int[(PolyLog[2, 1 - v]\*Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)/(a + b\*x)\*(c + d\*x), x], x] /; FreeQ[{g, h}, x] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0]

#### Rule 6610

Int[(u\_)\*PolyLog[n\_, v\_], x\_Symbol] := With[{w = DerivativeDivides[v, u\*v, x]}, Simp[w\*PolyLog[n + 1, v], x] /; !FalseQ[w]] /; FreeQ[n, x]

#### Rule 2503

Int[Log[(e\_.)\*((f\_.)\*((a\_.) + (b\_.)\*(x\_)^p\_.))\*((c\_.) + (d\_.)\*(x\_)^q\_.)]^(r\_.)]^(s\_.)\*(u\_), x\_Symbol] := With[{g = Coeff[Simplify[1/(u\*(a + b\*x))], x, 0], h = Coeff[Simplify[1/(u\*(a + b\*x))], x, 1]}, -Simp[(Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s\*Log[-((b\*c - a\*d)\*(g + h\*x))/((d\*g - c\*h)\*(a + b\*x))])]/(b\*g - a\*h), x] + Dist[(p\*r\*s\*(b\*c - a\*d))/(b\*g - a\*h), Int[(Log[e\*(f\*(a + b\*x)^p\*(c + d\*x)^q]^r]^s - 1)\*Log[-((b\*c - a\*d)\*(g + h\*x))/((d\*g - c\*h)\*(a + b\*x))])]/((a + b\*x)\*(c + d\*x), x], x] /; NeQ[b\*g - a\*h, 0] && NeQ[d\*g - c\*h, 0] /; FreeQ[{a, b, c, d, e, f, p, q, r, s}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[s, 0] && EqQ[p + q, 0] && LinearQ[Simplify[1/(u\*(a + b\*x))], x]

#### Rule 2502

Int[Log[(e\_.)\*((c\_.) + (d\_.)\*(x\_)))/((a\_.) + (b\_.)\*(x\_))]\*(u\_), x\_Symbol] := With[{g = Coeff[Simplify[1/(u\*(a + b\*x))], x, 0], h = Coeff[Simplify[1/(u\*(a + b\*x))], x, 1]}, -Dist[(b - d\*e)/(h\*(b\*c - a\*d)), Subst[Int[Log[e\*x]/(1 - e\*x), x], x, (c + d\*x)/(a + b\*x)], x] /; EqQ[g\*(b - d\*e) - h\*(a - c\*e), 0] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b\*c - a\*d, 0] && LinearQ[Simplify[1/(u\*(a + b\*x))], x]

#### Rule 2489

```
Int[Log[(e_.)*((f_.)*((a_.) + (b_.)*(x_))^(p_.)*((c_.) + (d_.)*(x_))^(q_.))
^(r_.)]^(s_)/((g_.) + (h_.)*(x_)), x_Symbol] := Dist[d/h, Int[Log[e*(f*(a +
b*x)^p*(c + d*x)^q)^r]^s/(c + d*x), x], x] - Dist[(d*g - c*h)/h, Int[Log[e
*(f*(a + b*x)^p*(c + d*x)^q)^r]^s/((c + d*x)*(g + h*x)), x], x] /; FreeQ[{a
, b, c, d, e, f, g, h, p, q, r, s}, x] && NeQ[b*c - a*d, 0] && EqQ[p + q, 0
] && NeQ[b*g - a*h, 0] && NeQ[d*g - c*h, 0] && IGtQ[s, 1]
```

Rubi steps

$$\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^3}{(g + hx)^3} dx = \int \left( \frac{A^3}{(g + hx)^3} + \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^3} + \frac{3AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^3} \right) dx$$

$$= -\frac{A^3}{2h(g + hx)^2} + (3A^2B) \int \frac{\log(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^3} dx + (3AB^2) \int \frac{\log^2(e(a + bx)^n(c + dx)^{-n})}{(g + hx)^3} dx$$

$$= -\frac{A^3}{2h(g + hx)^2} - \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{2h(g + hx)^2} - \frac{3AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{2h(g + hx)^2}$$

$$= -\frac{A^3}{2h(g + hx)^2} - \frac{3A^2B \log(e(a + bx)^n(c + dx)^{-n})}{2h(g + hx)^2} - \frac{3AB^2 \log^2(e(a + bx)^n(c + dx)^{-n})}{2h(g + hx)^2}$$

$$= -\frac{A^3}{2h(g + hx)^2} - \frac{3A^2B(bc - ad)n}{2(bg - ah)(dg - ch)(g + hx)} + \frac{3A^2b^2Bn \log(a + bx)}{2h(bg - ah)^2} - \frac{3A^2B^2n \log^2(a + bx)}{2h(bg - ah)^2}$$

$$= -\frac{A^3}{2h(g + hx)^2} - \frac{3A^2B(bc - ad)n}{2(bg - ah)(dg - ch)(g + hx)} + \frac{3A^2b^2Bn \log(a + bx)}{2h(bg - ah)^2} - \frac{3A^2B^2n \log^2(a + bx)}{2h(bg - ah)^2}$$

$$= -\frac{A^3}{2h(g + hx)^2} - \frac{3A^2B(bc - ad)n}{2(bg - ah)(dg - ch)(g + hx)} + \frac{3A^2b^2Bn \log(a + bx)}{2h(bg - ah)^2} - \frac{3A^2B^2n \log^2(a + bx)}{2h(bg - ah)^2}$$

$$= -\frac{A^3}{2h(g + hx)^2} - \frac{3A^2B(bc - ad)n}{2(bg - ah)(dg - ch)(g + hx)} + \frac{3A^2b^2Bn \log(a + bx)}{2h(bg - ah)^2} - \frac{3A^2B^2n \log^2(a + bx)}{2h(bg - ah)^2}$$

$$= -\frac{A^3}{2h(g + hx)^2} - \frac{3A^2B(bc - ad)n}{2(bg - ah)(dg - ch)(g + hx)} + \frac{3A^2b^2Bn \log(a + bx)}{2h(bg - ah)^2} - \frac{3A^2B^2n \log^2(a + bx)}{2h(bg - ah)^2}$$

**Mathematica [F]** time = 8.24731, size = 0, normalized size = 0.

$$\int \frac{(A + B \log(e(a + bx)^n(c + dx)^{-n}))^3}{(g + hx)^3} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3, x]
```

```
[Out] Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(g + h*x)^3, x]
```



**Maple [F]** time = 3.369, size = 0, normalized size = 0.

$$\int \frac{1}{(hx + g)^3} \left( A + B \ln \left( \frac{e(bx + a)^n}{(dx + c)^n} \right) \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(h\*x+g)^3,x)

[Out] int((A+B\*ln(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(h\*x+g)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(h\*x+g)^3,x, algorithm="maxima")

[Out]  $\frac{1}{2}B^3 \log((dx + c)^n)^3 / (h^3x^2 + 2gh^2x + g^2h) + \frac{3}{2}(b^{2e}n \log(bx + a) / (b^2g^2h - 2abg^2h^2 + a^2h^3) - d^{2e}n \log(dx + c) / (d^2g^2h - 2cdg^2h^2 + c^2h^3) - (2abd^{2e}gn - a^2d^{2e}hn - (2cdgegn - c^{2e}hn) * b^2) * \log(hx + g) / ((d^2g^2h^2 - 2cdg^2h^3 + c^2h^4) * a^2 - 2(d^2g^3h - 2cdg^2h^2 + c^2g^2h^3) * ab + (d^2g^4 - 2cdg^3h + c^2g^2h^2) * b^2) + (bcen - aden) / ((d^2gh^2 - cgh^2) * a - (d^2gh^2 - c^2gh^2) * b + ((d^2gh^2 - ch^3) * a - (d^2gh^2 - c^2gh^2) * b) * x)) * A^2B / e - 3/2A^2B \log((bx + a)^n * e / (dx + c)^n) / (h^3x^2 + 2gh^2x + g^2h) - 1/2A^3 / (h^3x^2 + 2gh^2x + g^2h) + \text{integrate}(1/2(2B^3c * h * \log(e)^3 + 6A * B^2c * h * \log(e)^2 + 2(B^3d * h * x + B^3c * h) * \log((bx + a)^n)^3 + 6(B^3c * h * \log(e) + A * B^2c * h + (B^3d * h * \log(e) + A * B^2d * h) * x) * \log((bx + a)^n)^2 + 3(2A * B^2c * h - (d * g * n - 2c * h * \log(e)) * B^3 - ((h * n - 2h * \log(e)) * B^3d - 2A * B^2d * h) * x + 2(B^3d * h * x + B^3c * h) * \log((bx + a)^n)) * \log((dx + c)^n)^2 + 2(B^3d * h * \log(e)^3 + 3A * B^2d * h * \log(e)^2) * x + 6(B^3c * h * \log(e)^2 + 2A * B^2c * h * \log(e) + (B^3d * h * \log(e)^2 + 2A * B^2d * h * \log(e)) * x) * \log((bx + a)^n) - 6(B^3c * h * \log(e)^2 + 2A * B^2c * h * \log(e) + (B^3d * h * x + B^3c * h) * \log((bx + a)^n)^2 + (B^3d * h * \log(e)^2 + 2A * B^2d * h * \log(e)) * x + 2(B^3c * h * \log(e) + A * B^2c * h + (B^3d * h * \log(e) + A * B^2d * h) * x) * \log((bx + a)^n)) * \log((dx + c)^n) / (d^4x^4 + c^3gh + (3d^2gh^3 + ch^4) * x^3 + 3(d^2gh^2 + c^2gh^3) * x^2 + (d^3gh + 3c^2gh^2) * x), x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{B^3 \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right)^3 + 3AB^2 \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right)^2 + 3A^2B \log \left( \frac{(bx+a)^n e}{(dx+c)^n} \right) + A^3}{h^3x^3 + 3gh^2x^2 + 3g^2hx + g^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B\*log(e\*(b\*x+a)^n/((d\*x+c)^n)))^3/(h\*x+g)^3,x, algorithm="fricas")

```
[Out] integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e/
(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(h^3*x^3 + 3
*g*h^2*x^2 + 3*g^2*h*x + g^3), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(h*x+g)**3,x)
```

[Out] Timed out

**Giac [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(h*x+g)^3,x, algorithm="giac
")
```

[Out] Timed out

# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26     If[FreeQ[result,Integrate] && FreeQ[result,Int],
27       "C",
28       "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```

```

38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46   If[AtomQ[expn],
47     1,
48     If[ListQ[expn],
49       Max[Map[ExpnType,expn]],
50       If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52           ExpnType[expn[[1]]],
53           If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55               1,
56               Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]]],
58           If[Head[expn]===Plus || Head[expn]===Times,
59             Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60           If[ElementaryFunctionQ[Head[expn]],
61             Max[3,ExpnType[expn[[1]]]],
62           If[SpecialFunctionQ[Head[expn]],
63             Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64           If[HypergeometricFunctionQ[Head[expn]],
65             Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66           If[AppellFunctionQ[Head[expn]],
67             Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68           If[Head[expn]===RootSum,
69             Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
70           If[Head[expn]===Integrate || Head[expn]===Int,
71             Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
72           9]]]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp,Log,
78     Sin,Cos,Tan,Cot,Sec,Csc,
79     ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
80     Sinh,Cosh,Tanh,Coth,Sech,Csch,
81     ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
82 },func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94 },func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
99
100

```

```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

## 4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```

```

119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'`^`') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'`+`') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150     max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152     max(8,apply(max,map(ExpnType,[op(expn)]))) else
153     9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159   member(func,[
160     exp,log,ln,
161     sin,cos,tan,cot,sec,csc,
162     arcsin,arccos,arctan,arccot,arcsec,arccsc,
163     sinh,cosh,tanh,coth,sech,csch,
164     arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168   member(func,[
169     erf,erfc,erfi,
170     FresnelS,FresnelC,
171     Ei,Ei,Li,Si,Ci,Shi,Chi,
172     GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173     EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181   member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

### 4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```



```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124 else:
125     return "C"

```

#### 4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34             #since this estimate of leaf count is bit lower than

```

```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```